Idyllic Description of Quarkyonic Matter

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(Confinement and Symmetry from Vacuum to QCD Phase Diagram, Benasques, Spain, Feb. 2025)

Recent work in collaboration with
Y. Fujimoto T. Kojo; V. Koch, V. Vovchenko,G. Miller
M. Bluhm and M. Nahrgang
See earlier work with
R. Pisarski, Y. Hidaka, S. Reddy, K. Jeon, D. Duarte,
S.Hernandez, , K. Fukushima, M. Praszalowicz, M.
Marczenko, K. Redlich and C. Sasaki

Mass and radii of observed neutron stars and data from neutron star collisions give
an excellent determination of the equation of state of strongly interacting matter

Such equations of state must be hard
The sound velocity squared is greater than or of the order of 1/3 at only a few times nuclear matter density
This is NOT what one expects from a 1st or 2nd order phase transition
Relativistic degrees of freedom appear to be important

Neutron Star Matter and Some Conjectures on Scale Invariance

How is equation of state determined?

One equates the outward force of matter arising from pressure inward force of gravity.
This gives a general relativistic equation of hydrostatic equilibrium.

For a specific equation of state, one obtains a relationship between radii and neutron star masses

Equations of state may be characterized by two dimensionless numbers

Sound velocity:

$$v_s^2 = \frac{dP}{de}$$

and the trace of the stress energy tensor scaled by the energy density

$$\Delta = \frac{1}{3} - \frac{P}{e}$$

$$P = -\frac{dE}{dV}$$

In a scale invariant theory at zero temperature:

$$E \sim (N/V)^{4/3} V \sim N^{4/3} V^{-1/3}$$

$$P = \frac{1}{3}\frac{E}{V} = \frac{1}{3} e$$

$$v_s^2 = \frac{dP}{de} = \frac{1}{3}$$

$$\Delta = \frac{1}{3} - \frac{p}{e} = 0$$

The trace of the stress energy tensor is taken to be a measure of scale invariance. It is anomalous in QCD.

$$T^\mu_\mu = -\frac{\beta(g)}{g}(E^2 - B^2) + m_q(1+\gamma_q)\overline{\psi}\psi$$

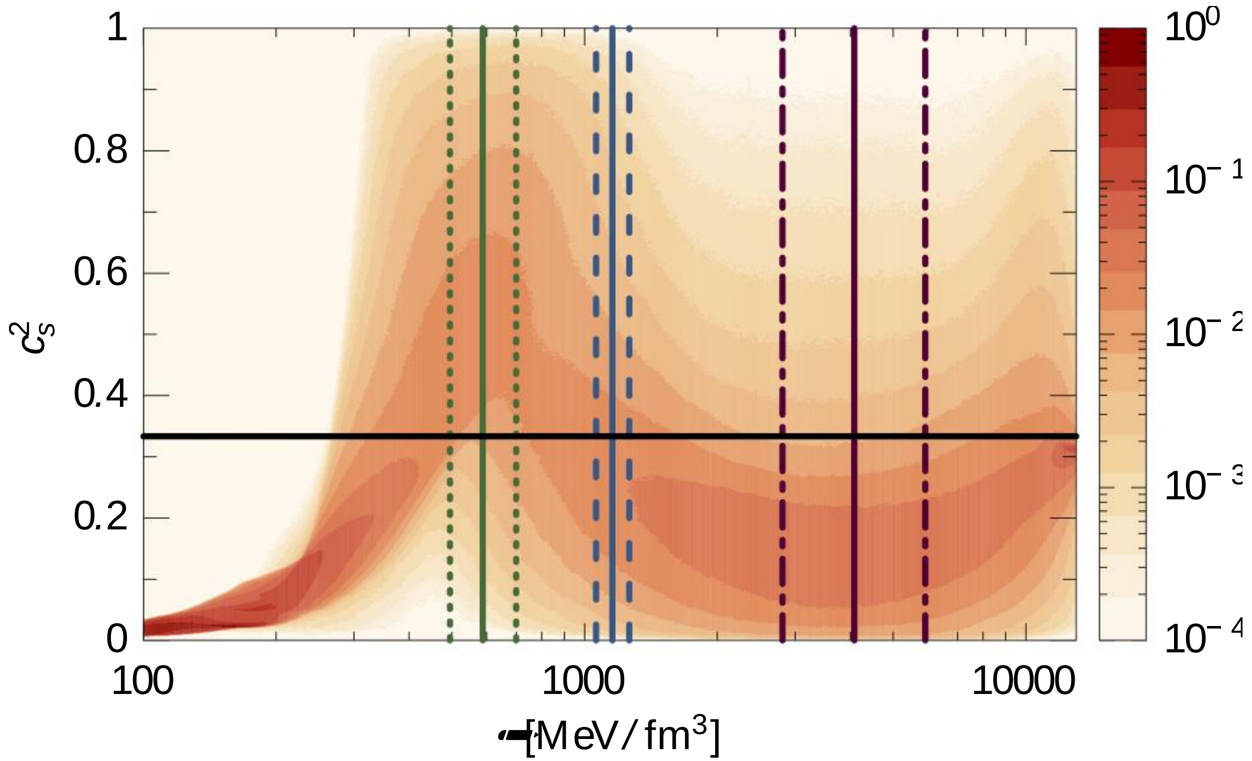
In this equation, the beta function of QCD is negative, and the fermion term is from quarks. The fermion term vanishes in the chiral limit.

