Winter School on Ultracold Quantum Many-body Systems
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PROPAGATION OF SOUND IN SUPERFLUID ATOMIC GASES

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Propagation of **sound** is an **ubiquitous** feature characterizing many body systems.

In **classical** gases sound propagates as a consequence of **collisions** ensuring the achievement of **hydrodynamic** regime

At **low temperature**, in the quantum world, sound exhibits novel features

Major questions addressed in these lectures

- **How does superfluidity affect the propagation of sound ?**

- **How many sounds can propagate in a superfluid ?**

- Superfluid density cannot be derived from thermodynamic functions at equilibrium. **Can the superfluid density be extracted from measurement of sound velocity**

- **What happens** to superfluid fraction and to sound propagation **if Galilean (or Translation) invariance is broken ?**

PLAN OF THE LECTURES

Lecture 1. **Superfluids at finite tempertaure: a tale of two sounds**

Lecture 2. **Dynamical breaking of Galilean invariance and propagation of sound at T=0**

Lecture 3. **Spontaneous breaking of translational invariance and supersolidity: another tale of two sounds**

Sound in a classical gas

From classical Bolzmann equation, plus **local equilibrium** imposed by collisions, one derives linearized hydrodynamic equations for density, velocity field and temperature

$$\frac{\partial}{\partial t}\rho + \rho\vec{\nabla}\cdot\vec{v} = 0$$

$$\frac{\partial}{\partial t}\vartheta + \frac{\vartheta}{c_v}\vec{\nabla}\cdot\vec{v} = 0$$

$$\frac{\partial}{\partial t}\vec{v} + \vec{\nabla}P = 0 \quad \text{with} \quad \mathrm{P} = \frac{\rho\theta}{m}$$

Sound wave solutions are fixed by linear dispersion

$\omega^2 = q^2 c^2$ with sound velocity fixed by

adiabatic compressibility $\mathrm{c}^2 = \dfrac{1}{\kappa_s \rho}$

Result for **adiabatic** sound is not trivial from the many-body point of view since the adiabatic compressibility $\kappa_S = \frac{3}{5}\frac{m}{\rho k_B T}$ differs from the isothermal compressibility

$\kappa_T = \frac{m}{\rho k_B T}$ which fixes the compressibility sum rule

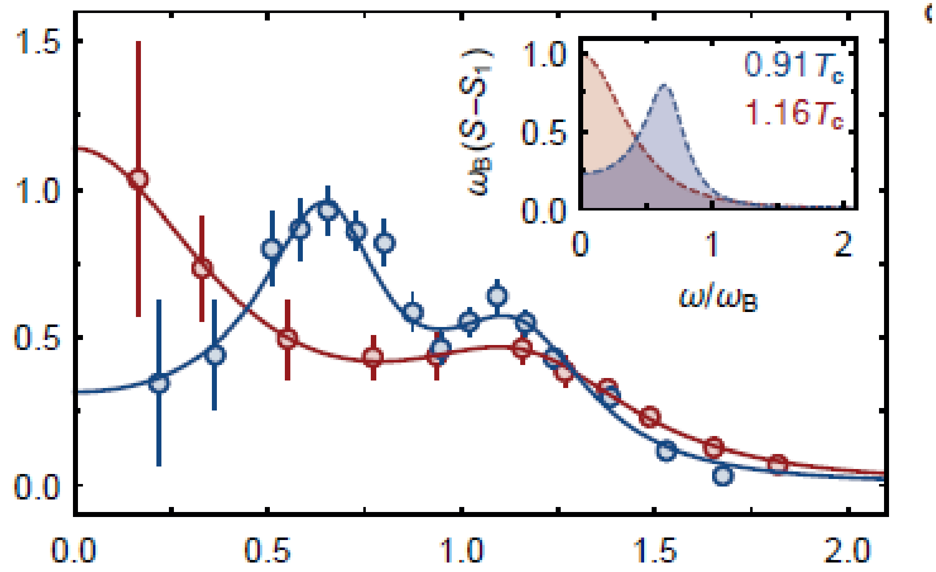
$$\lim_{q\to 0}\int S(q,\omega)\frac{1}{\omega}d\omega = \frac{1}{2}\kappa_T \quad \text{where}$$

$S(q,\omega)$ is dynamic structure factor

⇒ classical sound does not exhaust compressibility sum rule. Occurrence of additional low energy mode (of diffusive nature) is crucial to fulfill the compressibility sum rule !

What happens when one lowers temperature and enters the quantum superfluid regime ? **Diffuse mode transforms into a novel undamped mode, called second sound**

Dynamic structure factor measured in a 2D Bose gas below (**blue**) and above (**red**) the critical temperature (Christodoulou, …Hadzibabic, 2021)



Below Tc one finds **two resonances** (first and second sound)
Above Tc the **low frequency signal** has **diffusive** nature.

Landau developed theory of sounds propagating in a uniform superfluid at finite temperature.
Theory is based on the assumption that the system is composed of **two coupled fluids** (normal and superfluid fluids) whose motion gives rise to two sounds in the hydrodynamic collisional regime.

$\omega\tau << 1$

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

$$\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0$$

Irrotationality of superfluid flow

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla\mu(n) = 0$$

$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P = 0$$

$$\rho = mn = \rho_S + \rho_N$$

$$\vec{j} = \rho_S\vec{v}_S + \rho_N\vec{v}_N$$

s, P, μ are density and temperature dependent entropy, pressure and chemical potential. Related by Gibbs-Duhem relation

$$\rho d\mu = -msdT + mdP$$

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\vec{j}) = 0$$

~~$$\frac{\partial}{\partial t}s + \vec{\nabla}(s\vec{v}_N) = 0$$~~

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla\mu(n) = 0$$

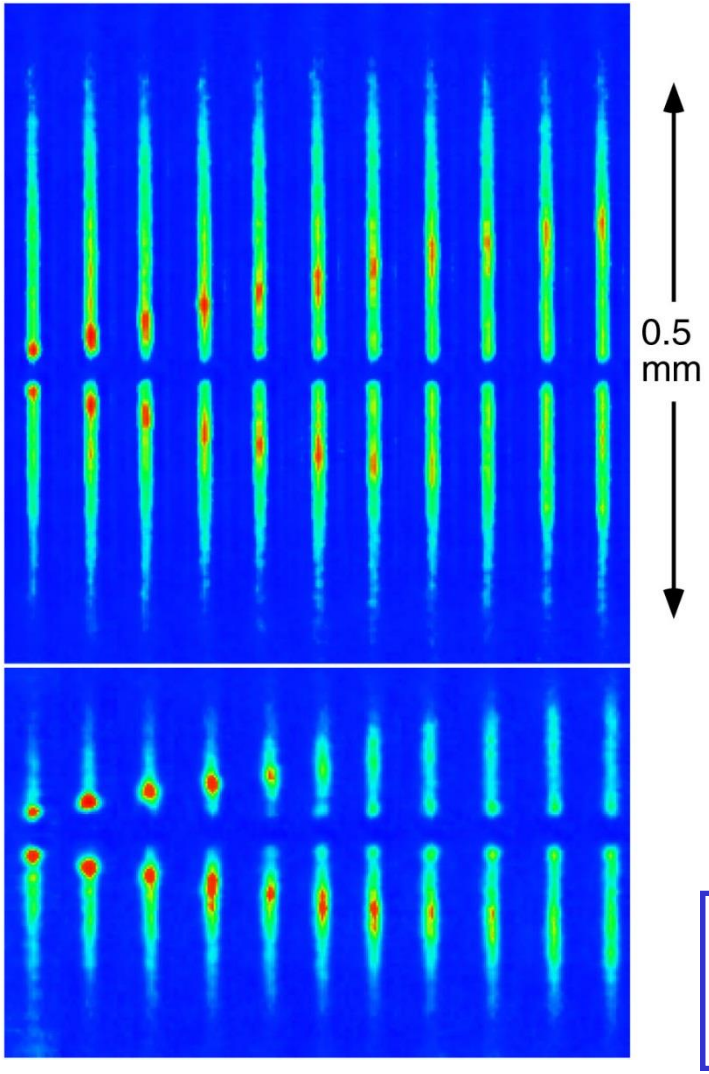
$$\frac{\partial}{\partial t}\vec{j} + \vec{\nabla}P = 0$$

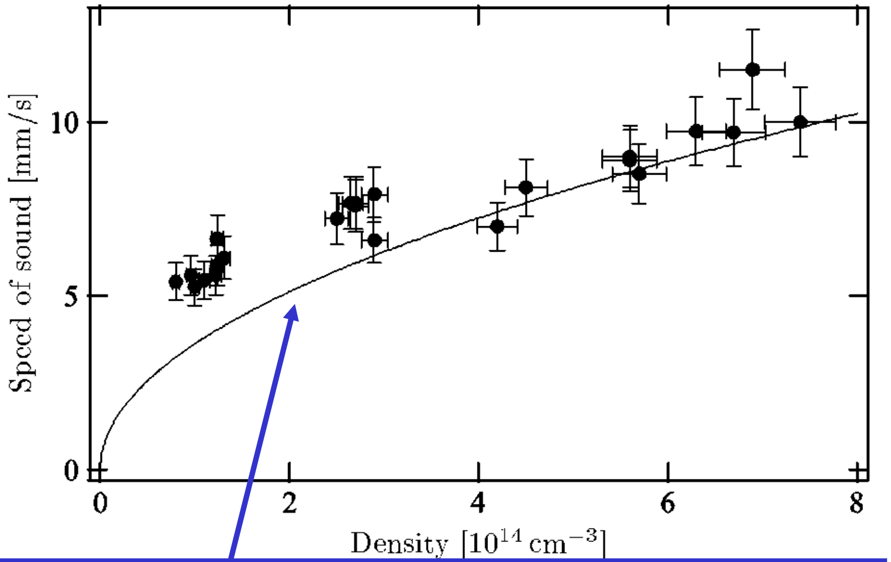
equivalent at T=0 as a consequence of Gibbs-Duhem relation $\rho d\mu = m dP$

At T=0: $\rho = \rho_S$, $\vec{j} = \rho\vec{v}_S$
and eqs. reduce to
T=0 irrotational superfluid
HD equations

*At T=0 irrotational hydrodynamics follows from superfluidity (role of the phase of the order parameter). Quite successful to describe the macroscopic dynamic behavior of trapped superfluid atomic gases (Bose and Fermi) (**expansion**, **collective oscillations**)*

T=0 Bogoliubov sound
(wave packet propagating in a dilute BEC, Mit 97)





sound velocity as a function of central density

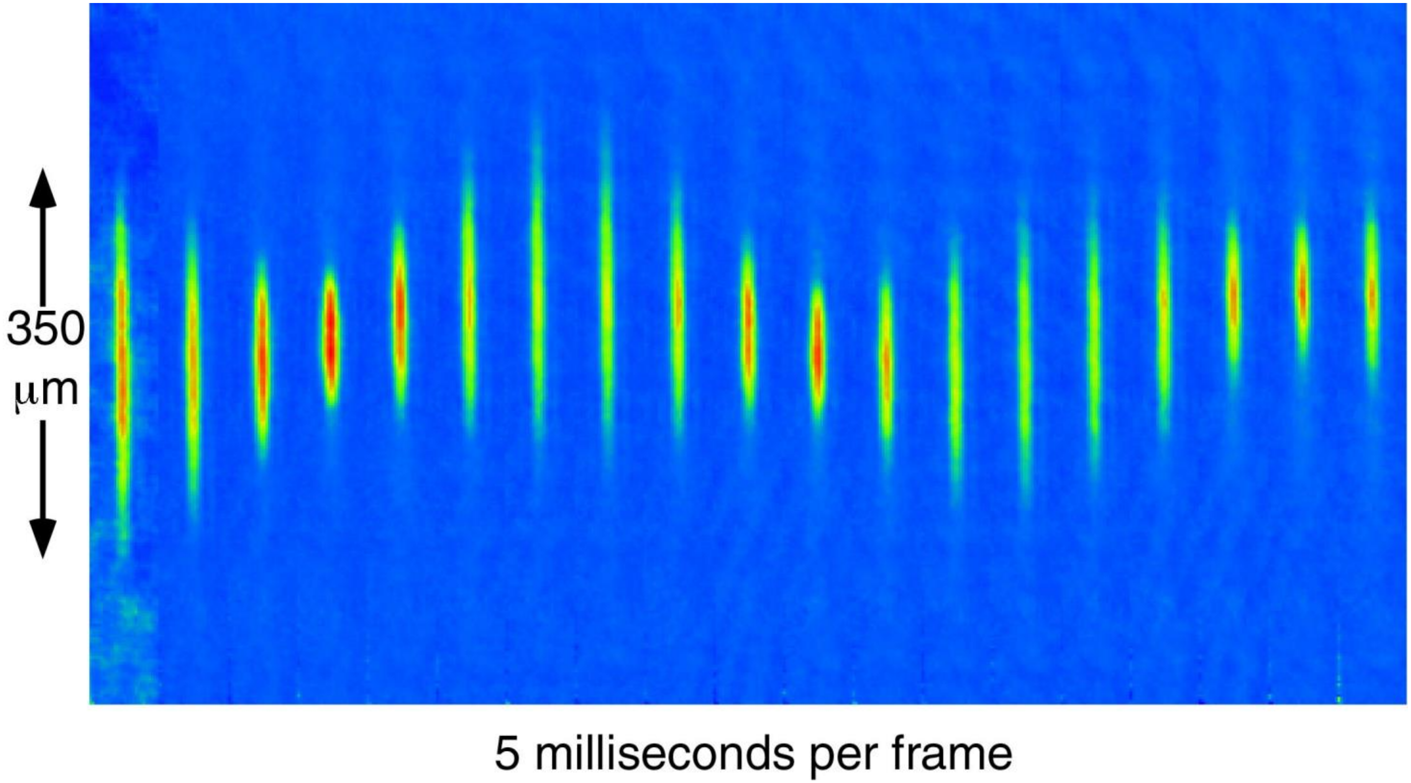
$$c = \sqrt{gn/2m}$$

factor 2 accounts for harmonic radial trapping (Zaremba, 98)

T=0 Collective oscillations in dilute BEC (axial compression mode) : checking validity of **hydrodynamic theory** of superfluids in **trapped gases**

Exp (Mit, 1997) $\omega = 1.57\omega_z$

HD Theory (S.S. 1996): $\omega = \sqrt{5/2}\,\omega_z = 1.58\omega_z$



SOLVING THE HYDRODYNAMIC EQUATIONS OF SUPERFLUIDS

AT FINITE TEMPERATURE

In **uniform matter** Landau equations of two fluid hydrodynamics gives rise to two solutions below the critical temperature:

First sound: superfluid and normal fluids move **in phase**

Second sound: superfluid and normal fluids move **out of phase**.

