Non-Hermitian dynamics and nonreciprocity of optically coupled nanoparticles

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Quantum Nanophotonics, Benasque, 26/03/2025

https://www.deliclab.at

Der Wissenschaftsfonds.

MARIE CURIE ACTIONS

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Multiparticle experiment

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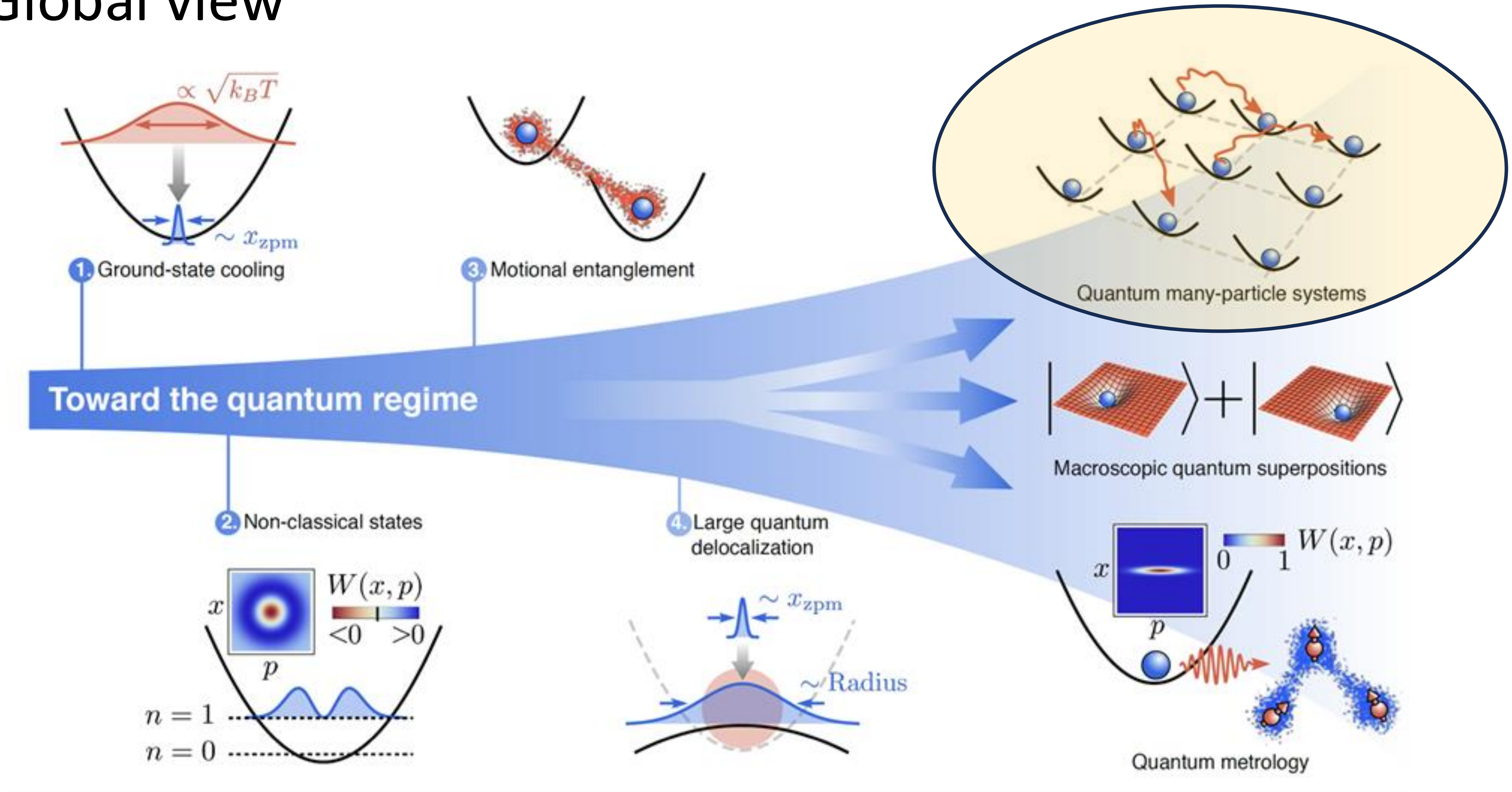
Levitated optomechanics

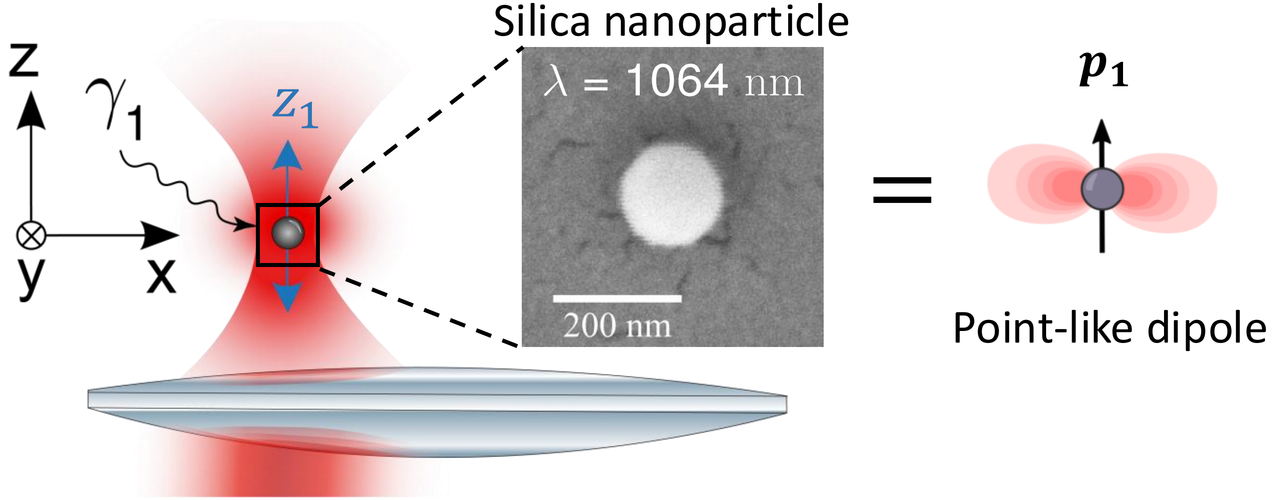
Global view



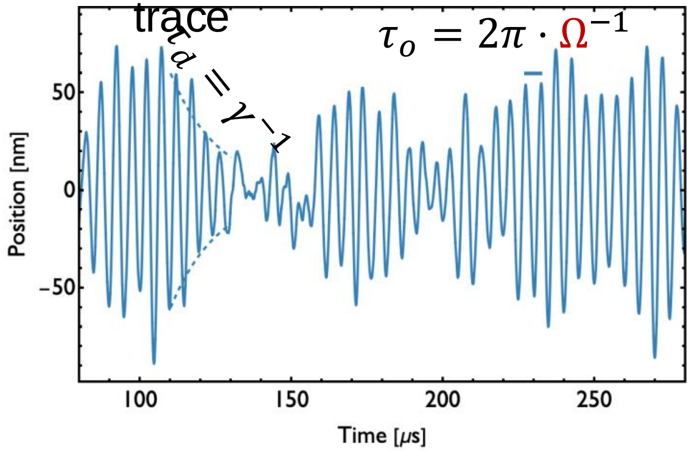
Gonzalez-Ballestero et al., Science 374, eabg3027 (2021)

Optically levitated nanoparticles

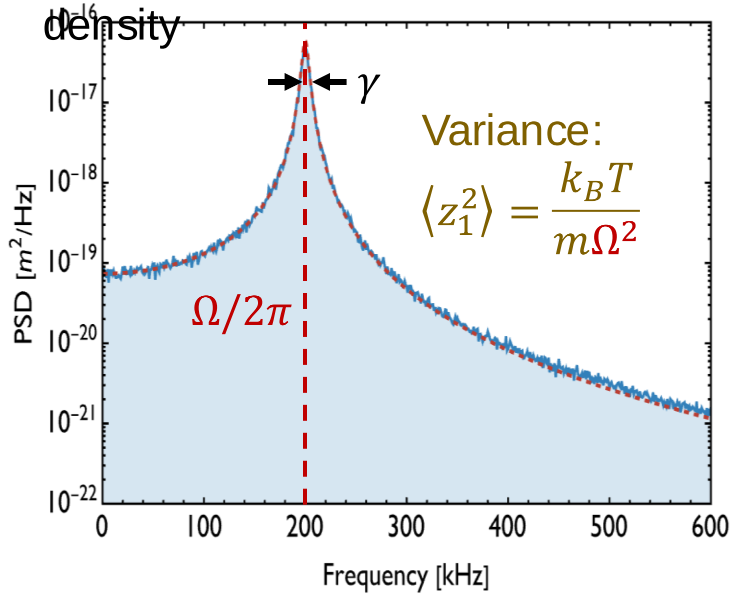
Mechanical oscillator **controlled** by light