If we take matrix elements of single particle states

$$< p \mid T^\mu_\mu \mid p > \sim p^2 = m^2 \geq = 0$$

In the chiral limit, this implies E > B, as we expect for massive quarks, except for the pion, which is very tightly. bound

For dilute systems, the trace anomaly is positive, as it is at high density for a quark gas. In general, we expect it to be positive, except possibly for small effects due to a pion condensate, if it exists.



L. McLerran, M. Marczenko, K. Redlich and C. Sasaki

Y. Fujimoto, K. Fukushima,
K. Murase

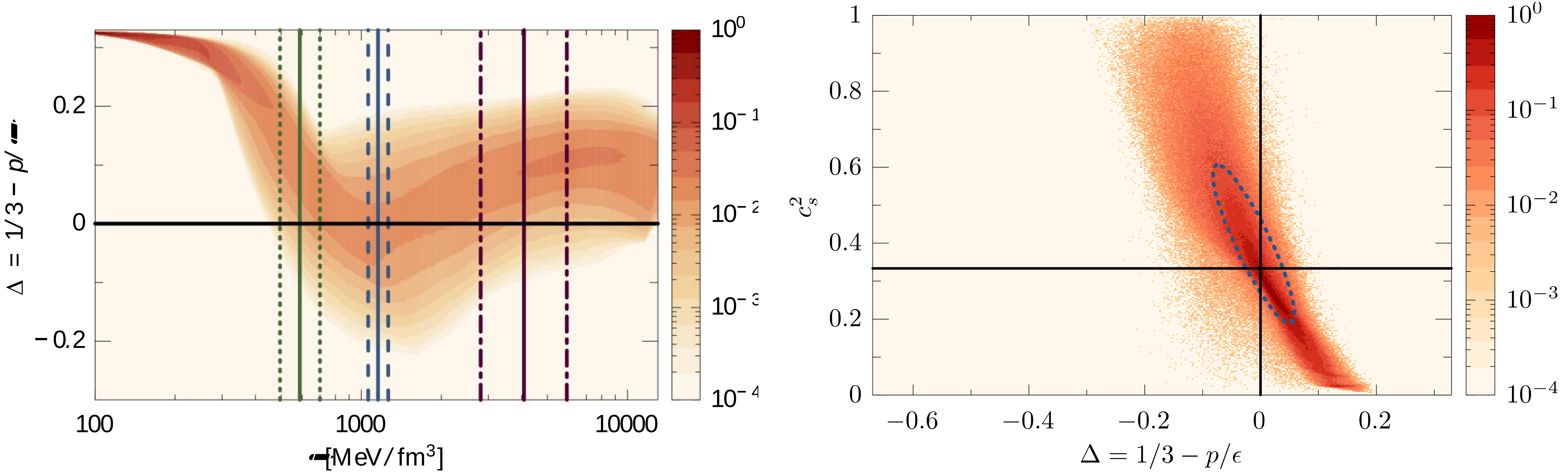
Tews, Carlson, Gandolfi and Reddy; Kojo; Annala, Gorda, Kurkela and Vuorinen

As a result of LIGO experiments, and more precise measurement of neutron star masses and radii, the equation of state of nuclear matter at a few times nuclear matter density is tightly constrained

Sound velocity approaches and perhaps exceeds

$$v_s^2 = 1/3$$

at a few times nuclear matter density



Also, the trace anomaly is approaching zero at highest densities in neutrons stars, where also the sound velocity squared approaches 1/3

Matter is strongly interacting and conformal:
Probably some form of quark matter

Sound velocity of order one has important consequences

For zero temperature Fermi gas:

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2$$

where the baryon chemical potential includes the effects of nucleon mass

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \; \frac{\delta n_B}{n_B}$$

So if the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon number chemical potential of order the nucleon mass

For nuclear matter densities

$$\mu_B - M \sim \frac{\Lambda^2_{QCD}}{2M} \sim 100 \; MeV$$

Large sound velocities will require very large intrinsic energy scales, and a partial occupation of available nucleon phase space because density is not changing much while Fermi energy changes a lot

Relation between quark and nucleon Fermi momenta

$$m_q = m_N / N_c$$

$$\mu_q = \mu_N / N_c$$

$$k_q^2 = \mu_q^2 - m_q^2 = \mu_n^2 / N_c^2 - M_N^2 / N_c^2 = k_N^2 / N_c^2$$

For 2 flavors of nucleons

$$n_B^N = 4 \int^{k_N} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2} k_N^3$$

For two flavors of quarks

$$n_q^N = \frac{1}{N_c} 4N_c \int^{k_q} \frac{d^3k}{(2\pi)^3} = \frac{2}{3\pi^2} k_q^3$$

To get any baryons from the quarks at the bottom of the shell of nucleons, need the quark fermi momentum to be of the order of the QCD scale, so that the nucleons in the shell are relativistic. Naturally driven to the conformal behaviour. The chemical potential of the quarks must jump up from a small value to a typical QCD scale

If there is a continuous transition then the baryon density will have to remain approximately fixed, so the chemical potential will change by of order N_c. The sound velocity is changing for a very non-relativistic system to a very relativistic one.

$$\epsilon_B = M_N \Lambda_{QCD}^3 \sim N_c \Lambda_{QCD}^4$$

$$\epsilon_Q \sim \Lambda_{QCD} n_q \sim N_c \Lambda_{QCD}^4$$

$$P \sim k_F^5 / M_N$$

The pressure on the other hand must jump by order N_c squared

$$P \sim \epsilon_q$$

Energy density and density fixed, but pressure and chemical potential jump.

A first order phase transition has pressure and chemical potential fixed, but energy density and density jump

Does Quarkyonic Matter Have Correct Properties to Describe Such Matter?

