Symmetry defects & gauging for MPS with MPU symmetries

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(based on arXiv: 2502.20257 w/ A. Bochniak & J.I. Cirac)

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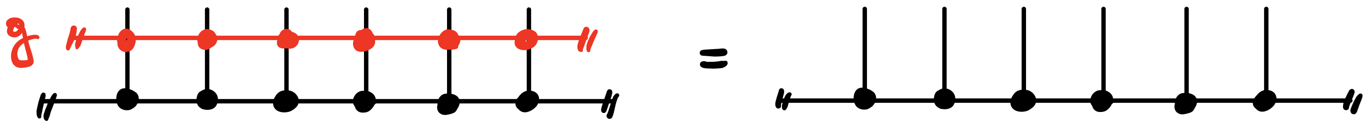
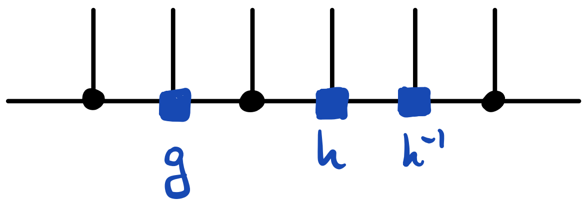
In one slide

Consequences of MPU $\sqrt{}$ finite group symmetry for MPS



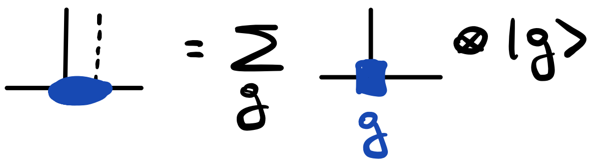
↳ System of defect tensors



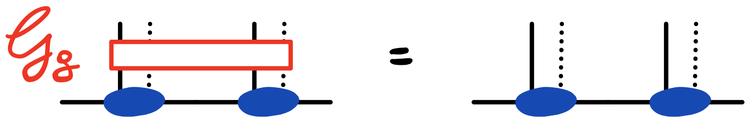
$g, h \in G$

↳ Gauging procedure (Global sym. $\xrightarrow{\text{add d.o.f.}}$ Local sym.)

Gauged tensor



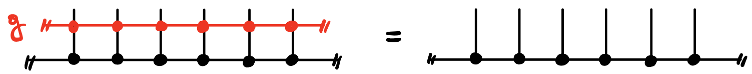
Locally invariant wavefunction



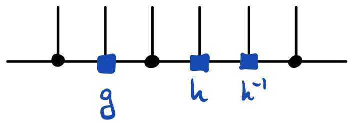
Motivation: entanglement struct. of SPTs, gauge theories, mapping btw. phases

Outline

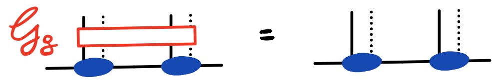
I. Preliminaries



II. Defect systems



III. Gauging



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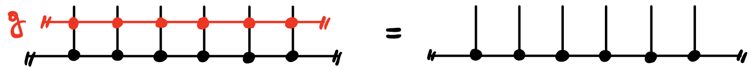

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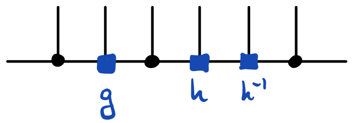

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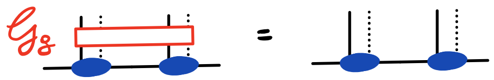
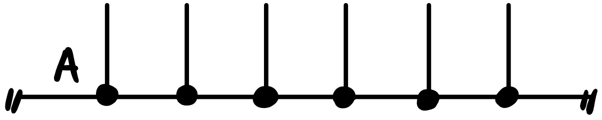
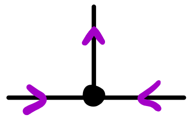
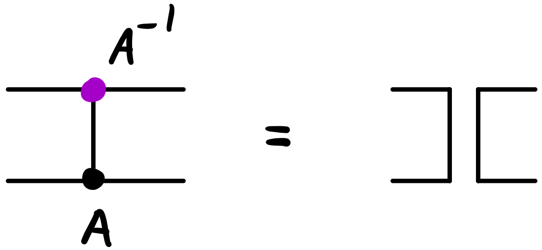


Matrix Product States



Injective

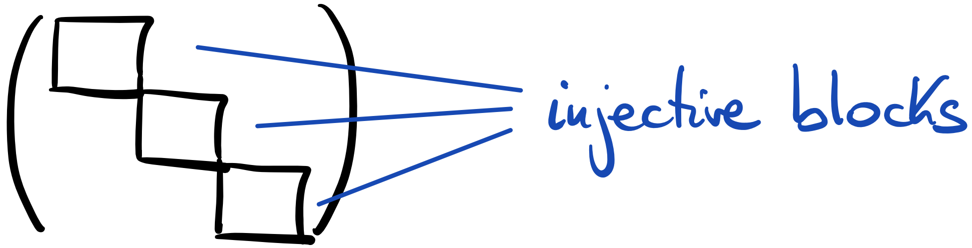




(basic building block)

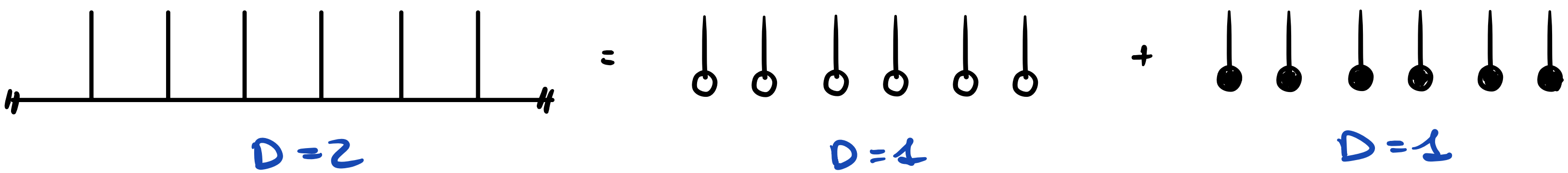
(think u.g.g.s.)

