





Exploring many-body physics

with arrays of Rydberg atoms (I)



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The context: "many-body problem"

Goal: Understand ensembles of *strongly* interacting quantum particles









superfluidity

superconductivity

magnetism

neutron star

Questions: phase diagram, excitation, dynamics, ...

The equation to solve:
$$i\hbar \frac{\partial \Psi}{\partial t} = H_{tot}\Psi$$

$$H_{tot} = \sum_{i=1}^{N} -\frac{\hbar^2}{2m_i} \nabla_i^2 + \sum_{i=1}^{N} \sum_{j \neq i} \frac{q_i q_j}{r_{ij}} + \frac{\mu_B^2}{r_{ij}^3} \mathbf{s}_i \cdot \mathbf{s}_j \qquad \text{Very, very, very, very}$$
well known...

Difficulty: exponential scaling of dim $\mathcal{H} \sim d^N$ Record *ab-initio* for s = $\frac{1}{2}$: $N \lesssim 50$

The context: "many-body problem"

Goal: Understand ensembles of *strongly* interacting quantum particles



The context: "many-body problem"



One approach: many-body physics with synthetic quantum systems





Int. J. Theo. Phys. 21 (1982)



Quantum simulation

Georgescu, Rev. Mod. Phys. (2014)

Well-controlled quantum systems implementing many-body Hamiltonians = quantum simulator

Larger tunability than "real" systems (geometry, interactions...)

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New types of probe & methods (e.g. out-of-equilibrium)

A new way to look at many-body using quantum information concepts (entanglement...)

Engineering with individual quantum systems (examples)









Trapped ions

Atoms in optical lattices

Atoms in tweezer arrays

Supercond. Circuits IBM, Google...

Scalable: beyond 100 particles ; potential > 1000

Addressability: local manipulations and measurement

 $\langle \sigma^{\alpha}_i \rangle, \langle \sigma^{\alpha}_i \sigma^{\beta}_j \rangle, \ldots$

Programmable: controlled geometry, interactions...

Engineering with individual quantum systems (examples)



Scalable: beyond 100 particles ; potential > 1000

Addressability: local manipulations and measurement $\langle \sigma_i^{\alpha} \rangle, \langle \sigma_i^{\alpha} \sigma_j^{\beta} \rangle, ...$

Programmable: controlled geometry, interactions...



Lecture 1: Arrays of atoms & "Rydbergology" Rydberg Interactions and spin models Engineering many-body Hamiltonians

Lecture 2: Examples of quantum simulations in and out-of-equilibrium: quantum magnetism

Outline – Lecture 1

- 1. Arrays of individual atoms in optical tweezers
- 2. Basics of Rydberg physics and their interaction
- 3. Interaction between Rydberg atoms and spin models
 - "Natural": Ising and XY Hamiltonians
 - Hard-core bosons and *t J* model
 - Floquet engineering of XYZ models

A single Rb atom in an optical tweezer



Grangier (2001) Sortais (2007)

A single Rb atom in an optical tweezer Grangier (2001) 1 mK Sortais (2007) \bigcirc **Fluorescence** 1 µm 780 nm Reservoir = laser-cooled Rb atoms T ~ 100 μK 100 fluorescence 80 1 atom Non-deterministic 60 40 single-atom source 0 atom 20 0 8.5 9.0 9.5 10.0 10.5 11.0 11.5 Times (s)

Single-atom trapping zoo (2024)



sciencenotes.org



Atoms in arrays of optical tweezers (single-shot images)

1D





2D

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L. da Vinci

Barredo, Nature 2016; Schymik, PRA 2020, 2022; PRAppl. 2021

Atoms in arrays of optical tweezers (single-shot images)

1D

2D



Barredo, Nature 2016; Schymik, PRA 2020, 2022; PRAppl. 2021

Atoms in arrays of optical tweezers (single-shot images)



Barredo, Nature 2016; Schymik, PRA 2020, 2022; PRAppl. 2021

arXiv:2412.14647

Also: Weiss, Nature (2018); Ahn, Opt. Exp (2016)

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"Rydberg atom" = a highly excited atom (e.g. Rb)



Browaeys, Barredo, Lahaye, J. Phys. B 2016

principal quantum number n

"Rydberg atom" = a highly excited atom (e.g. Rb)





\Rightarrow Exaggerated properties:

- strong interaction
- strong coupling to fields (DC, MW)

Interactions between Rydberg atoms



Interactions between "real" Rydberg atoms



 $R = 10 \ \mu \mathrm{m} \Rightarrow V_{\mathrm{int}}/h \sim 1 - 10 \ \mathrm{MHz} \Rightarrow \mathsf{timescales} < \mu \mathsf{sec}$

A fruitful idea: the Rydberg blockade

D. Jaksch, PRL **85**, 2208 (2000) M. D. Lukin, PRL **87**, 037901 (2001)



If $\hbar\Omega \ll V(R)$: no excitation of $|rr\rangle \Rightarrow$ blockage

A fruitful idea: the Rydberg blockade

D. Jaksch, PRL **85**, 2208 (2000) M. D. Lukin, PRL **87**, 037901 (2001)



Blockade ⇒ entanglement and gates!!

