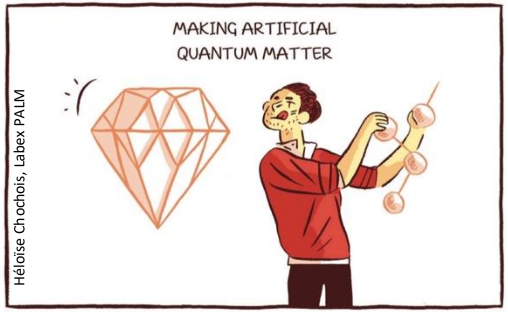
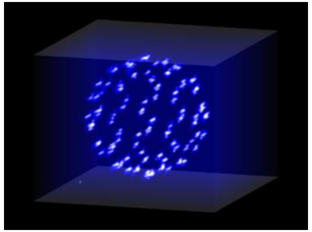
# Exploring many-body physics with arrays of Rydberg atoms (II)

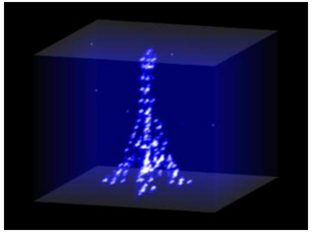



Héloïse Chochois, Labex PALM

**Antoine Browaeys**

*Laboratoire Charles Fabry,*
*Institut d'Optique, CNRS, FRANCE*





Benasque Workshop, february 24-25, 2025

# Interactions between Rydberg atoms and spin models


Browaeys & Lahaye, Nat.Phys. (2020)

**van der Waals**

2-atom energy

$|ns, ns\rangle$

$\Delta E$

$$\Delta E \sim \frac{C_6}{R^6}$$

$|gg\rangle$

**Resonant dipole**

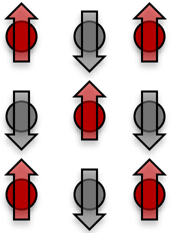
2-atom energy

$|ns, n'p\rangle$ **V** $|n'p, ns\rangle$

$$V \sim \frac{C_3}{R^3}$$

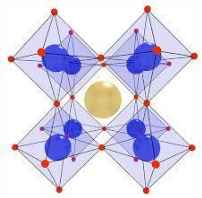
$|gg\rangle$

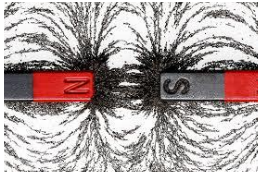
Spin 1/2



**Ising model**

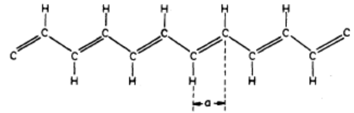
$$\hat{H} = \sum_{i \neq j} J_{ij} \hat{n}_i \hat{n}_j$$





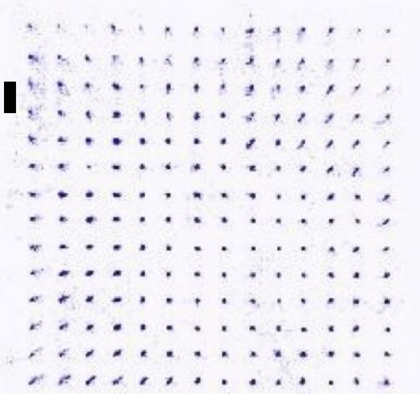

**XY model**

$$\hat{H} = \sum_{i \neq j} J_{ij} \left( \hat{\sigma}_i^+ \hat{\sigma}_j^- + \hat{\sigma}_i^- \hat{\sigma}_j^+ \right)$$



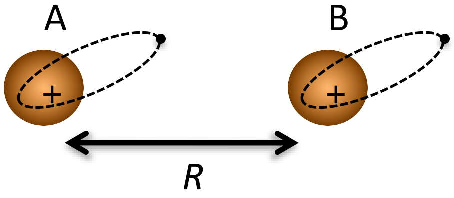
# Combining arrays of atoms and Rydberg interactions

5 µm



+

**Rydberg interactions**




Van der Waals $\frac{C_6}{R^6}$

resonant $\frac{C_3}{R^3}$

Quantum Ising
$s = 1/2$

Hardcore boson

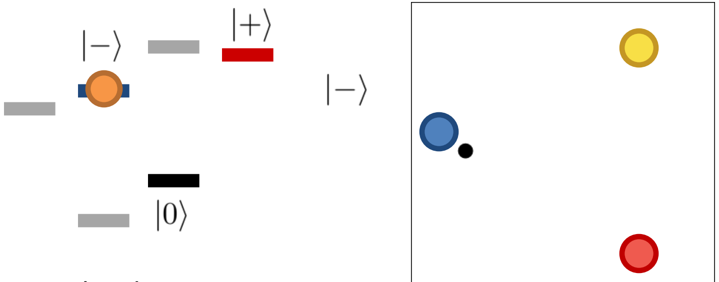
Bosons/ Fermions
Softcore potential

XY, $s = 1/2$
$\frac{1}{R^3}, \frac{1}{R^6}$

XYZ
Heisenberg
$s = 1/2$
Floquet

t- J model

Spin-orbit coupling




Lienhard, PRX 2021

# The program

Lecture 1: Arrays of atoms & "Rydbergology"
Rydberg Interactions and spin models
Engineering many-body Hamiltonians

Lecture 2: Examples of quantum simulations in
and out-of-equilibrium: quantum magnetism

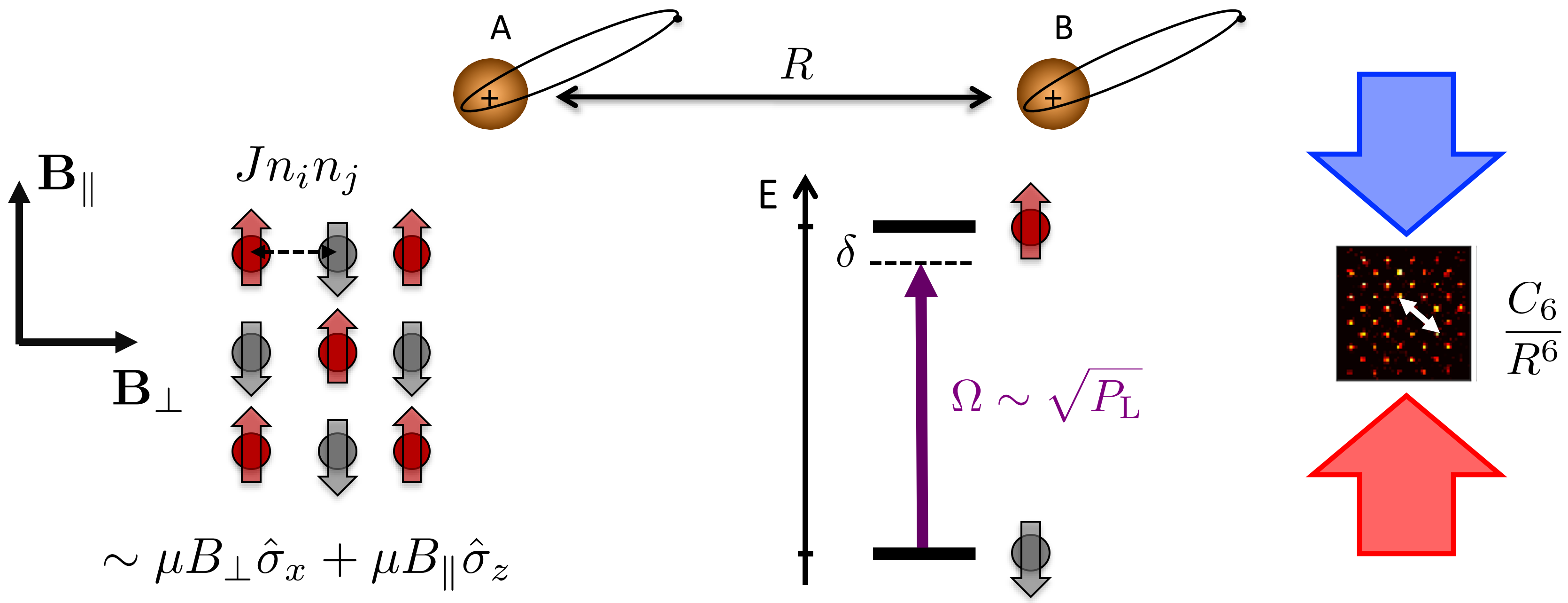
# Outline – Lecture 3

1. Studying the ground state of quantum magnets
   - Ising model in 2D
   - Dipolar XY model in 2D

2. Out-of-equilibrium dynamics
   - Quench dynamics in Ising model: thermalization or not…
   - Quench spectroscopy: measuring dispersion relation of quasi-particles

3. Outlook: what we did not discuss… &beyond

# From van der Waals interaction to spin models…




**Transverse Field Ising model:**

$$H = \frac{\hbar\Omega}{2}\sum_i \hat{\sigma}_x^i + \hbar\delta \sum_i \hat{\sigma}_z^i + \sum_{i<j} \frac{C_6}{R_{ij}^6}\hat{n}_i \hat{n}_j$$

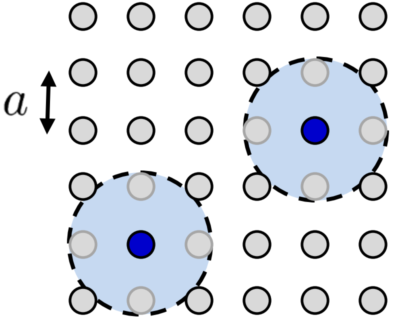
Laser: $B_\perp$ $B_\parallel$ spin-spin interactions

Controlled parameters:

From negligible to dominant interactions

# 2D Ising anti-ferromagnet on a square

## Nearest neighb. interaction

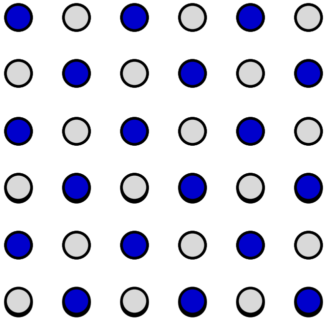



$$\frac{C_6}{a^6} \sim \Omega$$

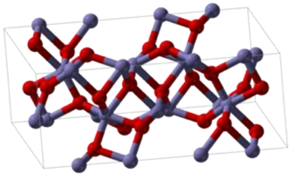
○ $= |g\rangle = |\downarrow\rangle$

● $= |r\rangle = |\uparrow\rangle$

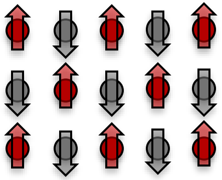
## Anti-ferromagnetic ground state



Ex of antiferromagnets:
MnO, FeO, CoO, NiO, $FeCl_2$...