Time domain: position trace



Frequency domain: power spectral density



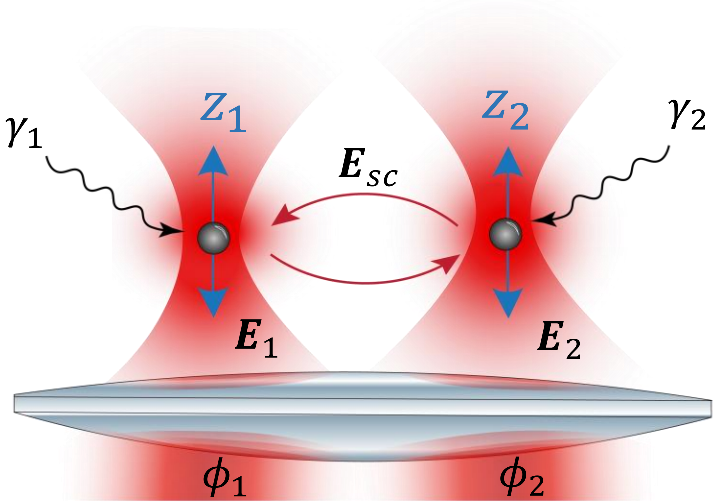
Particle dynamics in a tweezer: $U_{tw,1} = -\frac{\alpha_{0,1}}{2} |\boldsymbol{E}_{tw}|^2 \simeq \frac{m\Omega^2 z_1^2}{2}$

Ω (in all directions): → tweezer intensity
γ: collisions with gas molecules → pressure

Position readout via the phase of the backscattered light (heterodyne detection)

Optically levitated nanoparticles

Mechanical oscillators **coupled** by light



R=105 nm
λ=1064 nm

Dielectric (nonabsorbing) subwavelength ($R \ll \lambda$) nanoparticles

Light-induced force: $\boldsymbol{F}_i = -\frac{1}{2}\mathrm{Re}\{\boldsymbol{p}_i^*.\nabla_i \boldsymbol{E}_{tot}\}$

Particle's (optical) electric dipole

$$\boldsymbol{p}_i = \left(\alpha_i + i\frac{k^3\alpha_i^2}{6\pi\epsilon_0}\right)\boldsymbol{E}_{tot}(\boldsymbol{r}_i)$$

Polarizability   Radiative loss

Total electric field

$$\boldsymbol{E}_{tot} = \boldsymbol{E}_i + \boldsymbol{E}_{sc}$$

$\gamma_i \simeq$ 0.45 kHz @ 0.6 mbar: damping
$\boldsymbol{E}_i$ tweezer fields → $\Omega_i \simeq$ 30 kHz
$\boldsymbol{E}_{sc}$ scattered fields
ϕ_i tweezer phases
ω_i tweezer frequencies

Total force applied to particle i :

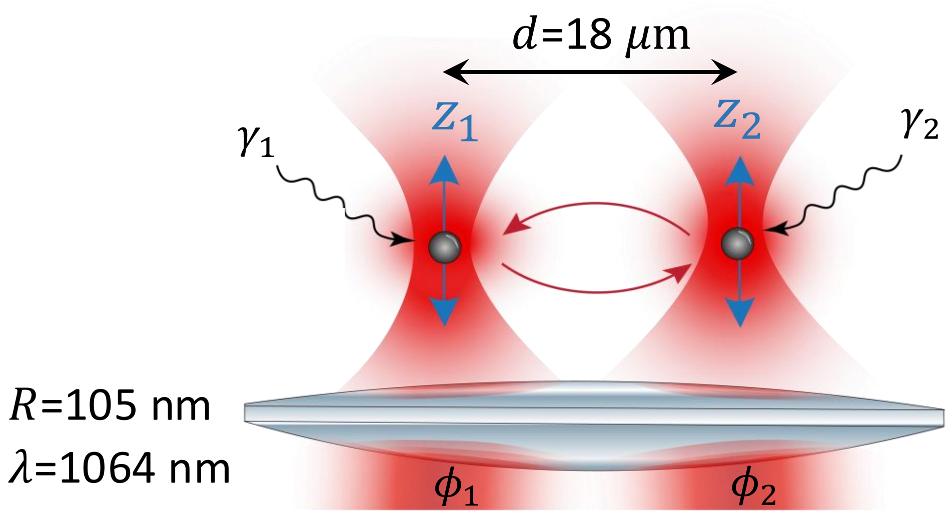
$$\boldsymbol{F}_i = \alpha_{0,i}\mathrm{Re}\left\{\left(1 - i\frac{k^3\alpha_{0,i}}{6\pi\epsilon_0}\right)\boldsymbol{E}_i^*.\nabla_i\boldsymbol{E}_i + \nabla_i(\boldsymbol{E}_i^*.\boldsymbol{E}_{sc})\right\}$$

tweezer force   scattering force   optical binding interaction

stems from interference!

Dipole-dipole interactions

Experimental control



$\theta \rightarrow$ laser polarization
$d \rightarrow$ tweezer distance
$\phi_{ji} = \phi_j - \phi_i$ tweezer phase difference
$\omega_{ji} = \omega_j - \omega_i$ tweezer detuning

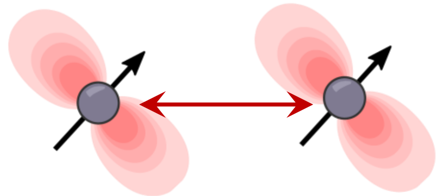
Full (**nonlinear**) expression for the force along z:

$$F_{ij} \simeq 2m\Omega g \cos(kd + \phi_{ji}) \times \frac{\sin(k(z_j - z_i))}{k}$$

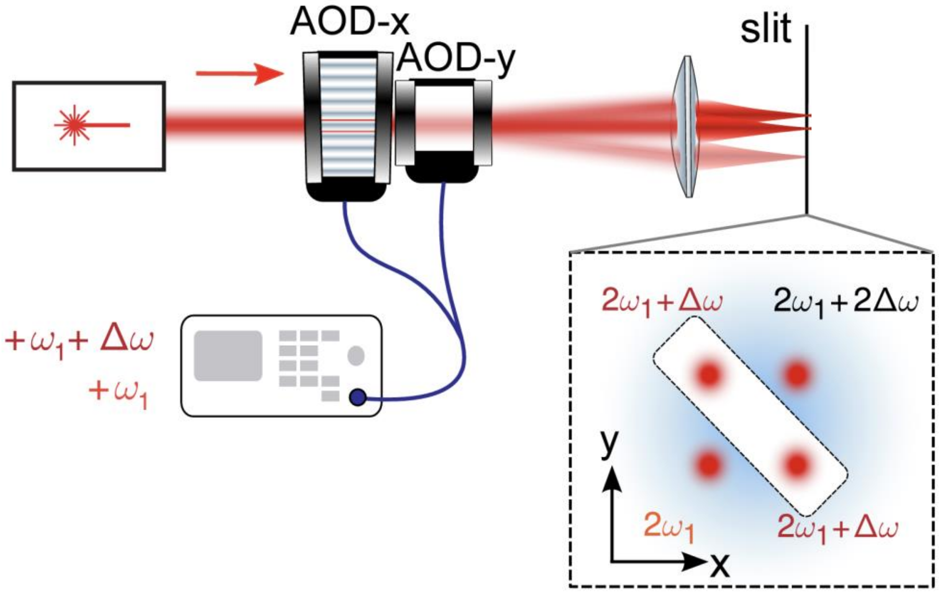
relative tweezer phase

tweezer distance

$g \propto \cos^2(\theta)$ laser polarization angle



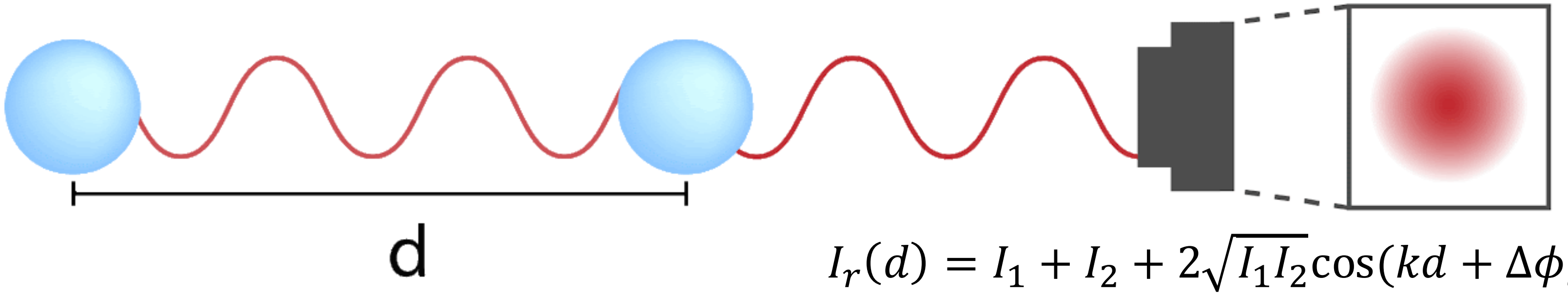
Trap engineering with Acousto Optic Deflectors (AODs)



