Renormalization group using tensor networks

Lecture 2: The joy of disentangling

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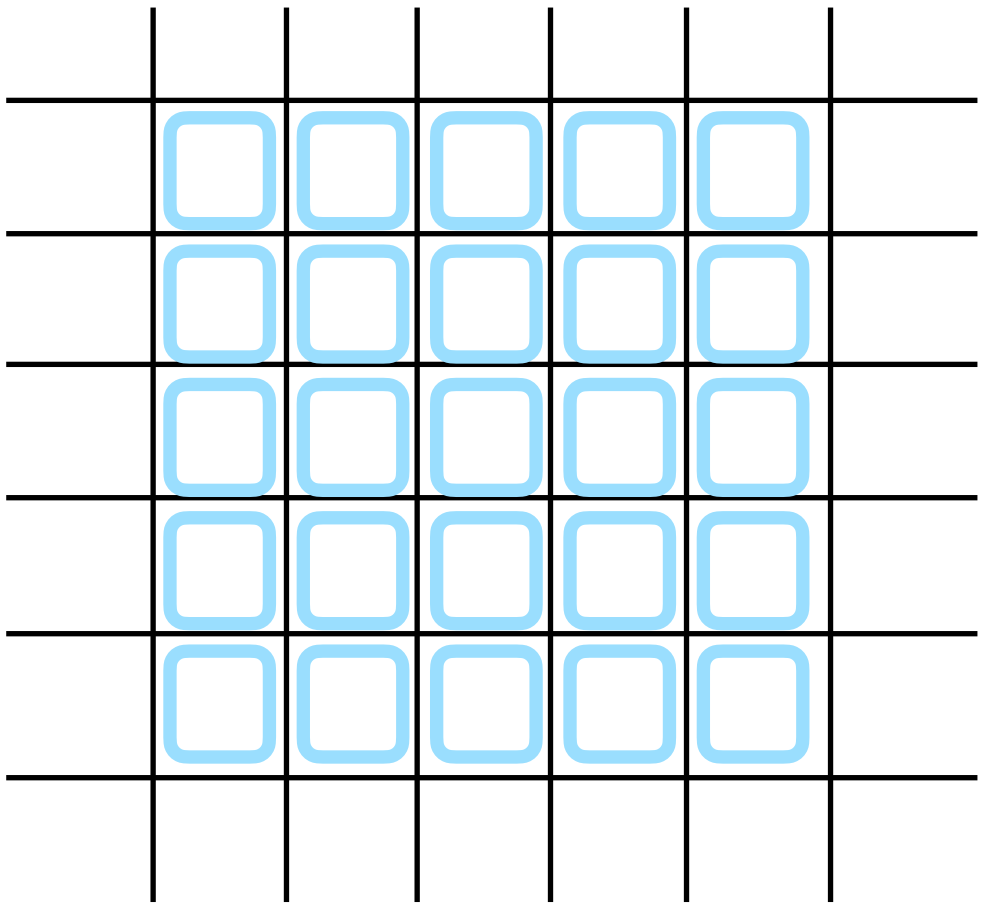
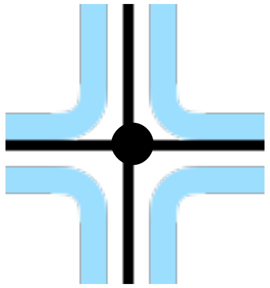
Last time:
Tensor network RG algorithm without disentangling (TRG or HOTRG)
=> after severalRG steps you will get



$A \approx$ 

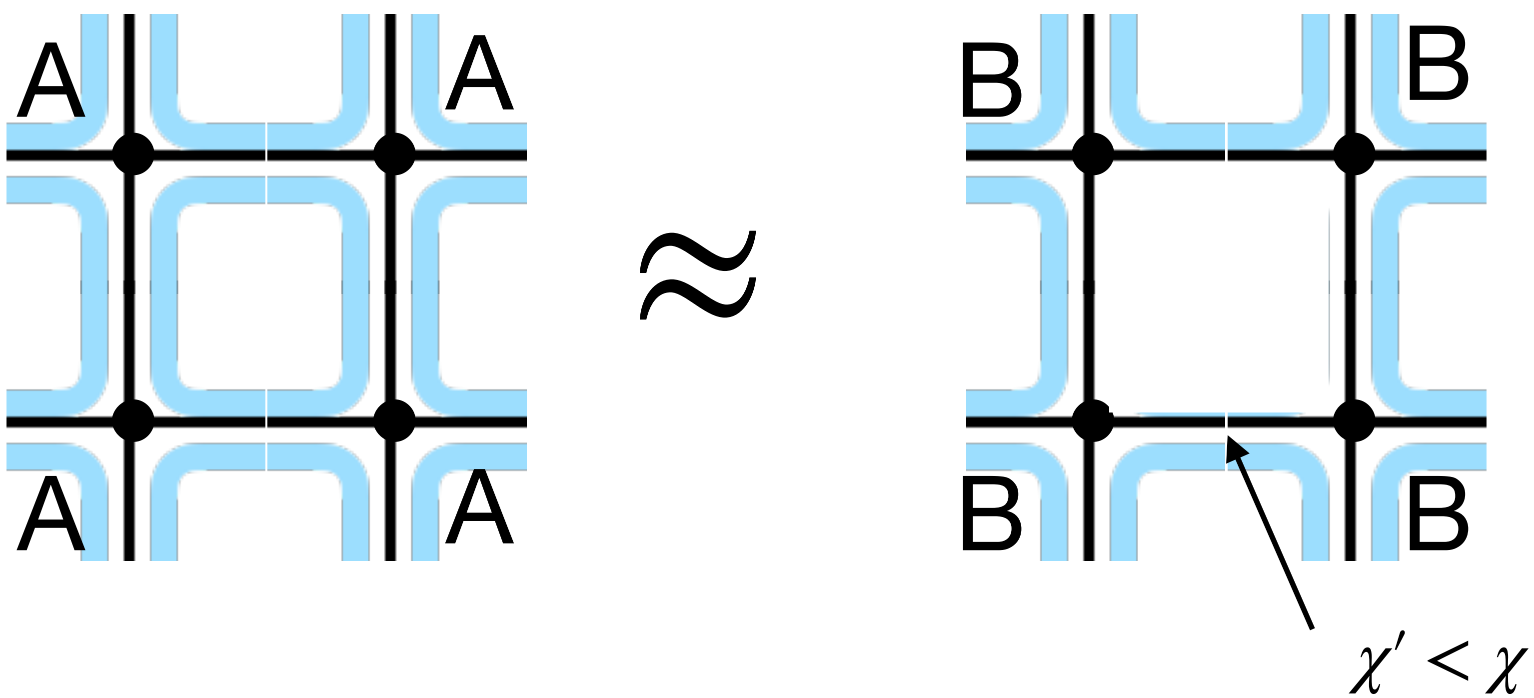
“CDL pollution”