Ordinary nucleon matter if relativistic has typical density:

$$n_B \sim (k_F^N)^3 \sim M_N^3 \sim 100 \ \rho_{NM}$$

$$n_Q \sim (k_F^Q)^3 \sim (k_F^N/N_c)^3 \sim \Lambda_{QCD}^3 \sim \rho_{NM}$$

Quarks are confined up to a high density in QCD:

$$M_{Debye}^2 \sim \frac{\alpha_{'tHooft}}{N_c} \ \mu_Q^2$$

Confinement disappears when

$$M_{Debye} \sim \Lambda_{QCD}$$

$$\mu_Q \sim \sqrt{N_c}\Lambda_{QCD}$$

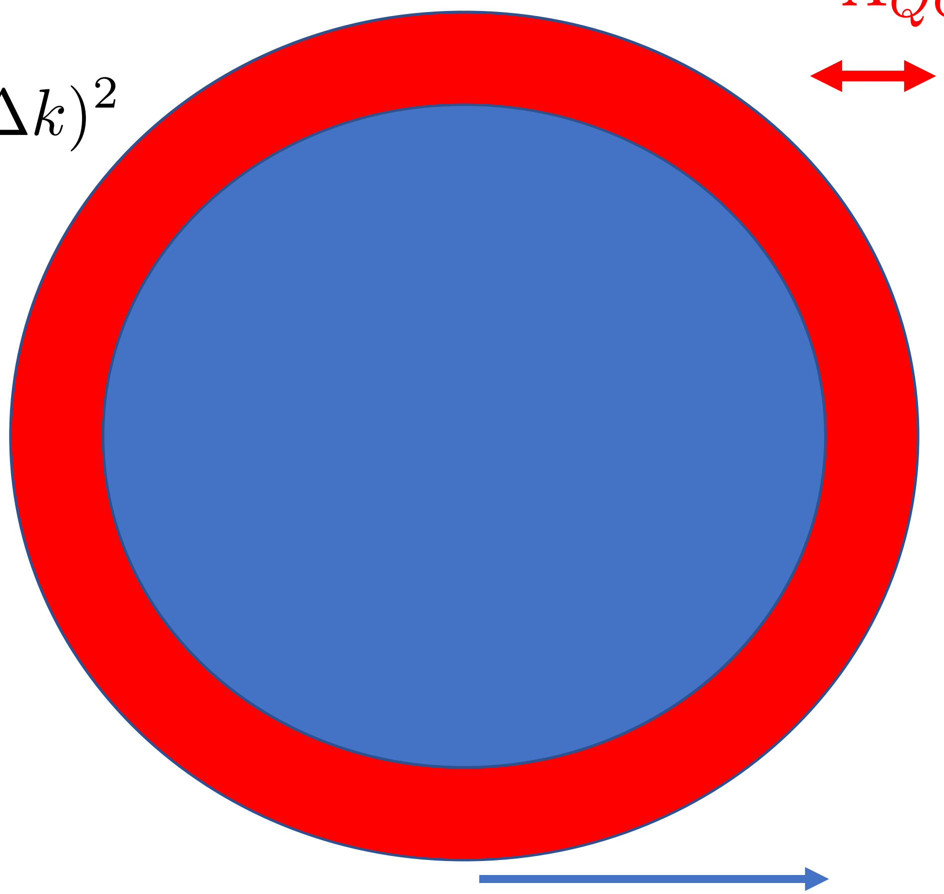
Matter inside of neutron stars is probably confined, but also probably not entirely baryonic

Fermi Surface is Non-perturbative

$$n^Q_{Baryon} \sim k_Q^3 \sim \frac{1}{N_c^3} k_N^3 = \frac{1}{N_c^3} n^N_{baryon}$$

$$\rho^N_{surface} \sim k_F (\Delta k)^2$$

$\sim \Lambda_{QCD}$



$\mu_{quark} >> \Lambda_{QCD}$

Fermi Surface
Interactions sensitive to infrared
Degrees of freedom: baryons, mesons and glueballs

Fermi Sea: Dominated by exchange interactions which are less sensitive to IR.
Degrees of freedom are quarks

Pisarski and LDM, Reddy and LDM

Saturation criteria due to Kojo
(KFC?)

An Explicit Quantum Mechanical Theory of Quarkyonic Matter

T. Kojo; Y. Fujimoto., LM and T. Kojo

Let occupation number density for nucleons and quarks be

$$f_q, f_N \qquad 0 \leq f_q, f_N \leq 1$$

A duality relation (nucleons are composed of quarks)

$$f_q(k) = \int [dp]\ K(k - p/N_c) f_N(p)$$

$$\int [dk]\ K(k) = 1$$

First: Free theory of nucleon and quarks (except for duality relation)

This is a solvable theory with non-trivial solution with two phases:
A nucleonic phase and a quarkyonic phase

A simple choice of kernel and explicit duality

$$f_q(k) = \int \ [dp] \ K(k - p/N_c) f_N(p)$$

$$K(k) = \frac{1}{4\pi\Lambda^2} \frac{e^{\frac{-|k|}{\Lambda}}}{|\ k\ |}$$

$$\left\{-\nabla_K^2 + \frac{1}{\Lambda^2}\right\} K(k) = \frac{1}{\Lambda^2}\delta(k)$$

$$(-\nabla_k^2 + \frac{1}{\Lambda^2}) f_q(k) = \frac{N_c^3}{\Lambda^2} f_N(N_c k)$$

