Anomalous transport from lattice QCD

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Confinement and symmetry from vacuum to QCD phase diagram

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Anomalous transport from lattice QCD

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in collaboration with:

Bastian Brandt, Eduardo Garnacho, Gergely Markó,
Leon Sandbote, Dean Valois

Fundament

top-10 most cited hep-ph theoretical papers from last 20 years

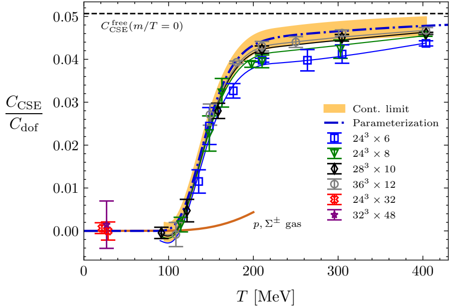
Kharzeev, McLerran, Warringa, *The Effects of topological charge change in heavy ion collisions: 'Event by event P and CP violation'*, Nucl.Phys.A 803 (2008) 227-253

Fukushima, Kharzeev, Warringa, *The Chiral Magnetic Effect*, Phys.Rev.D 78 (2008) 074033

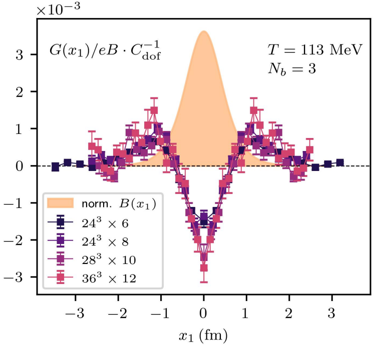
Appetizer

first fully non-perturbative determination
of in-equilibrium anomalous transport coefficients

chiral separation effect



(local) chiral magnetic effect



Brandt, Endrődi, Garnacho, Markó, JHEP 02 142

Brandt, Endrődi, Garnacho, Markó, Valois, 2409.17616

Brandt, Endrődi, Garnacho, Markó, JHEP 09 092

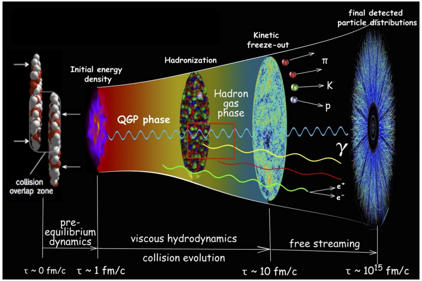
Outline

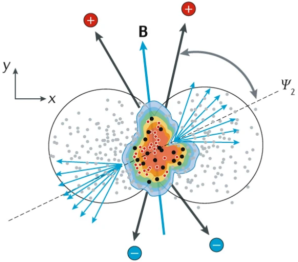
- introduction: anomalous transport phenomena
- in-equilibrium chiral magnetic effect
- in-equilibrium chiral separation effect
- local in-equilibrium chiral magnetic effect
- out-of-equilibrium chiral magnetic effect
- summary

Introduction

Quarks and gluons in extreme conditions

- heavy ion collisions $T \lesssim 10^{12}\,^{\circ}C = 200$ MeV, $n \lesssim 0.12$ fm^{-3}
  $B \lesssim 10^{19}$ G $= 0.3$ GeV$^2/e$





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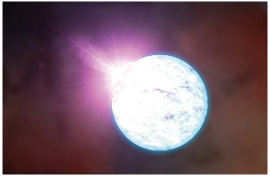
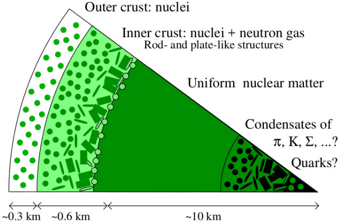
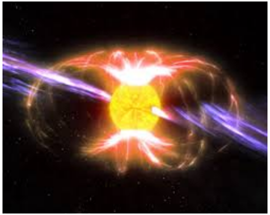
- neutron stars $T \lesssim 1$ MeV, $n \lesssim 2$ fm^{-3}
  magnetars $B \lesssim 10^{15}$ G







Lattimer, Nature Astronomy 2019

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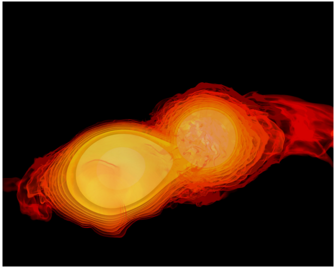

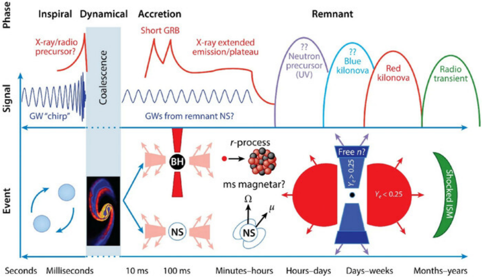

astro.uni-frankfurt.de/rezzolla

Quarks and gluons in extreme conditions

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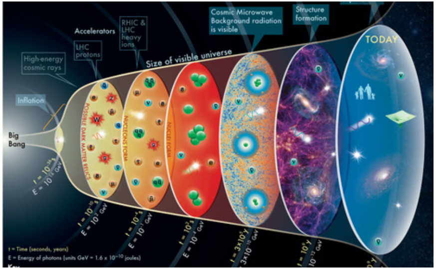
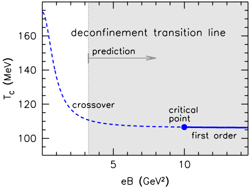
- eary universe, QCD epoch $T \lesssim 200$ MeV,   standard scenario: $n \approx 0$
  B from electroweak epoch Vachaspati '91 Enqvist, Olesen '93