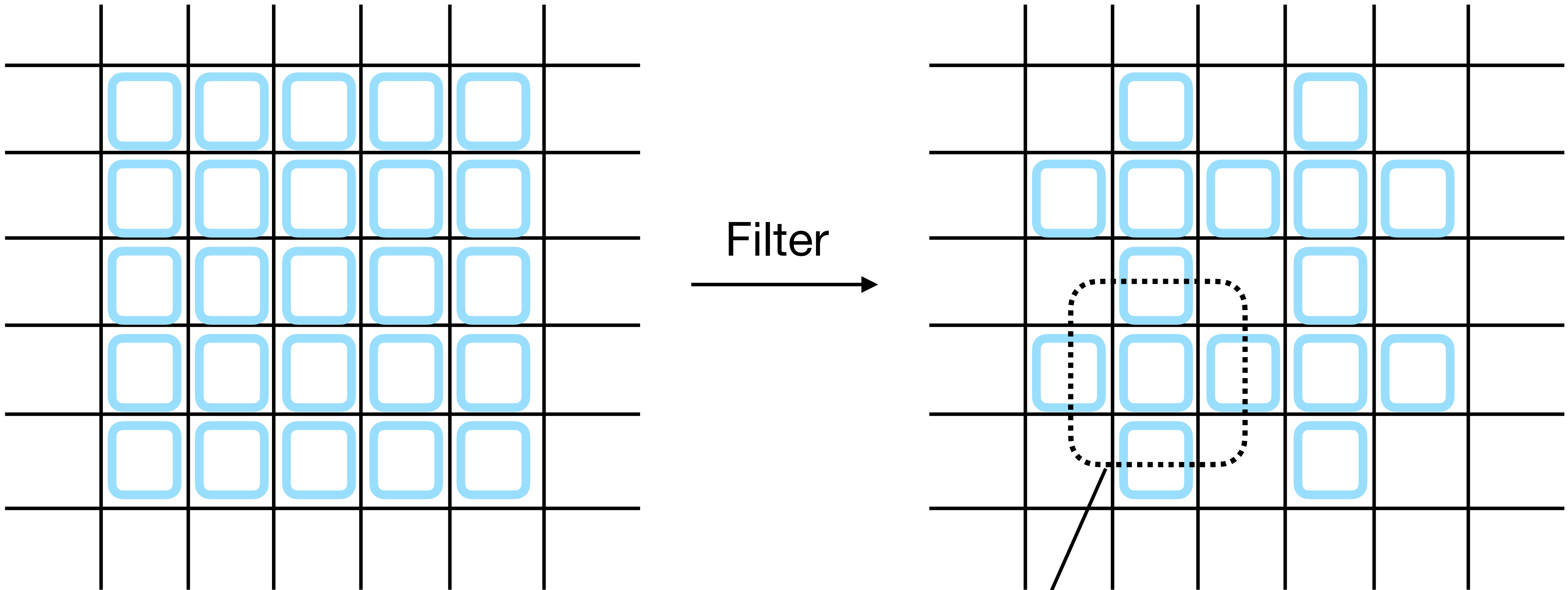
(factorization only approximate)

Need to clean up!

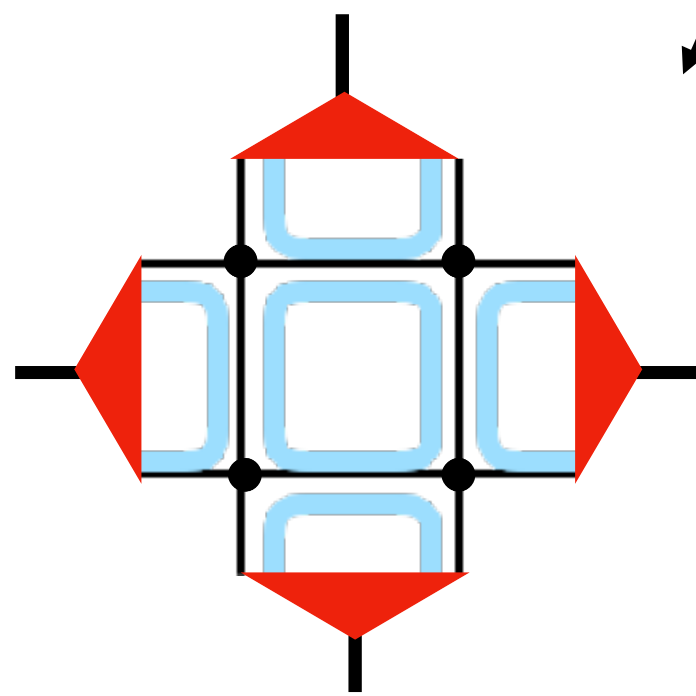
Approach 1: Entanglement filtering

TEFR - Gu Wen 2009
Loop-TNR (Yang, Gu, Wen 2017)
TNR+ (Bal et al 2017)
Gilt (Hauru, Delcamp, Mizera 2018)





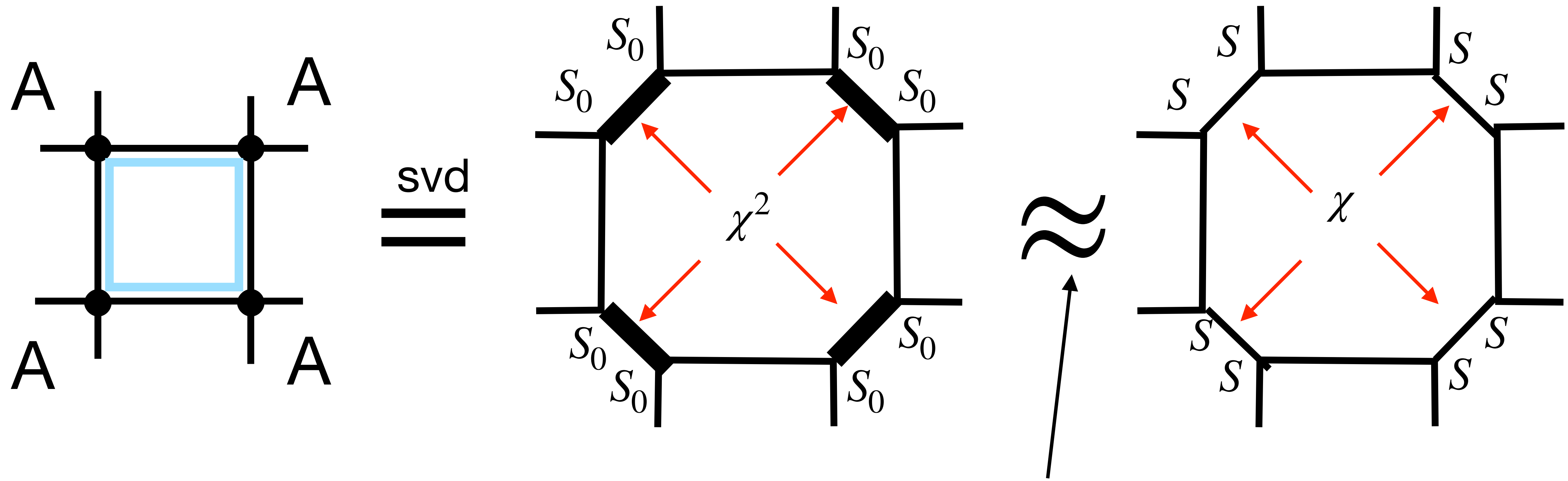
Reconnect on the plaquettes complementary to the filtered ones

