Non-Efimovian (universal) clusters
&
2D quantum anomaly

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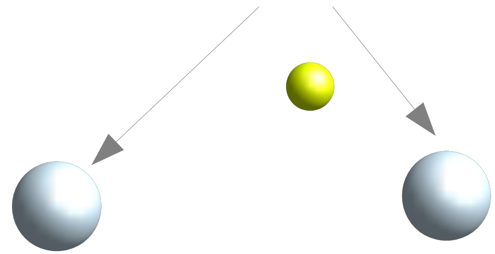
Identical fermions



$\Longrightarrow l=1$

$$R \ll a \Longrightarrow \widetilde{U}_{eff}(R) \approx \frac{\hbar^2}{MR^2}\underbrace{\left(2-0.16\frac{M}{m}\right)}_{\beta}$$

$\beta < -1/4, \quad M/m > 13.6$

$$\chi(R) \propto \sqrt{R}\sin\left(\sqrt{-1/4-\beta}\ \log R/r_3\right)$$

"Fall of a particle to the center in R^{-2} potential". Infinite number of zeros of the wavefunction. Infinite number of trimer states. Efimov effect

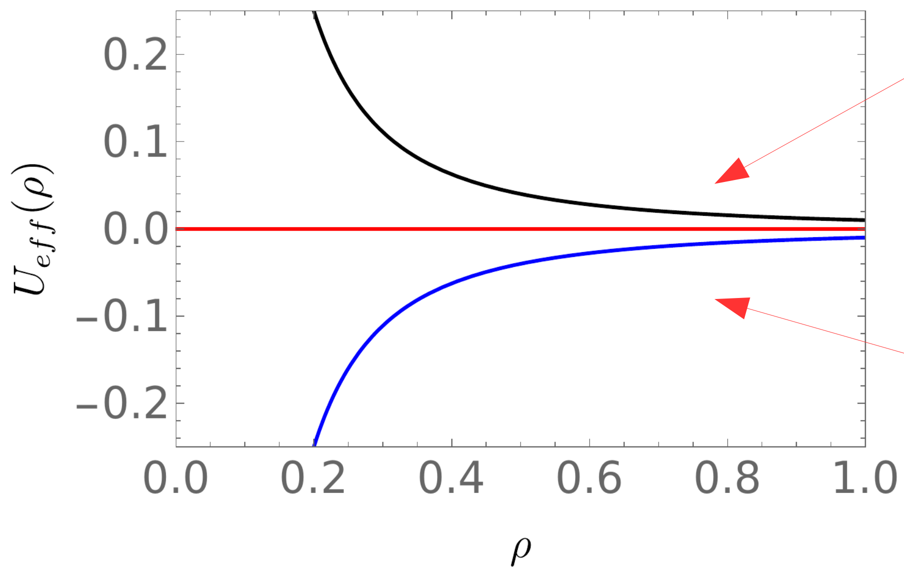
Yb-Li, Dy-Li, Er-Li,...?

$\beta > -1/4, \quad M/m < 13.6$

$$\chi(R) \propto R^{1/2+\sqrt{\beta+1/4}}$$

"Universal" regime in the sense that one needs no three-body parameter. Fermi statistics wins over the induced attraction

Homonuclear mixtures, K-Li, K-Dy, Li-Cr, K-Yb, K-Er,...?



$$U_{eff}(\rho) \propto \frac{1}{\rho^2}$$

Valid only in the scale-invariant region $\rho \ll |a|$

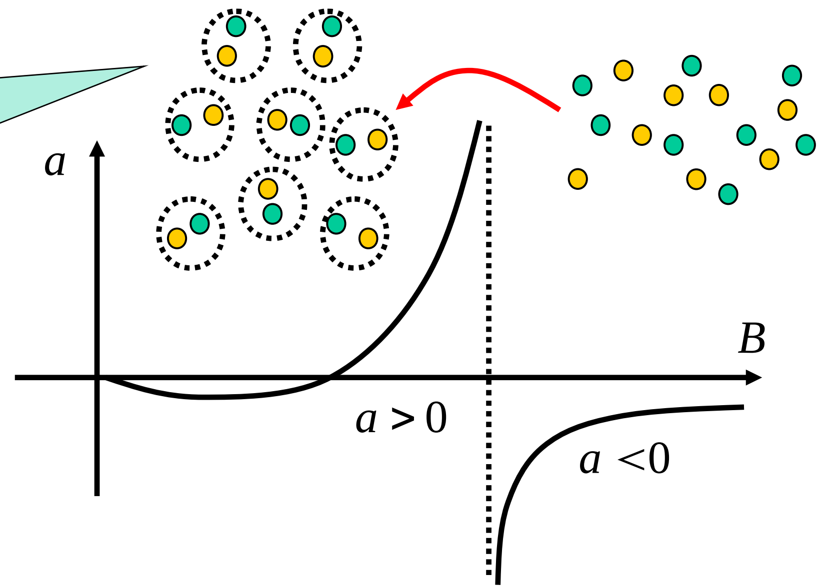
Consequence of the fact that for $a = \infty$
the two-body interaction is scaling invariant

$$\frac{(r\psi(r))'}{r\psi(r)} = -\frac{1}{a} = 0 \quad \Longleftrightarrow \quad \psi(r) \underset{r\to 0}{\to} C\left(\frac{1}{r} + o(r^0)\right)$$

Finite *a* ⇨
- Efimovian → loss resonances (« loss features »)
- Non-Efimovian → universality of few-body observables and universal bound states... and long lifetime

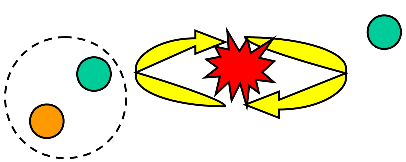
2003: BCS-BEC crossover, molecules

Bose gas of dimers ("BEC side" of the resonance)



Two-component Fermi gas ("BCS side" of the resonance)

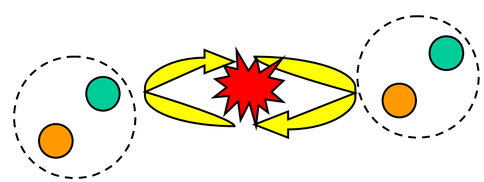
Few-body problem in this case is non-efimovian



$$a_{ad} = 1.2\, a$$

Skorniakov,

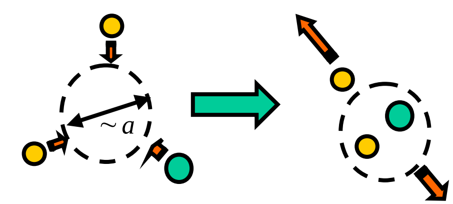
Ter-Martirosian (1957)



$$a_{dd} = 0.6\, a$$

DSP, Salomon,

Shlyapnikov (2003)

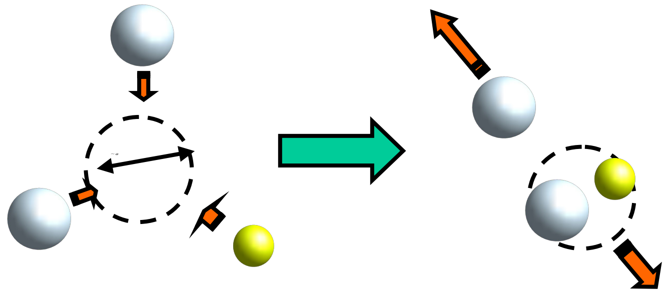


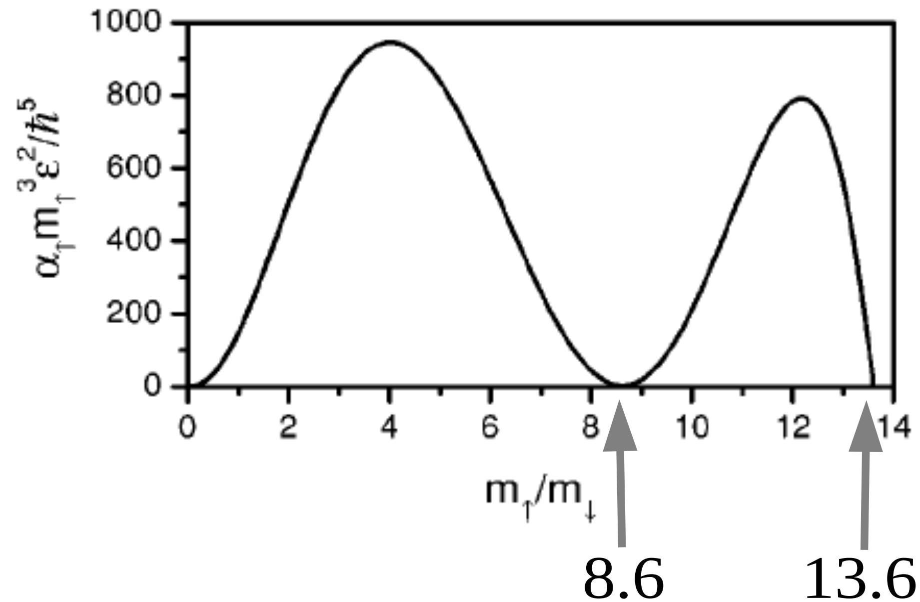
$$\alpha_{rec} = 148 \frac{\hbar a^4}{m} \cdot \frac{\bar{\epsilon}}{\epsilon_0}$$