In systems characterized by **small compressibility,** (like liquid He4 and strongly interacting Fermi gas)

second sound reduces to entropy wave.
Corresponding velocity fixed by superfluid density, hence providing unique possibility to **measure superfluid density**)

$$c_2^2 = \frac{1}{m}\frac{\rho_s T s^2}{\rho_n C_P}$$

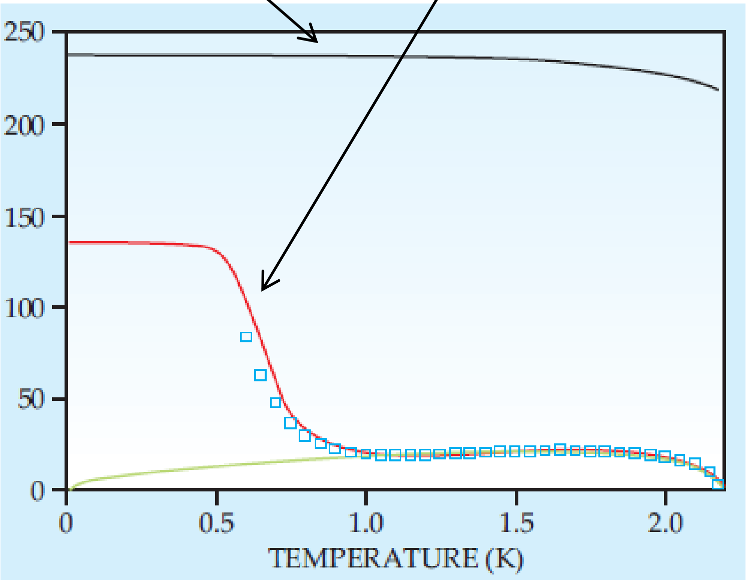
entropy

Specific heat

First and second sound velocities in **superfluid liquid He**

$$c_1^2 = \frac{1}{m}\left(\frac{\partial P}{\partial n}\right)_S$$

$$c_2^2 = \frac{1}{m}\frac{\rho_s T s^2}{\rho_n C_P}$$



Liquid He

(experiment, Peshkov 1946)

Propagation of sound in the 3D Fermi gas at unitarity

Thermodynamics and Universality of 3D Fermi gas at unitarity

Absence of interaction parameters implies that thermodynamics obeys universal law (Ho, 2004)

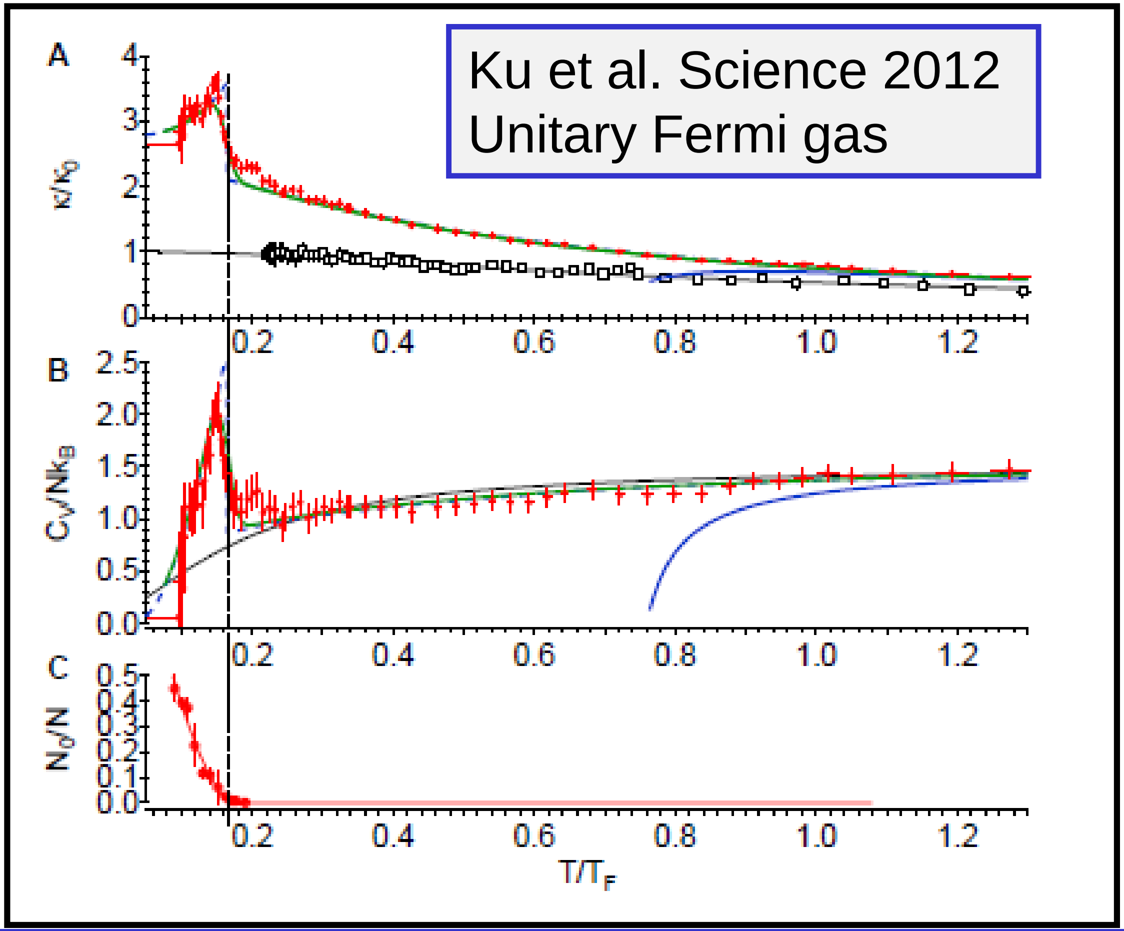
$$\frac{P}{k_B T}\lambda_T^3 = f_p(\mu / k_B T)$$

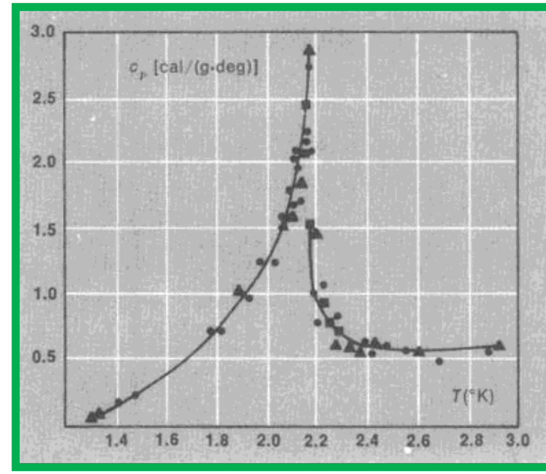
where $\lambda_T = \sqrt{2\pi\hbar^2 / mk_B T}$ is thermal wave-length and $f_p(x)$ is **dimensionless, universal function** (applies to quantum gases and neutron matter).

All thermodynamic functions (entropy, compressibilities, specific heats etc.) can be expressed in terms of the universal function

Calculation of $f_p(x)$ requires however non trivial many-body approaches at finite T.

Universal function $f_p(x)$ and thermodynamic functions now **available experimentally** in a wide range of temperatures





Superfluid He4

Experimental determination of critical temperature

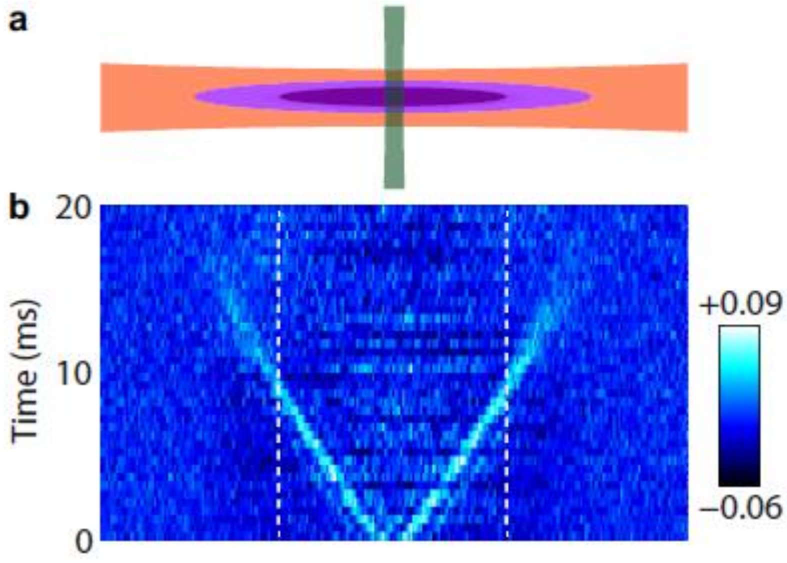
$$T_C / T_F = 0.167(13)$$

(determined by peak in specific heat and onset of BEC) in agreement with many-body predictions (Burowski et al. 2006; Haussmann et al. (2007); Goulko and Wingate 2010)

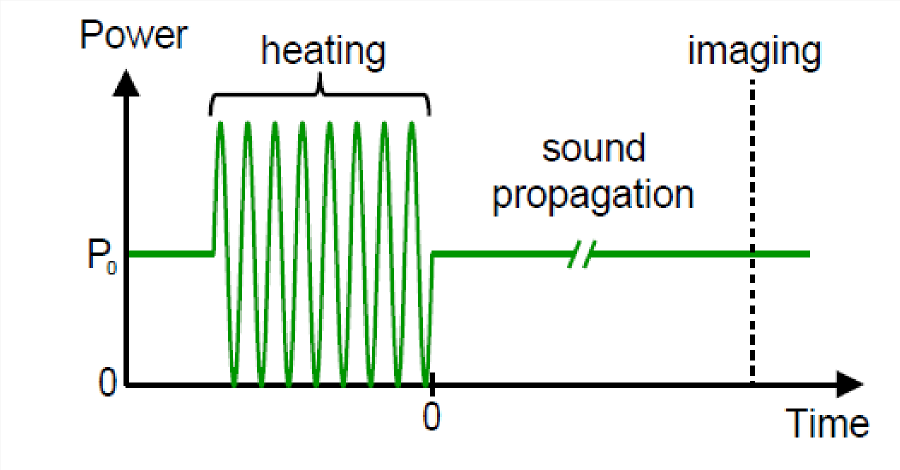
Measurement of **first** and **second sound** and determination of the **superfluid density** in a strongly **interacting Fermi gas**

Innsbruck- Trento collaboration (Sidorenkov et al., Nature 2013)

To excite **first sound** one suddenly turns on a repulsive (green) laser beam in the center of the trap [similar tecnhnique used at Mit (1998) and Utrecht (2009) to generate Bogoliubov sound in dilute BEC



To excite **second sound** one keeps the repulsive (green) laser power constant with the exception of a short time modulation producing local heating in the center of the trap

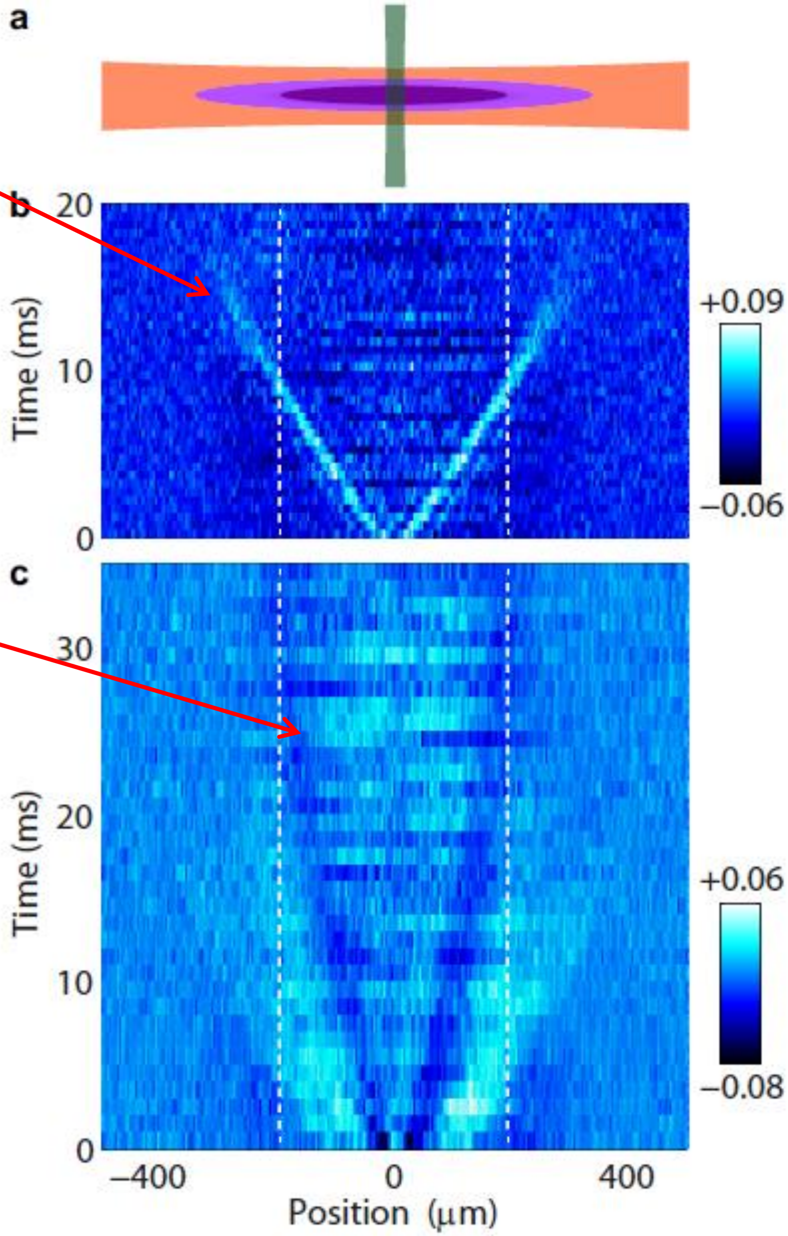


The average laser power is kept constant to limit the excitation of pressure waves (first sound)

First sound
propagates also beyond the boundary between the superfluid and the normal parts

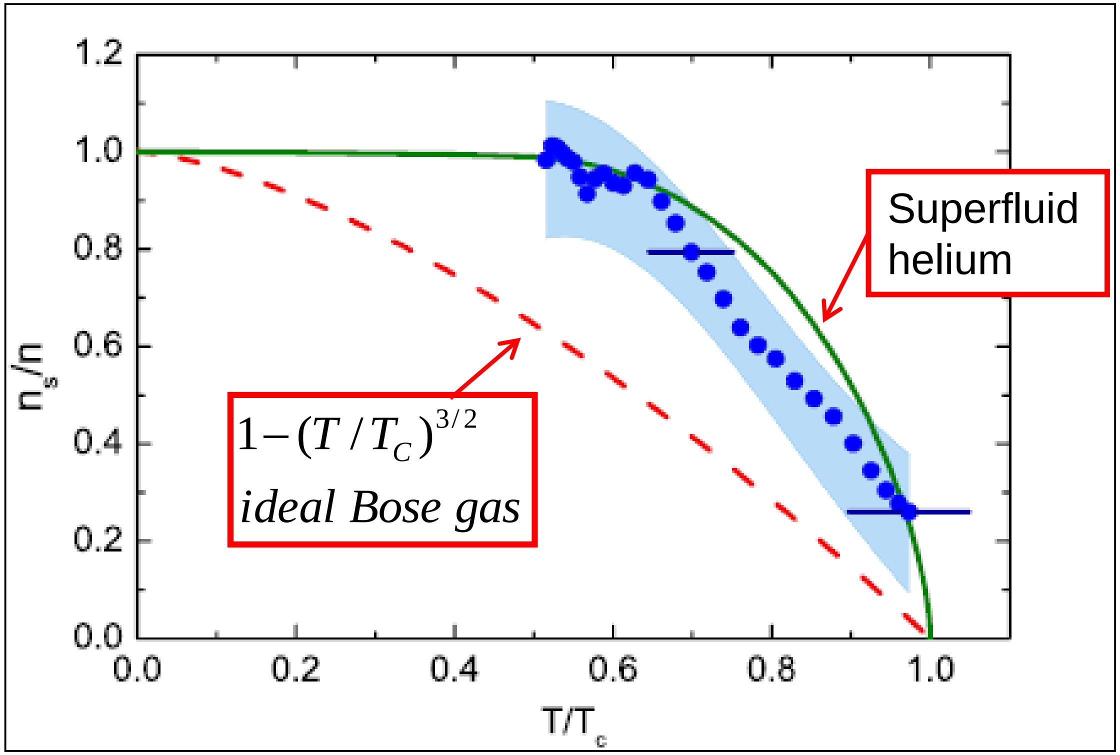
Second sound
propagates only within the region of co-existence of the super and normal fluids.

Second sound is basically an isobaric wave, but signal is visbile because of small, but **finite thermal expansion**.