$A' =$ 

no CDL in A'

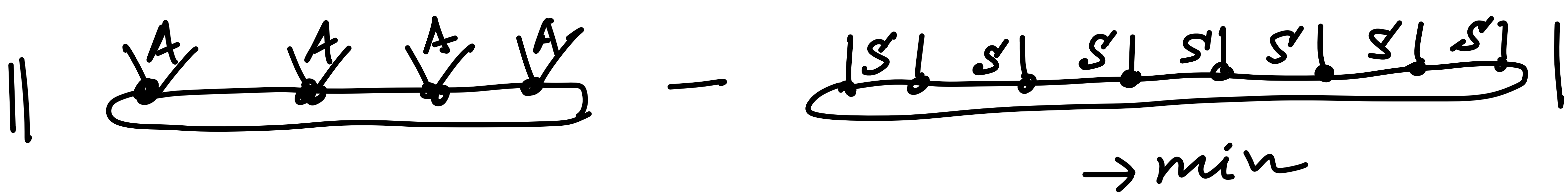
Loop-TNR

Yang,Gu,Wen 2017

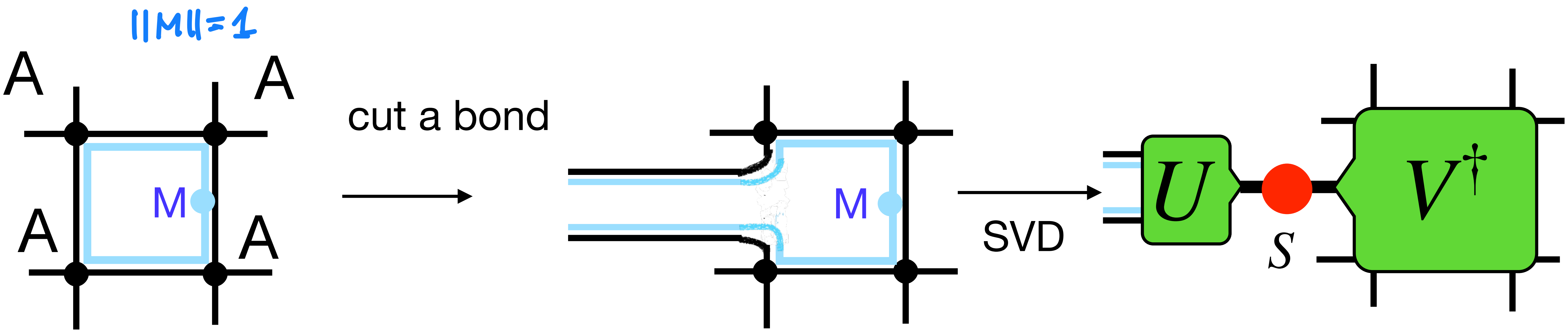


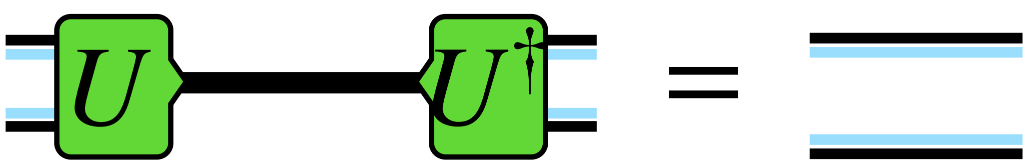
should be possible if there is CDL pollution

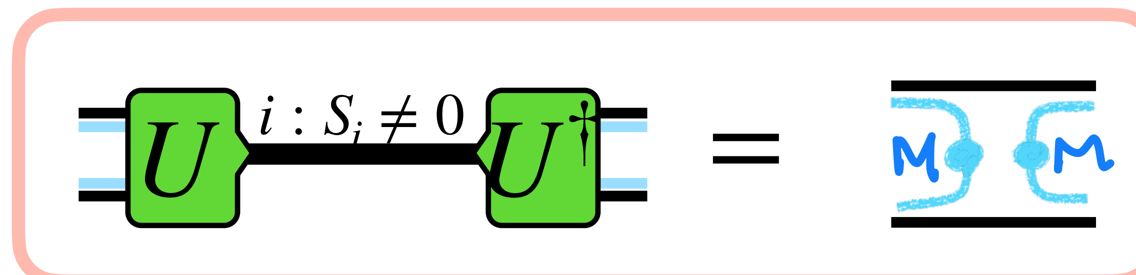
Optimize the whole loop contraction (standard variational MPS) starting from the truncated SVD as the initial approximation



