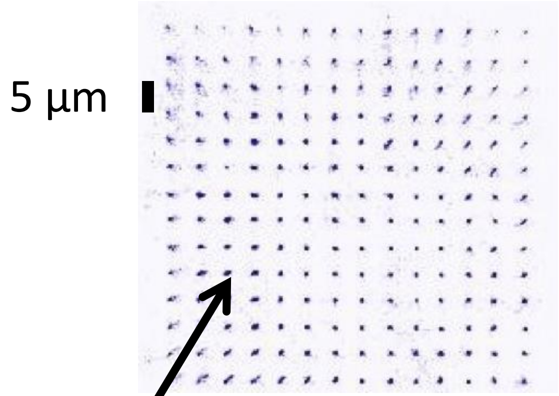
Many-body physics with Rydberg arrays – Lecture 2

Lecture 1: Many-body problem and quantum simulation
Arrays of atoms & “Rydbergology”
Interactions between atoms

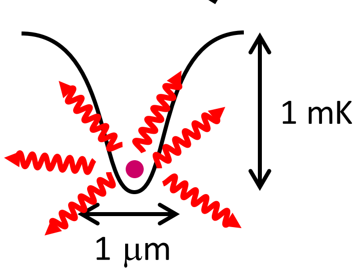
Lecture 2: Rydberg Interactions and spin models
Engineering many-body Hamiltonians

Lecture 3: Examples of quantum simulations in
and out-of-equilibrium: quantum magnetism

Combining arrays of atoms and Rydberg interactions

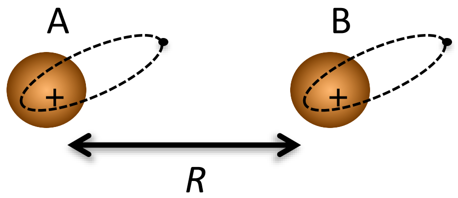


Addressable!!



+

Rydberg interactions



Van der Waals

$$\frac{C_6}{R^6}$$

resonant

$$\frac{C_3}{R^3}$$

Quantum simulation (mainly spin models)

Quantum information processing

A fruitful idea: the Rydberg Blockade

VOLUME 85, NUMBER 10 PHYSICAL REVIEW LETTERS 4 SEPTEMBER 2000

Fast Quantum Gates for Neutral Atoms

D. Jaksch, J. I. Cirac, and P. Zoller
Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

S. L. Rolston
National Institute of Standards and Technology, Gaithersburg, Maryland 20899

R. Côté[1] and M. D. Lukin[2]

VOLUME 87, NUMBER 3 PHYSICAL REVIEW LETTERS 16 JULY 2001

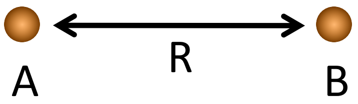
Dipole Blockade and Quantum Information Processing in Mesoscopic Atomic Ensembles

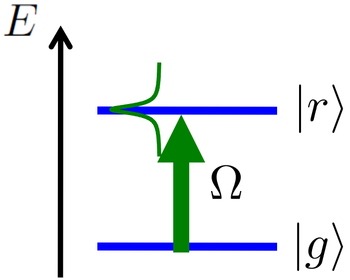
M. D. Lukin,[1] M. Fleischhauer,[1,2] and R. Cote[3]
[1]*ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138*
[2]*Fachbereich Physik, Universität Kaiserslautern, D-67663 Kaiserslautern, Germany*
[3]*Physics Department, University of Connecticut, Storrs, Connecticut 06269*

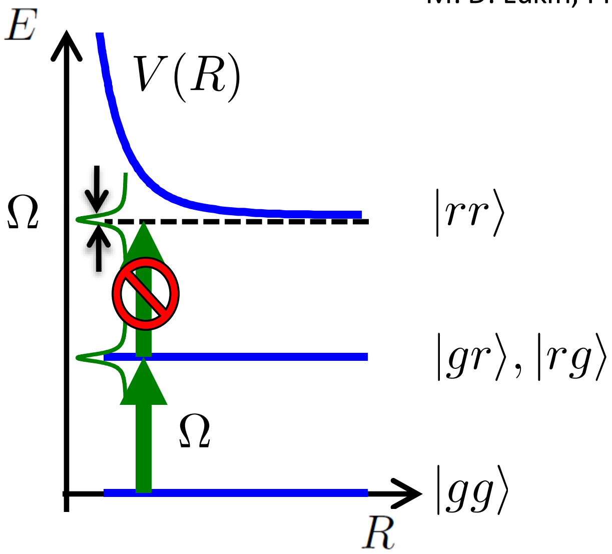
L. M. Duan, D. Jaksch, J. I. Cirac, and P. Zoller
Institut für Theoretische Physik, Universität Innsbruck, A-6020 Innsbruck, Austria
(Received 7 November 2000; published 26 June 2001)

A fruitful idea: the Rydberg Blockade

D. Jaksch, PRL **85**, 2208 (2000)
M. D. Lukin, PRL **87**, 037901 (2001)



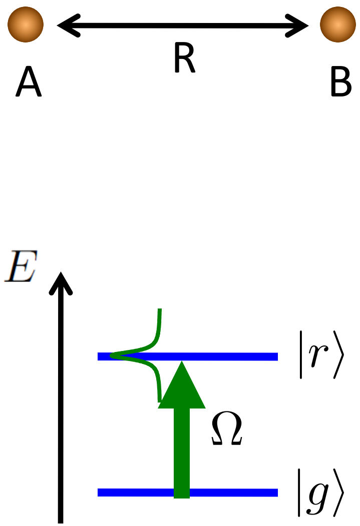


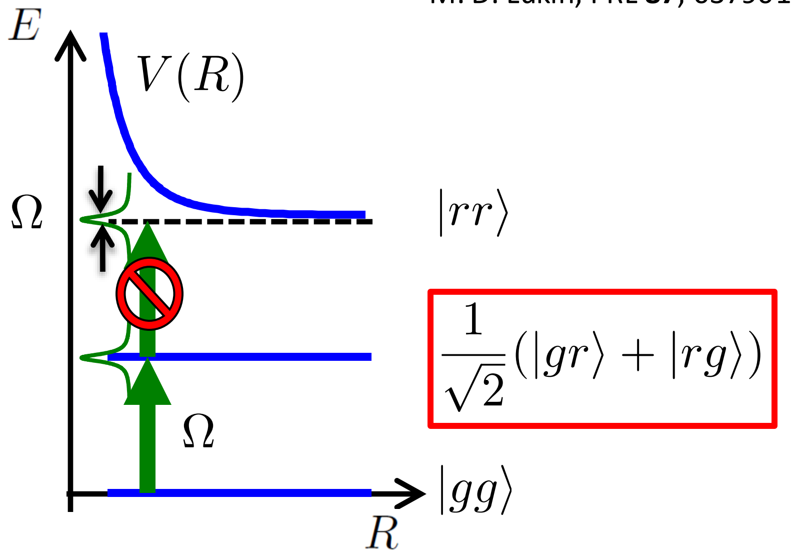


If $\hbar\Omega \ll V(R)$: no excitation of $|rr\rangle$ $\Rightarrow$ **blockage**

A fruitful idea: the Rydberg Blockade

D. Jaksch, PRL **85**, 2208 (2000)
M. D. Lukin, PRL **87**, 037901 (2001)

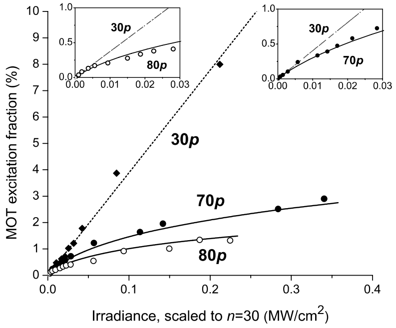




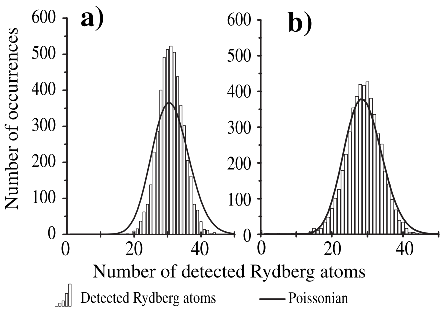
Blockade $\Rightarrow$ **entanglement and gates!!**

The first blockade experiments

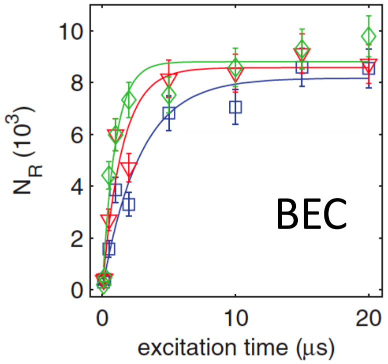
Atomic ensembles



Gould, PRL 2004
Weidemuller, PRL 2004
Martin, PRL 2004
Pillet, PRL 2006

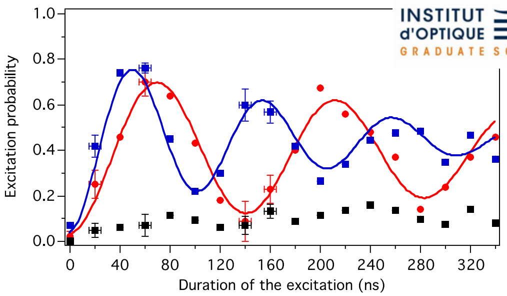


Raithel, PRL 2005



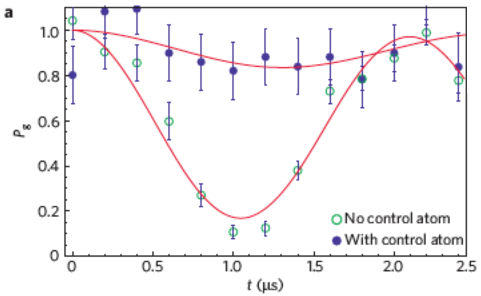
Pfau, PRL 2007

Individual atoms



Blockade + collective excitation √2

Gaétan *et al.*, Nat. Phys. **5**, 115 (2009)



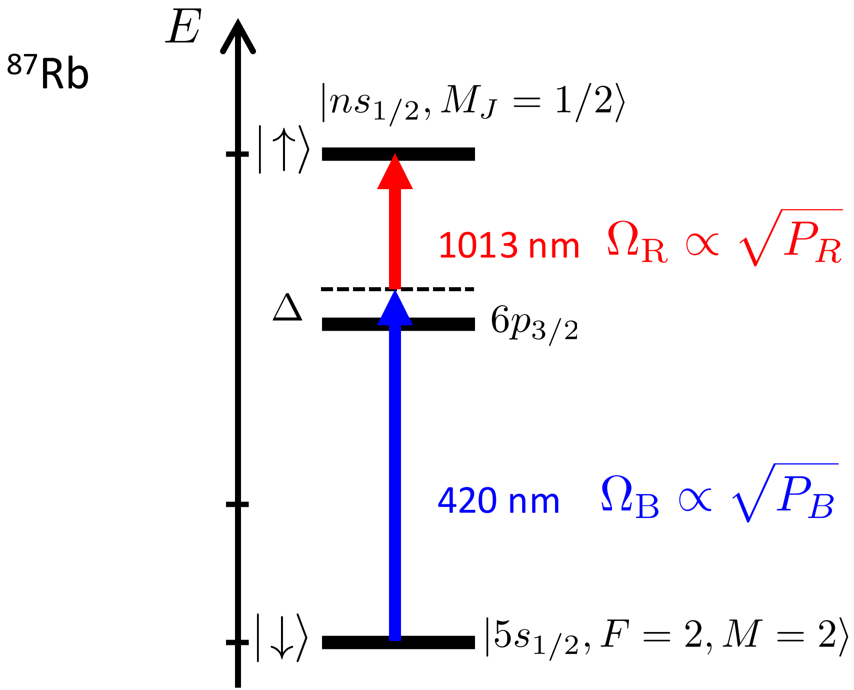
Blockade

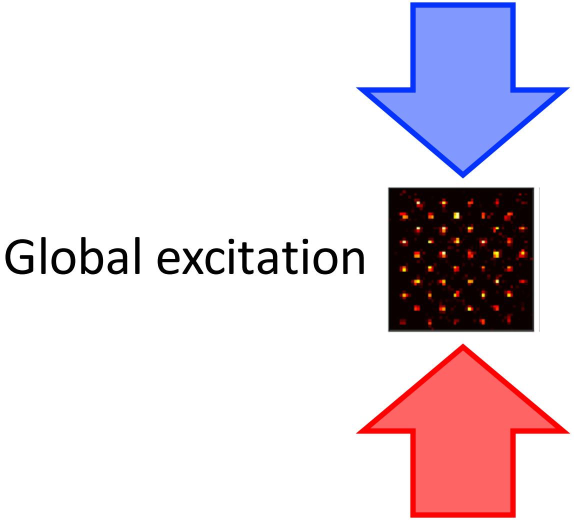
Urban *et al.*, Nat. Phys. **5**, 110 (2009)