From measurement of 1D second sound velocity + 3D reconstruction

one can obtain **3D superfluid fraction**



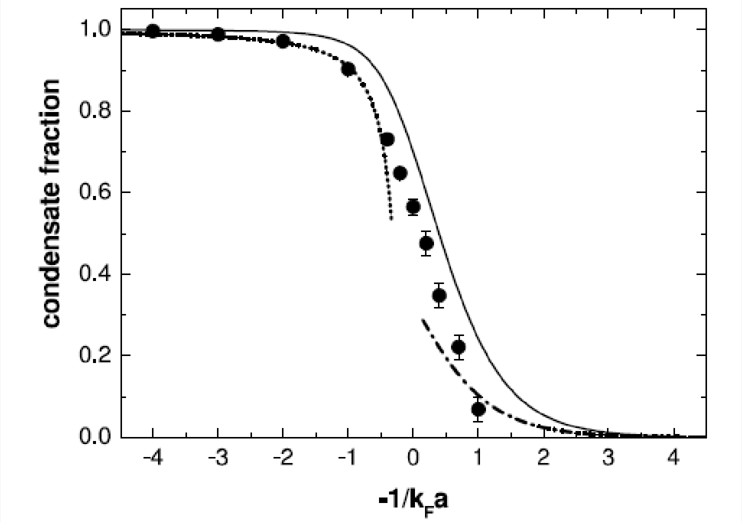
Sidorenkov et al., Nature 2013

Some comments:

- Superfluid fraction of **unitary Fermi gas** behaves similarly to **superfluid helium** (strongly interacting superfluid)

- Very **different** behavior compared to dilute **BEC gas.** New benchmark for **many-body calculations**

- Superfluid density **differs** significantly from condensate **fraction of pairs** (about 0.5 at T=0, Astrakharchik et al 2005)



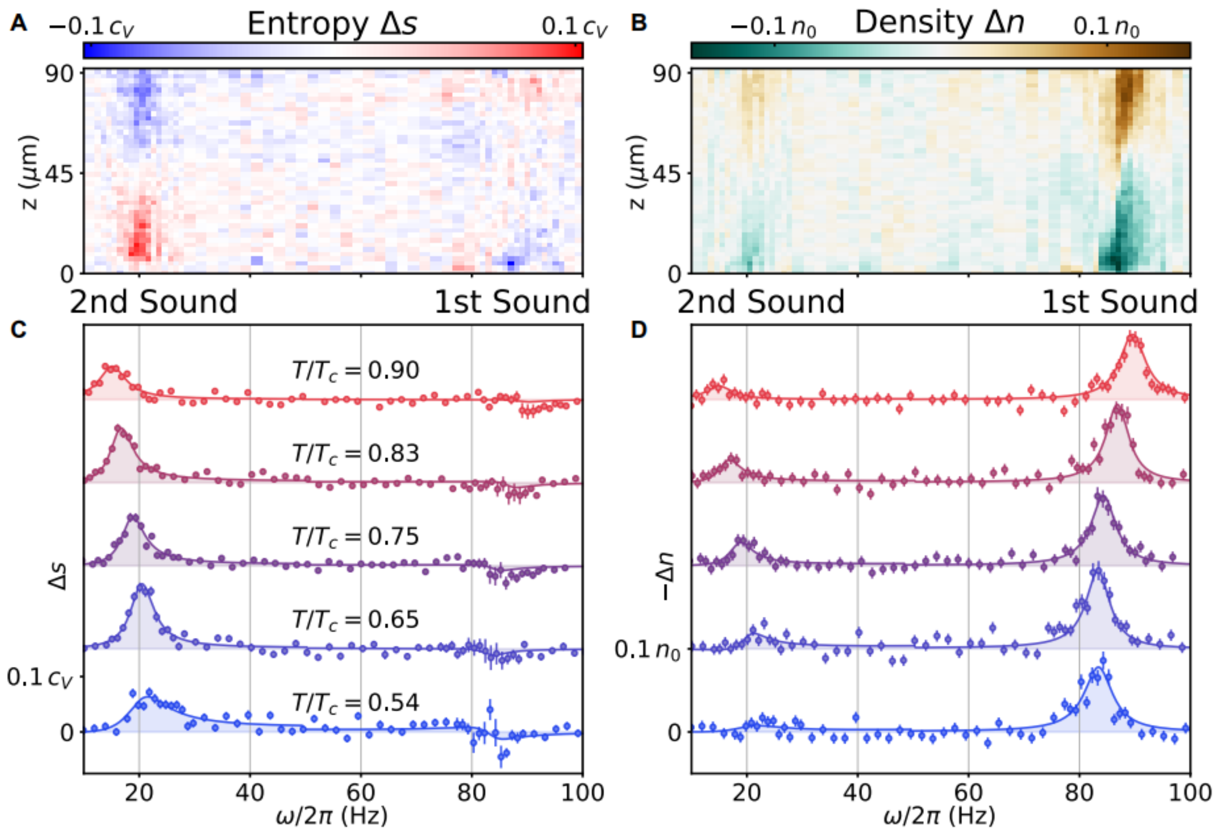
- Condensation **fraction of pairs** measurable by **fast ramping** of scattering length to BEC side (**bimoda**l distribution) (Jila 2004, Mit (2004, 2012))

More systematic experimental investigation of the propagation of first and second sound in the unitary Fermi gas recently obtained at MIT using **thermography** techniques (based on rf spectroscopy)

Thermography of two-fluid hydrodynamics in a strongly interacting Fermi gas
Zhenjie Yan, P. B. Patel, B. Mukherjee, Ch. J. Vale, R.J. Fletcher, and M.Zwierlein, Science 2024

By measuring time dependence of both local temperature and density
one obtains direct experimental evidence that
second sound is an entropy wave, to be compared with
isoentropic nature of first sound

$$\Delta s = c_V \left(\frac{\Delta T}{T} - \frac{2}{3} \frac{\Delta n}{n_0} \right).$$



Proof of entropy and density nature of second and first sound, respectively

(Zhenjie Yan et al. Science, 2024)

Can second sound propagate in a weakly interacting Bose gas ?

Weakly interacting **3D Bose gas** is highly **compressible** and behaves **differently** from **Helium** and **Unitary Fermi gas**

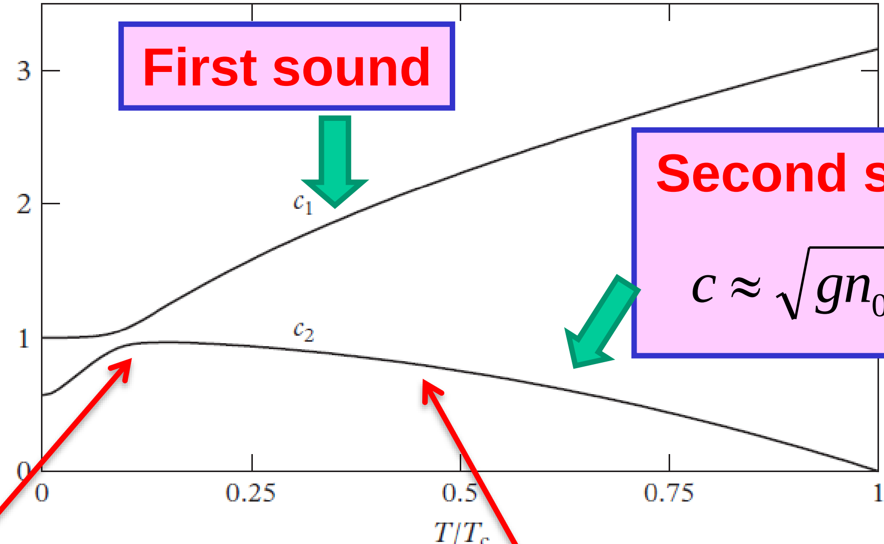
- **Superfluid density** coincides with BEC condensate except at very small T and near transition
- **First sound**: oscillation of **thermal** component
- **Second sound**: oscillation of the **condensate**

Theory:
Griffin, Nikuni, Zaremba, Pitaevskii, Stringari,….

First sound

Second sound

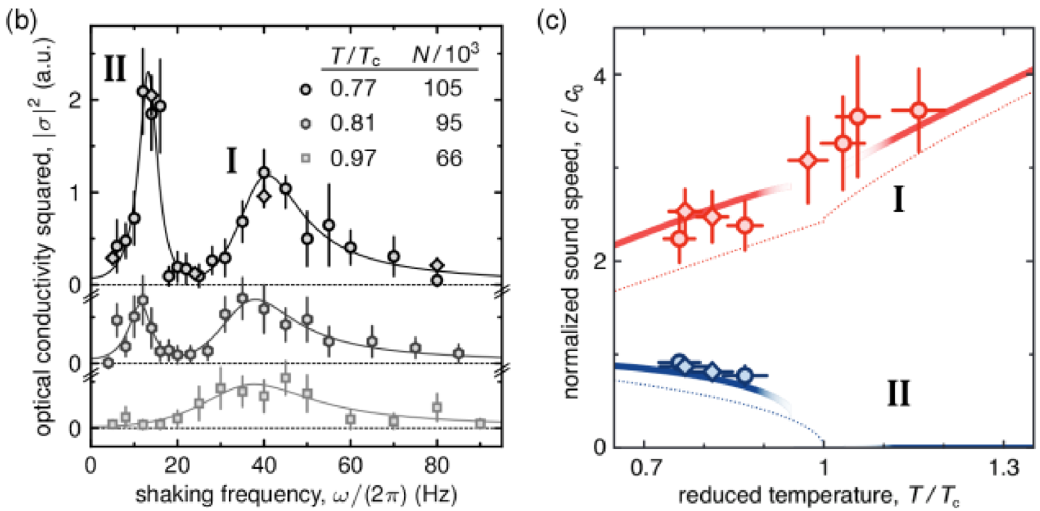
$$c \approx \sqrt{g n_0(T)/m}$$



Hybridization between the two sounds
(Lee and Yang,1959, Vernay et al. 2015)

Continuation of T=0 Bogoliubov sound
First measurement by Meppelink et al. 2009

Theoretical predictions for first and sound velocities in **3D BEC gas** confirmed in Cambridge using a 39K with **large scattering length** to ensure HD collisional regime



Hilker …Hadzibabic et al., PRL 2022

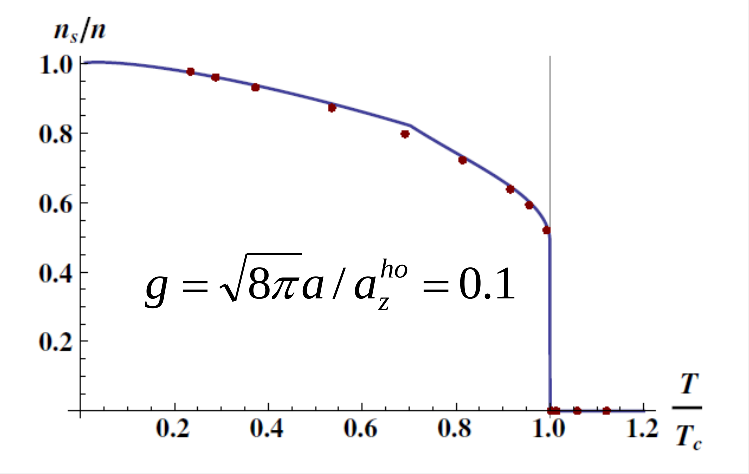
What happens to second sound in a 2D Bose gas ?

2D weakly interacting Bose gas

- **Absence of Bose-Einstein Condensation** at finite T (Hohenberg-Mermin-Wagner theorem)

- **Superfluid density** exhibits a **jump** at the Berezinskii - Kosterlitz - Thouless (BKT) transition while all thermodynamic functions are continuous (phase transition of infinite order)

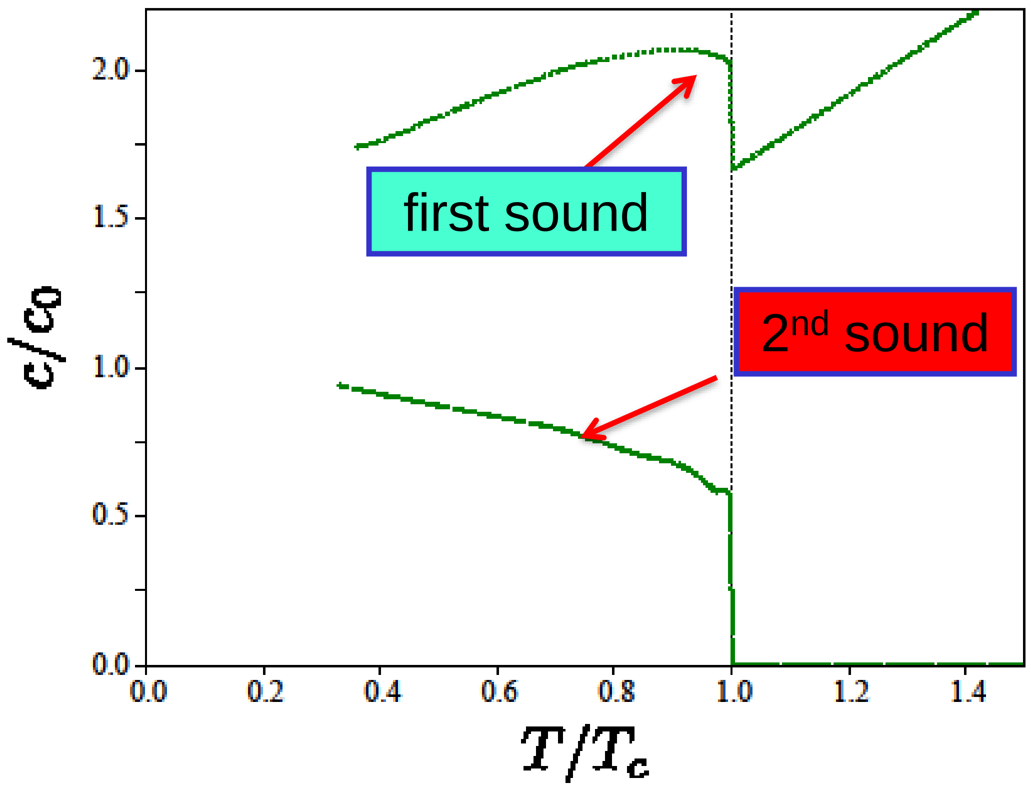
Temperature dependence of **superfluid density** (Prokofeev and Svistunov 2001)



- Nelson-Kosterlitz relationship (1977) $k_B T_C = \pi\hbar^2 n_S / 2m$ between critical temperature and superfluid density at the transition

Prediction for second sound in a 2D Bose gas

As a consequence of discontinuity of superfluid density **both first** and **second** sound in a 2D Bose gas are **discontinuous** at the BKT transition (T. Ozawa and S.S, PRL 2014)



Tomoki Ozawa

First measurement of **sound in 2D Bose gas at finite T**

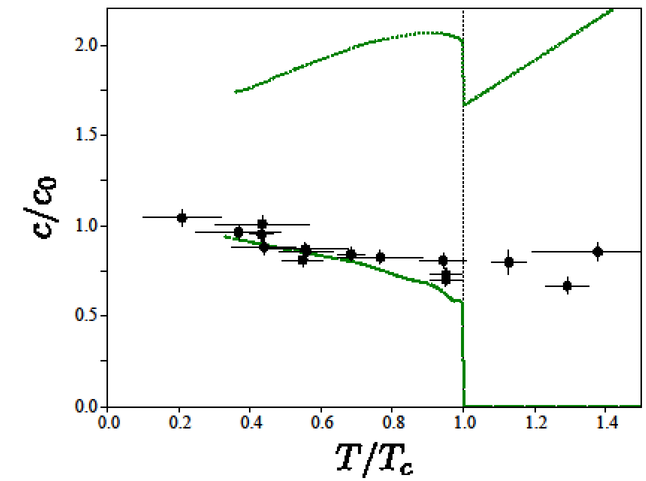
PHYSICAL REVIEW LETTERS **121**, 145301 (2018)