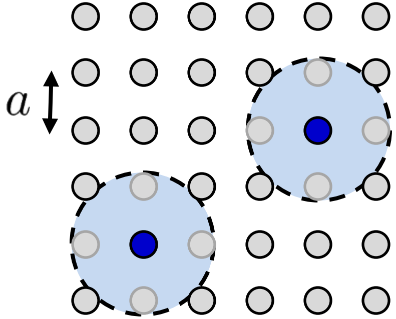


AFM (Néel) ordering ($Z_2$ phase)



# 2D Ising anti-ferromagnet on a square

## Nearest neighb. interaction




$$\frac{C_6}{a^6} \sim \Omega$$

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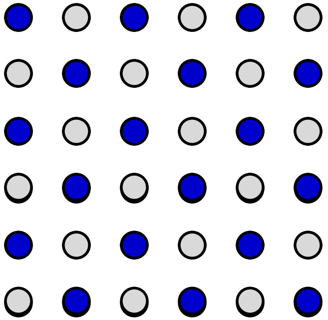
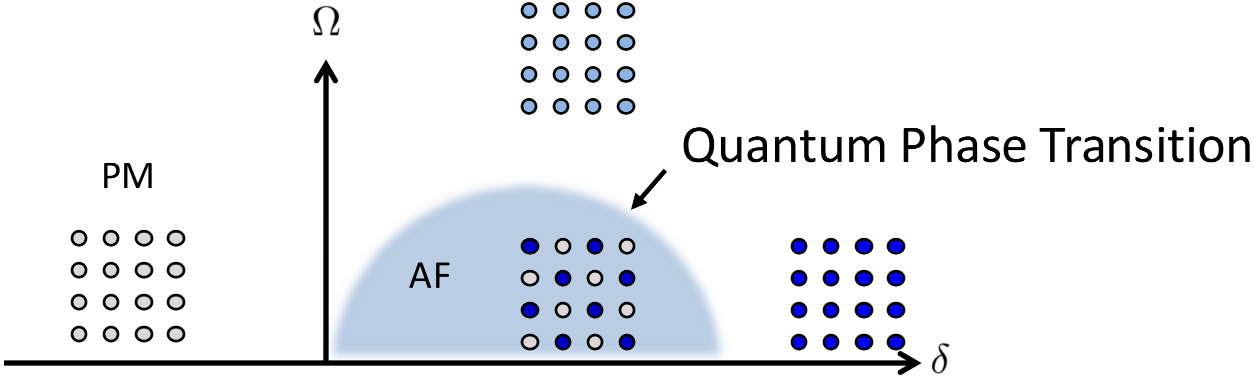
## Anti-ferromagnetic ground state



**2D phase diagram**
(1970)

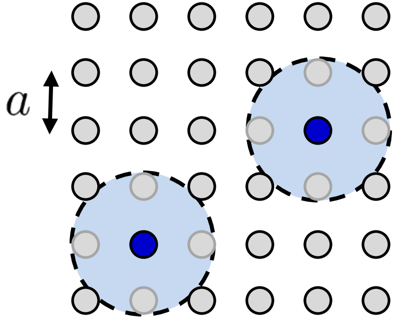



Known by Quantum Monte-Carlo

Never implemented and measured in 2D… (approximation in material)

# 2D Ising anti-ferromagnet on a square

## Nearest neighb. interaction

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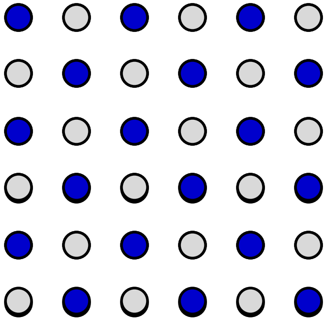
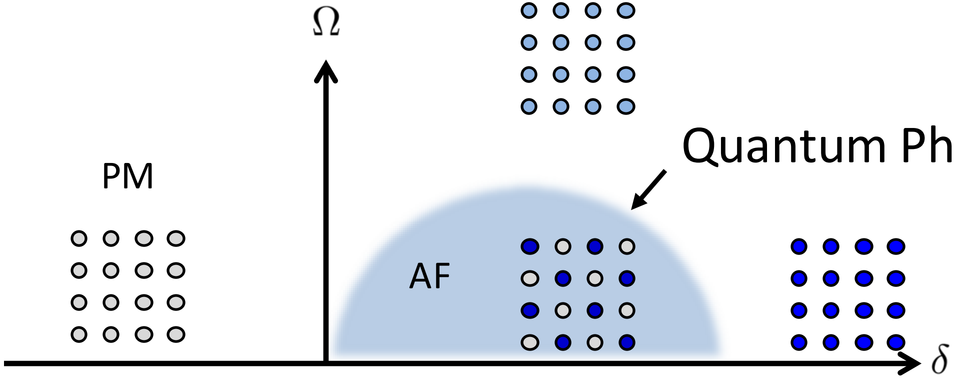
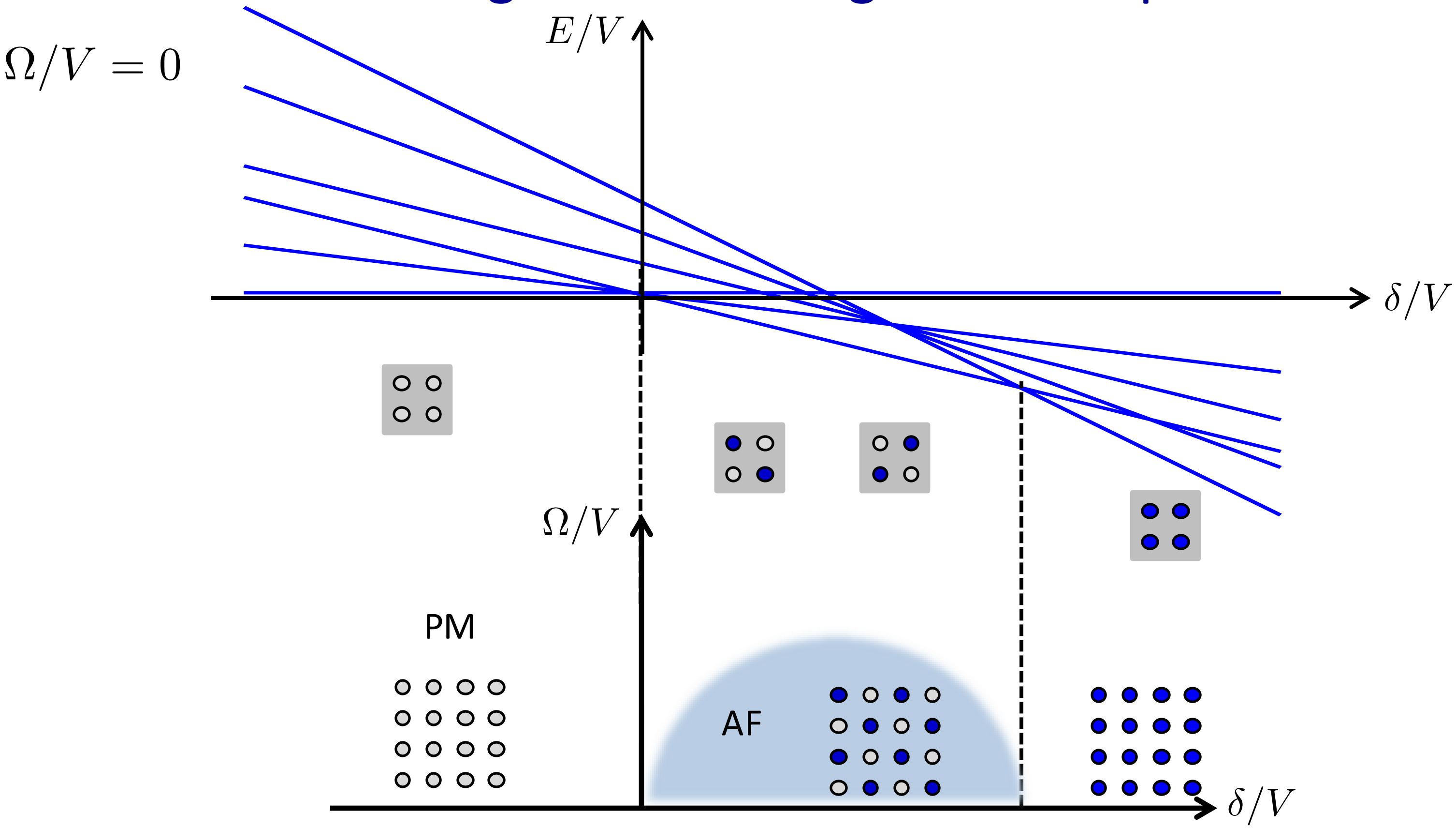