Gilt (Graph Independent Loop Truncation) Hauru, Delcamp, Mizera 2018

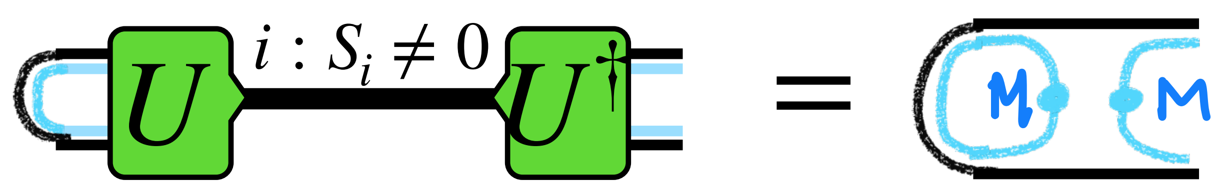






Key Eq

1. Insert into the cut bond



2. Effect: $M \to M^2$

3. Iterate: $M \to diag(1,0,0,...)$

4. Truncate

For more details about Gilt, see talks by

Xinliang Lyu

Gilt for 3D Ising

Nikolay Ebel

Newton method

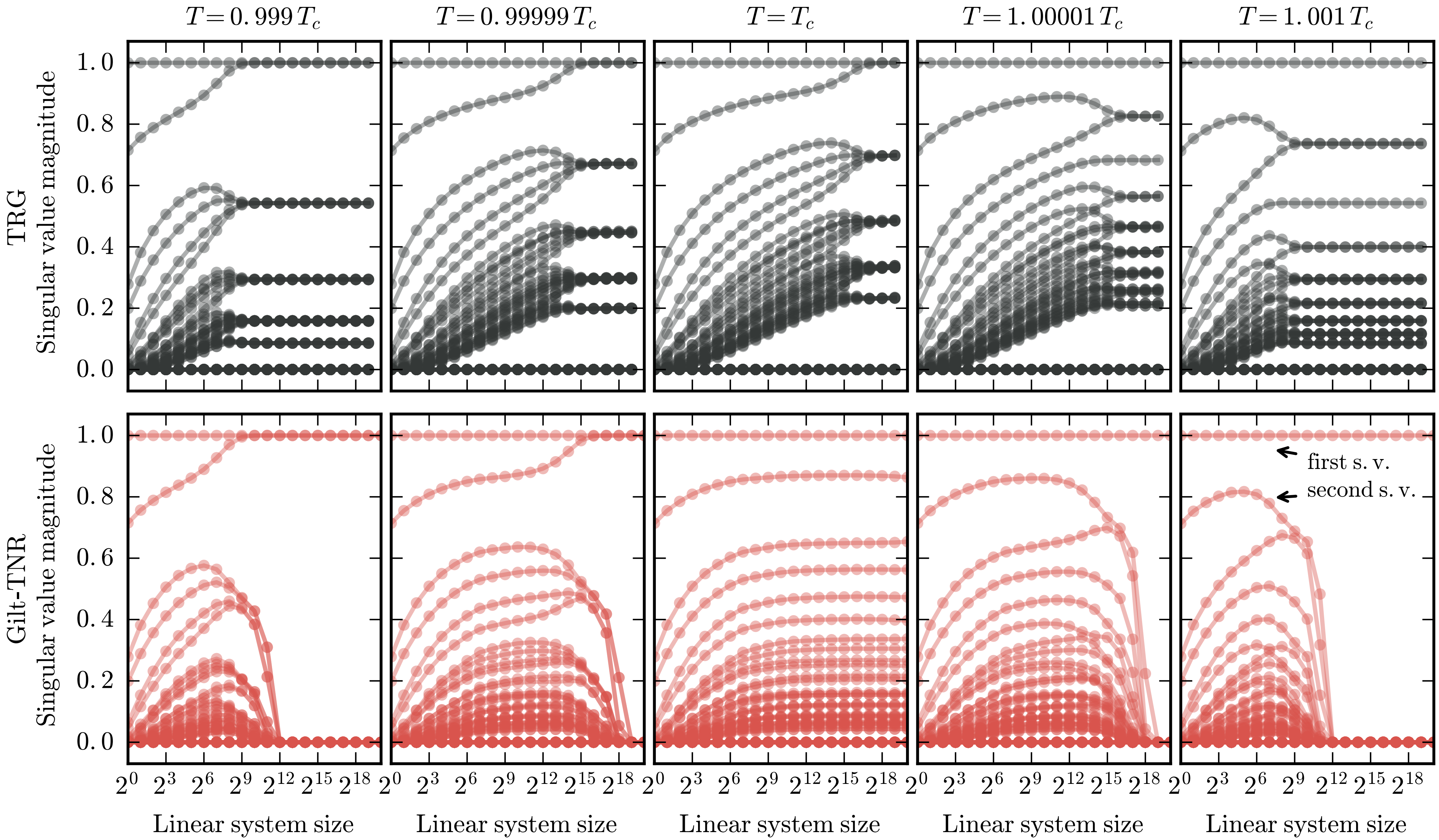
Approach 2: Disentanglers

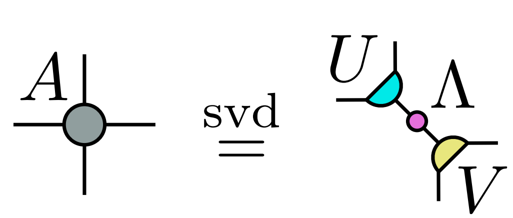
TNR (Evenbly-Vidal 2014)