Featured in Physics

Sound Propagation in a Uniform Superfluid Two-Dimensional Bose Gas

J. L. Ville, R. Saint-Jalm, É. Le Cerf, M. Aidelsburger,* S. Nascimbène, J. Dalibard, and J. Beugnon†
Laboratoire Kastler Brossel, Collège de France, CNRS, ENS-PSL University, Sorbonne Université,
11 Place Marcelin Berthelot, 75005 Paris, France

- No jump at the BKT transition
- No evidence for first sound
- Strong damping at finite T



Why ? System is not in HD collisional regime
(scattering length and 2D coupling constant are too small $(\omega\tau \approx 1)$)

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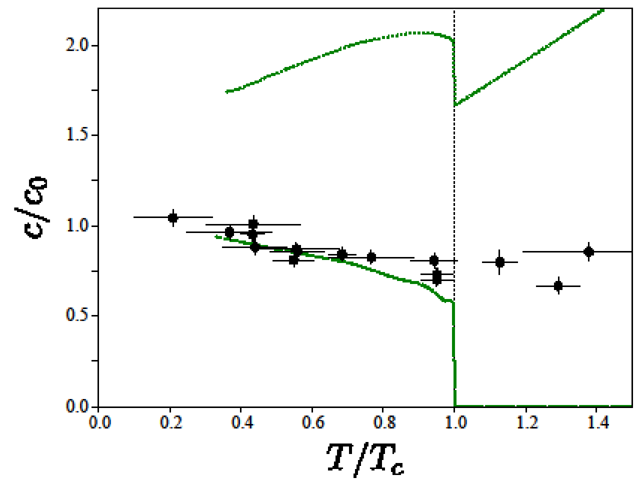

Why ? System is not in HD collisional regime
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PHYSICAL REVIEW LETTERS **121,** 145302 (2018)

Featured in Physics

Collisionless Sound in a Uniform Two-Dimensional Bose Gas

Miki Ota,[1] Fabrizio Larcher,[1,2] Franco Dalfovo,[1] Lev Pitaevskii,[1,3] Nick P. Proukakis,[2] and Sandro Stringari[1]
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Miki Ota

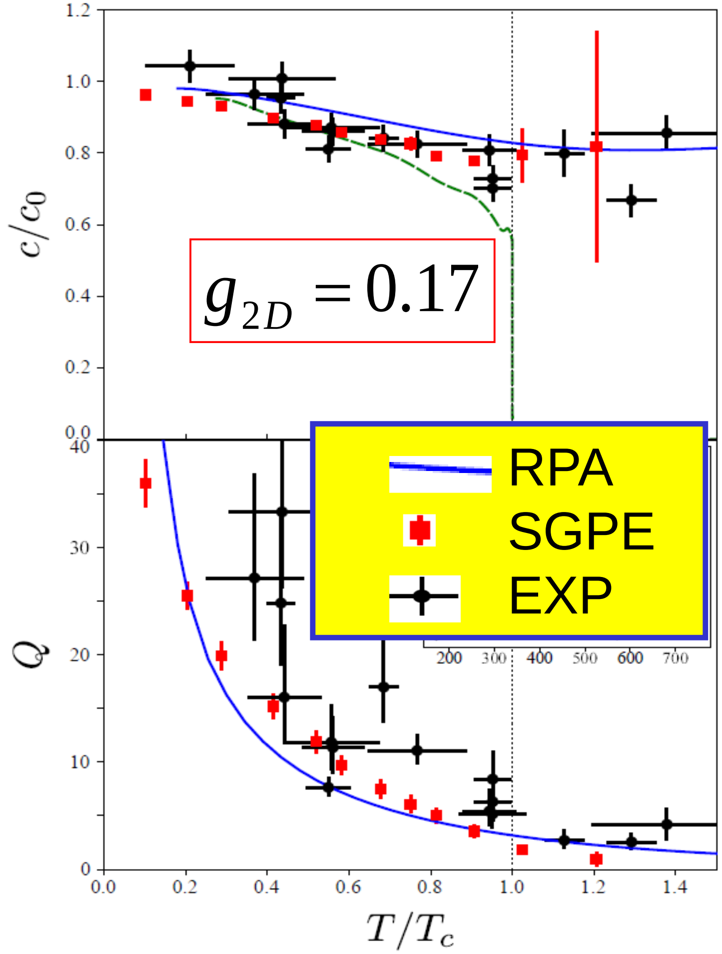
Good agreement between

RPA response function theory

$$\chi(k,\omega,T) = \frac{\chi_0(k,\omega,T)}{1 + g_{2D}\chi_0(k,\omega,T)}$$

(similar to Landau's theory of Fermi liquids) **and experiment** concerning **velocity** of collisionless sound and **Q factor**

Crucial role of **Landau damping**



Recent Observation of second sound in a 2D Bose gas

In order to observe second sound in a dilute Bose gas and the jump at the BKT transition it is crucial to **increase the role of interactions**, favoring the realization of the collisional hydrodynamic regime.

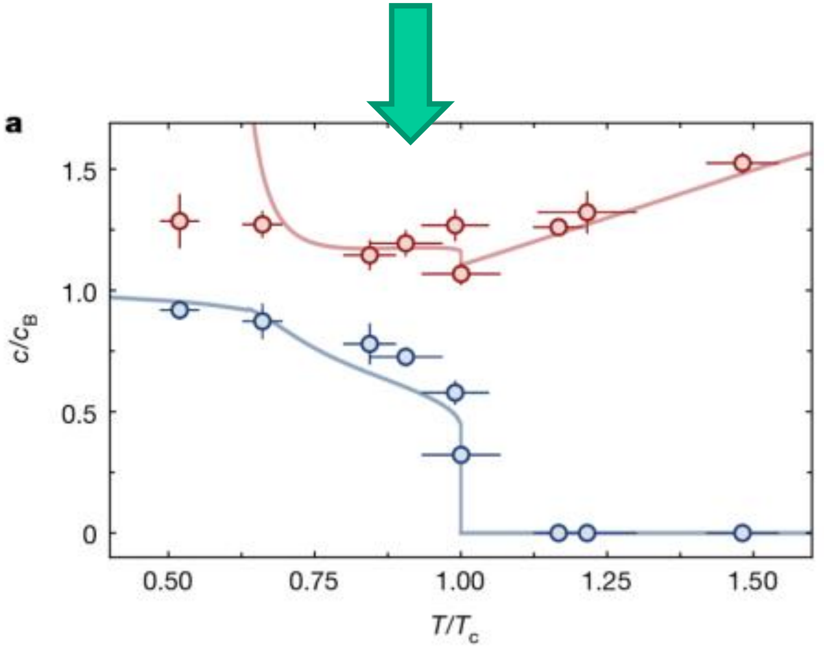
This was successfuly achieved by the **Cambridge team**
(Christodoulou et al. Nature 594, 191 (2021))
using a 39K gas with large values of scattering length

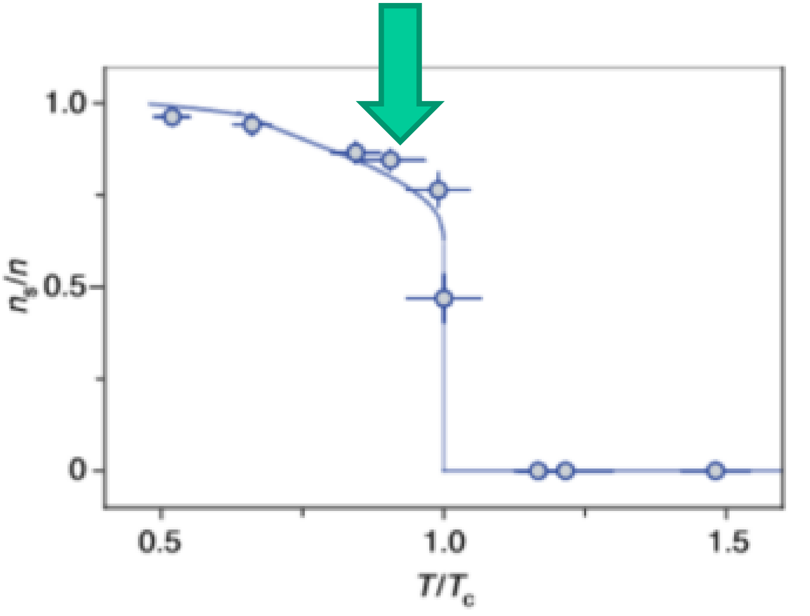
$$g = g_{2D} / (\hbar^2 / m) = \sqrt{8\pi}(a / a_z) = 0.64$$

as compared to value $g = 0.17$ of previous Paris experiment with Rb atoms

Measured values of **first** and **second** sound velocities

T-dependence of superfluid density extracted from measured sound velocities





First experimental confirmation (Christodoulou et al. Nature, 2021) of the predicted (Ozawa and S.S.,PRL 2014) **jump of second sound** velocity **at the BKT transition**

PLAN OF THE LECTURES

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Lecture 3. **Spontaneous breaking of translational invariance and supersolidity: another tale of two sounds**

- In my first Lecture I have implicitly assumed that at T=0 $\rho_s = \rho$ (superfluid density coincides with the total density) and that the sound velocity approaches the hydrodynamic value $mc^2 = \kappa^{-1}$

- This is true only if
  - system is **Galilean invariant**
  - fluid moves with **velocity smaller than Landau critical value**

$$v_{cr} = \min_{\vec{p}} \frac{\varepsilon(\vec{p})}{p}$$

- If both conditions are satisfied Landau derived famous result for the normal density in terms of thermal distribution $N_{\vec{p}}(\varepsilon) = [\exp(\varepsilon(\vec{p})/k_B T) - 1]^{-1}$ of elementary excitations

$$\rho_n = -\frac{1}{3}\int \frac{dN_{\vec{p}}(\varepsilon)}{d\varepsilon} p^2 \frac{d\vec{p}}{(2\pi\hbar)^3}$$

➡ $\rho_n \to 0 \quad (\rho_s \to \rho) \quad \text{as } T \to 0$

- If $T \to 0$ and $\rho_n \to 0$ hydrodynamic equations approach the simple form

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}(\rho v_s) = 0$$

$$m\frac{\partial}{\partial t}\vec{v}_S + \nabla\mu(n) = 0$$

yielding phononic dispersion relation $\omega^2 = c^2 q^2$ with

$$mc^2 = \kappa^{-1}$$ and $$\kappa^{-1} = \rho\frac{d\mu}{d\rho}$$

Main question addressed in second and third Lectures:

What happens to superfluid density and to sound velocity **if Galilean invariance is broken** ?

Galilean invariance can be broken

Dynamically $[H, P_x] \neq 0$ (This lecture)

or

Spontaneously $[H, P_x] = 0$ (Supersolids, Next lecture)

We prove that **if Galilean invariance is broken dynamically** superfluid fraction is reduced with respect to total density and sound velocity is modified according to hydrodynamic relation

$$mc^2 = \frac{\rho_S}{\rho}\kappa^{-1}$$

Actually in SOC BEC gases relation for sound velocity should be replaced by

$$\mathrm{m}\, c_+ c_- = \frac{\rho_s}{\rho}\kappa^{-1}$$

(consequence of parity and time reversal violation)

In this lecture I will consider two examples where Galilean invariance is dynamically broken $[H, P_x] \neq 0$

1) Bose superfluid in the presence of an external 1D periodic potential $V(x) = V_0 \cos(2\pi x / L)$ causing **density modulations**

2) BEC gas with spin orbit coupling $h_0 = \frac{1}{2}[(-i\hbar\partial_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x$ where physical momentum $P_x = -i\hbar\partial_x - k_0\sigma_z$ is the sum of canonical momentum (commuting with Hamiltonian) and spin component (**not** commuting with Raman coupling)

In both cases **superfluid density** along x-direction **is reduced**

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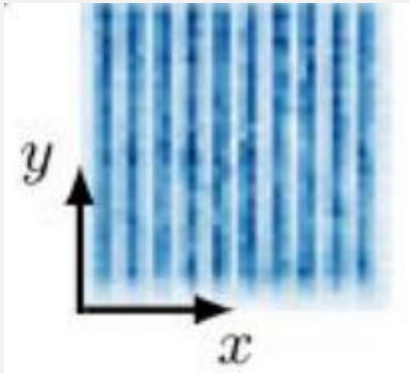
Main motivations to study periodically modulated superfluids

- Many experiments available in ultra cold atoms in the presence of **optical lattices** (e.g. Superfluid/Mott Insulator transition)
- Recent availability of **supersolid configurations** in ultracold atomic gases
- Fermi superfluidity in the **inner crust** of **neutron stars**
- Recent interest in **Leggett's bound** to superfluid fraction (relating quenching of superfluidity to density modulations)

The case of a dilute Bose-Einstein condensate confined in a box



- Application of the 1D periodic perturbation $V(x) = V_0 \cos(x 2\pi / d)$ gives rise to **stripes**

- In a dilute BEC gas, described by Gross-Pitaevskii theory one can prove that the **superfluid fraction** (along x) **coincides with Leggett's upper bound** (1970,1998)

$$f_{S,x}^{L} \equiv \frac{\rho_{S,x}}{\bar{\rho}} = \left(\frac{\bar{n}}{L} \int \frac{dx}{n(x)} \right)^{-1}$$

- On the other hand hydrodynamic theory of superfluids predicts the anisotropic result for the **sound velocities**, yielding result

$$mc_x^2 = f_{S,x} \kappa^{-1}$$

$$mc_y^2 = \kappa^{-1}$$

$$f_{S,x} = c_x^2 / c_y^2$$

for the superfluid fraction (avoiding determination of κ)

Exp/theory collaboration with Jean Dalibard's team at the Collège de France, has confirmed **consistency** of the determination of the superfluid density based on **independent** measurement of **Leggett's integral** and **sound velocities**

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Editors' Suggestion

Superfluid Fraction in an Interacting Spatially Modulated Bose-Einstein Condensate

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Santo Roccuzzo

Closely related paper

…IEW LETTERS **131,** 163401 (2023)

Featured in Physics

Observation of Anisotropic Superfluid Density in an Artificial Crystal

J. Tao,* M. Zhao,* and I. B. Spielman

Joint Quantum Institute, University of Maryland and National Institute of Standards and Technology, College Park, Maryland 20742, USA

Jean Dalibard + CdF team

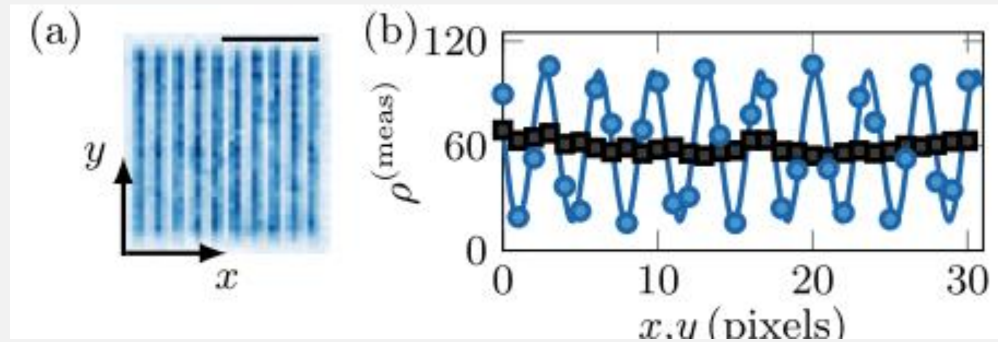
- Measurement of Leggett's integral

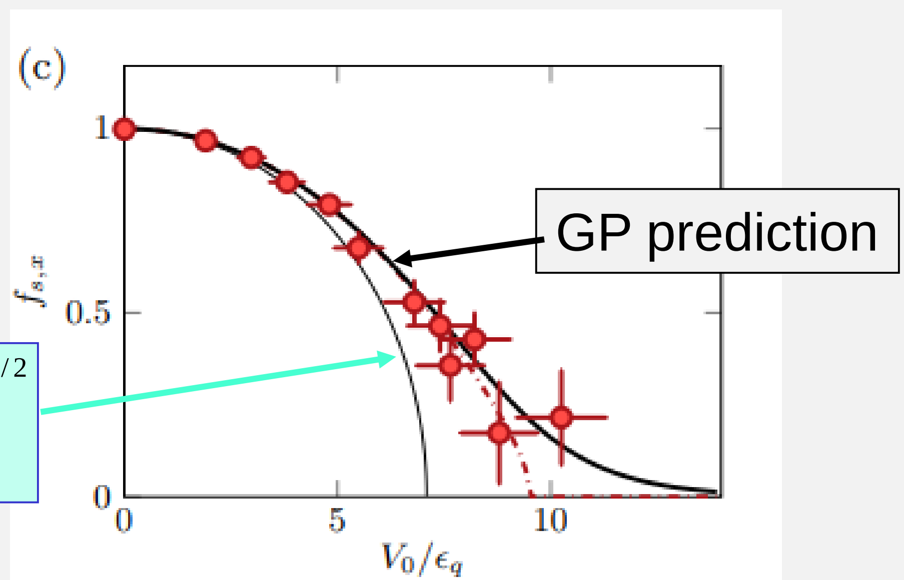
(Chauveau et al. PRL 133 (2023)

$$f_{S,x}^{L} \equiv \frac{\rho_{S,x}}{\bar{\rho}} = \left(\frac{\bar{n}}{L} \int \frac{dx}{n(x)} \right)^{-1}$$

