Tensor product space for studying the interaction of bipartite states of light with nanostructures

Phys. Rev. A **110**, 043516 (2024)

Lukas Freter, Benedikt Zerulla, Marjan Krstić, Christof Holzer, Carsten Rockstuhl, and Ivan Fernandez-Corbaton

Karlsruhe Institute of Technology

Karlsruhe Institute of Technology

A computational framework for quantum nanophotonics

A computational framework for quantum nanophotonics

- Algebraic approach: Hilbert spaces, operators, etc ...

A computational framework for quantum nanophotonics

- Algebraic approach: Hilbert spaces, operators, etc ...
- Benefits for classical fields / single photon case

A computational framework for quantum nanophotonics

- Algebraic approach: Hilbert spaces, operators, etc ...
- Benefits for classical fields / single photon case
- Extension to (entangled) biphotons

A computational framework for quantum nanophotonics

- Algebraic approach: Hilbert spaces, operators, etc ...
- Benefits for classical fields / single photon case
- Extension to (entangled) biphotons
- Compute the interaction between entangled biphoton pulses of light and relativistically moving objects

A computational framework for quantum nanophotonics

- Algebraic approach: Hilbert spaces, operators, etc ...
- Benefits for classical fields / single photon case
- Extension to (entangled) biphotons
- Compute the interaction between entangled biphoton pulses of light and relativistically moving objects
- Published formulas and public software

- $\mathbb{M}$: The Hilbert space of free solutions of Maxwell equations

[1] L. Gross, J. Math. Phys. **5**, 687–695 (1964).

[2] Y. B. Zel'dovich, Doklady Akademii Nauk SSSR (USSR) English translation currently published in a number of subject-oriented journals **163** (1965).

[3] I. Bialynicki-Birula and Z. Bialynicka-Birula, *Quantum electrodynamics*, (Pergamon, Oxford, UK, Jan. 1975).

- $\mathbb{M}$: The Hilbert space of free solutions of Maxwell equations
- $|f\rangle \in \mathbb{M}$, $|f\rangle \equiv \{\mathbf{E}_f(t,\mathbf{r}), \mathbf{B}_f(t,\mathbf{r})\}$

[1] L. Gross, J. Math. Phys. **5**, 687–695 (1964).

[2] Y. B. Zel'dovich, Doklady Akademii Nauk SSSR (USSR) English translation currently published in a number of subject-oriented journals **163** (1965).

[3] I. Bialynicki-Birula and Z. Bialynicka-Birula, *Quantum electrodynamics*, (Pergamon, Oxford, UK, Jan. 1975).

- $\mathbb{M}$: The Hilbert space of free solutions of Maxwell equations
- $|f\rangle \in \mathbb{M}$, $|f\rangle \equiv \{\mathbf{E}_f(t,\mathbf{r}), \mathbf{B}_f(t,\mathbf{r})\}$
- Scalar product[1]:

$$\langle f|g\rangle = \frac{1}{2\pi^2\hbar c_0}\int_{\mathbb{R}^3} d\mathbf{r} \int_{\mathbb{R}^3} d\bar{\mathbf{r}} \, \frac{\epsilon_0 \mathbf{E}_f^\dagger(t,\bar{\mathbf{r}})\mathbf{E}_g(t,\mathbf{r}) + \mathbf{B}_f^\dagger(t,\bar{\mathbf{r}})\mathbf{B}_g(t,\mathbf{r})/\mu_0}{|\mathbf{r}-\bar{\mathbf{r}}|^2}$$

[1] L. Gross, J. Math. Phys. **5**, 687–695 (1964).

[2] Y. B. Zel'dovich, Doklady Akademii Nauk SSSR (USSR) English translation currently published in a number of subject-oriented journals **163** (1965).

[3] I. Bialynicki-Birula and Z. Bialynicka-Birula, *Quantum electrodynamics*, (Pergamon, Oxford, UK, Jan. 1975).

- $\mathbb{M}$: The Hilbert space of free solutions of Maxwell equations
- $|f\rangle \in \mathbb{M}$, $|f\rangle \equiv \{\mathbf{E}_f(t,\mathbf{r}), \mathbf{B}_f(t,\mathbf{r})\}$
- Scalar product[1]:

$$\langle f|g\rangle = \frac{1}{2\pi^2\hbar c_0}\int_{\mathbb{R}^3} d\mathbf{r} \int_{\mathbb{R}^3} d\bar{\mathbf{r}} \; \frac{\epsilon_0 \mathbf{E}_f^\dagger(t,\bar{\mathbf{r}})\mathbf{E}_g(t,\mathbf{r}) + \mathbf{B}_f^\dagger(t,\bar{\mathbf{r}})\mathbf{B}_g(t,\mathbf{r})/\mu_0}{|\mathbf{r}-\bar{\mathbf{r}}|^2}$$

- Concise connection to physical quantities[2],[3]
  - $\langle f|f\rangle$: Number of photons in $|f\rangle$

[1] L. Gross, J. Math. Phys. **5**, 687–695 (1964).

[2] Y. B. Zel'dovich, Doklady Akademii Nauk SSSR (USSR) English translation currently published in a number of subject-oriented journals **163** (1965).

[3] I. Bialynicki-Birula and Z. Bialynicka-Birula, *Quantum electrodynamics*, (Pergamon, Oxford, UK, Jan. 1975).

- $\mathbb{M}$: The Hilbert space of free solutions of Maxwell equations
- $|f\rangle \in \mathbb{M}$, $|f\rangle \equiv \{\mathbf{E}_f(t,\mathbf{r}), \mathbf{B}_f(t,\mathbf{r})\}$
- Scalar product[1]:

$$\langle f|g\rangle = \frac{1}{2\pi^2\hbar c_0}\int_{\mathbb{R}^3} d\mathbf{r} \int_{\mathbb{R}^3} d\bar{\mathbf{r}} \; \frac{\epsilon_0 \mathbf{E}_f^\dagger(t,\bar{\mathbf{r}})\mathbf{E}_g(t,\mathbf{r}) + \mathbf{B}_f^\dagger(t,\bar{\mathbf{r}})\mathbf{B}_g(t,\mathbf{r})/\mu_0}{|\mathbf{r}-\bar{\mathbf{r}}|^2}$$

- Concise connection to physical quantities[2],[3]
  - $\langle f|f\rangle$: Number of photons in $|f\rangle$
  - $\langle f|\mathrm{H}|f\rangle$: Energy in $|f\rangle$,
  - $\langle f|\mathrm{P}_z|f\rangle$: Momentum in $|f\rangle$,
  - etc ...

[1] L. Gross, J. Math. Phys. **5**, 687–695 (1964).

[2] Y. B. Zel'dovich, Doklady Akademii Nauk SSSR (USSR) English translation currently published in a number of subject-oriented journals **163** (1965).

[3] I. Bialynicki-Birula and Z. Bialynicka-Birula, *Quantum electrodynamics*, (Pergamon, Oxford, UK, Jan. 1975).

- T: The transition matrix, or T-matrix of a material object[4]

[4] P. C. Waterman, Proc. IEEE **53**, 805–812 (1965).

[5] G. Gouesbet, Journal of Quantitative Spectroscopy and Radiative Transfer **230**, 247–281 (2019).

[6] M. I. Mishchenko, Journal of Quantitative Spectroscopy and Radiative Transfer **242**, 106692 (2020).

[7] T. Wriedt and J. Hellmers, Journal of Quantitative Spectroscopy and Radiative Transfer, X Conference on Electromagnetic and Light Scattering by Non-Spherical Particles **109**, 1536–1542 (2008).

[8] J. Hellmers and T. Wriedt, Journal of Quantitative Spectroscopy and Radiative Transfer **110**, 1511–1517 (2009).

- T: The transition matrix, or T-matrix of a material object[4]
- Linear light-matter interaction between object and illumination:

[4] P. C. Waterman, Proc. IEEE **53**, 805–812 (1965).

[5] G. Gouesbet, Journal of Quantitative Spectroscopy and Radiative Transfer **230**, 247–281 (2019).

[6] M. I. Mishchenko, Journal of Quantitative Spectroscopy and Radiative Transfer **242**, 106692 (2020).

[7] T. Wriedt and J. Hellmers, Journal of Quantitative Spectroscopy and Radiative Transfer, X Conference on Electromagnetic and Light Scattering by Non-Spherical Particles **109**, 1536–1542 (2008).

[8] J. Hellmers and T. Wriedt, Journal of Quantitative Spectroscopy and Radiative Transfer **110**, 1511–1517 (2009).

- T: The transition matrix, or T-matrix of a material object[4]
- Linear light-matter interaction between object and illumination:
  1. Decompose the illumination into multipolar fields: **a**
  2. $\mathbf{b} = \mathrm{T}\mathbf{a}$ are the multipolar coefficients of the scattered field
  3. Computation of scattering, absorption, total fields, etc ...

[4] P. C. Waterman, Proc. IEEE **53**, 805–812 (1965).

[5] G. Gouesbet, Journal of Quantitative Spectroscopy and Radiative Transfer **230**, 247–281 (2019).

[6] M. I. Mishchenko, Journal of Quantitative Spectroscopy and Radiative Transfer **242**, 106692 (2020).

[7] T. Wriedt and J. Hellmers, Journal of Quantitative Spectroscopy and Radiative Transfer, X Conference on Electromagnetic and Light Scattering by Non-Spherical Particles **109**, 1536–1542 (2008).

[8] J. Hellmers and T. Wriedt, Journal of Quantitative Spectroscopy and Radiative Transfer **110**, 1511–1517 (2009).