Postponed for a few slides

RG flow of tensors

Hauru, Delcamp, Mizera 2018

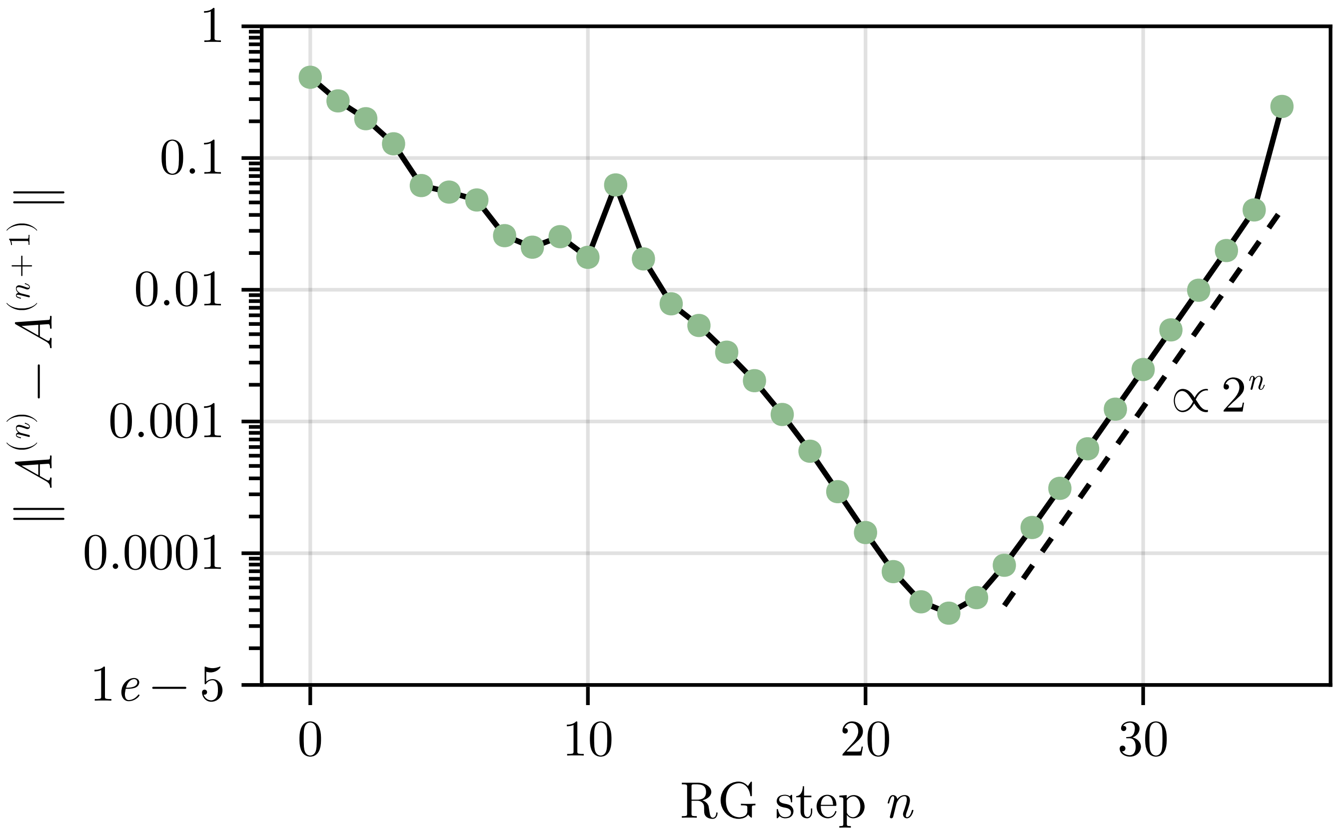




RG flow of tensors - after gauge-fixing

Ebel, Kennedy, S.R. 2408.10312

Gilt algorithm



Flow starting at T=Tc

Results for CFT scaling dimensions

Hauru, Delcamp, Mizera 2018

Exact	TRG $\chi = 120$	TNR $\chi = 24$	Loop-TNR $\chi = 24$	Gilt-TNR $\chi = 120$
0.125	0.124993	0.1250004	0.12500011	0.12500015
1	1.0002	1.00009	1.000006	1.00002
1.125	1.1255	1.12492	1.124994	1.12504
1.125	1.1255	1.12510	1.125005	1.12506
2	2.002	1.9992	1.9997	2.0002
2	2.002	1.99986	2.0002	2.0002
2	2.003	2.00006	2.0003	2.0003
2	2.002	2.0017	2.0013	2.0004

What about $\chi \to \infty$?

Issues:

- reliance on many-step optimization - is the RG map even continuous?

- RG map is inherently defined only for finite χ (optimize the error - no truncation, no error, nothing to optimize)

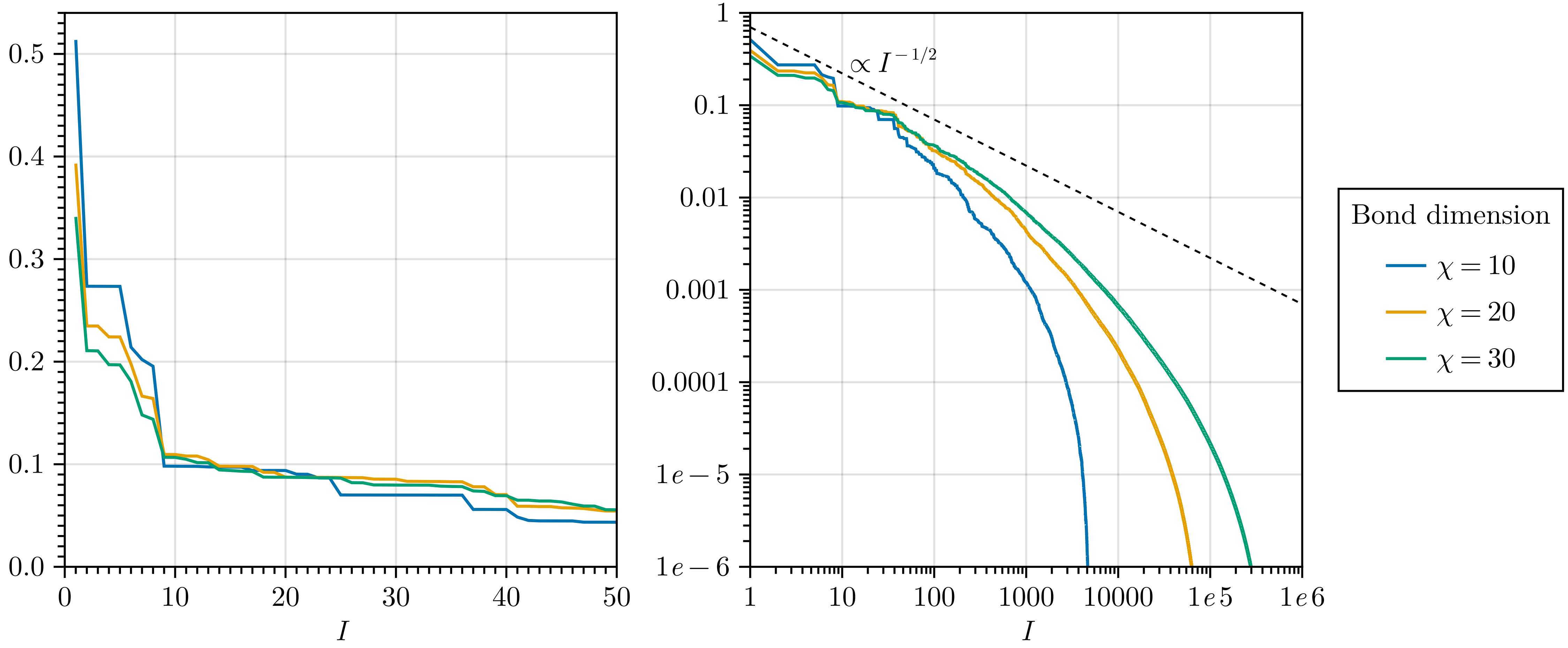
(could reduce this concern by optimizing the entanglement entropy, but not published work

- Does the fixed point tensor remain Hilbert-Schmidt in the $\chi = \infty$ limit? (if not probably did not disentangle enough)

Numerical results for tensor tails as χ increases

Ebel, Kennedy, S.R. 2408.10312

Gilt algorithm



Worrisome!

Precision reconstruction of rational CFT from exact fixed point tensor network

arXiv: 2311.18005

Gong Cheng,[1,2,*] Lin Chen,[3,*] Zheng-Cheng Gu,[4,†] and Ling-Yan Hung[5,‡]

[1] *Department of Physics, Virginia Tech, Blacksburg, VA 24060, USA*
[2] *Maryland Center for Fundamental Physics, University of Maryland, College Park, MD 20740, USA*
[3] *School of Physics and Optoelectronics, South China University of Technology, Guangzhou 510641, China*
[4] *Department of Physics, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong, China*
[5] *Yau Mathematical Sciences Center, Tsinghua University, Haidian, Beijing 100084, China*
(Dated: November 7, 2024)

Claim to have exact fixed point tensor for coarse-graining step RG (no disentangling)

This “fixed point tensor” is defined by cutting the CFT partition function into squares (actually triangles, but this does not matter)

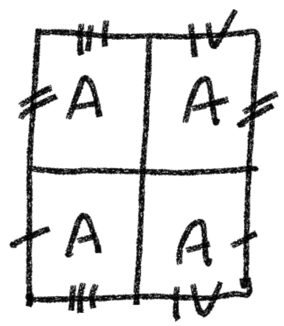
Their “fixed point tensor” is not Hilbert-Schmidt, by a simple argument:
its norm is given by a CFT partition function on a surface with conical defects - log divergent, regularization is required

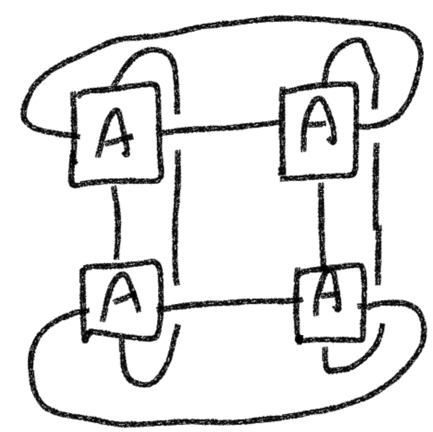
Litmus test: if someone tells you they have an exact fixed point of tensor RG but their map is only defined at the fixed point tensor but not in its neighborhood, they are probably wrong

Open problem 2

Can you cut CFT partition function into squares exactly?