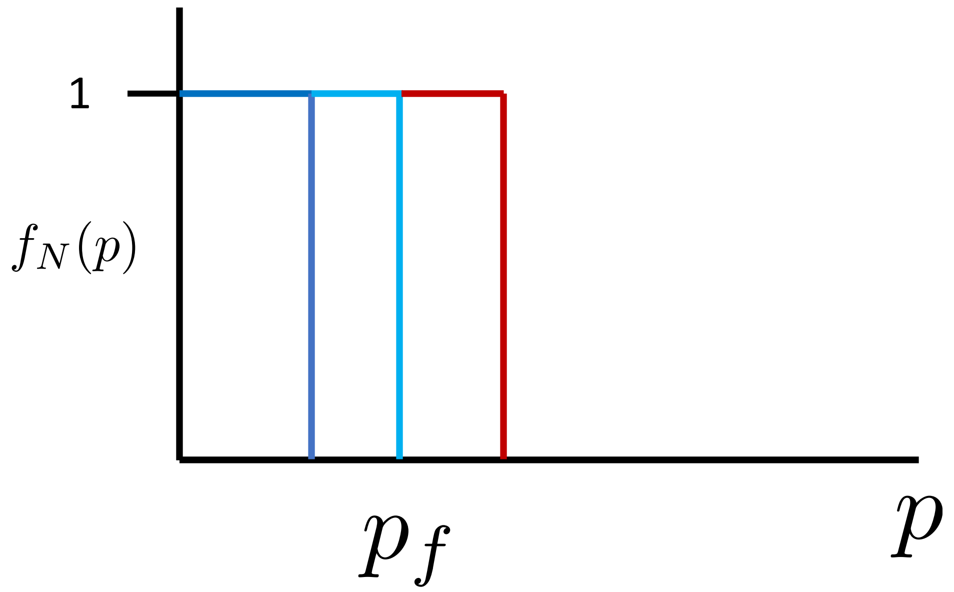
$$\epsilon = \int \ [dp]\sqrt{p^2 + M_n^2} f_N(p) = N_c \int \ [dk] \ E_q(k) f_q(k)$$

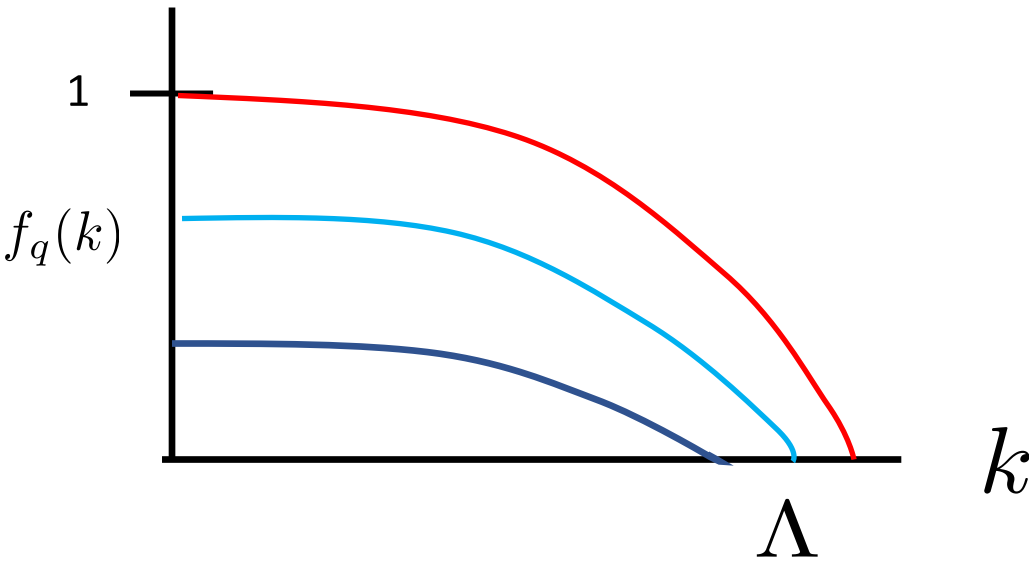
$$E_q(k) = \sqrt{k^2 + M_q^2} - \frac{M_q^2 \Lambda^2}{(k^2 + M_q^2)^{3/2}}$$

Free theory of quarks with modified kinetic energy term

Solution looks like:

Low density:





$$1 = \int [dp]\ K(k - p/N_c) f_N(p)$$

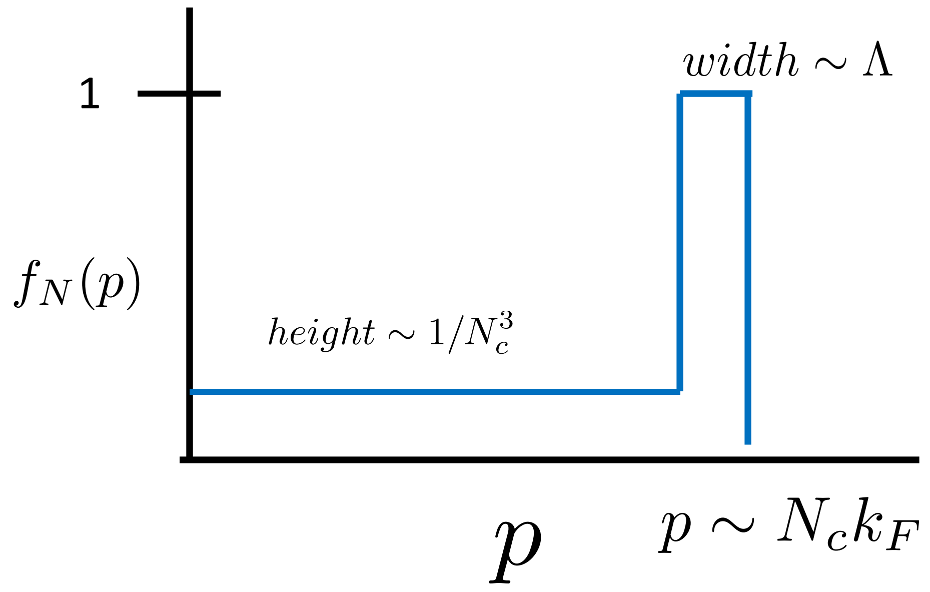
$$\sim K(0) \int [dp]\ f_N(p) = \kappa \frac{n_n}{\Lambda^3}$$