Block-injective $\quad A^i \sim$



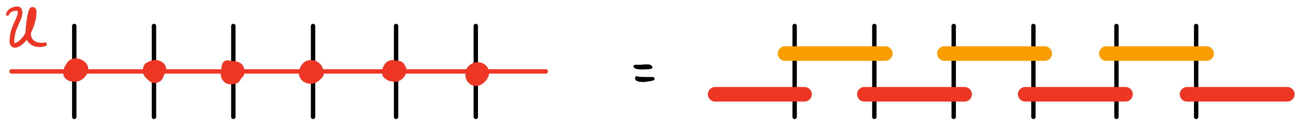
(think SSB)

example: GHZ state $\quad |000\ldots0\rangle + |111\ldots1\rangle$

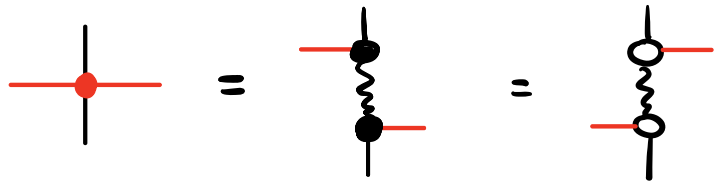


Matrix Product Unitaries

[Cirac et al. '17]
[Chen, Liu, Wen '11]



→ Two-layer circuit decomposition



(for simple MPU, reach by blocking)

Matrix Product Unitaries

[Cirac et al. '17]
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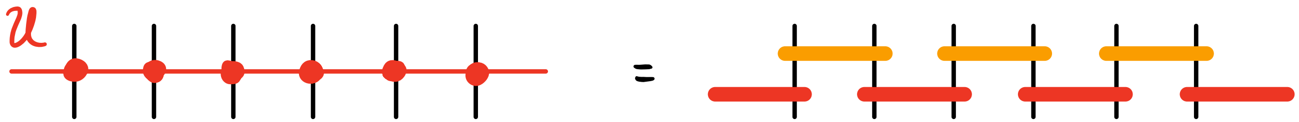

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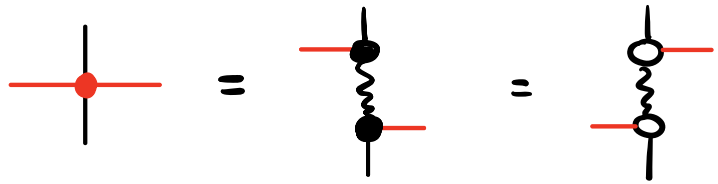
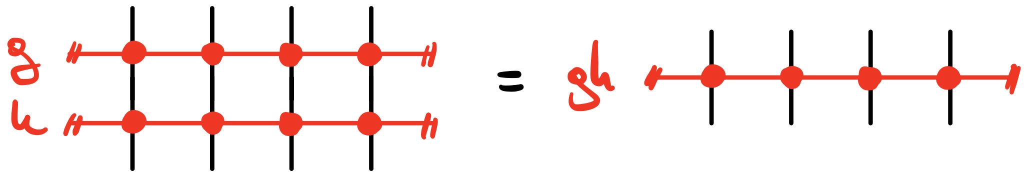
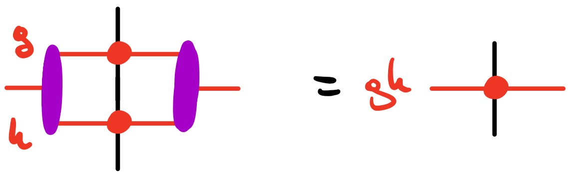


(for simple MPU, reach by blocking)

→ Group representation



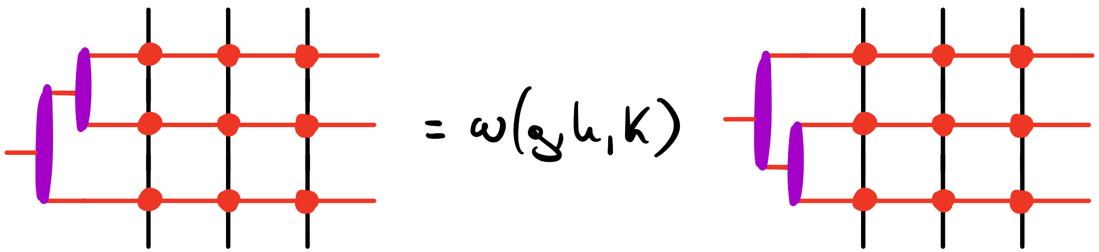
Fusion tensors



(defined up to scalar)

↕

3-cocycle



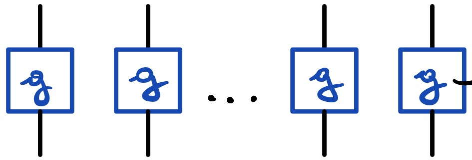
$[\omega] \in H^3(G, U(1))$

$[\omega] \neq 1 \Rightarrow$

$\Rightarrow$ anomaly

Examples of MPU

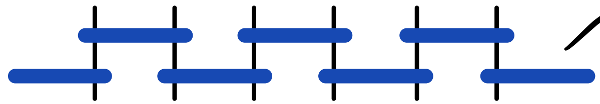
1.) Onsite D=1



U_g onsite representation of G

non anomalous

2.) CZ MPU ($\mathbb{Z}_2$)

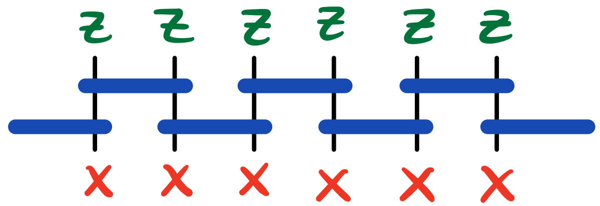


CZ gate

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

nonanomalous

3.) CZX ($\mathbb{Z}_2$)



anomalous!

$\Downarrow$

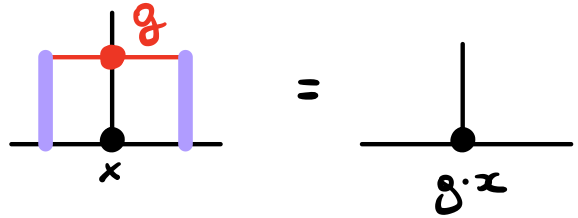
no injective invariant MPS (GHZ invariant, blocks permuted)

MPU acting on MPS

[Garre-Rubio, Lootens, Molnár '22]

$\boxed{G \curvearrowright X = \{x\} \text{ (injective blocks)}}$

Action tensors



(defined up to scalar)

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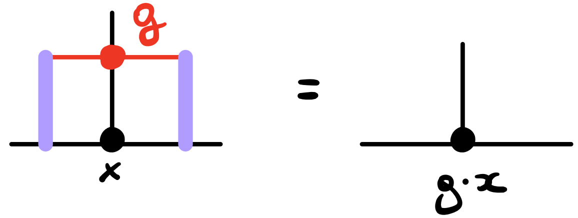
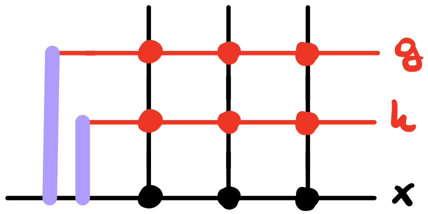
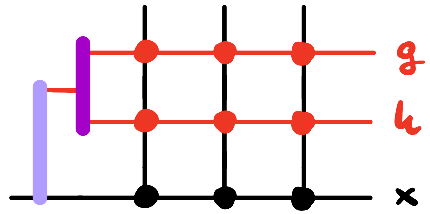


(defined up to scalar)

L-symbols



$= L^{x}_{g,h}$

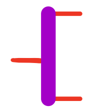


related to ω $\qquad L^{x}_{g,hk} L^{x}_{h,k} = \omega(g,h,k) L^{k\cdot x}_{g,h} L^{x}_{gh,k}$