Outline – Lecture 2

1. A bit of plumbing…: Rydberg excitation and detection
2. Interactions between Rydberg atoms and spin models
   - "Natural": Ising and XY Hamiltonians
   - Application to hardcore bosons, $t-J$ model
   - Engineered: XYZ (Floquet), Rydberg dressing

Coherent optical Rydberg excitation (n = 50 – 100)

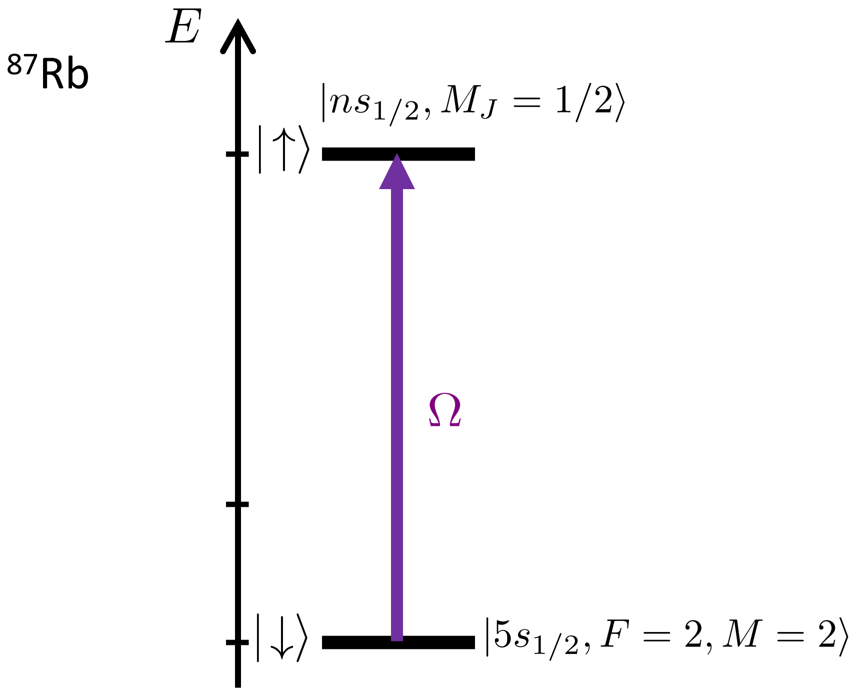


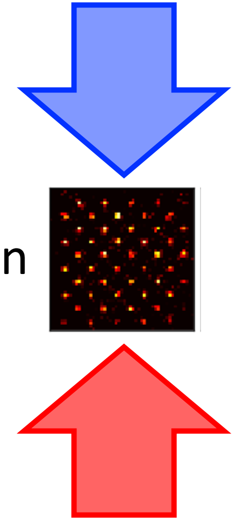


Effective Rabi frequency: $\Omega = \dfrac{\Omega_{\mathrm{R}}\Omega_{\mathrm{B}}}{2\Delta}$

Light-shift: $\delta_{\mathrm{eff}} = \delta - \left(\dfrac{|\Omega_B|^2}{4\Delta} - \dfrac{|\Omega_R|^2}{4\Delta} \right)$

Coherent optical Rydberg excitation (n = 50 – 100)



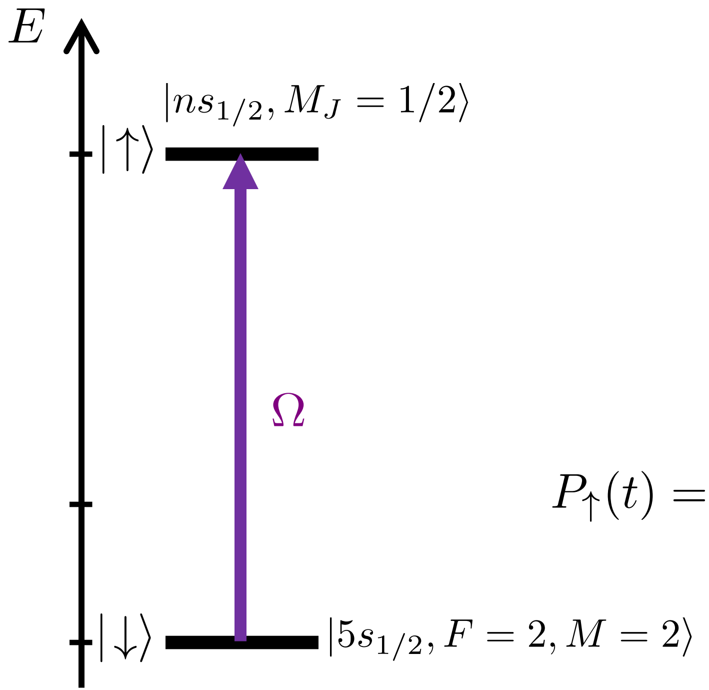


Effective Rabi frequency: $\Omega = \dfrac{\Omega_{\mathrm{R}}\Omega_{\mathrm{B}}}{2\Delta}$

Light-shift: $\delta_{\mathrm{eff}} = \delta - \left(\dfrac{|\Omega_B|^2}{4\Delta} - \dfrac{|\Omega_R|^2}{4\Delta} \right)$

Coherent optical Rydberg excitation ($n = 50 - 100$)

^{87}Rb

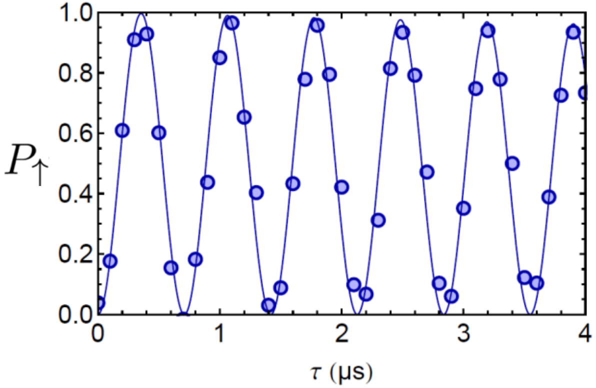


Detection of Rydberg = atom loss (fidelity > 96%)

Single atom $\Rightarrow$ repeat 100 times

Optical rabi oscillations ($\Omega = 0.5 - 5$ MHz)

$$P_\uparrow(t) = \sin^2 \frac{\Omega t}{2}$$

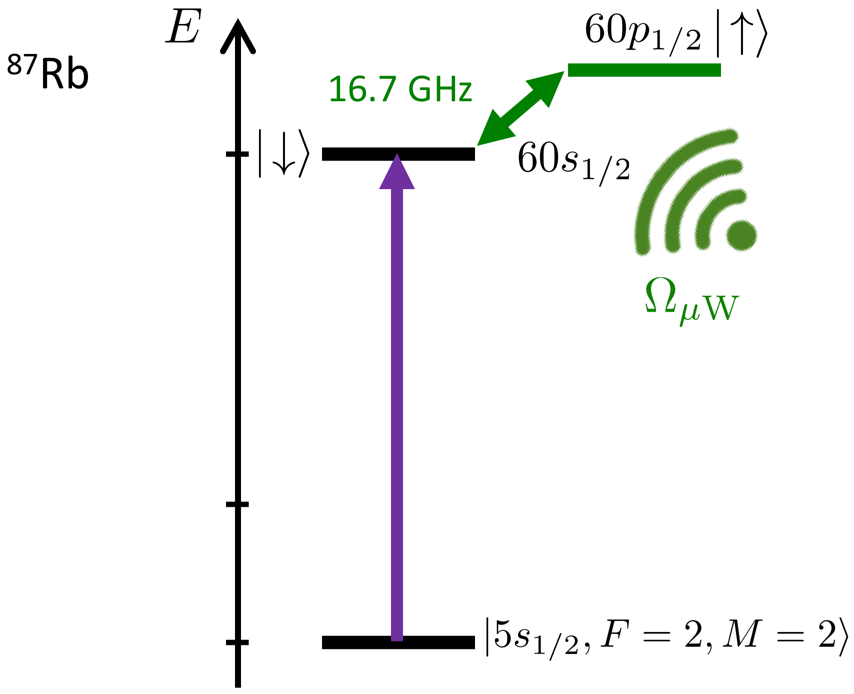


Effective Rabi frequency: $\Omega = \frac{\Omega_R \Omega_B}{2\Delta}$

$$|\psi(t)\rangle = \cos \frac{\Omega t}{2} |\downarrow\rangle - i \sin \frac{\Omega t}{2} e^{i\varphi} |\uparrow\rangle$$

Light-shift: $\delta_{\mathrm{eff}} = \delta - \left(\frac{|\Omega_B|^2}{4\Delta} - \frac{|\Omega_R|^2}{4\Delta} \right)$

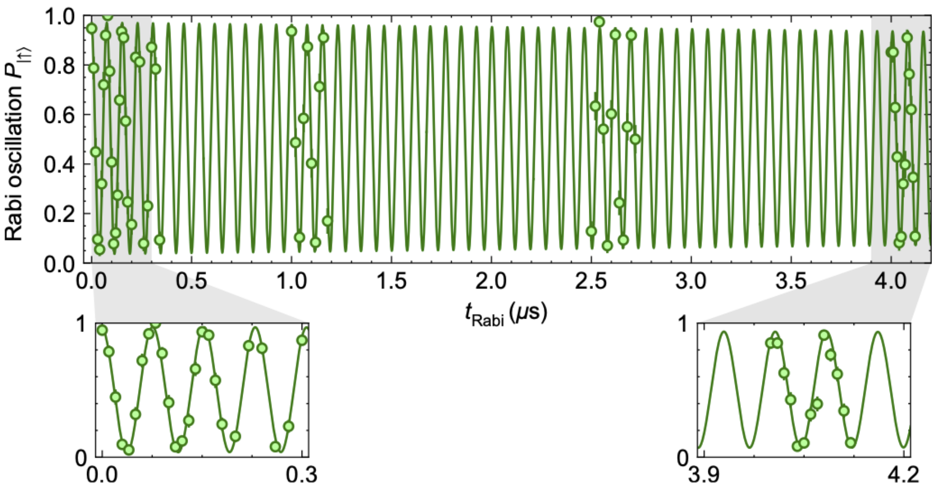
Coherent microwave manipulations (n = 50 – 100)



Detection of Rydberg state $|\uparrow\rangle$ = atom loss after de-excitation of $|\downarrow\rangle$