DSP (2003)

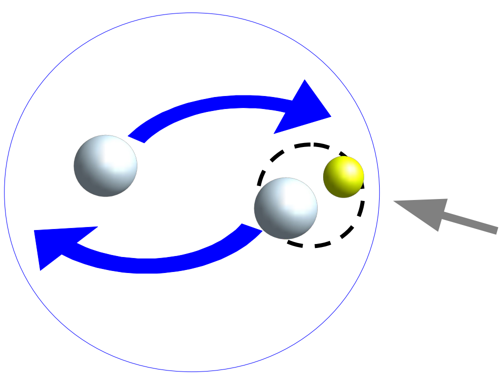
Heavy-heavy-light problem, magic mass ratios

3-body recombination to a weakly bound level DSP'03



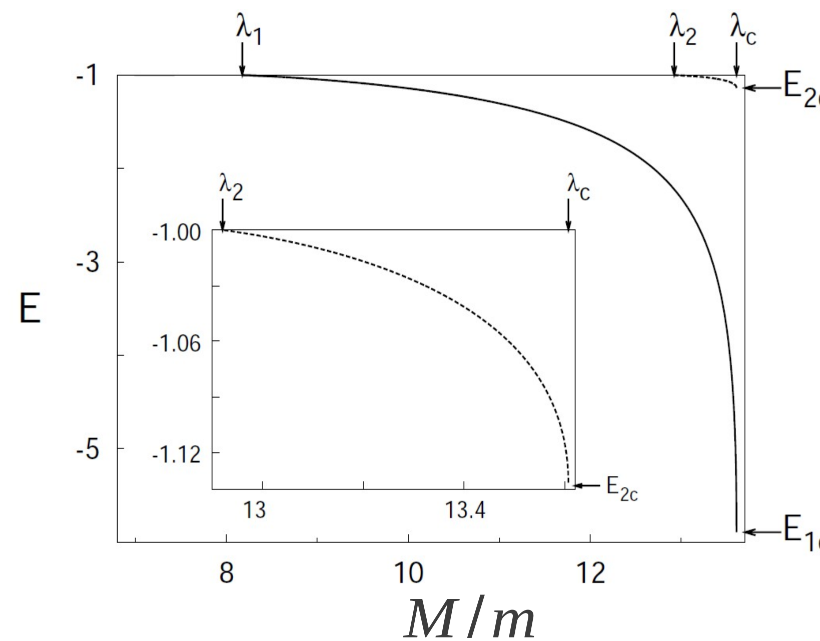


Emergence of a non-Efimovian trimer state for M/m>8.2 Kartavtsev&Malykh'06

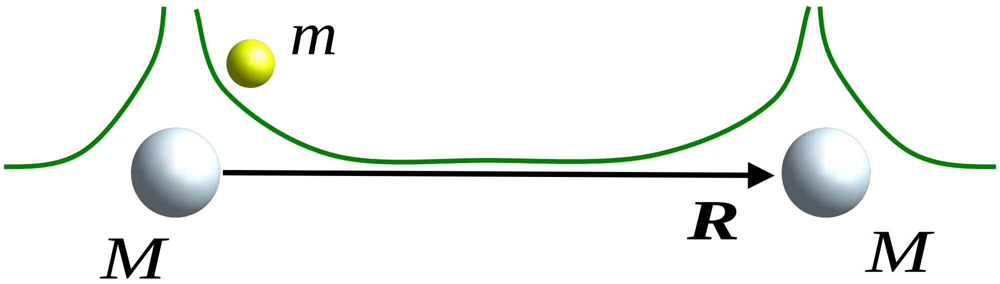


M/m<8.2 *p*-wave atom-dimer scattering resonance

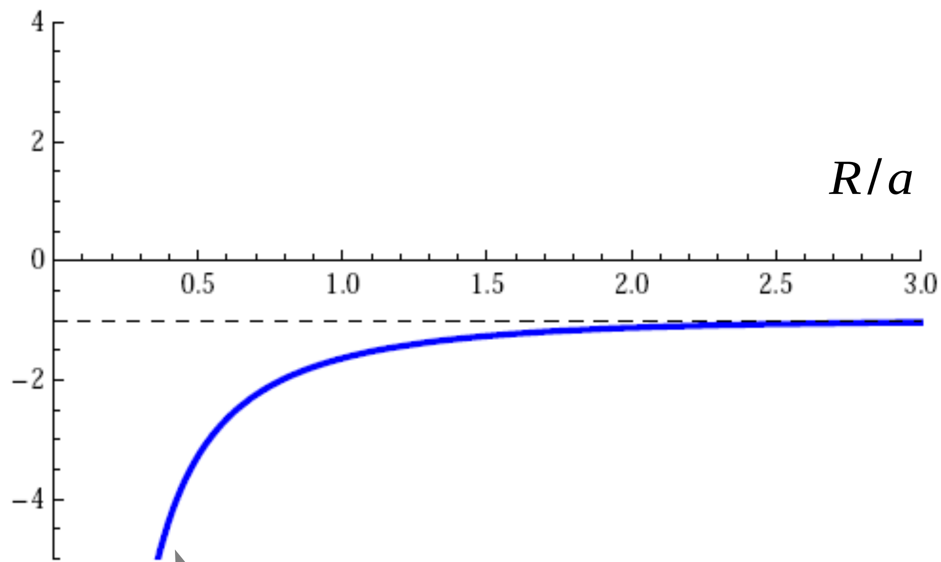
M/m>8.2 trimer state with *l*=1



Born-Oppenheimer picture



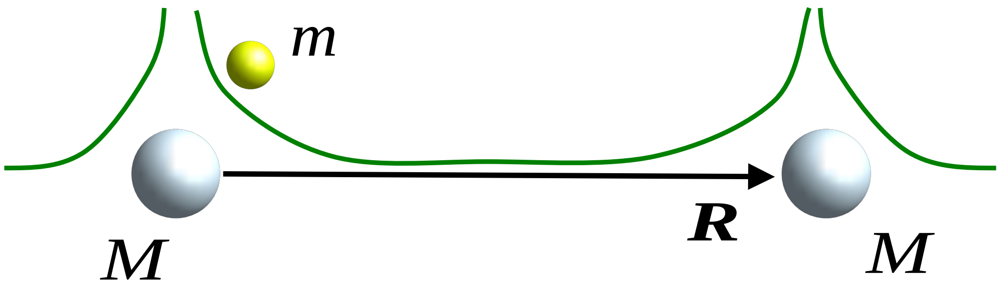
$$\frac{U_{eff}(R)}{\epsilon_0}$$

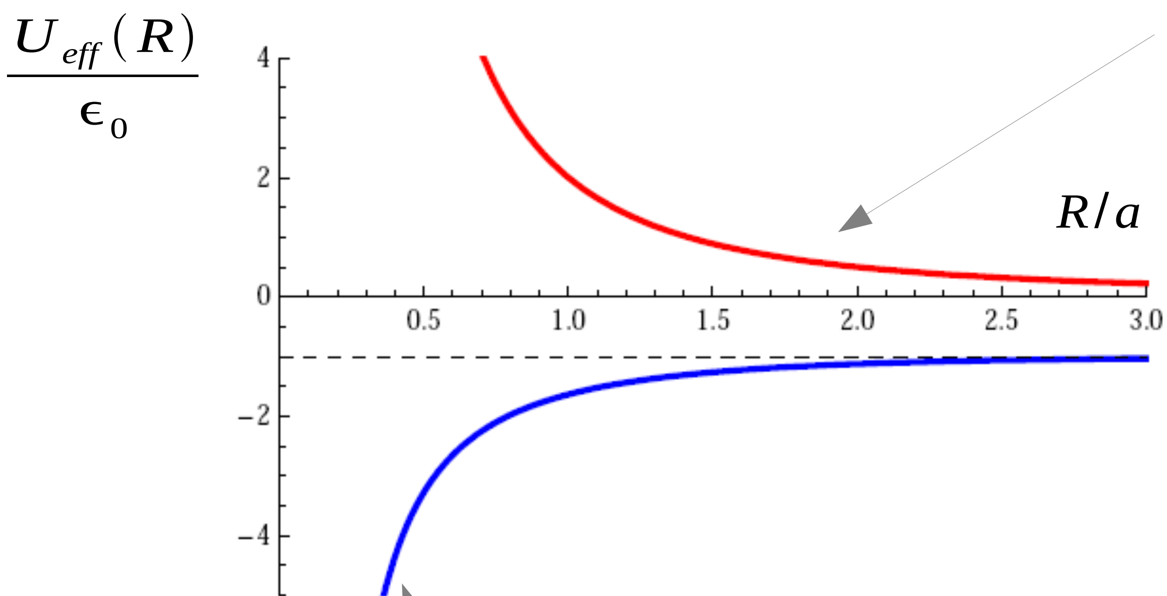


$$U_{eff}(R) \approx -\frac{\hbar^2}{2ma^2} = -|\epsilon_0|$$

$$U_{eff}(R) \approx -0.16\frac{\hbar^2}{mR^2}$$

Born-Oppenheimer picture





$$U_{centrifugal}(R)=\frac{\hbar^2 l(l+1)}{MR^2}$$

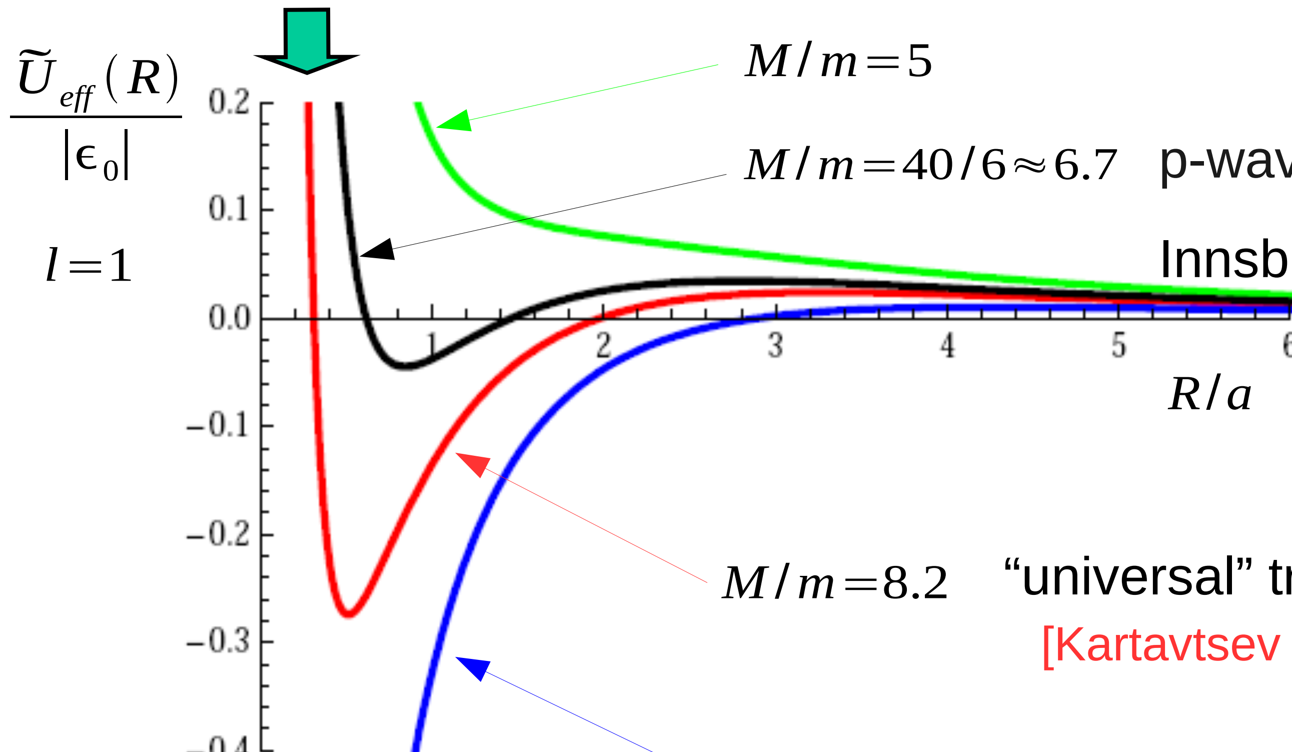
$$U_{eff}(R)\approx-\frac{\hbar^2}{2ma^2}=-|\epsilon_0|$$