I.e. find an exact **Hilbert-Schmidt** tensor A which represents the CFT torus partition function:

$Z_{CFT}(T^2) =$ 

$=$ 

E.g. for some exactly solvable CFT, like the 2D Ising?

Or maybe free (massless or massive) fermions or bosons?
(finding the disentanglers for a Gaussian theory may be doable)

Exact fixed point project

Tom Kennedy

Nikolay Ebel

Goals:

- Set up a tensor RG map with disentanglers given by explicit formulas, making sense for $\chi = \infty$
- Show that that map converges to high-T, low-T, and critical fixed points

Status:

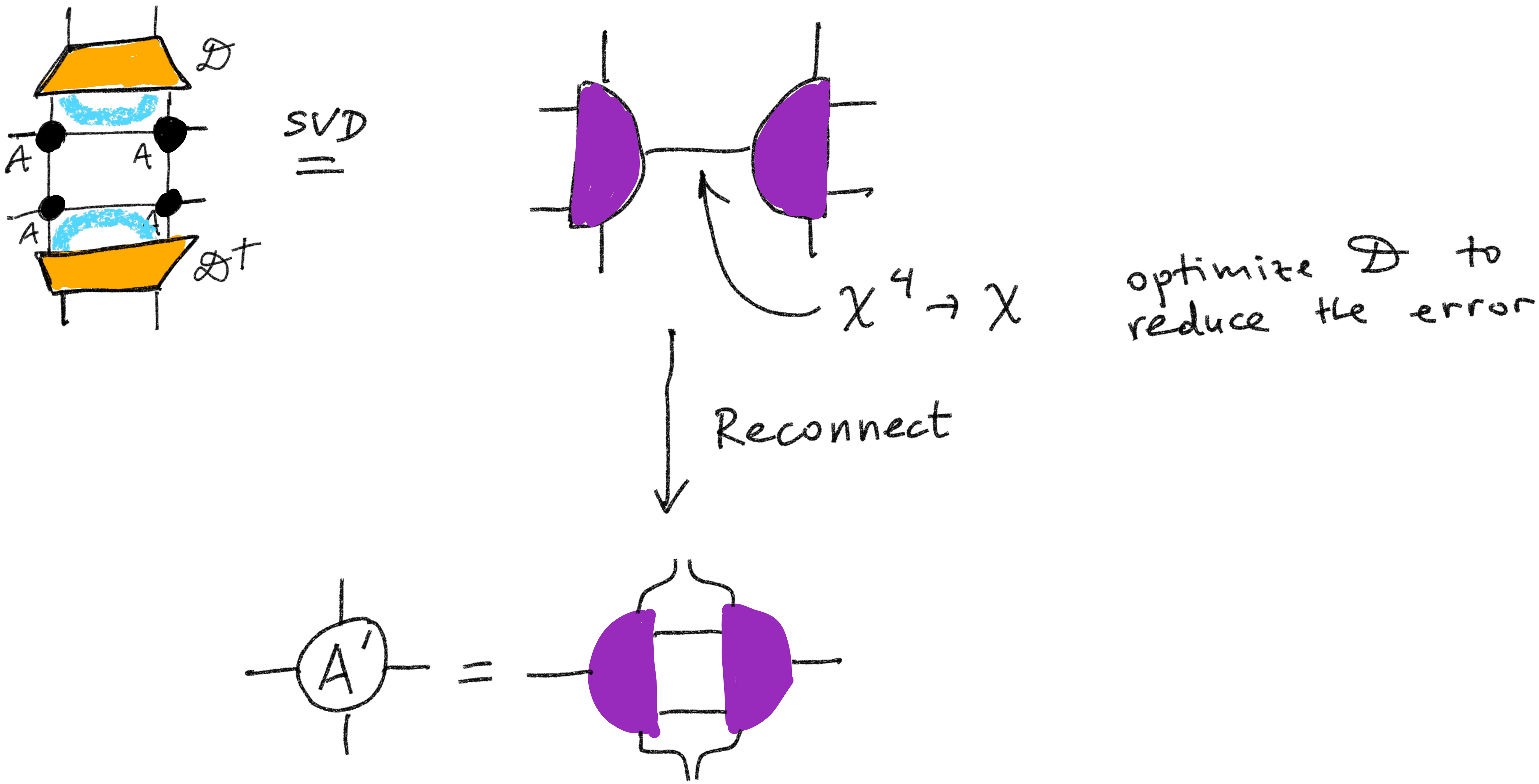
2D High-T, Low-T ✓ T. Kennedy, S.R., J.Statist.Phys. **187** (2022) 33
T. Kennedy, S.R., Annales Henri Poincaré 25, 773–841, (2024)

2D critical

3D High-T ✓ N. Ebel, Annales Henri Poincaré (2024), arXiv: 2408.10312

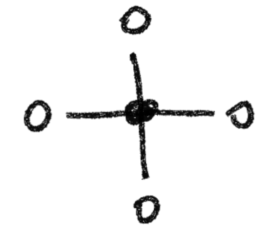
Approach 2: Disentanglers

TNR (Evenbly-Vidal 2014)



T. Kennedy, S.R., J.Statist.Phys. **187** (2022)

High-T rigorous result

A_{HT} :  $= 1 \quad (\chi = 1)$

Perturb: $A = A_{HT} +$  $\leftarrow$ can have arbitrary χ
$B_{0000} = 0$ WLOG

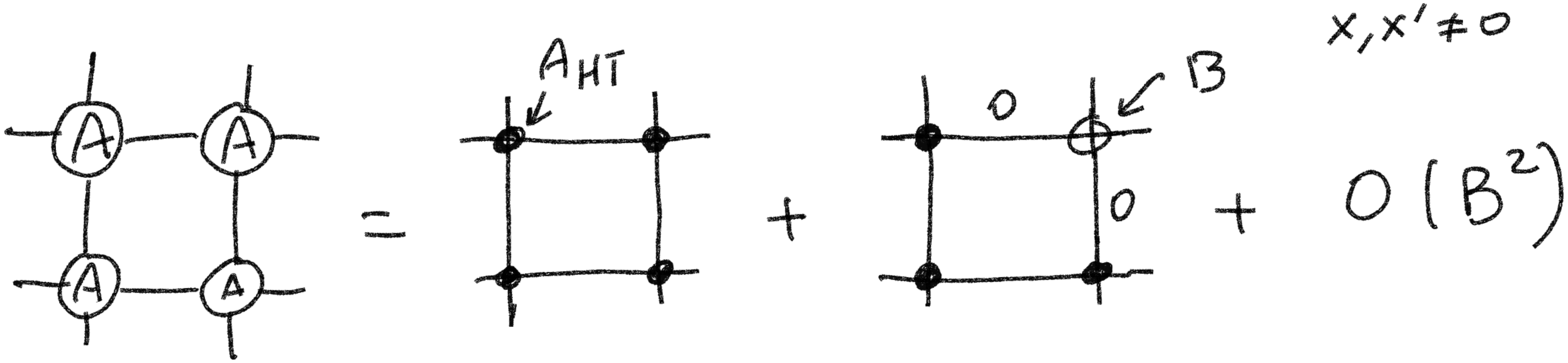
<u>Thm</u> $\exists$ exact tensor network RG transformation s.t.