- T: The transition matrix, or T-matrix of a material object[4]
- Linear light-matter interaction between object and illumination:
  1. Decompose the illumination into multipolar fields: $\mathbf{a}$
  2. $\mathbf{b} = \mathrm{T}\mathbf{a}$ are the multipolar coefficients of the scattered field
  3. Computation of scattering, absorption, total fields, etc ...
- T-matrices for different kinds of objects are differently obtained
  - Micro-particles: Generic Maxwell solvers

[4] P. C. Waterman, Proc. IEEE **53**, 805–812 (1965).

[5] G. Gouesbet, Journal of Quantitative Spectroscopy and Radiative Transfer **230**, 247–281 (2019).

[6] M. I. Mishchenko, Journal of Quantitative Spectroscopy and Radiative Transfer **242**, 106692 (2020).

[7] T. Wriedt and J. Hellmers, Journal of Quantitative Spectroscopy and Radiative Transfer, X Conference on Electromagnetic and Light Scattering by Non-Spherical Particles **109**, 1536–1542 (2008).

[8] J. Hellmers and T. Wriedt, Journal of Quantitative Spectroscopy and Radiative Transfer **110**, 1511–1517 (2009).

- T: The transition matrix, or T-matrix of a material object[4]
- Linear light-matter interaction between object and illumination:
  1. Decompose the illumination into multipolar fields: **a**
  2. $\mathbf{b} = \mathrm{T}\mathbf{a}$ are the multipolar coefficients of the scattered field
  3. Computation of scattering, absorption, total fields, etc ...
- T-matrices for different kinds of objects are differently obtained
  - Micro-particles: Generic Maxwell solvers
  - Molecules: Quantum-chemical calculations, such as (TD)-DFT

[4] P. C. Waterman, Proc. IEEE **53**, 805–812 (1965).

[5] G. Gouesbet, Journal of Quantitative Spectroscopy and Radiative Transfer **230**, 247–281 (2019).

[6] M. I. Mishchenko, Journal of Quantitative Spectroscopy and Radiative Transfer **242**, 106692 (2020).

[7] T. Wriedt and J. Hellmers, Journal of Quantitative Spectroscopy and Radiative Transfer, X Conference on Electromagnetic and Light Scattering by Non-Spherical Particles **109**, 1536–1542 (2008).

[8] J. Hellmers and T. Wriedt, Journal of Quantitative Spectroscopy and Radiative Transfer **110**, 1511–1517 (2009).

- T: The transition matrix, or T-matrix of a material object[4]
- Linear light-matter interaction between object and illumination:
  1. Decompose the illumination into multipolar fields: **a**
  2. $\mathbf{b} = \mathrm{T}\mathbf{a}$ are the multipolar coefficients of the scattered field
  3. Computation of scattering, absorption, total fields, etc ...
- T-matrices for different kinds of objects are differently obtained
  - Micro-particles: Generic Maxwell solvers
  - Molecules: Quantum-chemical calculations, such as (TD)-DFT
- T-matrix method very popular in engineering and physics

[4] P. C. Waterman, Proc. IEEE **53**, 805–812 (1965).

[5] G. Gouesbet, Journal of Quantitative Spectroscopy and Radiative Transfer **230**, 247–281 (2019).

[6] M. I. Mishchenko, Journal of Quantitative Spectroscopy and Radiative Transfer **242**, 106692 (2020).

[7] T. Wriedt and J. Hellmers, Journal of Quantitative Spectroscopy and Radiative Transfer, X Conference on Electromagnetic and Light Scattering by Non-Spherical Particles **109**, 1536–1542 (2008).

[8] J. Hellmers and T. Wriedt, Journal of Quantitative Spectroscopy and Radiative Transfer **110**, 1511–1517 (2009).

- T: The transition matrix, or T-matrix of a material object[4]
- Linear light-matter interaction between object and illumination:
  1. Decompose the illumination into multipolar fields: **a**
  2. $\mathbf{b} = \mathrm{T}\mathbf{a}$ are the multipolar coefficients of the scattered field
  3. Computation of scattering, absorption, total fields, etc ...
- T-matrices for different kinds of objects are differently obtained
  - Micro-particles: Generic Maxwell solvers
  - Molecules: Quantum-chemical calculations, such as (TD)-DFT
- T-matrix method very popular in engineering and physics
- Reference databases[5][6], software repositories[7][8]

---

[4] P. C. Waterman, Proc. IEEE **53**, 805–812 (1965).

[5] G. Gouesbet, Journal of Quantitative Spectroscopy and Radiative Transfer **230**, 247–281 (2019).

[6] M. I. Mishchenko, Journal of Quantitative Spectroscopy and Radiative Transfer **242**, 106692 (2020).

[7] T. Wriedt and J. Hellmers, Journal of Quantitative Spectroscopy and Radiative Transfer, X Conference on Electromagnetic and Light Scattering by Non-Spherical Particles **109**, 1536–1542 (2008).

[8] J. Hellmers and T. Wriedt, Journal of Quantitative Spectroscopy and Radiative Transfer **110**, 1511–1517 (2009).

- T: The transition matrix, or T-matrix of a material object[4]
- Linear light-matter interaction between object and illumination:
  1. Decompose the illumination into multipolar fields: $\mathbf{a}$
  2. $\mathbf{b} = \mathrm{T}\mathbf{a}$ are the multipolar coefficients of the scattered field
  3. Computation of scattering, absorption, total fields, etc ...
- T-matrices for different kinds of objects are differently obtained
  - Micro-particles: Generic Maxwell solvers
  - Molecules: Quantum-chemical calculations, such as (TD)-DFT
- T-matrix method very popular in engineering and physics
- Reference databases[5][6], software repositories[7][8]
- Challenge: Objects invading each other's circumscribing spheres

---

[4] P. C. Waterman, Proc. IEEE **53**, 805–812 (1965).

[5] G. Gouesbet, Journal of Quantitative Spectroscopy and Radiative Transfer **230**, 247–281 (2019).

[6] M. I. Mishchenko, Journal of Quantitative Spectroscopy and Radiative Transfer **242**, 106692 (2020).

[7] T. Wriedt and J. Hellmers, Journal of Quantitative Spectroscopy and Radiative Transfer, X Conference on Electromagnetic and Light Scattering by Non-Spherical Particles **109**, 1536–1542 (2008).

[8] J. Hellmers and T. Wriedt, Journal of Quantitative Spectroscopy and Radiative Transfer **110**, 1511–1517 (2009).

- Let us combine $\mathbb{M}$ and T ! ...

- Let us combine $\mathbb{M}$ and T ! ... not so fast

- Let us combine $\mathbb{M}$ and T ! ... not so fast
- T is a **monochromatic** operator $\mathbf{b} = \mathrm{T}\mathbf{a}$
  - Multipolar decomposition of **monochromatic** fields

- Let us combine $\mathbb{M}$ and T ! ... not so fast
- T is a **monochromatic** operator $\mathbf{b} = \mathrm{T}\mathbf{a}$
  - Multipolar decomposition of **monochromatic** fields
- Monochromatic fields are not members of $\mathbb{M}$
- Their number of photons $\langle f|f\rangle$ and energy $\langle f|\mathrm{H}|f\rangle$ diverge (∞)

- Let us combine $\mathbb{M}$ and T ! ... not so fast
- T is a **monochromatic** operator $\mathbf{b} = \mathrm{T}\mathbf{a}$
  - Multipolar decomposition of **monochromatic** fields
- Monochromatic fields are not members of $\mathbb{M}$
- Their number of photons $\langle f|f\rangle$ and energy $\langle f|\mathrm{H}|f\rangle$ diverge (∞)

Polychromatic T-matrix[a]

[a] M. Vavilin and I. Fernandez-Corbaton, JQSRT **314**, 108853 (2024).

- Let us combine $\mathbb{M}$ and T ! ... not so fast
- T is a **monochromatic** operator $\mathbf{b} = \mathrm{T}\mathbf{a}$
  - Multipolar decomposition of **monochromatic** fields
- Monochromatic fields are not members of $\mathbb{M}$
- Their number of photons $\langle f|f\rangle$ and energy $\langle f|\mathrm{H}|f\rangle$ diverge (∞)

Polychromatic T-matrix[a]

[a] M. Vavilin and I. Fernandez-Corbaton, JQSRT **314**, 108853 (2024).

- Re-base the T-matrix method to be inherently polychromatic

- Let us combine $\mathbb{M}$ and T ! ... not so fast
- T is a **monochromatic** operator $\mathbf{b} = \mathrm{T}\mathbf{a}$
  - Multipolar decomposition of **monochromatic** fields
- Monochromatic fields are not members of $\mathbb{M}$
- Their number of photons $\langle f|f\rangle$ and energy $\langle f|\mathrm{H}|f\rangle$ diverge (∞)

Polychromatic T-matrix[a]

[a] M. Vavilin and I. Fernandez-Corbaton, JQSRT **314**, 108853 (2024).

- Re-base the T-matrix method to be inherently polychromatic
- Base: Poincaré group of transformations. Includes Lorentz boosts:
  - Changes between inertial reference frames

- Let us combine $\mathbb{M}$ and T ! ... not so fast
- T is a **monochromatic** operator $\mathbf{b} = \mathrm{T}\mathbf{a}$
  - Multipolar decomposition of **monochromatic** fields
- Monochromatic fields are not members of $\mathbb{M}$
- Their number of photons $\langle f|f\rangle$ and energy $\langle f|\mathrm{H}|f\rangle$ diverge (∞)

Polychromatic T-matrix[a]

[a] M. Vavilin and I. Fernandez-Corbaton, JQSRT **314**, 108853 (2024).

