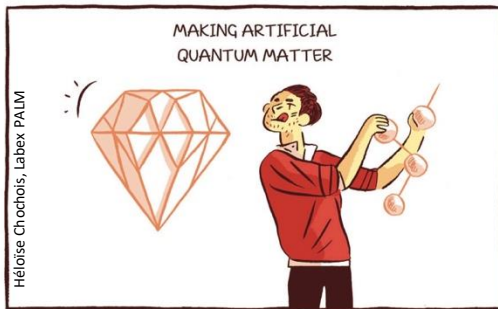


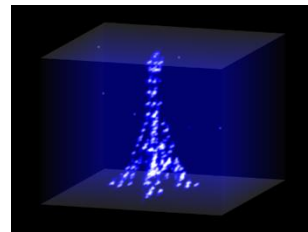
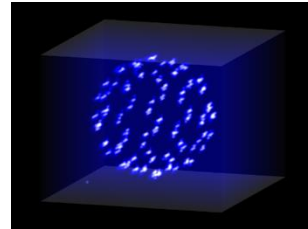
# Exploring many-body physics with arrays of Rydberg atoms



**Antoine Browaeys**

*Laboratoire Charles Fabry,  
Institut d'Optique, CNRS, FRANCE*

Benasque School, february 17-21, 2025



# The program

- Lecture 1: Many-body problem and quantum simulation  
Arrays of atoms & “Rydbergology”  
Interactions between atoms
- Lecture 2: Rydberg Interactions and spin models  
Engineering many-body Hamiltonians
- Lecture 3: Examples of quantum simulations in  
and out-of-equilibrium: quantum magnetism

# Outline – Lecture 1

1. Many-body physics and quantum simulation
2. Arrays of individual atoms in optical tweezers
3. Basics of Rydberg physics
4. Interaction between Rydberg atoms

# The context: “many-body problem”

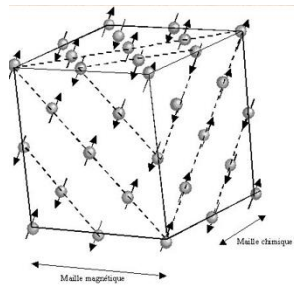
**Goal:** Understand ensembles of **strongly** interacting quantum particles



superfluidity



superconductivity



magnetism



neutron star

**Questions:** phase diagram, excitation, dynamics, ...

**The equation to solve:**  $i\hbar \frac{\partial \Psi}{\partial t} = H_{\text{tot}} \Psi$

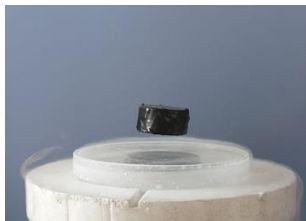
$$H_{\text{tot}} = \sum_{i=1}^N -\frac{\hbar^2}{2m_i} \nabla_i^2 + \sum_{i=1}^N \sum_{j \neq i} \frac{q_i q_j}{r_{ij}} + \frac{\mu_B^2}{r_{ij}^3} \mathbf{s}_i \cdot \mathbf{s}_j$$

Very, very, very well known...

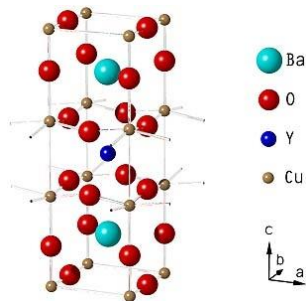
**Problem:**  $N \approx 10^{23}$  !!!

# The many-body problem: the art of modelling...

wikipedia



YBaCuO



Observe unexpected effects

Ex: high- $T_c$  superconductivity

**Microscopic understanding?**

Experiment on  
“real” system

simplify



**Too hard to calculate...**

Cook up a *model*

$$H_{\text{model}} = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + U \sum_i n_{i\downarrow} n_{i\uparrow}$$

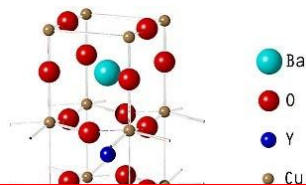
**Problem: exponential complexity**

2 d. of freedom (spin...)  $\psi_i = \begin{pmatrix} a \\ b \end{pmatrix}$

Many-body wavefunction:  $\Psi = \Psi(1, 2, \dots, N) \Rightarrow \Psi$  requires  $2^N$  numbers

Record *ab-initio* calculation (2025)  $N \sim 50 \Rightarrow 2^{50} \sim 10^{15} =$  **1000 Tb RAM !!**

# The many-body problem: the art of modelling...



Observe unexpected effects  
Ex: high- $T_c$  superconductivity

**Approximations possible!!**  
mean-field, perturbation theory, Monte-Carlo,  
variational methods: DFT, MPS, Neural Quantum States...

**But...** can be poorly controlled or not valid  
when *interactions dominate*

= **Strongly correlated** systems

Record *ab-initio* calculation (2025)  $N \sim 50 \Rightarrow 2^{50} \sim 10^{15} =$  **1000 Tb RAM !!**

Problems:

Many-body

$$H_{\text{model}} = -t \sum (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + U \sum_i n_{i\downarrow} n_{i\uparrow}$$

$$\sum_i n_{i\downarrow} n_{i\uparrow}$$

$$\begin{pmatrix} a \\ b \end{pmatrix}$$

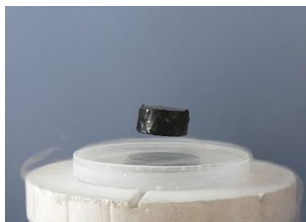
exponential complexity

2 d. of freedom (spin..)

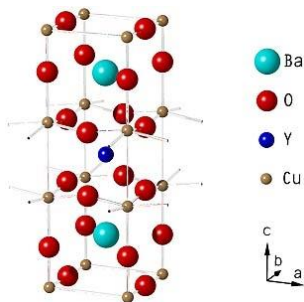
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# The many-body problem: the art of modelling...

wikipedia

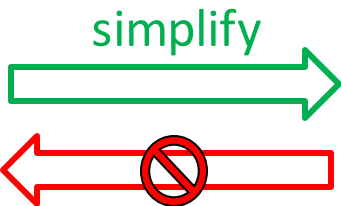


YBaCuO



Observe unexpected effects  
Ex: high- $T_c$  superconductivity  
**Microscopic understanding?**

Experiment on  
“real” system



**Too hard to calculate...**

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$$H_{\text{model}} = -t \sum_{\langle i,j \rangle, \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + U \sum_i n_{i\downarrow} n_{i\uparrow}$$



Measure on simulator:  
**Supercond. or not?**



**Quantum simulation =**  
Engineered system ruled  
by  $H_{\text{model}}$

# The original idea...



R.P. Feynman

*International Journal of Theoretical Physics, Vol. 21, Nos. 6/7, 1982*

## Simulating Physics with Computers

Richard P. Feynman

### 4. QUANTUM COMPUTERS—UNIVERSAL QUANTUM SIMULATORS

with it, with quantum-mechanical rules). For example, the spin waves in a spin lattice imitating Bose-particles in the field theory. I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world. But I don't know whether the general theory of this intersimulation of quantum systems has ever been worked out, and so I present that as another interesting problem: to work out the classes of different kinds of quantum mechanical systems which are really intersimulatable—which are equivalent—as has been done in the case of classical computers. It has been found that there is a kind of universal computer that can do anything, and it doesn't make much difference specifically how it's designed. The same way we should try to find



# Analog versus digital quantum simulation

## Analog

The platform implements directly  $H_{\text{model}}$

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar} \int_0^t H_{\text{mod}}(t') dt'\right) |\psi(0)\rangle$$


e.g.: Fermi Hubbard, spin models, electrons in B-fields...

**Non-universal**

Georgescu, Rev. Mod. Phys. (2014)

## Digital

$H_{\text{model}}$  synthesized digitally

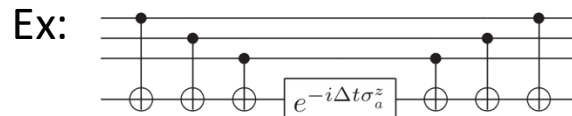
$$H_{\text{mod}} = \sum_{n=1}^N H_n$$


e.g. single & 2-qbit operations

$$e^{-iH_{\text{mod}}t} \approx$$

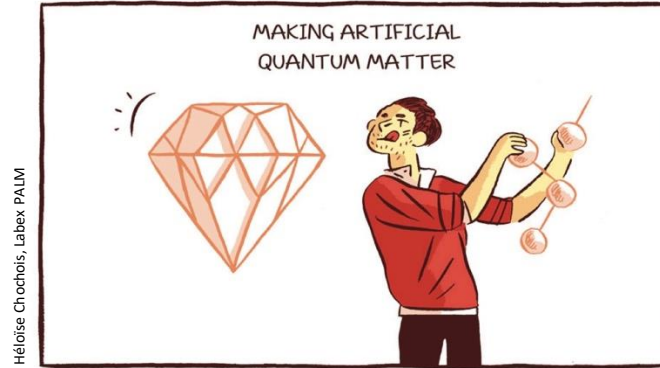
$$\left(e^{-iH_1 t/n} e^{-iH_2 t/n} \dots e^{-iH_3 t/n}\right)^n$$

= “universal” quantum simulation



$$H_{\text{mod}} = \sigma_1^z \otimes \sigma_2^z \otimes \sigma_3^z$$

# Analog Quantum Simulation with synthetic systems



Georgescu, Rev. Mod. Phys. (2014)

**Well-controlled** quantum systems implementing **many-body Hamiltonians**  
= quantum simulator

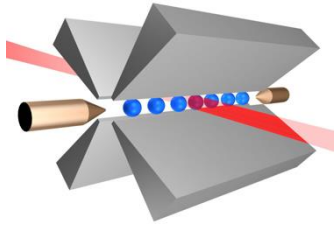
Larger tunability than “real” systems (geometry, interactions...)

Separate effects (impurities, role interactions...)

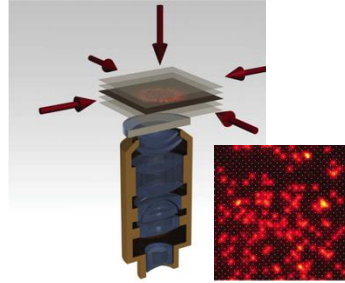
New types of probe & methods (e.g. out-of-equilibrium)

**A new way to look at many-body using quantum information concepts**  
(entanglement...)

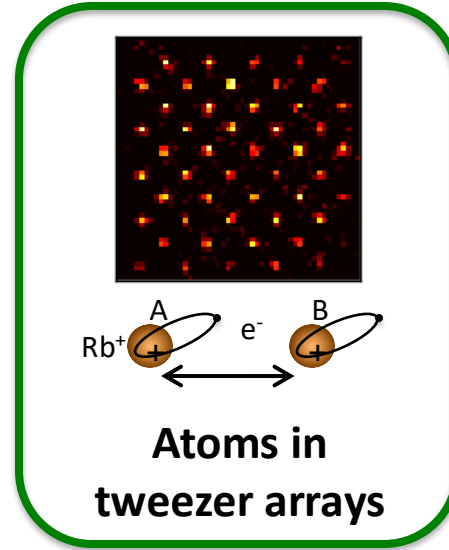
# Engineering with individual quantum systems (examples)



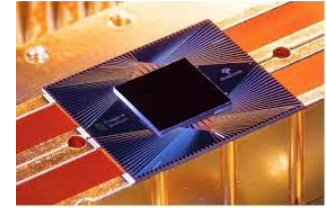
Trapped ions



Atoms in optical lattices



Atoms in tweezer arrays



Supercond. Circuits  
IBM, Google...