Magnetic fields impact on

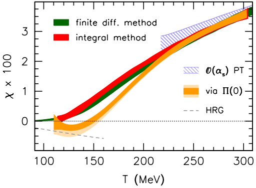
- many lattice groups D'Elia, Bonati et al. Braguta, Chernodub et al. Cea et al Alexandru et al. Ding et al. Buividovich et al. Yamamoto Detmold et al. and more
- phase diagram

  Endrődi, JHEP 07 (2015)



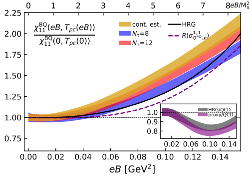
- equation of state

  Bali et al. JHEP 07 (2020)



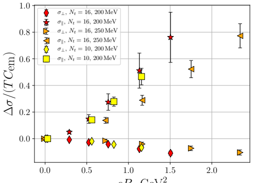
- fluctuations

  Ding et al. PRL 132 (2024)



- transport phenomena

  Astrakhantsev et al. PRD 102 (2020)



- anomalous transport phenomena

Anomalous transport

Anomalous transport

- usual transport:
  vector current due to electric field

$$\langle \boldsymbol{J} \rangle = \sigma \cdot \boldsymbol{E}$$

- chiral magnetic effect (CME)
  Fukushima, Kharzeev, Warringa, PRD 78 (2008)
  vector current due to chirality and magnetic field

$$\langle \boldsymbol{J} \rangle = \sigma_{\mathrm{CME}} \cdot \boldsymbol{B}$$

- chiral separation effect (CSE)
  Son, Zhitnitsky, PRD 70 (2004) Metlitski, Zhitnitsky, PRD 72 (2005)
  axial current due to baryon number and magnetic field

$$\langle \boldsymbol{J}_5 \rangle = \sigma_{\mathrm{CSE}} \cdot \boldsymbol{B}$$

Phenomenological and theoretical relevance

- experimental observation of CME in condensed matter systems
  Li, Kharzeev, Zhan et al., Nature Phys. 12 (2016)
- experimental searches for CME and related observables in heavy-ion collisions
  STAR collaboration, PRC 105 (2022)
- serves as indirect way to probe topological fluctuations and CP-odd domains in heavy-ion collisions
- recent reviews: Kharzeev, Liao, Voloshin, Wang, PPNP 88 (2016)
  Kharzeev, Liao, Tribedy, 2405.05427

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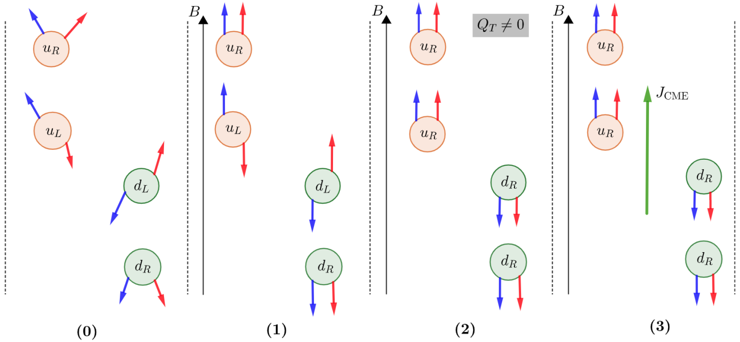
- disclaimer 2: this talk is about QCD and not chiral gauge theories (c.f. holography Rebhan et al. 2010)

General (handwaving) argument

- spin, momentum chiral magnetic effect



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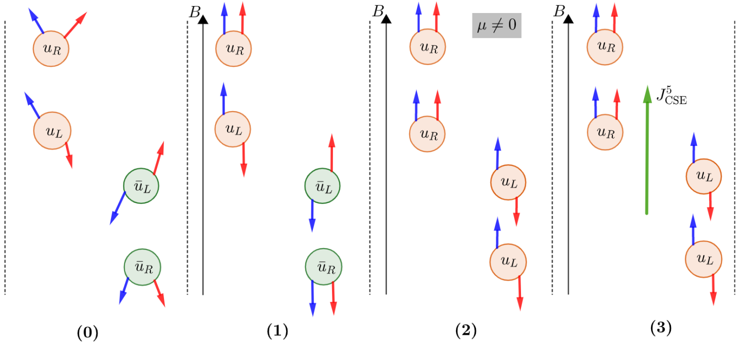

General (handwaving) argument – issues

- quantum theory requires ultraviolet regularization
- massless vs. massive fermions
- strong interactions between fermions
- in-equilibrium vs. out-of-equilibrium nature