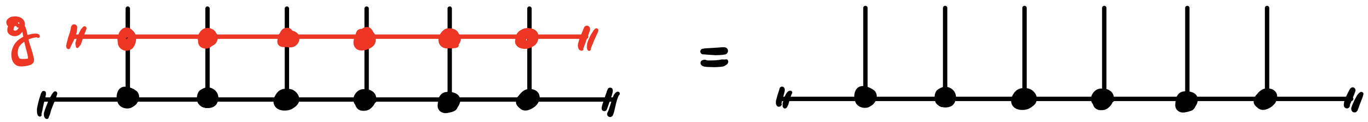
$L = 1$? $\quad (\Rightarrow \omega = 1)$

↳ "Block independence"

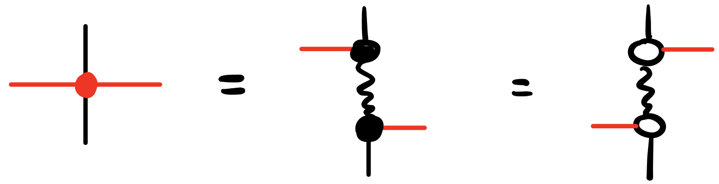
(in the injective case $\quad L^{x}_{g,h} \equiv L_{g,h} \xrightarrow{\text{absorb in}} = 1$)



Takeaways



1. We'll consider injective (u.g.g.s.) and noninjective (SSB) invariant MPS.

2. MPU tensors have nice splittings 

3. MPU group rep's come with some tensors 
and scalars $\omega(g,h,k)$

4. MPU-inv. MPS come with some tensors 
and scalars $L^{x}_{g,h}$

important info

Outline

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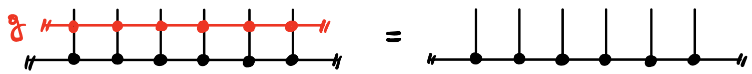

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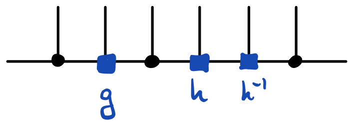

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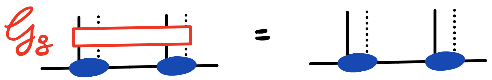
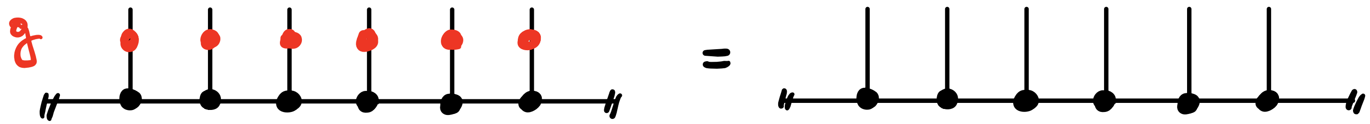
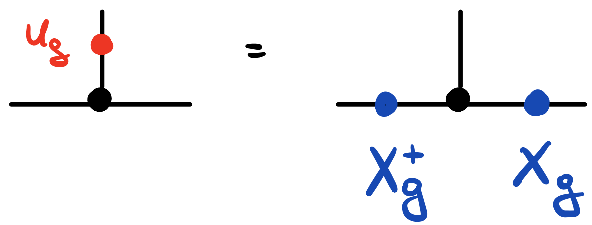


Refresher: Onsite case (D=1)

Injective invariant MPS



Virtual symmetry representation



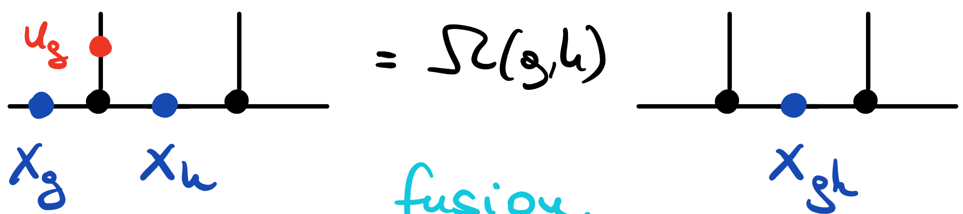
($g \to X_g$ may be projective)

$$X_g X_h = \Omega(g,h) X_{gh}$$

→ pair creation

Symmetry defects / twists

movement

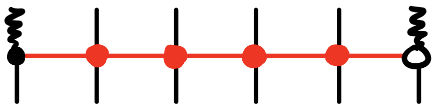


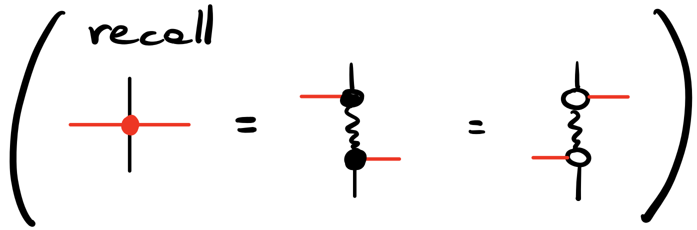
fusion

by local unitaries

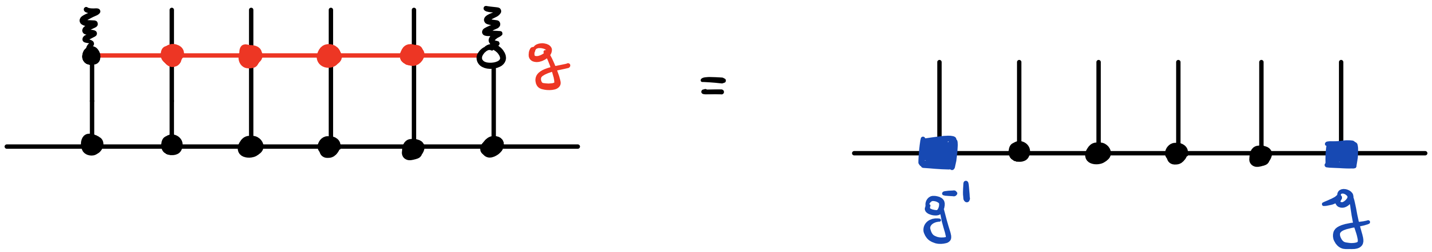
MPU defects (injective case)