**2D phase diagram**
(1970)



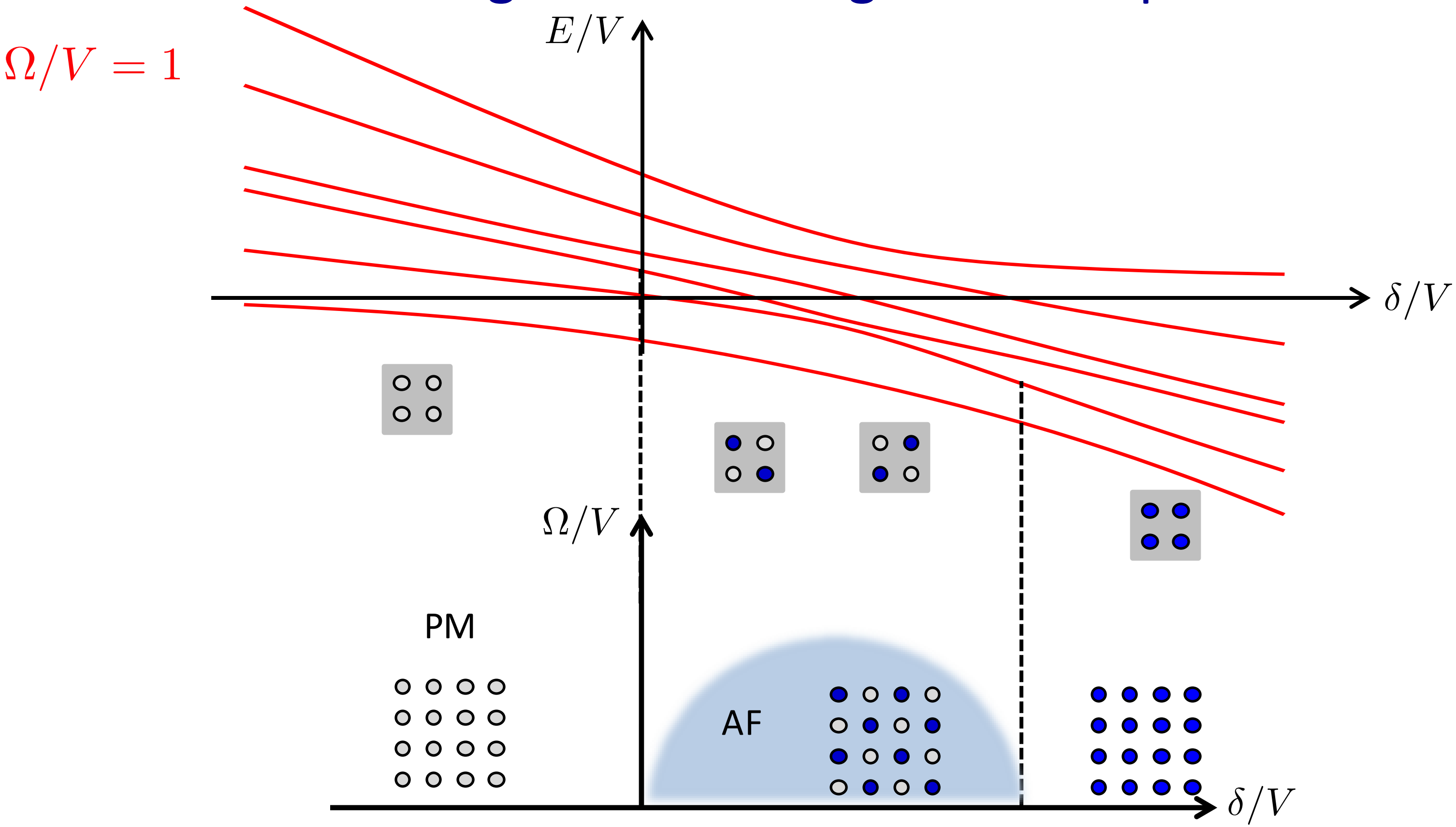

$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

# 2D Ising anti-ferromagnet on a square




$$H = -\hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

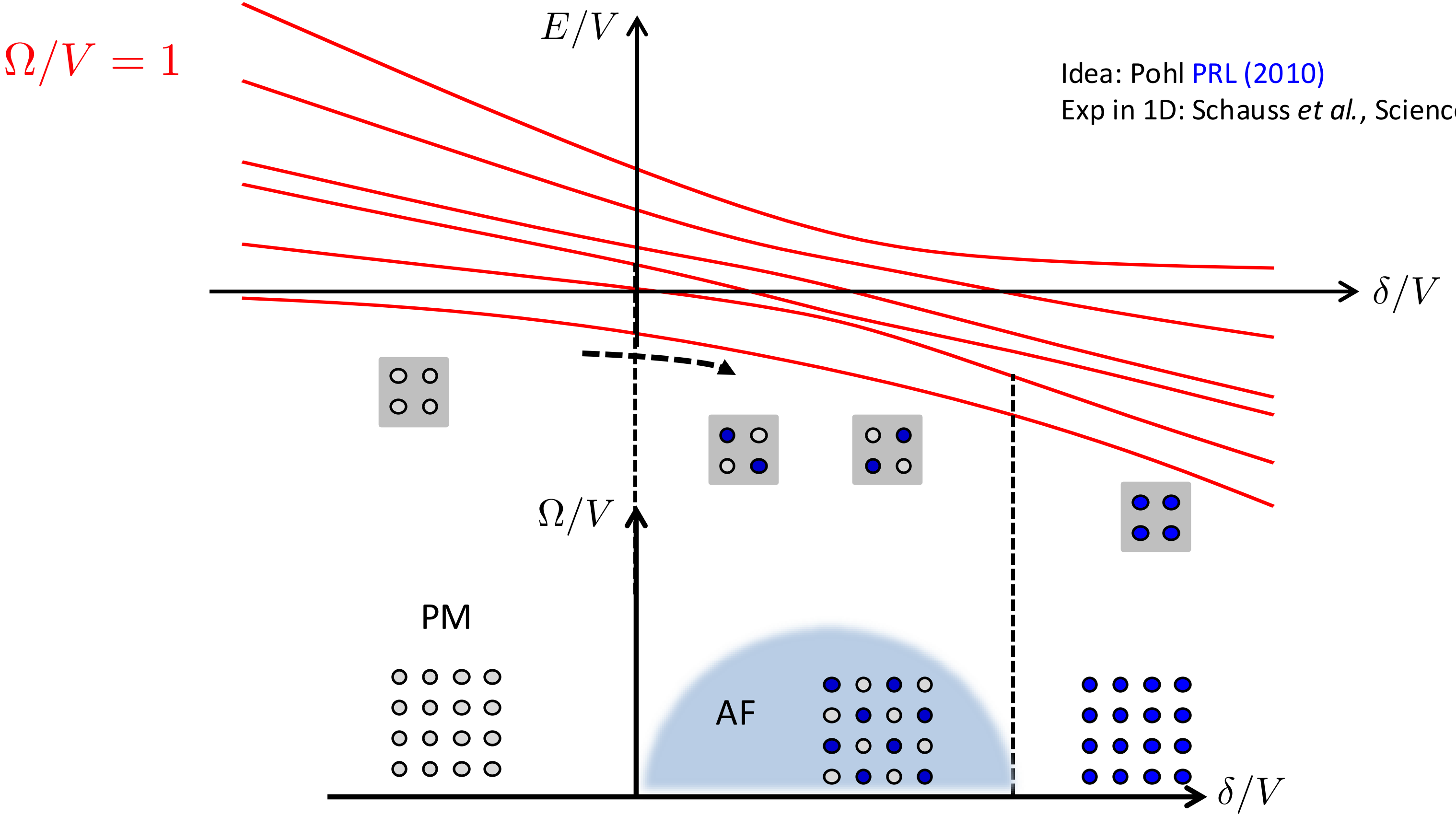
# 2D Ising anti-ferromagnet on a square




$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

# Adiabatic preparation of a 2D Ising anti-ferromagnet

Idea: Pohl PRL (2010)
Exp in 1D: Schauss *et al.*, Science (2015)




$$H = \frac{\hbar\Omega}{2}\sum_i \sigma_x^i - \hbar\delta\sum_i \hat{n}_i + \frac{V}{2}\sum_i \hat{n}_i\hat{n}_{i+1}$$

# Adiabatic preparation of a 2D Ising anti-ferromagnet

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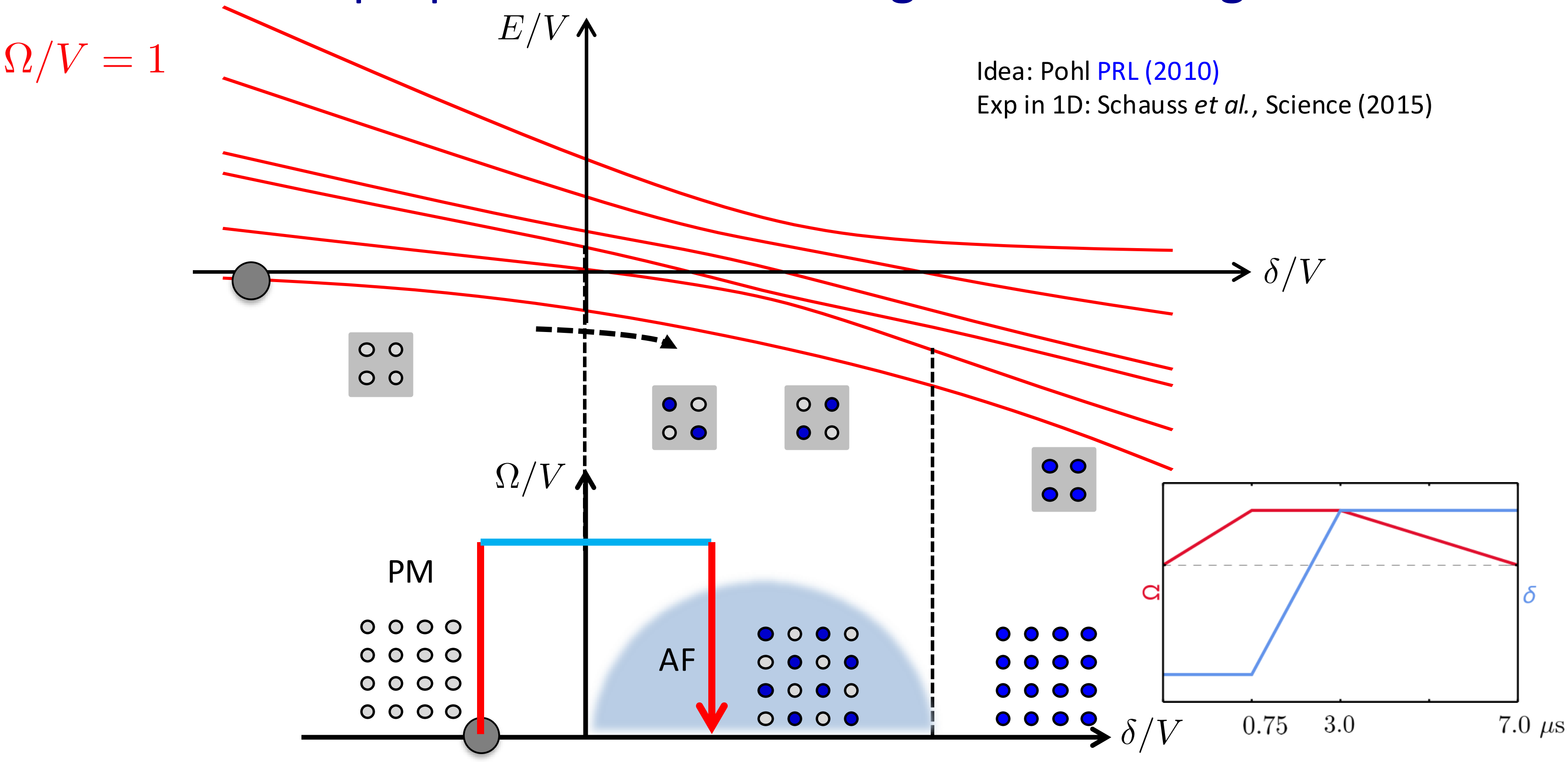


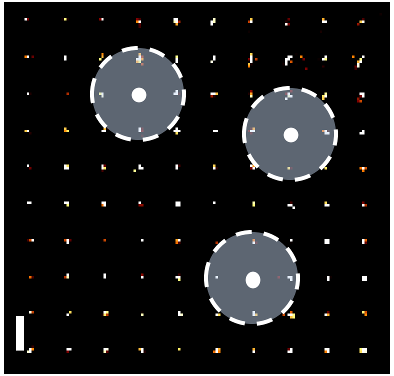

$$H = \sum_i \left( \frac{\hbar\Omega(t)}{2}\sigma_x^i - \hbar\delta(t)\hat{n}_i \right) + \sum_{i<j} \frac{C_6}{R_{ij}^6}\hat{n}_i\hat{n}_j$$

# Adiabatic preparation of an antiferromagnet on a square array

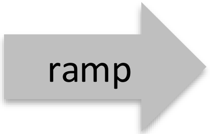
Scholl et al. Nature (2021)

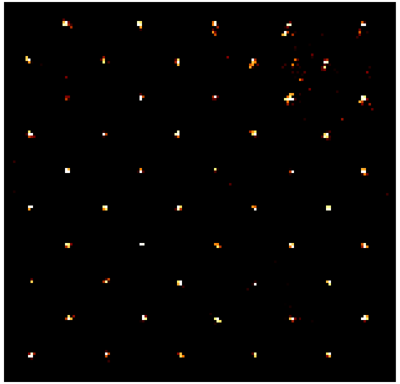
$$\frac{C_6}{a^6} \sim \Omega$$

$10 \times 10$




$\Omega(t), \delta(t)$



$= |f\rangle$ "bright"

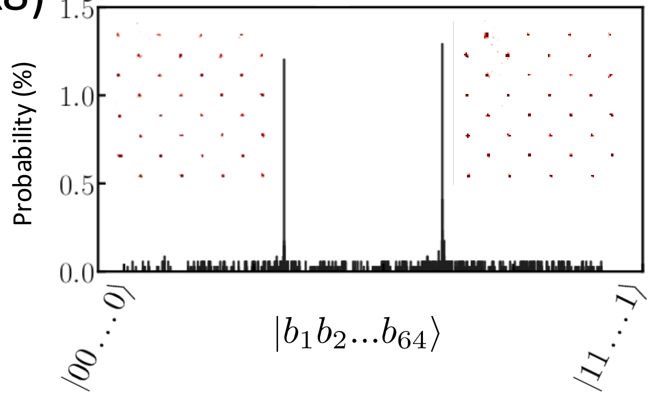
$= |r\rangle$ "dark"

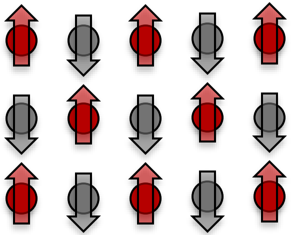
Perfect AF (Néel) ordering!

(proba < 1% )

(8x8)

$2^{64}$ states!!!



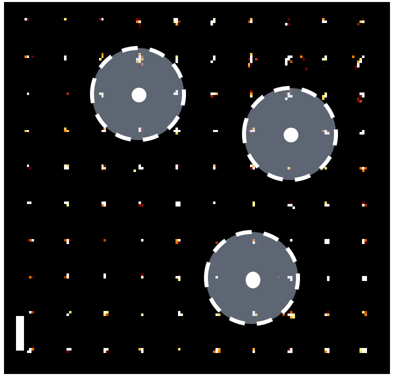



# Adiabatic preparation of an antiferromagnet on a square array

$\frac{C_6}{a^6} \sim \Omega$

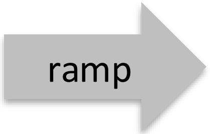
Scholl et *al.* Nature (2021)

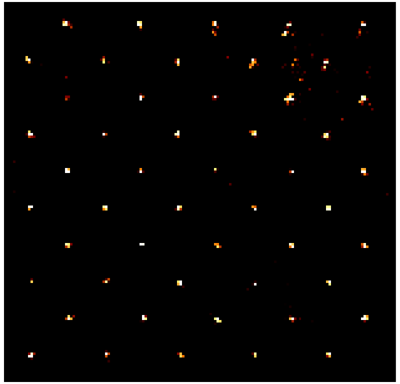
$10 \times 10$

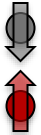


10 μm

$\Omega(t), \delta(t)$



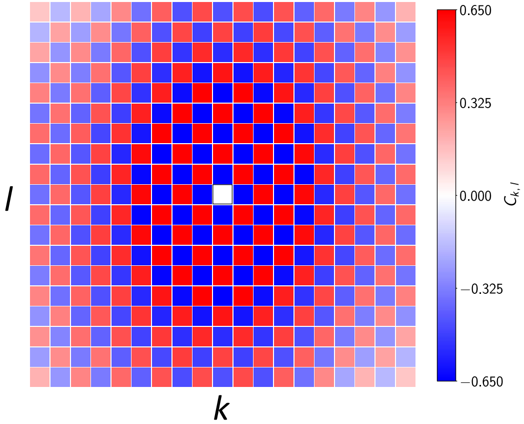



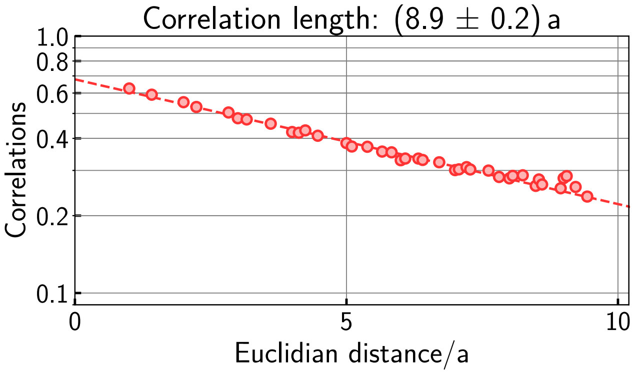


$= |f\rangle$ "bright"

$= |r\rangle$ "dark"