- Re-base the T-matrix method to be inherently polychromatic
- Base: Poincaré group of transformations. Includes Lorentz boosts:
  - Changes between inertial reference frames
  - Frequency changes as in e.g. the doppler effect

- Let us combine $\mathbb{M}$ and T ! ... not so fast
- T is a **monochromatic** operator $\mathbf{b} = \mathrm{T}\mathbf{a}$
  - Multipolar decomposition of **monochromatic** fields
- Monochromatic fields are not members of $\mathbb{M}$
- Their number of photons $\langle f|f\rangle$ and energy $\langle f|\mathrm{H}|f\rangle$ diverge (∞)

Polychromatic T-matrix[a]

[a] M. Vavilin and I. Fernandez-Corbaton, JQSRT **314**, 108853 (2024).

- Re-base the T-matrix method to be inherently polychromatic
- Base: Poincaré group of transformations. Includes Lorentz boosts:
  - Changes between inertial reference frames
  - Frequency changes as in e.g. the doppler effect

- All $|f\rangle \in \mathbb{M}$ are polychromatic
- Can be expanded with frequency integrals of monochromatic fields
  - Plane waves, multipolar fields, etc ...

$$\mathbf{E}(t,\mathbf{r}) = \sum_{\lambda=\pm1} \int \frac{d^3\mathbf{k}}{k} \, f_\lambda(\mathbf{k}) \; |\mathbf{k}\,\lambda\rangle$$

$\mathbf{E}(t,\mathbf{r}) = \sum_{\lambda=\pm 1} \int \frac{d^3\mathbf{k}}{k} \, f_\lambda(\mathbf{k}) \, |\mathbf{k}\,\lambda\rangle$, simpler expressions for the scalar product

$$\mathbf{E}(t,\mathbf{r}) = \sum_{\lambda=\pm 1}\int \frac{d^3\mathbf{k}}{k}\, f_\lambda(\mathbf{k})\, |\mathbf{k}\,\lambda\rangle$$, simpler expressions for the scalar product

$$\langle f|g\rangle = \sum_{\lambda=\pm 1}\int \frac{d^3\mathbf{k}}{k}\, f^*_\lambda(\mathbf{k}) g_\lambda(\mathbf{k}) = \sum_{\lambda=\pm 1}\int_0^\infty dk\, k \sum_{j=1}^{\infty}\sum_{m=-j}^{j} f^*_{jm\lambda}(k) g_{jm\lambda}(k)$$

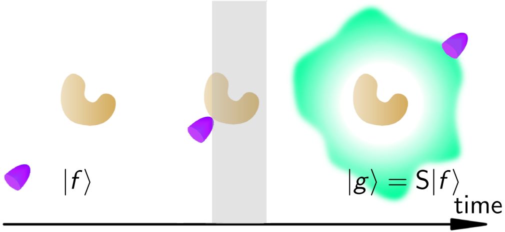
$$\mathbf{E}(t,\mathbf{r}) = \sum_{\lambda=\pm 1} \int \frac{d^3\mathbf{k}}{k}\, f_\lambda(\mathbf{k})\, |\mathbf{k}\,\lambda\rangle$$, simpler expressions for the scalar product

$$\langle f|g\rangle = \sum_{\lambda=\pm 1} \int \frac{d^3\mathbf{k}}{k}\, f^*_\lambda(\mathbf{k}) g_\lambda(\mathbf{k}) = \sum_{\lambda=\pm 1} \int_0^\infty dk\, k \sum_{j=1}^{\infty} \sum_{m=-j}^{j} f^*_{jm\lambda}(k) g_{jm\lambda}(k)$$, computationally friendly

$$\mathbf{E}(t,\mathbf{r}) = \sum_{\lambda=\pm 1} \int \frac{d^3\mathbf{k}}{k} f_\lambda(\mathbf{k}) \, |\mathbf{k}\,\lambda\rangle$$, simpler expressions for the scalar product

$$\langle f|g\rangle = \sum_{\lambda=\pm1} \int \frac{d^3\mathbf{k}}{k} f^*_\lambda(\mathbf{k}) g_\lambda(\mathbf{k}) = \sum_{\lambda=\pm1} \int_0^\infty dk\, k \sum_{j=1}^{\infty} \sum_{m=-j}^{j} f^*_{jm\lambda}(k) g_{jm\lambda}(k),$$ computationally friendly

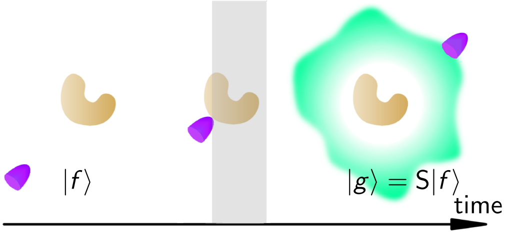
Light-matter interaction



$$\mathbf{E}(t,\mathbf{r}) = \sum_{\lambda=\pm1} \int \frac{d^3\mathbf{k}}{k} f_\lambda(\mathbf{k}) \, |\mathbf{k}\,\lambda\rangle$$, simpler expressions for the scalar product

$$\langle f|g\rangle = \sum_{\lambda=\pm1} \int \frac{d^3\mathbf{k}}{k} f^*_\lambda(\mathbf{k}) g_\lambda(\mathbf{k}) = \sum_{\lambda=\pm1} \int_0^\infty dk\, k \sum_{j=1}^{\infty} \sum_{m=-j}^{j} f^*_{jm\lambda}(k) g_{jm\lambda}(k),$$ computationally friendly

Light-matter interaction

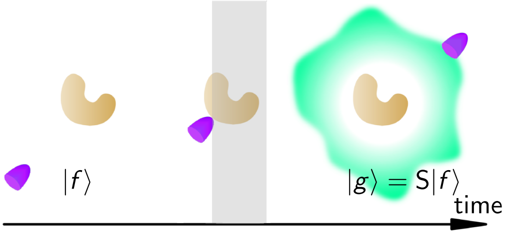


- $S = I + T$
- $|g\rangle = |f\rangle + T\,|f\rangle =$
- $|f\rangle + |h\rangle$

$$\mathbf{E}(t,\mathbf{r}) = \sum_{\lambda=\pm 1} \int \frac{d^3\mathbf{k}}{k} f_\lambda(\mathbf{k}) \, |\mathbf{k}\,\lambda\rangle$$, simpler expressions for the scalar product

$$\langle f|g\rangle = \sum_{\lambda=\pm1} \int \frac{d^3\mathbf{k}}{k} f_\lambda^*(\mathbf{k}) g_\lambda(\mathbf{k}) = \sum_{\lambda=\pm1} \int_0^\infty dk\, k \sum_{j=1}^{\infty} \sum_{m=-j}^{j} f_{jm\lambda}^*(k) g_{jm\lambda}(k),$$ computationally friendly

Light-matter interaction



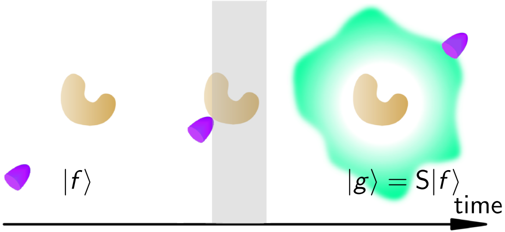
- $\mathrm{S} = \mathrm{I} + \mathrm{T}$
- $|g\rangle = |f\rangle + \mathrm{T}\,|f\rangle =$
- $|f\rangle + |h\rangle$

$$h_{jm\lambda}(k) = \int_0^\infty d\bar{k}\, \bar{k} \sum_{\bar\lambda=\pm1} \sum_{\bar{j}=1}^{\infty} \sum_{\bar{m}=-j}^{j} \mathrm{T}^{jm\lambda}_{\bar{j}\bar{m}\bar{\lambda}}(k,\bar{k}) f_{\bar{j}\bar{m}\bar{\lambda}}(\bar{k})$$

$$\mathbf{E}(t,\mathbf{r}) = \sum_{\lambda=\pm1}\int \frac{d^3\mathbf{k}}{k}\, f_\lambda(\mathbf{k})\, |\mathbf{k}\,\lambda\rangle$$, simpler expressions for the scalar product

$$\langle f|g\rangle = \sum_{\lambda=\pm1}\int \frac{d^3\mathbf{k}}{k}\, f^*_\lambda(\mathbf{k})g_\lambda(\mathbf{k}) = \sum_{\lambda=\pm1}\int_0^\infty dk\, k \sum_{j=1}^{\infty}\sum_{m=-j}^{j} f^*_{jm\lambda}(k)g_{jm\lambda}(k),$$ computationally friendly

Light-matter interaction



- $\mathrm{S} = \mathrm{I} + \mathrm{T}$
- $|g\rangle = |f\rangle + \mathrm{T}\,|f\rangle =$
- $|f\rangle + |h\rangle$

$$h_{jm\lambda}(k) = \int_0^\infty d\bar{k}\,\bar{k} \sum_{\bar{\lambda}=\pm1}\sum_{\bar{j}=1}^{\infty}\sum_{\bar{m}=-j}^{j} \mathrm{T}^{jm\lambda}_{\bar{j}\bar{m}\bar{\lambda}}(k,\bar{k}) f_{\bar{j}\bar{m}\bar{\lambda}}(\bar{k})$$

In most cases $\mathrm{T}(k,\bar{k}) = \frac{2}{k}\delta(k-\bar{k})\mathrm{T}(\bar{k})$, leveraging monochromatic T .

Changes of physical quantities during light matter interaction

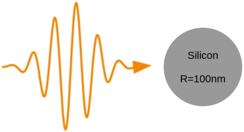
- Number of photons: $\langle f|f\rangle - \langle g|g\rangle = \langle f|\mathrm{I} - \mathrm{S}^{\dagger}\mathrm{S}|f\rangle$

Changes of physical quantities during light matter interaction

- Number of photons: $\langle f|f\rangle - \langle g|g\rangle = \langle f|\mathrm{I} - \mathrm{S}^\dagger\mathrm{S}|f\rangle$
- Energy: $\langle f|\mathrm{H}|f\rangle - \langle g|\mathrm{H}|g\rangle = \langle f|\mathrm{H} - \mathrm{S}^\dagger\mathrm{H}\mathrm{S}|f\rangle$, etc ...