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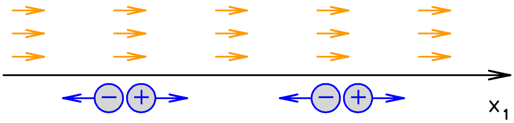
In-equilibrium vs. out-of-equilibrium

- example: charge transport due to electric field $\boldsymbol{E} \parallel \boldsymbol{e}_1$



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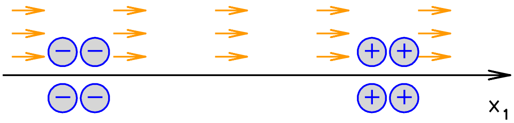

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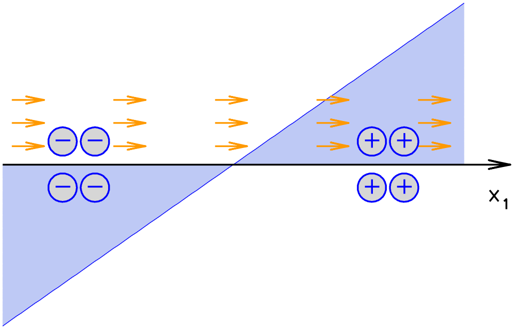

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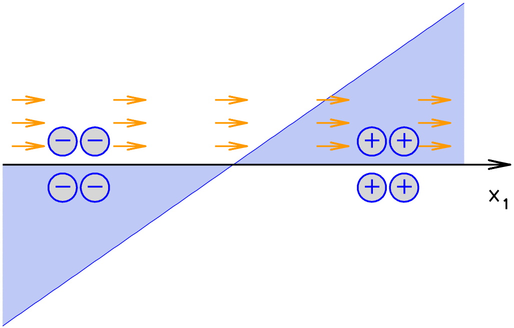

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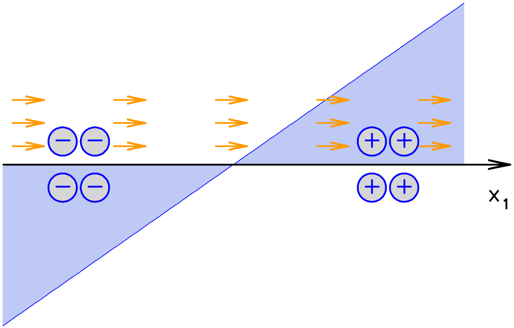

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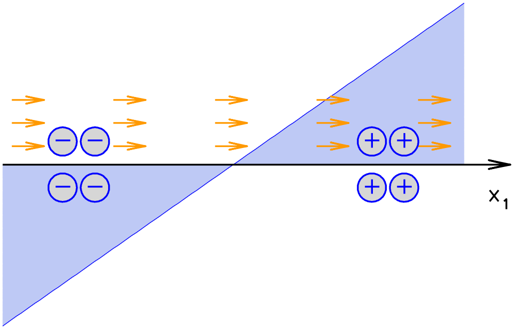

- out-of equilibrium linear response:
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- leading to an equilibrium distribution (electric polarization/susceptibility)
- same story can be told for CME

No currents in equilibrium

- **Bloch's theorem**: Bohm Phys. Rev. 75 (1949) N. Yamamoto, PRD 92 (2015)
  persistent electric currents do not exist in ground state of quantum systems
- applies to conserved currents
- applies to global (spatially averaged) currents
- applies in the thermodynamic limit ($V \to \infty$)

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- in-equilibrium CSE is possible
- in-equilibrium local CME currents are possible

Chiral magnetic effect in equilibrium

CME and inconsistencies

- parameterize chiral imbalance $J_{05} = \int \bar{\psi}\gamma_0\gamma_5\psi$ by a chiral chemical potential μ_5
  Fukushima, Kharzeev, Warringa, PRD 78 (2008)
- CME for weak chiral imbalance ($\boldsymbol{B} = B\boldsymbol{e}_3$)

$$\langle J_3 \rangle = \sigma_{\mathrm{CME}}\, B = C_{\mathrm{CME}}\, \mu_5 B + \mathcal{O}(\mu_5^3)$$

- from Bloch's theorem it follows that in equilibrium

$$C_{\mathrm{CME}} = 0 \; \checkmark$$

- several results in the literature give incorrectly

$$C_{\mathrm{CME}} = \frac{1}{2\pi^2} \; ⚡$$

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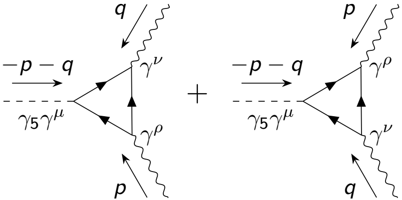
careful regularization is required

Perturbation theory

- triangle diagram

$$\Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =$$



- gives in-equilibrium CME coefficient

$$C_{\mathrm{CME}} = \lim_{p,q\to 0} \frac{1}{q_1} \Gamma^{023}_{AVV}(p+q,p,q)$$

- also gives the axial anomaly 🔗 Peskin-Schroeder 19.2

$$\langle \partial_\mu J_5^\mu \rangle \;\sim\; (p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) A_\nu A_\rho$$