$C_{kl} \sim \langle n_k n_l \rangle - \langle n_k \rangle \langle n_l \rangle$



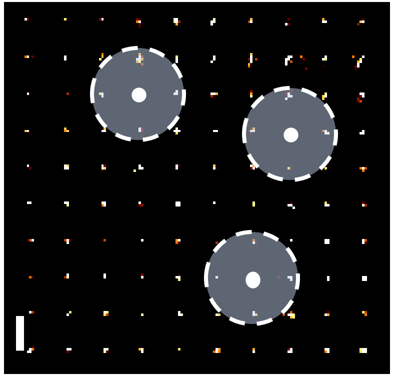




Also: Lukin Nature 2021

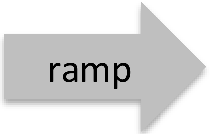
# Classical simulation of the preparation

$$\frac{C_6}{a^6} \sim \Omega$$

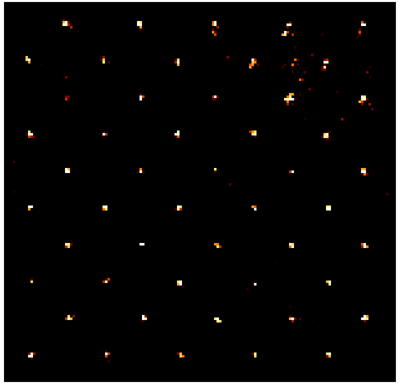
10 × 10




$\Omega(t), \delta(t)$

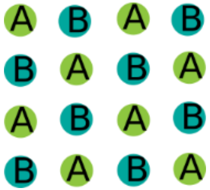



Scholl et *al.* Nature (2021)



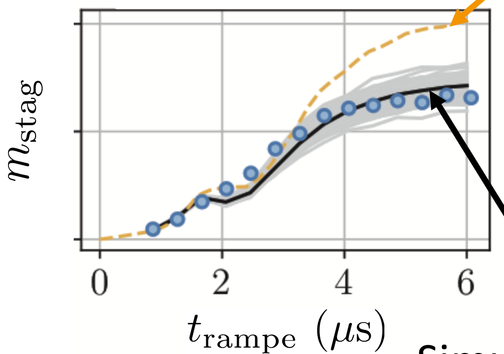
$= |f\rangle$ "bright"

$= |r\rangle$ "dark"




Dynamics of magnetization

$$m_{\text{stag}} = \langle |n_A - n_B| \rangle$$

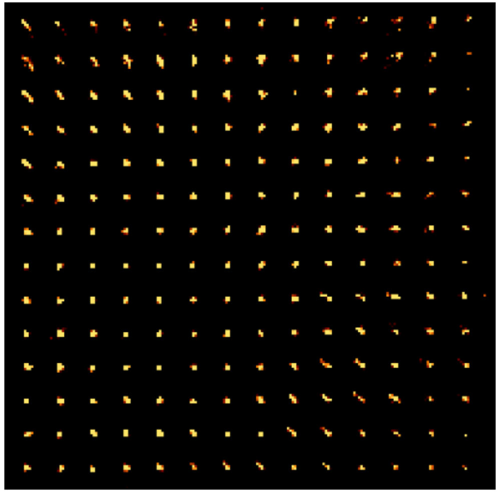



State of-the-art simulation (2021):
MPS limited to 10 x 10
(14 days on super supercomputer!!)

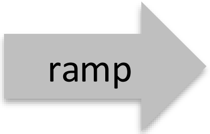
# But we can push the atom number… by a lot…

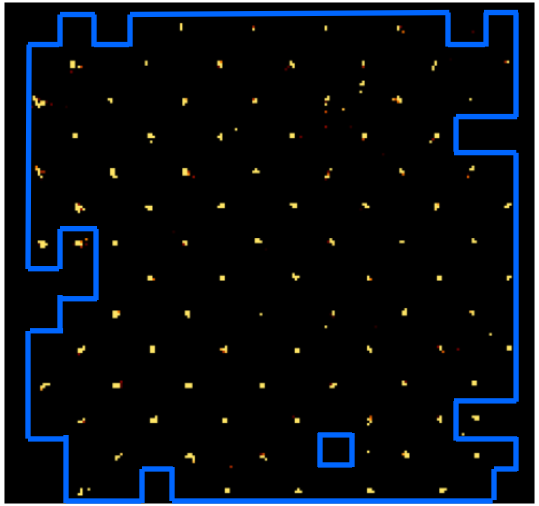
Scholl et *al.* Nature (2021)

14x14



$\Omega(t), \delta(t)$



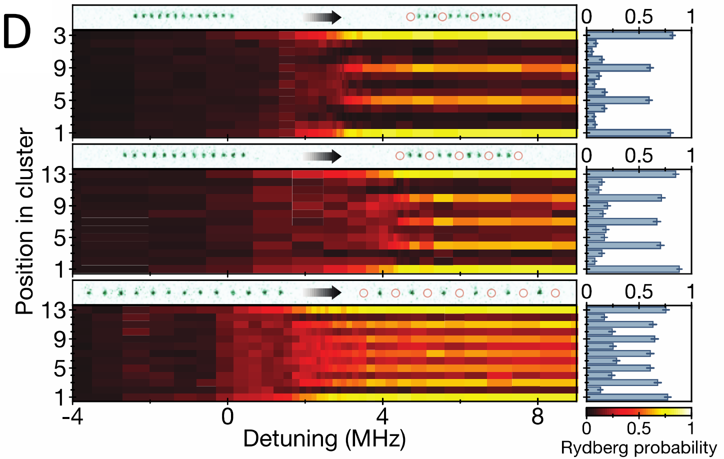



Antiferromagnetic cluster:
182 atomes

Since 2022: more elaborate numerical methods …!!

# Ising model in other geometries

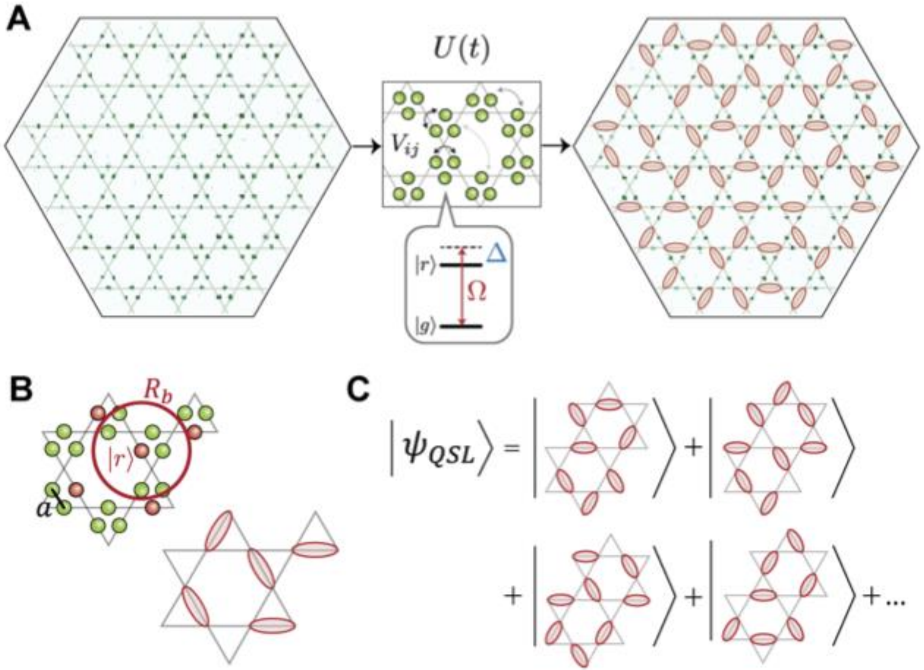
1D




Bernien, Nature 2017

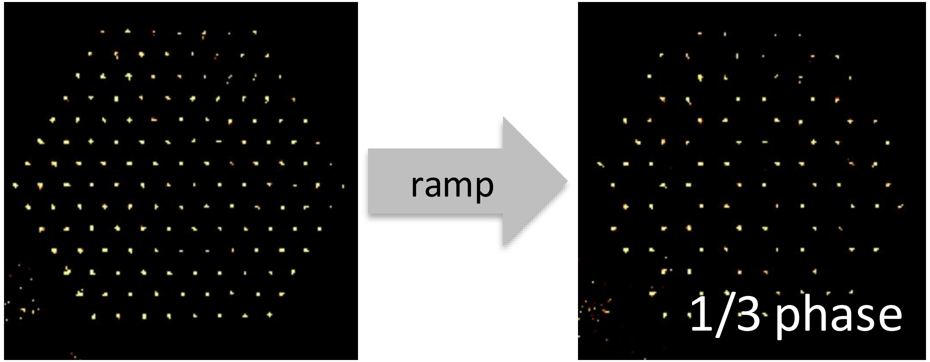
## Ruby lattice: spin liquid?

Lukin, Science 2021



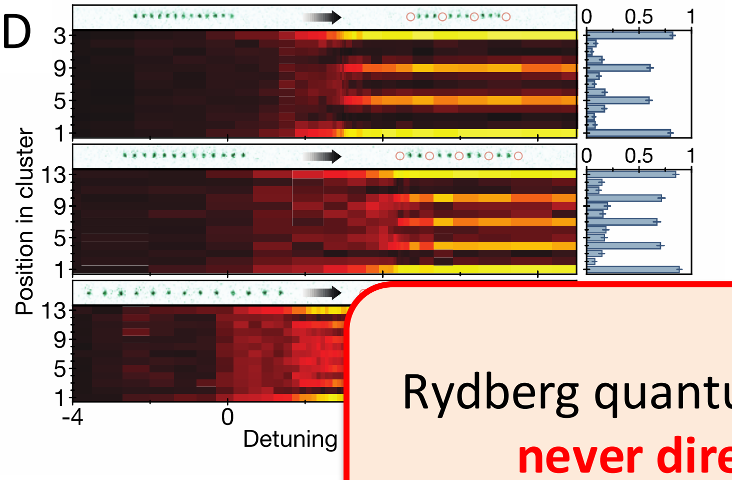

## Triangle (frustration)

Scholl et *al.* Nature (2021)



# Ising model in other geometries

1D

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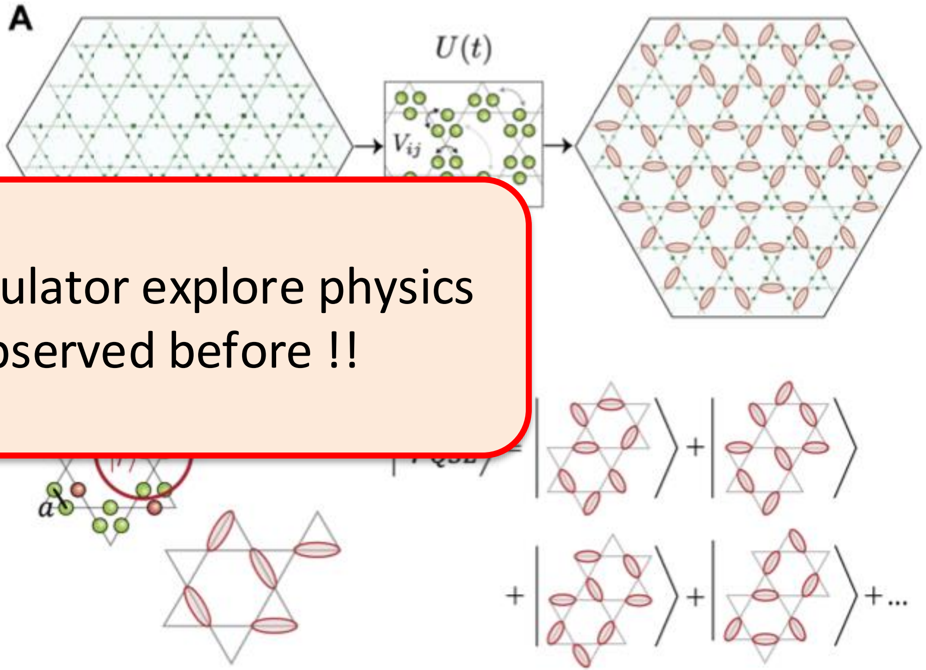
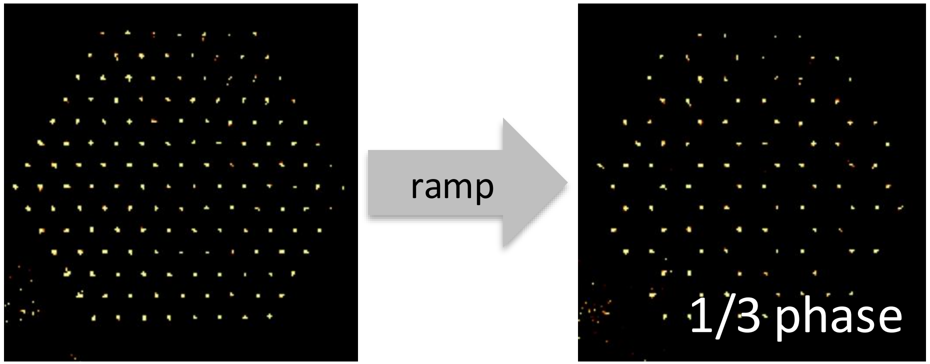
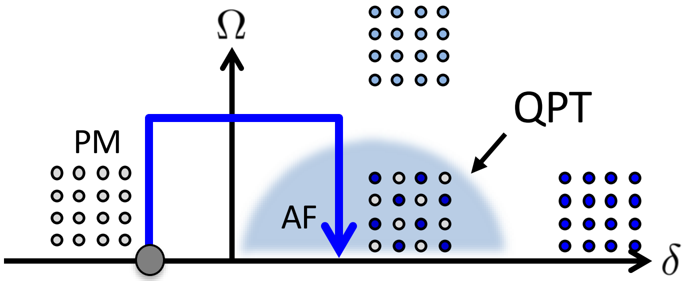



Rydberg quantum simulator explore physics **never directly** observed before !!