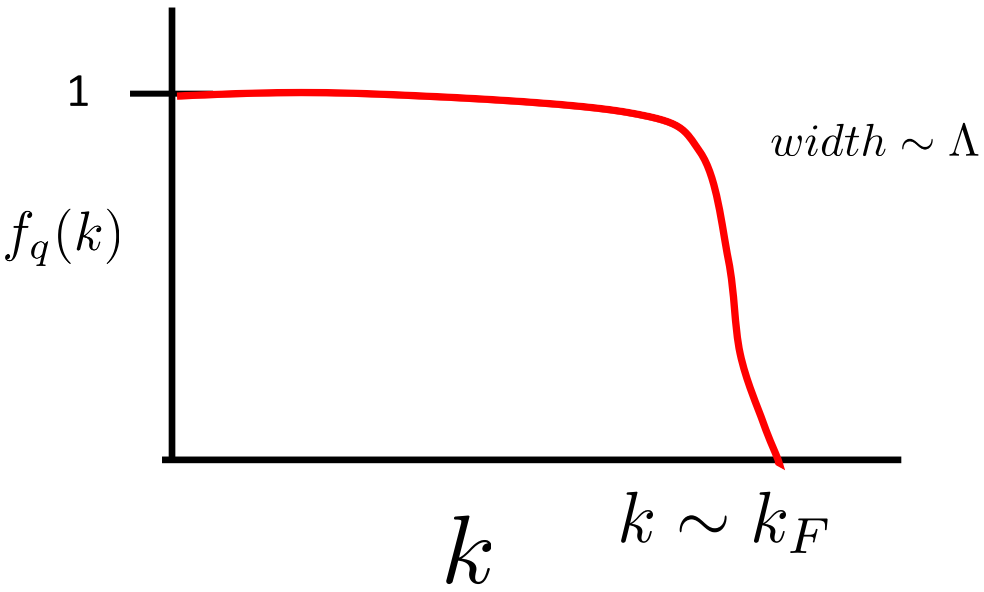
Width of quark distribution determined by intrinsic confinement scale of quarks inside of nucleons

Λ

$$n_N^{crit} \sim \Lambda^3$$

High Density:





At high densities, a thin shell of saturated baryon matter forms surrounded by an underoccupied distribution of nucleons

The quarks make a filled sea of nucleons with an underoccupied tail where the shell of nucleons are not Pauli blocked

Koch, LDM, Miller, and Vovchenko

Might Quarkyonic Matter be Relevant for Nuclear Matter?

General Considerations on the Transmutation Density to Quarkyonic Matter

Transmutation Criterium:

Saturation criteria due to Kojo

$$1 = f_Q(0) = \int^{k_f} \frac{d^3p}{(2\pi)^3} \; K(p/N_c) \; f_N(p) \sim \frac{1}{4} n_N \; K(0)$$

Using a Gaussian model for the distribution of quarks inside a hadron, and only including valence quarks

$$\phi_{gauss} = 8\pi^{3/2} R^3 e^{-p^2 R^2}$$

Using the RMS charge radius as R

$$R = \sqrt{\frac{2}{3}} r_{RMS}$$

Find transmutation density of twice nuclear matter for measured charge radius
But this ignores the contribution of quark-antiquark pairs: the meson cloud

Determination from observed quark distributions in. deep inelastic scattering:

LDM and Miller

Transverse momentum distribution functions were determined by de Teramond et. al. These distributions functions describe measured integrated valence and sea quark distribution functions, and electromagnetic form factors

$$\frac{dn_Q(k)}{d^3k} = \frac{x}{E} \mid \psi(x, k_T) \mid^2$$

$$\int \frac{dx d^2 k_T}{(2\pi)^3} \mid \psi(x, k_T) \mid^2 = 1$$

$$x = \frac{k^+ + E_k}{p^+ + E_p}$$

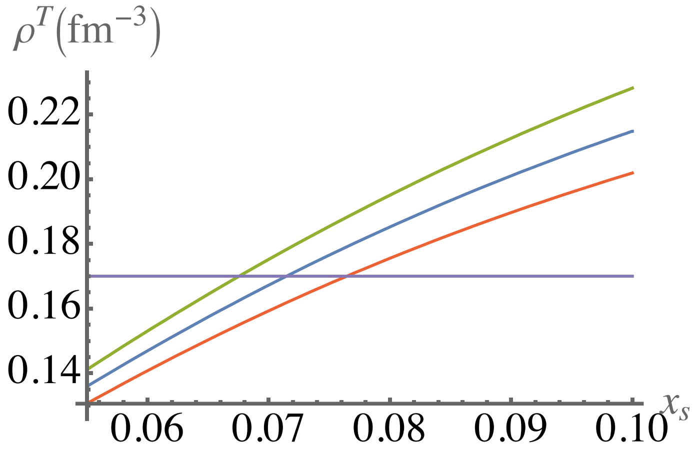
So that for a hadron at rest and a parton with zero momentum