Single atom ⇒ repeat 100 times

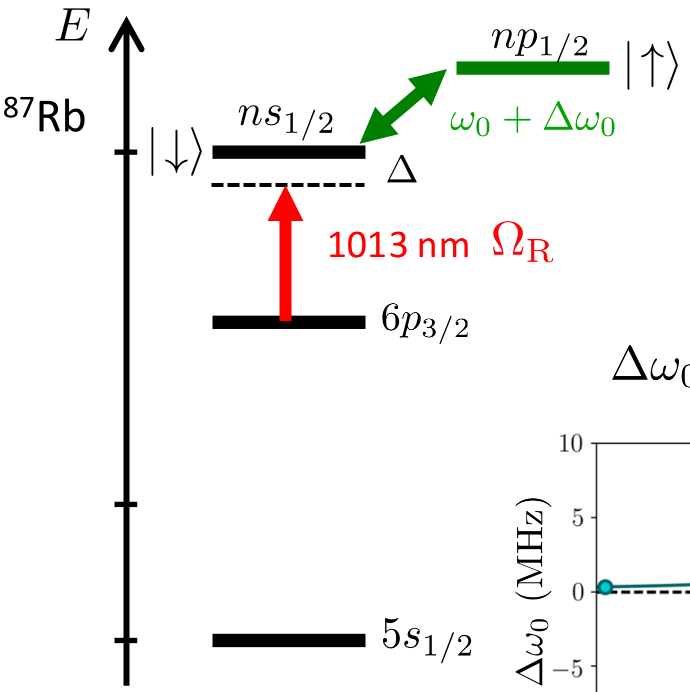
Microwave Rabi oscillations



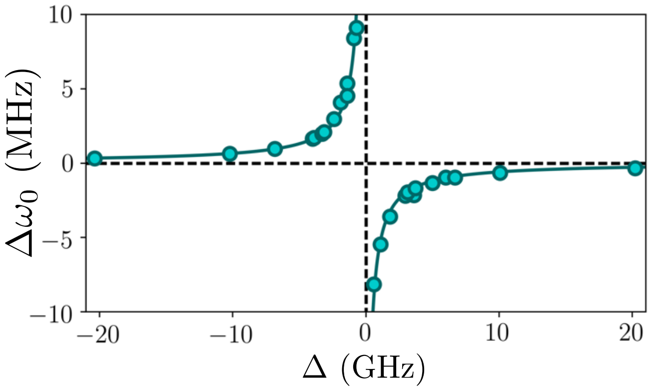
D. Barredo *et al.*,
PRL **114**, 113002 (2015)

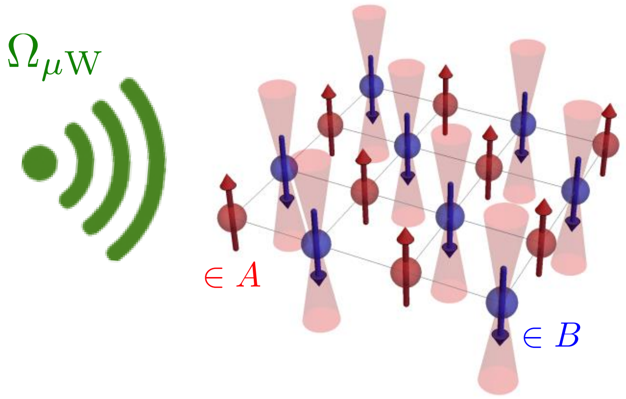
Addressable manipulation in the array with local light-shifts

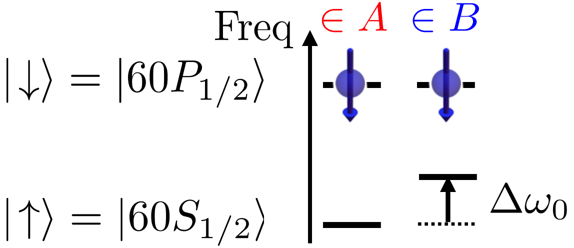
Microwave manipulations are ***global*** ($\lambda \sim$ cm)



$$\Delta\omega_0 \approx \frac{\Omega_R^2}{4\Delta}$$







de Léséleuc, PRL **119**, 053202 (2017)

Outline – Lecture 2

1. A bit of plumbing…: Rydberg excitation and detection

2. Interactions between Rydberg atoms and spin models

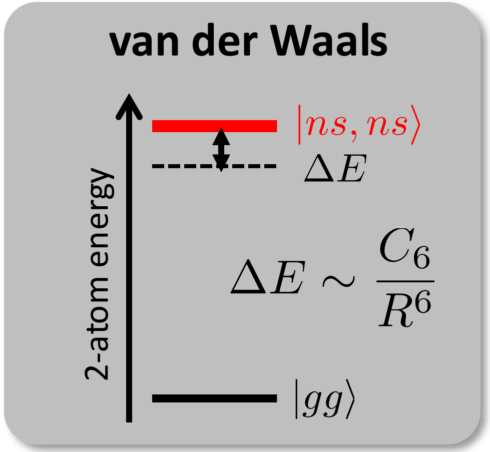
   - "Natural": Ising and XY Hamiltonians

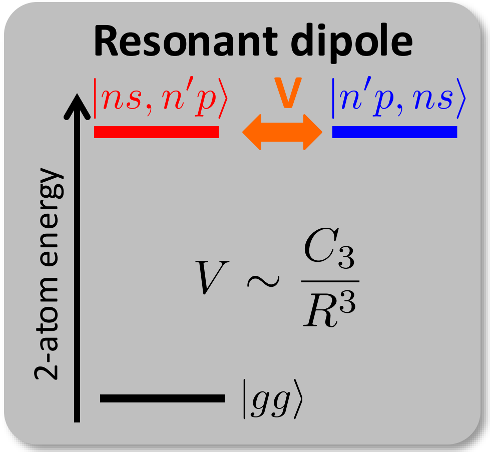
   - Application to hardcore bosons, $t-J$ model

   - Engineered: XYZ (Floquet), Rydberg dressing

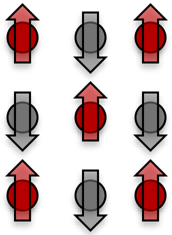
Interactions between Rydberg atoms and spin models

Browaeys & Lahaye, Nat.Phys. (2020)





Spin 1/2



Ising model

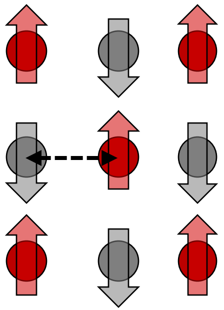
$$\hat{H} = \sum_{i \neq j} J_{ij} \hat{n}_i \hat{n}_j$$

XY model

$$\hat{H} = \sum_{i \neq j} J_{ij} \left(\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+ \right)$$

Spin models: one of the "simplest" many-body systems

Interacting spin ½ particles on a lattice:

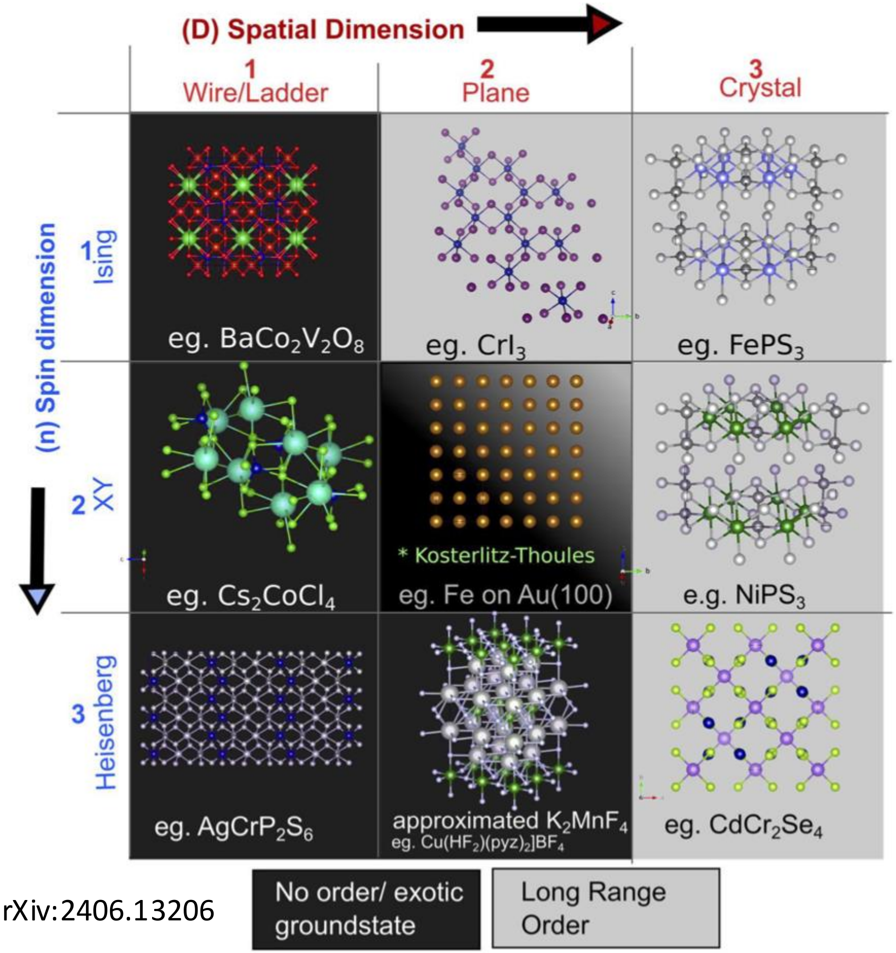


$$\hat{H}_{ij} = J\,\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$

Heisenberg

Ising $$\hat{H}_{\text{Ising}} = \sum_{i \neq j} J_{ij} \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)}$$

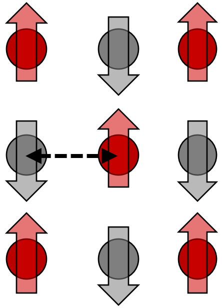
XY model $$\hat{H}_{\text{XY}} = \sum_{i \neq j} J_{ij} \left(\hat{\sigma}_i^{+} \hat{\sigma}_j^{-} + \hat{\sigma}_i^{-} \hat{\sigma}_j^{+} \right)$$



arXiv:2406.13206

Spin models: one of the "simplest" many-body systems

Interacting spin ½ particles on a lattice:



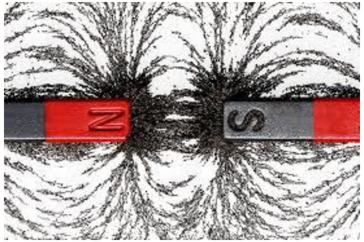
$$\hat{H}_{ij} = J\,\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$

Heisenberg

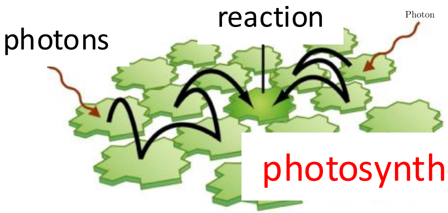
Ising $$\hat{H}_{\text{Ising}} = \sum_{i\neq j} J_{ij}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}$$

XY model $$\hat{H}_{\text{XY}} = \sum_{i\neq j} J_{ij}\left(\hat{\sigma}_i^+\hat{\sigma}_j^- + \hat{\sigma}_i^-\hat{\sigma}_j^+\right)$$

Magnetism



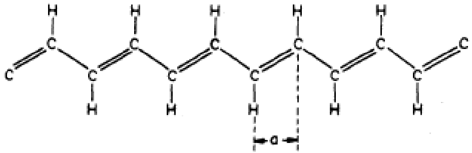
Transport of excitations



photosynthesis

Light scattering

excitons

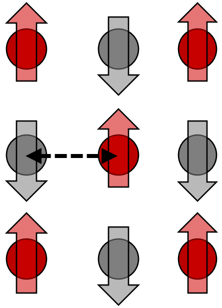


Spin models = generic systems to study many-body questions:

Quantum phase transition, out-of equilibrium, topology, entanglement…

Spin models: one of the "simplest" many-body systems

Interacting spin ½ particles on a lattice:



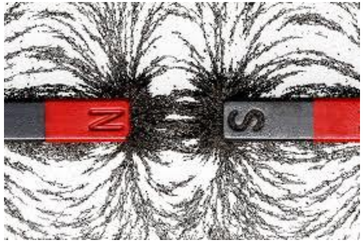
$$\hat{H}_{ij} = J\,\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j$$