Need a spatial truncation of the symmetry

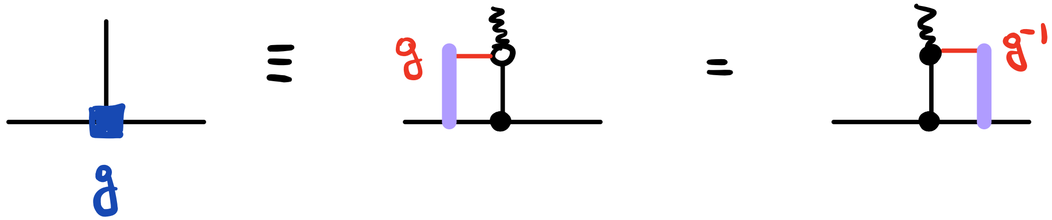
$U_L^g \equiv$ 



This time, defects sit on sites

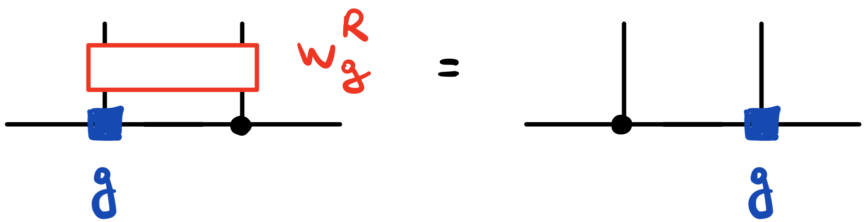


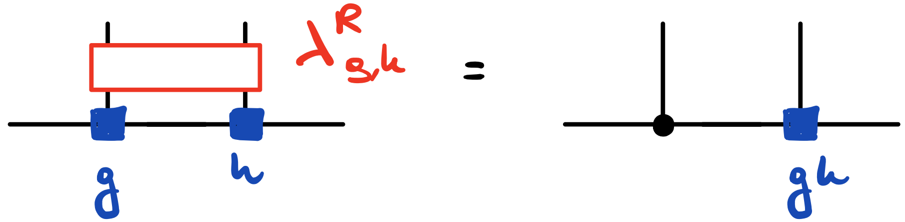
They are defined in terms of known tensors



MPU defects (injective case)

Movement and fusion achieved by 2-body unitaries





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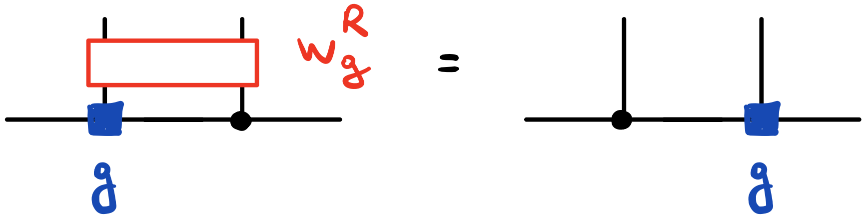
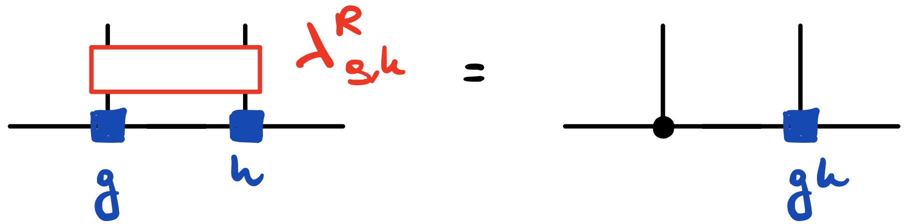
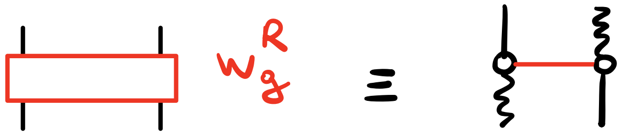
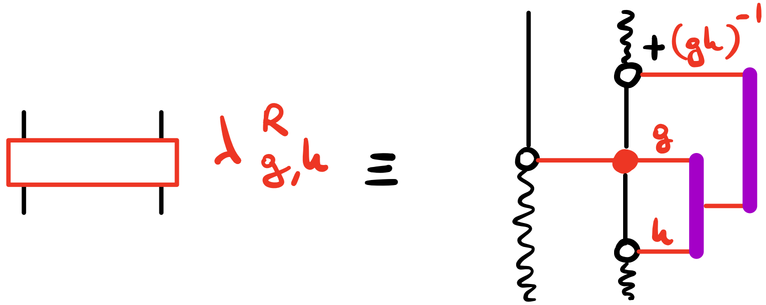




given in terms of known tensors





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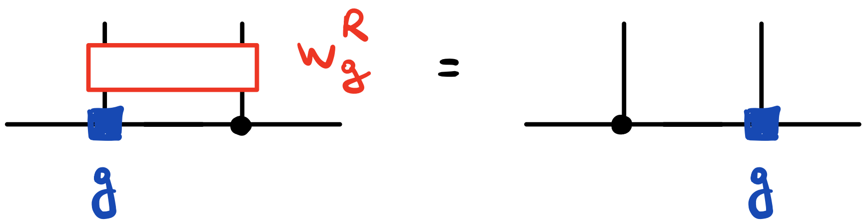

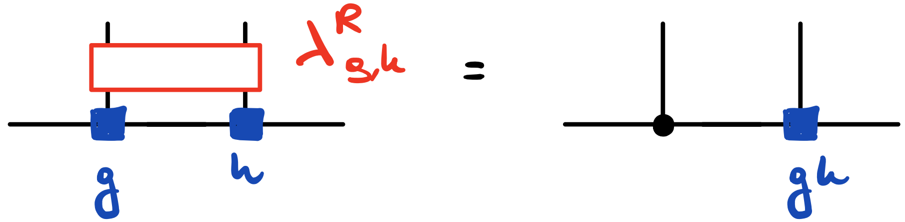

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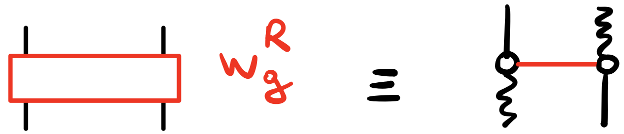
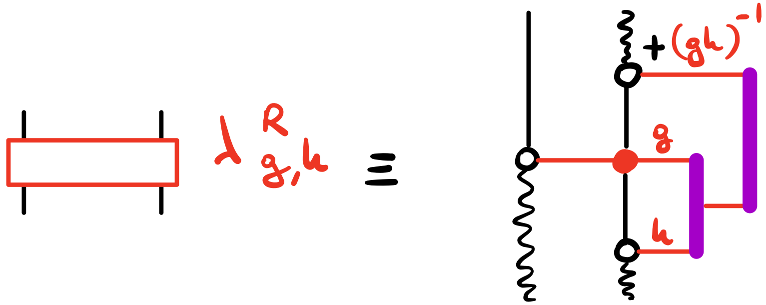
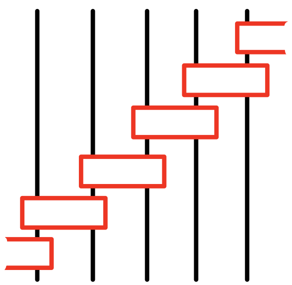
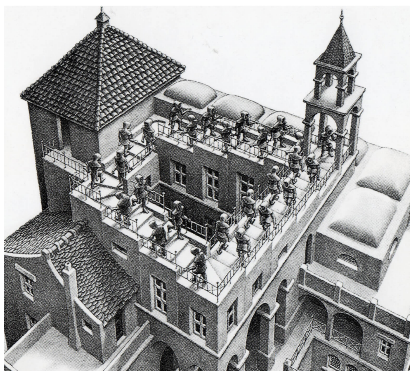




MPU =



" Escher product "