$$U_{eff}(R)\approx-0.16\frac{\hbar^2}{mR^2}$$

$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial R^2}+\tilde{U}_{eff}(R)\right]\chi(R)=E\,\chi(R)$$

$$\frac{\hbar^2}{MR^2}\left(l(l+1)-0.16\frac{M}{m}\right)$$

$$\tilde{U}_{eff}(R)=U_{eff}(R)+|\epsilon_0|+\frac{\hbar^2 l(l+1)}{MR^2}$$



$M/m=5$

$M/m=40/6\approx 6.7$ p-wave scattering resonance !

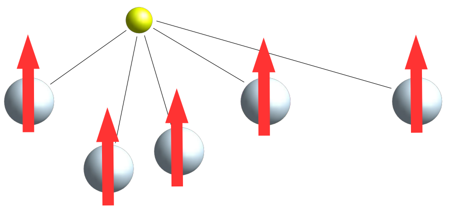
Innsbruck exp: [Jag et al'14]

$M/m=8.2$ “universal” trimer state at threshold [Kartavtsev & Malykh'06]

$M/m>13.6$ many trimer (Efimov) states [Efimov'73]

(N+1)-body problem

How many heavy fermions can be bound by a single light atom?



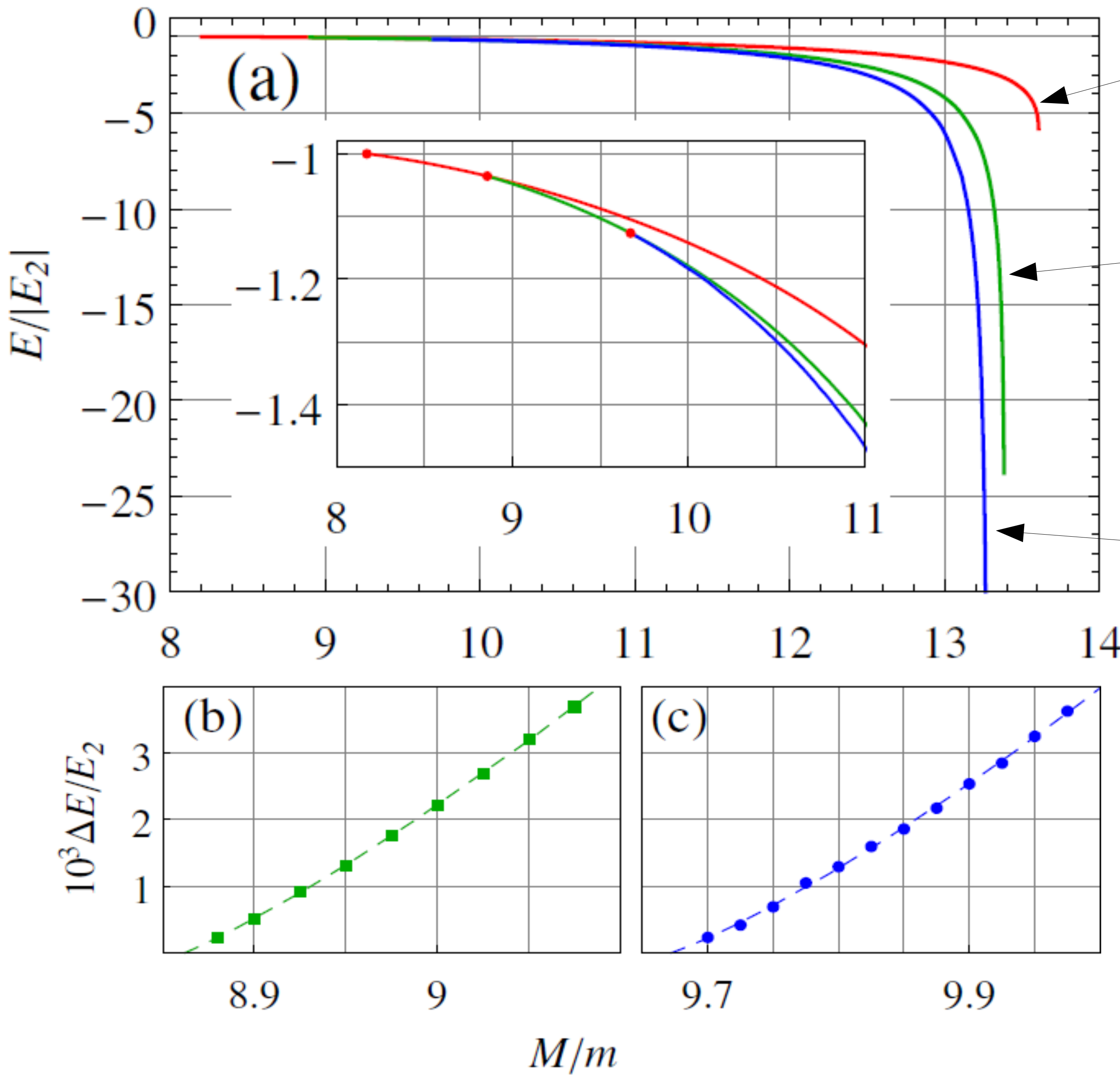
Kinetic energy of the heavy atoms ~1/*M*

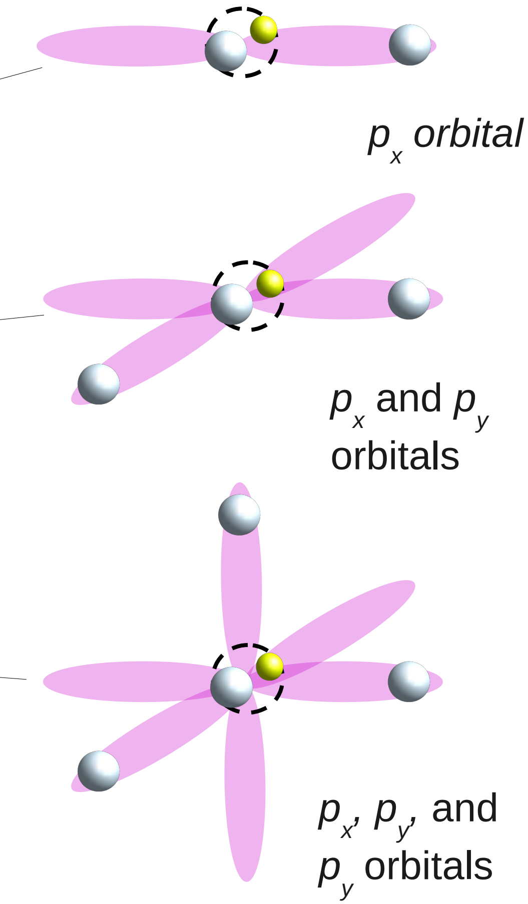
competes with

Attractive exchange potential of the light atom ~1/*m*

Parameters of the free-space zero-range N+1-body problem:

- space dimension *D*
- number of heavy atoms *N*
- mass ratio *M/m*
- dimer size *a* (can be used as the length unit)





pentamer = closed *p*-shell

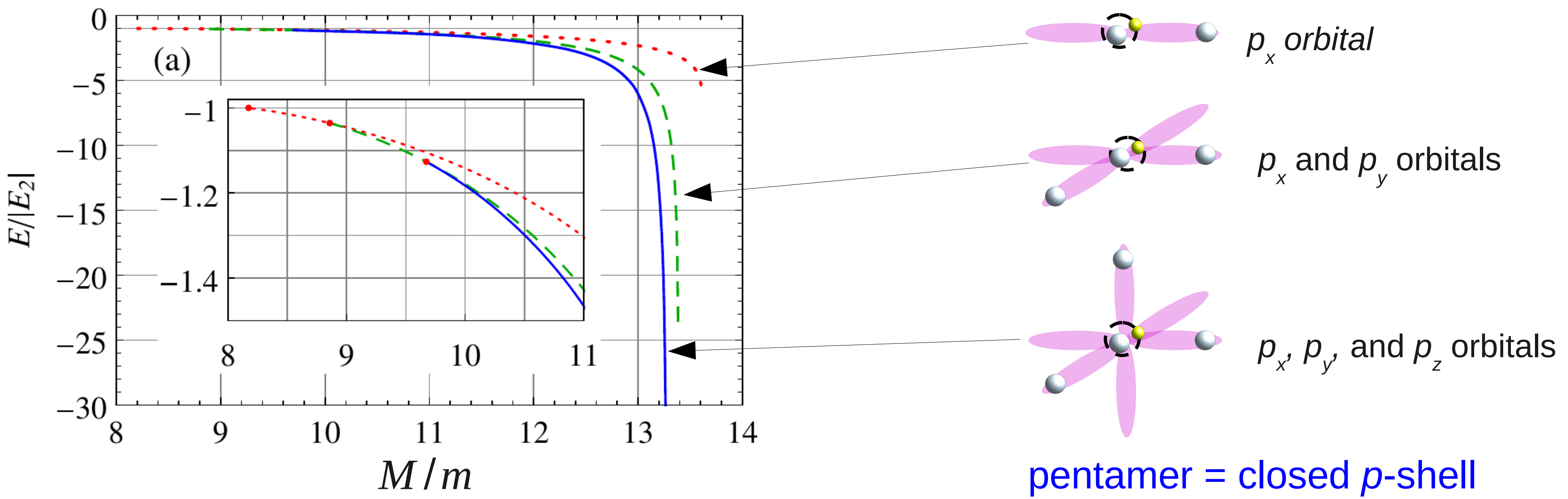
CONJECTURE:

No hexamer!

(requires justification)

3D trimer, tetramer, pentamer,...

	Symmetry L^{π}	appear at $M/m>$	Efimovian for $M/m>$
2+1 trimer	1^-	8.173 Kartavtsev&Malykh'06	13.607 Efimov'73
3+1 tetramer	1^+	8.862(1) Blume'12, Bazak&DSP'17	13.384 Castin,Mora&Pricoupenko'10
4+1 pentamer	0^-	9.672(6) Bazak&DSP'17	13.279(2) Bazak&DSP'17
N+1-mer	?	?	?



pentamer = closed *p*-shell

Physics at *a*=infinity (& zero range)

Small-hyperradius behavior of the (*N*+1)-body wave function:

$$\left[-\frac{\partial^2}{\partial R^2}-\frac{3N-1}{R^2}\frac{\partial}{\partial R}+\frac{s^2-(3N/2-1)^2}{R^2}\right]\Psi(R)=0$$

$$\Psi(R)\propto R^{-3N/2+1\pm s}$$

$$s^2>0 \quad (s>0)$$

$$\Psi(R)\propto R^{-3N/2+1+s}$$

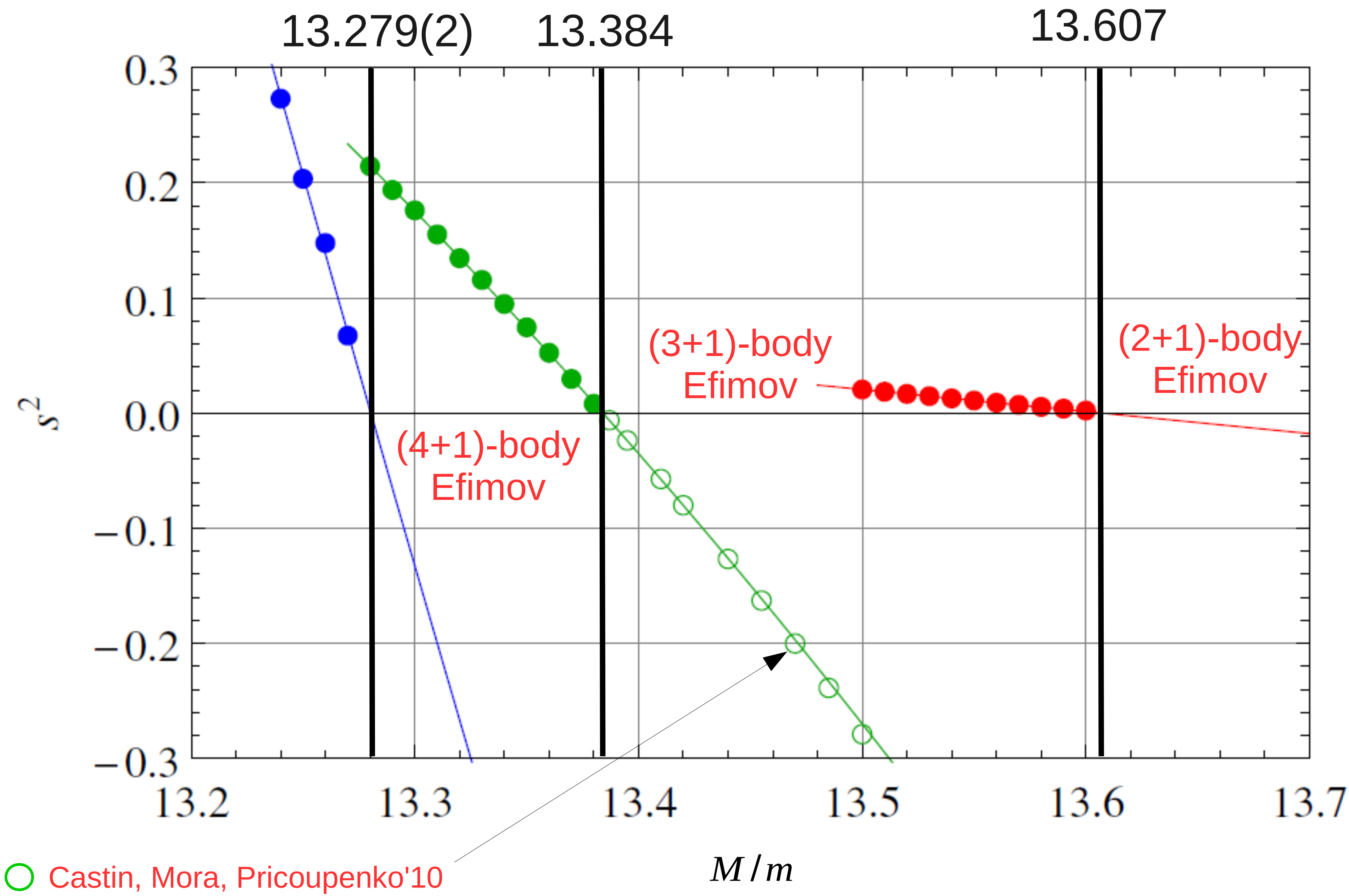
"Universal" regime in the sense that one needs no three-body parameter

Non-Efimovian regime

$$s^2<0 \quad (s=is_0)$$

$$\Psi(R)\propto R^{-3N/2+1}\sin(s_0 \ln R/R_0)$$

"Fall of a particle to the center in R^{-2} potential". Infinite number of zeros of the wave function. Infinite number of trimer states. Efimov effect



13.279(2)
13.384
13.607
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
s^2
(3+1)-body
Efimov
(2+1)-body
Efimov
(4+1)-body
Efimov
13.2
13.3
13.4
13.5
13.6
13.7
M/m
Castin, Mora, Pricoupenko'10

A few words about low dimensions

$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial R^2}+\tilde{U}_{eff}(R)\right]\chi(R)=E\chi(R)$$

3D: $\tilde{U}_{eff}(R)=U_{eff}(R)+|\epsilon_0|+\frac{\hbar^2 l(l+1)}{MR^2}$ ⟹ $l=1\rightarrow(M/m)_c=8.2$