Heisenberg

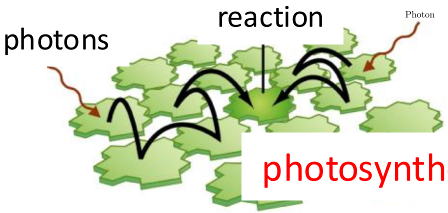
Ising $$\hat{H}_{\mathrm{Ising}} = \sum_{i \neq j} J_{ij} \hat{\sigma}_z^{(i)} \hat{\sigma}_z^{(j)}$$

XY model $\hat{H}_{\mathrm{XY}}$

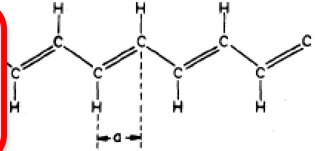
Magnetism



Transport of excitations



Light scattering

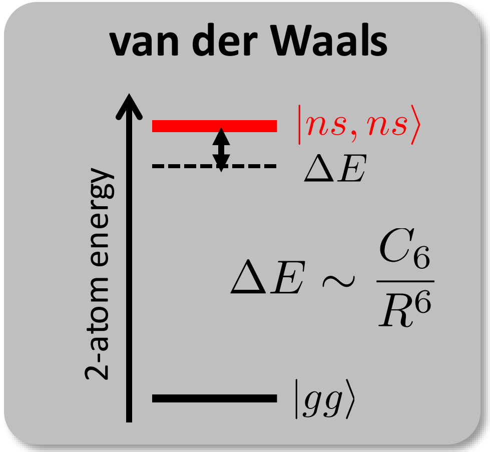


Use control over synthetic quantum systems

Spin models = generic systems to study many-body questions:

Quantum phase transition, out-of equilibrium, topology, entanglement…

From van der Waals interaction to spin models...



$C_6 \propto n^{11}$ $\Rightarrow$ switchable interaction

Ground state: $n = 5$

Rydberg: $n = 50$

$\times 10^{11}$

Ising - like!!

$$\hat{H}_{\text{int}} = \frac{C_6}{R^6} \hat{n}_1 \hat{n}_2 \sim J\, \hat{\sigma}_1^z \hat{\sigma}_2^z$$

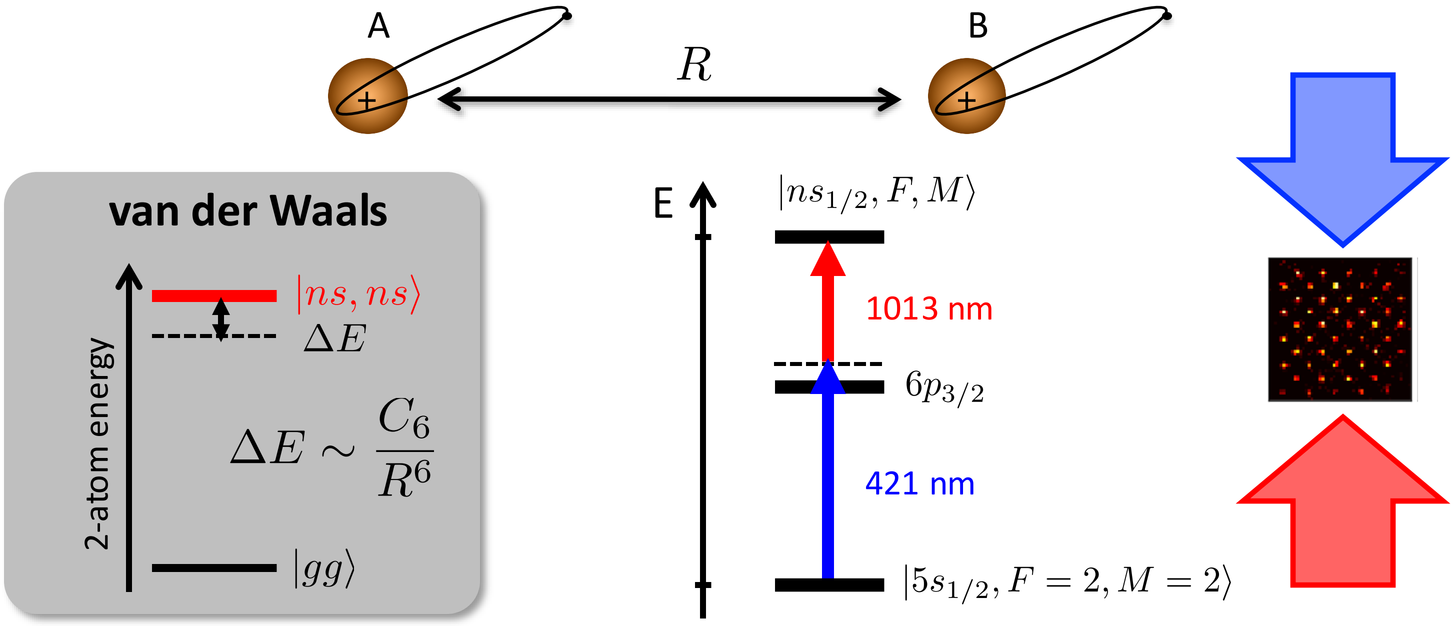
Rydberg $n_{1,2} = 1$

Ground state $n_{1,2} = 0$

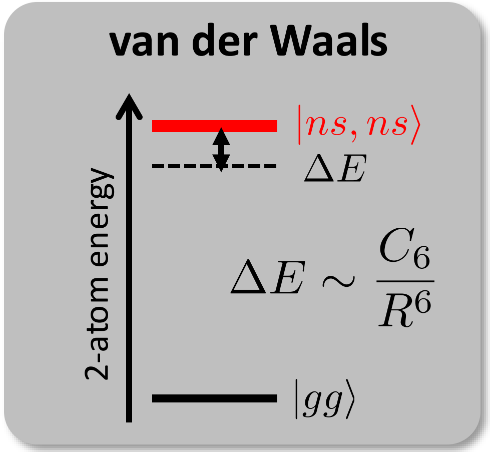
"Equivalent" to interaction between spins

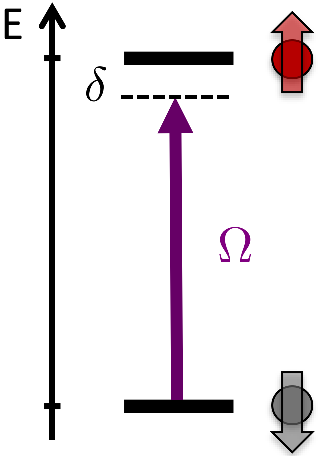
$U_{\text{int}} = J$ $U_{\text{int}} = -J$

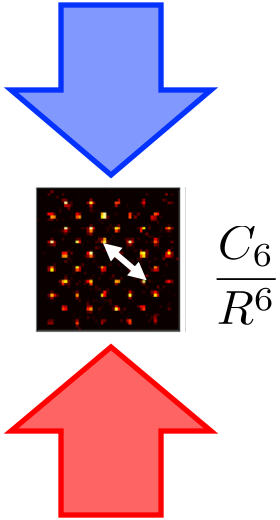
From van der Waals interaction to spin models...



From van der Waals interaction to spin models…

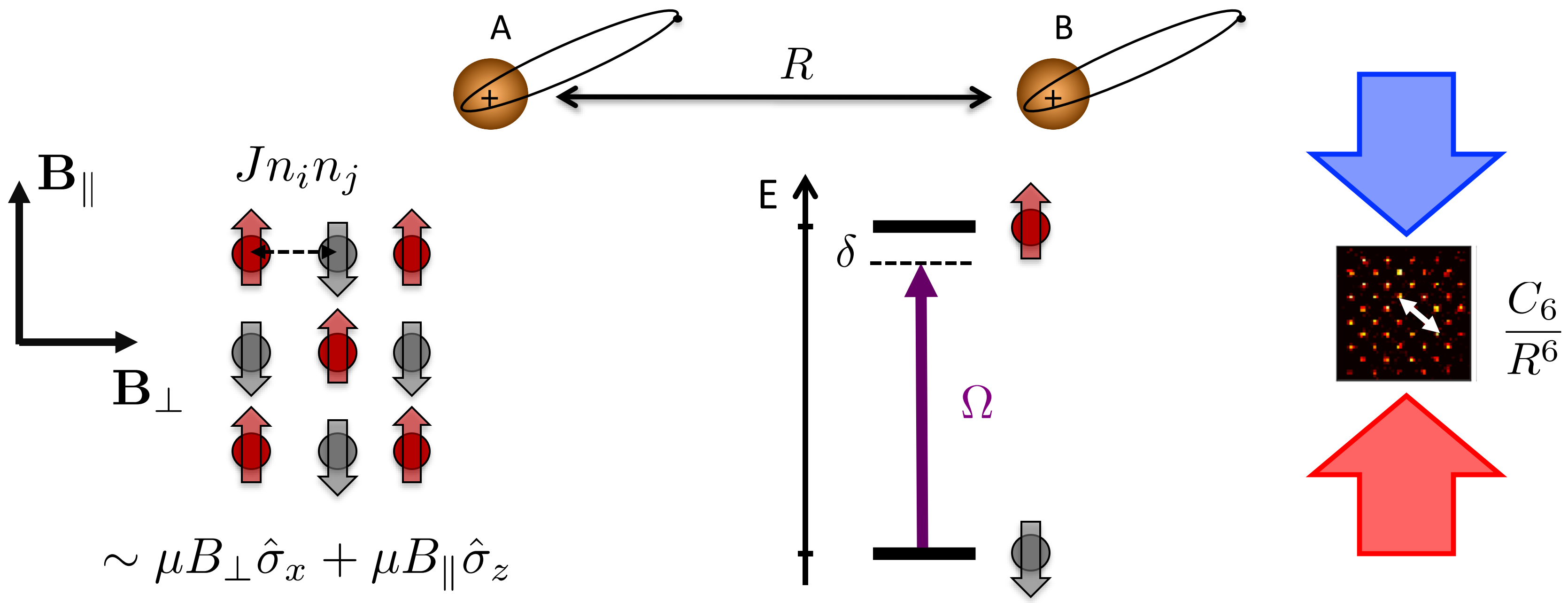






$$H = \frac{\hbar\Omega}{2}\sum_i \hat{\sigma}_x^i + \hbar\delta\sum_i \hat{\sigma}_z^i + \sum_{i<j}\frac{C_6}{R_{ij}^6}\hat{n}_i\hat{n}_j$$

From van der Waals interaction to spin models...



Transverse Field Ising model:

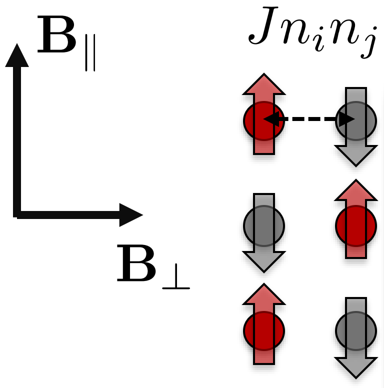
$$H = \frac{\hbar\Omega}{2}\sum_i \hat{\sigma}_x^i + \hbar\delta\sum_i \hat{\sigma}_z^i + \sum_{i<j}\frac{C_6}{R_{ij}^6}\hat{n}_i\hat{n}_j$$

Laser: $B_{\perp}$ $B_{\parallel}$ spin-spin interactions

Controlled parameters:

From negligible to dominant interactions

From van der Waals interaction to spin models…

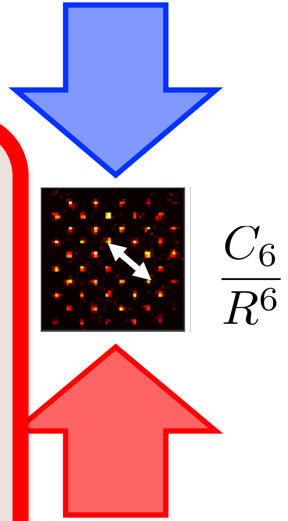


$\sim \mu B_{\perp}\hat{\sigma}_x +$

Quantum simulation:

Emulate a system by another one

Similar equations lead to same solutions!!