Triangle (frustratio



# Use failure of adiabaticity to study quantum phase transition




**Adiabaticity criteria**:

$$H(t) = (1 - \lambda(t))H_0 + \lambda(t)H_{\mathrm{MB}}$$

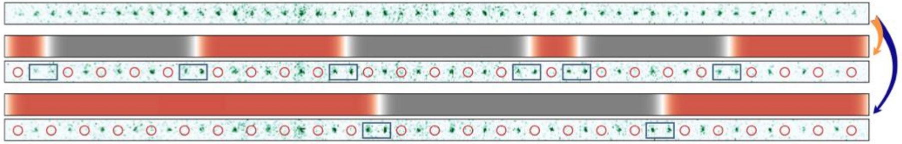
$$|\langle\psi_1(t)|\frac{dH}{dt}|\psi_0(t)\rangle| \ll \frac{\Delta E(t)^2}{\hbar}$$

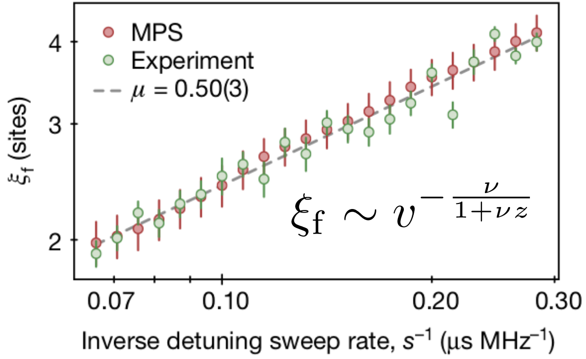
**But**...gaps close at the QPT!!

**Sweeping too fast** $\Rightarrow$ create defects

1D: Keesling, Nature (2019), 2D: arXiv.2012.12281

$R_b \sim a$ 51 atoms





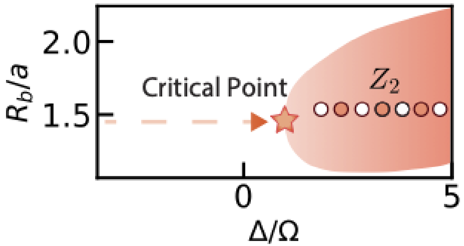

Kibble-Zurek mechanism:

statistics of defects $\Rightarrow$ critical exponent

$\nu_{1D}$ = 0.50(3) ($\nu_{MF}$ = 1/3)

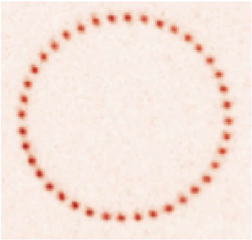
$\nu_{2D,square}$ = 0.62(4) ($\nu_{MF}$ = 1/2)

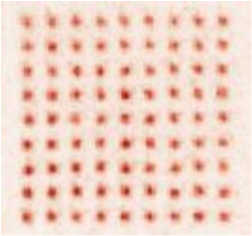
# Studying quantum phase transition in 1D and 2D

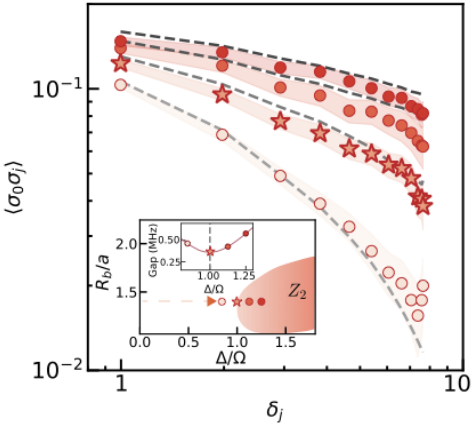


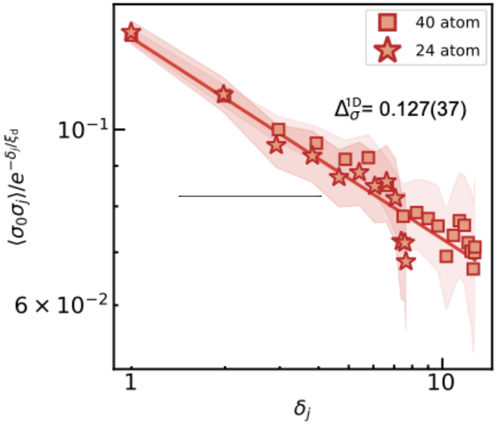

Far from QPT: $\langle \sigma_i^z \sigma_j^z \rangle \sim \exp[-|i-j|/\xi]$

Close to QPT (criticality): $\langle \sigma_i^z \sigma_j^z \rangle \sim |i-j|^{-\Delta}$








$$\Delta_{\text{th}}^{\text{2D}} = 0.518149$$
$$\Delta_{\text{exp}}^{\text{2D}} = 0.59(9)$$

Conformal field theory: $\Delta_{\text{th}}^{\text{1D}} = 1/8$

Fang *et al.*, arXiv:2402.15376

# Outline – Lecture 3

1. Studying the ground state of quantum magnets
   - Ising model in 2D
   - Dipolar XY model in 2D

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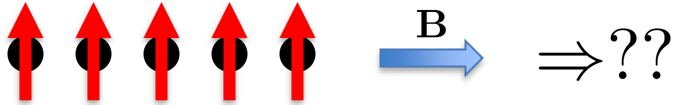
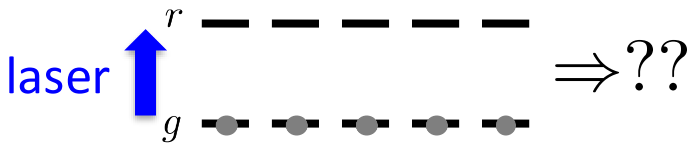
   - Quench dynamics in Ising model: thermalization or not…
   - Quench spectroscopy: measuring dispersion relation of quasi-particles

3. Outlook: what we did not discuss… & beyond

# Quench in Ising Hamiltonian with Rydberg simulator

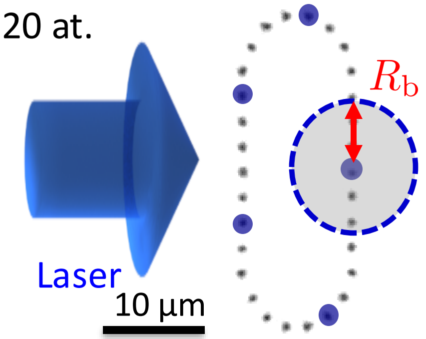
Labuhn *et al.*, Nature 2016



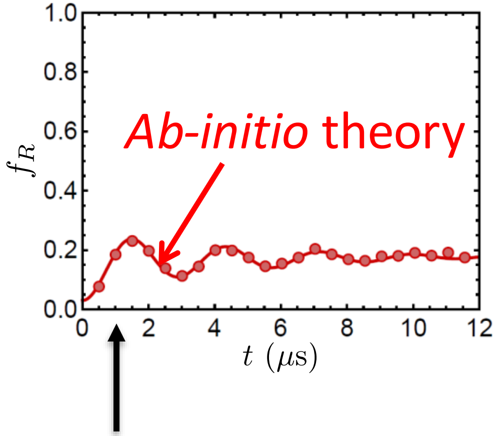

## 1D with periodic boundaries

20 at.



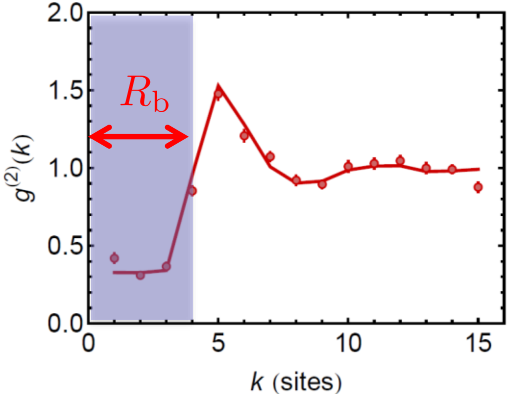

"Magnetization"

$$f_r = \frac{\langle N_r \rangle}{N}$$




## Spin-spin correlation

$$\sim \langle n_j n_{j+k} \rangle$$

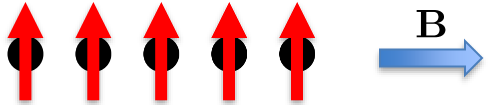



1 Rydberg atom
= hard sphere $R_b$

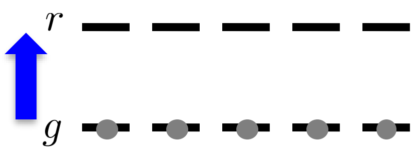
Schauss, Nature 2012
Lesanovsky, PRA 2012
Petrosyan, PRA 2013

# Quench in Ising Hamiltonian with Rydberg simulator

Labuhn *et al.*, Nature 2016



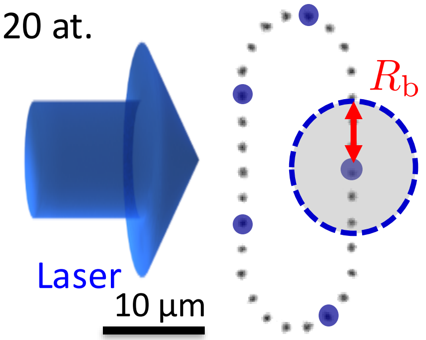

$\Rightarrow ??$




$\Rightarrow ??$

## 1D with periodic boundaries

20 at.