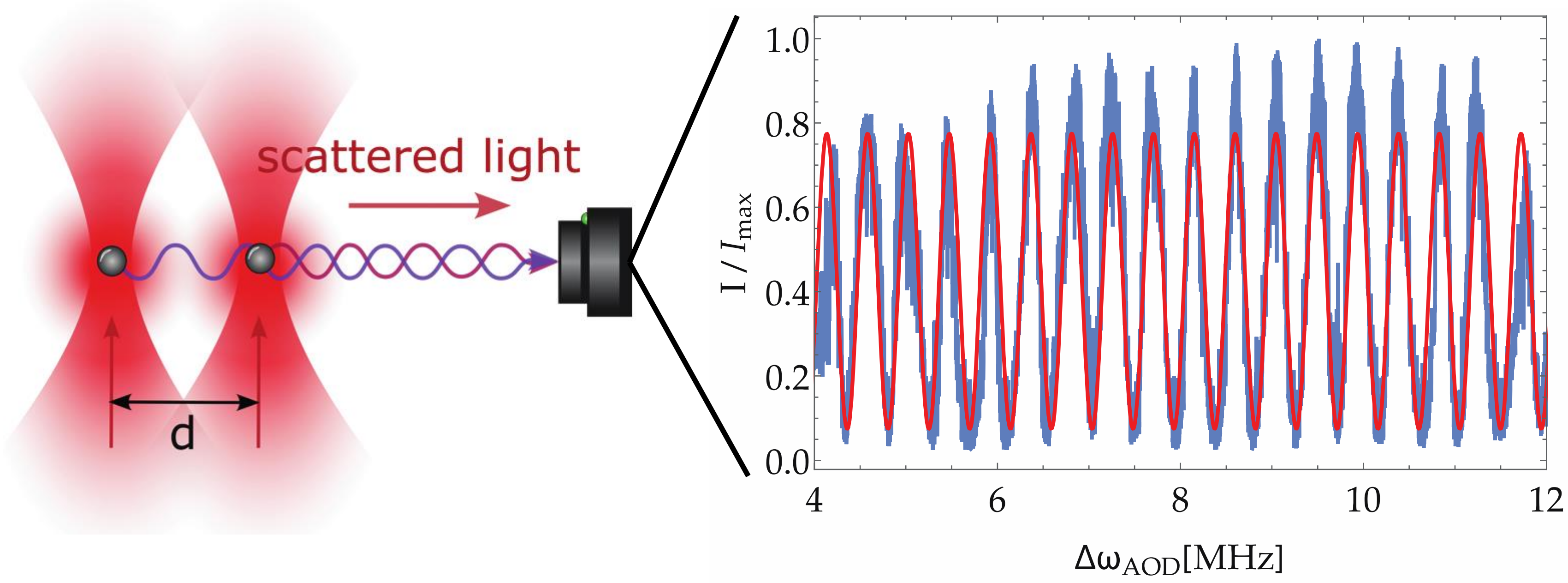
M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Relative distance calibration

Interferometric imaging



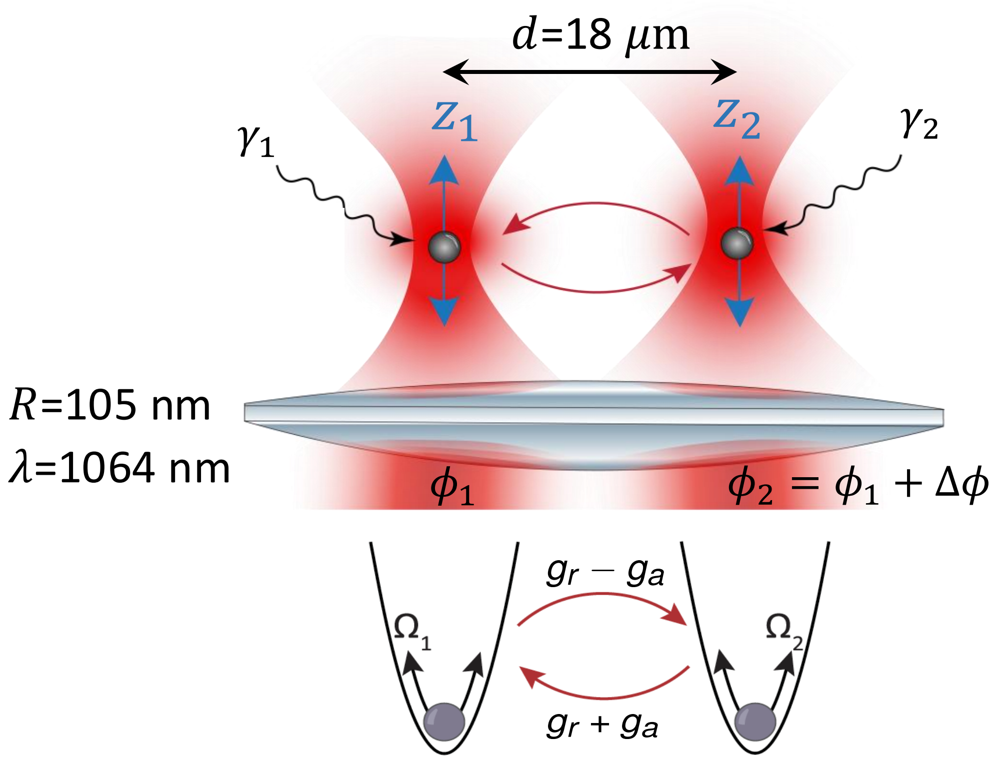
$$I_r(d) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(kd + \Delta\phi_{lr})$$



M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Dipole-dipole interactions

Nonreciprocity



Linearized interaction: $\sin\left(k(z_j - z_i)\right) \simeq k(z_j - z_i)$

$$\ddot{z}_1 + \gamma_1 \dot{z}_1 + \Omega_1^2 z_1 = 2\Omega(g_r - g_a)(z_2 - z_1) + \xi_{th,1}(t)$$

$$\ddot{z}_2 + \gamma_2 \dot{z}_2 + \Omega_2^2 z_2 = 2\Omega(g_r + g_a)(z_1 - z_2) + \xi_{th,2}(t)$$