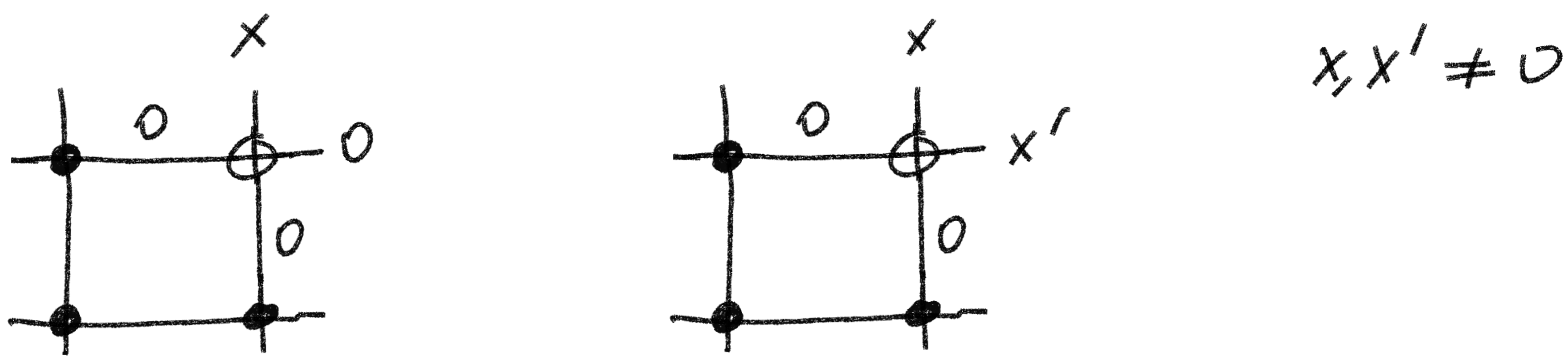
$A' = A_{HT} + B'$

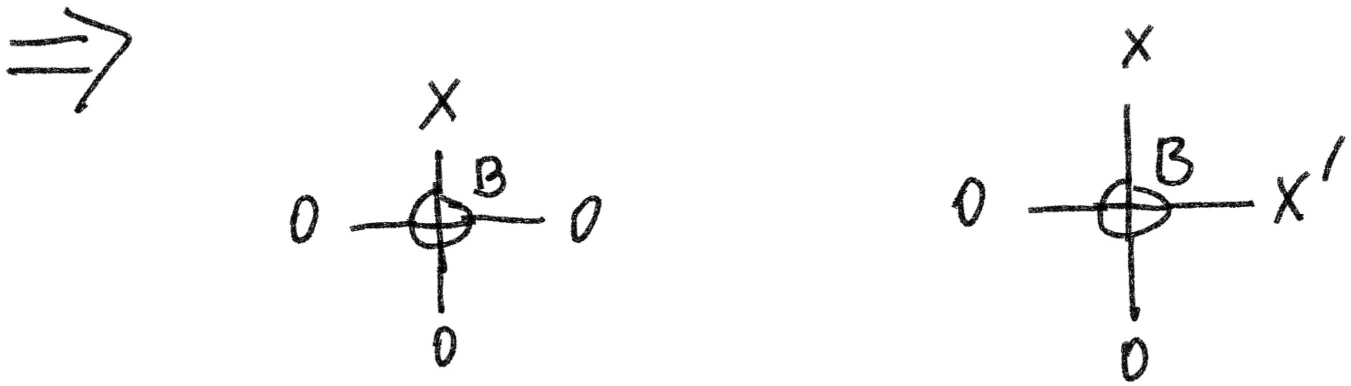
$\|B'\| < \lambda \|B\| \quad (\lambda < 1)$

(for $\|B\|$ sufficiently small)

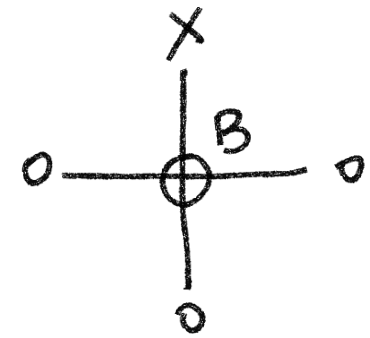
- any bond dimension (even infinite)
- error controlled in the HS norm

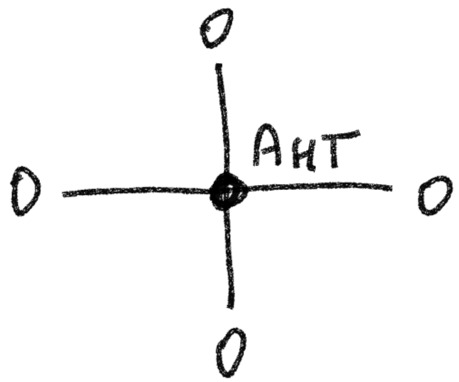




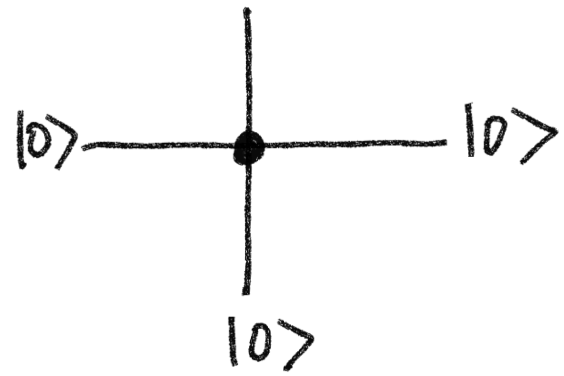


dangerous
(passed to the next step
w/out reducing in size)