Changes of physical quantities during light matter interaction

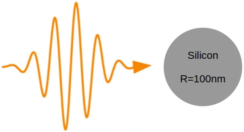
- Number of photons: $\langle f|f\rangle - \langle g|g\rangle = \langle f|\mathrm{I} - \mathrm{S}^\dagger \mathrm{S}|f\rangle$
- Energy: $\langle f|\mathrm{H}|f\rangle - \langle g|\mathrm{H}|g\rangle = \langle f|\mathrm{H} - \mathrm{S}^\dagger \mathrm{HS}|f\rangle$, etc ...



- Pulse $\Delta t \approx$10 fs, λ_0=380 nm

Changes of physical quantities during light matter interaction

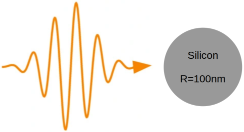
- Number of photons: $\langle f|f\rangle - \langle g|g\rangle = \langle f|\mathrm{I} - \mathrm{S}^\dagger \mathrm{S}|f\rangle$
- Energy: $\langle f|\mathrm{H}|f\rangle - \langle g|\mathrm{H}|g\rangle = \langle f|\mathrm{H} - \mathrm{S}^\dagger \mathrm{H}\mathrm{S}|f\rangle$, etc ...



- Pulse $\Delta t \approx$10 fs, λ_0=380 nm
- Pulse Energy
  $\langle f|\mathrm{H}|f\rangle = 1 \times 10^{-3}$ J
- Pulse Momentum
  $\langle f|\mathrm{P}_z|f\rangle = 3.3 \times 10^{-12}$ kg m s^{-1}

Changes of physical quantities during light matter interaction

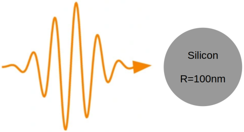
- Number of photons: $\langle f|f\rangle - \langle g|g\rangle = \langle f|\mathrm{I} - \mathrm{S}^\dagger\mathrm{S}|f\rangle$
- Energy: $\langle f|\mathrm{H}|f\rangle - \langle g|\mathrm{H}|g\rangle = \langle f|\mathrm{H} - \mathrm{S}^\dagger\mathrm{H}\mathrm{S}|f\rangle$, etc ...



- Pulse $\Delta t \approx 10$ fs, $\lambda_0 = 380$ nm
- Pulse Energy
  $\langle f|\mathrm{H}|f\rangle = 1 \times 10^{-3}$ J
- Pulse Momentum
  $\langle f|\mathrm{P}_z|f\rangle = 3.3 \times 10^{-12}$ kg m s^{-1}
- Transferred to sphere:
  - 9.15×10^{-6} J,
    6.8×10^{-14} kg m s^{-1}

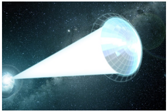
Changes of physical quantities during light matter interaction

- Number of photons: $\langle f|f\rangle - \langle g|g\rangle = \langle f|\mathrm{I} - \mathrm{S}^{\dagger}\mathrm{S}|f\rangle$
- Energy: $\langle f|\mathrm{H}|f\rangle - \langle g|\mathrm{H}|g\rangle = \langle f|\mathrm{H} - \mathrm{S}^{\dagger}\mathrm{H}\mathrm{S}|f\rangle$, etc ...



- Pulse $\Delta t \approx$10 fs, λ_0=380 nm
- Pulse Energy
  $\langle f|\mathrm{H}|f\rangle$ =1×10^{-3} J
- Pulse Momentum
  $\langle f|\mathrm{P}_z|f\rangle$ =3.3×10^{-12} kg m s^{-1}
- Transferred to sphere:
  - 9.15×10^{-6} J,
    6.8×10^{-14} kg m s^{-1}

www.astronomy.com

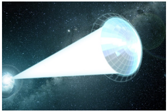


Relativistic speeds[a]

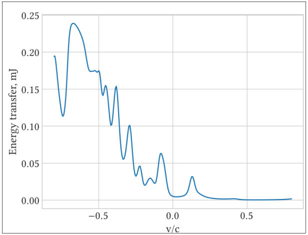
Changes of physical quantities during light matter interaction

- Number of photons: $\langle f|f\rangle - \langle g|g\rangle = \langle f|\mathrm{I} - \mathrm{S}^\dagger \mathrm{S}|f\rangle$
- Energy: $\langle f|\mathrm{H}|f\rangle - \langle g|\mathrm{H}|g\rangle = \langle f|\mathrm{H} - \mathrm{S}^\dagger \mathrm{HS}|f\rangle$, etc ...

www.astronomy.com



Relativistic speeds[a]

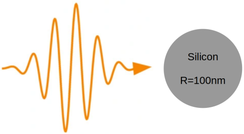


- Pulse $\Delta t \approx 10$ fs, $\lambda_0 = 380$ nm
- Pulse Energy $\langle f|\mathrm{H}|f\rangle = 1 \times 10^{-3}$ J
- Pulse Momentum $\langle f|\mathrm{P}_z|f\rangle = 3.3 \times 10^{-12}$ kg m s^{-1}
- Transferred to sphere:
  - 9.15×10^{-6} J, 6.8×10^{-14} kg m s^{-1}

[a] M. Vavilin, J. D. Mazo-Vásquez, and I. Fernandez-Corbaton, Phys. Rev. Res. **7**, 013132 (2025), M. R. Whittam et al., Scientific Reports **14**, 16812 (2024).

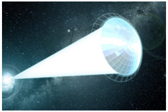
Changes of physical quantities during light matter interaction

- Number of photons: $\langle f|f\rangle - \langle g|g\rangle = \langle f|\mathrm{I} - \mathrm{S}^\dagger\mathrm{S}|f\rangle$
- Energy: $\langle f|\mathrm{H}|f\rangle - \langle g|\mathrm{H}|g\rangle = \langle f|\mathrm{H} - \mathrm{S}^\dagger\mathrm{H}\mathrm{S}|f\rangle$, etc ...

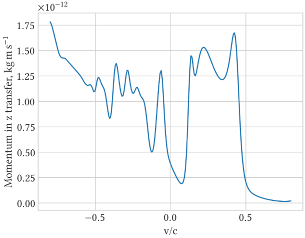


- Pulse $\Delta t \approx$10 fs, λ_0=380 nm
- Pulse Energy
  $\langle f|\mathrm{H}|f\rangle$ =1×10^{-3} J
- Pulse Momentum
  $\langle f|\mathrm{P}_z|f\rangle$ =3.3×10^{-12} kg m s^{-1}
- Transferred to sphere:
  - 9.15×10^{-6} J,
    6.8×10^{-14} kg m s^{-1}

www.astronomy.com



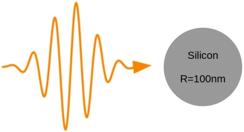
Relativistic speeds[a]



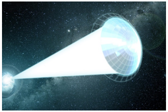
[a] M. Vavilin, J. D. Mazo-Vásquez, and I. Fernandez-Corbaton, Phys. Rev. Res. **7**, 013132 (2025), M. R. Whittam et al., Scientific Reports **14**, 16812 (2024).

Changes of physical quantities during light matter interaction

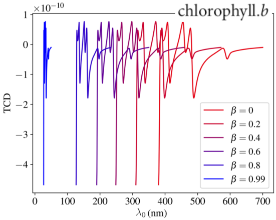
- Number of photons: $\langle f|f\rangle - \langle g|g\rangle = \langle f|\mathrm{I} - \mathrm{S}^\dagger \mathrm{S}|f\rangle$
- Energy: $\langle f|\mathrm{H}|f\rangle - \langle g|\mathrm{H}|g\rangle = \langle f|\mathrm{H} - \mathrm{S}^\dagger \mathrm{H}\mathrm{S}|f\rangle$, etc ...



www.astronomy.com



Relativistic speeds[a]



- Pulse $\Delta t \approx$10 fs, λ_0=380 nm
- Pulse Energy $\langle f|\mathrm{H}|f\rangle$ =1×10^{-3} J
- Pulse Momentum $\langle f|\mathrm{P}_z|f\rangle$ =3.3×10^{-12} kg m s^{-1}
- Transferred to sphere:
  - 9.15×10^{-6} J, 6.8×10^{-14} kg m s^{-1}

[a] M. Vavilin, J. D. Mazo-Vásquez, and I. Fernandez-Corbaton, Phys. Rev. Res. **7**, 013132 (2025), M. R. Whittam et al., Scientific Reports **14**, 16812 (2024).

Let us talk about quantum.

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle$

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse
- $|f\rangle = \sqrt{N}|\hat{f}\rangle$. Normalized bosonic mode $|\hat{f}\rangle$. As $\left(a^\dagger_{\hat{f}}\right)^N|0\rangle$.

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse
- $|f\rangle = \sqrt{N}\,|\hat{f}\rangle$. Normalized bosonic mode $|\hat{f}\rangle$. As $\left(a^{\dagger}_{\hat{f}}\right)^{N}|0\rangle$.
- Two entangled photons?

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse
- $|f\rangle = \sqrt{N}|\hat{f}\rangle$. Normalized bosonic mode $|\hat{f}\rangle$. As $\left(a^\dagger_{\hat{f}}\right)^N|0\rangle$.
- Two entangled photons? Then it does not work

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

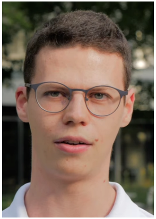
Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse
- $|f\rangle = \sqrt{N}\,|\hat{f}\rangle$. Normalized bosonic mode $|\hat{f}\rangle$. As $\left(a^\dagger_{\hat{f}}\right)^N |0\rangle$.
- Two entangled photons? Then it does not work , however[9] ...

