On-shell amplitude techniques for the standard-model effective field theory

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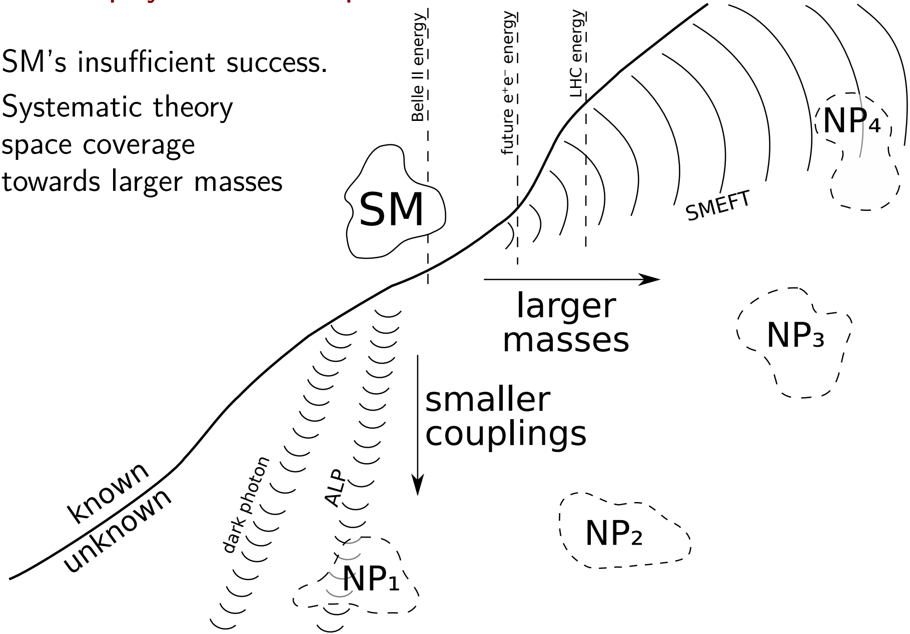
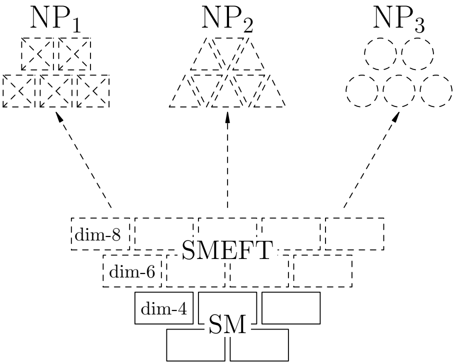
Particle physics landscape

SM's insufficient success.

Systematic theory space coverage towards larger masses

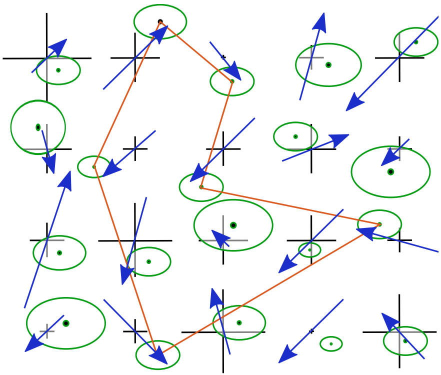


Taking the SM to higher dimensions



- using established bricks (fields and symmetries)
- extension organised by relevance (dimension)
- including all deformations (theory space coverage)

Isolating subtle patterns of new physics



array of sensitive observables

- precise SM&EFT predictions
- precise measurements

$\rightarrow$ correlate deviations

Building LHC's legacy

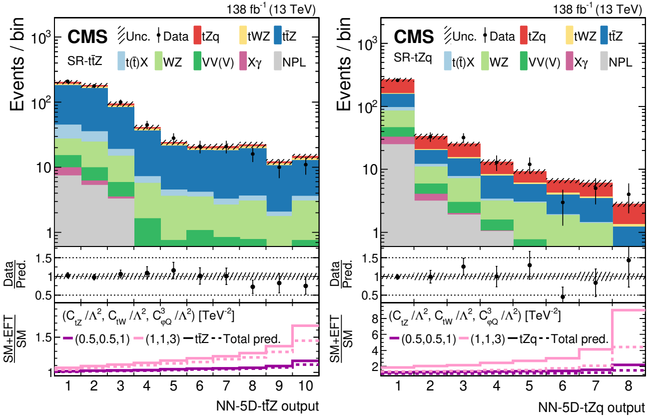
- Improved sensitivity (multidimensionally)
- Global picture (limited assumptions)
- Precise interpretation (better EFT predictions)
- Models' connection (matching, charting)
- New understanding (on-shell techniques)