Full (**nonlinear**) expression for the force along z:

$$\boldsymbol{F}_{ij} \simeq 2m\Omega g \cos\left(kd + \phi_{ji}\right) \times \frac{\sin\left(k(z_j - z_i)\right)}{k}$$

relative tweezer phase

tweezer distance

Different interference phases lead to **nonreciprocal interactions**

Particle 1: $kd + \Delta\phi$

Particle 2: $kd - \Delta\phi$

$$g\cos(kd \pm \Delta\phi) = g_r \mp g_a$$

$$g_r = g.\cos(\Delta\phi)\cos(kd)$$

$$g_a = g.\sin(\Delta\phi)\sin(kd)$$

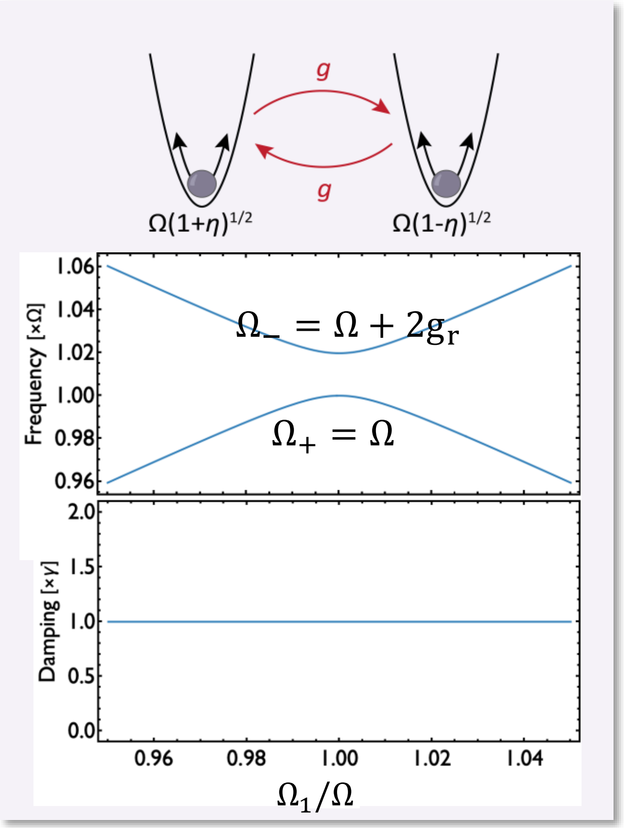
M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Full quantum theory: H. Rudolph et al, PRL **133**, 233603 (2024)

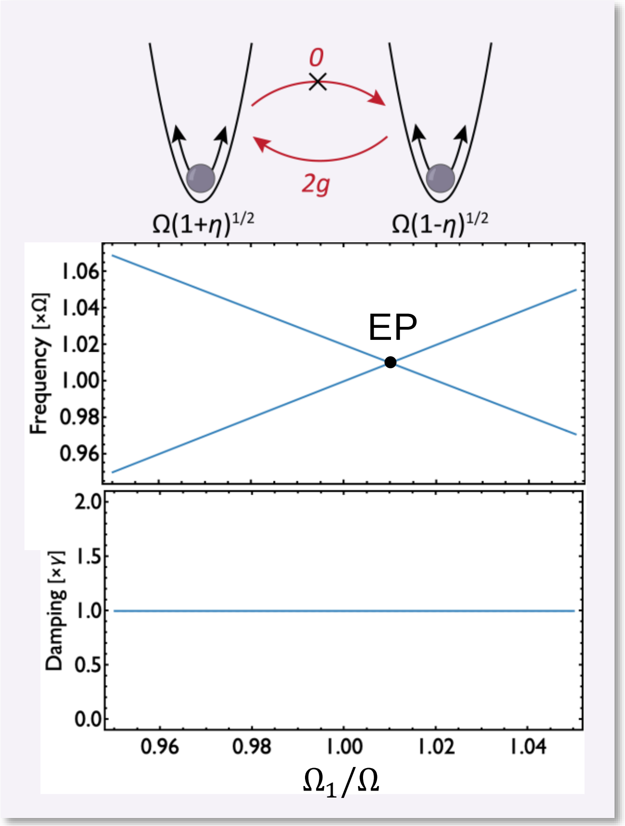
Different interaction regimes

Fully reciprocal, unidirectional, purely antireciprocal

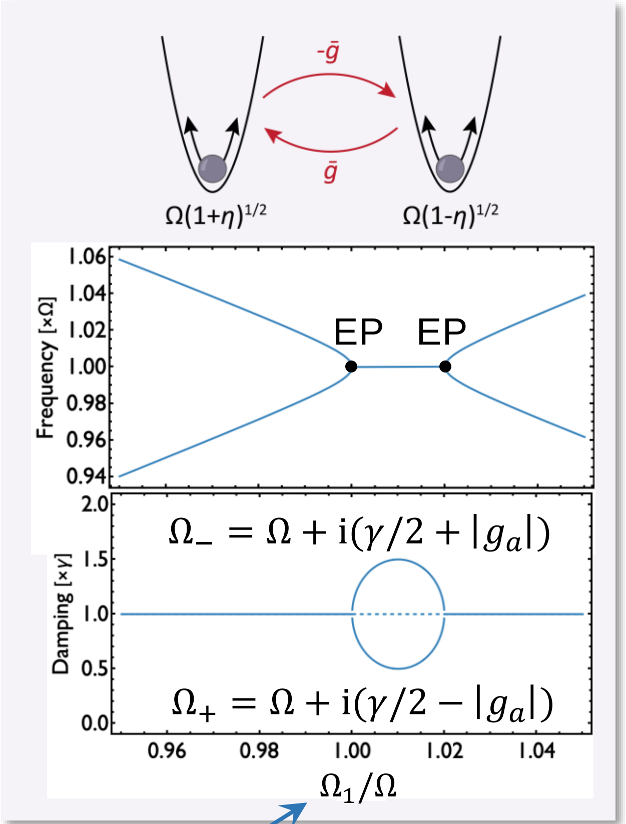
Reciprocal (conservative)



Unidirectional



Anti-reciprocal (non-conservative)



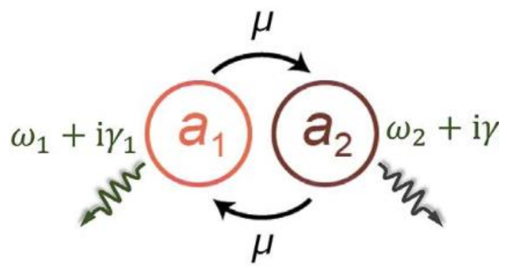
Normal mode eigenfrequencies:

$$\Omega_\pm = \bar{\Omega} + g_r + i\frac{\gamma}{2} \mp \sqrt{g_r^2 - g_a^2}$$

Ratio between oscillation frequencies

J. Rieser et al, Science 377, 987 (2022)

Non-Hermitian dynamics

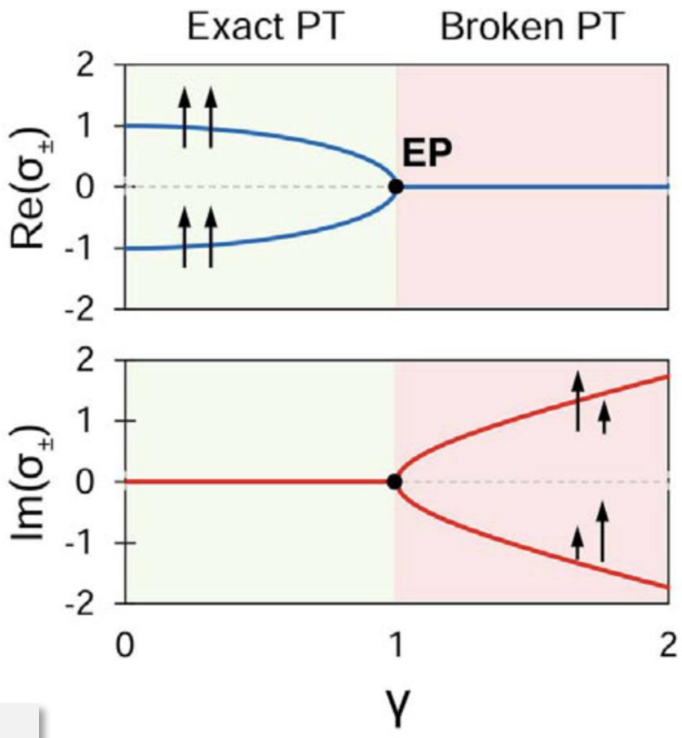


Complex Hamiltonian H: $i\hbar \frac{\partial \psi}{\partial t} = H\psi$

Balanced gain & loss → Parity-Time reversal (PT) symmetry allowing for real eigenvalues

Variation of parameters of H → PT symmetry breaking at the Exceptional Point (EP)

PT symmetry breaking transition strongly affects system dynamics



Here: Non-Hermitian dynamics **stems from nonreciprocal interaction!**

Carl M. Bender & Stefan Boettcher, PRL 80, 5243 (1998)