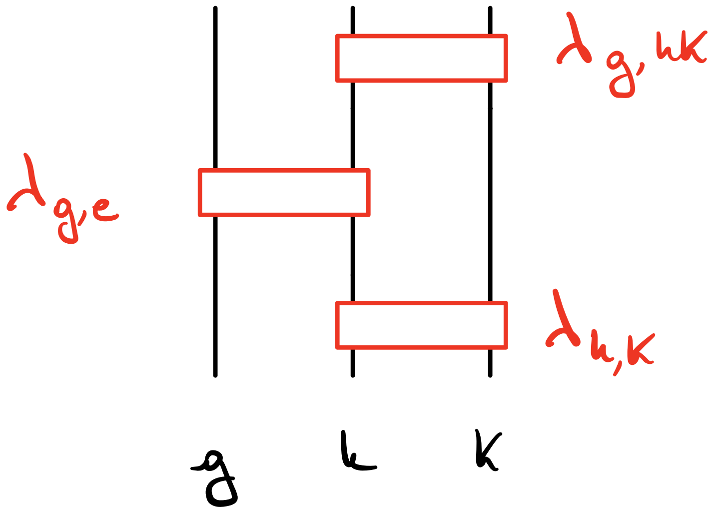


"Ascending and descending", 1960

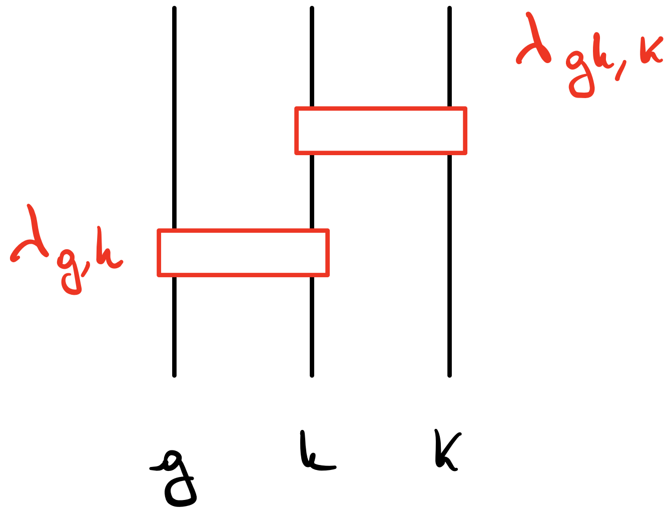
$(w_g^R \equiv \lambda^R_{g,e})$

Side note

The fusion operators associate up to a scalar, which happens to be the anomaly cocycle :



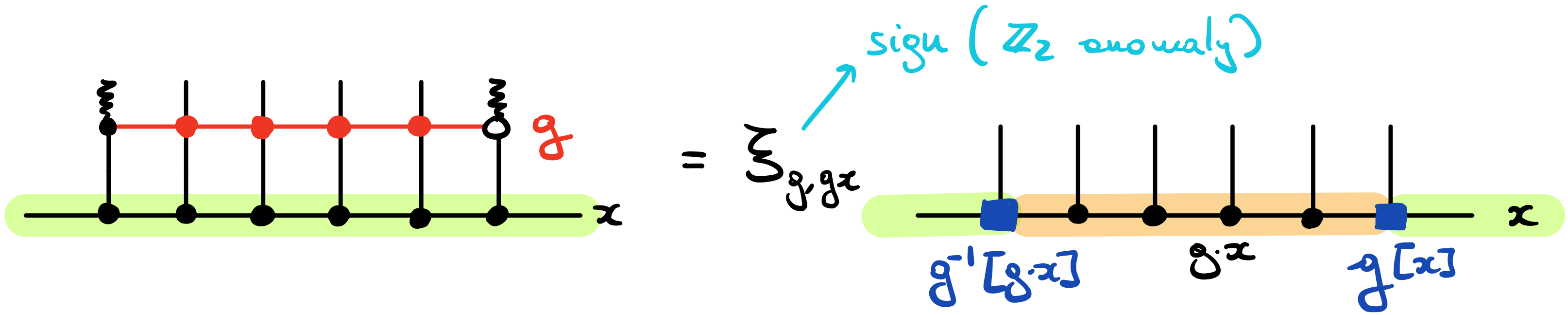
$= \omega(g,h,k)$

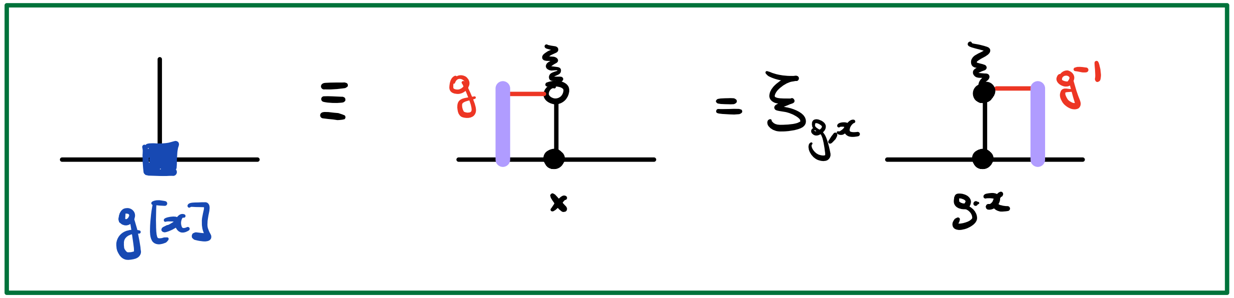


MPU defects (non-injective case)

[Haegeman et al. '12]
[Garre-Rubio, Schuch, '24]

Defects can be domain walls btw. g.s.