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse
- $|f\rangle = \sqrt{N}\,|\hat{f}\rangle$. Normalized bosonic mode $|\hat{f}\rangle$. As $\left(a^{\dagger}_{\hat{f}}\right)^{N}|0\rangle$.
- Two entangled photons? Then it does not work , however[9] ...

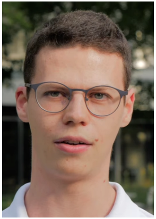


Lukas Freter

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse
- $|f\rangle = \sqrt{N}|\hat{f}\rangle$. Normalized bosonic mode $|\hat{f}\rangle$. As $\left(a^\dagger_{\hat{f}}\right)^N|0\rangle$.
- Two entangled photons? Then it does not work , however[9] ...



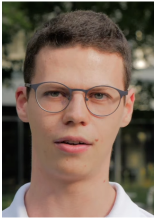
Lukas Freter

- $\mathbb{M}_2 \equiv \mathbb{M} \otimes \mathbb{M}$

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse
- $|f\rangle = \sqrt{N}\,|\hat{f}\rangle$. Normalized bosonic mode $|\hat{f}\rangle$. As $\left(a^\dagger_{\hat{f}}\right)^N |0\rangle$.
- Two entangled photons? Then it does not work , however[9] ...



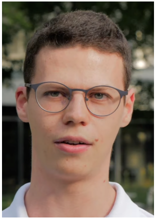
Lukas Freter

- $\mathbb{M}_2 \equiv \mathbb{M} \otimes \mathbb{M}$
- $|\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{k}\lambda\rangle \otimes |\bar{\mathbf{k}}\bar{\lambda}\rangle + |\bar{\mathbf{k}}\bar{\lambda}\rangle \otimes |\mathbf{k}\lambda\rangle\right)$

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse
- $|f\rangle = \sqrt{N}|\hat{f}\rangle$. Normalized bosonic mode $|\hat{f}\rangle$. As $\left(a^\dagger_{\hat{f}}\right)^N |0\rangle$.
- Two entangled photons? Then it does not work , however[9] ...



Lukas Freter

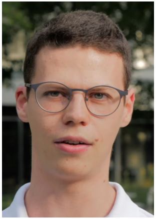
- $\mathbb{M}_2 \equiv \mathbb{M} \otimes \mathbb{M}$
- $|\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{k}\lambda\rangle \otimes |\bar{\mathbf{k}}\bar{\lambda}\rangle + |\bar{\mathbf{k}}\bar{\lambda}\rangle \otimes |\mathbf{k}\lambda\rangle\right)$

$$|f\rangle_2 = \sum_{\lambda\bar{\lambda}} \int \frac{\mathrm{d}^3 k}{k} \int \frac{\mathrm{d}^3 \bar{k}}{\bar{k}} \mathrm{f}_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) |\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle$$

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

Let us talk about quantum. Hilbert space $\mathbb{M}$ formalism valid for

- Single photons $\langle f|f\rangle = 1$
- Classical fields: $N = \langle f|f\rangle \approx 10^{12}$ in a femtosecond pulse
- $|f\rangle = \sqrt{N}|\hat{f}\rangle$. Normalized bosonic mode $|\hat{f}\rangle$. As $\left(a^\dagger_{\hat{f}}\right)^N|0\rangle$.
- Two entangled photons? Then it does not work , however[9] ...



Lukas Freter

- $\mathbb{M}_2 \equiv \mathbb{M} \otimes \mathbb{M}$
- $|\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle = \frac{1}{\sqrt{2}}\left(|\mathbf{k}\lambda\rangle \otimes |\bar{\mathbf{k}}\bar{\lambda}\rangle + |\bar{\mathbf{k}}\bar{\lambda}\rangle \otimes |\mathbf{k}\lambda\rangle\right)$

$$|f\rangle_2 = \sum_{\lambda\bar{\lambda}} \int \frac{d^3k}{k} \int \frac{d^3\bar{k}}{\bar{k}} f_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) |\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle$$

$${}_2\langle f|g\rangle_2 = \sum_{\lambda\bar{\lambda}} \int \frac{d^3k}{k} \int \frac{d^3\bar{k}}{\bar{k}} \left[\frac{f_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) + f_{\bar{\lambda}\lambda}(\bar{\mathbf{k}}, \mathbf{k})}{\sqrt{2}}\right]^* \left[\frac{g_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) + g_{\bar{\lambda}\lambda}(\bar{\mathbf{k}}, \mathbf{k})}{\sqrt{2}}\right]$$

[9] L. Freter, B. Zerulla, M. Krstic, C. Holzer, C. Rockstuhl, and I. Fernandez-Corbaton, Phys. Rev. A **110**, 043516 (2024).

$$|f\rangle_2 = \sum_{\lambda\bar{\lambda}} \int \frac{\mathrm{d}^3 k}{k} \int \frac{\mathrm{d}^3 \bar{k}}{\bar{k}} \mathrm{f}_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) |\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle$$

$$|f\rangle_2 = \sum_{\lambda\bar{\lambda}} \int \frac{\mathrm{d}^3 k}{k} \int \frac{\mathrm{d}^3 \bar{k}}{\bar{k}} \mathsf{f}_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) |\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle$$

Separable vs. entangled states

$$|f\rangle_2 = \sum_{\lambda\bar{\lambda}} \int \frac{\mathrm{d}^3 k}{k} \int \frac{\mathrm{d}^3 \bar{k}}{\bar{k}} \mathrm{f}_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) |\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle$$

Separable vs. entangled states

Separable: $\mathrm{f}_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) = \mathrm{a}_{\lambda}(\mathbf{k})\mathrm{b}_{\bar{\lambda}}(\bar{\mathbf{k}})$

$$|f\rangle_2 = \sum_{\lambda\bar{\lambda}} \int \frac{\mathrm{d}^3 k}{k} \int \frac{\mathrm{d}^3 \bar{k}}{\bar{k}} \mathsf{f}_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) |\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle$$

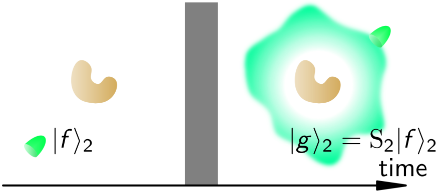
Separable vs. entangled states

Separable: $\mathsf{f}_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) = \mathsf{a}_{\lambda}(\mathbf{k})\mathsf{b}_{\bar{\lambda}}(\bar{\mathbf{k}})$ | Entangled: $\mathsf{f}_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) \neq \mathsf{a}_{\lambda}(\mathbf{k})\mathsf{b}_{\bar{\lambda}}(\bar{\mathbf{k}})$

$$|f\rangle_2 = \sum_{\lambda\bar{\lambda}} \int \frac{d^3k}{k} \int \frac{d^3\bar{k}}{\bar{k}} f_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) |\mathbf{k}\lambda, \bar{\mathbf{k}}\bar{\lambda}\rangle$$

Separable vs. entangled states

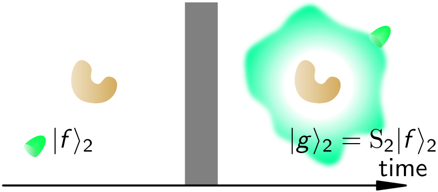
Separable: $f_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) = a_\lambda(\mathbf{k}) b_{\bar{\lambda}}(\bar{\mathbf{k}})$ | Entangled: $f_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) \neq a_\lambda(\mathbf{k}) b_{\bar{\lambda}}(\bar{\mathbf{k}})$



$$|f\rangle_2 = \sum_{\lambda\bar\lambda} \int \frac{d^3k}{k} \int \frac{d^3\bar{k}}{\bar{k}} f_{\lambda\bar\lambda}(\mathbf{k},\bar{\mathbf{k}})|\mathbf{k}\lambda,\bar{\mathbf{k}}\bar\lambda\rangle$$

Separable vs. entangled states

Separable: $f_{\lambda\bar\lambda}(\mathbf{k},\bar{\mathbf{k}}) = a_\lambda(\mathbf{k})b_{\bar\lambda}(\bar{\mathbf{k}})$ | Entangled: $f_{\lambda\bar\lambda}(\mathbf{k},\bar{\mathbf{k}}) \neq a_\lambda(\mathbf{k})b_{\bar\lambda}(\bar{\mathbf{k}})$



$$g_{jm\lambda\bar{j}\bar{m}\bar\lambda}(k,\bar{k}) = \sum_{ln\sigma\bar{l}\bar{n}\bar\sigma} \int dq\ q \int d\bar{q}\ \bar{q}\ \mathrm{S}^{jm\lambda\bar{j}\bar{m}\bar\lambda}_{ln\sigma\bar{l}\bar{n}\bar\sigma}(k,\bar{k},q,\bar{q}) f_{ln\sigma\bar{l}\bar{n}\bar\sigma}(q,\bar{q})$$

$$S_2 = S \otimes S + N_2$$

$$S_2 = S \otimes S + N_2$$

- Separable response $S \otimes S$
- Response to one part of the state independent of the other part

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part
- Non-separable N_2: $N_{2,\lambda\bar{\lambda}\sigma\bar{\sigma}}(\mathbf{k}, \bar{\mathbf{k}}, \mathbf{q}, \bar{\mathbf{q}}) \neq A_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) B_{\sigma\bar{\sigma}}(\mathbf{q}, \bar{\mathbf{q}})$
- Response to one part of the state depends on the other part