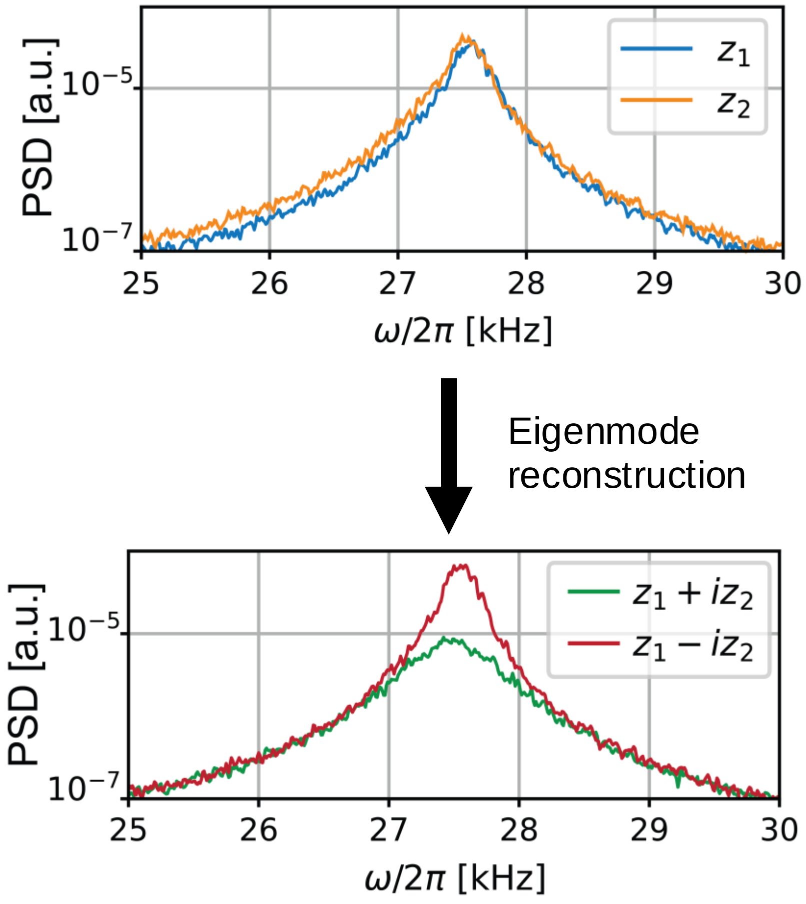
Kosmas V. Kepesidis et al, NJP 18, 095003 (2016)

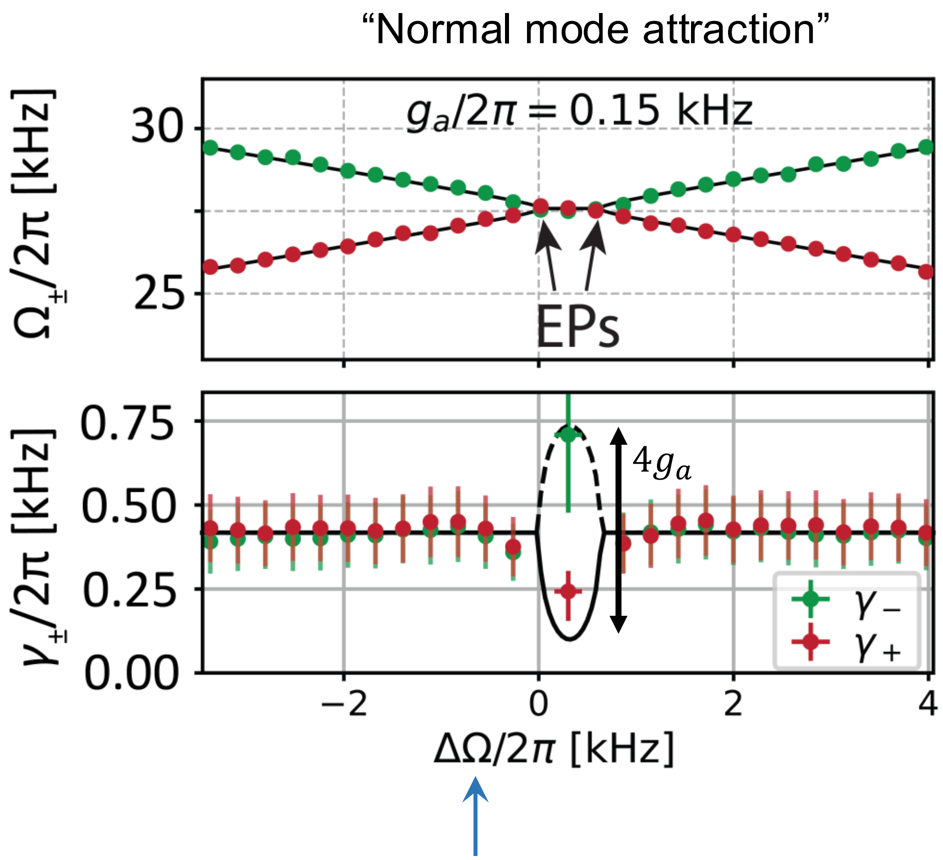
B. Peng et al., Science 346, 328-332 (2014)

Mohammad-Ali Miri, Andrea Alù, Science 363, eaar7709 (2019)

Purely anti-reciprocal interactions

Non-Hermitian dynamics / linear theory



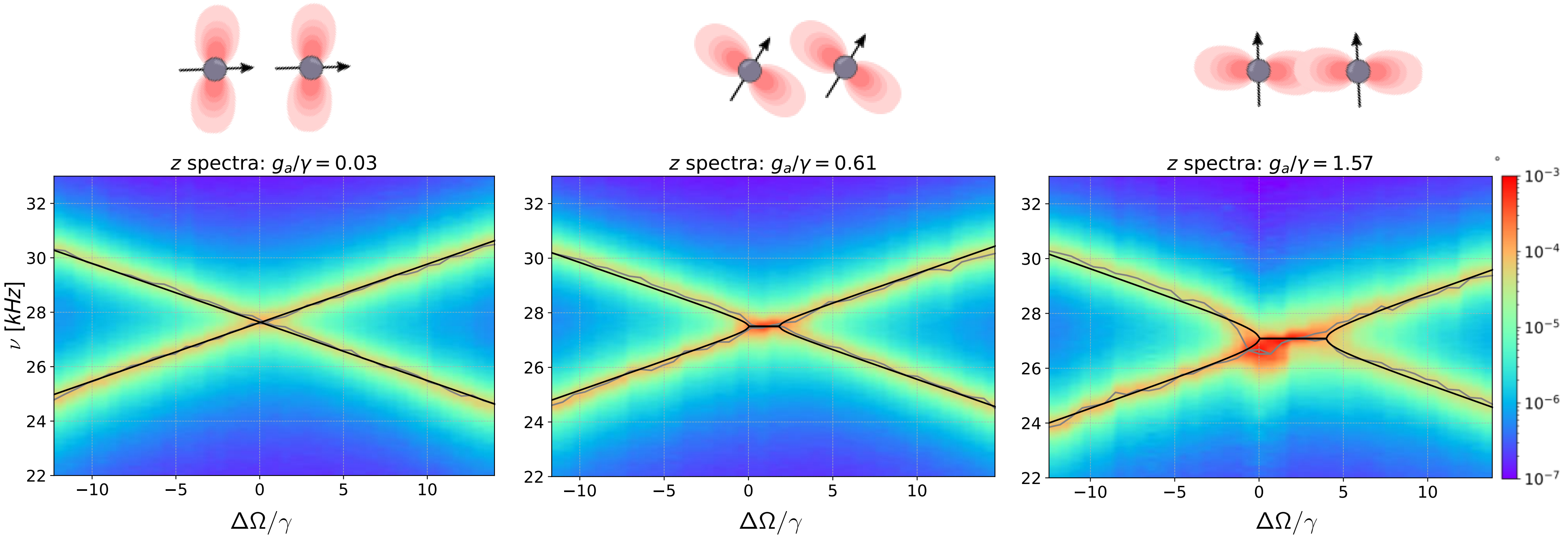


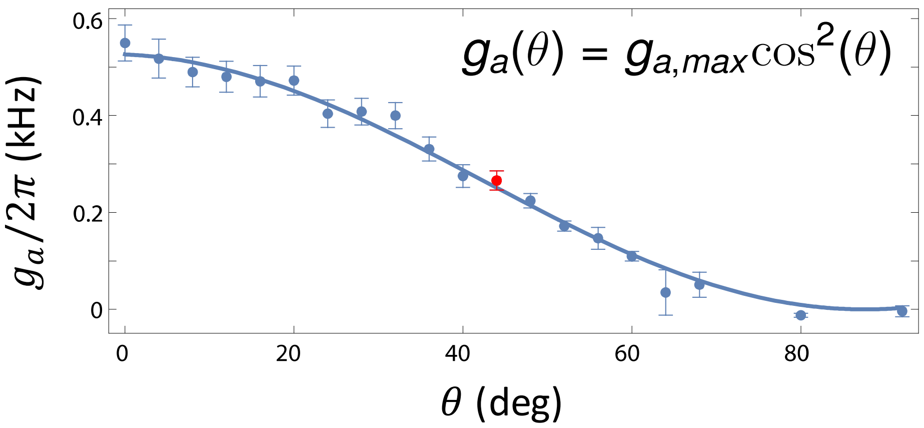
Mechanical detuning: difference between the oscillation frequencies