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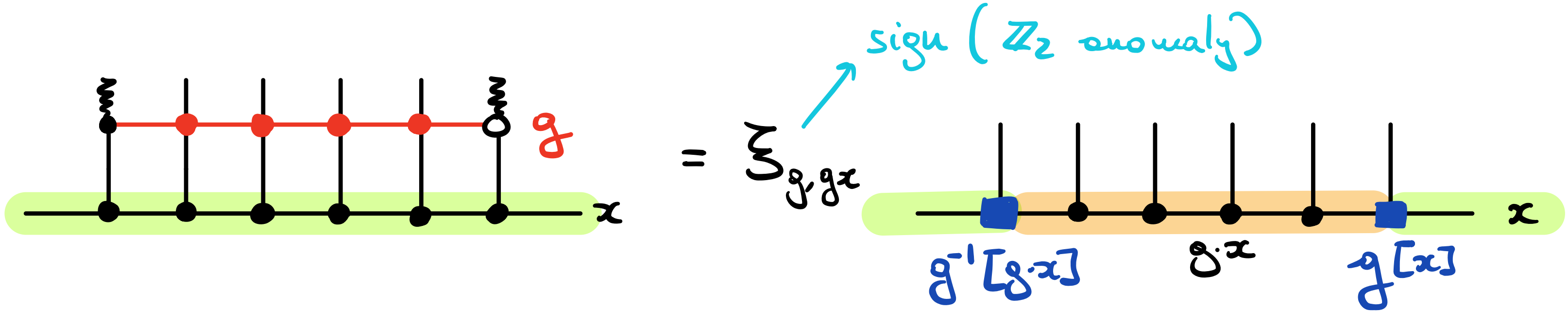
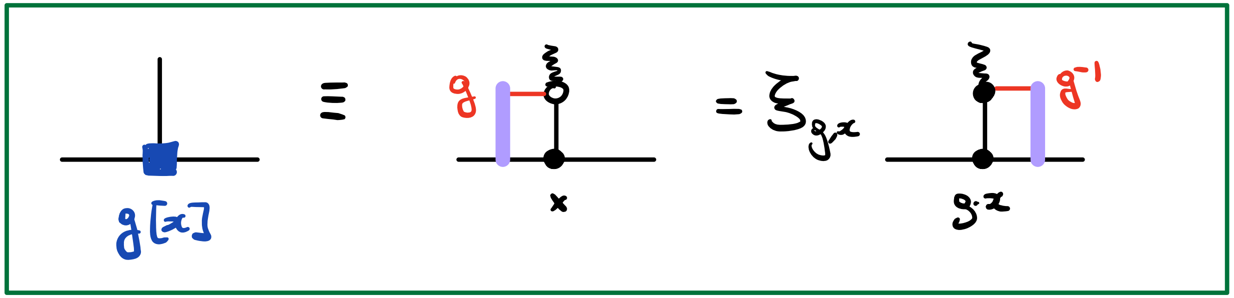
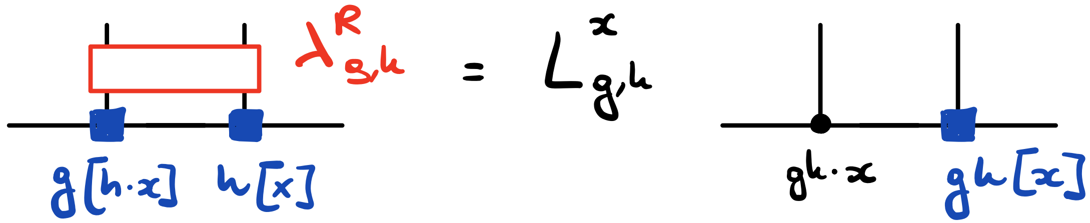


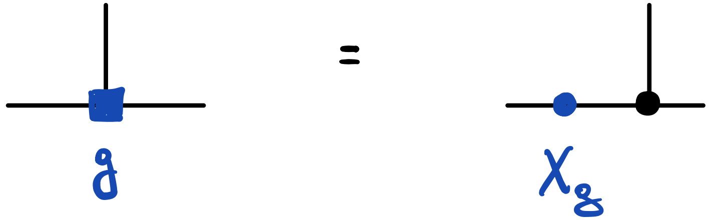


Fusion may give rise to "non-trivializable" phases



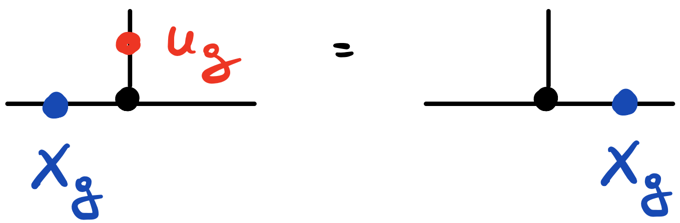
Example: Onsite case

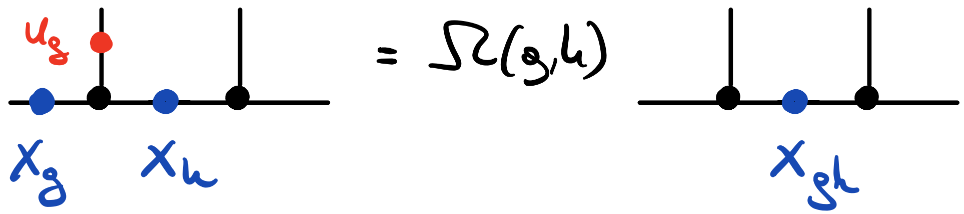
Defect tensors



Fusion operator

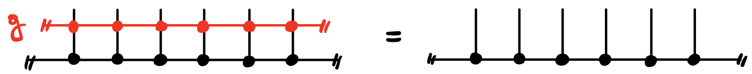
$$\lambda^{R}_{g,h} \equiv \frac{1}{\Omega(g,h)} \left(u_g \otimes \mathbb{1} \right)$$



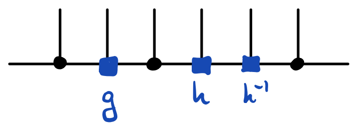


Outline

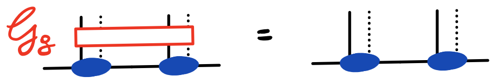
I. Preliminaries



II. Defect systems



III. Gauging



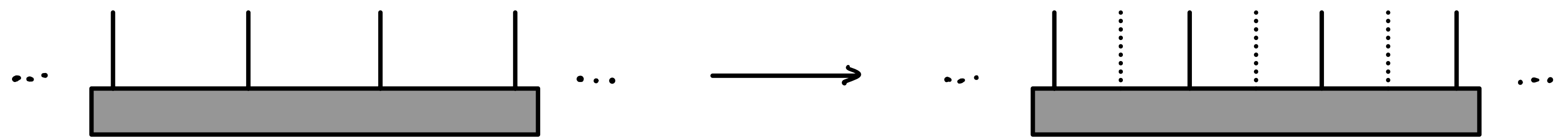
Time to gauge

Gauging ~ localizing the symmetry by adding gauge d.o.f. $\Rightarrow$

$\Rightarrow$ new (related) theory!

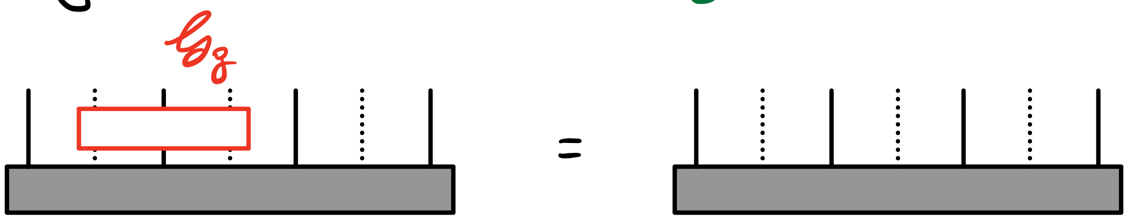
→ Traditionally performed on Lagrangian / Hamiltonian (think QED)

→ Can also be implemented on the state



$\mathbb{C}G = \text{span}\{|g\rangle\}$

Local symmetries become gauge constraints ($\Rightarrow$ Gauss's law)