SMEFT progresses

ML optimisation for SMEFT

$tZ + X$ process in the three-lepton signal region

1. discriminate $t\bar{t}Z$, tZj signals and backgrounds
2. train SM vs. $(c_{tZ}, c_{tW}, c^3_{\phi q})$ from reweighted samples

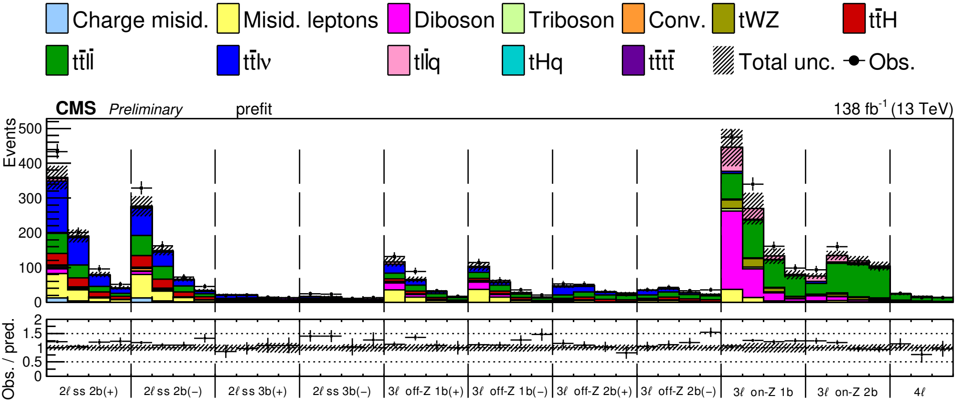


Beyond signal and background processes

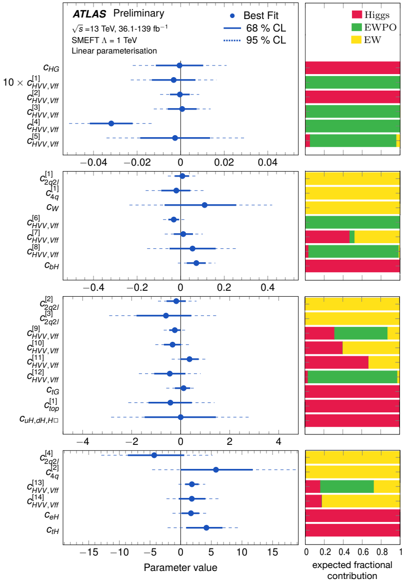
[CMS '20, '23]
[see also 4ℓ in ATLAS '21]

- leptons+b's+jets final state, p_T bins, 178 data points
- contains tth, ttZ, ttW, tZq, tHq, diboson, etc.
- 26 top operator contributions from reweighting
- towards publication of 26D likelihood

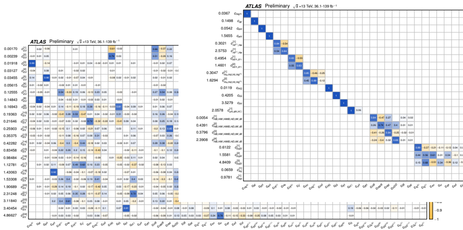
[Valsecchi LHCP23]



ATLAS Higgs+diboson+EWPO combination

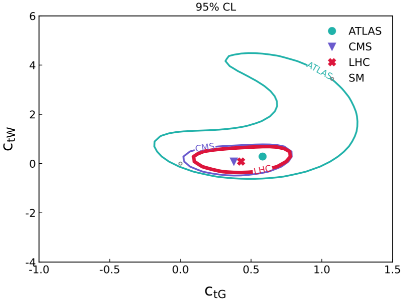


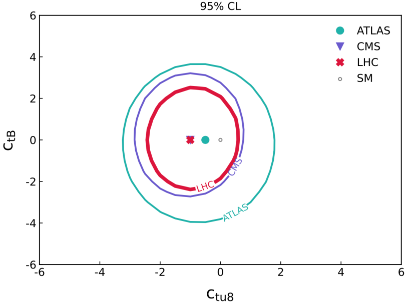
- Higgs '21 STXS combination
- diboson $WW, WZ, 4\ell, Zjj$
- Z pole from LEP+SLC
- principal component analysis removing flat directions
- fit results for 22 eigen-vectors
- lin results



ATLAS+CMS top combination

- full likelihoods: · $4t$ ($n\ell$ ATLAS), $4t$ ($n\ell$ CMS),
  · $tt\gamma$ (1ℓ CMS), $tt\gamma$ (2ℓ CMS),
  · ttZ ($n\ell$ ATLAS)
- 700^+ bin, $\sim$20 processes
- 8 operators, lin & quad, also in tth and ttW
- uncorrelated systematics