Transverse Field Ising model:

$$H = \frac{\hbar\Omega}{2}\sum_i \hat{\sigma}_x^i + \hbar\delta\sum_i \hat{\sigma}_z^i + \sum_{i<j}\frac{C_6}{R_{ij}^6}\hat{n}_i\hat{n}_j$$

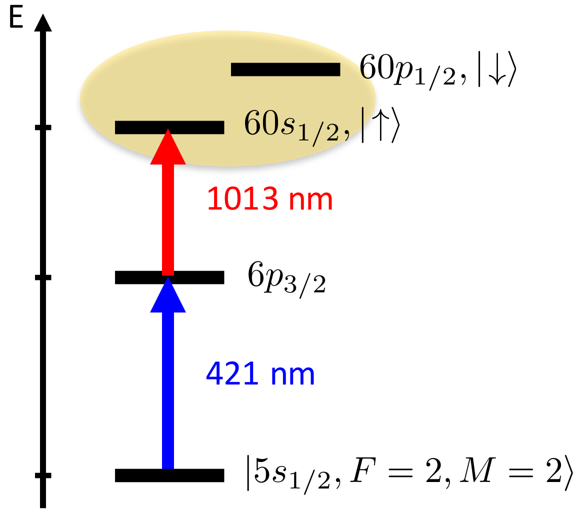
Laser: $B_{\perp}$ $B_{\parallel}$ spin-spin interactions

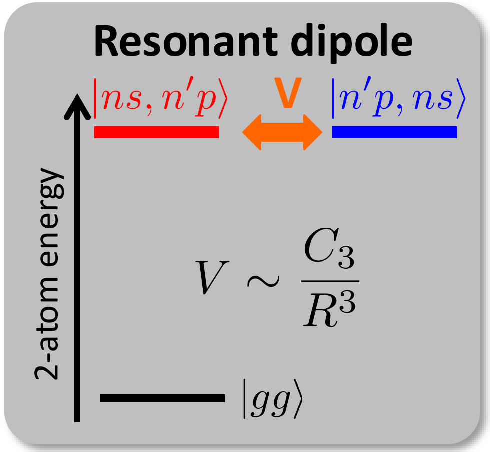
Controlled parameters:

From negligible to dominant interactions

Resonant interaction between Rydbergs and XY spin model

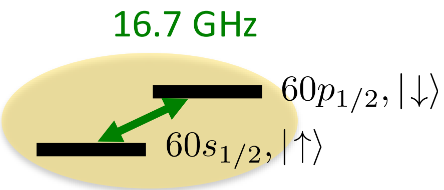
Browaeys & Lahaye, Nat.Phys. (2020)
Barredo PRL (2015), de Léséleuc, PRL (2017)

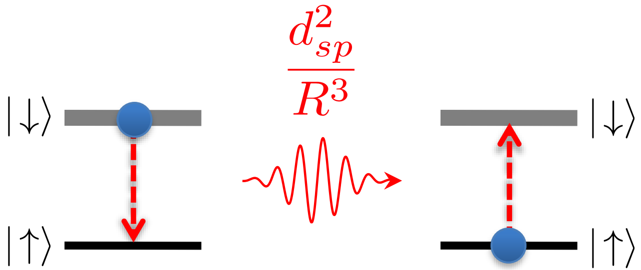




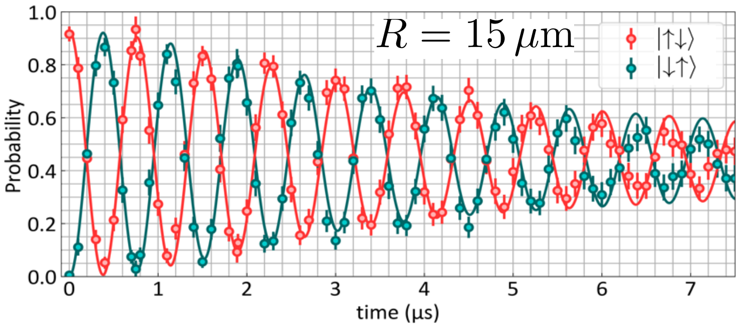
Resonant interaction between Rydbergs and XY spin model

Browaeys & Lahaye, Nat.Phys. (2020)
Barredo PRL (2015), de Léséleuc, PRL (2017)





Non radiative “exchange” of excitation

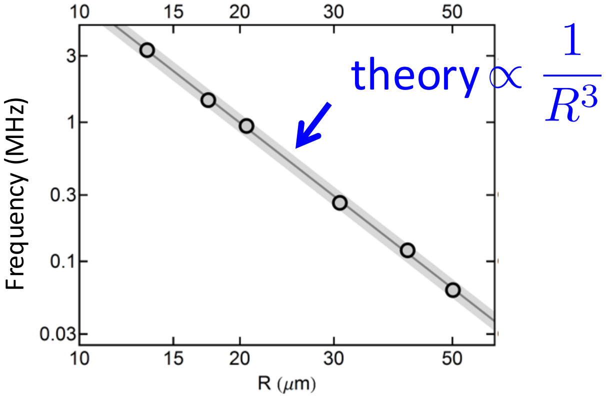


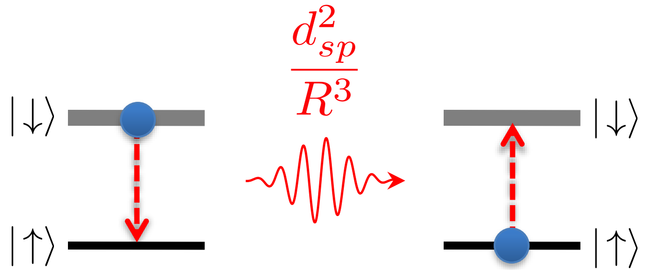
$$\hat{H}_{\mathrm{XY}} = \frac{C_3}{R_{ij}^3}(\hat{\sigma}_i^+\hat{\sigma}_j^- + \hat{\sigma}_i^-\hat{\sigma}_j^+)$$

$$= 2\frac{C_3}{R_{ij}^3}(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y)$$

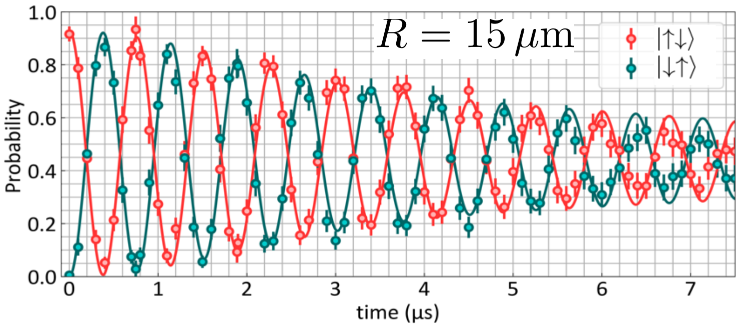
Resonant interaction between Rydbergs and XY spin model

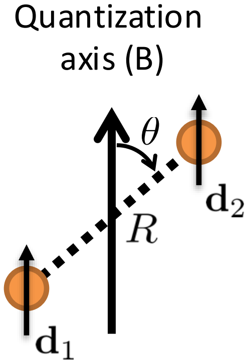
Browaeys & Lahaye, Nat.Phys. (2020)
Barredo PRL (2015), de Léséleuc, PRL (2017)

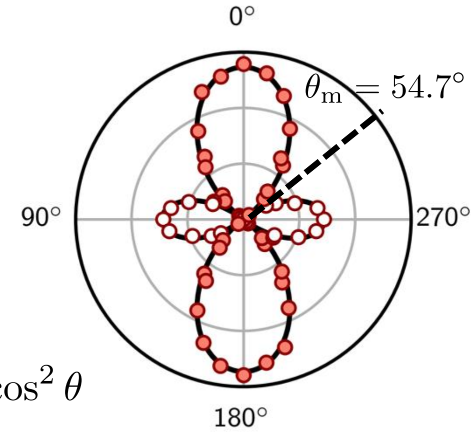




Non radiative “exchange” of excitation







$$C_3(\theta) \propto 1 - 3\cos^2\theta$$

$$\hat{H}_{XY} = \frac{C_3}{R_{ij}^3}(\hat{\sigma}_i^+\hat{\sigma}_j^- + \hat{\sigma}_i^-\hat{\sigma}_j^+)$$

$$= 2\frac{C_3}{R_{ij}^3}(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y)$$

Outline – Lecture 2

1. A bit of plumbing…: Rydberg excitation and detection

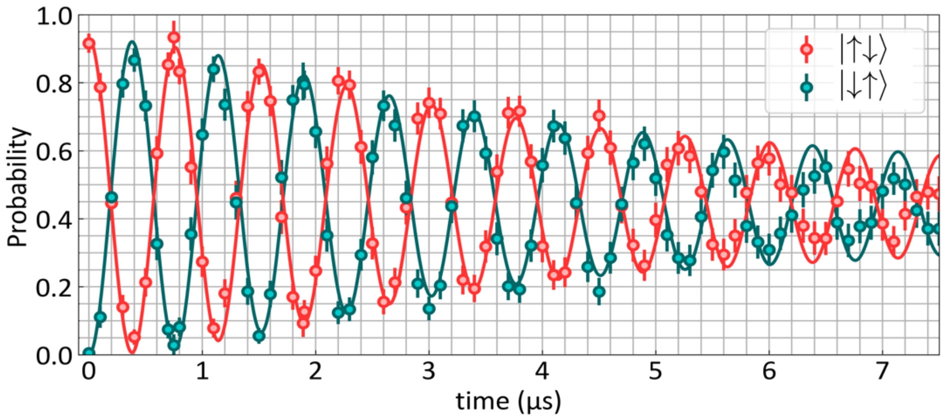
2. Interactions between Rydberg atoms and spin models

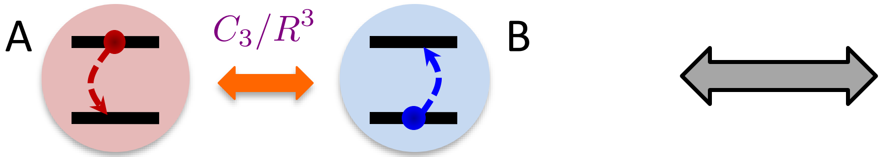
- "Natural": Ising and XY Hamiltonians

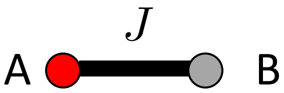
- Application to hardcore bosons, $t - J$ model

- Engineered: XYZ (Floquet), Rydberg dressing

XY spin model and transport of excitations



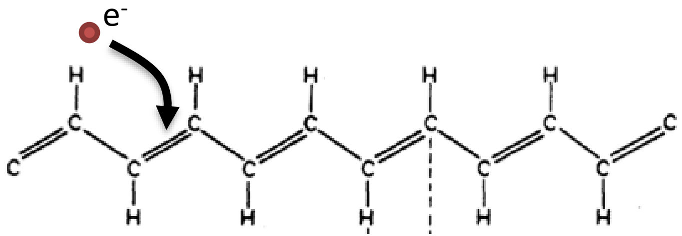




Particle hopping

$$J|A\rangle\langle B|$$

The Su-Schrieffer-Heeger model



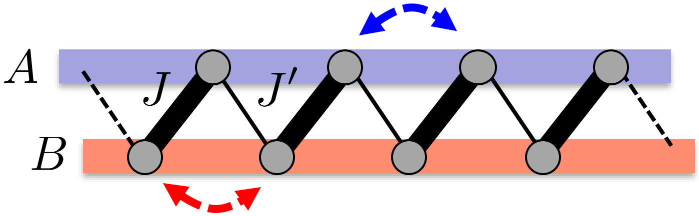
Electronic transport in polyacetylene

PRL **42**, 1698 (1979)