This is actually exact (not Born-Oppenheimer) number

different

2D: $\tilde{U}_{eff}(R)=U^{2D}_{eff}(R)+|\epsilon_0|+\frac{\hbar^2(l^2-1/4)}{MR^2}$ ⟹ Rough guess:

$$(M/m)^{2D}_c\approx\frac{l^2-1/4}{l(l+1)}(M/m)^{3D}_c=3.1$$

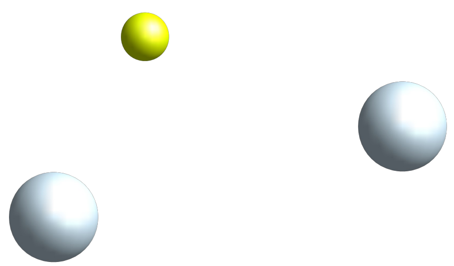
Exact ratio $(M/m)^{2D}_c=3.3$ [Pricoupenko & Pedri'10]

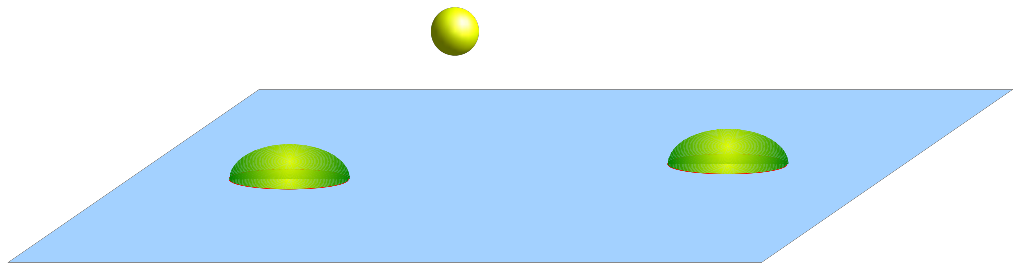
Centrifugal force weaker in 2D → p-wave resonance for smaller mass ratio!

… and in 1D $(M/m)^{1D}_c=1$ exactly!

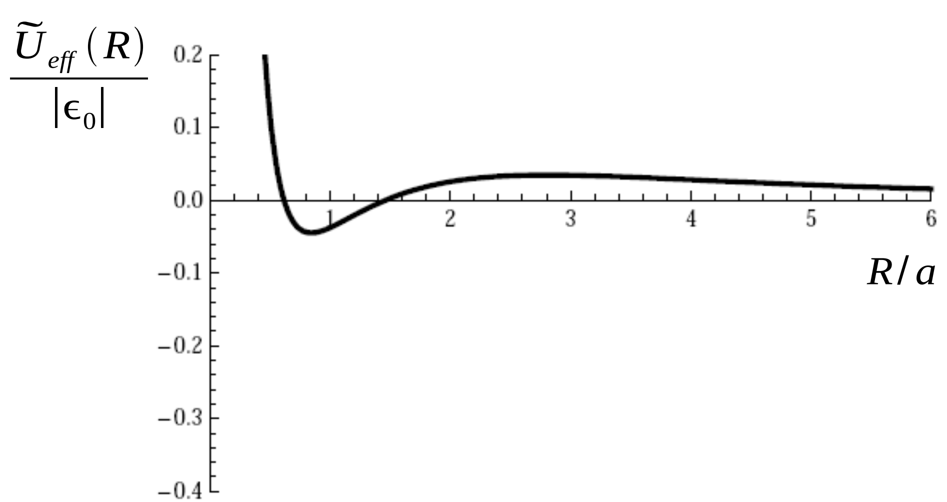
Can we make a bound Li-K-K trimer state?

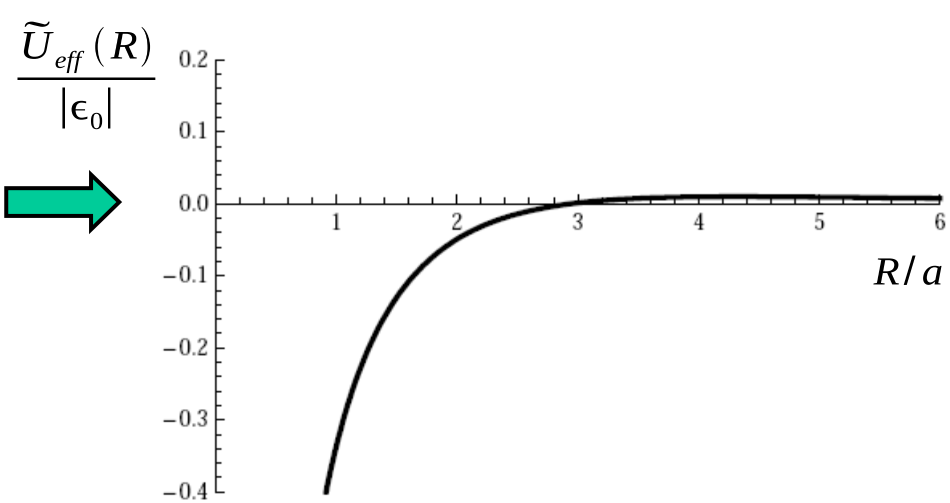
[Levinsen et al'09]

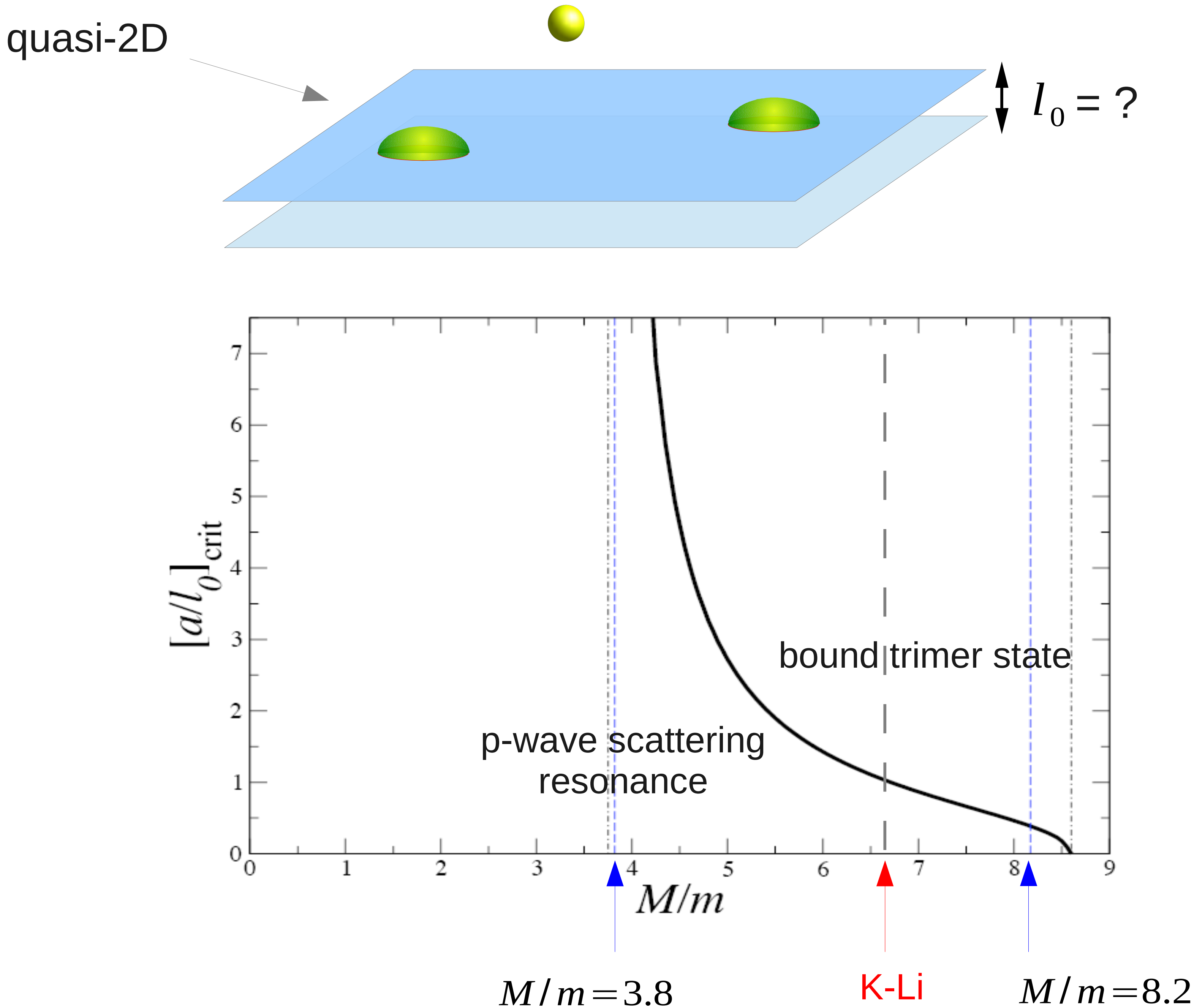




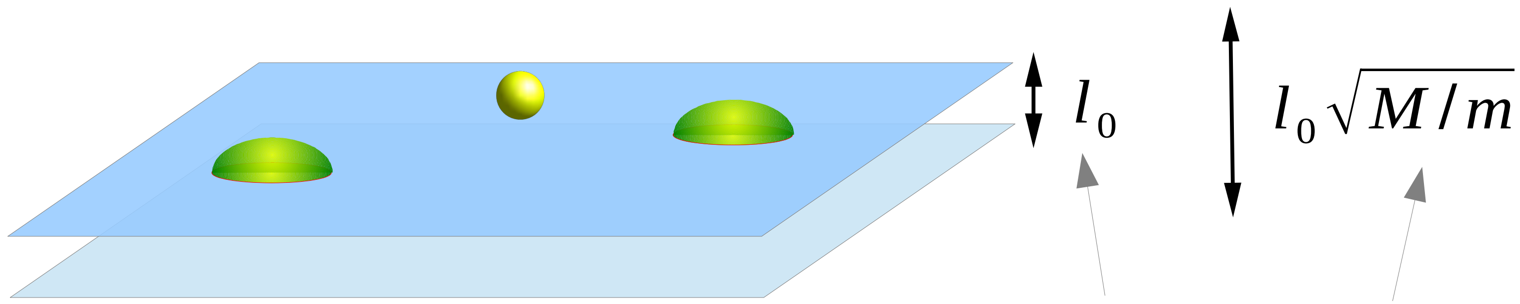
$$U_{centrifugal}(R) = \frac{\hbar^2 l(l+1)}{MR^2} \quad \Rightarrow \quad U_{centrifugal}(R) = \frac{\hbar^2 (l^2 - 1/4)}{MR^2}$$