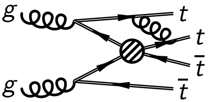


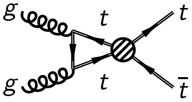
[Degrande, GD, Maltoni, Mimasu, Vryonidou, Zhang '20]

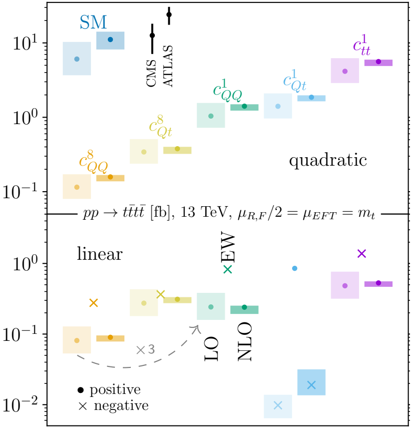
SMEFT at one loop

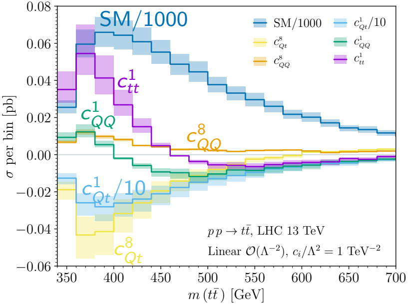
Better accuracy
and uncertainties

New sensitivities
... and degeneracies





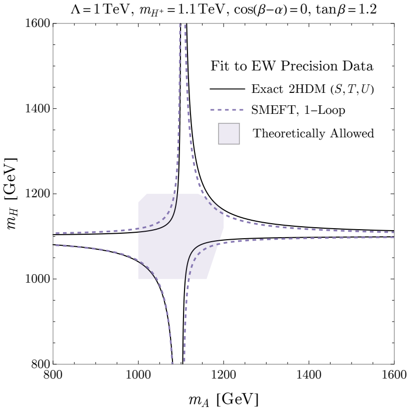


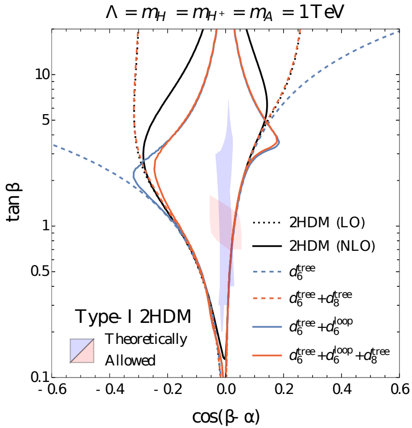


... to be resolved with more
differential measurements

Matching at one loop: the 2HDM

[Das Bakshi, Dawson, Fontes, Homiller '24]





- EWPO constraints arising first at one-loop
  - mild impact so far; more important with new Z pole?
- more accurate large-$\tan\beta$ description
  - from Yukawa operators; probed with new Higgs measurements

On-shell amplitude techniques

beside Hilbert series, field geometry, double copy, etc.

On-shell amplitudes

bypass unphysical fields, operators, Lagrangians

avoid gauge and field redefinition redundancies

e.g. graviton Feynman rules [De Witt '67]

3 pt.

$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\begin{aligned}&\mathrm{Sym}[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\&\qquad-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})],\end{aligned}$$

171 terms vs. $\left([12]^3/[23][31]\right)^2$

4 pt.

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow$$

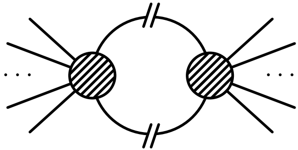
$$\begin{aligned}&\mathrm{Sym}[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})\\&+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})\\&+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})\\&-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})\\&-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})\\&+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})\\&\qquad-P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})].\end{aligned}$$

2850 terms vs. $[12]^4\langle 34\rangle^4/stu$

Analyticity and unitarity

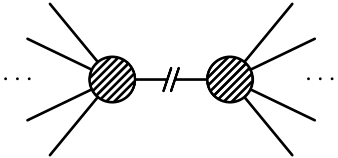
loops cut into lower loops

+ rational terms



trees cut into smaller trees

+ contact terms



recursive construction from the simplest amplitudes

or more direct extraction of various quantities

On-shell techniques for SMEFT

- operator enumeration

[Shadmi, Weiss '18], [Ma, Shu, Xiao '19], [Falkowski '19], [GD, Machado '19]
[Li, Ren, et al. '20, '20], [Harlander, Kempkens, Schaaf '23]