Regularization sensitivity – anomaly

- naive regularization

$$(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q)A_\nu A_\rho = mP_5(p,q) \lightning$$

- Pauli-Villars regularization
  (regulator particles $s = 1,2,3$ with $c_s = \pm 1$ and $m_s \to \infty$)

$$(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q)A_\nu A_\rho = mP_5(p,q) + \sum_{s=1}^{3} c_s m_s P^{\nu\rho}_{5,s}(p,q)A_\nu A_\rho$$

$$\xrightarrow{m_s\to\infty} mP_5(p,q) + \frac{\epsilon^{\alpha\beta\nu\rho}F_{\alpha\nu}F_{\beta\rho}}{16\pi^2} \checkmark$$

Regulator sensitivity – CME

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$$C_{\mathrm{CME}} = \lim_{p,q,p+q\to 0} \frac{1}{q_1} \Gamma^{023}_{AVV}(p+q,p,q) = \frac{1}{2\pi^2} \ \lightning$$

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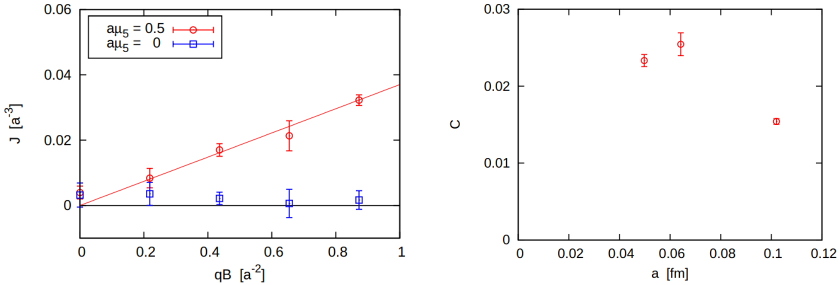
- in equilibrium, C_{CME} vanishes due to anomalous contribution

CME in equilibrium – lattice simulations

Regularization sensitivity on the lattice: quenched Wilson

- seminal lattice determination of $\langle J_3 \rangle$ at $B \neq 0$, $\mu_5 \neq 0$ A. Yamamoto, PRL 107 (2011)



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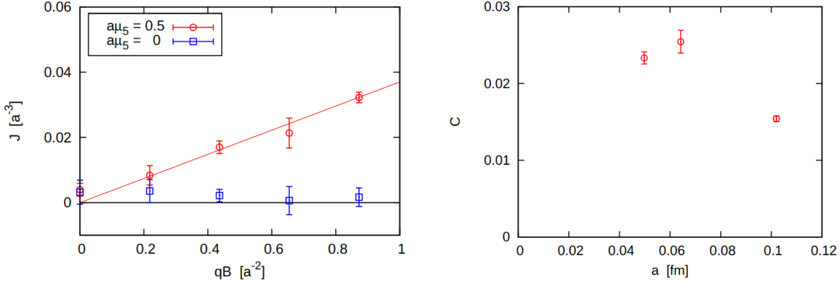

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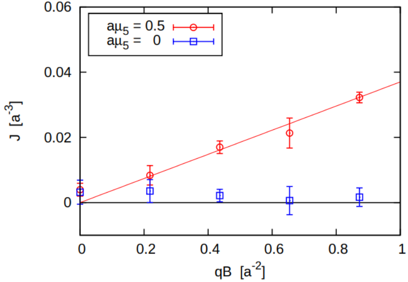

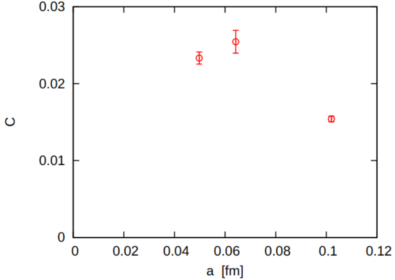

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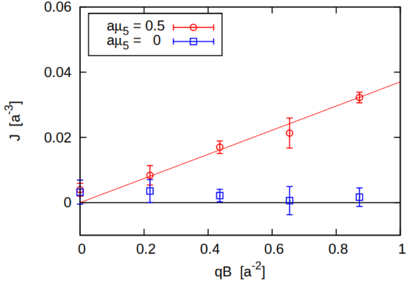

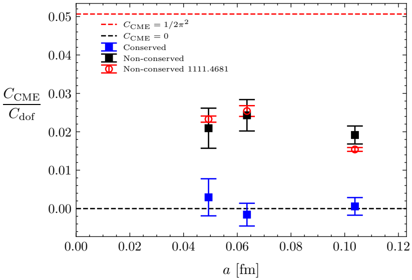

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- conserved current: $C_{\rm CME} = 0$ ✓ Brandt, Endrődi, Garnacho, Markó, JHEP 09 (2024)

More on lattice currents

- conserved vector current $\langle J_3 \rangle = \langle \mathrm{Tr}(\Gamma_3 M^{-1}) \rangle$

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$$C_{\mathrm{CME}} \cdot B = \left. \frac{\partial \langle J_3 \rangle}{\partial \mu_5} \right|_{\mu_5 = 0} = \langle \mathrm{Tr}(\Gamma_{45} M^{-1}) \mathrm{Tr}(\Gamma_3 M^{-1}) \rangle - \langle \mathrm{Tr}(\Gamma_{45} M^{-1} \Gamma_3 M^{-1}) \rangle$$