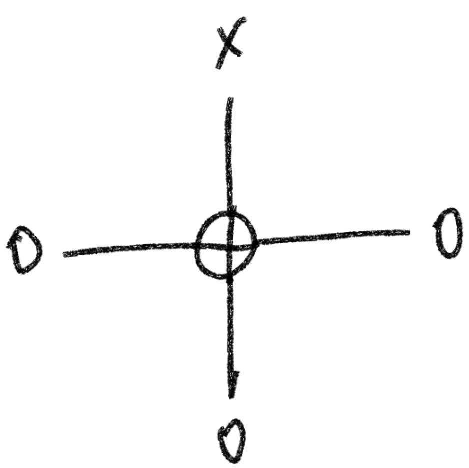
$b_x :=$ 



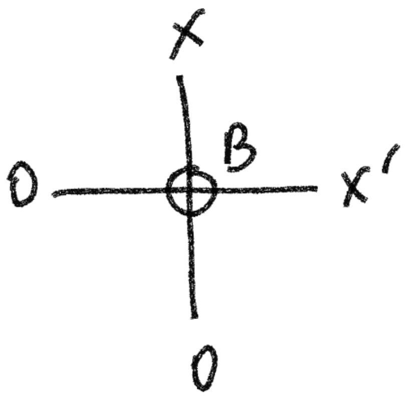
$$|0'\rangle = |0\rangle + \sum_x b_x |x\rangle$$



gauge transformation

=> 

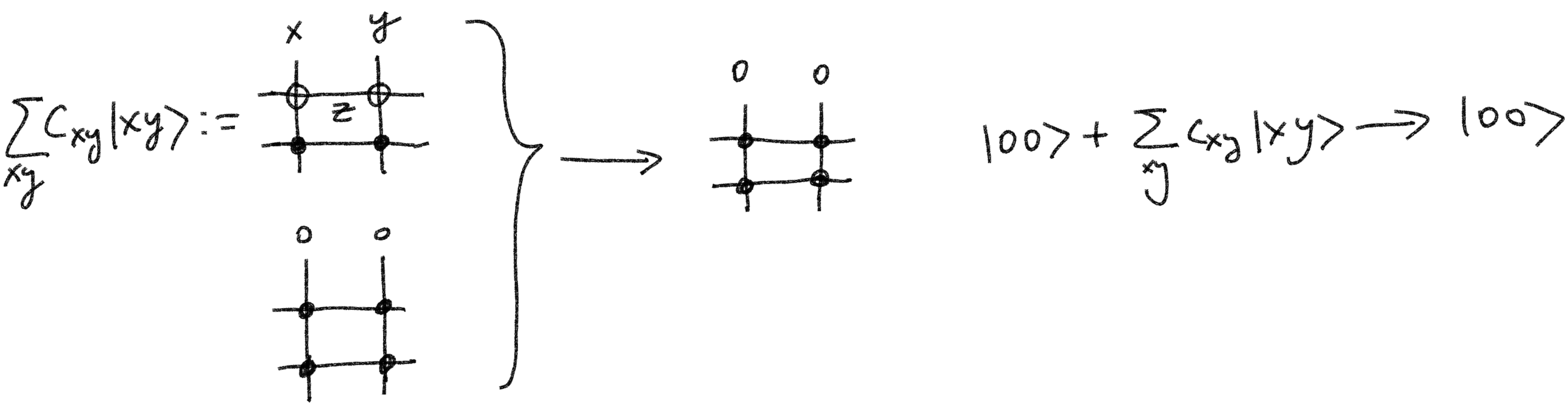
$= O(\varepsilon^2)$ $(\varepsilon = \|B\|)$



to remove this need disentangling

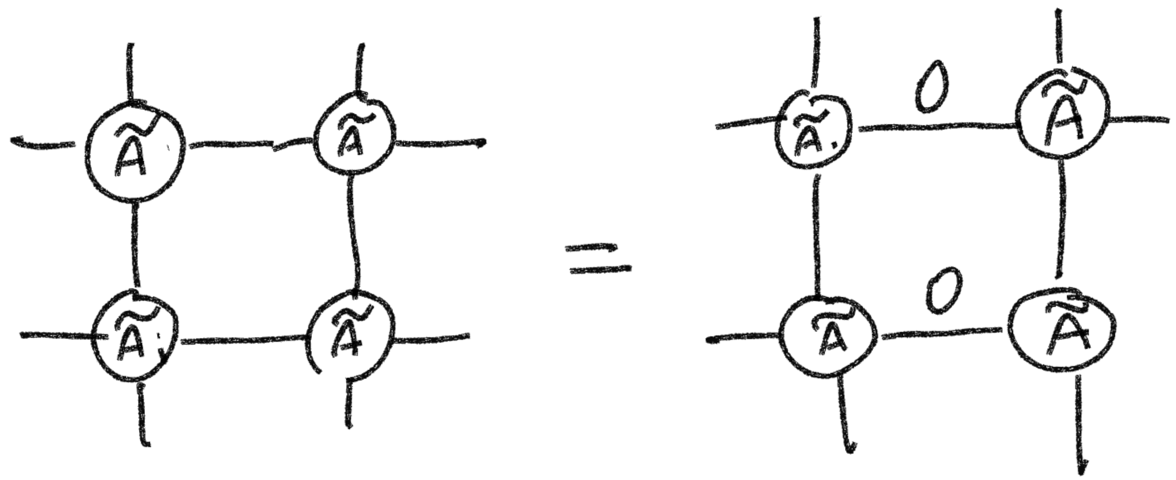
(Today) use a rigorous version of Evenbly-Vidal TNR

T. Kennedy, S.R., J.Statist.Phys. **187** (2022) 33



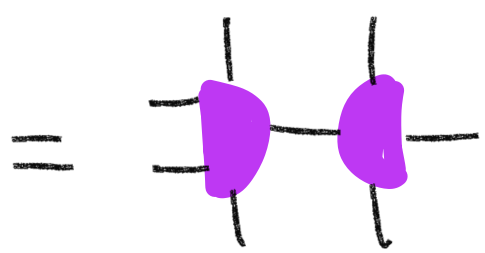
For a rigorous version of Loop-TNR see N. Ebel, Ann. H. Poincaré (2024), 2408.10312

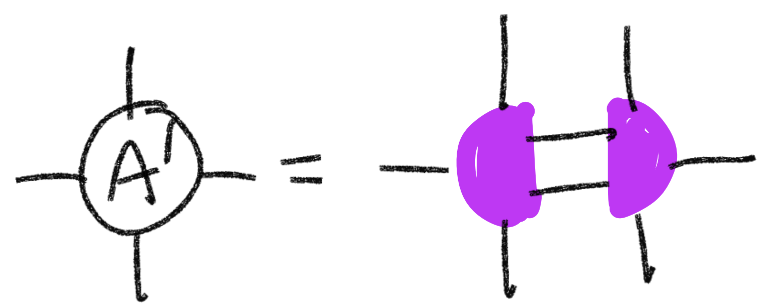
After disentangling:



$$+\ O(\varepsilon^3)$$

When we split and reconnect we get:





$$\Rightarrow\ A' = A_{HT} + O(\varepsilon^{3/2})$$

∎

Omissions

- gauge fixing

- extracting conformal data from the fixed point tensor
  - transfer matrix
  - lattice dilatation operator
  - linearized RG

- Newton method search

- 3D results

See talks by Xinliang and Nikolay