A fruitful idea: the Rydberg blockade

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1st demonstrations of controlled Rydberg interactions



Nat. Phys. 2009



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Interactions between Rydberg atoms and spin models







 $C_6 \propto n^{11} \Rightarrow$ switchable interaction Ground state: n = 5Rydberg: n = 50 > $\times 10^{11}$

 $\hat{H}_{int} = \frac{C_6}{R^6} \hat{n}_1 \hat{n}_2 \sim J \hat{\sigma}_1^z \hat{\sigma}_2^z$ Rydberg $n_{1,2} = 1$

Rydberg $n_{1,2} = 1$ Ground state $n_{1,2} = 0$







$$H = \frac{\hbar\Omega}{2} \sum_{i} \hat{\sigma}_x^i + \hbar\delta \sum_{i} \hat{\sigma}_z^i + \sum_{i < j} \frac{C_6}{R_{ij}^6} \hat{n}_i \hat{n}_j$$



Transverse Field Ising model:

$$\begin{split} H &= \frac{\hbar\Omega}{2}\sum_{i}\hat{\sigma}_{x}^{i} + \hbar\delta\sum_{i}\hat{\sigma}_{z}^{i} + \sum_{i < j}\frac{C_{6}}{R_{ij}^{6}}\hat{n}_{i}\hat{n}_{j} \\ \text{Laser:} \quad B_{\perp} \qquad B_{\parallel} \qquad \text{spin-spin interactions} \end{split}$$



Transverse Field Ising model:

$$H = \frac{\hbar\Omega}{2} \sum_{i} \hat{\sigma}_{x}^{i} + \frac{\hbar\delta\sum_{i} \hat{\sigma}_{z}^{i}}{i} + \frac{\sum_{i < j} \frac{C_{6}}{R_{ij}^{6}} \hat{n}_{i} \hat{n}_{j}}{B_{\parallel}}$$
 From Easer: $B_{\perp} = \frac{B_{\parallel}}{B_{\parallel}}$ spin-spin interactions

Controlled parameters:

From negligible to dominant interactions



Transverse Field Ising model:

$$H = \frac{\hbar\Omega}{2} \sum_{i} \hat{\sigma}_{x}^{i} + \frac{\hbar\delta\sum_{i} \hat{\sigma}_{z}^{i}}{i} + \frac{\sum_{i < j} \frac{C_{6}}{R_{ij}^{6}} \hat{n}_{i} \hat{n}_{j}}{B_{\parallel}} \quad \text{From}$$
Laser: $B_{\perp} \qquad B_{\parallel} \qquad \text{spin-spin interactions}$

Controlled parameters: From negligible to dominant

interactions

Browaeys & Lahaye, Nat.Phys. (2020) Barredo PRL (2015), de Léséleuc, PRL (2017)





Browaeys & Lahaye, Nat.Phys. (2020) Barredo PRL (2015), de Léséleuc, PRL (2017)



Browaeys & Lahaye, Nat.Phys. (2020) Barredo PRL (2015), de Léséleuc, PRL (2017)





Browaeys & Lahaye, Nat.Phys. (2020) Barredo PRL (2015), de Léséleuc, PRL (2017)







Non radiative "exchange" of excitation



XY spin model and transport of excitations





 $J|A\rangle\langle B|$

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The Su-Schrieffer-Heeger model

Electronic transport in polyacetylene PRL 42, 1698 (1979)

Now, considered as simplest example of topological model

Asboth, arXiv:1509.02295 Cooper, arXiv:1803.00249

The Su-Schrieffer-Heeger model



Asboth, arXiv:1509.02295 Cooper, arXiv:1803.00249

Implementation of SSH spin chain with Rydberg atoms

Déléseleuc, Science 365, 775 (2019)



Model: tight-binding dimerization: J > J'