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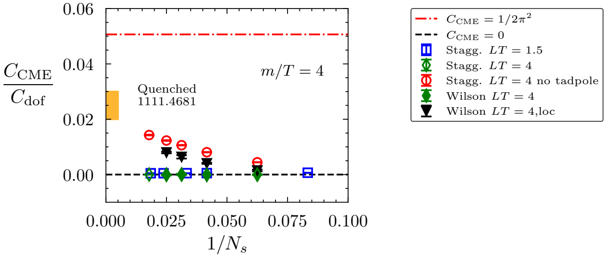
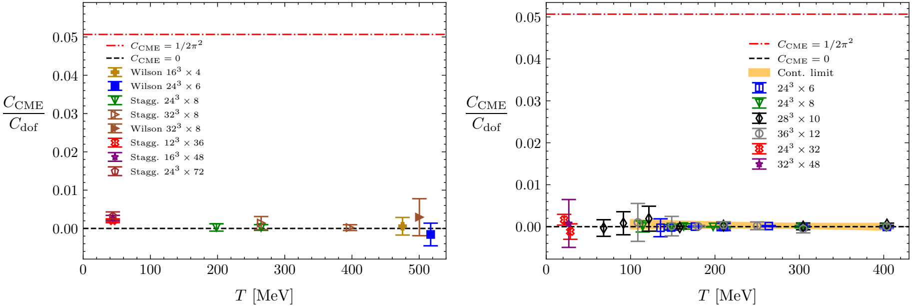


CME in equilibrium – final result

- quenched, heavier-than-physical Wilson quarks
- full QCD simulations with dynamical staggered quarks
  at physical quark masses, extrapolated to the continuum limit
  employing the conserved electric current

- global CME current vanishes in equilibrium
  Brandt, Endrődi, Garnacho, Markó, JHEP 09 (2024)



Chiral separation effect in equilibrium

Chiral separation effect

- axial current due to magnetic field and baryon density
  Son, Zhitnitsky, PRD 70 (2004) Metlitski, Zhitnitsky, PRD 72 (2005)
- parameterize baryon density $J_0 = \int \bar{\psi}\gamma_0\psi$ by chemical potential μ
- CSE for small density ($\boldsymbol{B} = B\boldsymbol{e}_3$)

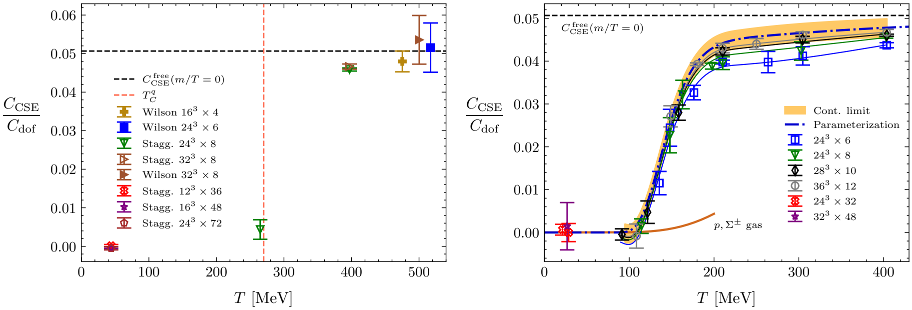
$$\langle J_{35}\rangle = \sigma_{\mathrm{CSE}}\, B = C_{\mathrm{CSE}}\, \mu B + \mathcal{O}(\mu^3)$$

- Bloch's theorem allows in-equilibrium CSE $(\partial_\nu J_{\nu 5} \neq 0)$
- regularization less intricate, but conserved vector current on lattice is important
- previous lattice efforts Puhr, Buividovich, PRL 118 (2017)
  Buividovich, Smith, von Smekal, PRD 104 (2021)

CSE in equilibrium – final result

- quenched, heavier-than-physical Wilson quarks
- full QCD simulations with staggered quarks
  at physical quark masses, extrapolated to the continuum limit
  employing the conserved electric current

  Brandt, Endrődi, Garnacho, Markó, JHEP 02 (2024)

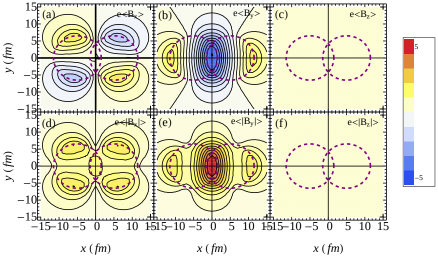


- comparison to baryon gas model at low T

Local chiral magnetic effect

Inhomogeneous magnetic fields

- up to now: homogeneous magnetic background
- off-central heavy-ion collisions: inhomogeneous fields Deng et al., PRC 85 (2012)



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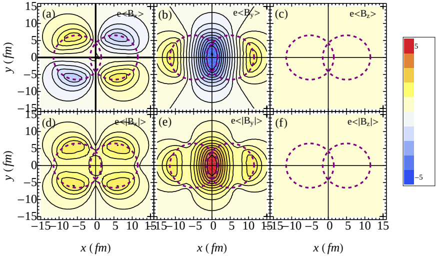