M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Experiment: control of interactions

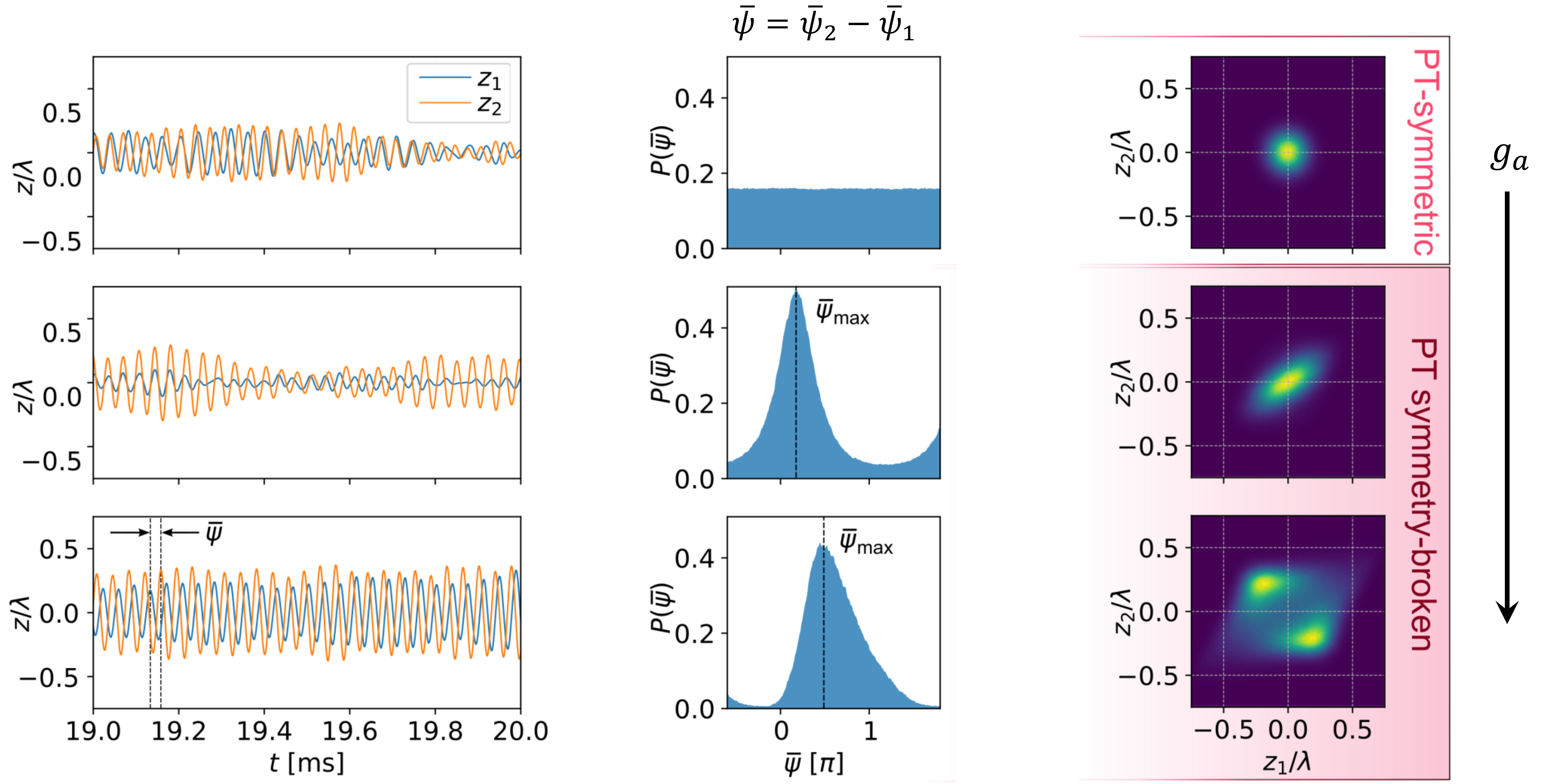
Magnitude of interactions: trap polarization





Motional correlations

Oscillation amplitude and mechanical relative phase



M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Purely anti-reciprocal interactions

Non-Hermitian dynamics / full theory

Nonlinear model for the complex amplitudes of motion:

$$\dot{a}_j = \left(\pm\frac{\Delta\Omega}{2} - \frac{\gamma}{2}\right)a_j + i\beta\left|a_j\right|^2 a_j + \sqrt{\gamma n_{th}}\xi_j(t) + ig_a(a_2 - a_1)f(2kz_{zp}|a_2 - a_1|)$$

trap potential anharmonicity (negligible effect)

thermal noise

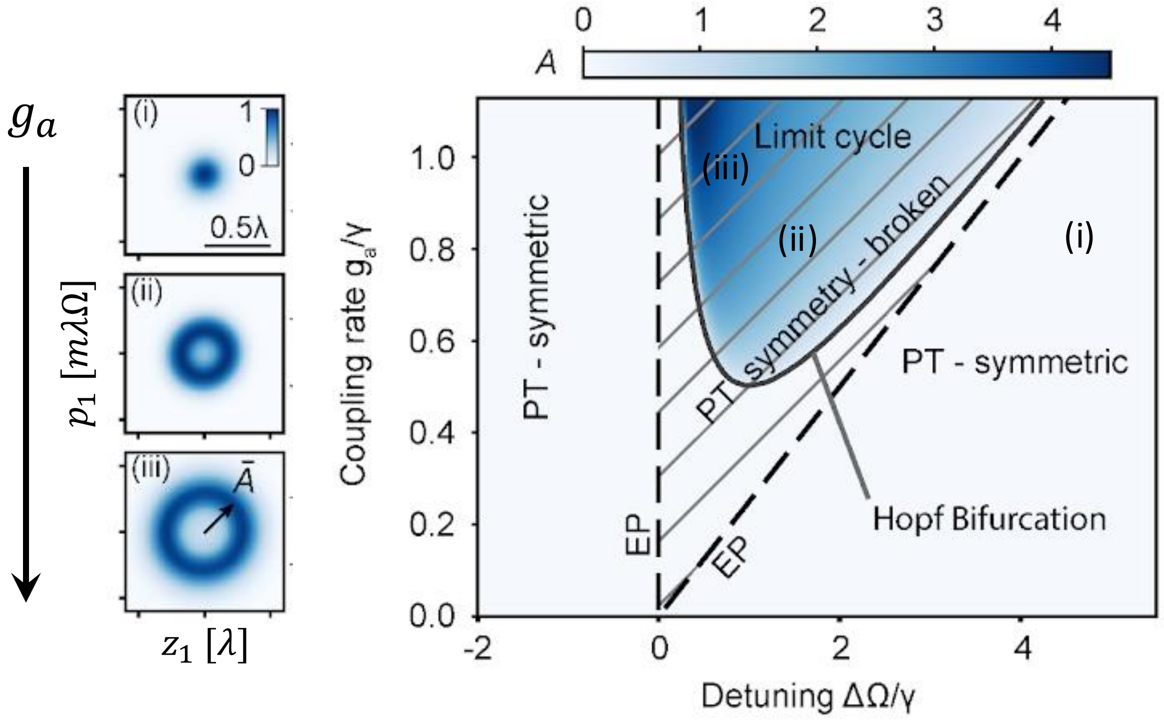
non-linear interaction

$$f(x) = \frac{2J_1(x)}{x} \simeq 1 - \frac{x^2}{8}$$

Can be rewritten for the new collective variables:

$A = 2k\sqrt{\frac{\hbar}{m\Omega}(|a_1|^2 + |a_2|^2)}$ collective amplitude

$\psi = \arg(a_2 a_1^*)$ relative mechanical phase

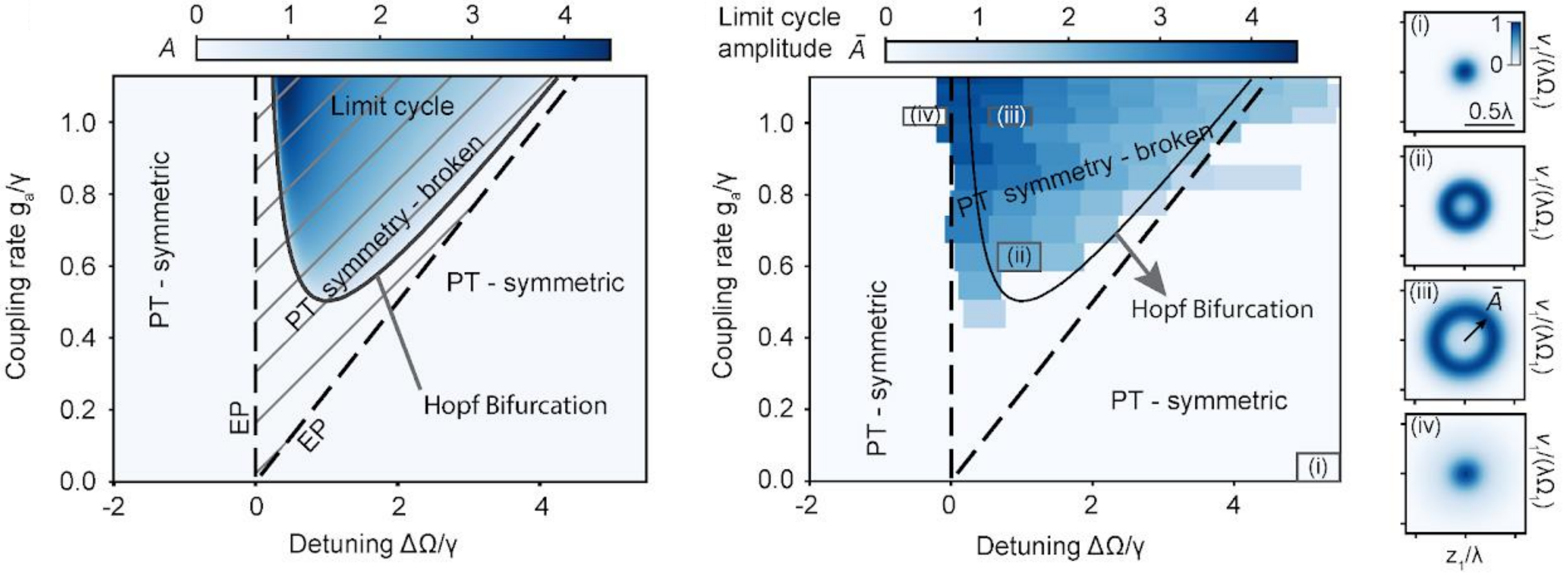


Interaction nonlinear stabilizes the motion into a limit cycle
Interpretation:
gain + saturation = mechanical lasing