- kinematics characterisation

[Aoude, Machado '19], [GD, Kitahara, Shadmi, Weiss '19], [GD et al. '20]
[Balkin et al. '21], [Dong, Ma, Shu, Zheng '21, '22], [De Angelis '22]
[Bradshaw, Chang, Chen, Liu, Luty '22, '23], [Liu, Ma, Shadmi, Waterbury '23]

- anomalous dimensions

[Cheung, Shen '15], [Azatov et al. '16], [Bern et al. '19, '20], [Jiang et al. '20], [Elias Miró et al. '20, '21]
[Baratella et al. '20, '20, '21][Accettulli Huber, De Angelis '21], [Delle Rose et al. '22], [Baratella '22]
[Machado, Renner, Sutherland '22], [Chala '23]

- matching to UV theories [De Angelis, GD '23]

Operator enumeration

Massless helicity spinors

[Mangano, Parke '91]
[Dreiner, Haber, Martin '08]
[Helvang, Huang '13]
[Dixon '13]
[Schwartz '14]
[Cheung '17]

As square and angle brackets

$$u_{i^+} = \begin{pmatrix} 0 \\ i] \end{pmatrix}, \qquad u_{i^-} = \begin{pmatrix} i\rangle \\ 0 \end{pmatrix} \qquad \text{for massless particle } i$$

Rewritting momenta

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} \equiv p^i_{\alpha\dot\alpha} = \epsilon_{\alpha\beta}\ {}^{\beta} i\rangle[i_{\dot\alpha} \qquad \text{2-by-2 matrix of rank 1}$$

Trivialising $p_i^2 = \det(p^i_{\alpha\dot\alpha}) = \langle ii\rangle[ii]/2 = 0$

$$\langle ii\rangle = \epsilon_{\alpha\beta}\ i\rangle^\alpha i\rangle^\beta = 0, \qquad [ii] = \epsilon^{\dot\alpha\dot\beta}\ i]_{\dot\alpha} i]_{\dot\beta} = 0$$

Massless three points

fully determined by little-group covariance

$$\mathcal{M}(1^{h_1},2^{h_2},3^{h_3}) = g\begin{cases}[12]^{h_1+h_2-h_3}\quad[23]^{h_2+h_3-h_1}\quad[31]^{h_3+h_1-h_2} & \text{for } h>0\\ \langle12\rangle^{-h_1-h_2+h_3}\langle23\rangle^{-h_2-h_3+h_1}\langle31\rangle^{-h_3-h_1+h_2} & \text{for } h<0\end{cases}$$

up to a constant coefficient

f^+f^+s $[12]$

v^+v^+s $[12]^2$

$f^+f^-v^+$ $[13]^2/[12]$

$v^+v^+v^-$ $[12]^3/[23][31]$

$t^+t^+t^-$ $\left([12]^3/[23][31]\right)^2$

$$[g] = 1-|h|$$

$h \equiv \sum h_i$

Massless higher-point contact terms

Multiple independent structures for given helicities

non-vanishing Lorentz invariants $\left(s_{ij} \equiv 2\, p_i \cdot p_j,\ \epsilon_{ijkl} \equiv \epsilon_{\mu\nu\rho\sigma}\, p_i^\mu p_j^\nu p_k^\rho p_l^\sigma\right)$

solving · little-group covariance

· momentum conservation

· Schouten identity $[12][34] - [13][24] + [14][23] = 0$

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solving · little-group covariance
· momentum conservation
· Schouten identity $[12][34] - [13][24] + [14][23] = 0$

e.g. GR-SM-EFT up to dim-8:

[GD, Machado '19]

$$
\begin{aligned}
t^+t^+t^+t^+ &: \quad [12]^4[34]^4 + [13]^4[24]^4 + [14]^4[23]^4 \\
t^+t^+v^+v^+ &: \quad [12]^4[34]^2,\ [12]^2[13][14][24][23] \\
t^+v^+f^+f^- &: \quad [12]^2[13][124\rangle \qquad \times\text{polynomial}(s_{ij}, \epsilon_{ijkl}) \\
t^+f^+f^+f^+f^+ &: \quad [12][13][14][15] \\
\ldots & \qquad \ldots
\end{aligned}
$$

also from Hilbert series: [Ruhdorfer et al. '19]

Kinematics characterisation

Massive spinors

[Arkani-Hamed, Huang, Huang '17]

Two massless for one massive

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} \;=\; q^i\rangle[q^i + k^i\rangle[k^i \;=\; i^J\rangle[i_J \qquad \text{with } \begin{matrix} k_i^2 = 0 = q_i^2, \;\; J = 1,2 \\ 2k^i \cdot q^i = m_i^2 \end{matrix}$$