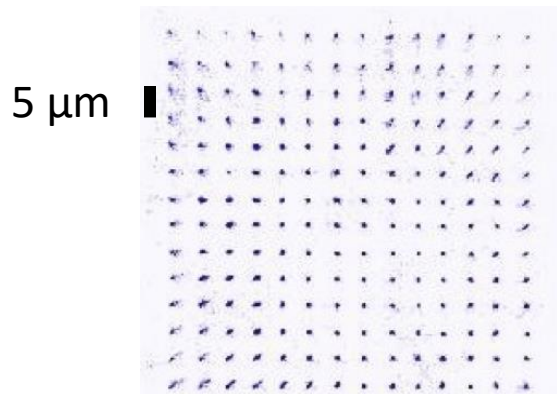
**Scalable:** beyond 100 particles ; potential > 1000

**Addressability:** local manipulations and measurement

$$\langle \sigma_i^\alpha \rangle, \langle \sigma_i^\alpha \sigma_j^\beta \rangle, \dots$$

**Programmable:** controlled geometry, interactions...

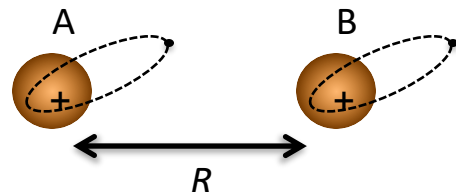
# These lectures: combining arrays of atoms and Rydberg interactions



Addressable!!

+

## Rydberg interactions



Van der Waals

resonant

$$\frac{C_6}{R^6}$$

$$\frac{C_3}{R^3}$$

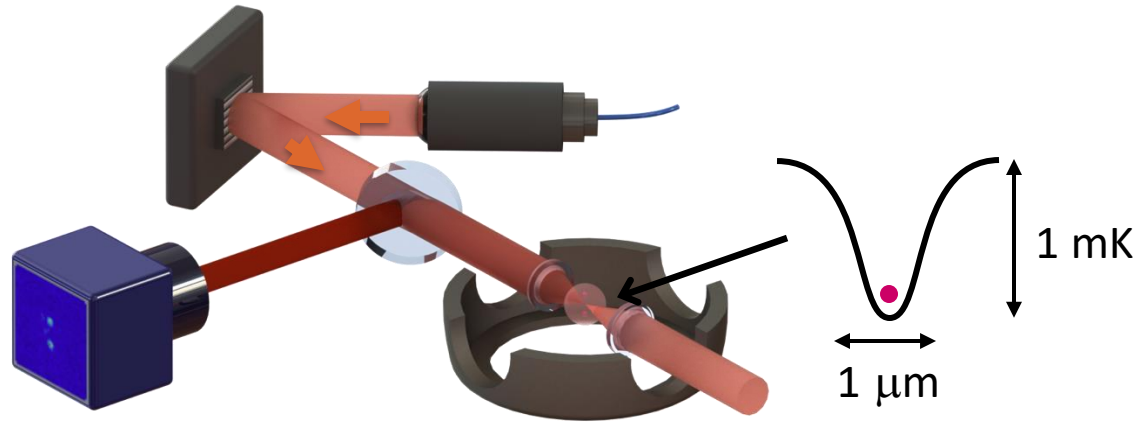
**Quantum simulation (mainly spin models)**

Quantum information processing

# Outline – Lecture 1

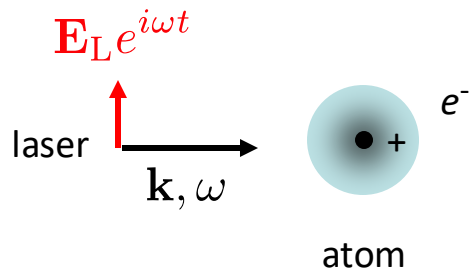
1. Many-body physics and quantum simulation
2. Arrays of individual atoms in optical tweezers
3. Basics of Rydberg physics
4. Interaction between Rydberg atoms

# A single Rb atom in an optical tweezer

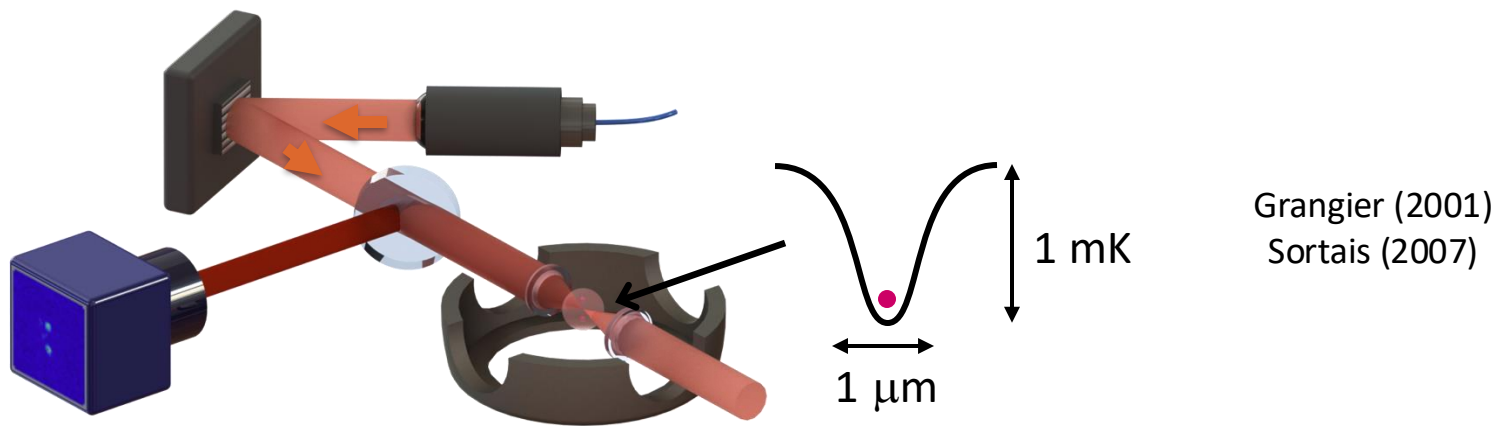


Grangier (2001)  
Sortais (2007)

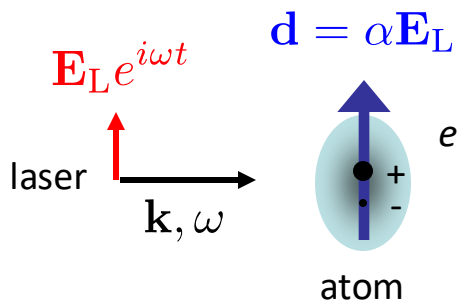
## Dipole force



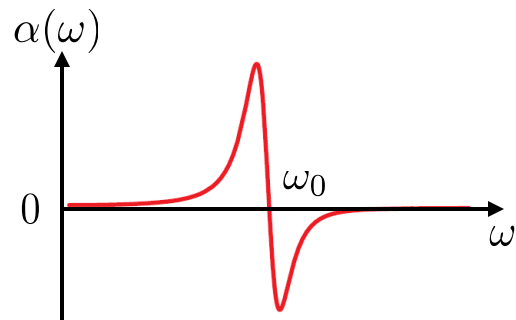
# A single Rb atom in an optical tweezer



## Dipole force



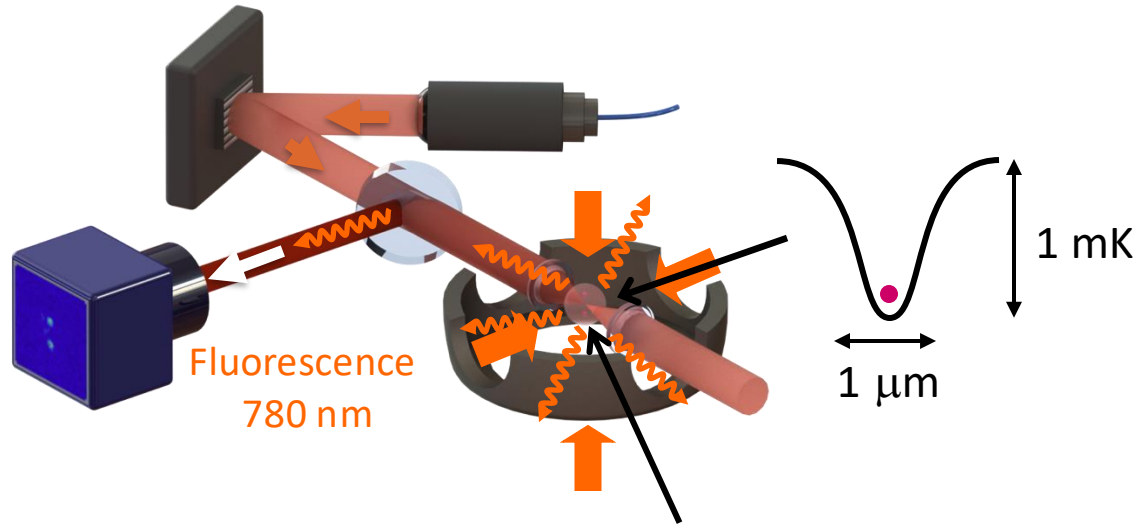
$$U = -\frac{1}{2} \langle \mathbf{d} \cdot \mathbf{E}_L \rangle$$
$$= -\frac{1}{2} \alpha \langle \mathbf{E}_L^2 \rangle$$



$\omega < \omega_0 \Rightarrow$  high-intensity seeker

Ex: 1 mW on 1 μm  $\Rightarrow$  **Trap depth = 1 mK**  $\Rightarrow$  Laser cooled atoms...