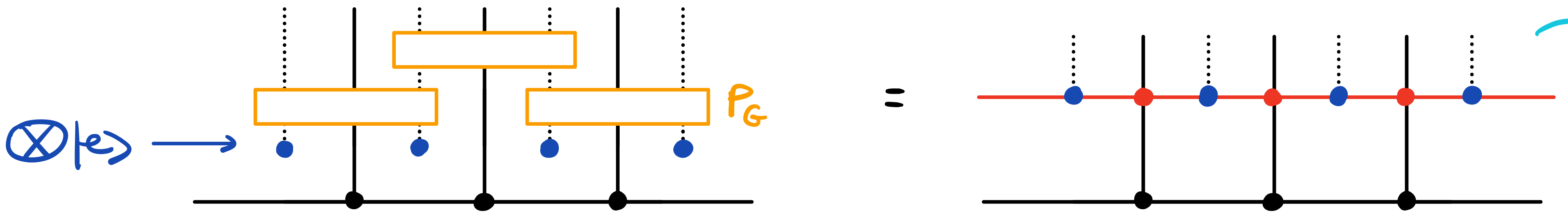


$$P_G^j = \frac{1}{|G|}\sum_g \mathcal{G}_g^j$$

Projection vs Promotion (onsite)

[Haegeman et al. '14]

→ Prescription 1: $\otimes$ with gauge dof, project on $\mathcal{H}_{\text{gauge inv.}}$

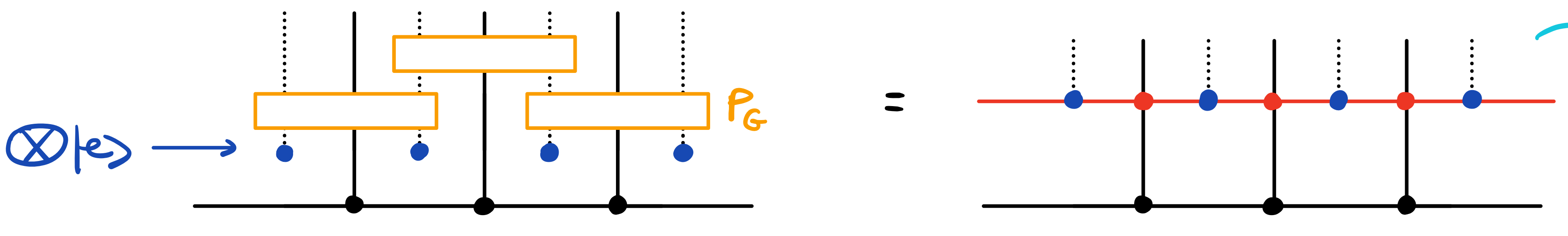


the MPO in Pepe's talk!

Projection vs Promotion (onsite)

[Haegeman et al. '14]

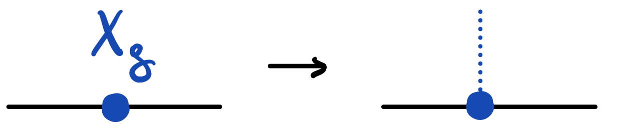
→ Prescription 1: ⊗ with gauge dof, project on $\mathcal{H}_{\text{gauge inv.}}$



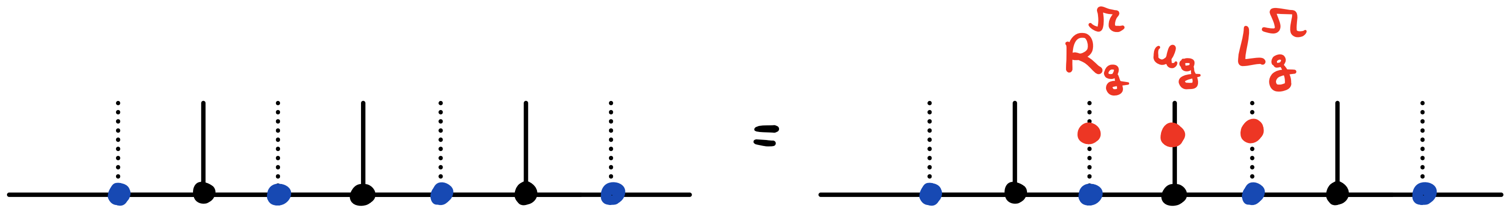
the MPO in Pepe's talk!

[Kull et al. '17]

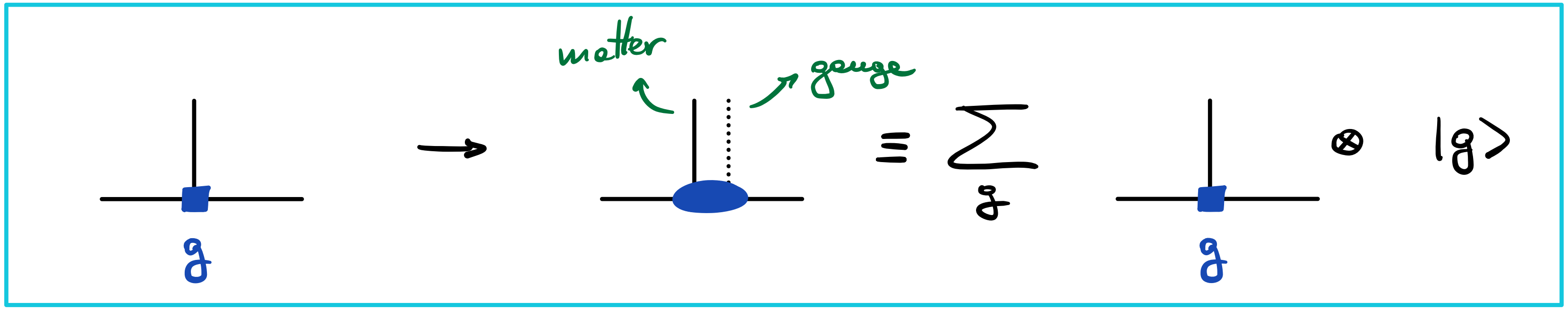
→ Prescription 2: promote the virtual defects to gauge dof



$$= \sum_g X_g \otimes |g\rangle$$

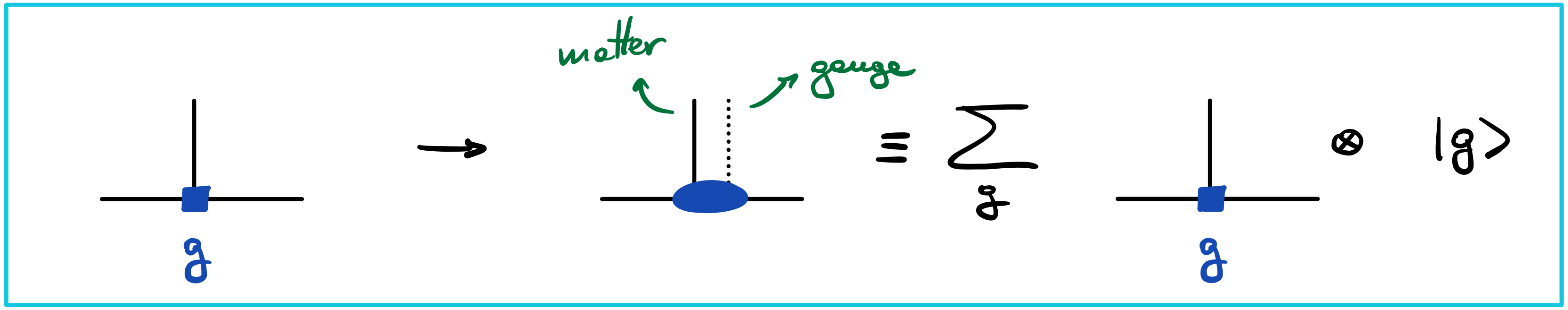


Promoting our defect tensors



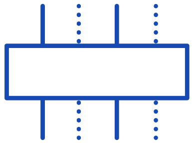
(analogous for noninjective)