Now, considered as simplest example of topological model

Asboth, arXiv:1509.02295
Cooper, arXiv:1803.00249

The Su-Schrieffer-Heeger model



Model: tight-binding

dimerization: $J > J'$

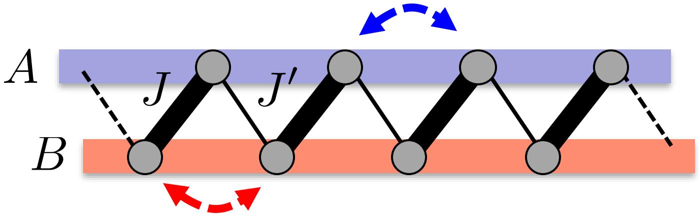
$J'' = 0$: **chiral symmetry** $\Rightarrow$ symmetric **single particle** spectrum

Asboth, arXiv:1509.02295

Cooper, arXiv:1803.00249

Implementation of SSH spin chain with Rydberg atoms

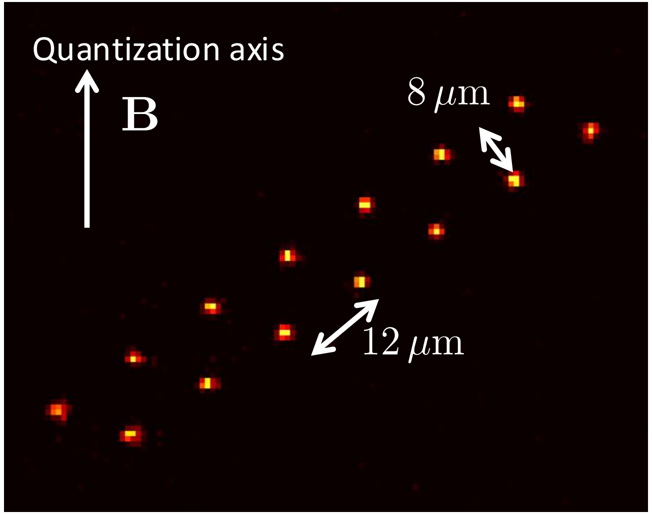
Déléseleuc, Science **365**, 775 (2019)

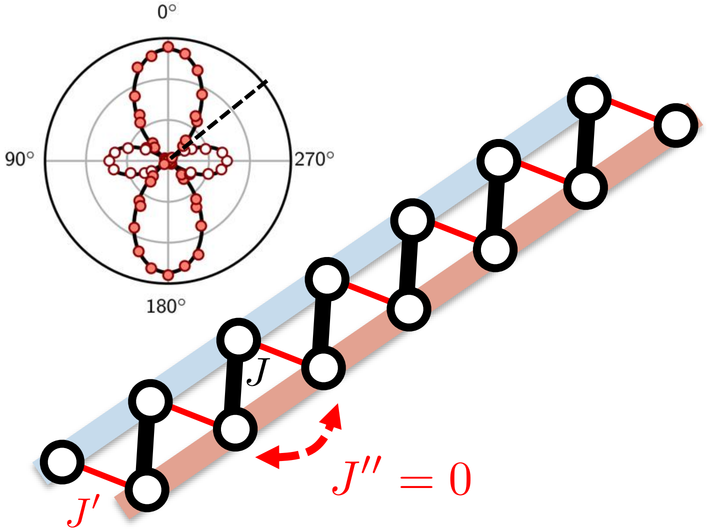


Model: tight-binding

dimerization: $J > J'$

$J'' = 0$: **chiral symmetry** $\Rightarrow$ symmetric **single particle** spectrum



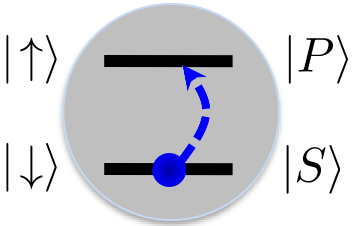


Asboth, arXiv:1509.02295

Cooper, arXiv:1803.00249

Spin excitations interact: hard core bosons

Spin excitation = "particle"

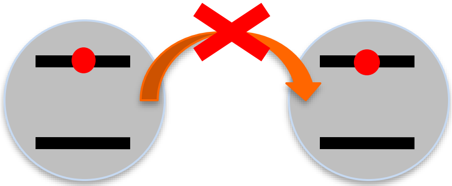


$$\hat{\sigma}^+ \rightarrow \hat{b}^\dagger \;,\; b^\dagger|0\rangle = |1\rangle$$

$$\hat{\sigma}^- \rightarrow \hat{b} \;,\; b|1\rangle = |0\rangle$$

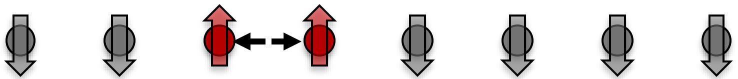
$$[\hat{b}_i, \hat{b}_j^\dagger] = \delta_{ij}$$

Atom cannot carry 2 excitations $\Rightarrow$ excitations = **hard-core bosons**



On-site interaction $U \rightarrow \infty$

$$H_\mathrm{B} = \sum_{i,j} J_{ij}(b_i^\dagger b_j + b_i b_j^\dagger), \quad b_i^{\dagger 2} = 0$$



$\Rightarrow$The first symmetry protected topological phase...

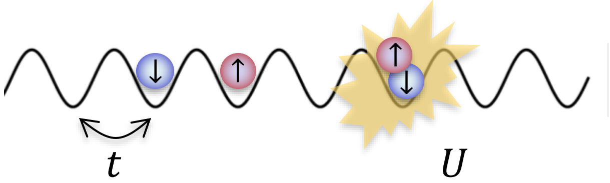
Predicted in 2012

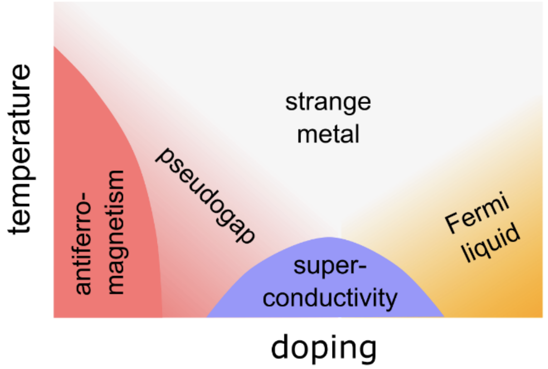
Déléseleuc, Science **365**, 775 (2019)

XY model beyond two levels: Doped magnet and $t-J-V$ model

Hubbard model

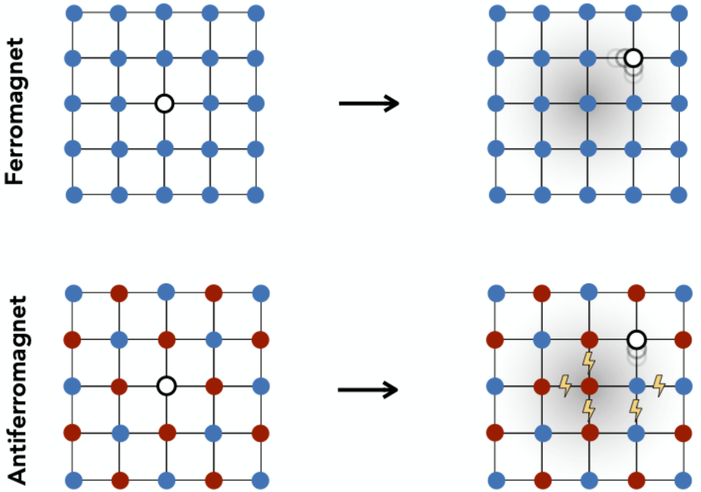
$$H_{\mathrm{FH}} = -t \sum_{\langle i,j\rangle,\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + U \sum_{i} n_{i\downarrow} n_{i\uparrow}$$





Doping = 0 + $U \gg t \Rightarrow H_{\mathrm{FH}} = \frac{4t^2}{U} \sum_{\langle i,j\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$

Doping $\neq$ 0: hole motion coupled to magnetic background



$t-J$ model

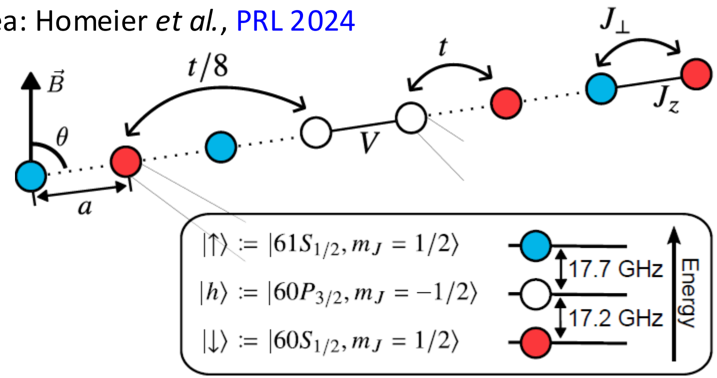
Auerbach, Wiley 1994

$$H_{\mathrm{FH}} = -t \sum_{\langle i,j\rangle,\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} + J \sum_{\langle i,j\rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{n_i n_j}{4} \right) + \mathcal{O}(t^3/U^2)$$

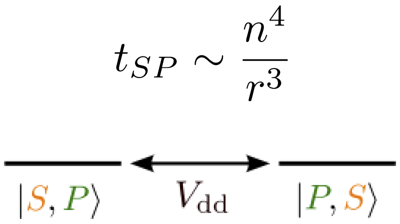
L. Homeier, thesis

$t-J-V$ model using 3 Rydberg states

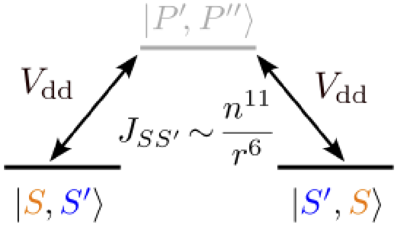
Idea: Homeier *et al.*, PRL 2024



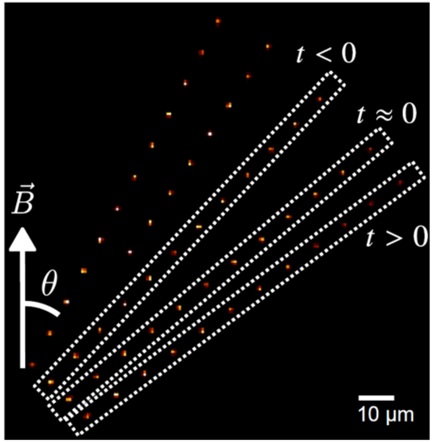
First-order exchange



2nd-order exchange



Tunability: vary θ and r



$$\hat{H}_{tJV} = \hat{H}_t + \hat{H}_J + \hat{H}_V$$

$$\hat{H}_t = -\sum_{i<j} \sum_{\sigma=\downarrow,\uparrow} \frac{t_\sigma}{r_{ij}^3} \left(\hat{a}^\dagger_{i,\sigma} \hat{a}^\dagger_{j,h} \hat{a}_{i,h} \hat{a}_{j,\sigma} + \text{ h.c. } \right)$$

Resonant dip.-dip. *S, P*

$$\hat{H}_J = \sum_{i<j} \frac{1}{r_{ij}^6} \left[J^z \hat{S}^z_i \hat{S}^z_j + \frac{J_\perp}{2} \left(\hat{S}^+_i \hat{S}^-_j + \text{h.c.} \right) \right]$$

vdW *S,S'*: diag. (J_z) and off-diag. ($J_\perp$)

$$\hat{H}_V = \sum_{i<j} \frac{V}{r_{ij}^6} \hat{n}^h_i \hat{n}^h_j$$

vdW *PP*: interaction between holes

M. Qiao *et al.*, arXiv:2501.08233

Outline – Lecture 2

1. A bit of plumbing…: Rydberg excitation and detection

2. Interactions between Rydberg atoms and spin models

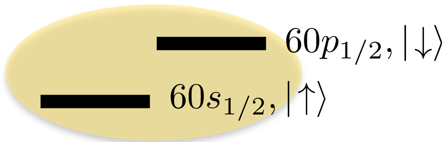
   - "Natural": Ising and XY Hamiltonians