N=10^5 atoms a box of L= 40 microns

Due to large period of density modulations (3.94 microns) in-situ density distribution is measurable, accounting for finite optical resolution



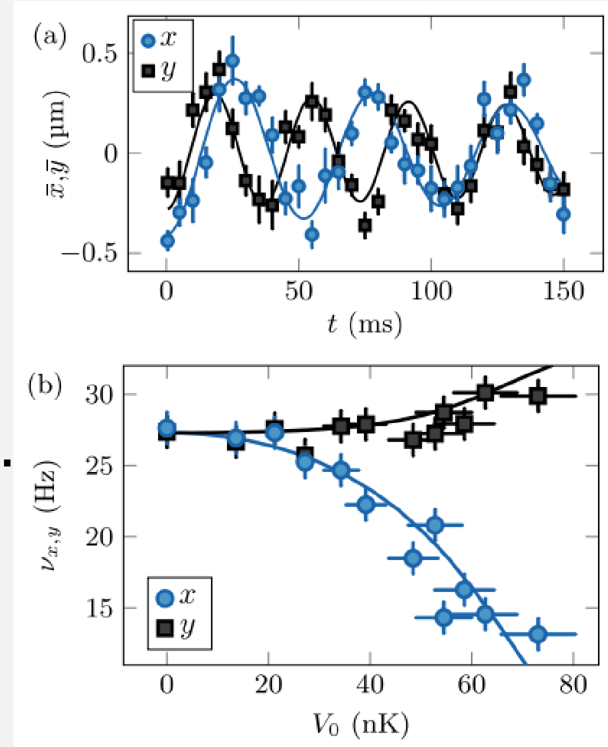


$$f_{S,x}^{LDA} = \left(1 - \frac{V_0}{\mu} \right)^{1/2}$$

- Measurement of sound velocities

(Chauveau et al. PRL (2023))

Sound is excited by suddenly removing a weak linear perturbation generated along x or y and measuring the time evolution the center of mass of the cloud.



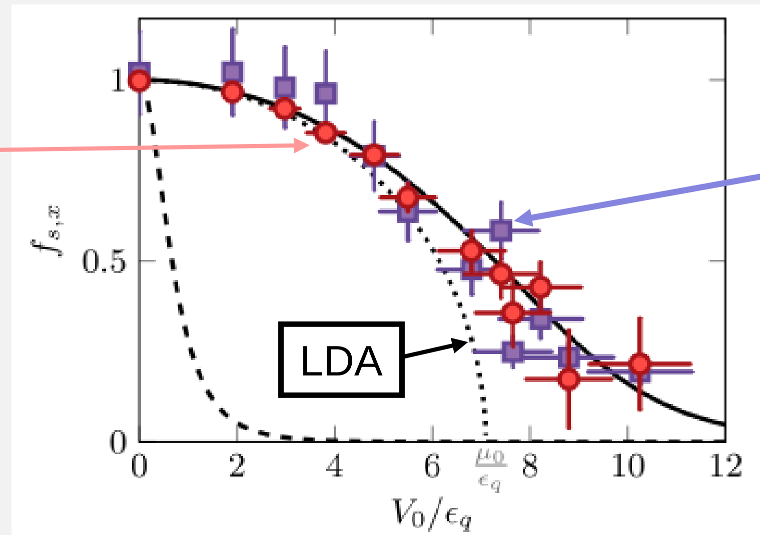
The speed of sound is determined by the HD relation $c_{x,y} = 2L\upsilon_{x,y}$

- Excellent agreement with theory predictions based on TDGP equation (full lines)

Comparison between experimental results for superfluid fraction obtained using

Leggett's approach and ratio of HD sound velocities

$$f_{S,x}^{L} = \left(\frac{\bar{n}}{L} \int \frac{dx}{n(x)} \right)^{-1}$$



$$f_{S,x} = \frac{c_x^2}{c_y^2}$$

provides a **consistent** understanding of the suppression of the superfluid fraction in the presence of a periodic potential, in agreement with the predictions of GP theory

Chauveau et al. PRL 133, 226003 (2023)

Validity of Leggett's bound $f_{S,x}^{L} = \left(\frac{\bar{n}}{L} \int \frac{dx}{n(x)} \right)^{-1}$, as a measure of superfluid fraction,

is however limited to dilute Bose gas and to factorized density profiles $n(x, y) = f(x) f(y)$

Important deviations between Leggett's bound and actual value of superfluid fraction take place

- in **Fermi superfluids** (relevant for neutron stars)

- if **density** profile is **not factorized** (e.g. triangular optical lattice, isotropic disorder)

- in systems **violating Galilean invariance** (e.g. spin-orbit coupled superfluids)

Measurement of sound velocity is better option

In this lecture I will consider two examples where Galilean invariance is dynamically broken $[H, P_x] \neq 0$

1) Bose superfluid in the presence of an external 1D periodic potential $V(x) = V_0 \cos(2\pi x / L)$ causing **density modulations**

2) BEC gas with spin orbit coupling $h_0 = \frac{1}{2}[(-i\hbar\partial_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x$ where physical momentum $P_x = -i\hbar\partial_x - k_0\sigma_z$ is the sum of canonical momentum (commuting with Hamiltonian) and spin component (**not** commuting with Raman coupling)

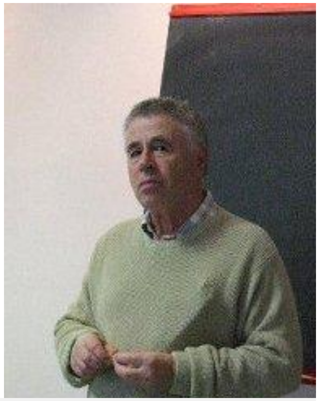
In both cases **superfluid density** along x-direction **is reduced**

Why Spin-Orbit Coupled BEC Gases?

- Give rise to artificial gauge fields opening perspectives for novel quantum effects in neutral systems

- Spin orbit coupling breaks Galilean invariance with crucial consequence on dynamic and superfluid behavior even in configurations of uniform density

- Emergence of a supersolid phase where translational invariance is broken sponateneously, with the consequent emergence of a novel class of Goldstone modes (next Lecture)



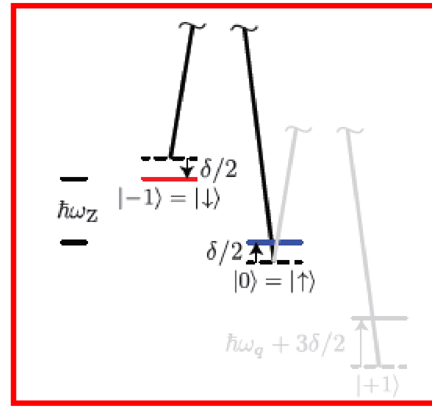
Lev Pitaevskii

Yun li

Giovanni Martone

Simplest realization of (1D) spin-orbit coupling in s=1/2 Bose-Einstein condensates (Spielman, Nist, 2009)

Two detuned and polarized laser beams + non linear Zeeman field provide Raman transitions between two spin states, giving rise to new s.p. Hamitonian



$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

$p_x = -i\hbar\partial_x$ is **canonical** momentum

k_0 is laser wave vector difference

Ω is strength of Raman coupling

$\delta = \Delta\omega_L - \omega_Z$ is effective Zeeman field

Symmetry properties of spin-orbit Hamiltonian

$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

- Hamiltonian is **translational** invariant: ⇨ uniform ground state unless crystalline order is broken spontaneously (**supersolidity**, see next lecture)

- **Violation** of **parity** and **time** reversal symmetry ⇨ breaking of symmetry $\omega(q) = \omega(-q)$ in excitation spectrum

- **Violation** of **Galilean** invariance (physical momentum $P_x = mv_x = (p_x - k_0\sigma_z)$ does not commute with the Hamiltonian): ⇨ **suppression** of **superfluidity**

Are two body **interactions** relevant ?

Crucial effects show up in

- Novel **dynamic and superfluid** features (this lecture)
- Emergence of new **supersolid** phase (next lecture)

Role of two-body interactions

Interactions in 1D SO coupled s=1/2 BECs (T=0) discussed by
Ho and Zhang (PRL 2011), Yun Li, Pitaevskii, Stringari (PRL 2012), …..

$$H = \sum_i h_0(i) + \sum_{\alpha,\beta} \frac{1}{2} \int d\vec{r}\, g_{\alpha\beta} n_\alpha n_\beta$$

$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x + \frac{1}{2}\delta\sigma_z$$

- We assume $g_{\uparrow\uparrow} g_{\downarrow\downarrow} > g_{\uparrow\downarrow}^2$ which ensures **phase mixing** in the absence of Raman coupling

- Interactions are treated within **mean field** approximation (s=1/2 coupled Gross-Pitaevskii equations)

- Setting $k_0 = 0$ (**no momentum transfer** by lasers) yields **Rabi coupled spin mixtures**

Gross Pitaevskii equations in presence of SO coupling

$$i\hbar\frac{\partial}{\partial t}\Psi_\uparrow = \left(-\frac{\hbar^2}{2m}(\nabla_x + k_0)^2 + \nabla_\perp^2] + \frac{1}{2}\delta + g_{\uparrow\uparrow}|\Psi_\uparrow|^2 + g_{\uparrow\downarrow}|\Psi_\downarrow|^2\right)\Psi_\uparrow - \frac{\hbar\Omega}{2}\Psi_\downarrow$$

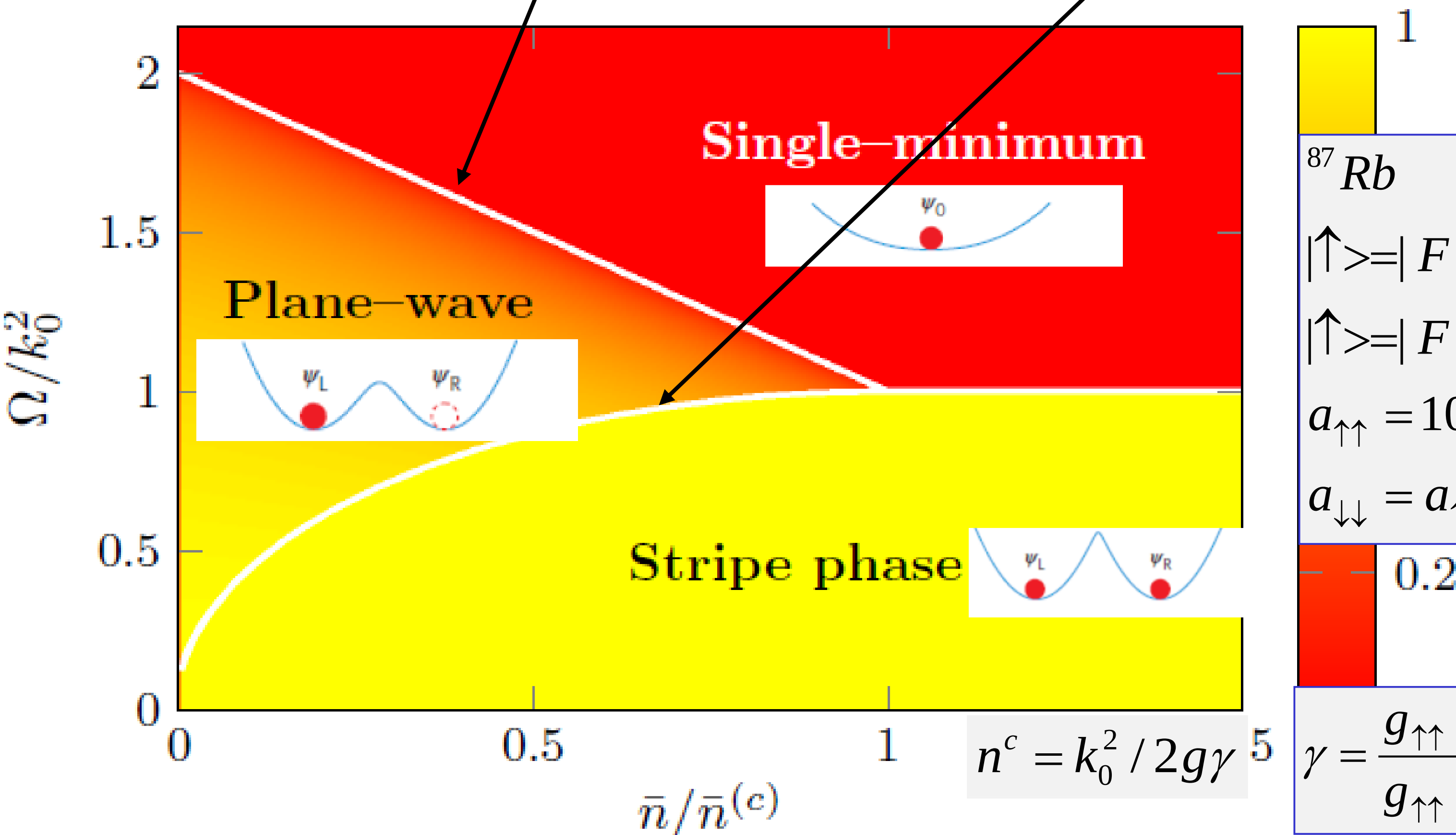
$$i\hbar\frac{\partial}{\partial t}\Psi_\downarrow = \left(-\frac{\hbar^2}{2m}(\nabla_x - k_0)^2 + \nabla_\perp^2] - \frac{1}{2}\delta + g_{\downarrow\downarrow}|\Psi_\downarrow|^2 + g_{\uparrow\downarrow}|\Psi_\downarrow|^2\right)\Psi_\downarrow - \frac{\hbar\Omega}{2}\Psi_\uparrow$$

Interplay between modified single particle Hamiltonian and two-body interactions give rise to

- Novel dynamic and superfluid properties (this lecture)
- Emergence of a novel supersolid phase (next lecture)

Quantum phase diagram predicted by SOC Hamiltonian at zero temperature

- transition between plane wave and single minimum phases is actually crossover if $g_{\uparrow\uparrow} \neq g_{\downarrow\downarrow}$
- phase transition between plane wave and stripe phase is first order and fixed by interactions



^{87}Rb

$|\uparrow\rangle = |F=1, m_F=0\rangle$

$|\uparrow\rangle = |F=1, m_F=-1\rangle$

$a_{\uparrow\uparrow} = 101.41 a_B$

$a_{\downarrow\downarrow} = a_{\uparrow\updownarrow} = 100.94 a_B$

$n^c = k_0^2 / 2g\gamma$

$$\gamma = \frac{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} - 2g_{\uparrow\downarrow}}{g_{\uparrow\uparrow} + g_{\downarrow\downarrow} + 2g_{\uparrow\downarrow}}$$

Superfluid density and propagation of sound in spin orbit coupled gases (uniform density phase)