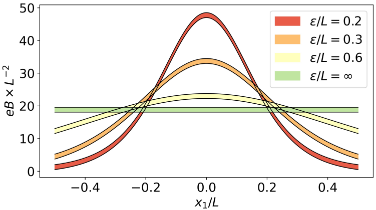

- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ Dunne, hep-th/0406216
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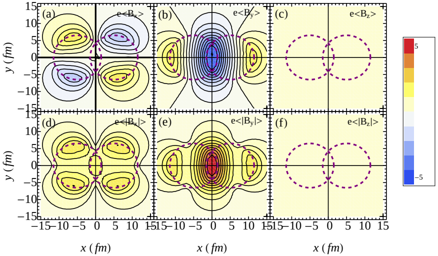

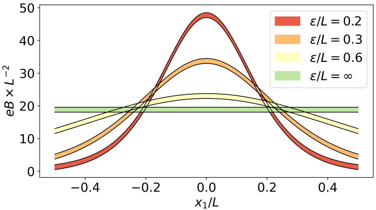

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  with $\epsilon \sim 0.6$ fm
- impact on thermodynamic observables in QCD and phase diagram
  🔗 Brandt, Endrődi, Markó, Valois, JHEP 07 (2024)

Local currents

- response for weak μ_5 for homogeneous B

$$\langle J_3 \rangle = C_{\rm CME}\, \mu_5 B$$

Local currents

- response for weak μ_5 for homogeneous and inhomogeneous B

$$\langle J_3 \rangle = C_{\mathrm{CME}}\, \mu_5 B \qquad\qquad \langle J_3(x_1) \rangle = \mu_5 \underbrace{\int \mathrm{d}x_1'\, C_{\mathrm{CME}}(x_1 - x_1') B(x_1')}_{G(x_1)}$$

- Bloch's theorem allows local currents if $\int \mathrm{d}x_1 \langle J_3(x_1) \rangle = 0$

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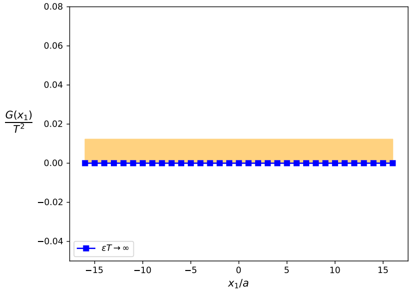

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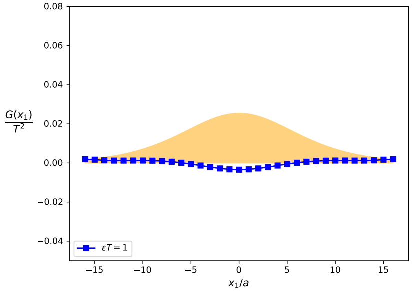

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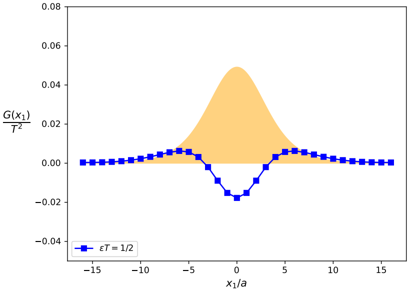

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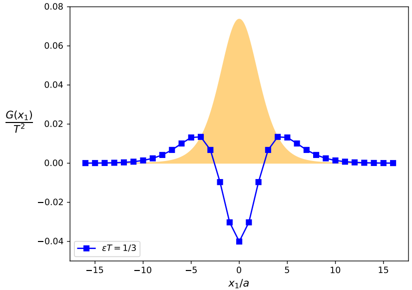

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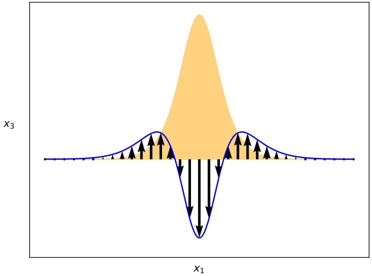

Local currents in QCD

- full QCD simulations with staggered quarks
  at physical quark masses, extrapolated to the continuum limit
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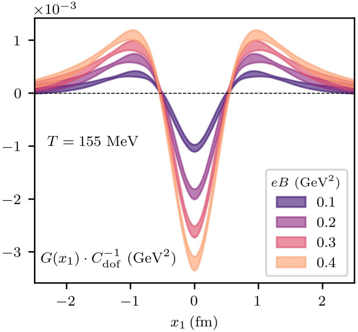
- non-trivial localized CME signal Brandt, Endrődi, Garnacho, Markó, Valois, 2409.17616



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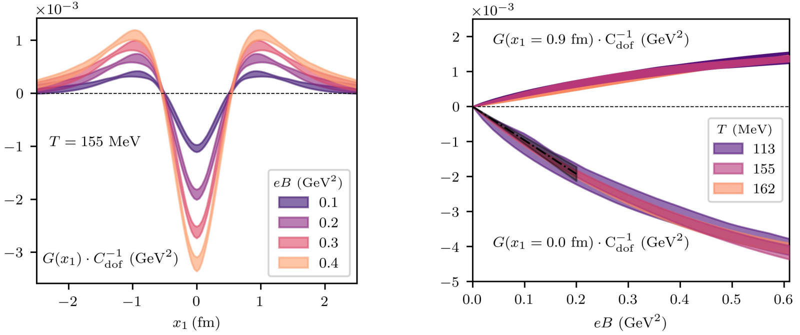