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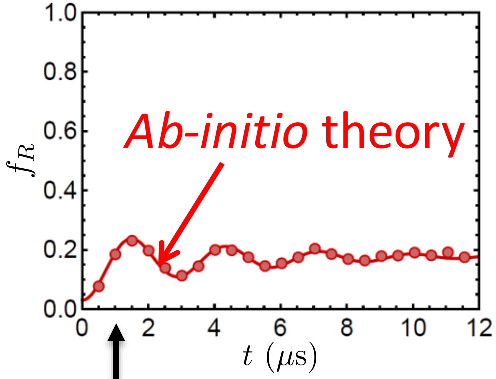


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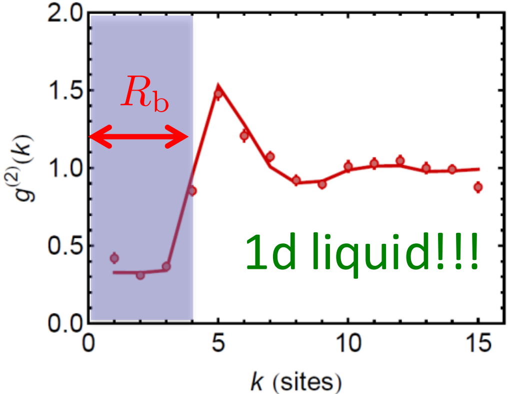
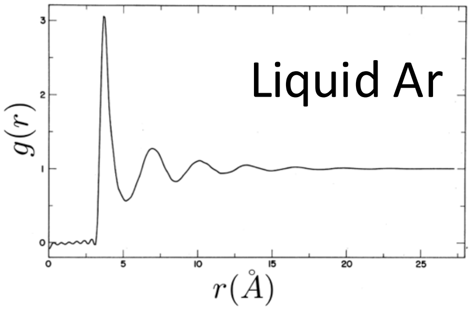
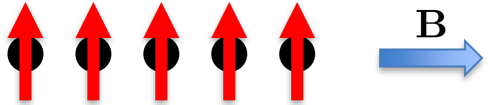


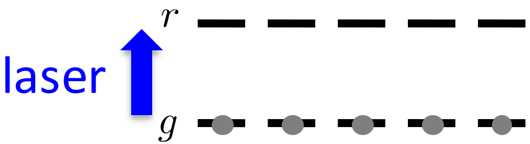

Schauss, Nature 2012
Lesanovsky, PRA 2012
Petrosyan, PRA 2013

# Quench in Ising Hamiltonian with Rydberg simulator

Labuhn *et al.*, Nature 2016

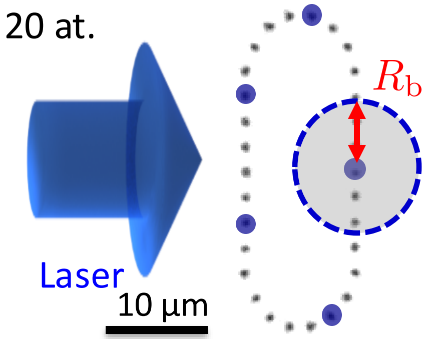



$\Rightarrow ??$



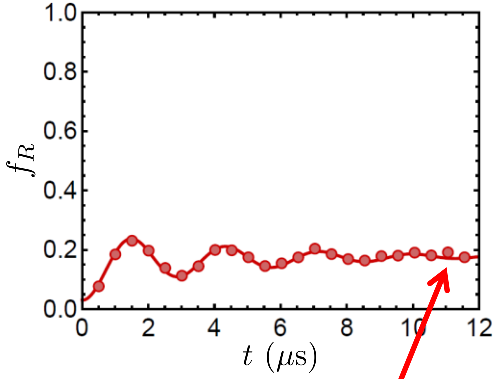

$\Rightarrow ??$

## 1D with periodic boundaries




“Magnetization”

$$f_r = \frac{\langle N_r \rangle}{N}$$

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Schauss, Nature 2012
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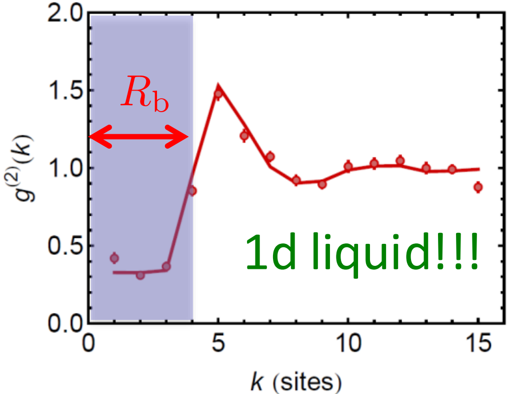
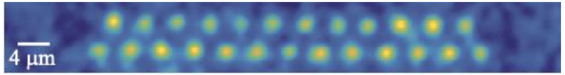



Thermalization??

Kim,... Ahn, PRL 2018



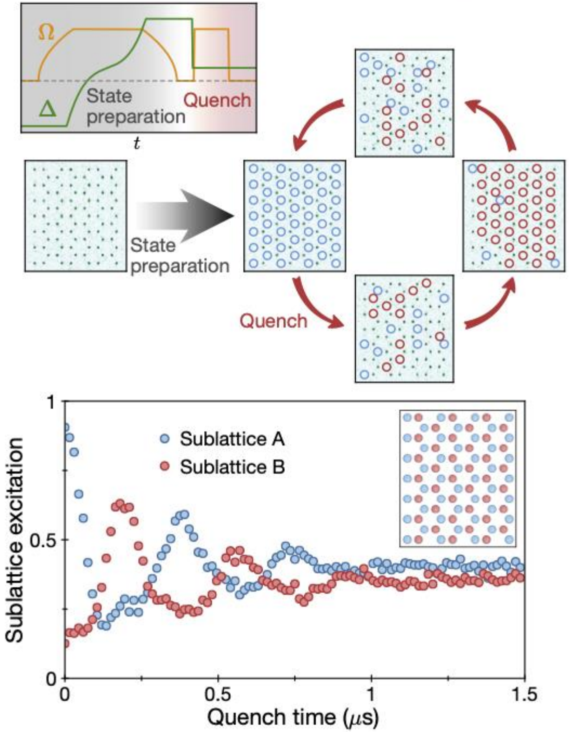
# Thermalization of closed Many-Body systems

Question: do closed systems always reach equilibrium?

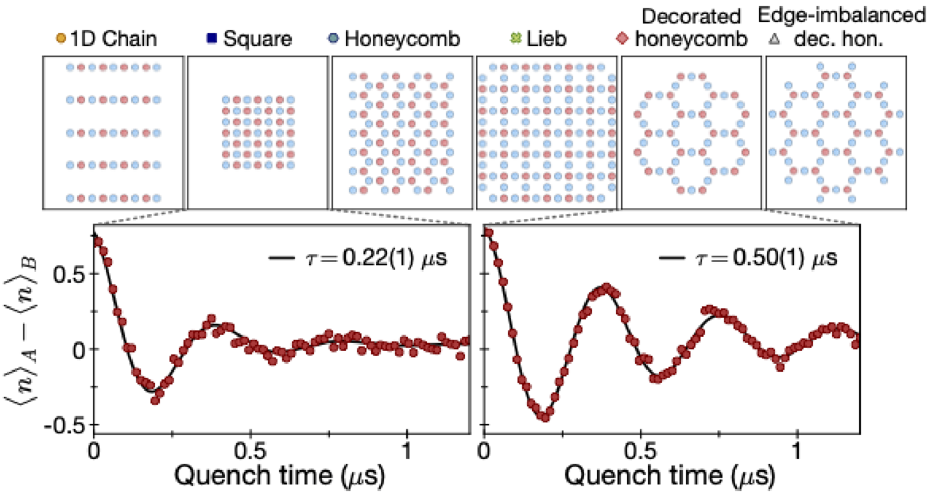
Answer: it depends… ETH, many-body localization and Quantum Scars

Quantum scarrs in 2D (**1D:** Lukin Nature 2019)

Bluvstein…Lukin, Science 2021




Scarrs depends on geometry




Blockade constraint breaks ergodicity

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# Elementary excitations of *dipolar* XY model: spin waves


Büchler *et al.*, PRL 109, 025303 (2012)
Roscilde *et al.*, arXiv:2303.00380

**Linear spin wave theory + Bogolubov:**

$$H_{\mathrm{XY}} = -J \sum_{i<j} \frac{a^3}{r_{ij}^3} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y)$$

$\sigma^+ \to \hat{b}_{\mathbf{q}}^\dagger$ ⇒ $\sigma^- \to \hat{b}_{\mathbf{q}}$

Ground state — Spin wave

$$H_{\mathrm{XY}} \approx E_0 + \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}}\, a_{\mathbf{q}}^\dagger a_{\mathbf{q}}$$

$$\omega_{\mathbf{q}} = J_0 \sqrt{1 - J_{\mathrm{q}}/J_0} \quad \text{with} \quad J_{\mathbf{q}} = \frac{J a^\alpha}{N} \sum_{i \neq j} (\pm 1)^{|i-j|} \frac{e^{i\mathbf{q}\cdot\mathbf{r}_{ij}}}{r_{ij}^\alpha}$$

FM ($+$), AFM ($-$)

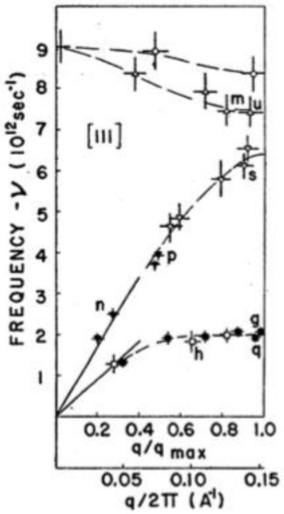
| | | N.N. | | Dipolar |
|---|---|---|---|---|
| Ferro | $J > 0$ | $\omega(\mathbf{q}) \propto \lvert\mathbf{q}\rvert$ | ⇒ | $\omega(\mathbf{q}) \propto \sqrt{\lvert\mathbf{q}\rvert}$ |
| Antiferro | $J < 0$ | $\omega(\mathbf{q}) \propto \lvert\mathbf{q}\rvert$ | | $\omega(\mathbf{q}) \propto \lvert\mathbf{q}\rvert$ |

Manousakis RMP 1991

# “Quench spectroscopy” of dipolar XY magnets

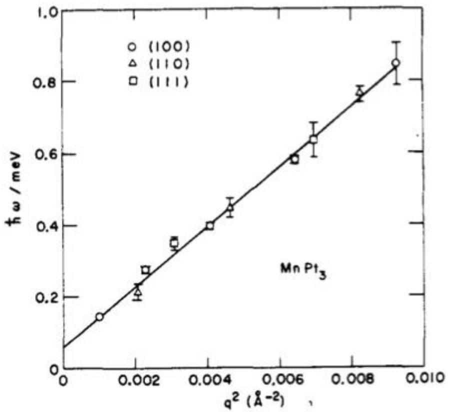
**Usually:** Excite system above ground state & measure dynamics

## Neutron scattering




phonons

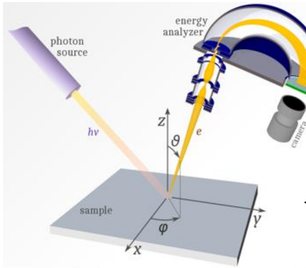
Brockhouse PR 1958



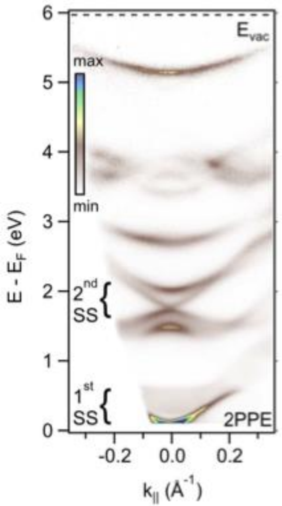

Magnons

Antonini Sol. St. Comm. 1972

## ARPES

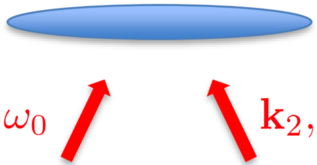



Wikipedia

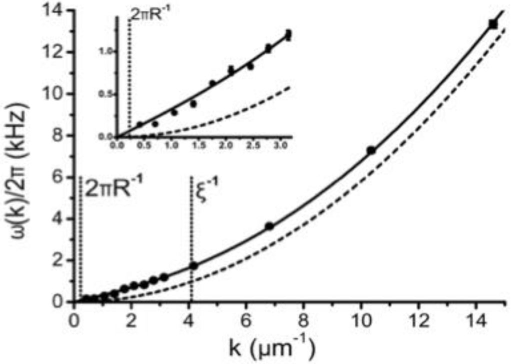



## Bragg spectroscopy

$\mathbf{k} = \mathbf{k}_2 - \mathbf{k}_1$



$\mathbf{k}_1, \omega_0$ $\mathbf{k}_2, \omega_0 + \omega(k)$




Steinhauer PRL 2002

Phonons in BEC

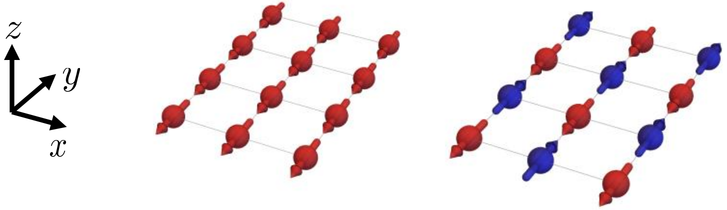
# "Quench spectroscopy" of dipolar XY magnets

**Usually:** Excite system above ground state & measure dynamics

**Here:** Measure dynamics after quench preparing MEAN-FIELD ground state

"Mean-field ground state = true ground state + spin waves"

Villa, Despres & Sanchez-Palencia, PRA 2019
Roscilde *et al.*, PRB 2018, arXiv:2303.00380




Classical FM / AFM in $(xy)$
= MEAN-FIELD ground state
= easy to prepare state (product state)

***t*-Structure factor:**

$$S_{zz}(\mathbf{q},t) \propto \sum_{i,j} \langle \sigma_i^z \sigma_j^z \rangle(t)\ e^{i\mathbf{q}\cdot(\mathbf{r}_j - \mathbf{r}_i)}$$

$$= 1 - \frac{J_\mathbf{q}}{2J_0} + \frac{J_\mathbf{q}}{2J_0} \cos(2\omega_\mathbf{q} t)$$