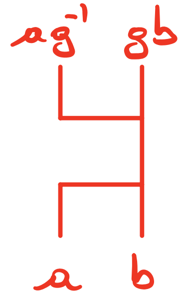
Promoting our defect tensors



(analogous for noninjective)

For any defect system, [Seifnashri, '23] gives a local representation

$\mathcal{G}_g$  $\equiv \sum_{a,b} \underbrace{(\lambda_{ag^{-1},gb})^{\dagger} \lambda_{ab}}_{\text{matter}} \otimes \underbrace{|ag^{-1}gb\rangle\langle ab|}_{\text{gauge}}$



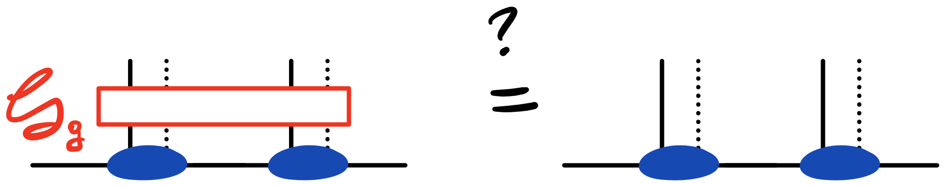
such that, whenever the λ's associate up to ω, we have

$$[P_G^j, P_G^{j+1}] = 0 \iff \omega = 1$$

i.e., the Gauss laws commute iff the anomaly is trivial.

(recurrent observation)

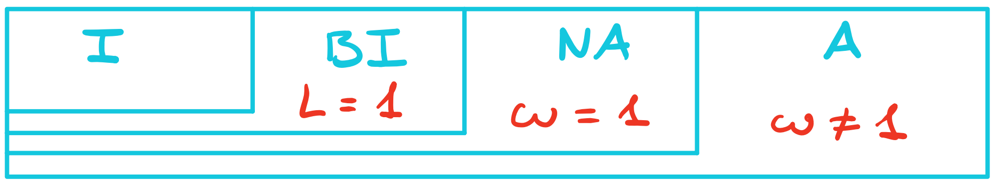
Gauge invariance of the gauged MPS



Phase factors appear! We have to assume block independence

BI: L^x_{gh} can be made $= 1$

(In particular, no issue in the injective case)

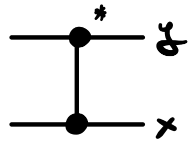


→ BI can be violated even when $D=1$ (we give a characterization)

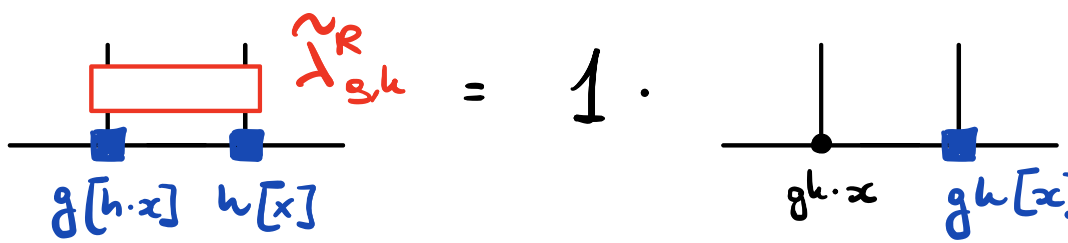
→ We still have the projectors: if the anomaly is trivial we can try to go the projective route.

State-level gauging

→ Alternatively, we can modify the local symmetry rep. to ensure our candidate MPS is gauge invariant...

→ Condition: local orthogonality  $\propto \delta_{x,y}$ (~ RG fixed pt)

→ Intuition: modify $\lambda \to \tilde{\lambda}$



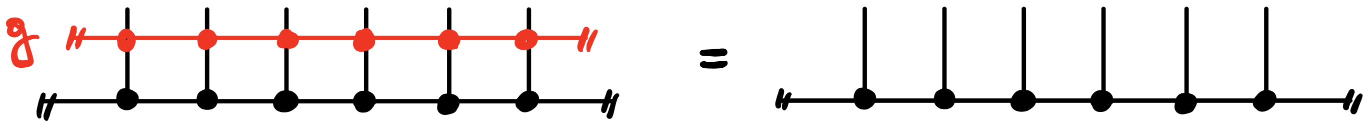
→ Drawback: $\tilde{\lambda}$'s don't associate, P_G don't commute

(but they do on a subspace...)

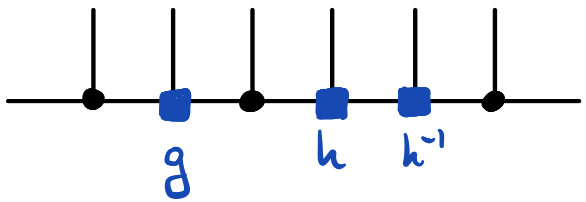
Does this have any nontrivial physical meaning?

In one slide

Consequences of MPU $\sqrt{}$ finite group symmetry for MPS



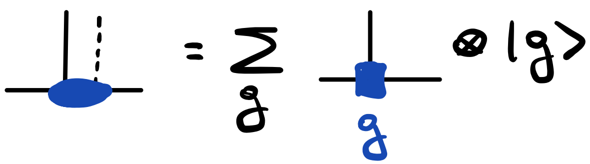
↳ System of defect tensors



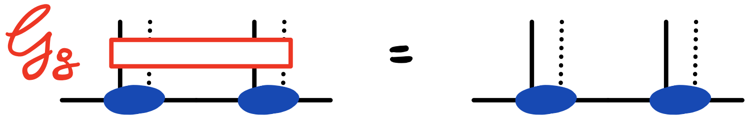
$g, h \in G$

↳ Gauging procedure (Global sym. $\xrightarrow{\text{add d.o.f.}}$ Local sym.)

Gauged tensor



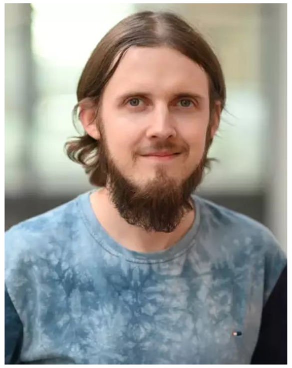
Locally invariant wavefunction



provided BI holds, otherwise: projective, state-level ...

Thank you for your attention!

And thanks to my collaborators



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