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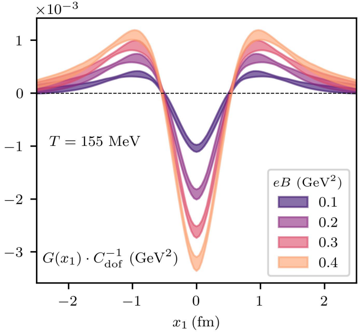

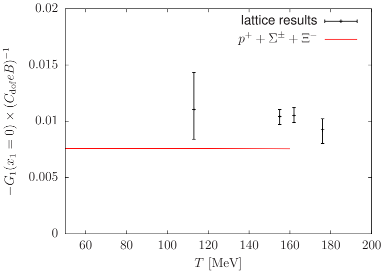

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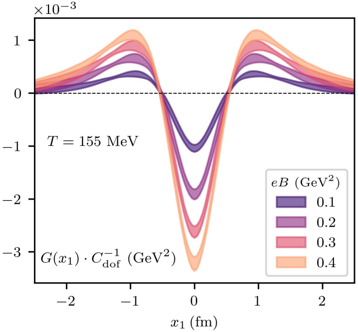
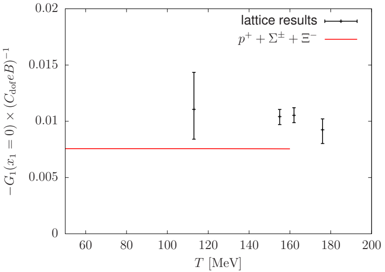




- may guide experimental efforts to detect CME

Out-of-equilibrium phenomena

Out-of-equilibrium transport in real time

- linear response theory for time-dependent chirality perturbation $\delta\mu_5(t)$ in static magnetic field B
- retarded correlator

$$G^R_{\mathrm{CME}}(t) = i\Theta(t)\left\langle [J_{45}(t), J_3(0)] \right\rangle$$

- spectral function

$$\rho_{\mathrm{CME}}(\omega) = \frac{1}{\pi}\mathrm{Im}\,\widetilde{G}^R_{\mathrm{CME}}(\omega)$$

- Kubo formula for out-of-equilibrium CME coefficient

$$C^{\mathrm{neq.}}_{\mathrm{CME}} \cdot B = \lim_{\omega\to 0} \frac{\rho_{\mathrm{CME}}(\omega)}{\omega}$$

Out-of-equilibrium transport from Euclidean time

- Euclidean correlator and spectral function

$$G_{\mathrm{CME}}(x_4) = \int_0^\infty \mathrm{d}\omega \, \frac{\rho_{\mathrm{CME}}(\omega)}{\omega} \, K(x_4,\omega) \,, \qquad K(x_4,\omega) = \frac{\omega \, \cosh[\omega(x_4 - 1/(2T))]}{\sinh[\omega/(2T)]}$$

- ill-posed problem with long history
  Backus-Gilbert, MEM, Tikhonov, Gaussian processes, ...
- alternative for a first estimate: midpoint 🔗 Buividovich, PRD 110 (2024)

$$K(x_4 = 1/(2T),\omega) = \frac{\omega}{\sinh[\omega/(2T)]} \xrightarrow{T\to 0} \pi^2 T \cdot \delta(\omega)$$

  pinches out the CME coefficient at low T

$$C_{\mathrm{CME}}^{\mathrm{neq.\,mp}} \cdot B = G_{\mathrm{CME}}(x_4 = 1/(2T))/T$$

Out-of-equilibrium CME and static CSE

- for free fermions (1-loop perturbation theory)
  Brandt, Endrődi, Garnacho, Markó, Valois, 2502.01155

$$\rho_{\mathrm{CME}}(\omega) = B\left[\alpha(m/T)\,\omega\,\delta(\omega) + \beta(m/T)\,\Theta(\omega^2 - m^2)\right]$$
$$\rho_{\mathrm{CSE}}(\omega) = B\left[\alpha(m/T)\,\omega\,\delta(\omega)\right]$$

- midpoint estimate

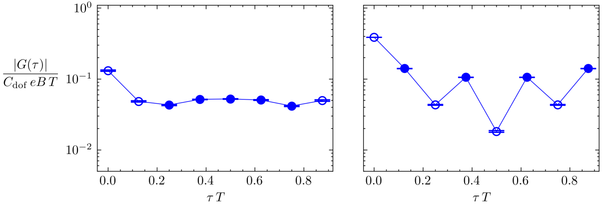
$$C_{\mathrm{CME}}^{\mathrm{neq.\,mp}}(m/T) = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}p\,\left[1 + \cosh(\sqrt{p^2 + (m/T)^2})\right]^{-1}.$$