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part
- Non-separable N_2: $N_{2,\lambda\bar{\lambda}\sigma\bar{\sigma}}(\mathbf{k}, \bar{\mathbf{k}}, \mathbf{q}, \bar{\mathbf{q}}) \neq A_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) B_{\sigma\bar{\sigma}}(\mathbf{q}, \bar{\mathbf{q}})$
- Response to one part of the state depends on the other part
- E.g: Second harmonic generation, and other non-linear effects

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part
- Non-separable N_2: $N_{2,\lambda\bar{\lambda}\sigma\bar{\sigma}}(\mathbf{k}, \bar{\mathbf{k}}, \mathbf{q}, \bar{\mathbf{q}}) \neq A_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) B_{\sigma\bar{\sigma}}(\mathbf{q}, \bar{\mathbf{q}})$
- Response to one part of the state depends on the other part
- E.g: Second harmonic generation, and other non-linear effects
- High intensity typically needed to observe N_2 effects

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- Then, this leads to the definition of T_2 as:

$$\mathsf{S}_2 = \mathsf{S} \otimes \mathsf{S} = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T}) = \mathrm{I} \otimes \mathrm{I} + \underbrace{\mathsf{T} \otimes \mathrm{I} + \mathrm{I} \otimes \mathsf{T} + \mathsf{T} \otimes \mathsf{T}}_{\mathsf{T}_2}$$

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- Then, this leads to the definition of T_2 as:

$$\mathsf{S}_2 = \mathsf{S} \otimes \mathsf{S} = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T}) = \mathrm{I} \otimes \mathrm{I} + \underbrace{\mathsf{T} \otimes \mathrm{I} + \mathrm{I} \otimes \mathsf{T} + \mathsf{T} \otimes \mathsf{T}}_{\mathsf{T}_2}$$

- Sanity checks

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- Then, this leads to the definition of T_2 as:

$$\mathsf{S}_2 = \mathsf{S} \otimes \mathsf{S} = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T}) = \mathrm{I} \otimes \mathrm{I} + \underbrace{\mathsf{T} \otimes \mathrm{I} + \mathrm{I} \otimes \mathsf{T} + \mathsf{T} \otimes \mathsf{T}}_{\mathsf{T}_2}$$

- Sanity checks
- If S is unitary $\implies$ S_2 is unitary

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- Then, this leads to the definition of T_2 as:

$$\mathsf{S}_2 = \mathsf{S} \otimes \mathsf{S} = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T}) = \mathrm{I} \otimes \mathrm{I} + \underbrace{\mathsf{T} \otimes \mathrm{I} + \mathrm{I} \otimes \mathsf{T} + \mathsf{T} \otimes \mathsf{T}}_{\mathsf{T}_2}$$

- Sanity checks
- If S is unitary $\implies$ S_2 is unitary
- One obtains the Hong-Ou-Mandel effect
  - Thanks Gabriel! (Molina-Terriza)

When $S_2 \approx S \otimes S$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $S_2 = (\mathrm{I} + \mathrm{T}) \otimes (\mathrm{I} + \mathrm{T})$

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $\mathsf{S}_2 \approx \mathsf{S} \otimes \mathsf{S}$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $\mathsf{S}_2 = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T})$
- Interaction of entangled biphoton states with nanostructures

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $S_2 \approx S \otimes S$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $S_2 = (\mathrm{I} + \mathrm{T}) \otimes (\mathrm{I} + \mathrm{T})$
- Interaction of entangled biphoton states with nanostructures
    - Study entanglement evolution

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $\mathbf{S}_2 \approx \mathbf{S} \otimes \mathbf{S}$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $\mathsf{S}_2 = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T})$
- Interaction of entangled biphoton states with nanostructures
  - Study entanglement evolution
- Lorentz boosts for $\mathbb{M}_2$ readily obtained from those for $\mathbb{M}$

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $\mathbf{S}_2 \approx \mathbf{S} \otimes \mathbf{S}$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $\mathsf{S}_2 = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T})$
- Interaction of entangled biphoton states with nanostructures
  - Study entanglement evolution
- Lorentz boosts for $\mathbb{M}_2$ readily obtained from those for $\mathbb{M}$
- Satellite quantum communication with entangled biphoton pulses

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $\mathbf{S}_2 \approx \mathbf{S} \otimes \mathbf{S}$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $\mathsf{S}_2 = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T})$
- Interaction of entangled biphoton states with nanostructures
  - Study entanglement evolution
- Lorentz boosts for $\mathbb{M}_2$ readily obtained from those for $\mathbb{M}$
- Satellite quantum communication with entangled biphoton pulses

1. Obtain the T-matrix of your system. Check software repositories.

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $\mathsf{S}_2 \approx \mathsf{S} \otimes \mathsf{S}$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $\mathsf{S}_2 = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T})$
- Interaction of entangled biphoton states with nanostructures
  - Study entanglement evolution
- Lorentz boosts for $\mathbb{M}_2$ readily obtained from those for $\mathbb{M}$
- Satellite quantum communication with entangled biphoton pulses

1. Obtain the T-matrix of your system. Check software repositories.
2. Specify your (entangled) illumination $|f\rangle_2 \equiv \mathsf{f}_{jm\lambda,\bar{j}\bar{m}\bar{\lambda}}(k, \bar{k})$

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $\mathbf{S}_2 \approx \mathbf{S} \otimes \mathbf{S}$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $\mathsf{S}_2 = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T})$
- Interaction of entangled biphoton states with nanostructures
  - Study entanglement evolution
- Lorentz boosts for $\mathbb{M}_2$ readily obtained from those for $\mathbb{M}$
- Satellite quantum communication with entangled biphoton pulses

1. Obtain the T-matrix of your system. Check software repositories.
2. Specify your (entangled) illumination $|f\rangle_2 \equiv \mathsf{f}_{jm\lambda,\bar{j}\bar{m}\bar{\lambda}}(k, \bar{k})$
3. Apply $\mathrm{S}_2 |f\rangle_2$, and study light-matter interaction effects

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $\mathbf{S}_2 \approx \mathbf{S} \otimes \mathbf{S}$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $\mathrm{S}_2 = (\mathrm{I} + \mathrm{T}) \otimes (\mathrm{I} + \mathrm{T})$
- Interaction of entangled biphoton states with nanostructures
  - Study entanglement evolution
- Lorentz boosts for $\mathbb{M}_2$ readily obtained from those for $\mathbb{M}$
- Satellite quantum communication with entangled biphoton pulses

1. Obtain the T-matrix of your system. Check software repositories.
2. Specify your (entangled) illumination $|f\rangle_2 \equiv \mathrm{f}_{jm\lambda,\bar{j}\bar{m}\bar{\lambda}}(k, \bar{k})$
3. Apply $\mathrm{S}_2 |f\rangle_2$, and study light-matter interaction effects

- **treams** T-matrix code[10]: https://github.com/tfp-photonics/treams
  - Native support for clusters of spheres

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $S_2 \approx S \otimes S$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $S_2 = (\mathrm{I} + \mathrm{T}) \otimes (\mathrm{I} + \mathrm{T})$
- Interaction of entangled biphoton states with nanostructures
  - Study entanglement evolution
- Lorentz boosts for $\mathbb{M}_2$ readily obtained from those for $\mathbb{M}$
- Satellite quantum communication with entangled biphoton pulses

1. Obtain the T-matrix of your system. Check software repositories.
2. Specify your (entangled) illumination $|f\rangle_2 \equiv f_{jm\lambda,\bar{j}\bar{m}\bar{\lambda}}(k, \bar{k})$
3. Apply $\mathrm{S}_2 |f\rangle_2$, and study light-matter interaction effects

- **treams** T-matrix code[10]: https://github.com/tfp-photonics/treams
  - Native support for clusters of spheres
- **T-matrix database**[11]: https://tmatrix.scc.kit.edu/
  - Access through any github account.
  - Pre-computed T-matrices. Number should grow with time.

---

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

When $S_2 \approx S \otimes S$ is a good approximation

- Knowledge of T in $\mathbb{M}$ is sufficient to obtain $S_2 = (\mathrm{I} + \mathrm{T}) \otimes (\mathrm{I} + \mathrm{T})$
- Interaction of entangled biphoton states with nanostructures
  - Study entanglement evolution
- Lorentz boosts for $\mathbb{M}_2$ readily obtained from those for $\mathbb{M}$
- Satellite quantum communication with entangled biphoton pulses

1. Obtain the T-matrix of your system. Check software repositories.
2. Specify your (entangled) illumination $|f\rangle_2 \equiv \mathrm{f}_{jm\lambda,\bar{j}\bar{m}\bar{\lambda}}(k, \bar{k})$
3. Apply $\mathrm{S}_2 |f\rangle_2$, and study light-matter interaction effects

- **treams** T-matrix code[10]: https://github.com/tfp-photonics/treams
  - Native support for clusters of spheres
- **T-matrix database**[11]: https://tmatrix.scc.kit.edu/
  - Access through any github account.
  - Pre-computed T-matrices. Number should grow with time.

Thank you for your time!

[10] D. Beutel, I. Fernandez-Corbaton, and C. Rockstuhl, Computer Physics Communications **297**, 109076 (2024).

[11] N. Asadova et al., Journal of Quantitative Spectroscopy and Radiative Transfer **333**, 109310 (2025).