Quench $\Rightarrow$ pairs $(q, -q)$

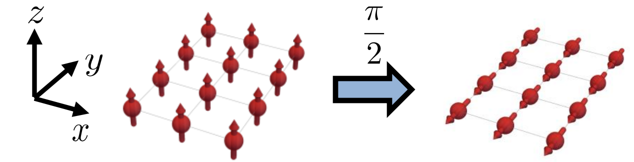
LSW theory

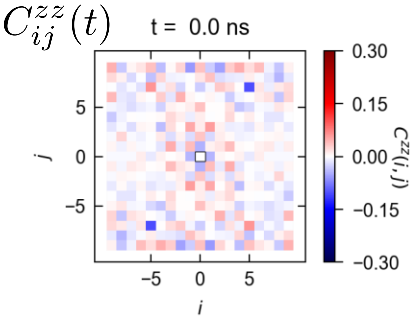
**Related work**: ultra-cold atoms in lattices, ions, superconducting circuits…

# Quench spectroscopy: measuring the "dispersion relation" FM / AFM

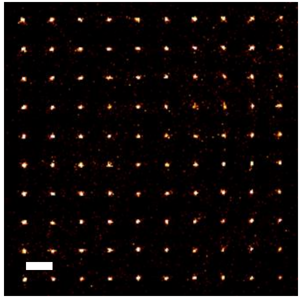
C. Cheng *et al.*, arXiv:2311.11726

**FM**



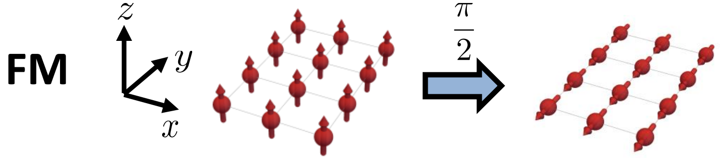

10 x 10

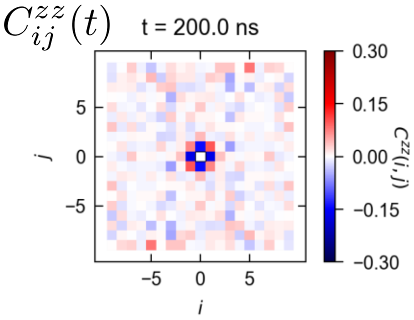


15 μm

# Quench spectroscopy: measuring the "dispersion relation" FM / AFM

C. Cheng *et al.*, arXiv:2311.11726

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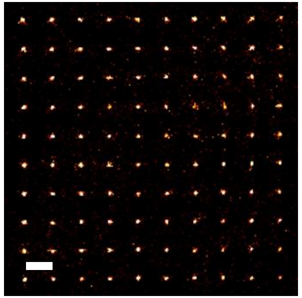

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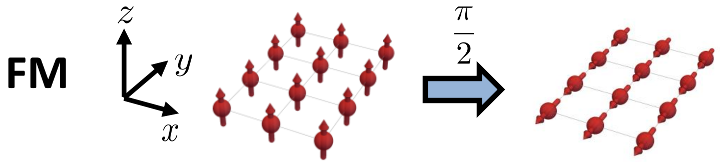


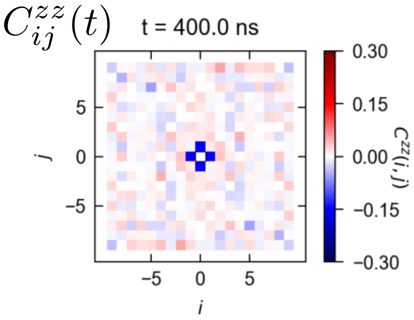


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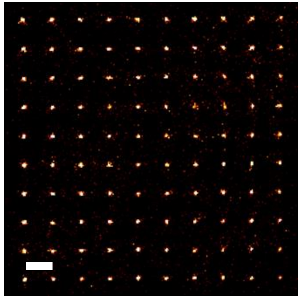

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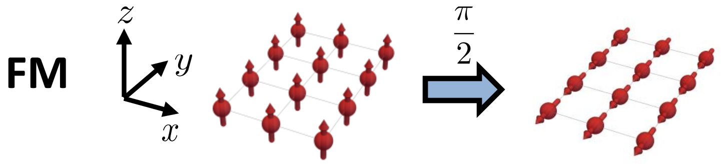


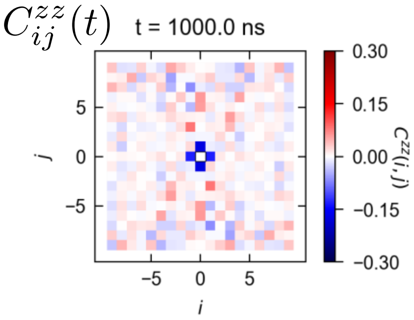


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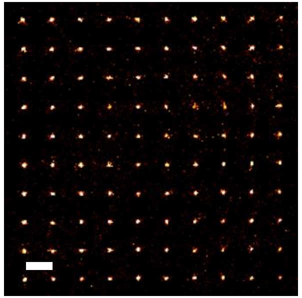

15 µm

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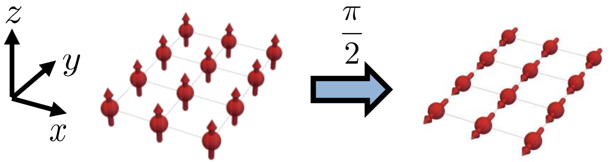


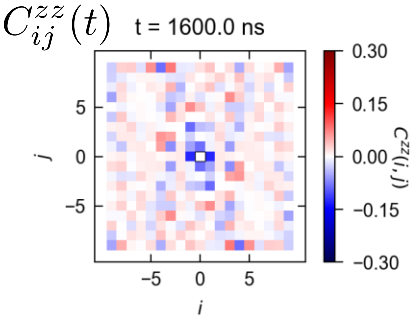


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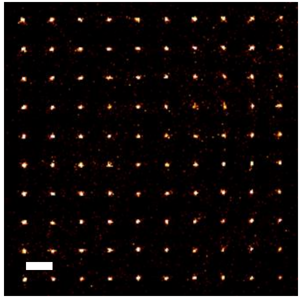

15 µm

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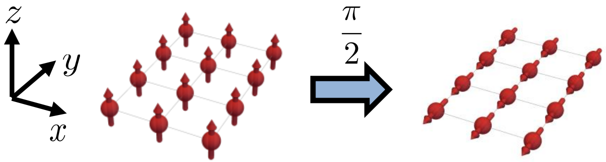


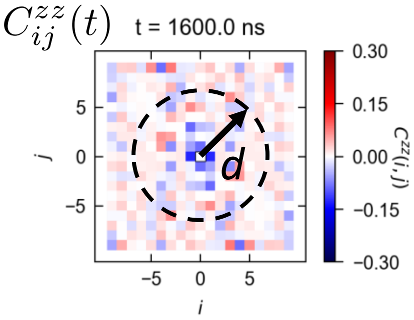


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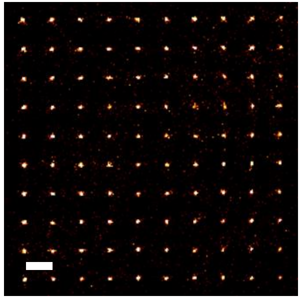

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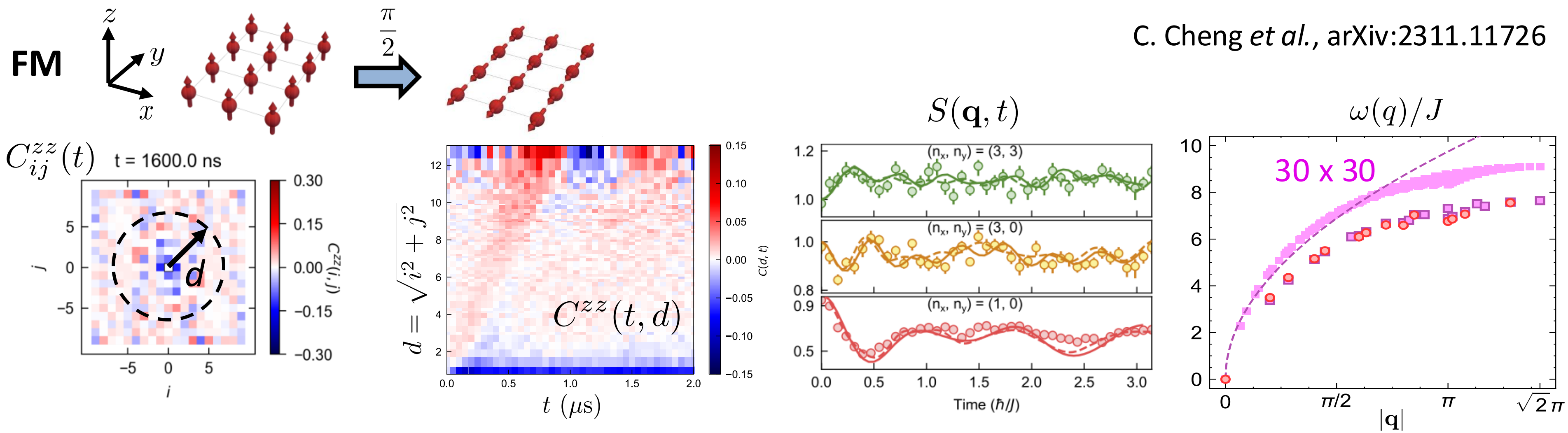
C. Cheng *et al.*, arXiv:2311.11726



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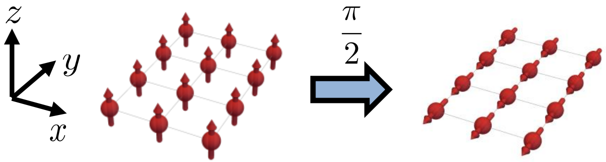
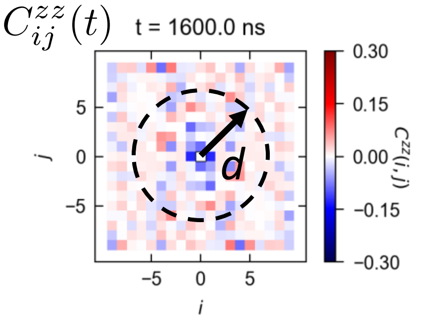
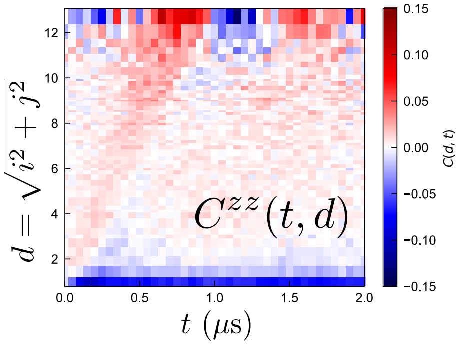
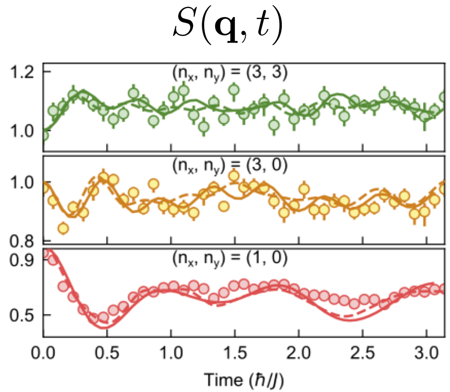
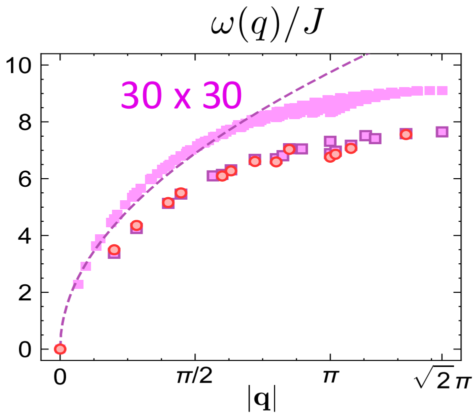


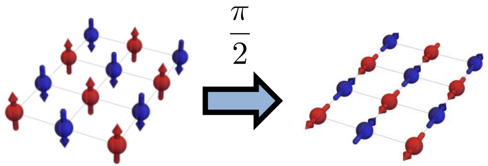




**AFM**

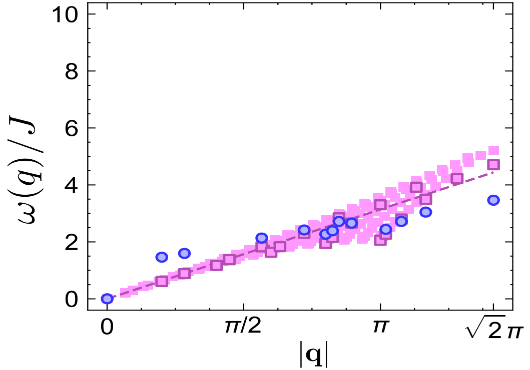
$J_{60S-60P} > 0$

FM coupling




**Highest excited** state $H_{\mathrm{XY}}$ = **ground state** $-H_{\mathrm{XY}}$

Same dynamics $-H_{\mathrm{XY}}/H_{\mathrm{XY}}$




$1/r^3$ Interaction modifies dispersion!!