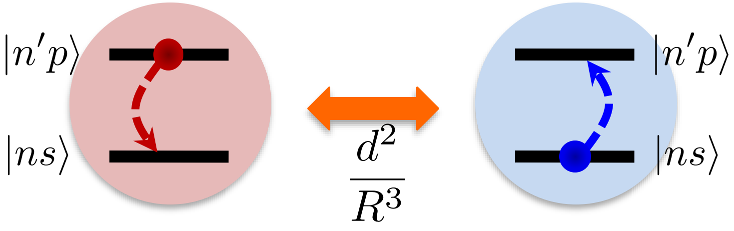
   - Application to hardcore bosons, $t-J$ model

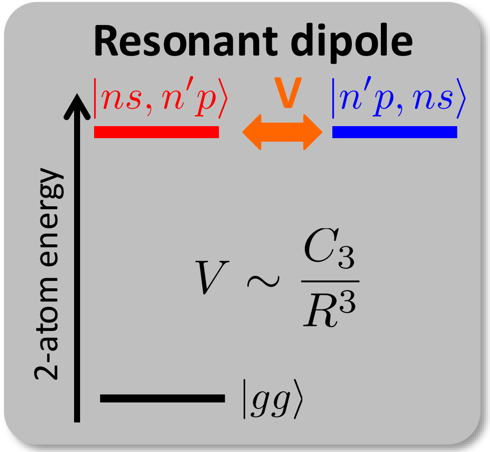
   - Engineered: XYZ (Floquet), Rydberg dressing

Interactions between Rydberg atoms and spin models

Browaeys & Lahaye, Nat.Phys. (2020)







XY model

$$\hat{H} = \sum_{i \neq j} J_{ij} \left(\hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+ \right)$$

Interactions between Rydberg atoms and spin models

Browaeys & Lahaye, Nat.Phys. (2020)

nt dipole

V $|n'p, ns\rangle$

$\frac{C_3}{R^3}$

$g\rangle$

$|n'p\rangle$

$|ns\rangle$

del

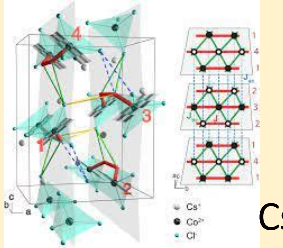
$\sum_{i\neq j} (\ldots_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+)$

Extend to more general XYZ spin models

$$\hat{H}_{\mathrm{XYZ}} = \sum_{i\neq j} J_{ij}^x\, \sigma_i^x \sigma_j^x + J_{ij}^y\, \sigma_i^y \sigma_j^y + J_{ij}^z\, \sigma_i^z \sigma_j^z$$

A. Abragham,
Principle of Nuclear Magnetism (1983)

XXZ



Cs_2CoCl_4

Whitlock, J. Phys. B 2017

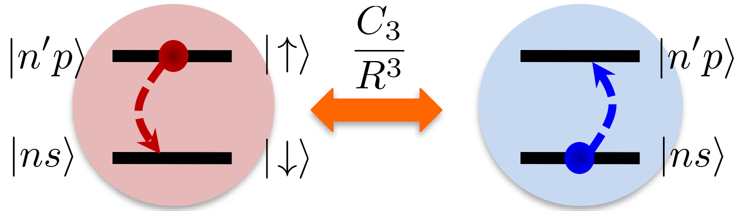
Heisenberg

$$\sum_{i\neq j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

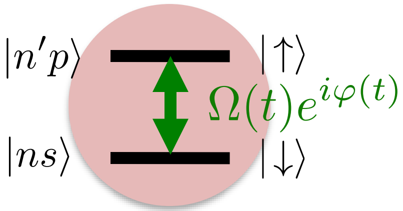
Engineering the XYZ model with microwaves

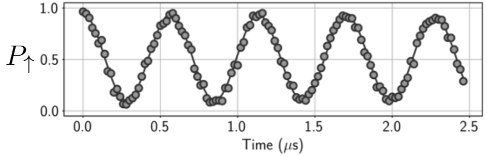
Combine:

Naturally occuring XY interaction



Microwave driving



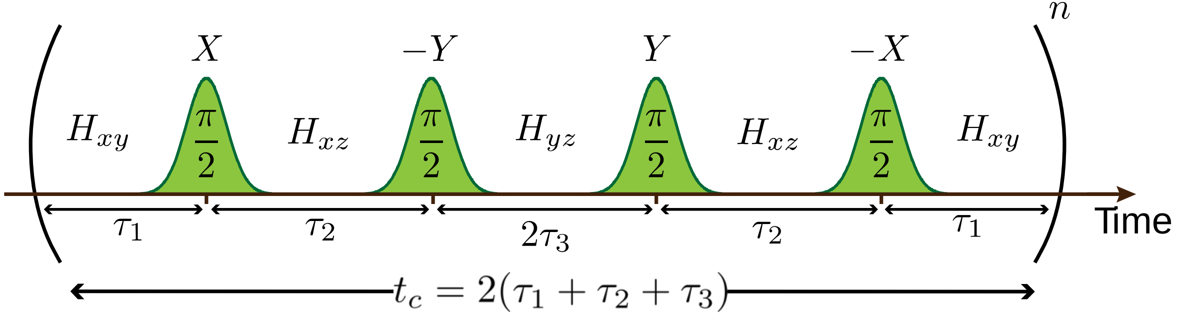


XY model + external (resonant) microwave field:

$$\hat{H}_{\mathrm{driv}} = \sum_{i\neq j} \frac{C_3}{R_{ij}} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y) + \frac{\hbar\Omega(t)}{2} \sum_i \cos\varphi(t)\, \hat{\sigma}_i^x + \sin\varphi(t)\, \hat{\sigma}_i^y$$

XYZ model with microwaves: Floquet engineering

Microwave pulse sequence $\Omega(t)$:



(infinitely short pulses)

$\frac{C_3}{R_{ij}^3} t_c \ll 1$ $\Rightarrow$ averaged hamiltonian: $H_{\text{av}} = \frac{1}{t_c} \int_0^{t_c} H(t)\, dt$

$$\Rightarrow H_{\text{av}} = 2 \sum_{i \neq j} \frac{C_3}{R_{ij}^3} \left(\frac{\tau_1 + \tau_2}{t_c} \sigma_i^x \sigma_j^x + \frac{\tau_1 + \tau_3}{t_c} \sigma_i^y \sigma_j^y + \frac{\tau_2 + \tau_2}{t_c} \sigma_i^z \sigma_j^z \right)$$

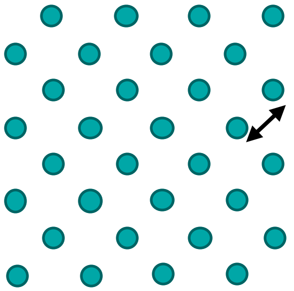
$\Rightarrow$ Programmable XYZ Hamiltonians!

Vandersypen, RMP 2006
Choi *et al.*, PRX **10**, 031002 (2020)

Heisenberg XXX engineering in 2D

Scholl, PRX Quantum 2022

32 atoms

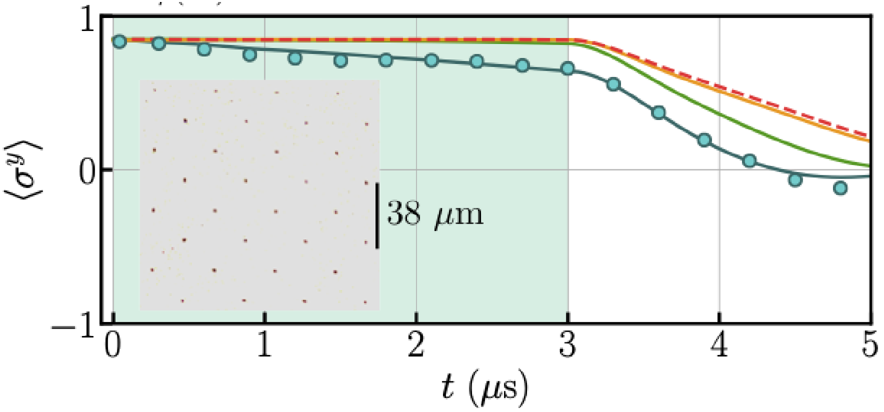


$$\hat{H}_{\text{Heis.}} = \sum_{i \neq j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

y-magnetization nearly conserved

$$|\downarrow\rangle = |75S\rangle; |\uparrow\rangle = |75P\rangle$$

Initial state: $(|\rightarrow\rangle_y)^{\otimes N}$

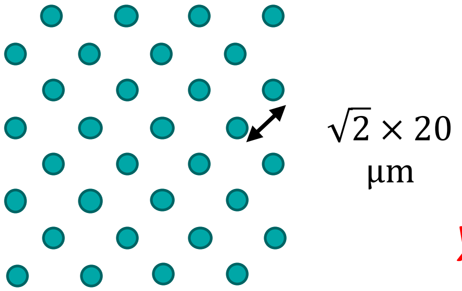


Expt: cloud of atoms
Geier, Science 2021

Heisenberg XXX engineering in 2D

Scholl, PRX Quantum 2022

32 atoms



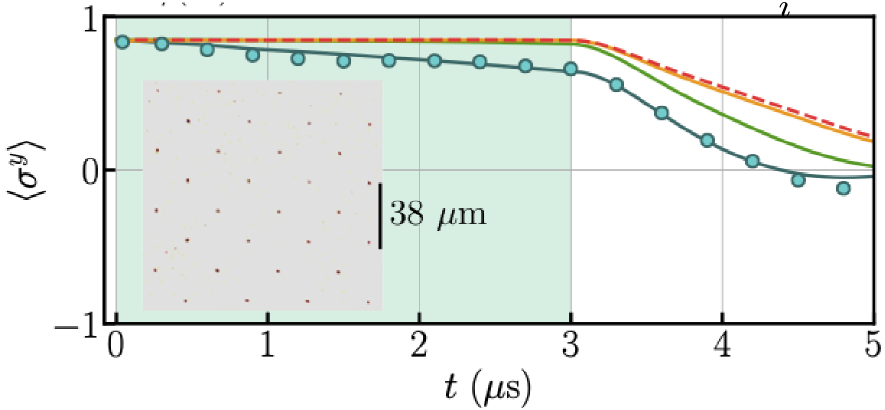
$$\hat{H}_{\text{Heis.}} = \sum_{i \neq j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

y-magnetization nearly conserved

SU(2) symmetry: $[\hat{H}_{\text{Heis.}}, \sum_i \mathbf{S}_i] = 0$

$|\downarrow\rangle = |75S\rangle; |\uparrow\rangle = |75P\rangle$

Initial state: $(|\rightarrow\rangle_y)^{\otimes N}$

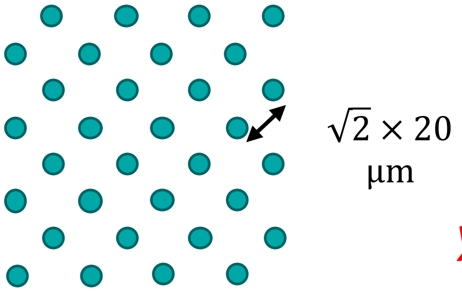


Expt: cloud of atoms
Geier, Science 2021

Heisenberg XXX engineering in 2D

Scholl, PRX Quantum 2022

32 atoms



$$\hat{H}_{\text{Heis.}} = \sum_{i \neq j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