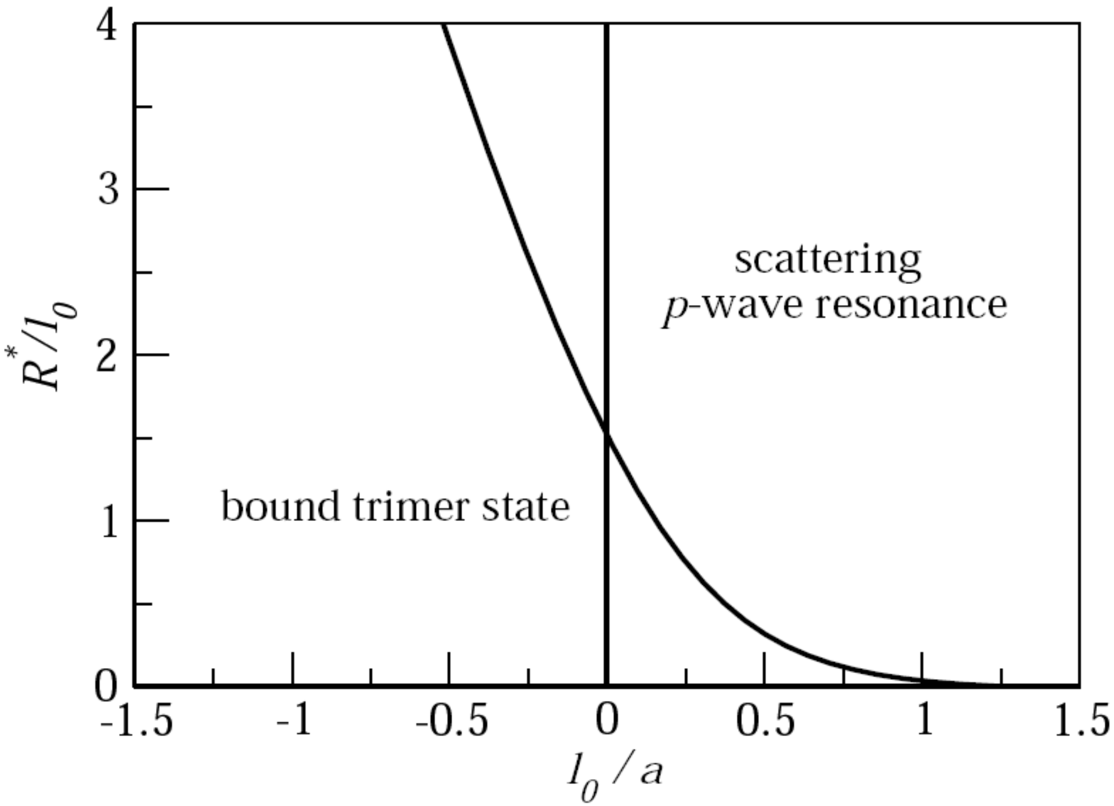



quasi-2D
$l_0 = ?$
$[a/l_0]_{crit}$
p-wave scattering resonance
bound trimer state
M/m
$M/m = 3.8$
K-Li
$M/m = 8.2$

Quasi-2D – quasi-2D case $\omega_{Li} = \omega_K$



Oscillator length for K, for Li



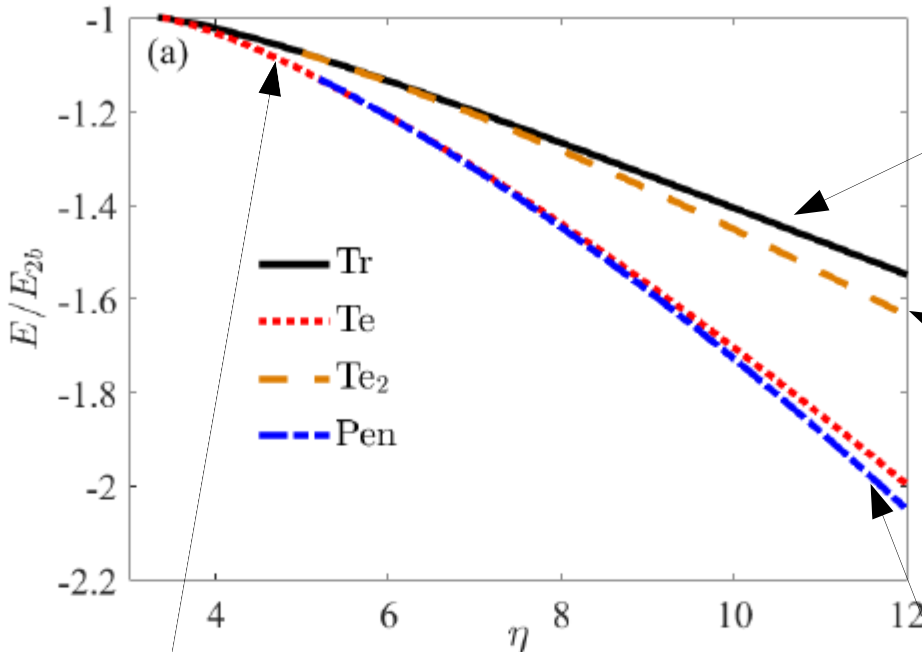
[Levinsen et al'09]

Expect similar effect for Li-Cr and K-Dy!

Exact ratio for the trimer formation in 2D $(M/m)_c^{2D} = 3.3$ [Pricoupenko & Pedri'10]

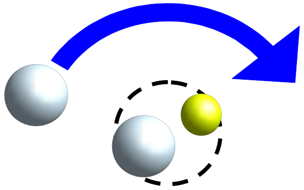
2D trimer, tetramer, pentamer...

[Liu & Peng & Cui'22]



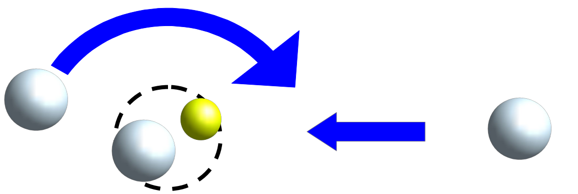
L=1 trimer $(M/m)_c = 3.33$

[Pricoupenko & Pedri'10]



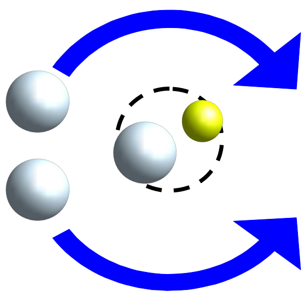
L=1 tetramer $(M/m)_c^{2D} = 5.0$

[Levinsen & Parish'13]



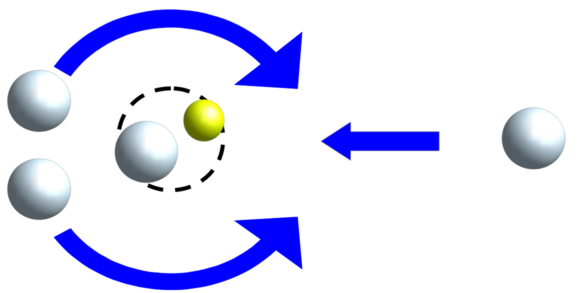
L=0 tetramer $(M/m)_c = 3.38$

[Liu & Peng & Cui'22]