- Leggett's bound useless in this case

- **Baym** approach to normal density well elucidates the role of the excitation spectrum of elementary excitations

- Result for normal density is consistent with prediction for superfluid density based on phase twist approach

$$\rho_n + \rho_s = \rho$$

Yi-Cai Zhang et al. PRA 2016
Hong Kong-Trento collaboration

Definition of **normal** (non superfluid) density (T=0)

(G. Baym, The microscopic description of superfluidity, 1969):

$$\frac{\rho_n}{\rho} = \lim_{q\to 0} \frac{1}{N}\sum_n \frac{|<0|J_x^T(q)|n>|^2}{E_n - E_0} + (q \to -q)$$

Macroscopic static response to transverse current

$$J_x^T(q) = \sum_k (p_{k,x} - k_0\sigma_{k,z})e^{iqy_k}$$

- When $q \to 0$ the current operator approaches total momentum

$$J_x^T \to P_x \equiv \sum (p_{k,x} - k_0\sigma_{k,z})$$

- Non commutativity of P_x with H (violation of **Galilean** invariance) is consequence of **spin term** ⇨ $\rho_n \neq 0$ even at $T = 0$

- Effect is compatible with translational invariance (canonical momentum commutes with Hamiltonian)

- Effect is absent along y direction (tensor nature of superfluidity)

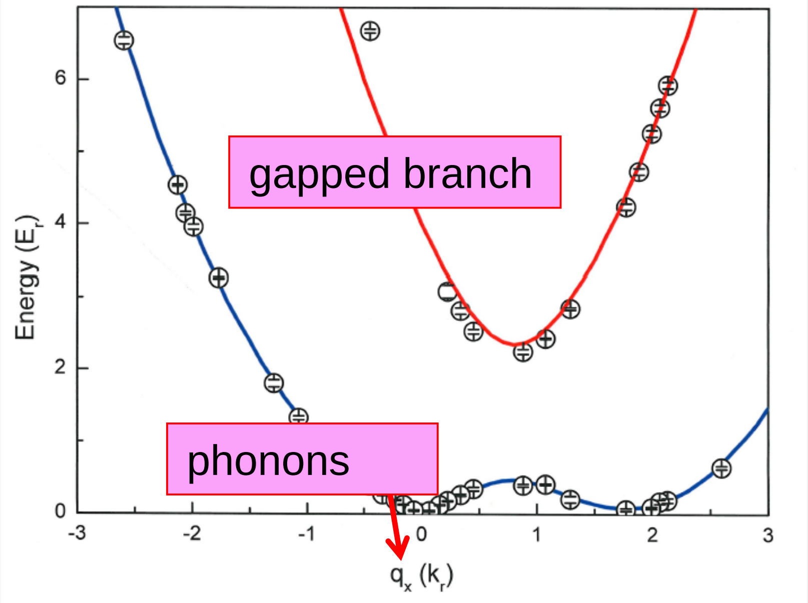
To calculate normal density

$$\frac{\rho_n}{\rho} = \lim_{q\to 0} \frac{1}{N} \sum_n \frac{|<0|J_x^T(q)|n>|^2}{E_n - E_0} + (q \to -q)$$

one needs knowledge of spectrum of elementary excitations

Two branches in the excitation spectrum of spinor BEC's

- Due to Raman coupling **only one branch is gapless** in PW and SM phases (one Goldstone mode)
- phonon behavior at small q



Exp: Si-Cong Ji et al., PRL 2015;
Khamehchi et al, PRA 2014
Theory: Martone et al., PRA 2012

Using rigorous sum rule arguments it is possible to show that phonon dispersion relation is fixed by the law

$$\omega_{\pm} = q_{\pm} c_{\pm} \quad \text{with} \quad c_{+} c_{-} = \frac{\rho_s}{\rho} \kappa^{-1}$$

(Yi-Cai Zhang et al. PRA 2016)

with $c_{\pm}$ sound velocity propagating parlallel (antiparallel) to x-direction and superfluid density given by ($g_{\uparrow\uparrow} = g_{\downarrow\downarrow} \equiv g \neq g_{\uparrow\downarrow}$)

Plane Wave Phase

$$\Omega \leq \Omega_c$$

$$\frac{\rho_s}{\rho} = \frac{\Omega_c(\Omega_c^2 - \Omega^2)}{\Omega_c^3 + 2g_{ss} n \Omega^2}$$

$$g_{ss} = (g - g_{\uparrow\downarrow})/2$$

Single Minimum Phase

$$\Omega \geq \Omega_c$$

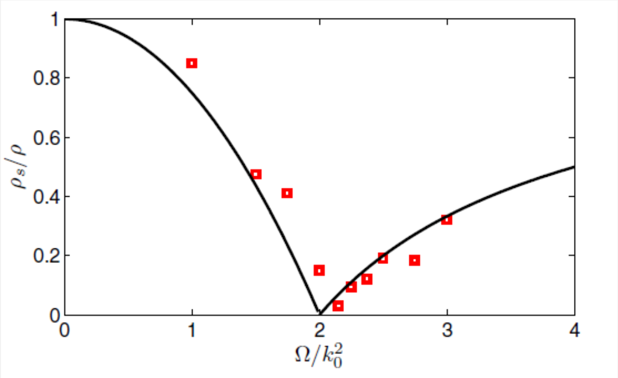
$$\frac{\rho_s}{\rho} = \frac{\Omega - \Omega_c}{\Omega + 2g_{ss} n}$$

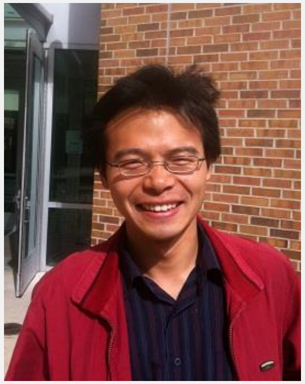
where $\hbar\Omega_c = 4E_R - 2g_{ss}n$ is value of Raman coupling at the transition

Strong reduction of the superfluid fraction at zero temperature despite the absence of the density modulations

(dramatic consequence of the breaking of Galilean invariance)

The experimental results (Si-Cong Ji et al, PRL2015) for the sound velocities c_+ and c_- along the x direction in ^{87}Rb (in addition to the knowledge of the compressibility) can be used to provide the value of the superfluid density as a function of Raman coupling, in good agreement with theory prediction (Yi-Cai Zhang et al. PRA2016, Hong Kong-Trento collaboration)





Shizhong Zhang

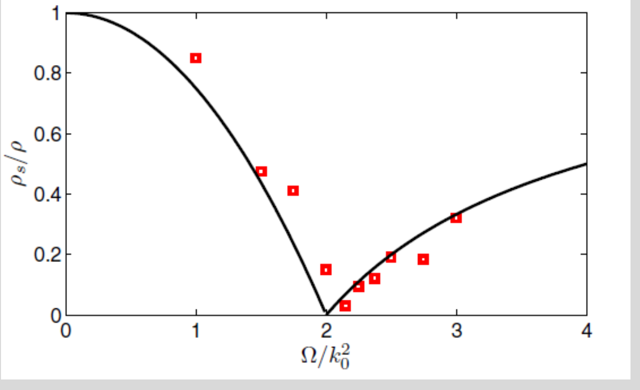
The additional measurement of the sound velocity along the transverse direction, would permit to extract the value of the superfluid fraction avoiding the determination of the compressibility

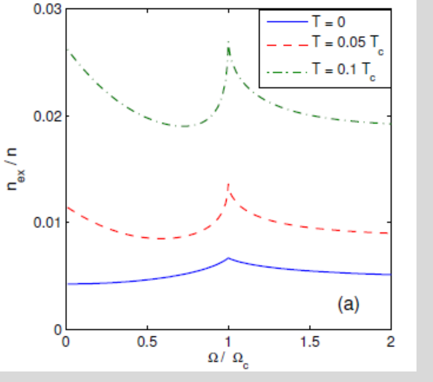
$$\frac{\rho_s}{\rho} = \frac{c_x^+ c_x^-}{c_y^2}$$

Approach applicable **also to spin asymmetic** configurations ^{39}K

Superfluid density vs Bose-Einstein condensation

- **Superfluid density** strongly **suppressed** near the phase transition between the plane wave and zero-momentum phase
- **BEC fraction** is instead practically unperturbed (**quantum depletion** always remains very small, less than 1%)





Superfluid density

(Yi-Cai Zhang et al., PRA 2016)

Quantum depletion

(W. Zheng et al. JPhysB 2013)

PLAN OF THE LECTURES

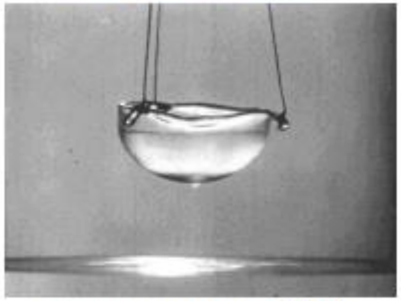
Lecture 1. **Superfluids at finite tempertaure: a tale of two sounds**

Lecture 2. **Dynamical breaking of Galilean invariance and propagation of sound at T=0**

Lecture 3. **Spontaneous breaking of translational invariance and supersolidity: another tale of two sounds**

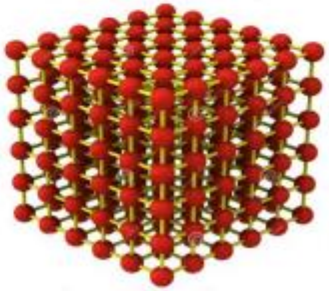
What is a supersolid?

SUPERFLUID



Spontaneous U(1) symmetry breaking

SOLID



Spontaneous translational symmetry breaking

SUPERSOLID

Spontaneous and simultaneous breaking of both symmetries

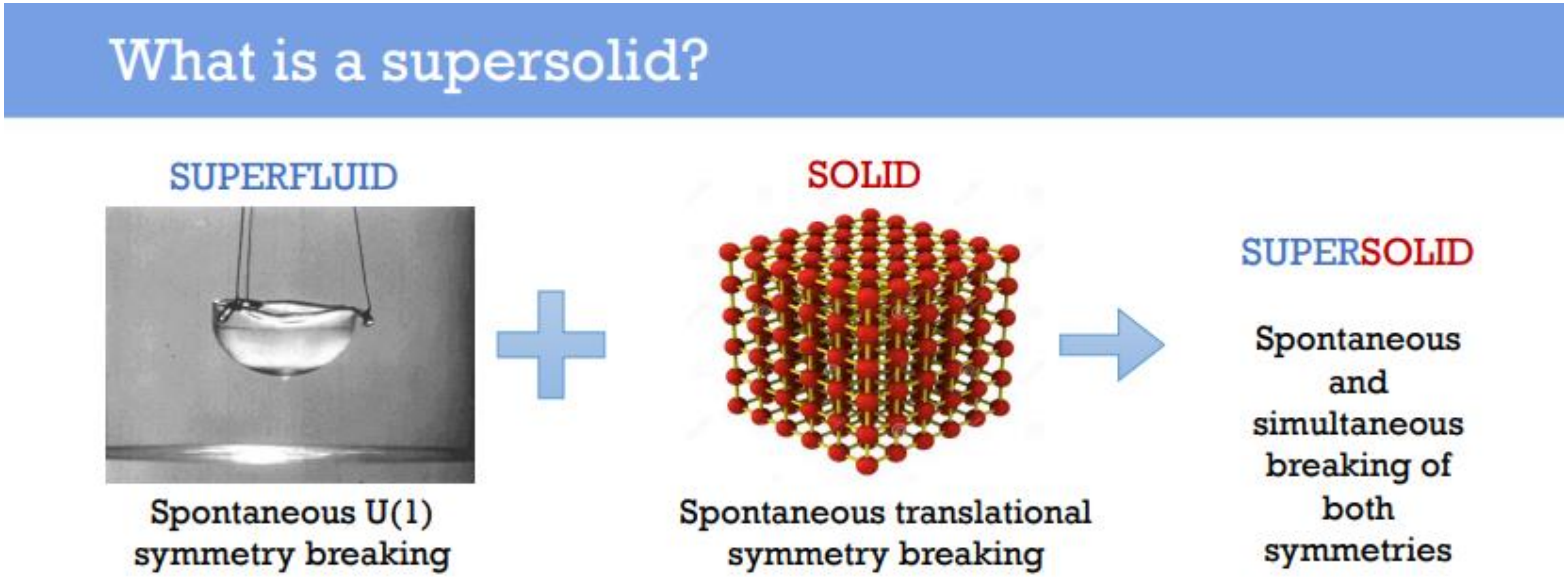
Can a solid be superfluid ?

(Leggett 1970)

Unsuccessful experiments in solid helium

Kim and Chan (Nature, 2004) 

Kim and Chan (PRL 2012) 



Can a gas behave like a crystal?

Recent experimental realization of supersolidity in ultracold atomic gases 

Ultra-cold atomic gases have recently become successful platforms for supersolidity

- **Bec in optical resonators** (ETH 2017)
- **Spin-orbit coupled BEC's** (MIT 2017)
- **Dipolar gases** (Florence/Pisa, Stuggart, Innsbruck, 2019)
- **Polariton condensates** (Lecce, 2024)

Key signatures associated with supersolidity:

- **Spontaneous density modulations**
- **Phase coherence**
- **Superfluid rotational effects**
- **Novel Goldstone modes (new sound waves)**

Recent overview papers on supersolids (2023)

IOP Publishing Reports on Progress in Physics
Rep. Prog. Phys. **86** (2023) 026401 (90pp) https://doi.org/10.1088/1361-6633/aca814

Review

Dipolar physics: a review of experiments with magnetic quantum gases

Lauriane Chomaz[1,2,*], Igor Ferrier-Barbut[3,4], Francesca Ferlaino[1,5], Bruno Laburthe-Tolra[6,7], Benjamin L Lev[8] and Tilman Pfau[3]

nature reviews physics https://doi.org/10.1038/s42254-023-00648-2

Perspective Check for updates

Supersolidity in ultracold dipolar gases

Alessio Recati & Sandro Stringari

epl A LETTERS JOURNAL EXPLORING THE FRONTIERS OF PHYSICS EPS Società Italiana di Fisica edp

PERSPECTIVE • OPEN ACCESS

Bose-Einstein condensates with Raman-induced spin-orbit coupling: An overview

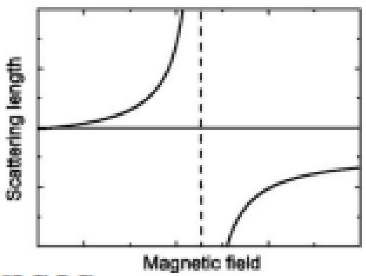
To cite this article: Giovanni Italo Martone 2023 *EPL* **143** 25001

- Sound in supersolid dipolar gases

Interaction in ultracold dipolar gases

Contact interaction

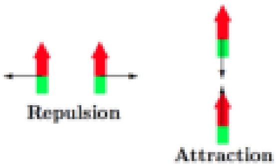
$$V_c(\boldsymbol{r}) = g\,\delta(\boldsymbol{r}), \quad g = \frac{4\pi\hbar^2 \boldsymbol{a}}{m}$$



Tunable via
Feshbach resonances

Dipole-dipole interaction

$$V_{\rm dd}(\boldsymbol{r}) = \frac{3\hbar^2 a_{\rm dd}}{m}\,\frac{1-3\cos^2\theta}{r^3}, \quad \boldsymbol{a}_{\rm dd} = \frac{\mu_0\mu^2 m}{12\pi\hbar^2}$$



- Long-ranged
- Anisotropic
- Need for confinement along *z*-direction

Interaction in ultracold dipolar gases

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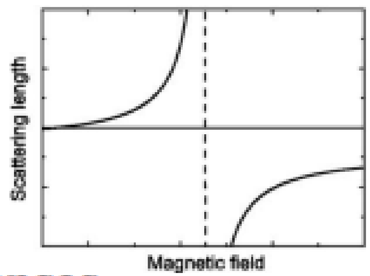