Spin s from $2s$ symmetrized spin $1/2$

left implicit, e.g. $\langle 1^I 3^J\rangle[2^K 3^{J'}] + (J \leftrightarrow J')$ written as $\langle \mathbf{13}\rangle[\mathbf{23}]$

Leading high-energy limit is just *unbolding*

Three-point examples:

ffs $[\mathbf{12}],\ \langle\mathbf{12}\rangle$

vvs $\langle\mathbf{12}\rangle^2,\ \langle\mathbf{12}\rangle[\mathbf{12}],\ [\mathbf{12}]^2$

ssv $[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \equiv [\mathbf{3}(p_\mathbf{1}-p_\mathbf{2})\mathbf{3}\rangle$

ffv $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle,\ \langle\mathbf{13}\rangle[\mathbf{23}],\ [\mathbf{13}]\langle\mathbf{23}\rangle,\ [\mathbf{13}][\mathbf{23}]$

... counting by spin irreps addition

Three-point example: W^+W^-Z

- 8 combinations of $\begin{matrix}\langle\mathbf{12}\rangle\\ [\mathbf{12}]\end{matrix} \otimes \begin{matrix}\langle\mathbf{23}\rangle\\ [\mathbf{23}]\end{matrix} \otimes \begin{matrix}\langle\mathbf{31}\rangle\\ [\mathbf{31}]\end{matrix}$

- one non-trivial relation between them:

$$m_1\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + m_2\langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + m_3[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle$$
$$= m_1[\mathbf{12}]\,[\mathbf{13}]\langle\mathbf{23}\rangle + m_2[\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + m_3\langle\mathbf{12}\rangle[\mathbf{13}]\,[\mathbf{23}]$$

7 combinations expected from angular momentum
$3 \otimes 3 \otimes 3 = 1 \oplus 3 \oplus 3 \oplus 3 \oplus 5 \oplus 5 \oplus 7$

- combinations growing like E^3 and E^2
can only arise at the non-renormalisable (tree) level

Three-point example: W^+W^-Z

C is $\mathbf{1} \leftrightarrow \mathbf{2}$

P is $\cdot] \leftrightarrow \cdot\rangle$

$$\mathcal{M}(\mathbf{1}_W, \mathbf{2}_W, \mathbf{3}_Z) \ni$$

$$\big\{[\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\; m_Z/m_W + \langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}]$$
$$+ \langle\mathbf{12}\rangle\,[\mathbf{13}]\,[\mathbf{23}]\; m_Z/m_W + [\mathbf{12}]\,\langle\mathbf{13}\rangle[\mathbf{23}] + [\mathbf{12}]\,[\mathbf{13}]\,\langle\mathbf{23}\rangle\big\}\;/m_Z m_W$$

$$\langle\mathbf{12}\rangle(\langle\mathbf{13}\rangle[\mathbf{23}] \pm [\mathbf{13}]\langle\mathbf{23}\rangle)/m_Z\Lambda$$

$$[\mathbf{12}](\langle\mathbf{13}\rangle[\mathbf{23}] \pm [\mathbf{13}]\langle\mathbf{23}\rangle)/m_Z\Lambda$$

$$\langle\mathbf{12}\rangle\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle/\Lambda^2$$

$$[\mathbf{12}][\mathbf{13}][\mathbf{23}]/\Lambda^2$$

- one unique renormalisable structure
- for identical vectors ($m_Z/m_W \to 1$):
  - no fully symmetric combination $\to$ ZZZ vanishes
  - only fully antisymmetric combinations $\to$ $W^a W^b W^c$ requires ϵ_{abc}

Four-point example: *ffZh*

- Twelve independent structures: [GD, Kitahara, Shadmi, Weiss '19]

$$\mathcal{M}(\mathbf{1}_f,\mathbf{2}_f,\mathbf{3}_Z,\mathbf{4}_h) \ni \begin{matrix} [\mathbf{13}]\langle\mathbf{23}\rangle \\ \langle\mathbf{13}\rangle[\mathbf{23}] \end{matrix} \quad \begin{matrix} [\mathbf{13}][\mathbf{23}] \\ \langle\mathbf{13}\rangle\langle\mathbf{23}\rangle \\ [\mathbf{12}]\langle\mathbf{3}(\mathbf{1}\pm\mathbf{2})\mathbf{3}] \\ \langle\mathbf{12}\rangle\langle\mathbf{3}(\mathbf{1}\pm\mathbf{2})\mathbf{3}] \end{matrix} \quad \begin{matrix} [\mathbf{312}\rangle[\mathbf{13}] \\ \langle\mathbf{321}]\langle\mathbf{23}\rangle \\ [\mathbf{321}\rangle[\mathbf{23}] \\ \langle\mathbf{312}]\langle\mathbf{13}\rangle \end{matrix} \quad \times \text{poly}(s_{ij})$$

- Counted by Hilbert series numerators: [Gráf, Henning, Lu, Melia, Murayama '22] [Bradshaw, Chang, Chen, Liu, Luty '22, '23]

$$\mathrm{H}_{ffZh}(d) = \frac{2d^5 + 6d^6 + 4d^7}{(1-d^2)^2}$$

$\rightarrow$ fully characterised kinematics, beyond lowest operator dim.