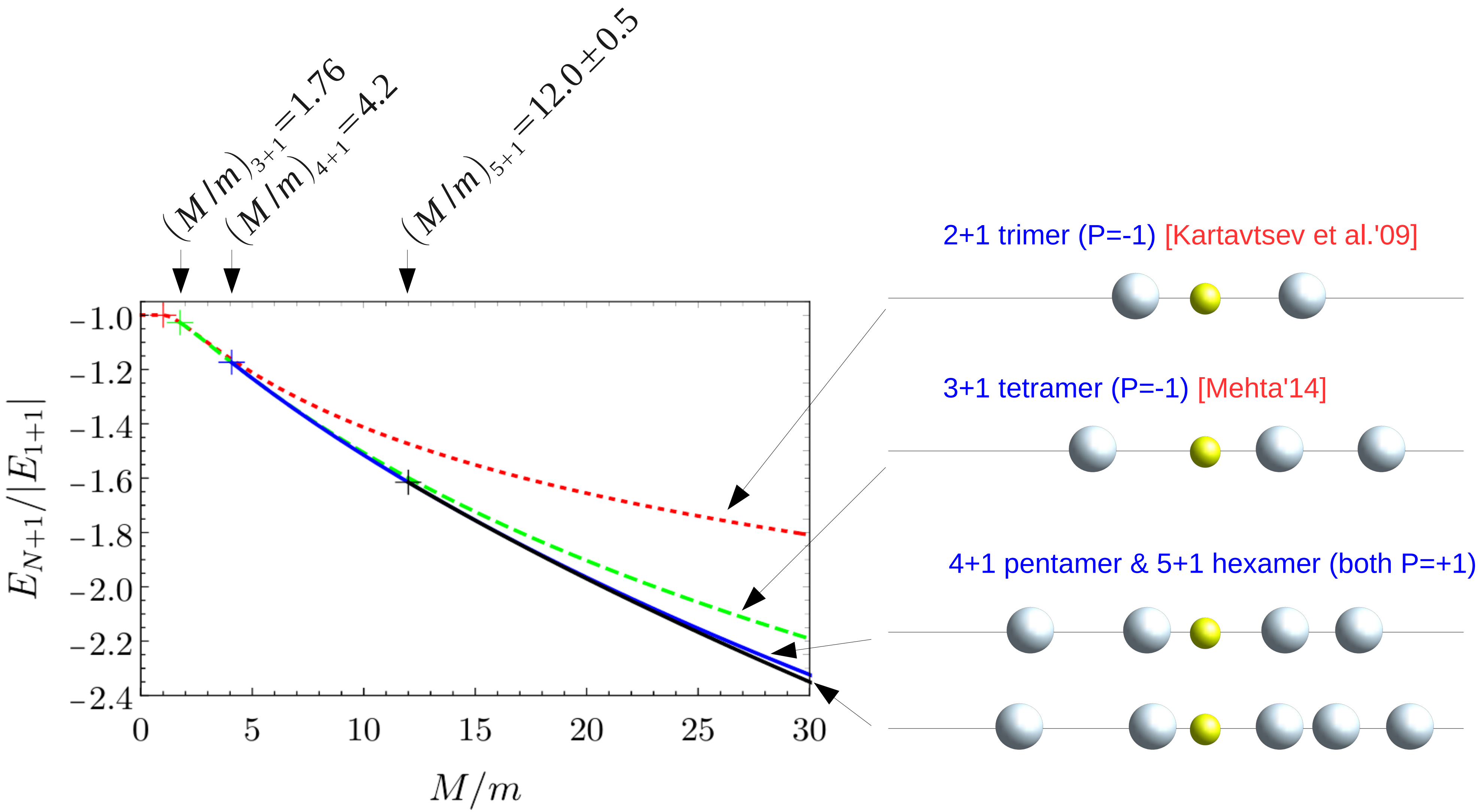
L=0 pentamer $(M/m)_c = 5.14$

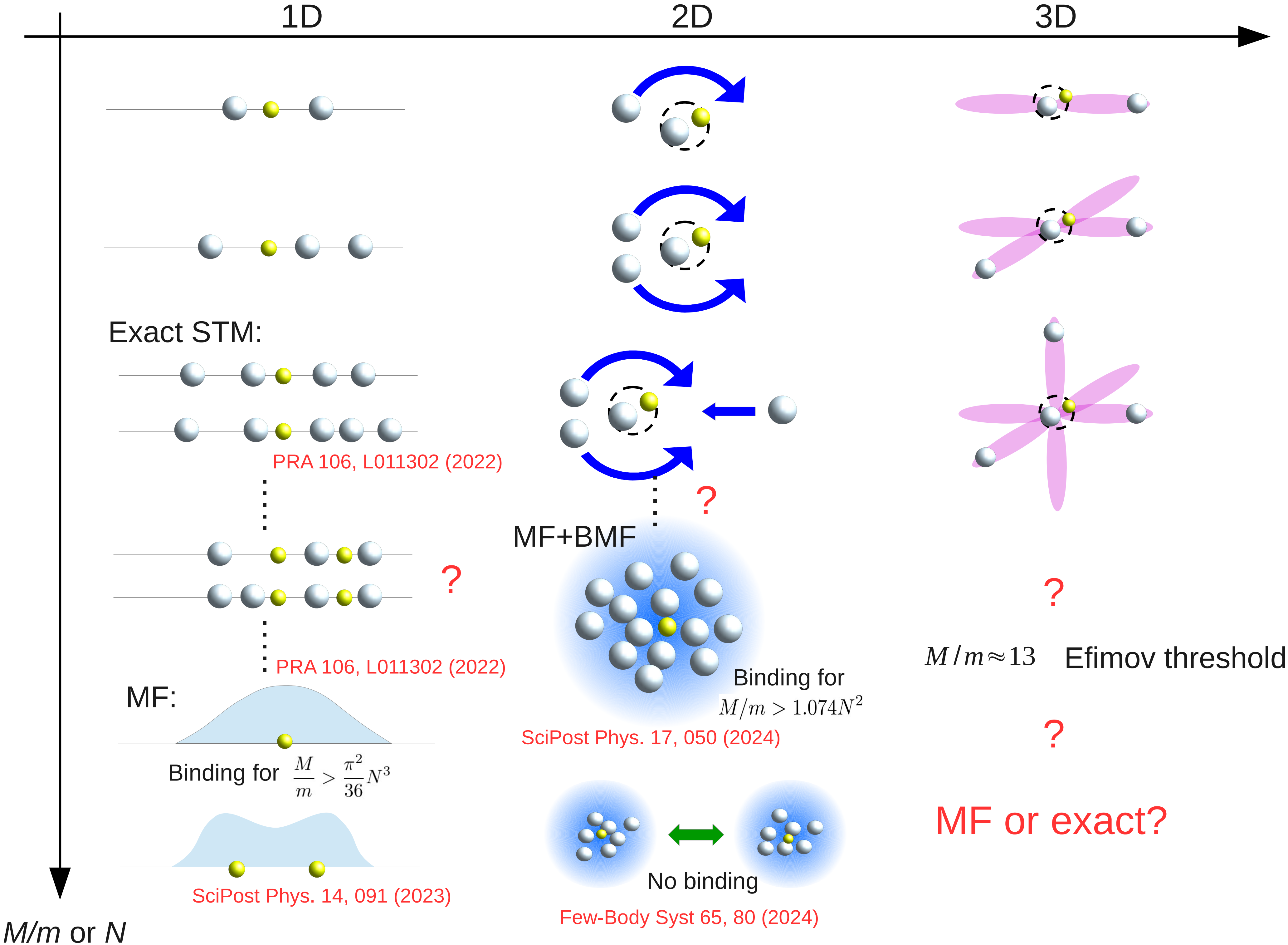
[Liu & Peng & Cui'22]



1D trimer, tetramer...(exact)

A. Tononi, J. Givois, DSP, Phys. Rev. A **106**, L011302 (2022)





1D
2D
3D
Exact STM:
PRA 106, L011302 (2022)
?
PRA 106, L011302 (2022)
MF:
Binding for $\frac{M}{m} > \frac{\pi^2}{36} N^3$
SciPost Phys. 14, 091 (2023)
M/m or N
?
MF+BMF
Binding for $M/m > 1.074N^2$
SciPost Phys. 17, 050 (2024)
No binding
Few-Body Syst 65, 80 (2024)
?
$M/m \approx 13$ Efimov threshold
?
MF or exact?

Pitaevskii-Rosch scaling symmetry

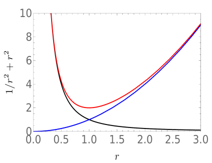
Classical (mean-field) Gross-Pitaevskii energy functional for 2D bosons (similar for 2D Fermi mixtures)

$$E_{\mathrm{MF}}(\Psi,\Psi^*) = (1/2)\int d^2\rho\,[|\nabla_\rho \Psi(\rho,t)|^2 + g|\Psi(\rho,t)|^4]$$

$$V(\rho) = g\delta^2(\rho) \quad : \quad V(\lambda\rho) = \lambda^{-2} V(\rho)$$

dimensionless

$$V(r) = \beta/r^2$$ - other example (any dimension)



$$E_{\mathrm{MF}}(\Psi,\Psi^*) = (1/2)\int d^2\rho\,[|\nabla_\rho \Psi(\rho,t)|^2 + g|\Psi(\rho,t)|^4 + \omega^2\rho^2|\Psi(\rho,t)|^2]$$

Undamped breathing mode with frequency 2ω independent of interaction

[Pitaevskii'1996;Pitaevskii&Rosch'1997]

The model $E_{\mathrm{MF}}(\Psi,\Psi^*)$ (with $V(\rho) = g\delta^2(\rho)$) does not survive quantization, but remains a good approximation in some cases (spoiler : too good)!