$$x = E_0/M$$

$$\frac{dn_Q(0)}{d^3k} = \mid \psi(x_0, 0_T) \mid^2 \frac{1}{M}$$

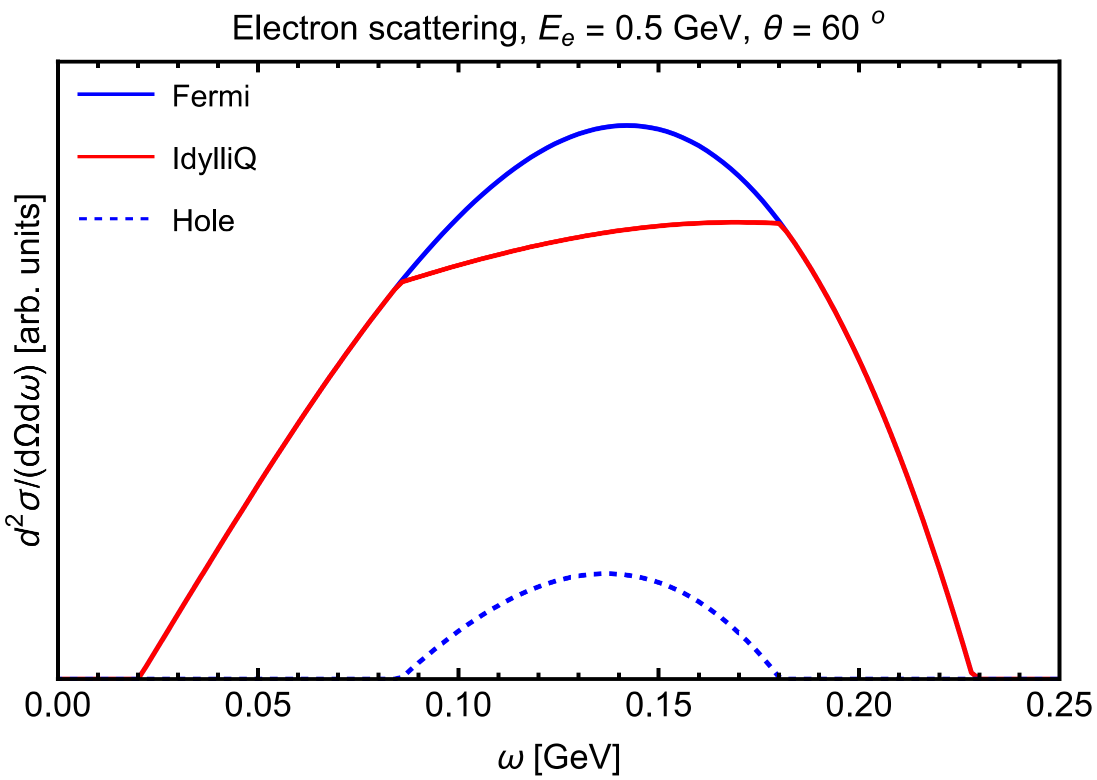
Typical x for valence quarks is about 1/6, because glue carries ½ the momentum, and less than 1/10 for quark-antiquark pairs

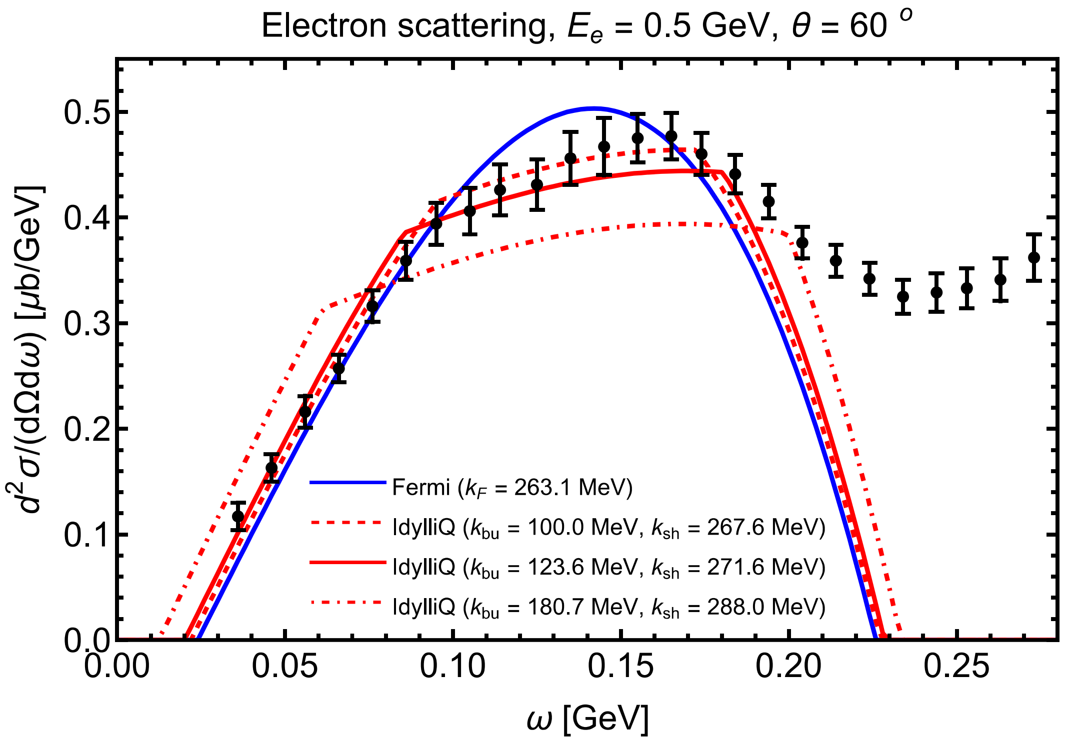
Find that for valence quarks alone the transmutation density would be about twice nuclear matter. Sea quark contribution has large uncertainties since it is sensitive to the value of x used.