EW symmetry from perturbative unitarity

[GD, Kitahara, Shadmi, Weiss '19]

S-Matrix Derivation of the Weinberg Model[1]

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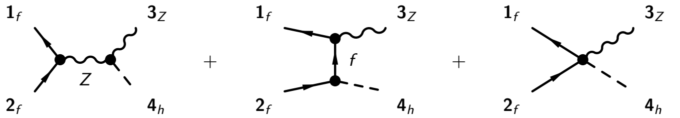
Received June 18, 1973

[Llewellyn-Smith '73]
[Joglekar '73]
[Conwall et al. '73, '74]

$f\,f\,Z\,h \sim$



factorisable s, t, u channels — contact terms

$$\xrightarrow[\text{energy}]{\text{high}} \begin{cases} \dfrac{[12]}{m_Z}\left(c_{ffZ}^{\text{left}} - c_{ffZ}^{\text{right}}\right)\left(c_{ffh}^{\text{right}} - c_{ZZh}\dfrac{m_f}{2m_Z}\right) \\ \dfrac{\langle 12\rangle}{m_Z}\left(c_{ffZ}^{\text{left}} - c_{ffZ}^{\text{right}}\right)\left(c_{ffh}^{\text{left}} - c_{ZZh}\dfrac{m_f}{2m_Z}\right) \end{cases}$$

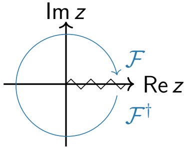
Anomalous dimensions & selection rules

Anomalous dimensions

[adapted from: Caron-Huot, Wilhelm '16]

- In a massless theory, any $(\log \mu^2)$ comes with a $(-\log s_I)$
- A dilation $z^{D/2}$ with $D \equiv \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$ captures all Mandelstam logs in a single $(-\log z)$ and disregards logs of s_I/s_J ratios
- Form factors $\mathcal{F} \equiv {}_{\text{out}}\langle p_1, \dots p_m|\mathcal{O}(q)|0\rangle_{\text{in}}$ have all $s_I \equiv (\sum_{i\in I} p_i^\mu)^2$ Mandelstams positive

momentum influx



- Dilated form factors $\hat{\mathcal{F}}(z) \equiv z^{D/2}\mathcal{F}$ only have singularities at positive z's

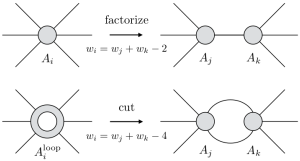
at $\sum_k \alpha_k m_k^2 / \sum_I \alpha_I s_I$ in Feynman parameterisation

[Cheung, Shen '15]

Non-renormalisation

vanishing tree helicity amp. $\Rightarrow$ vanishing one-loop divergences

define (anti)holomorphic weights $\overleftrightarrow{w} \equiv n \mp h$
renormalisable trees: $\overleftrightarrow{w}^{\text{tree}}_{\text{SM}} \geq 4$ for $n \geq 4$
(except for e.g. Yukawa amps)
from cut: $\overleftrightarrow{w}^{\text{loop}}_{\text{EFT}} = \overleftrightarrow{w}^{\text{tree}}_{\text{EFT}} + \overleftrightarrow{w}^{\text{tree}}_{\text{SM}} - 4$
so $\overleftrightarrow{w}^{\text{loop}}_{\text{EFT}} \geq \overleftrightarrow{w}^{\text{tree}}_{\text{EFT}}$