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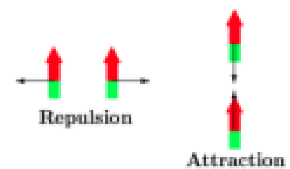
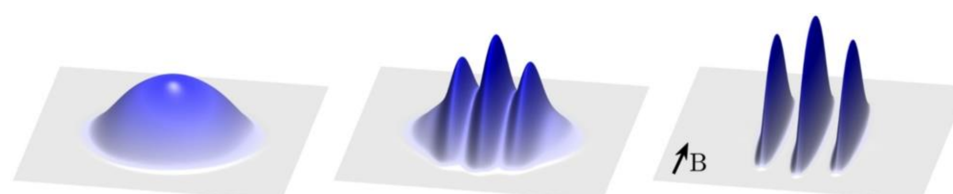


- Long-ranged
- Anisotropic
- Need for confinement along *z*-direction

Relevant dimensionless parameter $\varepsilon_{dd} \equiv a_{dd} / a$
provides relative weight of dipolar vs short range force.
It drives the transition from the superfluid to the new phases exhibited by dipolar gases



superfluid *supersolid* *crystal*

Mean field collapse and beyond mean field effects

Using the contact + dipole-dipole interaction in the **mean field** approach yields collapse for large values of $\varepsilon_{dd} \equiv a_{dd} / a$ due to the negative component in the dipole force

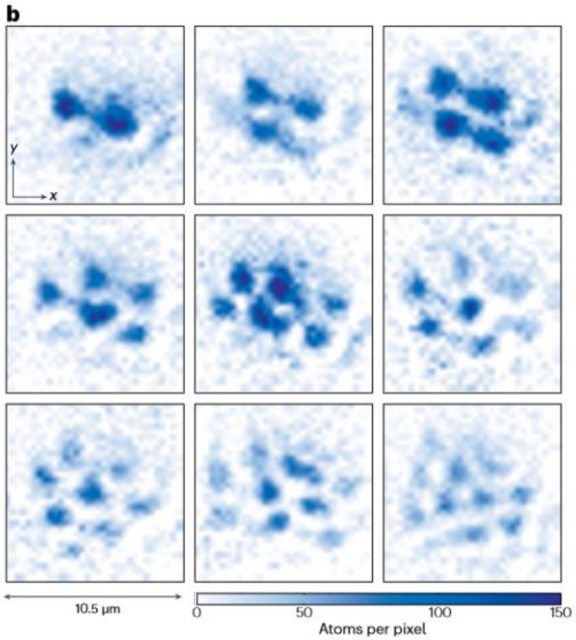
Collapse can be avoided including quantum fluctuation effects (accounting for the Lee-Huang-Yang correction to the equation of state) which provide a stabilizing positve term in the extended Gross Pitaeveskii equation (Lima and Pelster 2012)

$$i\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \Big[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{ho}}(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2 + \int d\mathbf{r}' V_{dd}(\mathbf{r}-\mathbf{r}')|\Psi(\mathbf{r}',t)|^2 + \gamma(\varepsilon_{dd})|\Psi(\mathbf{r},t)|^3\Big]\Psi(\mathbf{r},t),$$

A spectacular consequence of beyond mean effects was the possibility of realizing self bound droplets (small pieces of a quantum liquid)

Experimental realization of self-bound droplets

By tuning the dimensionless ratio $\varepsilon_{dd} \equiv a_{dd} / a$ to large values the Stuttgart team (Ferrier-Barbut et al PRL 2016) was able to generate experimentally a configuration of incoherent crystal of **self bound** droplets of interacting dipolar atoms



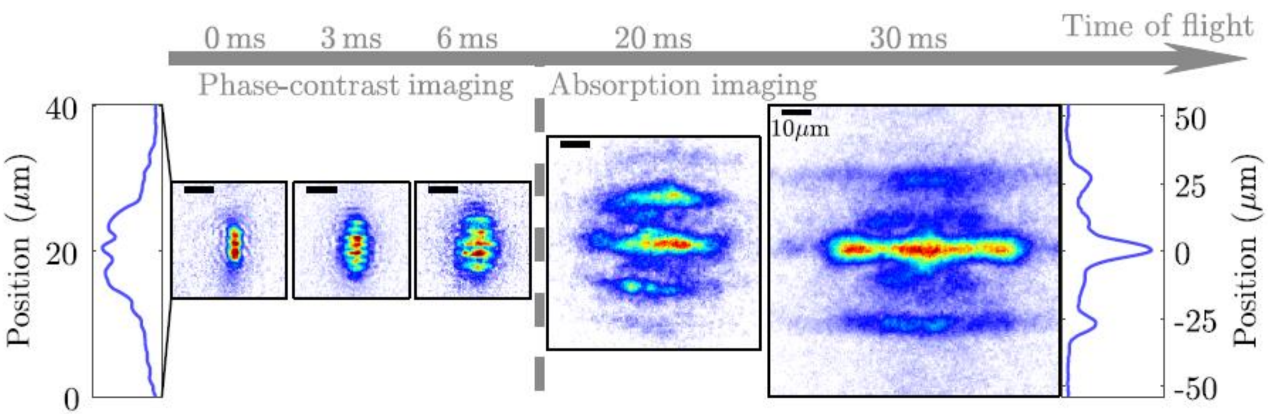
[self bound droplets were also as predicted (Petrov 2015) and later observed (Cabrera et al., Science 2018, Semeghini et al, PRL 2018) in BEC mixtures interacting with negative scattering length]

Differently from solid helium the **single sites of the crystal** are not single atoms, but droplets **containing a large number of atoms.**

These droplets are not however coherently coupled and hence the configuration **does not** correspond to a **supersolid**.

In 2019 **three experimental teams** (Pisa-Florence, Stuggart, Innsbruck) reported **evidence for supersolidity**, confirming **phase coherence of droplets** in interference experiments.

Tanzi, … Modugno, PRL 2019
Bottcher, … Pfau, PRX 2019
Chomaz, … Ferlaino PRX 2019



The last years have been characterized by extensive experimental and theoretical efforts
to explore the **superfluid** features of a supersolid dipolar gas

Focus has mainly concerned:

- Nature of **Goldstone** modes and role of superfluidity
- Realization of **Quantized vortices**

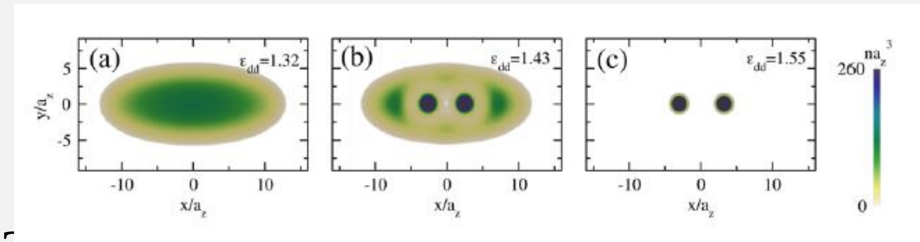
Goldstone modes in a supersolid

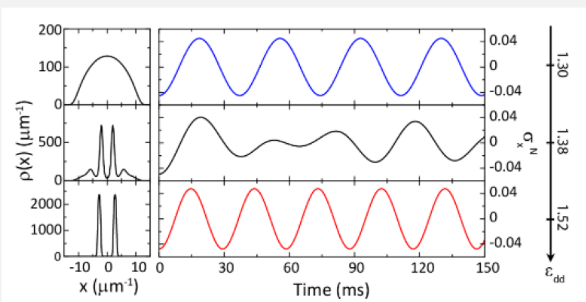
- In uniform matter spontaneous breaking of both phase and translational invariance) gives rise to **two Goldstone modes** resulting in the propagation of two different gapless phonons

- The nature of the two Goldstone modes is expected to be a combination of superfluid behavior (corresponding to flow of atoms between different clusters) without change of relative distance - Josephson like oscillation) and crystal behavior (corresponding to propagating oscillation of the relative distance between nearby clusters).

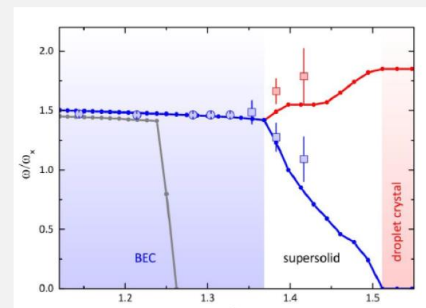
Measurement of the Goldstone modes has been already the object of experimental papers in a **supersolid dipolar gas** confined in harmonic trap (axial breathing modes)

- In elongated harmonic trap **supersolidity** is expected to cause **bifurcation** of the axial compression mode at the superfluid-supersolid transition.





Tanzi et al.
(Nature 2019)
Pisa-Florence-Trento



Lowest mode (**blu**) corresponds to a density oscillations, the position of peaks remaining unchanged (**superfluid** oscillation). Highest mode (**red**) corresponds to oscillation of relative distance between peaks (**crystal** oscillation). Pisa experiment confirms theory predictions

Similar experiments on Goldstone modes carried out in Stuttgart and Innsbruck

- Guo, … Pfau, Nature 2019
- Natale, … Ferlaino et al. PRL 2019

NEWS AND VIEWS (NATURE)
16 October 2019

Sounds of a supersolid detected in dipolar atomic gases for the first time

Ultracold gases of dipolar atoms can exhibit fluid and crystalline oscillations at the same time, illuminating the ways in which different kinds of sound propagate in the quantum state of matter known as a supersolid.

The explicit connection between the propagation of **sound** and the **superfluid fraction** is however **largely unexplored**

In Lecture 2 we have shown that the hydrodynamic relation

$$mc^2 = \frac{\rho_s}{\rho}\kappa^{-1}$$

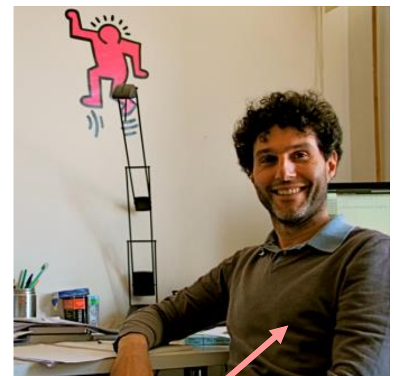
between the superfluid density and the sound velocity holds if translational (or Galilean) invariance is broken dynamically (only one Goldstone mode). The relation can be used to determine the superfluid density from the measurement of sound (at T=0).

The **relation cannot hold in a supersolid** because of the presence of additional **spontaneous breaking of translational symmetry**. In this case Goldstone theorem predicts **two gapless sounds**

Question:

Can we measure the superfluid fraction of a T=0 supersolid through the measurement of the two sound velocities ?

In a recent paper we have addressed the question of the link between the propagation of sound and the superfluid fraction in the case of a **dipolar supersolid** gas confined in a **ring geometry**



PHYSICAL REVIEW LETTERS **132,** 146001 (2024)

Sound, Superfluidity, and Layer Compressibility in a Ring Dipolar Supersolid

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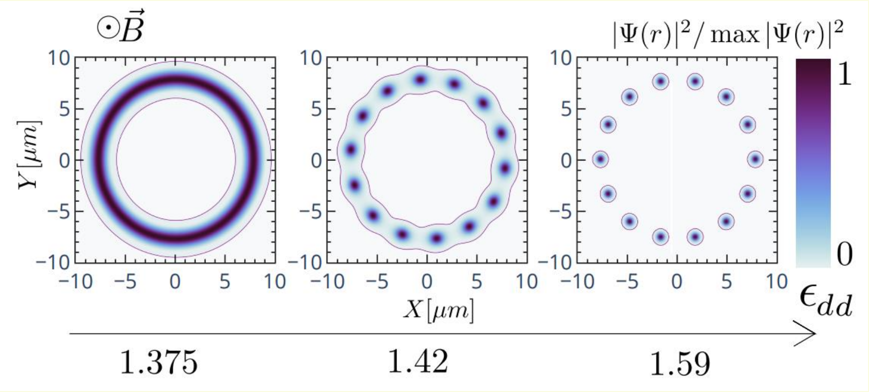
Ring geometry provides a promising configuration where the thermodynamic limit of a (1D) uniform confinement can be hopefully realized in dipolar gases, avoiding the tendency of droplets to accumulate near the walls of a box potential.
It can be naturally emplyed to host **permanent currents**

Atomic densities in the ring configuration

for different values of the relevant interaction parameter

$$\varepsilon_{dd} = a_{dd} / a$$

where $a_{dd} = \mu_0 \mu^2 / 12\pi\hbar^2$ is the dipolar length, and a is the s-wave scattering length.



For small values of ε_{dd} the system is in the uniform superfluid phase. By increasing ε_{dd} one enters the supersolid phase where droplets are formed over a sea of a superfluid gas. For even larger values of ε_{dd} one enters the crystal phase of well separated droplets

Protocol for exciting the Goldstone modes

By suddenly releasing a small periodic perturbation of the form

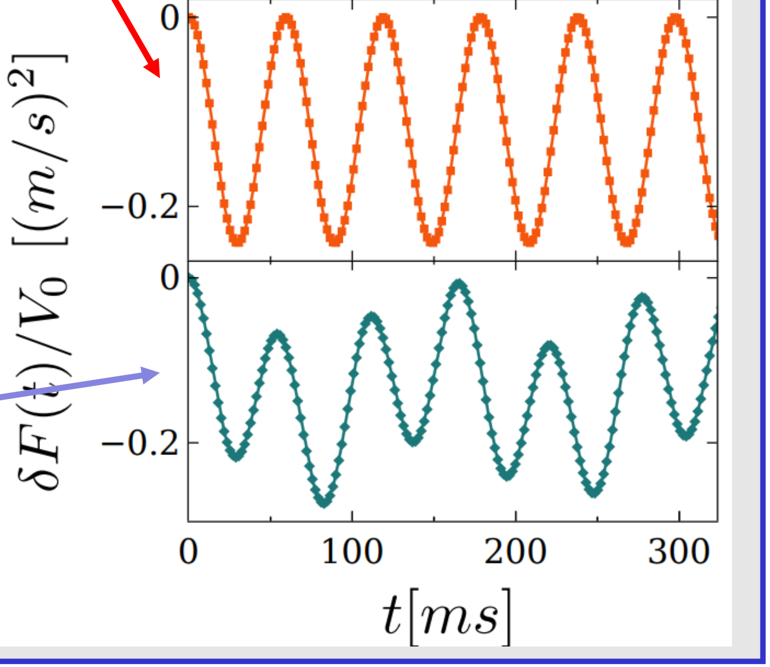
$$V(\varphi) = V_0 \cos(\varphi)$$

one explores the resulting time dependent oscillations of the quantity $F(t) = \langle \cos(\varphi) \rangle (t)$, obtained solving the extended GP eq.

In the superfluid phase one observes a single frequency of the excitation $\omega(k)$ spectrum of the elongated superfluid configuration, corresponding to

$$k = 2\pi / L$$

In the supersolid phase one instead observes a beating of two frequencies, corresponding to the excitation of the two Goldstone modes.



The observed signal has the form

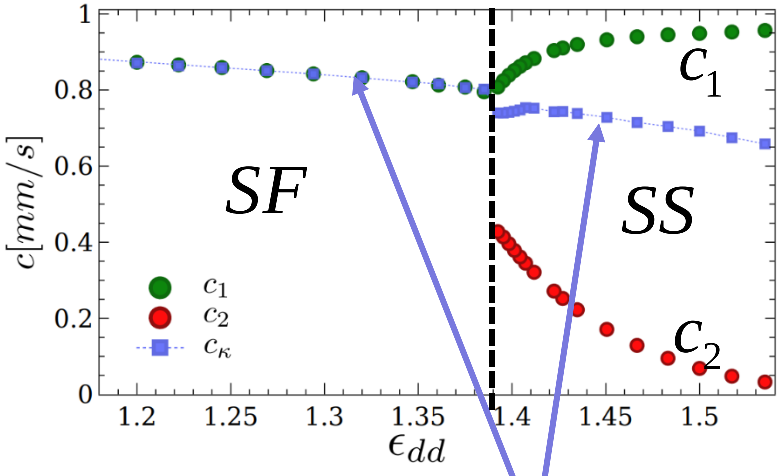
$$\delta F(t) = V_0[\chi_1(q)\cos(\omega_1(q)t) + \chi_2(q)\cos(\omega_2(q)t)]$$

with $\omega_1(q)$ and $\omega_2(q)$ approaching, for small q (and hence large L), the linear phonon dispersion

$$\omega_1(\mathrm{q}) = \mathrm{c}_1\mathrm{q} \text{ and } \omega_2(q) = c_2 q$$

with c_1 and c_2 the **first** and **second** sound velocities.