# A single Rb atom in an optical tweezer

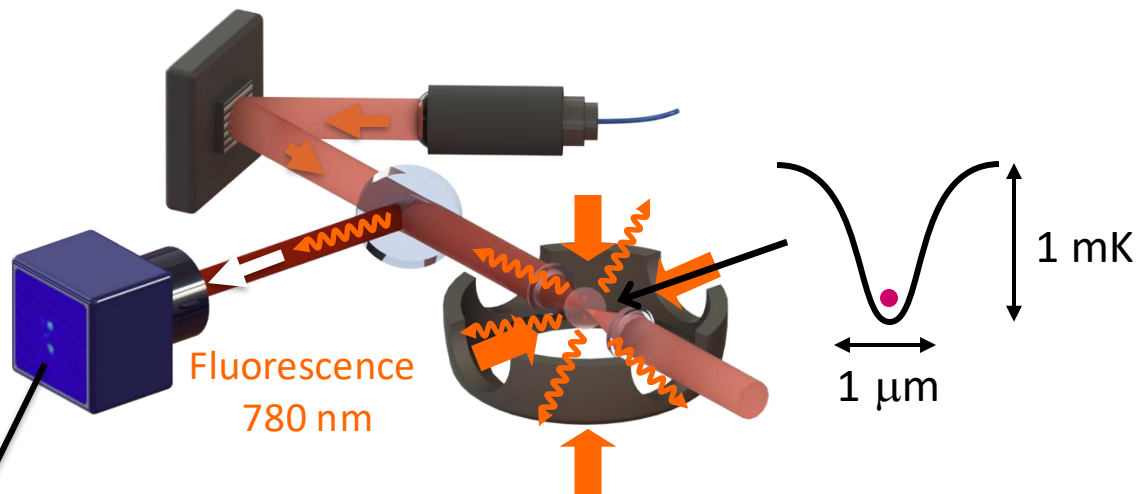


Grangier (2001)  
Sortais (2007)

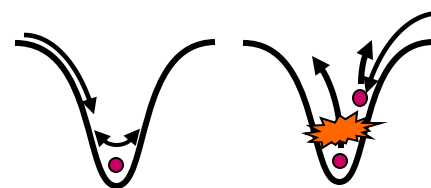
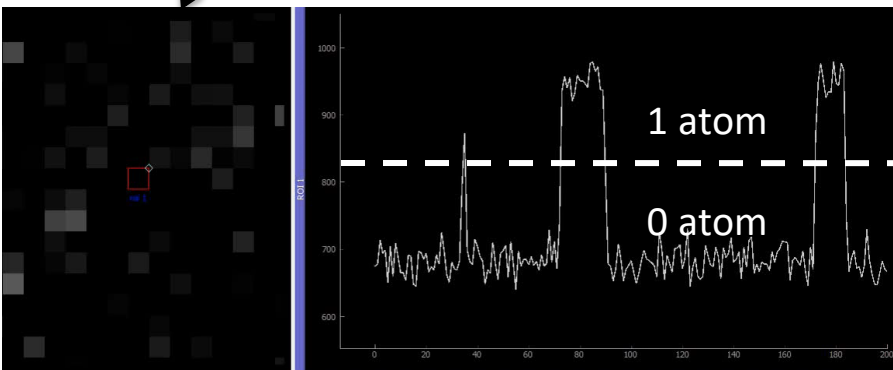
Reservoir = laser-cooled Rb atoms  
 $T \sim 100 \mu\text{K}$



# A single Rb atom in an optical tweezer



Grangier (2001)  
Sortais (2007)

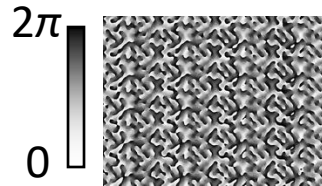
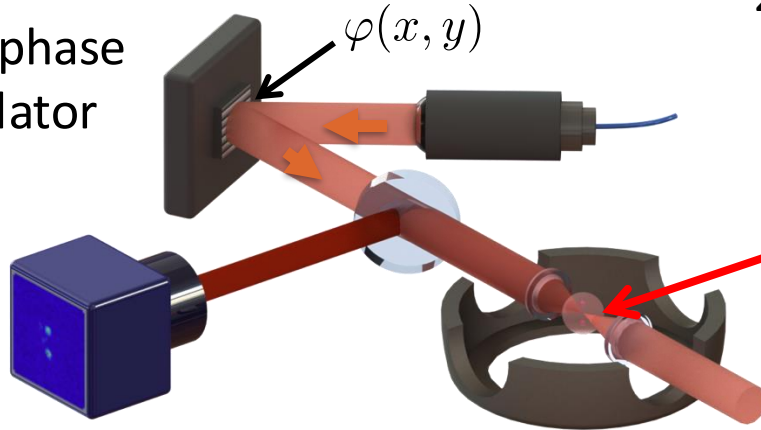


**Non-deterministic**  
single-atom source



# Atoms in arrays of optical tweezers

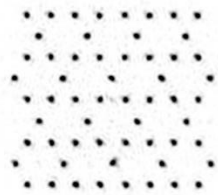
Spatial phase  
modulator



Phase mask

Nogrette, PRX (2014)

$$\left| \text{FT}[e^{i\varphi(x,y)}] \right|^2$$

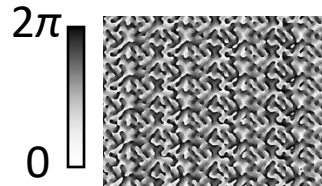
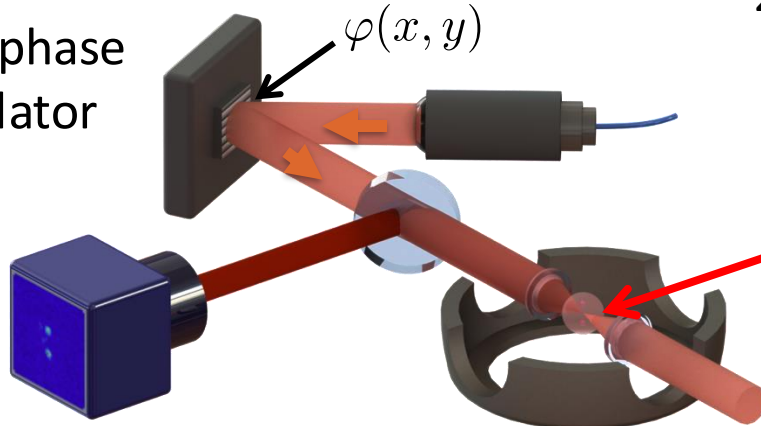


10  $\mu\text{m}$



# Atoms in arrays of optical tweezers

Spatial phase  
modulator

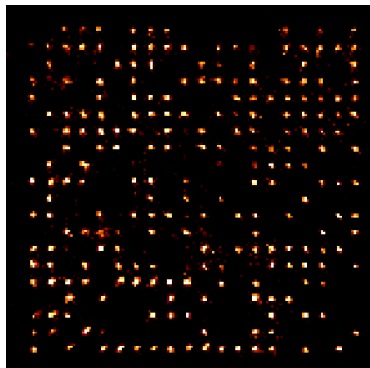


Phase mask

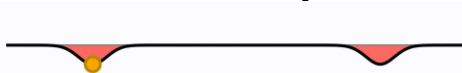
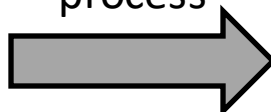
Nogrette, PRX (2014)

**First demo (1D):** Meschede, Nature (2006);  
Beugnon, Nat. Phys. (2007)

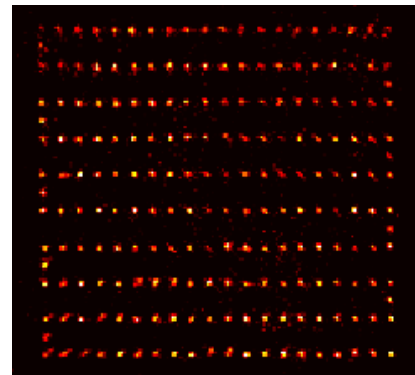
Initial configuration



Assembling  
process

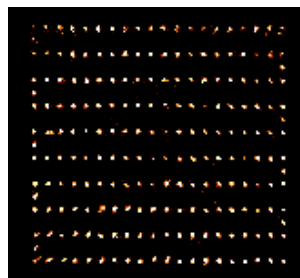


Assembled configuration

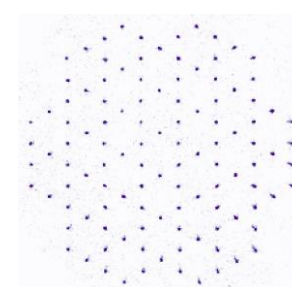
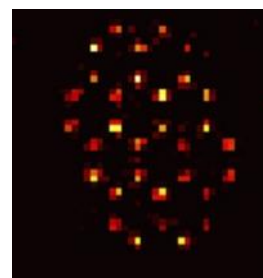
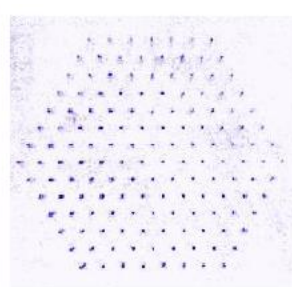
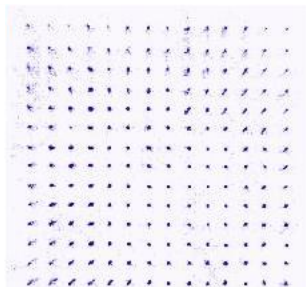


# Atoms in arrays of optical tweezers (single-shot images)

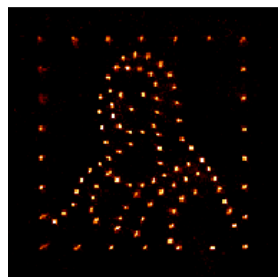
1D



2D



~100  $\mu\text{m}$

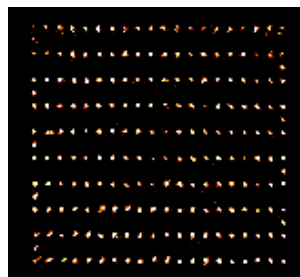


L. da Vinci



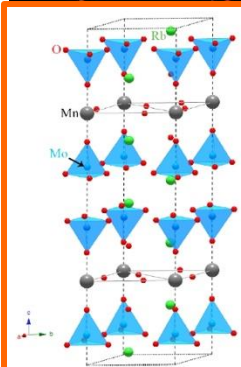
# Atoms in arrays of optical tweezers (single-shot images)