	$(w,\bar{w})$	F^3 $(0,6)$	$F^2\phi^2$ $(2,6)$	$F\psi^2\phi$ $(2,6)$	ψ^4 $(2,6)$	$\psi^2\phi^3$ $(4,6)$	$\bar{F}^3$ $(6,0)$	$\bar{F}^2\phi^2$ $(6,2)$	$\bar{F}\bar{\psi}^2\phi$ $(6,2)$	$\bar{\psi}^4$ $(6,2)$	$\bar{\psi}^2\phi^3$ $(6,4)$	$\bar{\psi}^2\psi^2$ $(4,4)$	$\bar{\psi}\psi\phi^2 D$ $(4,4)$	$\phi^4 D^2$ $(4,4)$	ϕ^6 $(6,6)$
F^3	$(0,6)$			×	×	×			×	×	×	×	×	×	×
$F^2\phi^2$	$(2,6)$				×	×				×	×	×			×
$F\psi^2\phi$	$(2,6)$									×				×	×
ψ^4	$(2,6)$	×	×			×	×	×	×	×	×	y^2		×	×
$\psi^2\phi^3$	$(4,6)$	×*									y^2				×
$\bar{F}^3$	$(6,0)$			×	×	×			×	×	×	×	×	×	×
$\bar{F}^2\phi^2$	$(6,2)$				×	×				×	×	×			×
$\bar{F}\bar{\psi}^2\phi$	$(6,2)$				×									×	×
$\bar{\psi}^4$	$(6,2)$	×	×	×	×	×	×	×			×	$\bar{y}^2$		×	×
$\bar{\psi}^2\phi^3$	$(6,4)$					$\bar{y}^2$	×*								×
$\bar{\psi}^2\psi^2$	$(4,4)$		×		$\bar{y}^2$	×		×		y^2	×			×	×
$\bar{\psi}\psi\phi^2 D$	$(4,4)$														×
$\phi^4 D^2$	$(4,4)$				×					×		×			×
ϕ^6	$(6,6)$	×*		×	×		×*		×	×		×			

$+$ angular momentum selection rules

[Jiang, Shu, Xiao, Zheng '20]

Non-interference

[Azatov, Contino, Machado, Riva '16]

massless tree four-point amplitudes involving transverse bosons do not overlap in helicity at dim-4 and dim-6

A_4	$\lvert h(A_4^{\mathrm{SM}})\rvert$	$\lvert h(A_4^{\mathrm{BSM}})\rvert$
$VVVV$	0	4,2
$VV\phi\phi$	0	2
$VV\psi\psi$	0	2
$V\psi\psi\phi$	0	2
$\psi\psi\psi\psi$	2,0	2,0
$\psi\psi\phi\phi$	0	0
$\phi\phi\phi\phi$	0	0

interference mass- or loop- suppressed, recovered in the azimuthal angle of decay products or through extra radiation

[Azatov, Elias-Miro, Reyimuaji, Venturini '17]
[Azatov, Barducci, Venturini '19]
[Panico, Riva, Wulzer '17]

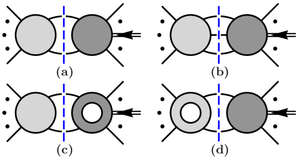
Non-renormalisation beyond one loop

[Bern, Parra-Martinez, Sawyer '19, '20]

$$\text{length}(\mathcal{O}_i) > \text{length}(\mathcal{O}_j) - \#\text{loops}$$

only maximal cut, between tree amplitudes, at minimal L order

	F^3	ϕ^2F^2	$F\phi\psi^2$	$D^2\phi^4$	$D\phi^2\psi^2$	ψ^4	$\phi^3\psi^2$	ϕ^6
F^3		$\times_1$	(2)	$\times_2$	$\times_2$	$\times_2$	$\times_3$	$\times_3$
ϕ^2F^2							(2)	$\times_2$
$F\phi\psi^2$							$\times_1$	$\times_3$
$D^2\phi^4$							$\times_1$	$\times_2$
$D\phi^2\psi^2$							$\times_1$	(3)
ψ^4							(2)	(4)
$\phi^3\psi^2$								(2)
ϕ^6								



UV–EFT matching

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06]

Positivity constraints

- Unitarity $S \cdot S^\dagger = \mathbb{I}$ with $S = \mathbb{I} + i\mathcal{A}$:

$\mathcal{A}^\dagger(+i\epsilon) \overset{\text{CPT}}{=} \mathcal{A}(-i\epsilon)$

sum over intermediate state X

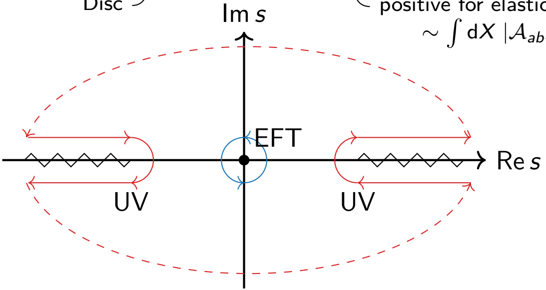
$$(\mathcal{A} - \mathcal{A}^\dagger)/i = \mathcal{A} \cdot \mathcal{A}^\dagger$$