J'' = 0 : chiral symmetry \Rightarrow symmetric single particle spectrum





Asboth, arXiv:1509.02295 Cooper, arXiv:1803.00249

Spin excitations interact: hard core bosons

Spin excitation = "particle"

$$\begin{array}{c|c} |\uparrow\rangle & & |P\rangle \\ |\downarrow\rangle & & |S\rangle \end{array}$$

$$\hat{\sigma}^{+} \rightarrow \hat{b}^{\dagger} , \ b^{\dagger}|0\rangle = |1\rangle$$
$$\hat{\sigma}^{-} \rightarrow \hat{b} , \ b|1\rangle = |0\rangle$$
$$[\hat{b}_{i}, \hat{b}_{j}^{\dagger}] = \delta_{ij}$$

Atom cannot carry 2 excitations \Rightarrow excitations = hard-core bosons



 $\Rightarrow The first symmetry protected topological phase...$ Predicted in 2012 Délésele

Déléseleuc, Science 365, 775 (2019)

Doped magnets and t - J model

Hubbard model



Doping = 0 +
$$U \gg t \Rightarrow H_{\rm FH} = \frac{4t^2}{U} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

















Homeier, thesis

.

Mapping onto three Rydberg states: t - J - V model





Tunability: vary θ and r

 $V_{\rm dd}$

 $|S', S\rangle$

$$\begin{split} \hat{H}_{tJV} &= \hat{H}_t + \hat{H}_J + \hat{H}_V \\ \hat{H}_t &= -\sum_{i < j} \sum_{\sigma = \downarrow, \uparrow} \frac{t_\sigma}{r_{ij}^3} \left(\hat{a}_{i,\sigma}^{\dagger} \hat{a}_{j,h}^{\dagger} \hat{a}_{i,h} \hat{a}_{j,\sigma} + \text{ h.c. } \right) \quad \text{Resonant dip.-dip. S, P} \\ \hat{H}_J &= \sum_{i < j} \frac{1}{r_{ij}^6} \left[J^z \hat{S}_i^z \hat{S}_j^z + \frac{J_\perp}{2} \left(\hat{S}_i^+ \hat{S}_j^- + \text{h.c. } \right) \right] \quad \text{vdW S, S': diag. } (J_z) \\ \text{and off-diag. } (J_\perp) \\ \hat{H}_V &= \sum_{i < j} \frac{V}{r_{ij}^6} \hat{n}_i^h \hat{n}_j^h \quad \text{vdW PP: interaction between holes} \end{split}$$



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Interactions between Rydberg atoms and spin models



Interactions between Rydberg atoms and spin models



Engineering the XYZ model with microwaves

Combine:

Naturally occuring XY interaction

Microwave driving





XY model + external (resonant) microwave field:

$$\hat{H}_{\rm driv} = \sum_{i \neq j} \frac{C_3}{R_{ij}} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y) + \frac{\hbar \Omega(t)}{2} \sum_i \cos \varphi(t) \,\hat{\sigma}_i^x + \sin \varphi(t) \,\hat{\sigma}_i^y$$

XYZ model with microwaves: Floquet engineering

Microwave pulse sequence $\Omega(t)$:



$$\frac{C_3}{R_{ij}^3} t_c \ll 1 \quad \Rightarrow \text{averaged hamiltonian:} \quad H_{\text{av}} = \frac{1}{t_c} \int_0^{t_c} H(t) \, dt$$

$$\Rightarrow H_{\rm av} = 2\sum_{i\neq j} \frac{C_3}{R_{ij}^3} \left(\frac{\tau_1 + \tau_2}{t_c} \,\sigma_i^x \sigma_j^x + \frac{\tau_1 + \tau_3}{t_c} \,\sigma_i^y \sigma_j^y + \frac{\tau_2 + \tau_2}{t_c} \,\sigma_i^z \sigma_j^z \right)$$

 \Rightarrow Programmable XYZ Hamiltonians!

Vandersypen, RMP 2006 Choi *et al.*, PRX **10**, 031002 (2020)

Scholl, PRX Quantum 2022



Geier, Science 2021

Scholl, PRX Quantum 2022



Geier, Science 2021

Scholl, PRX Quantum 2022



Geier, Science 2021

Scholl, PRX Quantum 2022



Limitations: finite MW pulse duration + imperfections

Conclusion: many variants of spin Hamiltonians



In various *addressable* geometries: 1D (OBC, PBC), 2D : square, triangle, Kagome...

Warning: mapping is only approximate (on top of uncontrolled parameters)...:

XY has small Ising; neglect quadrupolar interactions; not exactly 2 levels...

Hard to assess the impact...!!