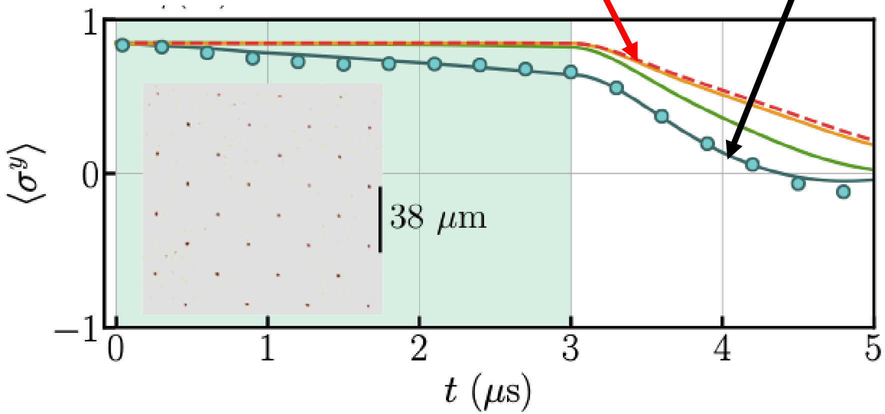
y-magnetization nearly conserved

$H_{\text{Heis.}} \rightarrow H_{\text{XX}}$ MACE simulation H_{driv}

Hazzard, PRL 2014

$|\downarrow\rangle = |75S\rangle; |\uparrow\rangle = |75P\rangle$

Initial state: $(|\rightarrow\rangle_y)^{\otimes N}$

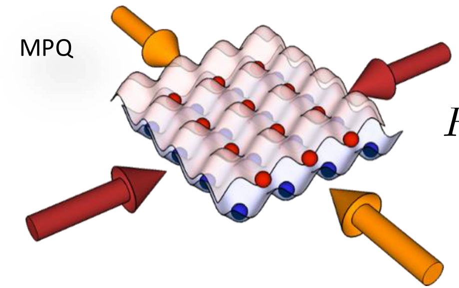


No adjustable parameter, includes MW imperfections

Expt: cloud of atoms
Geier, Science 2021

Limitations: finite MW pulse duration + imperfections

Tailoring the ground-state interaction by Rydberg dressing



$R \sim 0.5 - 1\,\mu\mathrm{m}$

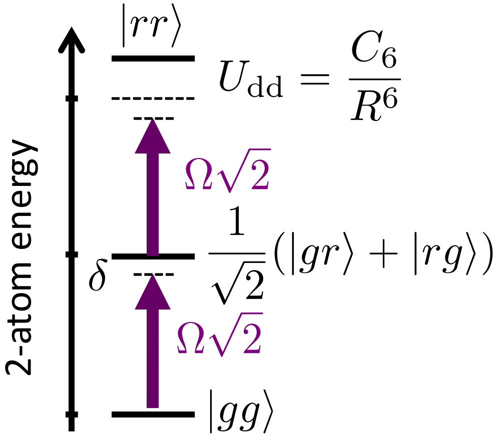
Atom in ground state $\Rightarrow$ no inter-site interaction

Solution: admix ground-state with Rydberg state

$$|\tilde{g}\rangle \approx |g\rangle + \frac{\Omega}{\delta}|r\rangle$$

interactions

Idea: Bouchoule &Moelmer PRA 2002
Pupillo et al. PRL 2010



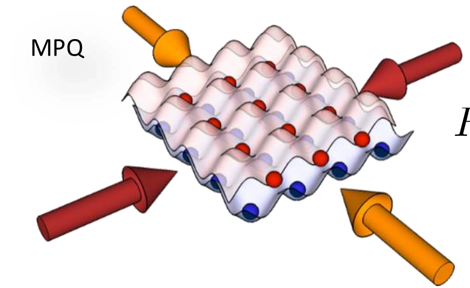
$$\frac{C_6}{R_{\mathrm{b}}^6} = \Omega$$

$$R \gg R_{\mathrm{b}} : E_{gg}^0 = 2 \times \frac{\hbar}{2}\left(\delta - \sqrt{\delta^2 + \Omega^2}\right)$$

$$R \ll R_{\mathrm{b}} : E_{gg} = \frac{\hbar}{2}\left(\delta - \sqrt{\delta^2 + 2\Omega^2}\right)$$

$$E_{gg} - E_{gg}^0 \approx \frac{\hbar\Omega^4}{8\delta^3}$$

Tailoring the ground-state interaction by Rydberg dressing



$R \sim 0.5 - 1\,\mu m$

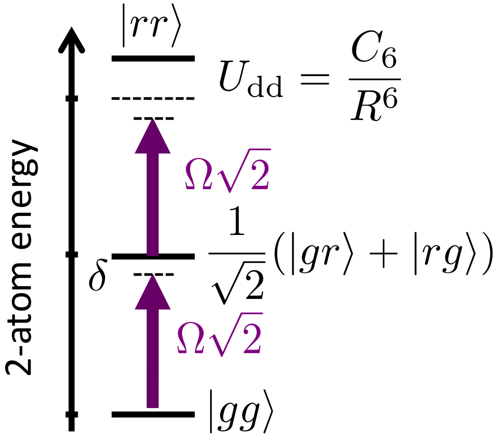
Atom in ground state $\Rightarrow$ no inter-site interaction

Solution: admix ground-state with Rydberg state

$$|\tilde{g}\rangle \approx |g\rangle + \frac{\Omega}{\delta}|r\rangle$$

interactions

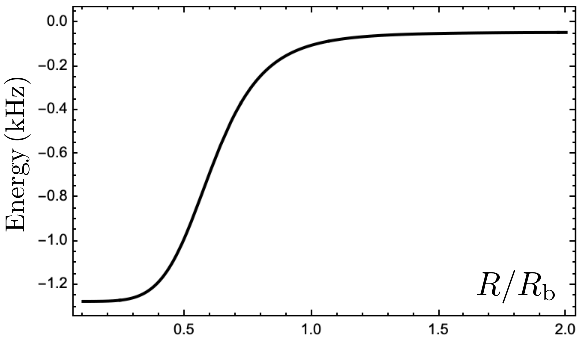
Idea: Bouchoule &Moelmer PRA 2002
Pupillo et al. PRL 2010



$$\hat{H} = \begin{pmatrix} 0 & \frac{\Omega}{\sqrt{2}} & 0 \\ \frac{\Omega}{\sqrt{2}} & \delta & \frac{\Omega}{\sqrt{2}} \\ 0 & \frac{\Omega}{\sqrt{2}} & 2\delta + U_{\mathrm{dd}} \end{pmatrix}_{|gg\rangle, |+\rangle, |rr\rangle}$$

$$\frac{C_6}{R_{\mathrm{b}}^6} = \Omega$$

$$E_{gg} - E_{gg}^0 \approx \frac{\hbar\Omega^4}{8\delta^3}$$

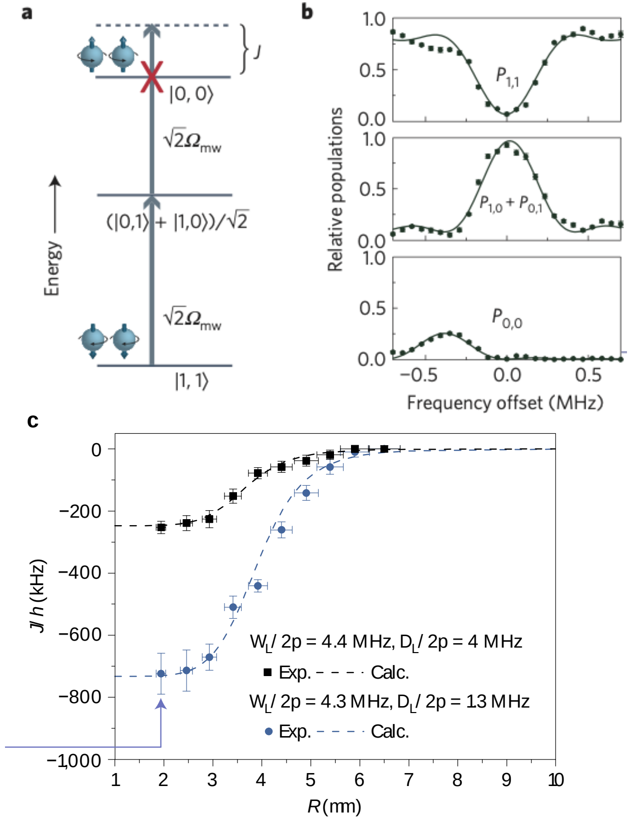


Softcore potential

Johnson & Rolston PRA 2010

Rydberg dressing: first experiment and recent developments

Two atoms in tweezers

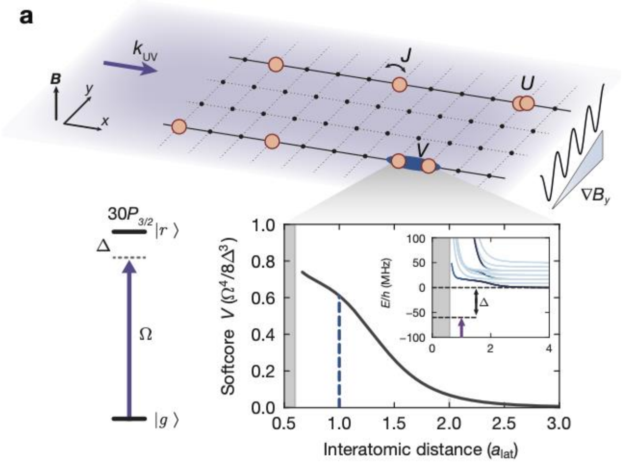


Jau *et al.*, Nat. Phys. **12**, 71 (2015)

For a long time: unwanted losses…

Zeiher (Nat. Phys. 2016) lattices
Porto (PRL 2016) ensemble + lattices

Rydberg –dressed 1D Bose-Hubbard arXiv:2405.20128



$$H = J \sum_{\langle i,j \rangle} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i(n_i - 1) + V \sum_i n_i n_{i+1}$$

Dressing!

Also: Spin squeezing with Rydberg dressed interactions

A. Kaufman arXiv:2303.10668, Nature (2023)
M. Schleier-Smith PRL **131**, 063401 (2023)

Conclusion: many variants of spin Hamiltonians

Quantum Ising
$s = 1/2$

Hardcore boson

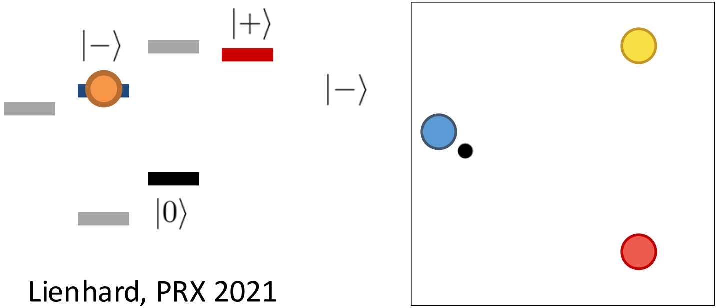
Bosons/ Fermions
Softcore potential

XY, $s = 1/2$
$\frac{1}{R^3}, \frac{1}{R^6}$

XYZ Heisenberg
$s = 1/2$
Floquet

t- J model

Spin-orbit coupling



Lienhard, PRX 2021

In various *addressable* geometries: 1D (OBC, PBC), 2D : square, triangle, Kagome…

Warning: mapping is only approximate (on top of uncontrolled parameters)…:

XY has small Ising; neglect quadrupolar interactions; not exactly 2 levels…

Hard to assess the impact…!!

The program

Lecture 1: Many-body problem and quantum simulation
Arrays of atoms & "Rydbergology"
Interactions between atoms

Lecture 2: Rydberg Interactions and spin models
Engineering many-body Hamiltonians

Lecture 3: Examples of quantum simulations in
and out-of-equilibrium: quantum magnetism