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2. Out-of-equilibrium dynamics
   - Quench dynamics in Ising model: thermalization or not…
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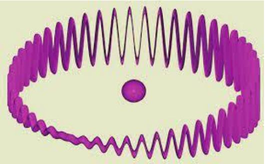
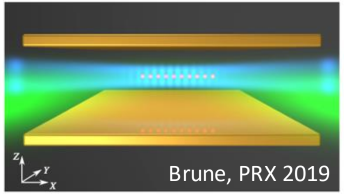
3. Outlook: what we did not discuss… & beyond

# Outlook: what we did not discuss…

**New developments**: circular Rydberg states ⇒ lifetimes > 50 s…






Brune (Paris): arXiv:2407.04109, Nat. Phys. 2022
Covey (Urbana Champaign)
Thompson (Princeton)
Meinert & Pfau (Stuttgart)…

overhead:
Rydberg trapping

High precision quantum simulation: validation of the simulation

Article

**Benchmarking highly entangled states on a 60-atom analogue quantum simulator**

https://doi.org/10.1038/s41586-024-07173-x

Received: 18 August 2023

Adam L. Shaw[1,5✉], Zhuo Chen[2,3,5], Joonhee Choi[1,4,5], Daniel K. Mark[2,5], Pascal Scholl[1], Ran Finkelstein[1], Andreas Elben[1], Soonwon Choi[2✉] & Manuel Endres[1✉]

First attempt of digital quantum simulation (and hybrid analog-digital)

**Variational simulation of the Lipkin-Meshkov-Glick model on a neutral atom quantum computer**

R. Chinnarasu,[1] C. Poole,[1] L. Phuttitarn,[1] A. Noori,[1,2] T. M. Graham,[1] S. N. Coppersmith,[3,1] A. B. Balantekin,[1] and M. Saffman[1,4]

arXiv:2501.06097

**Probing topological matter and fermion dynamics on a neutral-atom quantum computer**

Simon J. Evered[1,*], Marcin Kalinowski[1,*], Alexandra A. Geim[1], Tom Manovitz[1], Dolev Bluvstein[1], Sophie H. Li[1], Nishad Maskara[1], Hengyun Zhou[1,2], Sepehr Ebadi[1,3], Muqing Xu[1], Joseph Campo[2], Madelyn Cain[1], Stefan Ostermann[1], Susanne F. Yelin[1], Subir Sachdev[1], Markus Greiner[1], Vladan Vuletić[4], and Mikhail D. Lukin[1,†]

arXiv:2501.18554

# Digital quantum simulation: resource estimates…

Quantum Science and Techno. **7**, 045025 (2022)

Number of *perfect* gates to reproduce current *imperfect* analog simulation

| Gate | Gate Count | Depth |
|---|---|---|
| CNOT | $1.7 \times 10^5$ | $8.4 \times 10^3$ |
| $R_Z(\theta)$ | $6.8 \times 10^4$ | $6.7 \times 10^2$ |

TABLE I. Gate count and depth estimates for digital quantum simulation of the Hubbard model with $J\tau = 2.7$, $M = 100$ and $tJ = 10$.

*M* sites

| Gate | Gate Count | Depth |
|---|---|---|
| CNOT | $1.6 \times 10^3$ | $5.5 \times 10^2$ |
| $R_Z(\theta)$ | $2.1 \times 10^4$ | $3.5 \times 10^2$ |

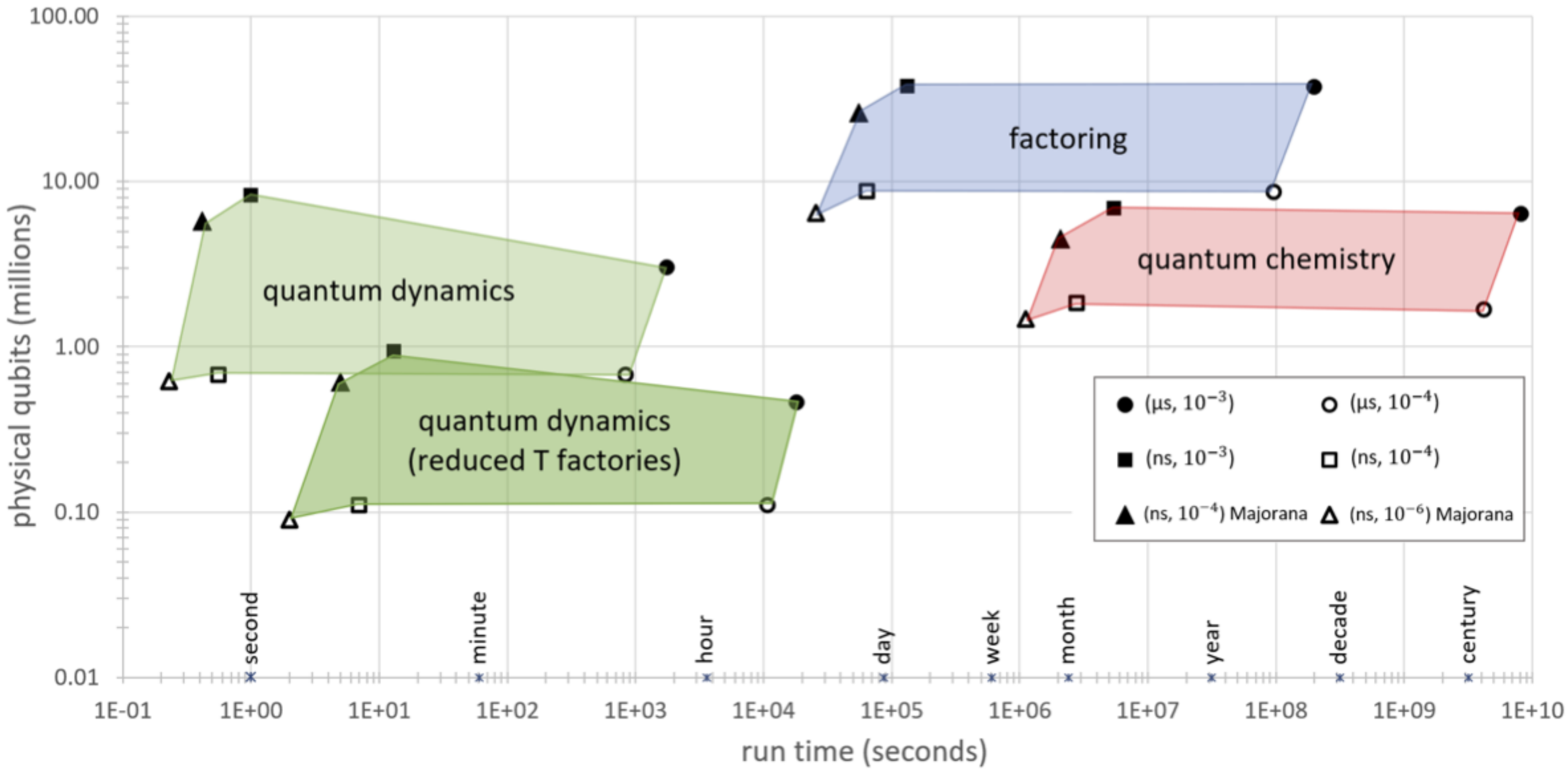
TABLE III. Gate count and depth estimates for digital quantum simulation of the nearest neighbour Ising model with $J\tau = 2.6$, $M = 100$, $tJ = 10$.

| Gate | Gate Count | Depth |
|---|---|---|
| CNOT | $6.9 \times 10^5$ | $1.4 \times 10^4$ |
| $R_Z(\theta)$ | $3.5 \times 10^5$ | $7.0 \times 10^3$ |

TABLE II. Gate count and depth estimates for digital quantum simulation of the long-range Ising model with $J\tau = 2.6$, $M = 100$ and $tJ = 10$.

Numbers explode when analog errors $\rightarrow 0$

# Digital quantum simulation: resource estimates…

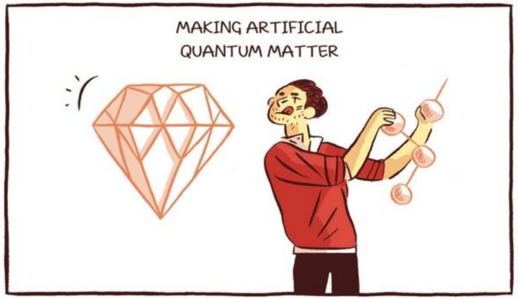



arXiv:2211.07629v1

# Many-body physics with synthetic systems or Quantum Simulation?

Experiments are imperfect… ⇒ **Not a pristine quantum simulation** of a model…

Study the noisy many-body system for itself…

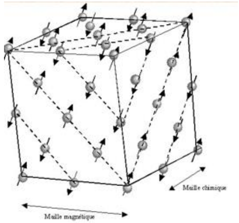



Héloïse Chochois, Labex PALM

**Use "toy many-body systems" to**

- Develop intuition ("simple to complex", noise…)
- Trigger new theoretical methods
- Generate "interesting" quantum states (squeezed…)

**Understand better "real" systems?**

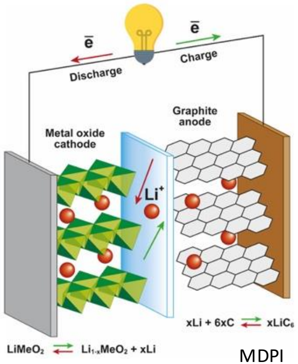



Strongly correlated matter

**Develop applications?**

Fraunhofer

Material



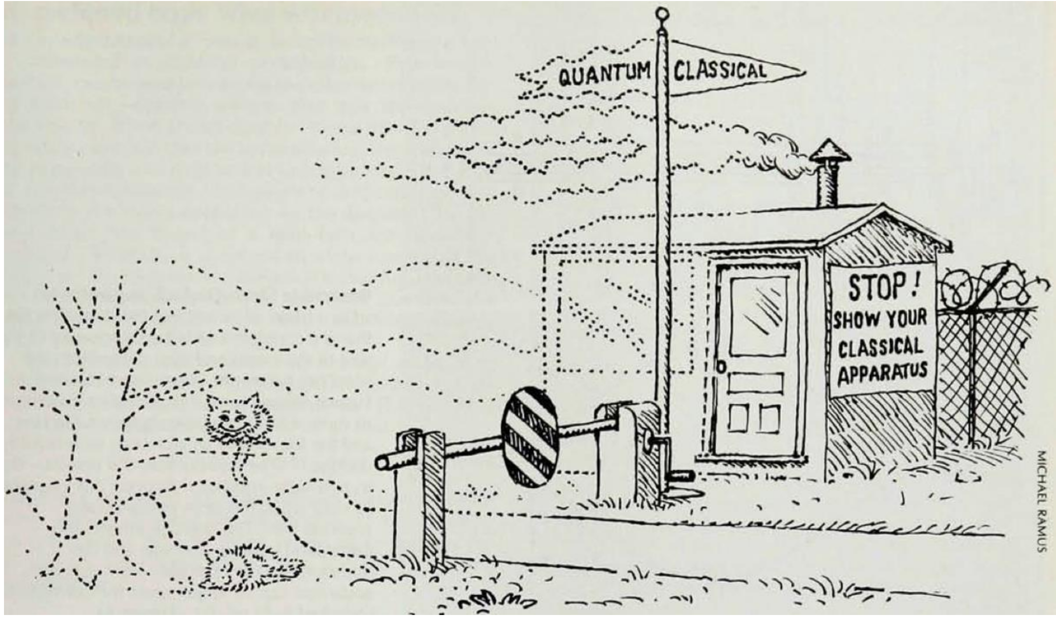

MDPI

Chemistry

Catalysis

…

# How large can a quantum system be?




Zurek, Physics Today 1991