1D



~100  $\mu\text{m}$

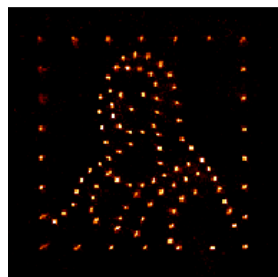
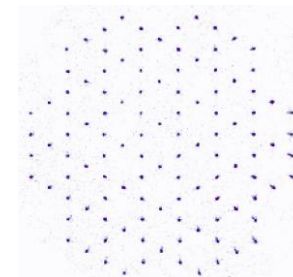
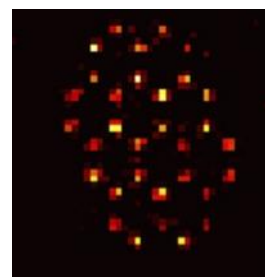
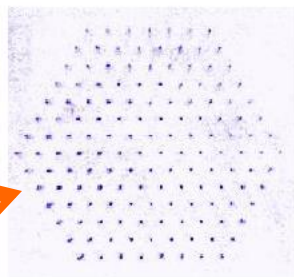
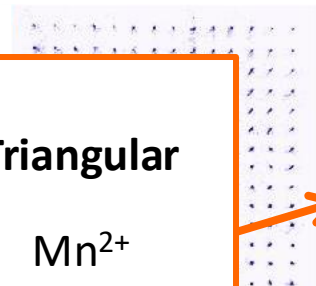
2D



Triangular

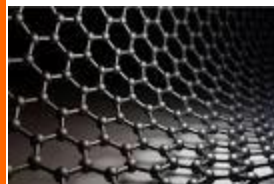
$\text{Mn}^{2+}$

$\text{Rb}_4\text{Mn}(\text{MoO}_4)_3$



L. da Vinci

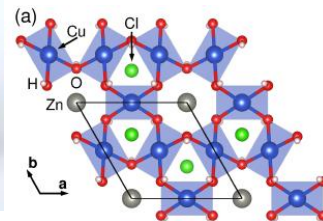
Hexagonal



graphene

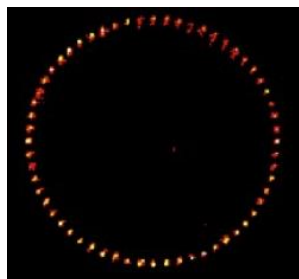
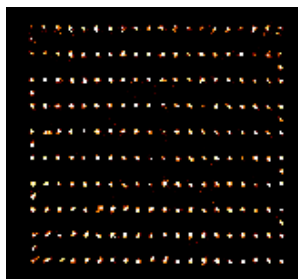
Kagome: Herbertsmithite

$\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$



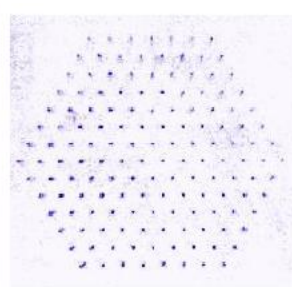
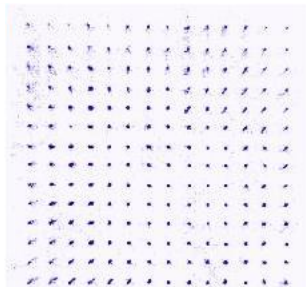
# Atoms in arrays of optical tweezers (single-shot images)

1D

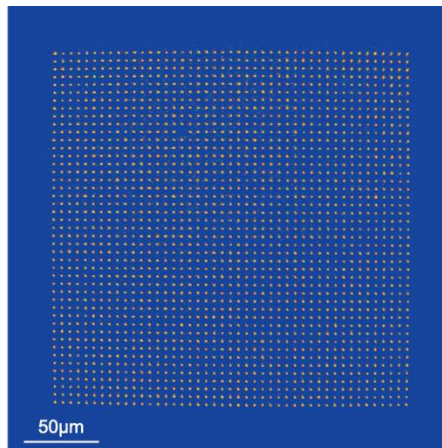


~100  $\mu\text{m}$

2D

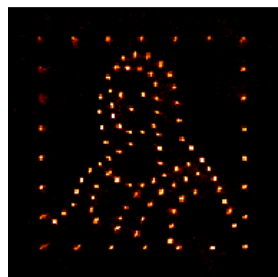
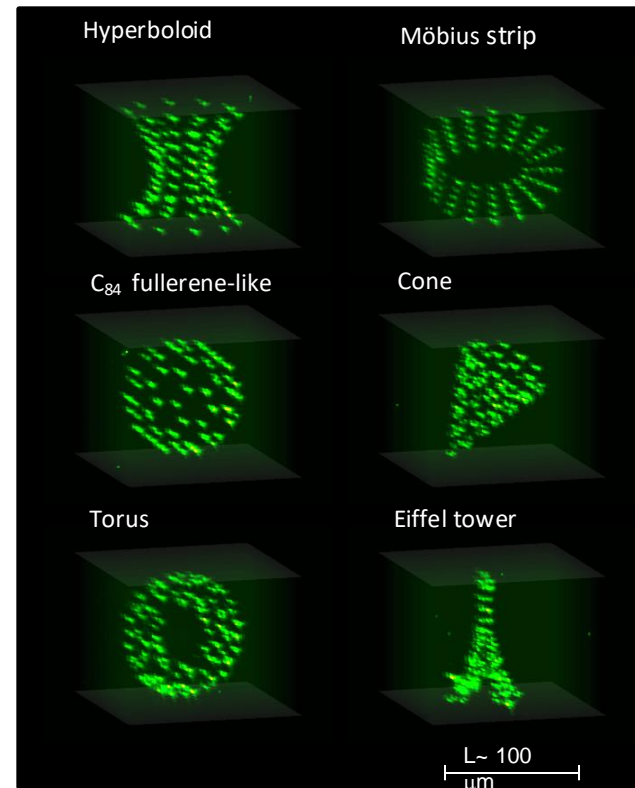


2024 atoms (AI + fast SLM)



3D

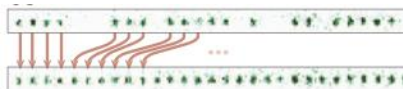
Barredo, Nature (2018)



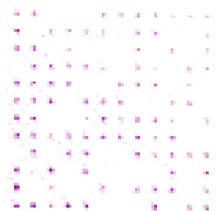
L. da Vinci

# Now a popular platform...with many developments

## Variants & new species



Lukin Science 2016

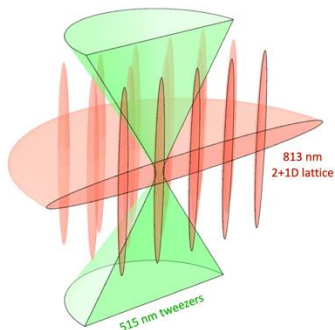


Yb, Sr

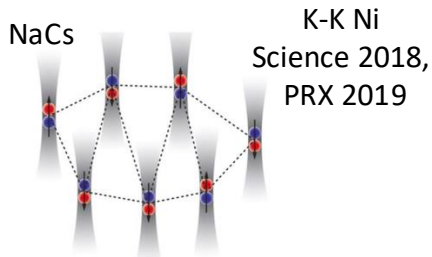
Endres,  
Kaufman,  
Thompson...

## Combining optical lattices + tweezer array

A. Kaufman Science 2022

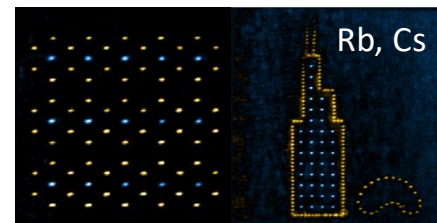


## Trapping molecules

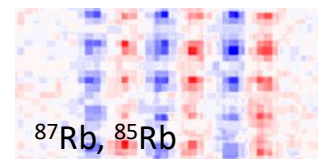


Ni, Doyle, Science 2019

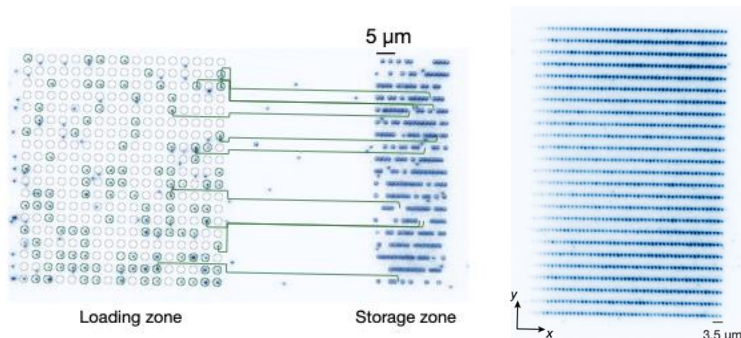
## Dual species arrays



H. Bernien PRX 2022



Zhan  
PRL 2022

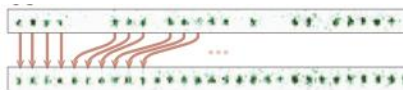


arXiv:2402.04994

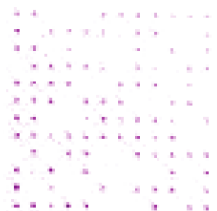


# Now a popular platform...with many developments

## Variants & new species



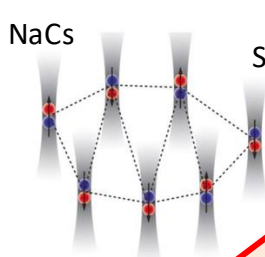
Lukin Science 2016



Yb, Sr

Endres,  
Kaufman,  
Thompson...

## Trapping molecules



NaCs

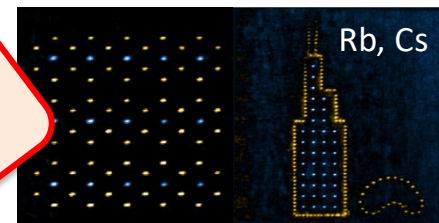
K-K Ni  
Science 2018,  
PRX 2019

CaF



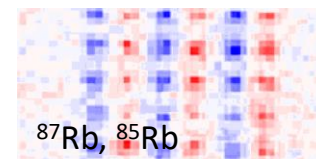
2019

## Dual species arrays



Rb, Cs

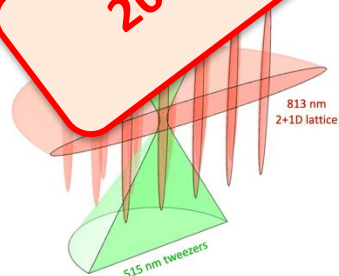
H. Bernien PRX 2022



$^{87}\text{Rb}$ ,  $^{85}\text{Rb}$

Zhan  
PRL 2022

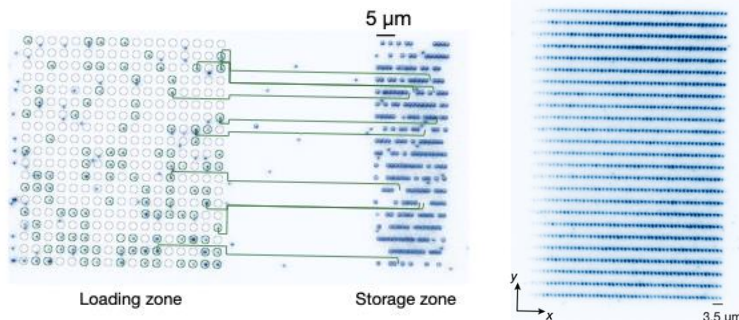
## Combining optical lattices + tweezer array



813 nm  
2+1D lattice

515 nm tweezers

A. Kaufman Science 2022



5  $\mu\text{m}$

Loading zone

Storage zone



3.5  $\mu\text{m}$

200+ in construction worldwide...