Results for the sound velocities as a function of ε_{dd} in a ring trap



The usual hydrodynamic result $c_{\kappa} = \sqrt{1/m\kappa}$ is consistent with the observed velocity **only** in the superfluid (SF) phase

Similar results obtained in the case of an infinite tube potential
Platt et al. Phys. Rev. A **110**, 023320 (2024)

Using hydrodynamic theory of 1D supersolids

[(Andreev and Lifschtz 1969) Josserand, Pomeau and Rica 2008, Yoo and Dorsey (2010), Hofmann and Zwerger (2021)]

one can relate sound velocities of the two sounds in terms of compressibility κ, layer compressibility modulus β and superfluid fraction $f_S = \rho_S / \rho$ (strain density coupling ignored)

$$c_{1,2}^2 = \frac{c_\kappa^2}{2}\left(1 + \beta\kappa \pm \sqrt{(1+\beta\kappa)^2 - 4 f_S \beta\kappa}\right)$$

with $c_\kappa^2 = \kappa^{-1} / m$

One can invert 1D supersolid HD result for the two sound velocities

$$c_{1,2}^2 = \frac{c_\kappa^2}{2}\left(1+\beta\kappa \pm \sqrt{(1+\beta\kappa)^2 - 4 f_S \beta\kappa}\right)$$

and express the superfluid fraction $f_S = \rho_S / \rho$ in terms of the two sound velocities !!

$$c_\kappa^2 f_S = \frac{c_1^2 c_2^2}{c_1^2 + c_2^2 - c_\kappa^2}$$

$$c_\kappa^2 = \kappa^{-1} / m$$

to be compared with HD relationship

$$c_\kappa^2 f_S = c^2$$

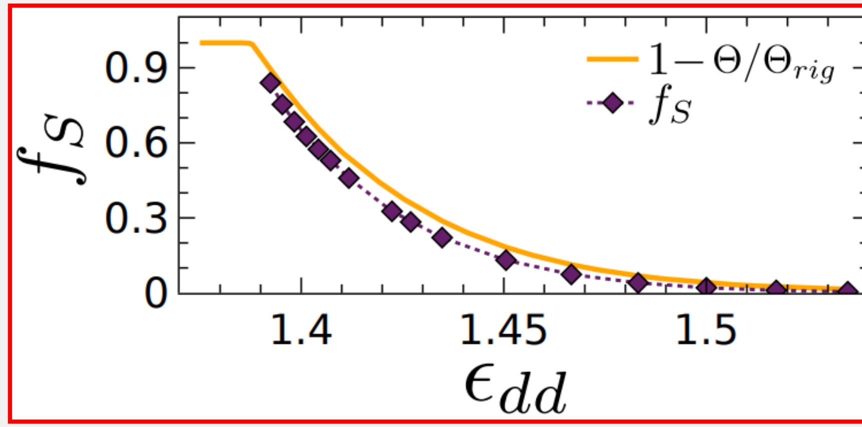
holding in the presence of a single Goldstone mode (see Lecture 2)

Using our Gross-Pitaevskii results for the sound velocities c_1, c_2 and $c_\kappa = \sqrt{1/m\kappa}$

we can extract the value of **the superfluid fraction** in the supersolid phase.

$$c_\kappa^2 f_S = \frac{c_1^2 c_2^2}{c_1^2 + c_2^2 - c_\kappa^2}$$

f_S decreases as one increases the value of ε_{dd} approaching the transition to the crystal phase of independent droples, while it increases to unity at the transition to the superfluid phase.

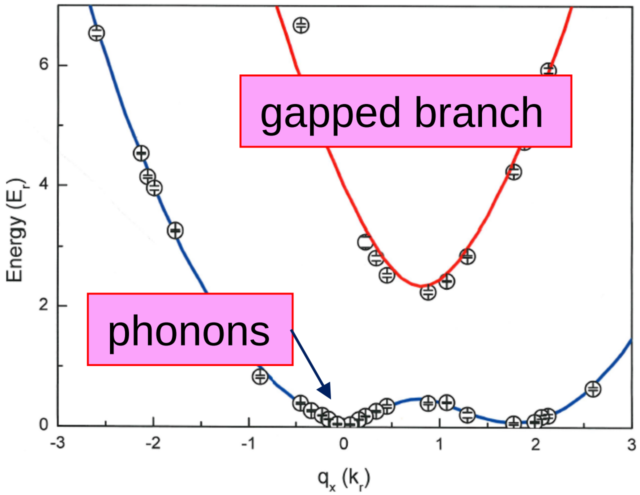


Excellent agreement with Leggett's prescription, based on evaluation of non classical moment of inertia

$$f_S^L = 1 - \frac{\Theta}{\Theta_{rig}}$$

Sound in supersolid spin orbit coupled BEC gases

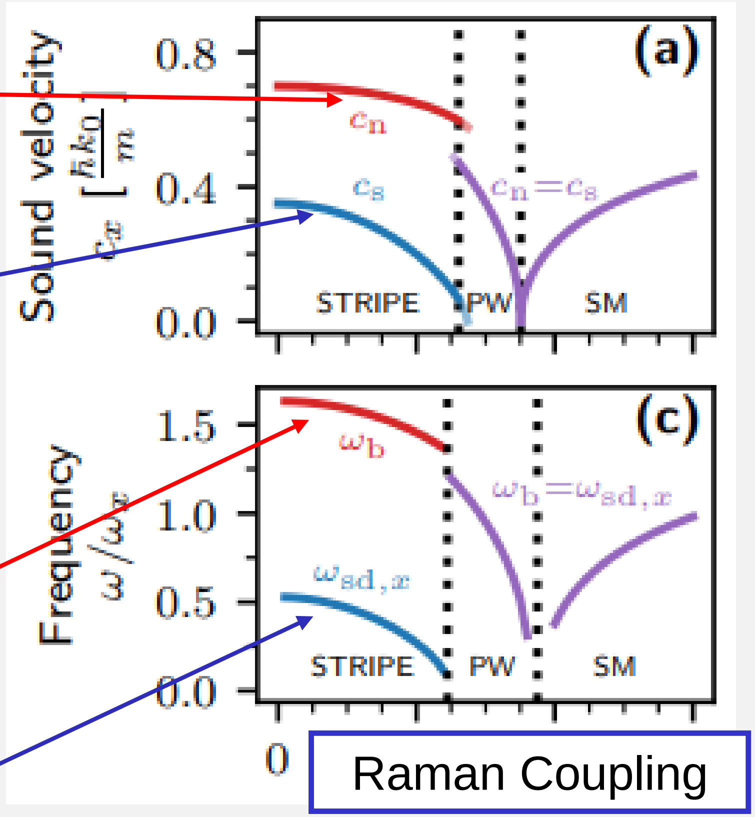
In the **non supersolid phase** the excitation spectrum of a spin orbit coupled BEC gas exhibits **a single phonon mode** propagating along the x-direction. The phonon has hybridized density and spin nature.
The second phonon mode is gapped because of the presence of Raman coupling.
(In the absence of Raman coupling a BEC mixture would exhibit two gapless phonon modes (reflecting the presence of two superfluids)



Spontaneous breaking of translational invariance causes the appearence of a **novel gapless branch** (see previous discussion in dipolar gases)

Differently from dipolar gases, in SOC BEC gases the novel gapless mode emerging from the spontaneous breaking of translational symmetry has a clear **spin nature**

- In **infinite matter** the **density** Goldstone mode is excited by e^{iqx}
- Instead the **spin** Goldstone mode is excited by spin operator $\sigma_z e^{iqx}$
  (Yun Li et al. PRL 2013)



- In **elongated harmonic trap** the **density** mode is excited by x^2
- the **spin** Goldstone mode is instead excited by spin dipole operator $x\sigma_z$
  (Geier et al. PRL 2021
  see also Chen et al. PRA2017)

$$g_{\uparrow\uparrow} = g_{\downarrow\downarrow}$$

Question:

What is the **interplay** between the **spin** degree of freedom and the **dynamic crystal** nature of the novel Goldstone mode ?

Collaboration with Wolfgang Ketterle (K.Geier et al PRL2022)

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What is the **interplay** between the **spin** degree of freedom and the **dynamic crystal** nature of the novel Goldstone mode ?

Collaboration with Wolfgang Ketterle (K.Geier et al PRL2022)

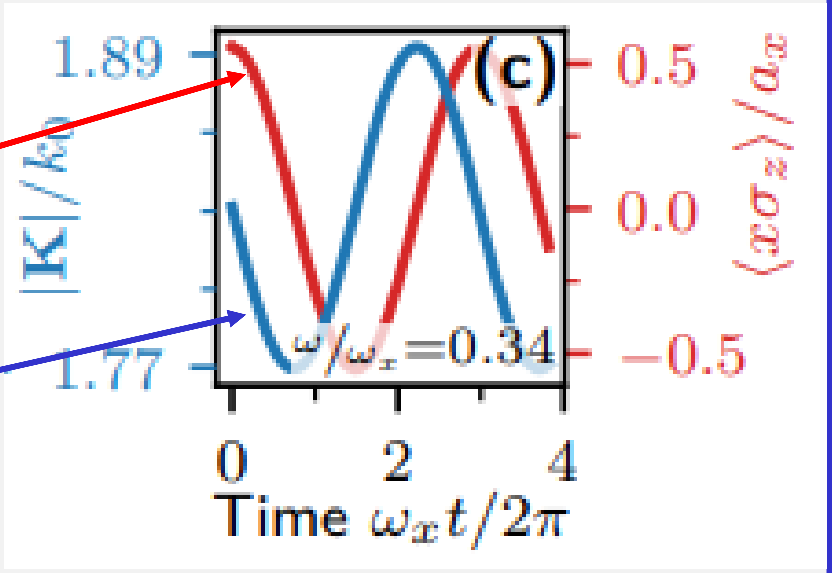
Kevin Geier

Giovanni Martone

Philipp Hauke

We have explored the **dynamic behavior of stripes**, following the application of a spin perturbation and solving the resulting time dependent oscillation of the spin-orbit BEC confined in a harmonic trap

By applying a spin perturbation of the type $\lambda x \sigma_z$ we find that not only the value $< x\sigma_z > (t)$ of the spin dipole mode oscillates with the frequency of the spin dipole mode, but also the distance $d = 2\pi / K$ between stripes oscillates with the same frequency.



Full confirmation of the supersolid nature of the stripe phase of SOC BEC gases

The crystal Goldstone mode in a highly **spin asymmetric** SOC Bec gas recently observed at ICFO by applying a spin perturbation

(Leticia Tarruell team, arXiv:2412.1386)

Probing supersolidity through excitations in a spin-orbit-coupled Bose-Einstein condensate

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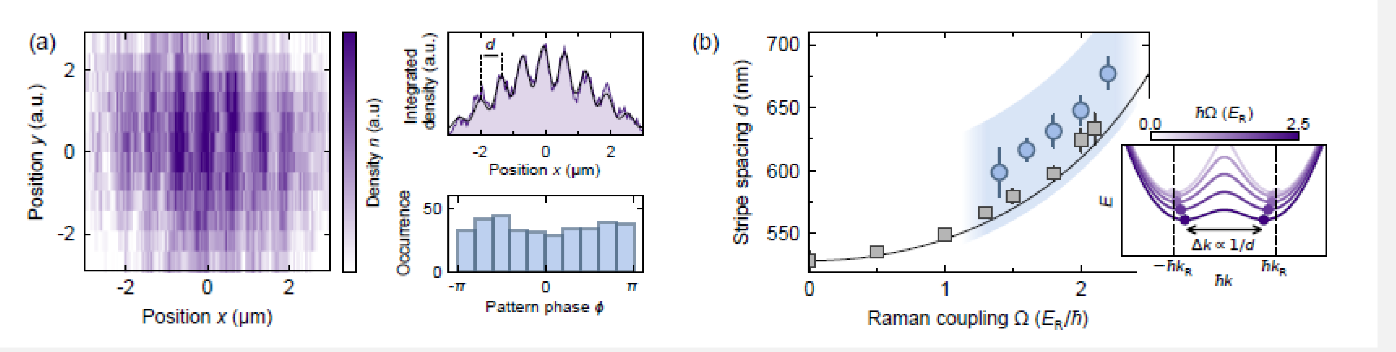
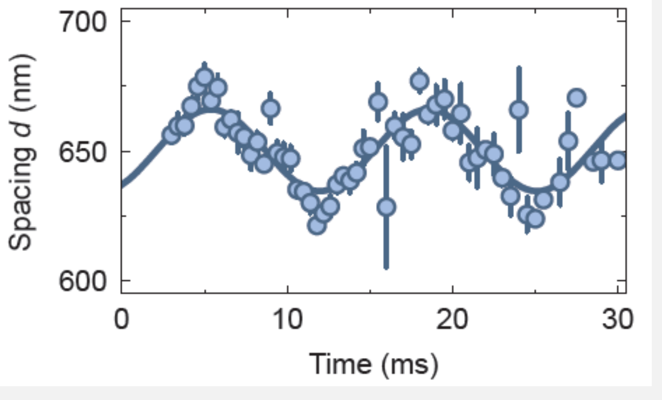
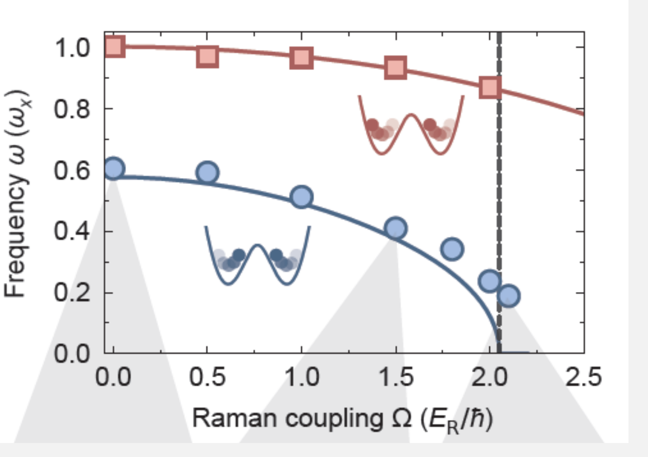
[3]*ICREA, Pg. Lluís Companys 23, 08010 Barcelona, Spain*

Observation of stripes in SOC BEC gas of 39K atoms:



observation of novel crystal Goldstone mode (**oscillation of stripes**), excited by fast ramping of Raman coupling.





PLAN OF THE LECTURES

Lecture 1. **Superfluids at finite tempertaure: a tale of two sounds**

Lecture 2. **Dynamical breaking of Galilean invariance and propagation of sound at T=0**

Lecture 3. **Spontaneous breaking of translational invariance and supersolidity: another tale of two sounds**

Related questions for theoretical and experimental research

- Behavior of **transverse (crystal)** sound in **2D** supersolids role of superfluidity and supersolid **hydrodynamics**

- Sound propagation in **ring** geometry containing **permanent currents** (measurement of **quantized ciriculation in Fermi superfluids** (collaboration with Roati team at LENS)

- Doppler effect in the presence of two sounds (**Dipolar upersolids** or **interacting binary mixtures**)

- **Josephson effect** and superfluidity in supersolids (see Biagioni, … Modugno, Nature 2024)

THANK YOU