Disc

positive for elastic+forward $\sim \int \mathrm{d}X \, |\mathcal{A}_{ab\to X}|^2$

- Analyticity:

controlled at 4 points



vanishes by Froissart for $n \geq 2$

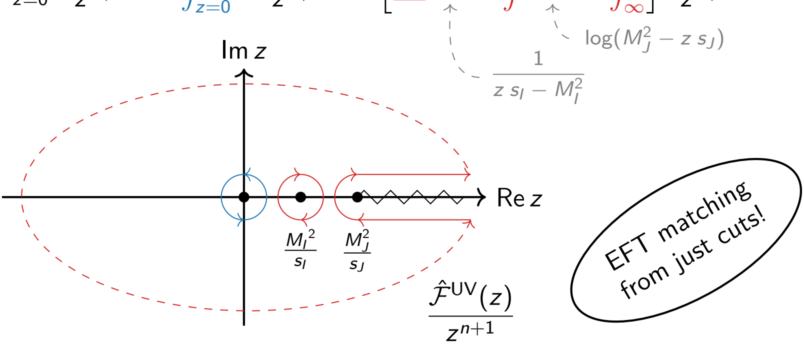
$$\underset{s=0}{\text{Res}} \frac{\mathcal{A}^{\text{EFT}}_{ab\to ab}(s)}{s^{n+1}} = \frac{1}{2\pi} \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}s}{s^{n+1}} \left[\text{Disc}\,\mathcal{A}^{\text{UV}}_{ab\to ab} + (-1)^n \,\text{Disc}\,\mathcal{A}^{\text{UV}}_{a\bar{b}\to a\bar{b}} \right] + C_\infty$$

≥ 0 ≥ 0

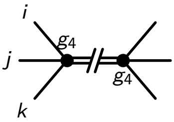
so $c_n \geq 0$ for n even ≥ 2

Dispersive EFT matching

- equate $\mathcal{F}^{\text{EFT}}$ and $\mathcal{F}^{\text{UV}}$ order by order in the zero-momentum expansion
- dilate (with $z^{D/2}$) and enforce $\underset{z=0}{\text{Res}} \dfrac{\hat{\mathcal{F}}^{\text{EFT}}(z)}{z^{n+1}} = \underset{z=0}{\text{Res}} \dfrac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}}$
- EFT: $\underset{z=0}{\text{Res}} \dfrac{\hat{\mathcal{F}}^{\text{EFT}}(z)}{z^{n+1}} = c_n \,\text{poly}_n(s_I)$ with $\mathcal{F}^{\text{EFT}}_{\text{tree}} = \sum_k c_k \,\text{poly}_k(s_I)$
- UV: $\underset{z=0}{\text{Res}} \dfrac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}} = \oint_{z=0} \text{d}z \, \dfrac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}} = \left[\sum \text{Res} + \int \text{Disc} + \int_\infty \right] \dfrac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}}$

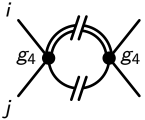


Simple toy $\Phi\phi^3$ example



$$: \underset{z=M^2/s_{ijk}}{\text{Res}} \frac{|\mathcal{A}(\phi\phi\phi \to \Phi)|^2}{z s_{ijk} - M^2} \frac{1}{z^{n+1}}$$

$$= \frac{g_4^2}{M^2} \left(\frac{s_{ijk}}{M^2} \right)^n$$



$$: \frac{1}{2\pi} \int_{M^2/s_{ij}}^{\infty} \frac{\mathrm{d}z}{z^{n+1}} \int \mathrm{dLIPS}\ |\mathcal{A}(\phi\phi \to \phi\Phi)|^2$$

$$= \frac{1}{2\pi} \int_{M^2/s_{ij}}^{\infty} \frac{\mathrm{d}z}{z^{n+1}} \frac{1}{8\pi} \left(1 - \frac{M^2}{z s_{ij}}\right) g_4^2$$

$$= \frac{g_4^2}{16\pi^2 n(n+1)} \left(\frac{s_{ij}}{M^2} \right)^n \qquad \text{for } n > 0$$

- all EFT orders obtained at once
- nothing to know about, or compute in, the EFT
- fewer legs and loops

On-shell amplitude techniques for the SMEFT

SMEFT is becoming the framework of choice
for collider data interpretations.

It allows to isolate subtle patterns of heavy new physics
and to encode the LHC legacy.

On-shell amplitude techniques bring new understanding and
facilitate computations:

to construct operator bases, and characterise kinematics to
arbitrary high dimension.

to understand the structure of anomalous dimensions, and to
match on UV models.