# Outline – Lecture 1

1. Many-body physics and synthetic quantum systems
2. Arrays of individual atoms: lattices and tweezers
3. **Basics of Rydberg physics**
4. Interaction between Rydberg atoms

# Rydberg atoms: the discovery

**1814** Joseph von Fraunhofer

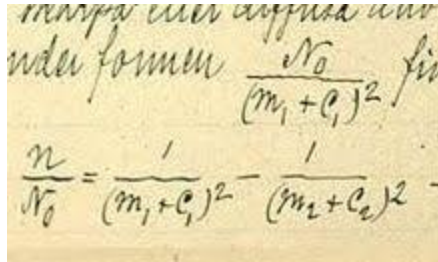


observation of dark lines in spectrum of the sun

**1888** “Rydberg formula”



Johannes Rydberg  
1854-1919

A photograph of a handwritten note on aged paper. The text includes the Rydberg formula: 
$$\frac{n}{N_0} = \frac{1}{(m_1 + c_1)^2} - \frac{1}{(m_2 + c_2)^2}$$
 and some other handwritten text.

$$\frac{1}{\lambda_{nm}} = R_H \left( \frac{1}{n^2} - \frac{1}{m^2} \right)$$

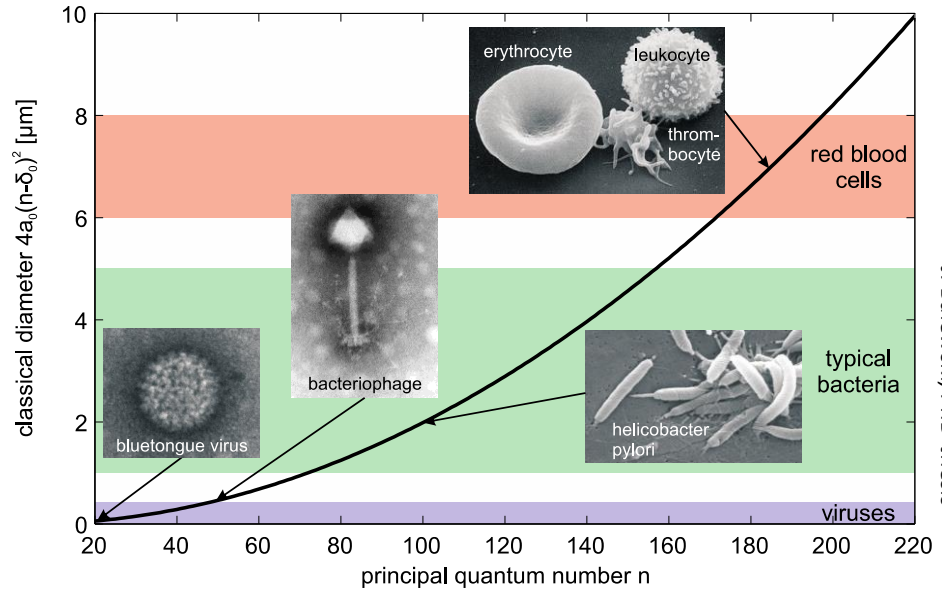
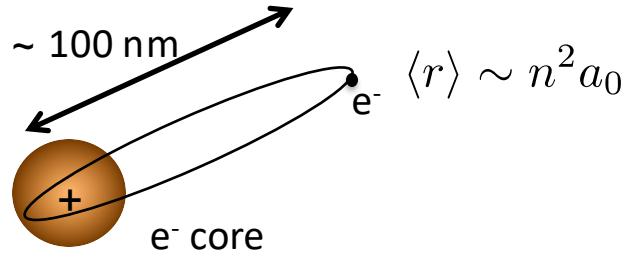
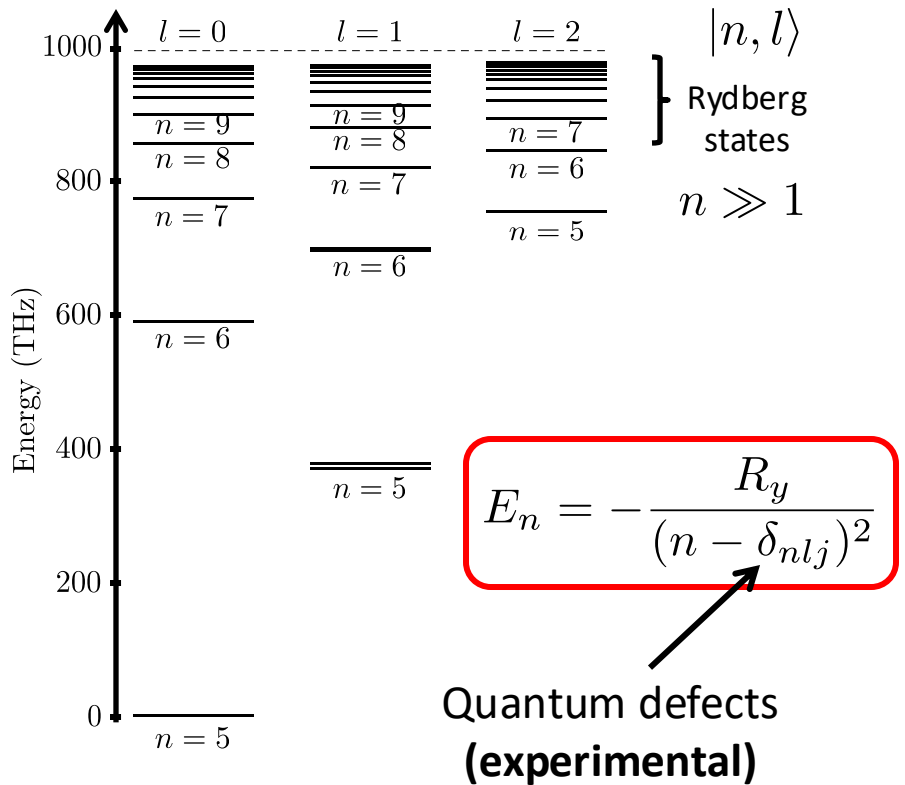
Idea of an infinite series  
⇒ **highly excited** states

# Examples: alkali atoms

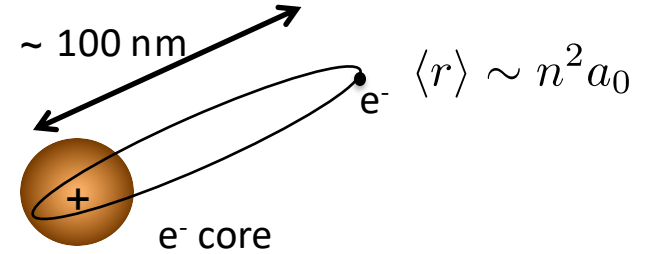
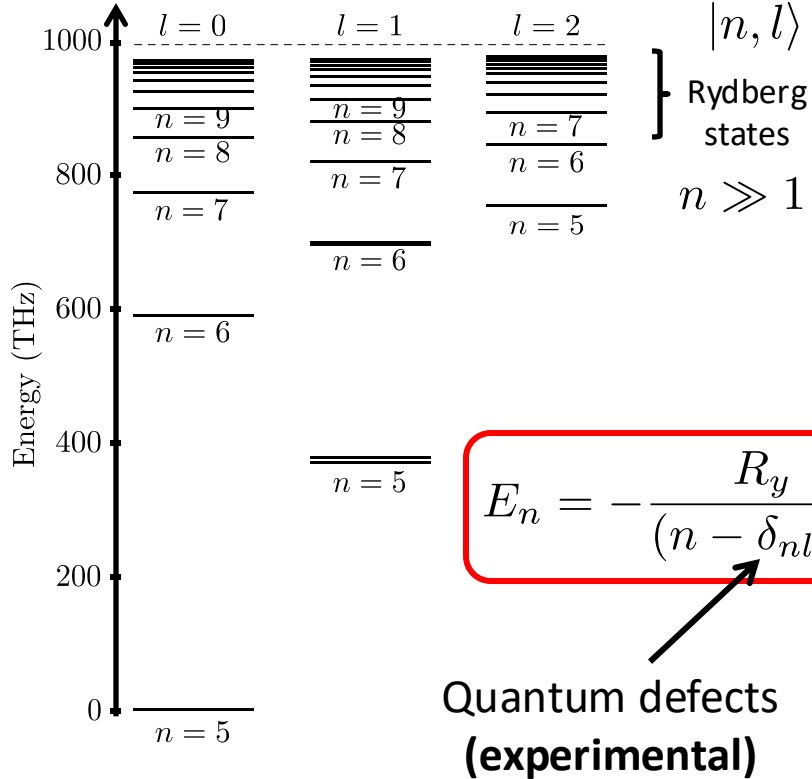
Alkali: 1 external electron  
 $1s^2 2s^2 \dots (n-1)p^6 ns$