M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Limit cycle phase diagram

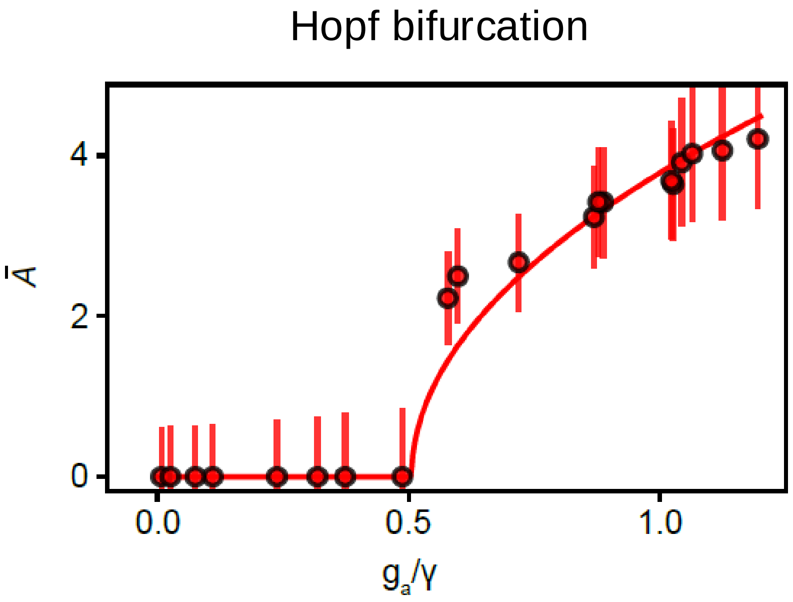
Phase diagram: experiment vs theory

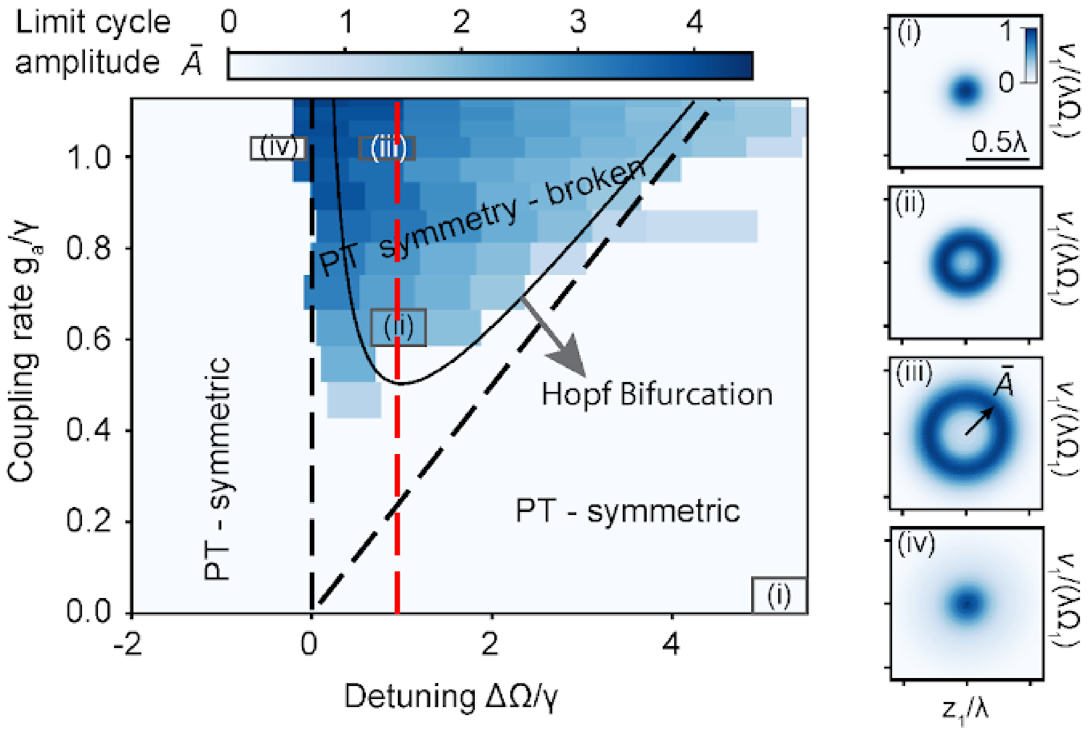


M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Limit cycle phase diagram

Phase diagram: experiment vs theory



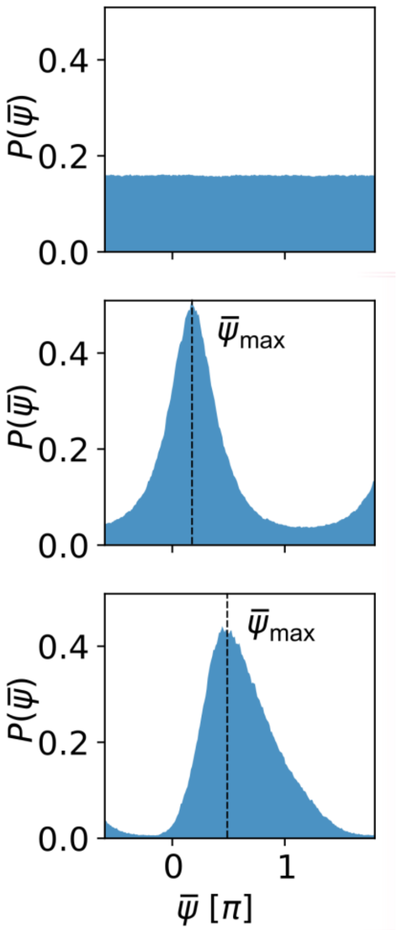


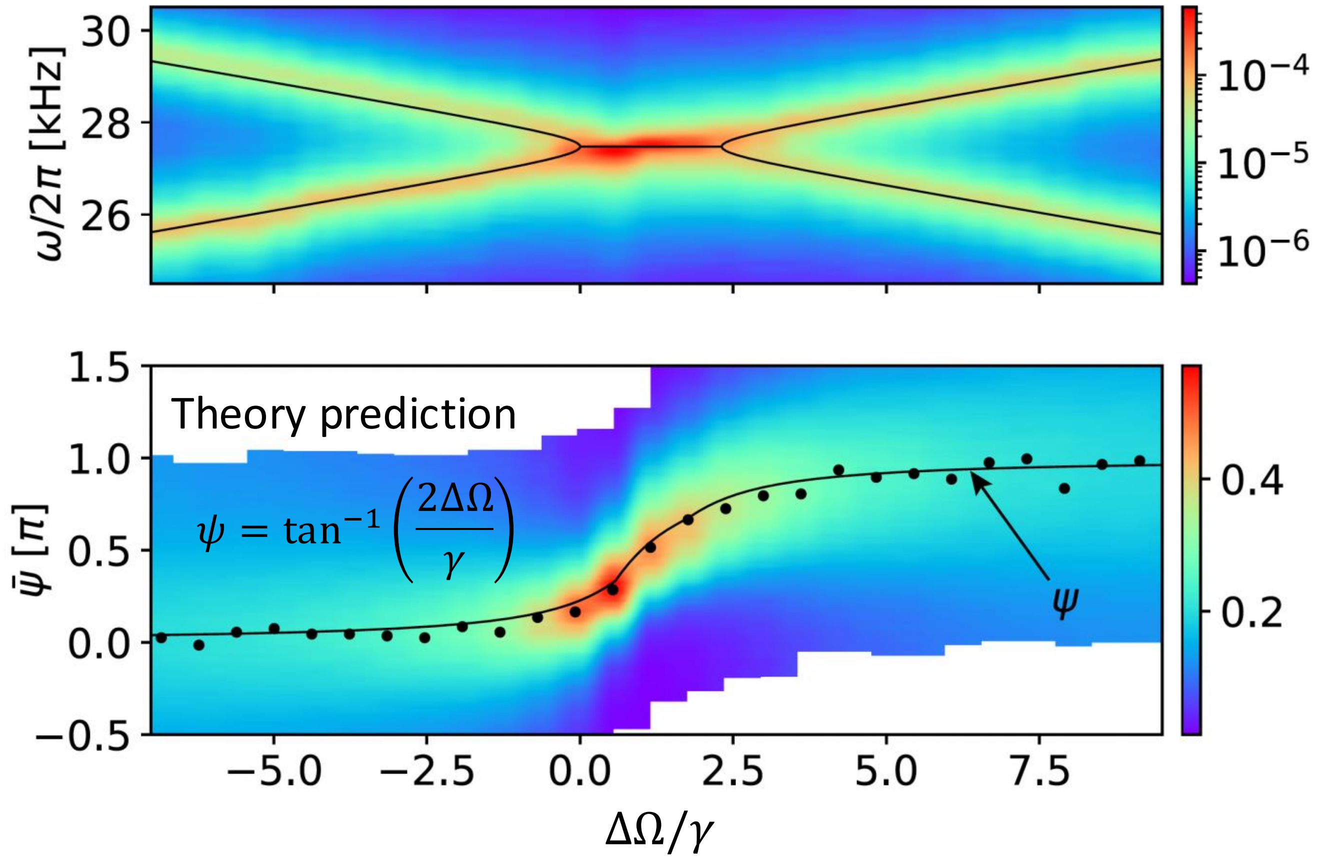
M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Motional correlations

Locking of mechanical relative phase

$\bar{\psi} = \bar{\psi}_2 - \bar{\psi}_1$

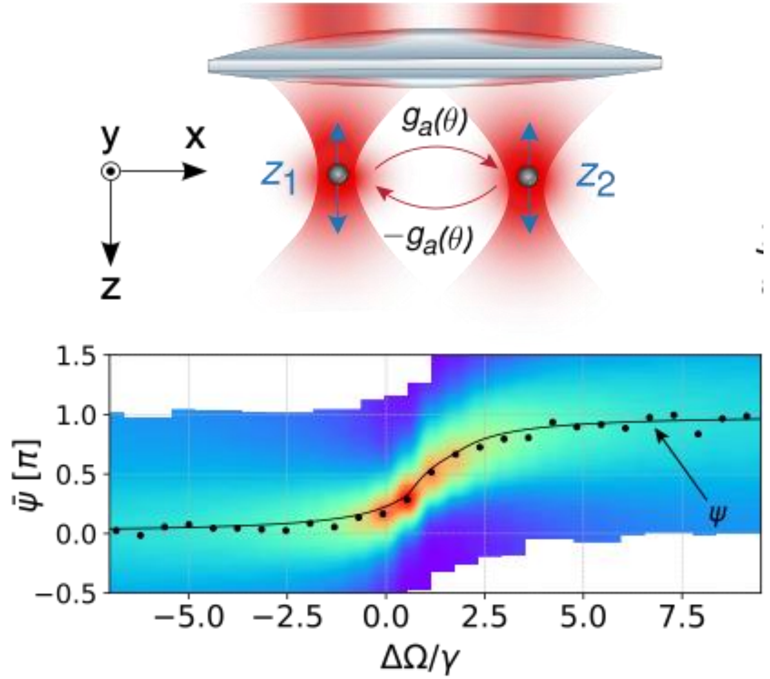




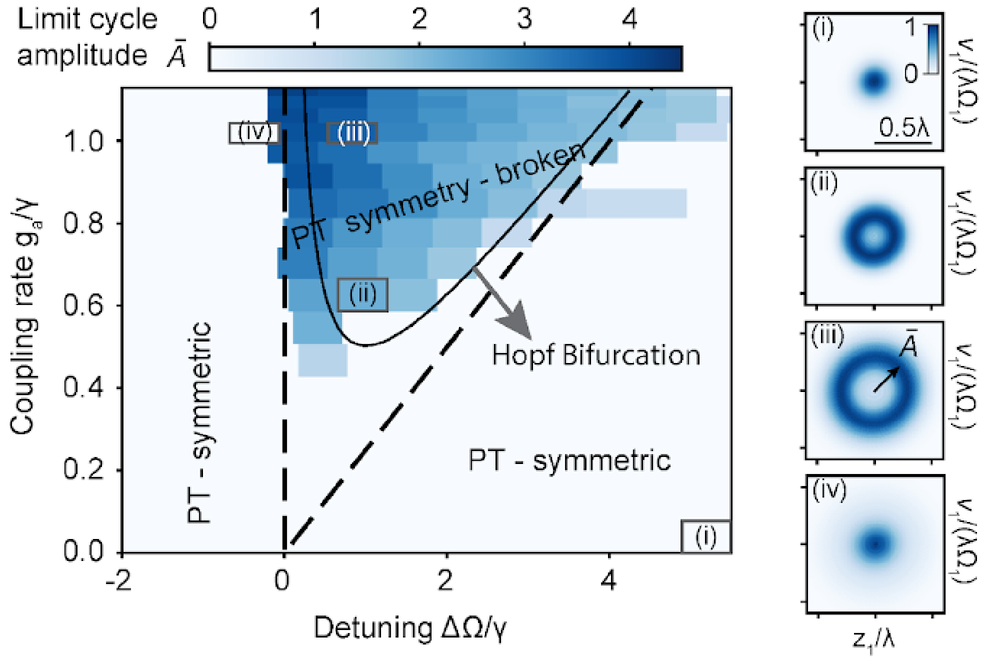
M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Conclusions

Non-Hermitian dynamics via light induced anti-reciprocal interaction



Phase locking in the motion of particles



Nonlinear dynamics resulting in mechanical lasing

M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

See also similar work by collaborators @ ISI Brno: V. Liska et al, *Nat. Phys.* (2024)

Outlook: time dependent interactions

Light induced interaction between two trapped nanoparticles

Stationary:

Reciprocal interaction:

J. Rieser et al, Science 377, 987 (2022)

Anti-Reciprocal interaction:

M. Reisenbauer, L. Egyed, H. Rudolph et al, *Nat. Phys.* (2024)

Full quantum theory: H. Rudolph et al, PRL **133**, 233603 (2024)

Time dependent:

Detuned interaction:

In preparation…

Der Wissenschaftsfonds.