One benefit of $\mathbb{M}_2$: Selection rules derived from symmetries, as in $\mathbb{M}$

One benefit of $\mathbb{M}_2$: Selection rules derived from symmetries, as in $\mathbb{M}$

If the scatterer is symmetric under $X \implies \langle h|\mathsf{S}|l\rangle = \langle h|XSX^{-1}|l\rangle$

One benefit of $\mathbb{M}_2$: Selection rules derived from symmetries, as in $\mathbb{M}$

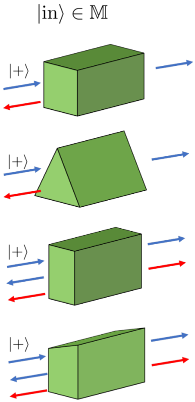
If the scatterer is symmetric under $X \implies \langle h|\mathsf{S}|l\rangle = \langle h|XSX^{-1}|l\rangle$

which can force $\langle h|\mathsf{S}|l\rangle = 0$, e.g. $\langle h|\mathsf{S}|l\rangle = -\langle h|\mathsf{S}|l\rangle$

One benefit of $\mathbb{M}_2$: Selection rules derived from symmetries, as in $\mathbb{M}$

If the scatterer is symmetric under $X \implies \langle h|\mathrm{S}|I\rangle = \langle h|XSX^{-1}|I\rangle$

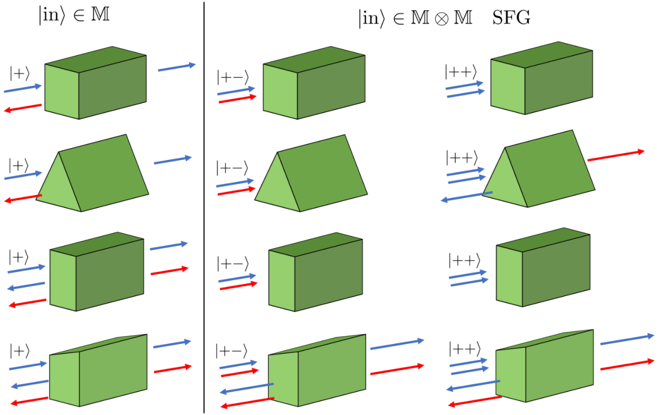
which can force $\langle h|\mathrm{S}|I\rangle = 0$, e.g. $\langle h|\mathrm{S}|I\rangle = -\langle h|\mathrm{S}|I\rangle$



One benefit of $\mathbb{M}_2$: Selection rules derived from symmetries, as in $\mathbb{M}$

If the scatterer is symmetric under $X \implies \langle h|\mathrm{S}|I\rangle = \langle h|XSX^{-1}|I\rangle$

which can force $\langle h|\mathrm{S}|I\rangle = 0$, e.g. $\langle h|\mathrm{S}|I\rangle = -\langle h|\mathrm{S}|I\rangle$



- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

[12] J. C. Schotland, A. Cazé, and T. B. Norris, Opt. Lett. **41**, 444–447 (2016).

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- Then, this leads to the definition of T_2 as:

$$\mathsf{S}_2 = \mathsf{S} \otimes \mathsf{S} = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T}) = \mathrm{I} \otimes \mathrm{I} + \underbrace{\mathsf{T} \otimes \mathrm{I} + \mathrm{I} \otimes \mathsf{T} + \mathsf{T} \otimes \mathsf{T}}_{\mathsf{T}_2}$$

[12] J. C. Schotland, A. Cazé, and T. B. Norris, Opt. Lett. **41**, 444–447 (2016).

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- Then, this leads to the definition of T_2 as:

$$\mathsf{S}_2 = \mathsf{S} \otimes \mathsf{S} = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T}) = \mathrm{I} \otimes \mathrm{I} + \underbrace{\mathsf{T} \otimes \mathrm{I} + \mathrm{I} \otimes \mathsf{T} + \mathsf{T} \otimes \mathsf{T}}_{\mathsf{T}_2}$$

- $\hat{\mathsf{S}}_2 = \mathrm{I} \otimes \mathrm{I} + \mathsf{T} \otimes \mathsf{T}$ can be found in literature[12]

[12] J. C. Schotland, A. Cazé, and T. B. Norris, Opt. Lett. **41**, 444–447 (2016).

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- Then, this leads to the definition of T_2 as:

$$\mathsf{S}_2 = \mathsf{S} \otimes \mathsf{S} = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T}) = \mathrm{I} \otimes \mathrm{I} + \underbrace{\mathsf{T} \otimes \mathrm{I} + \mathrm{I} \otimes \mathsf{T} + \mathsf{T} \otimes \mathsf{T}}_{\mathsf{T}_2}$$

- $\hat{\mathsf{S}}_2 = \mathrm{I} \otimes \mathrm{I} + \mathsf{T} \otimes \mathsf{T}$ can be found in literature[12]
- $\hat{\mathsf{S}}_2$ shown to be incorrect

[12] J. C. Schotland, A. Cazé, and T. B. Norris, Opt. Lett. **41**, 444–447 (2016).

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- Then, this leads to the definition of T_2 as:

$$\mathsf{S}_2 = \mathsf{S} \otimes \mathsf{S} = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T}) = \mathrm{I} \otimes \mathrm{I} + \underbrace{\mathsf{T} \otimes \mathrm{I} + \mathrm{I} \otimes \mathsf{T} + \mathsf{T} \otimes \mathsf{T}}_{\mathsf{T}_2}$$

- $\hat{\mathsf{S}}_2 = \mathrm{I} \otimes \mathrm{I} + \mathsf{T} \otimes \mathsf{T}$ can be found in literature[12]
- $\hat{\mathsf{S}}_2$ shown to be incorrect
- Assuming unitarity of S
  - implies unitarity of S_2, but not of $\hat{\mathsf{S}}_2$

[12] J. C. Schotland, A. Cazé, and T. B. Norris, Opt. Lett. **41**, 444–447 (2016).

- For low intensities, such as a source of entangled biphoton states

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2 \approx \mathrm{S} \otimes \mathrm{S}$$

- Then, this leads to the definition of T_2 as:

$$\mathsf{S}_2 = \mathsf{S} \otimes \mathsf{S} = (\mathrm{I} + \mathsf{T}) \otimes (\mathrm{I} + \mathsf{T}) = \mathrm{I} \otimes \mathrm{I} + \underbrace{\mathsf{T} \otimes \mathrm{I} + \mathrm{I} \otimes \mathsf{T} + \mathsf{T} \otimes \mathsf{T}}_{\mathsf{T}_2}$$

- $\hat{\mathsf{S}}_2 = \mathrm{I} \otimes \mathrm{I} + \mathsf{T} \otimes \mathsf{T}$ can be found in literature[12]
- $\hat{\mathsf{S}}_2$ shown to be incorrect
- Assuming unitarity of S
  - implies unitarity of S_2, but not of $\hat{\mathsf{S}}_2$
- One obtains the Hong-Ou-Mandel effect with S_2, but not with $\hat{\mathsf{S}}_2$
  - Thanks Gabriel! (Molina-Terriza)

[12] J. C. Schotland, A. Cazé, and T. B. Norris, Opt. Lett. **41**, 444–447 (2016).

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part
- Non-separable N_2: $N_{2,\lambda\bar{\lambda}\sigma\bar{\sigma}}(\mathbf{k}, \bar{\mathbf{k}}, \mathbf{q}, \bar{\mathbf{q}}) \neq A_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) B_{\sigma\bar{\sigma}}(\mathbf{q}, \bar{\mathbf{q}})$
- Response to one part of the state depends on the other part

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part
- Non-separable N_2: $N_{2,\lambda\bar{\lambda}\sigma\bar{\sigma}}(\mathbf{k}, \bar{\mathbf{k}}, \mathbf{q}, \bar{\mathbf{q}}) \neq A_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) B_{\sigma\bar{\sigma}}(\mathbf{q}, \bar{\mathbf{q}})$
- Response to one part of the state depends on the other part
- E.g: Second harmonic generation, and other non-linear effects

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part
- Non-separable N_2: $N_{2,\lambda\bar{\lambda}\sigma\bar{\sigma}}(\mathbf{k}, \bar{\mathbf{k}}, \mathbf{q}, \bar{\mathbf{q}}) \neq A_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) B_{\sigma\bar{\sigma}}(\mathbf{q}, \bar{\mathbf{q}})$
- Response to one part of the state depends on the other part
- E.g: Second harmonic generation, and other non-linear effects

$S_2 : \mathbb{M}_2 \to \mathbb{M}_2$ is linear $S_2 \left(\alpha|f\rangle_2 + \beta|g\rangle_2\right) = \alpha \mathsf{S}|f\rangle_2 + \beta \mathsf{S}|g\rangle_2$

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part
- Non-separable N_2: $N_{2,\lambda\bar{\lambda}\sigma\bar{\sigma}}(\mathbf{k}, \bar{\mathbf{k}}, \mathbf{q}, \bar{\mathbf{q}}) \neq A_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) B_{\sigma\bar{\sigma}}(\mathbf{q}, \bar{\mathbf{q}})$
- Response to one part of the state depends on the other part
- E.g: Second harmonic generation, and other non-linear effects

$S_2 : \mathbb{M}_2 \to \mathbb{M}_2$ is linear $S_2\left(\alpha|f\rangle_2 + \beta|g\rangle_2\right) = \alpha \mathsf{S}|f\rangle_2 + \beta \mathsf{S}|g\rangle_2$

Nonlinearities? Can e.g SHG fit ?

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part
- Non-separable N_2: $N_{2,\lambda\bar{\lambda}\sigma\bar{\sigma}}(\mathbf{k}, \bar{\mathbf{k}}, \mathbf{q}, \bar{\mathbf{q}}) \neq A_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) B_{\sigma\bar{\sigma}}(\mathbf{q}, \bar{\mathbf{q}})$
- Response to one part of the state depends on the other part
- E.g: Second harmonic generation, and other non-linear effects

$S_2 : \mathbb{M}_2 \to \mathbb{M}_2$ is linear $S_2\left(\alpha|f\rangle_2 + \beta|g\rangle_2\right) = \alpha \mathsf{S}|f\rangle_2 + \beta \mathsf{S}|g\rangle_2$

Nonlinearities? Can e.g SHG fit ?