|  |  |   |   |  |   |  |  |  |  |  |  |  |  |   |  |  |  |                            |
|--|--|---|---|--|---|--|--|--|--|--|--|--|--|---|--|--|--|----------------------------|
| 1<br>IA<br>1<br>H<br>Hydrogen<br>1.008 | 2<br>IIA<br>2<br>He<br>Helium<br>4.003 |   |   |  |   |  |  |  |  |  |  |  |  |   |  |  |  |                            |
| 3<br>Li<br>Lithium<br>6.941            | 4<br>Be<br>Beryllium<br>9.012          |   |   |  |   |  |  |  |  |  |  |  |  |   |  |  |  | 10<br>Ne<br>Neon<br>20.180 |
| 11<br>Na<br>Sodium<br>22.990           | 12<br>Mg<br>Magnesium<br>24.305        | 3<br>III B<br>3<br>B<br>Boron<br>10.811 | 4<br>IV B<br>4<br>C<br>Carbon<br>12.011 | 5<br>V B<br>5<br>N<br>Nitrogen<br>14.007 | 6<br>VI B<br>6<br>O<br>Oxygen<br>15.999 | 7<br>VII B<br>7<br>F<br>Fluorine<br>18.998 | 8<br>VIII<br>8<br>Fe<br>Iron<br>55.933 | 9<br>VIII<br>9<br>Co<br>Cobalt<br>58.933 | 10<br>VIII<br>10<br>Ni<br>Nickel<br>58.693 | 11<br>IB<br>11<br>Cu<br>Copper<br>63.546 | 12<br>IIB<br>12<br>Zn<br>Zinc<br>65.39 | 13<br>IIIA<br>13<br>Al<br>Aluminum<br>26.982 | 14<br>IVA<br>14<br>Si<br>Silicon<br>28.086 | 15<br>VA<br>15<br>P<br>Phosphorus<br>30.974 | 16<br>VIA<br>16<br>S<br>Sulfur<br>32.066 | 17<br>VIIA<br>17<br>Cl<br>Chlorine<br>35.453 | 18<br>VIIIA<br>18<br>Ar<br>Argon<br>39.948 |                            |
| 19<br>K<br>Potassium<br>39.098         | 20<br>Ca<br>Calcium<br>40.078          | 21<br>Sc<br>Scandium<br>44.956          | 22<br>Ti<br>Titanium<br>47.88           | 23<br>V<br>Vanadium<br>50.942            | 24<br>Cr<br>Chromium<br>51.996          | 25<br>Mn<br>Manganese<br>54.938            | 26<br>Fe<br>Iron<br>55.933             | 27<br>Co<br>Cobalt<br>58.933             | 28<br>Ni<br>Nickel<br>58.693               | 29<br>Cu<br>Copper<br>63.546             | 30<br>Zn<br>Zinc<br>65.39              | 31<br>Ga<br>Gallium<br>69.723                | 32<br>Ge<br>Germanium<br>72.61             | 33<br>As<br>Arsenic<br>74.922               | 34<br>Se<br>Selenium<br>78.972           | 35<br>Br<br>Bromine<br>79.904                | 36<br>Kr<br>Krypton<br>84.90               |                            |
| 37<br>Rb<br>Rubidium<br>84.468         | 38<br>Sr<br>Strontium<br>87.62         | 39<br>Y<br>Yttrium<br>88.906            | 40<br>Zr<br>Zirconium<br>91.224         | 41<br>Nb<br>Niobium<br>92.906            | 42<br>Mo<br>Molybdenum<br>95.95         | 43<br>Tc<br>Technetium<br>98.907           | 44<br>Ru<br>Ruthenium<br>101.07        | 45<br>Rh<br>Rhodium<br>102.906           | 46<br>Pd<br>Palladium<br>106.42            | 47<br>Ag<br>Silver<br>107.868            | 48<br>Cd<br>Cadmium<br>112.411         | 49<br>In<br>Indium<br>114.818                | 50<br>Sn<br>Tin<br>118.710                 | 51<br>Sb<br>Antimony<br>121.760             | 52<br>Te<br>Tellurium<br>127.6           | 53<br>I<br>Iodine<br>126.904                 | 54<br>Xe<br>Xenon<br>131.29                |                            |
| 55<br>Cs<br>Cesium<br>132.905          | 56<br>Ba<br>Barium<br>137.327          | 57-71<br>Lanthanide Series              | 72<br>Hf<br>Hafnium<br>178.49           | 73<br>Ta<br>Tantalum<br>180.948          | 74<br>W<br>Tungsten<br>183.85           | 75<br>Re<br>Rhenium<br>186.207             | 76<br>Os<br>Osmium<br>190.23           | 77<br>Ir<br>Iridium<br>192.22            | 78<br>Pt<br>Platinum<br>195.08             | 79<br>Au<br>Gold<br>196.967              | 80<br>Hg<br>Mercury<br>200.59          | 81<br>Tl<br>Thallium<br>204.383              | 82<br>Pb<br>Lead<br>207.2                  | 83<br>Bi<br>Bismuth<br>208.980              | 84<br>Po<br>Polonium<br>[208.982]        | 85<br>At<br>Astatine<br>209.987              | 86<br>Rn<br>Radon<br>222.018               |                            |
| 87<br>Fr<br>Francium<br>223.020        | 88<br>Ra<br>Radium<br>226.025          | 89-103<br>Actinide Series               | 104<br>Rf<br>Rutherfordium<br>[261]     | 105<br>Db<br>Dubnium<br>[262]            | 106<br>Sg<br>Seaborgium<br>[266]        | 107<br>Bh<br>Bohrium<br>[264]              | 108<br>Hs<br>Hassium<br>[269]          | 109<br>Mt<br>Meitnerium<br>[268]         | 110<br>Ds<br>Darmstadtium<br>[269]         | 111<br>Rg<br>Roentgenium<br>[272]        | 112<br>Cn<br>Copernicium<br>[277]      | 113<br>Uut<br>Ununtrium<br>unknown           | 114<br>Fl<br>Flerovium<br>[289]            | 115<br>Uup<br>Ununpentium<br>unknown        | 116<br>Lv<br>Livermorium<br>[293]        | 117<br>Uus<br>Ununseptium<br>unknown         | 118<br>Uuo<br>Ununoctium<br>unknown        |                            |

# “Rydberg atom” = a highly excited atom (e.g. Rb)



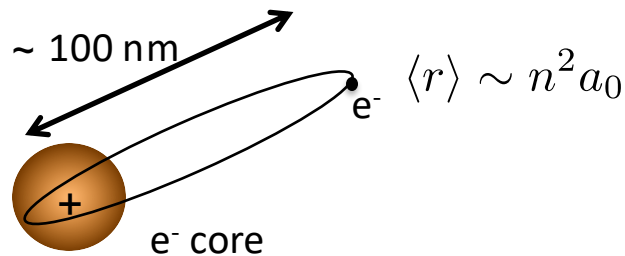
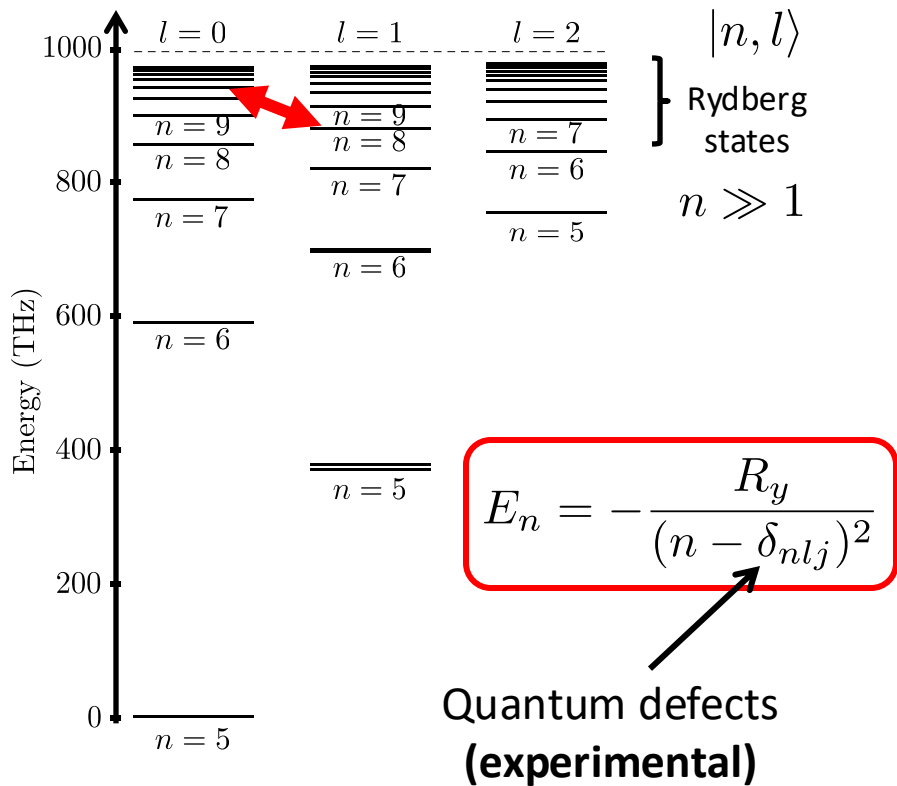
# “Rydberg atom” = a highly excited atom (e.g. Rb)



For Rb:  
 $n \geq 30$

| $L$ | $J$ | $\delta_{L,J}$ |
|-----|-----|----------------|
| 0   | 1/2 | 3.131          |
| 1   | 1/2 | 2.654          |
|     | 3/2 | 2.641          |
| 2   | 3/2 | 1.348          |
|     | 5/2 | 1.346          |
| 3   | 5/2 | 0.016          |
|     | 7/2 | 0.016          |

# “Rydberg atom” = a highly excited atom (e.g. Rb)



**Long lifetime:**  $\tau \sim n^3$   
 $\Rightarrow n > 60, \tau > 100 \mu\text{s}$

**Large transition dipole:**  
 $\langle n, l | \hat{D} | n, l \pm 1 \rangle \sim n^2 e a_0$

**Large polarizability:**  $\alpha \sim n^7$

$\Rightarrow$  **Exaggerated properties:**

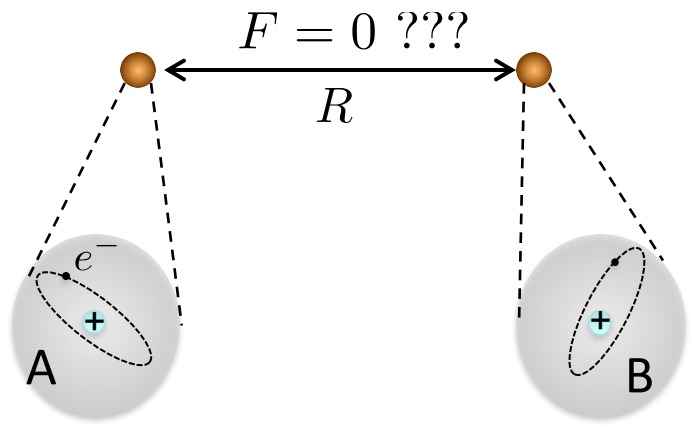
- strong interaction
- strong coupling to fields (DC, MW)

# Outline – Lecture 1

1. Many-body physics and synthetic quantum systems
2. Arrays of individual atoms: lattices and tweezers
3. Basics of Rydberg physics
4. Interaction between Rydberg atoms

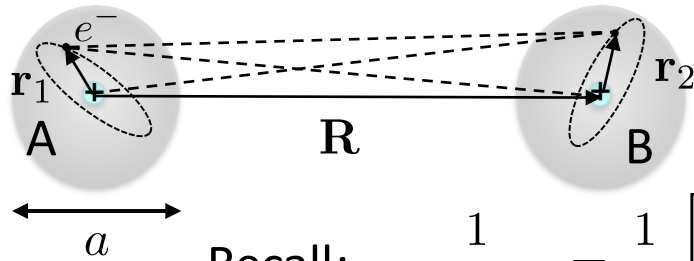


# Dipolar Interaction between atoms



# Dipolar Interaction between atoms

$$e^2 = \frac{q^2}{4\pi\epsilon_0}$$



$$H = \frac{p_1^2}{2m} - \frac{e^2}{r_1} + \frac{p_2^2}{2m} - \frac{e^2}{r_2} - \frac{e^2}{|\mathbf{R} - \mathbf{r}_1|} - \frac{e^2}{|\mathbf{R} + \mathbf{r}_2|} + \frac{e^2}{|\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1|} + \frac{e^2}{R}$$