Plausible that the transmutation density is below or less than that of nuclear matter

Experimental Constraint: Electron Scattering:
Quarkyonic Matter has a Fully Occupied Fermi Surface of Nucleons and an underoccupied sea





Hole in Fermi sea lowers cross section near maximum. Peak would be delta function if there was no Fermi momentum

Data is Coulomb corrected and extrapolated to nuclear matter. Energy chosen minimizes final state interactions

Constructing a Theory:

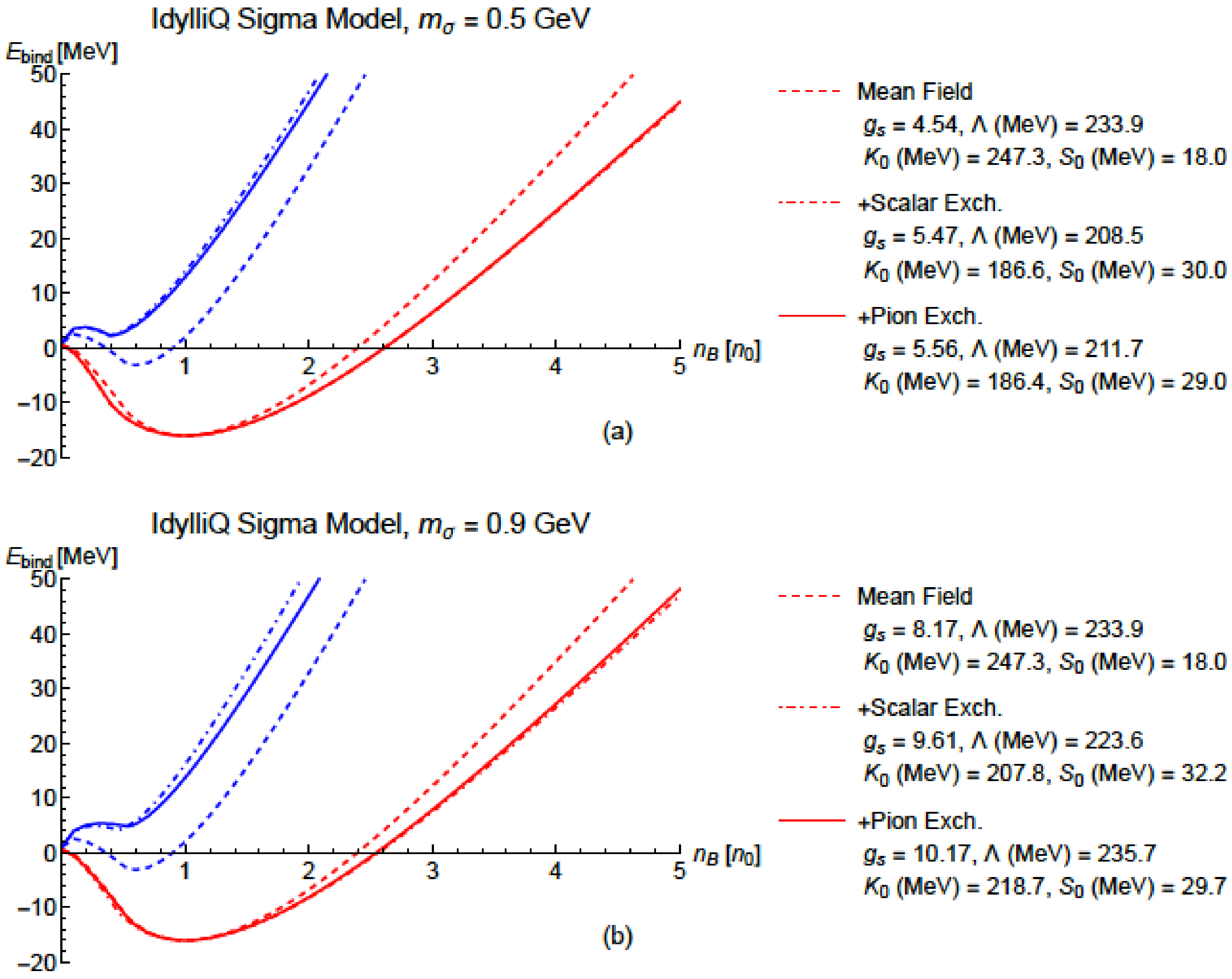
Include only pion and sigma meson interactions. First order correction to free theory of nucleon with constrain on quark occupations number.
Sigma meson interaction in mean field and include exchange interactions.
Note that mean field term is scale invariant at high Fermi momentum:

$$n_{scalar} \sim M \int^{k_f} \frac{d^3p}{2E(2\pi)^3} \sim k_f^2$$

$$\epsilon \sim \frac{g^2}{M^2} n_{scalar}^2 \sim k_f^4$$

Naturally matches to QCD approximate scale invariance.

Vector meson interactions are not scale invariant in mean field.



Reasonable values for nuclear matter. (Too large a hole in the Fermi sea for these parameters)

As isospin increases, minimum moves to lower density and almost disappears for neutron matter. Neutron matter is slightly unbound

How do we test this hypothesis?
Is it more or less true or is it false?