To « survive quantization » means to stay valid for the same quantum Lagrangian.

no survival → quantum anomaly (smoking gun = deviation from 2ω)

Quantum models featuring PR symmetry : 3D unitary (non-Efimovian) gases, 1D Tonks gas ($V_{1D}(x)=\infty\,\delta(x)$), $1/r^2$-models in any dimension

PRL 105, 095302 (2010) PHYSICAL REVIEW LETTERS week ending 27 AUGUST 2010

Example of a Quantum Anomaly in the Physics of Ultracold Gases

Maxim Olshanii,[1,2] Hélène Perrin,[2] and Vincent Lorent[2]

(2D bosons)

PRL 108, 185303 (2012) PHYSICAL REVIEW LETTERS week ending 4 MAY 2012

Quantum Anomaly, Universal Relations, and Breathing Mode of a Two-Dimensional Fermi Gas

Johannes Hofmann*

(2D BCS-BEC crossover)

QM Hamiltonian with the Bethe-Peierls bounday condition $\psi(\rho) \underset{\rho \to 0}{\to} C \ln\left(\frac{\rho}{a_{2D}}\right)$

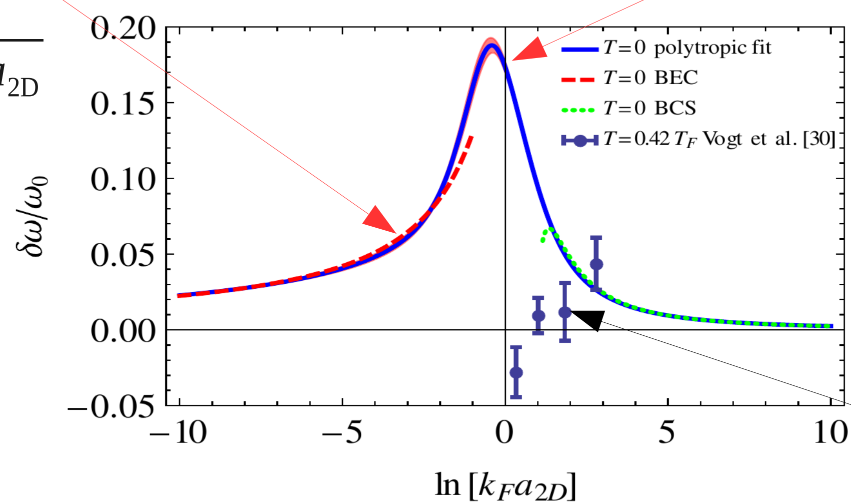
⇓

Hydrodynamic equations with local-density equation of state

Perturbative for bosons [Schick'1971,Popov'1972,Mora&Castin'2009]

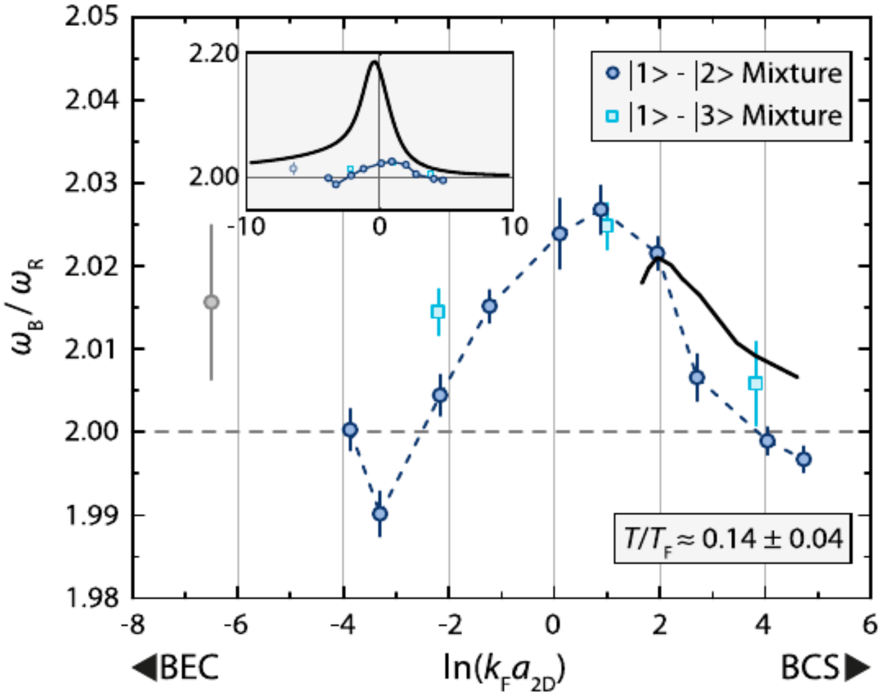
Monte-Carlo in the 2D BCS-BEC crossover [Bertaina&Giorgini'2011]

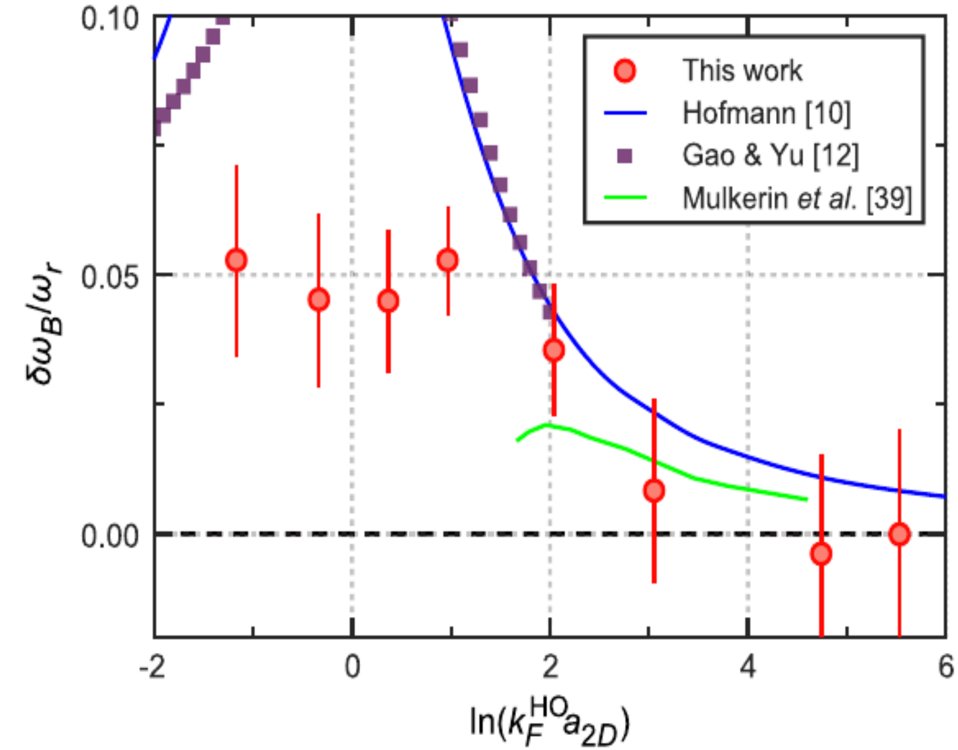
$$\frac{\delta\omega}{\omega} \approx -\frac{1}{4 \ln k_F a_{2D}}$$



[Vogt et al.'2012] (Cambridge)

FERMIONS



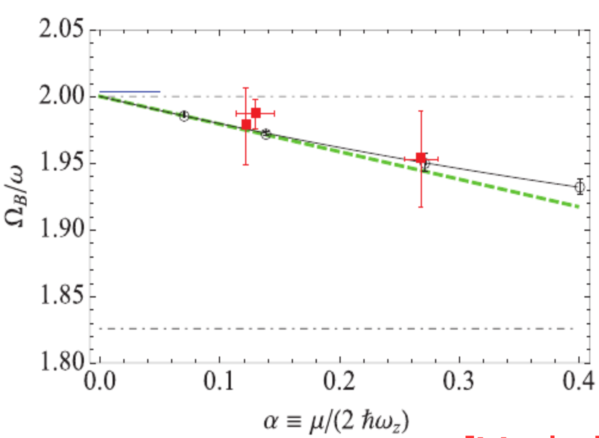


[Holten et al.'2018] (Heidelberg)

see references for theory works

[Peppler et al.'2018] (Melbourne)

BOSONS



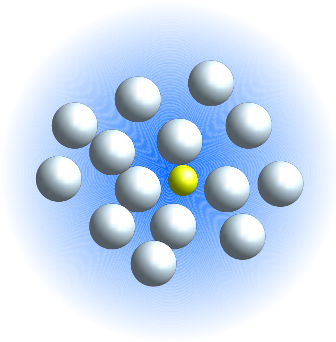
[Merloti et al.'2013] (Paris 13)

Problems :

- small relative shift
- finite-T
- third direction (2D-3D crossover)
- trap anosotropy and inharmonicity

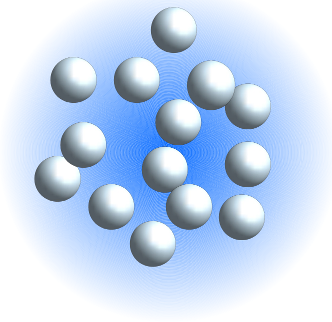
Self-bound

2D N+1 fermionic clusters



Self-bound

2D bosons



If no trap

MF predicts $2\omega = 0$ for the breathing mode frequency → small relative shift

Self-evaporation, « self-cleaning » → finite-T

relevant, not analyzed yet → third direction (2D-3D crossover)

No trap ! → trap anosotropy and inharmonicity

Problems :

- ~~small relative shift~~
- ~~finite-T~~
- third direction (2D-3D crossover)
- ~~trap anosotropy and inharmonicity~~

Next : 2D attractive bosons