Recall:  $\frac{1}{|\mathbf{R} - \mathbf{r}|} = \frac{1}{R} \left[ 1 - \frac{\mathbf{r} \cdot \mathbf{R}}{2R^2} - \frac{r^2}{2R^2} + \frac{3}{2} \left( \frac{\mathbf{r} \cdot \mathbf{R}}{R^2} \right)^2 \right] + \mathcal{O} \left( \frac{r^4}{R^4} \right)$

Dipole-dipole interaction:  $H_{\text{dd}} = \frac{1}{4\pi\epsilon_0 R^3} [\mathbf{d}_1 \cdot \mathbf{d}_2 - 3(\mathbf{d}_1 \cdot \mathbf{u})(\mathbf{d}_2 \cdot \mathbf{u})]$  ,  $\mathbf{u} = \frac{\mathbf{R}}{R}$   
 $a \ll R$

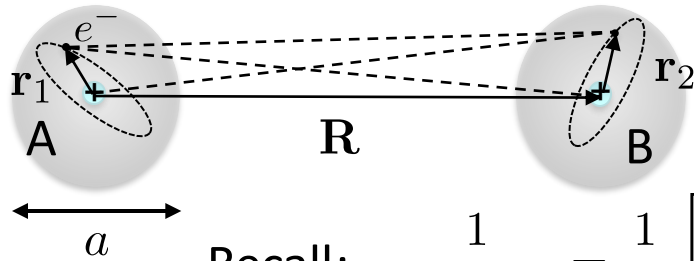
with dipoles:  $\mathbf{d}_{1,2} = q \mathbf{r}_{1,2}$

# Dipolar Interaction between atoms

$$e^2 = \frac{q^2}{4\pi\epsilon_0}$$

$$H = \frac{p_1^2}{2m} - \frac{e^2}{r_1} + \frac{p_2^2}{2m} - \frac{e^2}{r_2}$$

$$- \frac{e^2}{|\mathbf{R} - \mathbf{r}_1|} - \frac{e^2}{|\mathbf{R} + \mathbf{r}_2|} + \frac{e^2}{|\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1|} + \frac{e^2}{R}$$



Recall: 
$$\frac{1}{|\mathbf{R} - \mathbf{r}|} = \frac{1}{R} \left[ 1 - \frac{\mathbf{r} \cdot \mathbf{R}}{2R^2} - \frac{r^2}{2R^2} + \frac{3}{2} \left( \frac{\mathbf{r} \cdot \mathbf{R}}{R^2} \right)^2 \right] + \mathcal{O} \left( \frac{r^4}{R^4} \right)$$

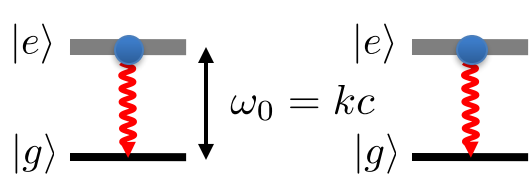
Dipole-dipole interaction: 
$$\hat{H}_{\text{dd}} = \frac{1}{4\pi\epsilon_0 R^3} \left[ \hat{\mathbf{d}}_1 \cdot \hat{\mathbf{d}}_2 - 3(\hat{\mathbf{d}}_1 \cdot \mathbf{u})(\hat{\mathbf{d}}_2 \cdot \mathbf{u}) \right], \quad \mathbf{u} = \frac{\mathbf{R}}{R}$$

$a \ll R$

with dipoles: 
$$\hat{\mathbf{d}}_{1,2} = q \hat{\mathbf{r}}_{1,2}$$

2 atom basis: 
$$\{|n, l, m\rangle \otimes |n', l', m'\rangle\}$$

# Interaction between atoms: A toy model

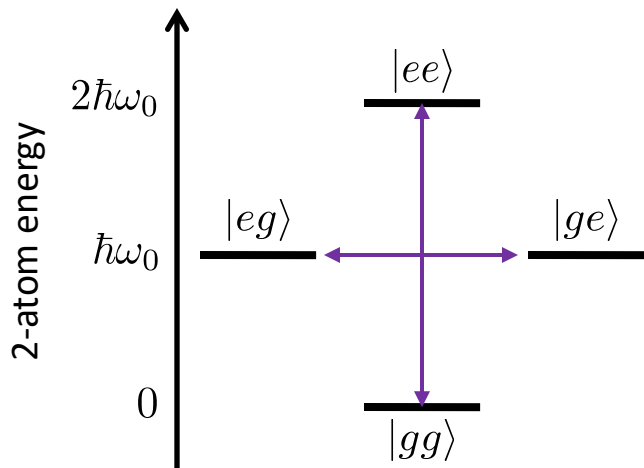


$$\Gamma = \frac{d_{eg}^2 k^3}{3\pi\epsilon_0}$$

Dipole interaction:  $\hat{H}_{dd} = \frac{1}{4\pi\epsilon_0} \frac{\hat{d}_{Az}\hat{d}_{Bz}}{R^3}$

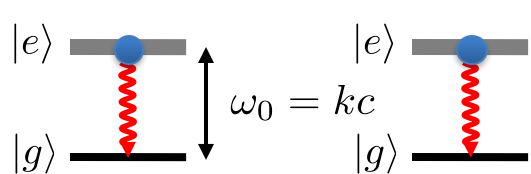
$d_{eg} = \langle e|\hat{d}_{Az}|g\rangle = \langle e|\hat{d}_{Bz}|g\rangle$

$$\hat{H}_{dd} = \begin{pmatrix} 0 & d_{eg}^2/R^3 & 0 & 0 \\ d_{eg}^2/R^3 & 2\hbar\omega_0 & 0 & 0 \\ 0 & 0 & \hbar\omega_0 & d_{eg}^2/R^3 \\ 0 & 0 & d_{eg}^2/R^3 & \hbar\omega_0 \end{pmatrix} \begin{matrix} |gg\rangle, |ee\rangle, |eg\rangle, |ge\rangle \end{matrix}$$



$$\frac{d_{eg}^2}{R^3} \ll \hbar\omega_0$$

# Interaction between atoms: A toy model

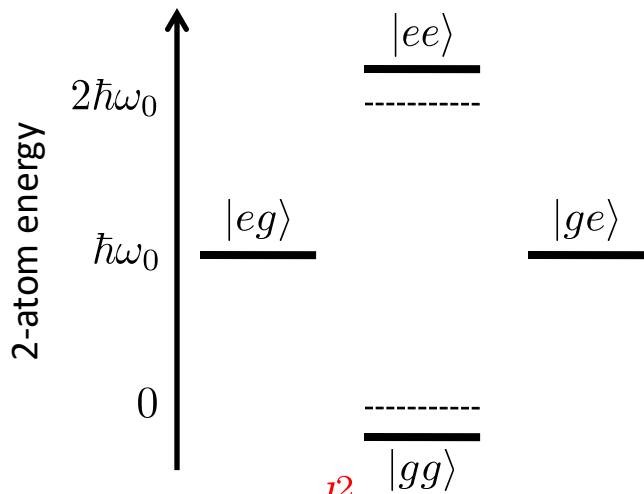


$$\Gamma = \frac{d_{eg}^2 k^3}{3\pi\epsilon_0}$$

Dipole interaction:  $\hat{H}_{dd} = \frac{1}{4\pi\epsilon_0} \frac{\hat{d}_{Az}\hat{d}_{Bz}}{R^3}$

$d_{eg} = \langle e|\hat{d}_{Az}|g\rangle = \langle e|\hat{d}_{Bz}|g\rangle$

$$\hat{H}_{dd} = \begin{pmatrix} 0 & d_{eg}^2/R^3 & 0 & 0 \\ d_{eg}^2/R^3 & 2\hbar\omega_0 & 0 & 0 \\ 0 & 0 & \hbar\omega_0 & d_{eg}^2/R^3 \\ 0 & 0 & d_{eg}^2/R^3 & \hbar\omega_0 \end{pmatrix} \begin{matrix} |gg\rangle, |ee\rangle, |eg\rangle, |ge\rangle \end{matrix}$$



$$\Delta E_{gg}^{(2)} = -\Delta E_{ee}^{(2)}$$

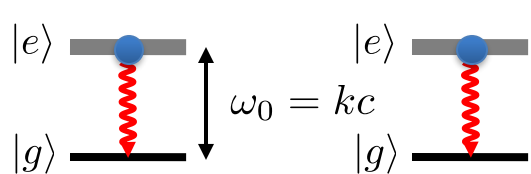
$$\approx -\frac{1}{2\hbar\omega_0} \left( \frac{d_{eg}^2}{R^3} \right)^2$$

$$= -\frac{9}{32} \frac{\Gamma}{\omega_0} \frac{\hbar\Gamma}{(kR)^6}$$

$$\frac{d_{eg}^2}{R^3} \ll \hbar\omega_0$$

**Van der Waals**

# Interaction between atoms: A toy model

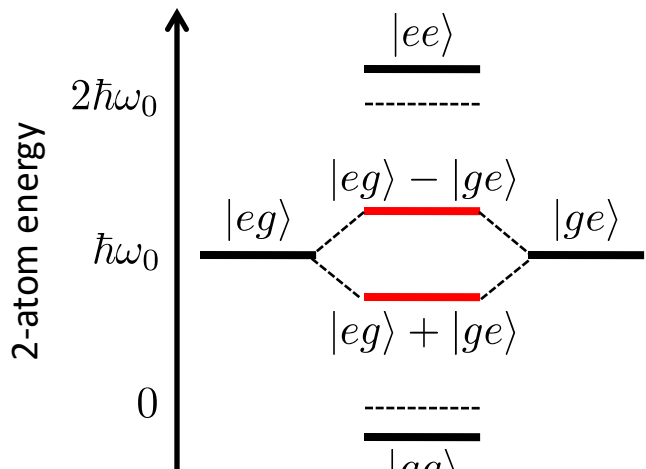


$$\Gamma = \frac{d_{eg}^2 k^3}{3\pi\epsilon_0}$$

Dipole interaction:  $\hat{H}_{dd} = \frac{1}{4\pi\epsilon_0} \frac{\hat{d}_{Az}\hat{d}_{Bz}}{R^3}$