How Quarkyonic Matter Might Ameliorate the Hyperon Problem

In an ordinary neutron resonance gas, when the baryon chemical potential exceeds the mass of the lowest mass hyperon, then hyperons enter. They are non-relativistic and there are many of them so they greatly soften the equation of state.

$$\mu_N = M_Y = M_Y - M_N + M_N = M_{squark} - M_{light-quark} + M_N$$

In quarkyonic matter, the neutrons make a filled d-quark sea with a half filled u-quark sea. The only available state is the ssu hyperon

$$\mu_N = M_{\Xi^0} = M_{\Xi^0} - M_N + M_N = 2M_{squark} - 2M_{light-quark} + M_N$$

The threshold is higher and there is on quark state. Some softening, but sets in at higher density, and is enhanced by multiple strange baryons.

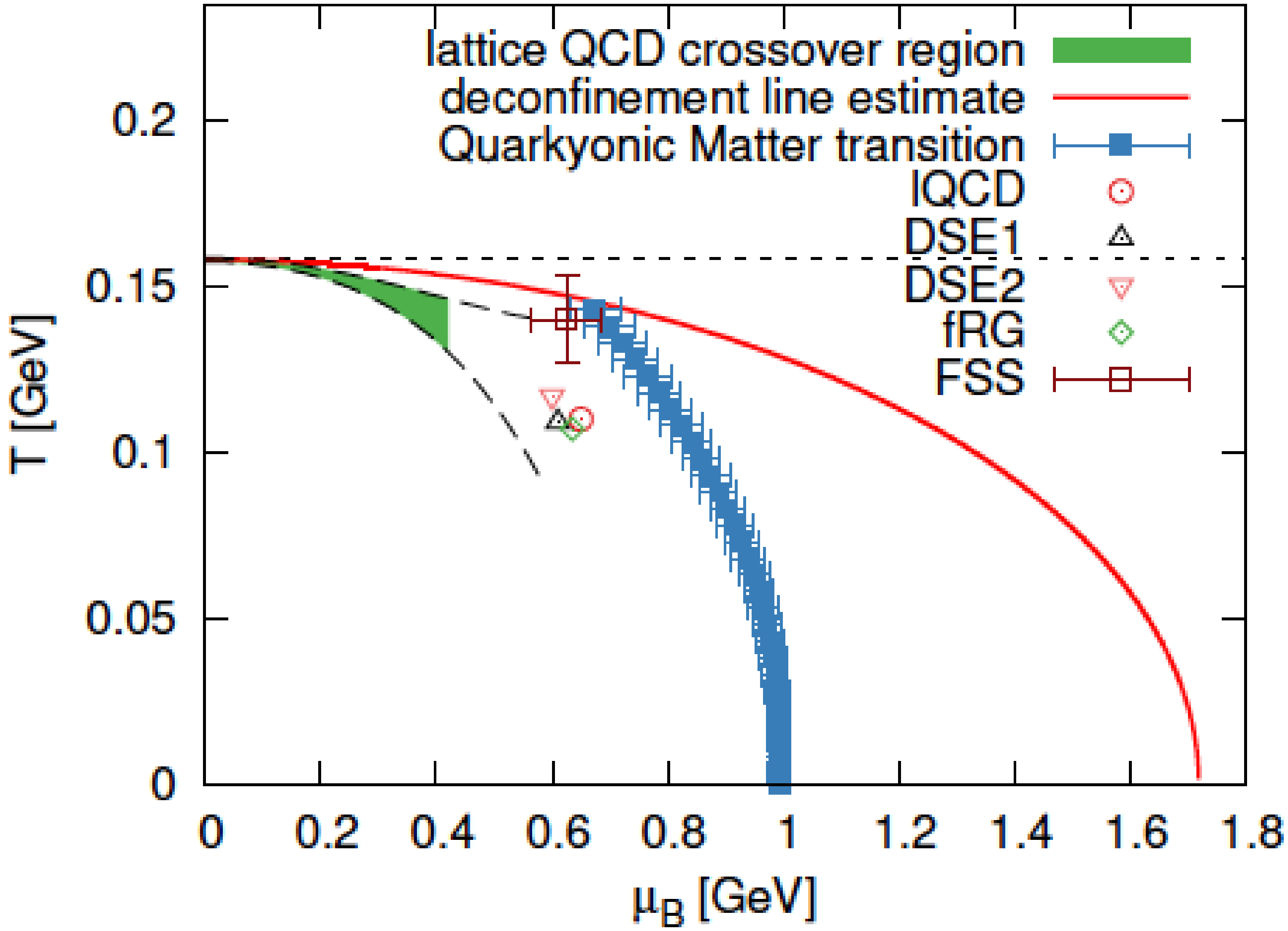
What About Finite Temperature?

One can determine the Quarkyonic boundary by the saturation condition. One most include the contributions form delta and baryon resonances, and a small contribution from mesons. The finite temperature Quarkyonic region is limited by the confinement boundary, which we is approximately by Debye screening:

$$m_D^2(T,\mu) = c_T N_c T^2 + c_\mu \mu^2$$

$$T_c(\mu)^2 = T_c^2 - \frac{c_\mu}{c_T}\frac{\mu^2}{N_c}$$

Our biggest uncertainty is the Quarkyonic transition density at zero temperature, which we take to be 1-3 times the density of nuclear matter



It is interesting that the intersections of the quarkyonic line with that of deconfinement occurs close to values estimated for the critical end point of chiral transition

Important Issues:

Is nuclear matter quarkyonic? How to determine experimentally? Theory of finite nuclei?

How to test theory from controllable computations? 1+1 d?
3+1 d Bosonic systems?

How to compute at finite T?

How is Quarkyonic matter related to this new region of Glozman and Cohen?