- we observe that

$$C_{\mathrm{CME}}^{\mathrm{neq.\,mp}}(m/T) = C_{\mathrm{CSE}}(m/T)$$

Lattice results

- correlators for quenched Wilson and dynamical staggered



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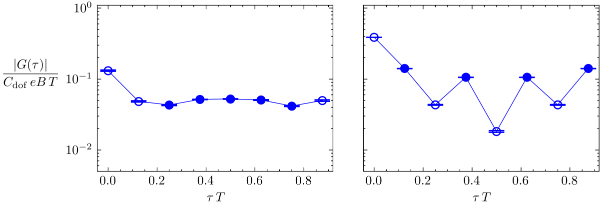

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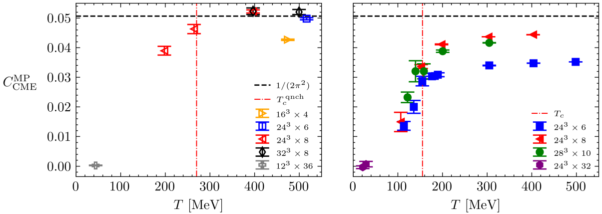

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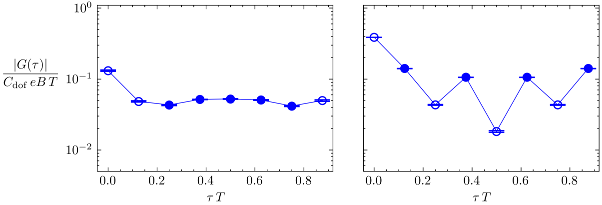

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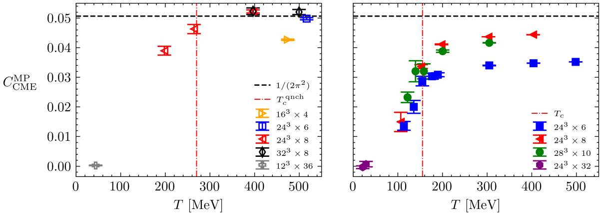
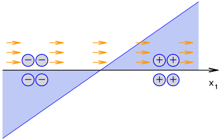


- qualitative agreement $C_{\rm CME}^{\rm neq.\,mp}(T) \approx C_{\rm CSE}$ seems to hold in QCD

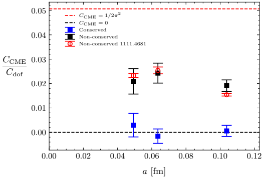
Summary

Summary

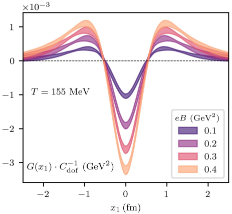
- CME subtleties:
  in- / out-of-equilibrium



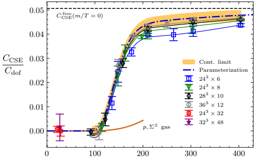
- careful regularization crucial
  in-equilibrium global CME vanishes



- in-equilibrium local CME in full QCD



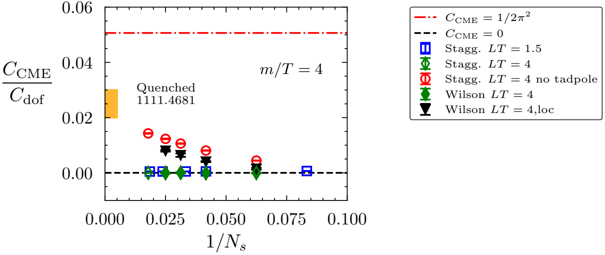
- in-equilibrium CSE in full QCD
  and first results for out-of-eq. CME



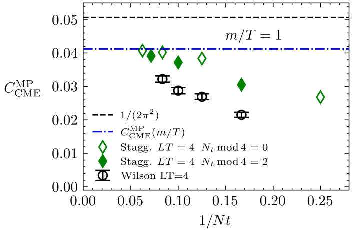
Backup

Benchmarks in the free case

- equilibrium CME



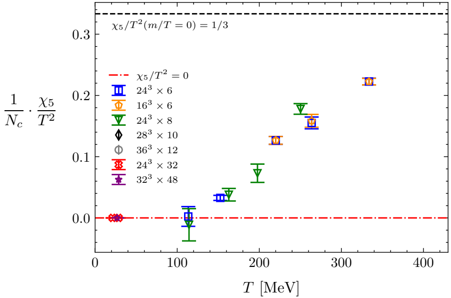
- CME midpoint estimate



Chiral density

- chiral density J_{05} is parameterized by chiral chemical potential μ_5

$$J_{05}(\mu_5) = \chi_5\,\mu_5 + \mathcal{O}(\mu_5^3), \qquad \chi_5 = \frac{T}{V}\,\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_5^2}\bigg|_{\mu_5=0}$$



Inhomogeneous chiral imbalance

- inhomogeneous $B(x_1)$ *and* inhomogeneous $\mu_5(x_1)$

$$\langle J_3(x_1)\rangle = \int \mathrm{d}x_1' \underbrace{\mathrm{d}x_1'' \, \chi_{\mathrm{CME}}(x_1 - x_1', x_1 - x_1'')\, B(x_1'')}_{H(x_1, x_1')} \mu_5(x_1')$$