$d_{eg} = \langle e|\hat{d}_{Az}|g\rangle = \langle e|\hat{d}_{Bz}|g\rangle$

$$\hat{H}_{dd} = \begin{pmatrix} 0 & d_{eg}^2/R^3 & 0 & 0 \\ d_{eg}^2/R^3 & 2\hbar\omega_0 & 0 & 0 \\ 0 & 0 & \hbar\omega_0 & d_{eg}^2/R^3 \\ 0 & 0 & d_{eg}^2/R^3 & \hbar\omega_0 \end{pmatrix} |gg\rangle, |ee\rangle, |eg\rangle, |ge\rangle$$



$$\frac{d_{eg}^2}{R^3} \ll \hbar\omega_0$$

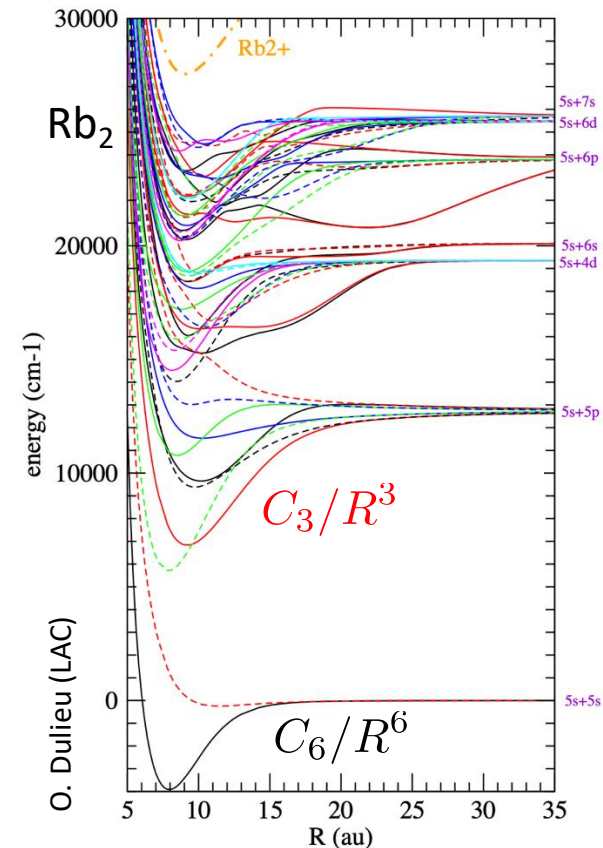
$$\begin{aligned} \Delta E_{gg}^{(2)} &= -\Delta E_{ee}^{(2)} \\ &\approx -\frac{1}{2\hbar\omega_0} \left( \frac{d_{eg}^2}{R^3} \right)^2 \\ &= -\frac{9}{32} \frac{\Gamma}{\omega_0} \frac{\hbar\Gamma}{(kR)^6} \end{aligned}$$

**Van der Waals**

$$\begin{aligned} E_{\pm} &= \pm \frac{1}{4\pi\epsilon_0} \frac{d_{eg}^2}{R^3} \\ &= \pm \frac{3}{4} \frac{\hbar\Gamma}{(kR)^3} \end{aligned}$$

**Resonant**

# Long-range interaction between *real* atoms

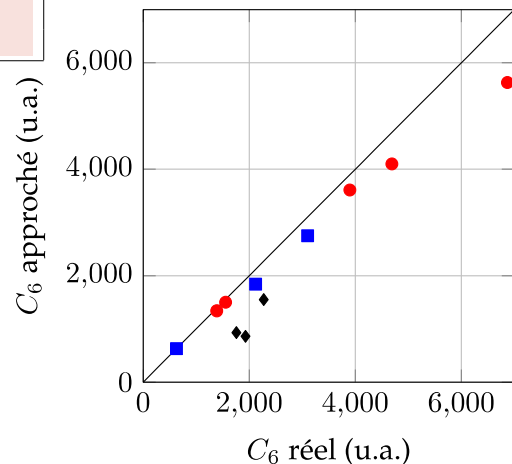


| système | Exp <sup>t</sup> |                     |                | Theory                |
|---------|------------------|---------------------|----------------|-----------------------|
|         | $C_6$ [u.a.]     | $\Gamma/2\pi$ [MHz] | $\lambda$ [nm] | $C_6$ approché [u.a.] |
| Li-Li   | 1389             | 5.87                | 671            | 1340                  |
| Na-Na   | 1556             | 9.80                | 589            | 1500                  |
| K-K     | 3897             | 6.04                | 767            | 3610                  |
| Rb-Rb   | 4691             | 6.07                | 780            | 4100                  |
| Cs-Cs   | 6870             | 5.22                | 852            | 5629                  |
| Mg-Mg   | 627              | 80.9                | 235            | 630                   |
| Ca-Ca   | 2121             | 34.6                | 423            | 1840                  |
| Sr-Sr   | 3103             | 32.0                | 461            | 2750                  |
| Er-Er   | 1760             | 29.7                | 401            | 930                   |
| Dy-Dy   | 2275             | 32.2                | 421            | 1550                  |
| Yb-Yb   | 1929             | 29                  | 399            | 860                   |

J. Dalibard, Collège de France, 2021

Van der Waals:  $C_6/R^6$

$$C_6 = -\frac{27 \hbar \Gamma}{16 k^6} \frac{\Gamma}{\omega_0}$$

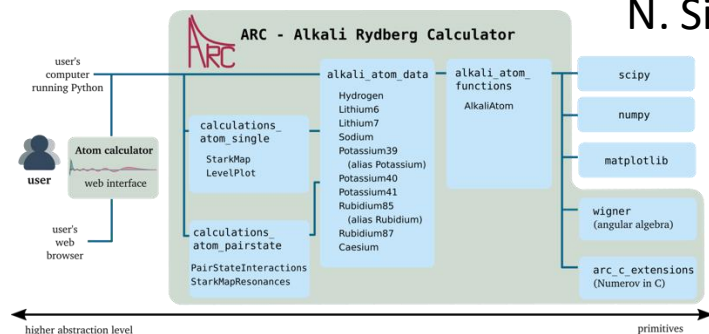
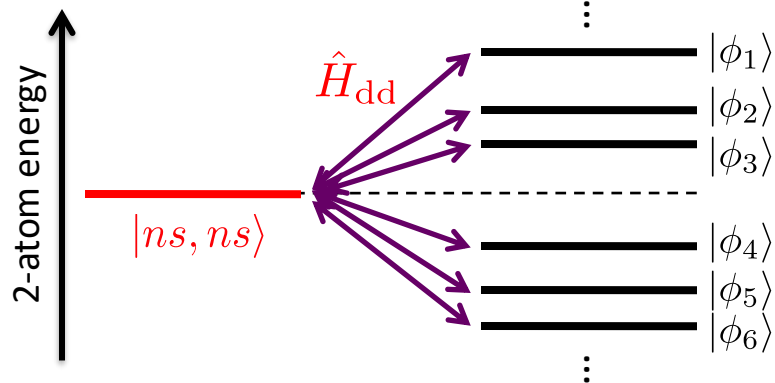


Useful for: scattering length (quantum gases), Rydberg physics...

# Interactions between Rydberg atoms

N. Sibalic

2-atom basis:  $\{|\phi_{nn'}\rangle = |n, l, m\rangle \otimes |n', l', m'\rangle\}$



<https://arc-alkali-rydberg-calculator.readthedocs.io/en/latest/>

[Docs](#) » Pairinteraction - A Rydberg Interaction Calculator

S. Weber

Van der Waals regime:

$$\Delta E_{ss}^{(2)} = \sum_{|\phi\rangle} \frac{|\langle\phi|\hat{H}_{dd}|ss\rangle|^2}{E_{ss} - E_{\phi}} = \frac{C_6}{R^6}, \quad C_6 \propto n^{11}$$

Resonant regime:

$$E_{\pm} = \pm \langle sp|\hat{H}_{dd}|ps\rangle = \pm \frac{1}{4\pi\epsilon_0} \frac{d_{sp}^2}{R^3} \propto n^4$$

## Pairinteraction - A Rydberg Interaction Calculator

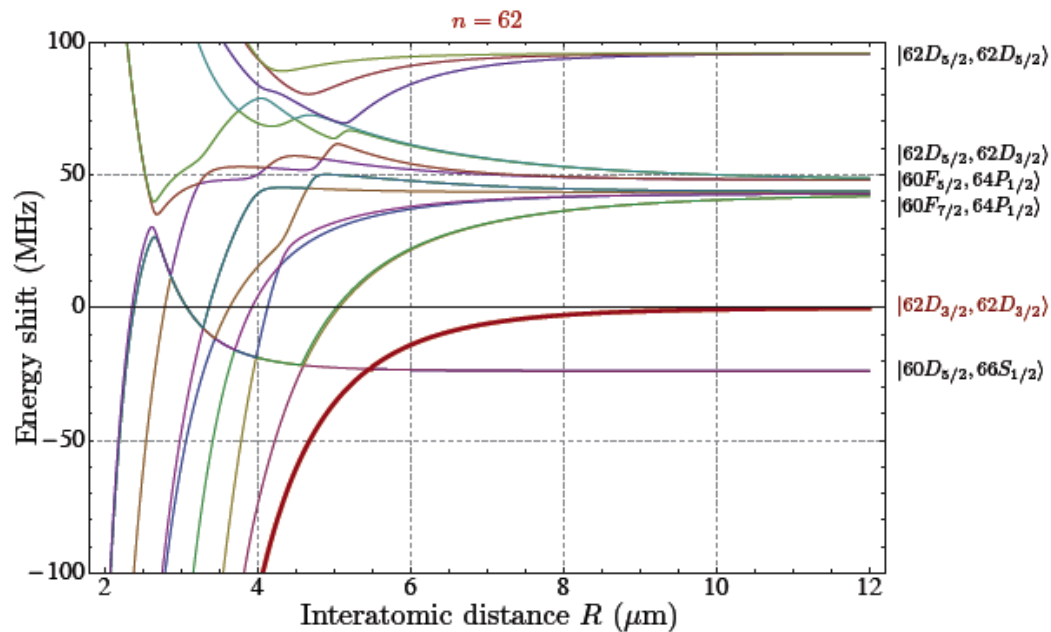
build passing
build passing
codecov 67%
python v0.9.5a0
arXiv 1612.08053  
 License: GPLV3

The *pairinteraction* software calculates properties of Rydberg systems. The software consists of a C++/Python library and a graphical user interface for pair potential calculations. For usage examples visit the [tutorials](#) section of the documentation. Stay tuned by [signing up](#) for the newsletter so whenever there are updates to the software or new publications about pairinteraction we can contact you. If you have a question that is related to problems, bugs, or suggests an improvement, consider raising an [issue](#) on [GitHub](#).

<https://pairinteraction.github.io/pairinteraction/sphinx/html/index.html>



# Interactions between “real” Rydberg atoms



$$R = 10 \mu\text{m} \Rightarrow V_{\text{int}}/h \sim 1 - 10 \text{ MHz} \Rightarrow \text{timescales} < \mu\text{sec}$$

# The program

Lecture 1: Many-body problem and quantum simulation  
Arrays of atoms & “Rydbergology”  
Interactions between atoms

**Lecture 2: Rydberg Interactions and spin models**  
**Engineering many-body Hamiltonians**

Lecture 3: Examples of quantum simulations in  
and out-of-equilibrium: quantum magnetism