For simplicity, assume $\mathsf{f}_{jm\lambda\bar{j}\bar{m}\bar{\lambda}}(k, \bar{k}) = \mathsf{a}_{jm\lambda}(k)\mathsf{a}_{\bar{j}\bar{m}\bar{\lambda}}(\bar{k})$

$$\mathrm{S}_2 = \mathrm{S} \otimes \mathrm{S} + \mathrm{N}_2$$

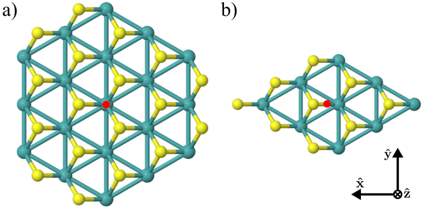
- Separable response $\mathrm{S} \otimes \mathrm{S}$
- Response to one part of the state independent of the other part
- Non-separable N_2: $N_{2,\lambda\bar{\lambda}\sigma\bar{\sigma}}(\mathbf{k}, \bar{\mathbf{k}}, \mathbf{q}, \bar{\mathbf{q}}) \neq A_{\lambda\bar{\lambda}}(\mathbf{k}, \bar{\mathbf{k}}) B_{\sigma\bar{\sigma}}(\mathbf{q}, \bar{\mathbf{q}})$
- Response to one part of the state depends on the other part
- E.g: Second harmonic generation, and other non-linear effects

$S_2 : \mathbb{M}_2 \to \mathbb{M}_2$ is linear $S_2\left(\alpha|f\rangle_2 + \beta|g\rangle_2\right) = \alpha \mathsf{S}|f\rangle_2 + \beta \mathsf{S}|g\rangle_2$

Nonlinearities? Can e.g SHG fit ?

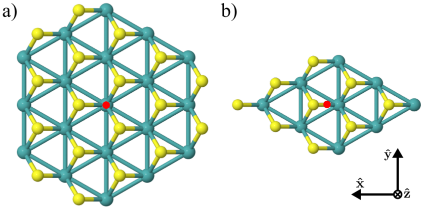
For simplicity, assume $\mathrm{f}_{jm\lambda\bar{j}\bar{m}\bar{\lambda}}(k, \bar{k}) = \mathrm{a}_{jm\lambda}(k)\mathrm{a}_{\bar{j}\bar{m}\bar{\lambda}}(\bar{k})$

- $\mathrm{g}_{jm\lambda\bar{j}\bar{m}\bar{\lambda}}(k, \bar{k})$ is linear with respect to $\mathrm{f}_{jm\lambda\bar{j}\bar{m}\bar{\lambda}}(k, \bar{k})$, but
- nonlinear w.r.t the single photon $\mathrm{a}_{jm\lambda}(k)$



Circular polarizations

SFG with C_n symmetry			
n	Incident	Tr.	Re.
1	$++$	$+,-$	$+,-$
2		x	x
3		$-$	$+$
≥ 4		x	x
1	$+-$	$+,-$	$+,-$
2		x	x
≥ 3		x	x

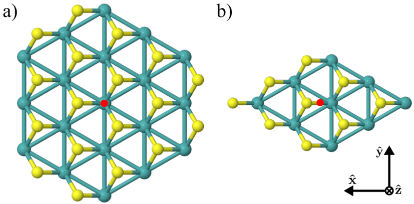


Circular polarizations

SFG with C_n symmetry			
n	Incident	Tr.	Re.
1	++	+, −	+, −
2		x	x
3		−	+
≥ 4		x	x
1	+−	+, −	+, −
2		x	x
≥ 3		x	x

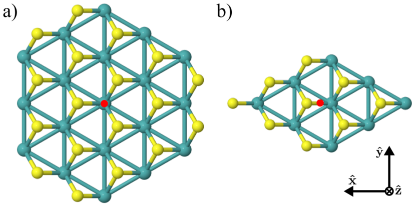
Linear polarizations: TE($\hat{\mathbf{y}}$) / TM($\hat{\mathbf{x}}$)

SFG with XZ mirror symmetry		
	Incident	Tr./ Re.
SFG	TE–TE	TM
	TM–TM	TM
	TE–TM	TE



Linear polarizations: TE($\hat{\mathbf{y}}$) / TM($\hat{\mathbf{x}}$)

SFG with XZ mirror symmetry		
	Incident	Tr./ Re.
SFG	TE–TE	TM
	TM–TM	TM
	TE–TM	TE

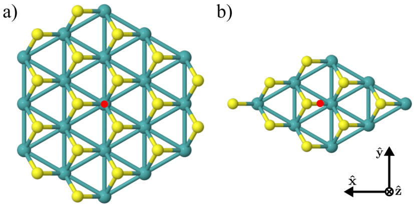


Linear polarizations: TE($\hat{\mathbf{y}}$) / TM($\hat{\mathbf{x}}$)

SFG with XZ mirror symmetry		
	Incident	Tr./ Re.
SFG	TE–TE	TM
	TM–TM	TM
	TE–TM	TE

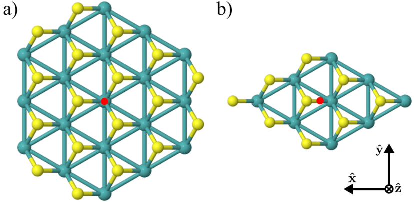
Rhomboid b)

Incident	Transmission (a.u.)		Reflection (a.u.)	
	TE	TM	TE	TM
TE–TE	9.11e-10	0.498	9.11e-10	0.498
TM–TM	1.42e-09	1	1.42e-09	1
TE–TM	0.383	2.44e-09	0.383	2.44e-09



Circular polarizations

SFG with C_n symmetry			
n	Incident	Tr.	Re.
1	++	$+,-$	$+,-$
2		x	x
3		$-$	$+$
≥ 4		x	x
1	+−	$+,-$	$+,-$
2		x	x
≥ 3		x	x

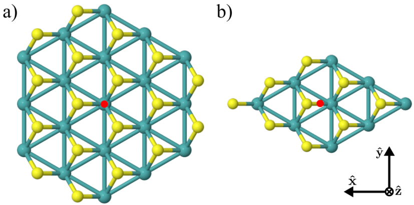


Circular polarizations

SFG with C_n symmetry			
n	Incident	Tr.	Re.
1	$++$	$+,-$	$+,-$
2		x	x
3		$-$	$+$
≥ 4		x	x
1	$+-$	$+,-$	$+,-$
2		x	x
≥ 3		x	x

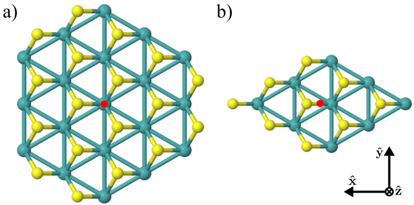
Rhomboid b)

Incident	Transmission (a.u.)		Reflection (a.u.)	
	$+$	$-$	$+$	$-$
$++$	0.160	0.254	0.254	0.160
$+-$	0.359	0.359	0.359	0.359



Circular polarizations

SFG with C_n symmetry			
n	Incident	Tr.	Re.
1	$++$	$+,-$	$+,-$
2		x	x
3		$-$	$+$
≥ 4		x	x
1	$+-$	$+,-$	$+,-$
2		x	x
≥ 3		x	x



Circular polarizations

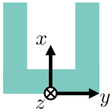
SFG with C_n symmetry			
n	Incident	Tr.	Re.
1	++	+, −	+, −
2		x	x
3		−	+
≥ 4		x	x
1	+−	+, −	+, −
2		x	x
≥ 3		x	x

“Hexagonal” with C_3 symmetry a)

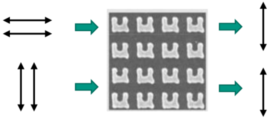
Incident	Transmission (a.u.)		Reflection (a.u.)	
	+	−	+	−
++	4.29e-06	0.139	0.139	4.29e-06
+−	3.67e-05	3.64e-05	3.64e-05	3.67e-05

SHG and THG in mirror symmetric object

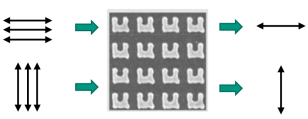
- TE/TM basis: $|\tau\rangle = \dfrac{|+\rangle + \tau\,|-\rangle}{\sqrt{2}}$ $\qquad |\tau = +1\rangle \equiv |\updownarrow\rangle$ $\qquad |\tau = -1\rangle \equiv |\leftrightarrow\rangle$
- Transformation under $\hat{M}_y : y \mapsto -y \qquad \hat{M}_y\,|\tau\rangle = \tau\,|\tau\rangle$



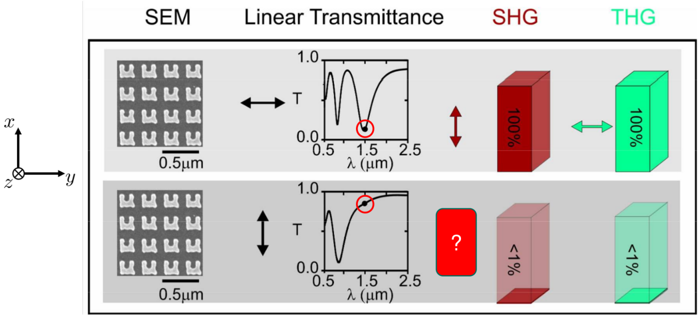
Second harmonic generation



Third harmonic generation



SHG in mirror symmetric scatterer



Optics Express 15, 5238-5247 (2007)