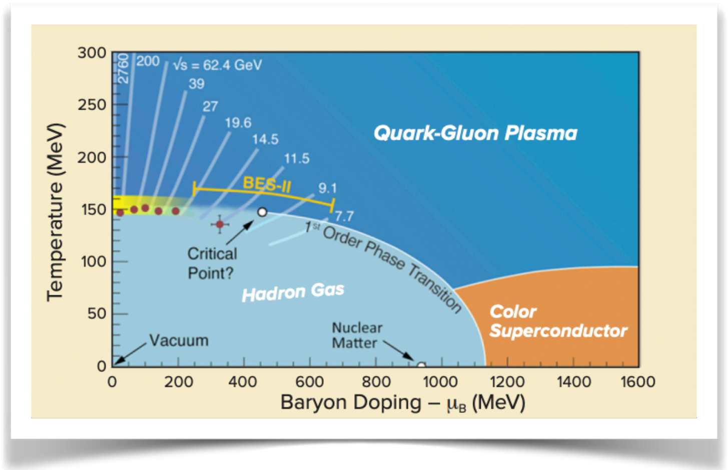


Mapping the QCD phase diagram : Connecting EoS to observables

Workshop on “Confinement and Symmetry from vacuum to QCD phase diagram”

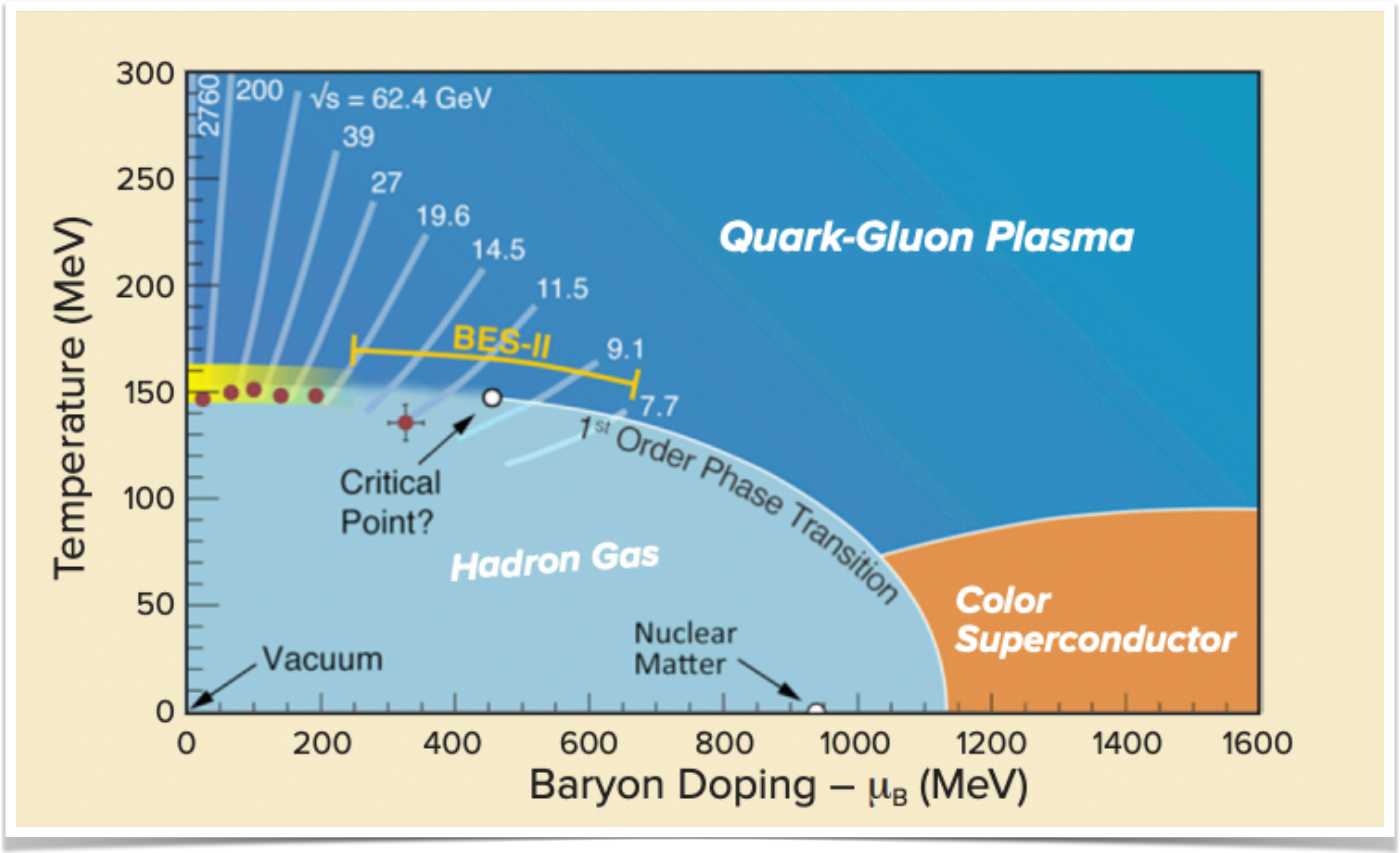
Maneesha Pradeep

University of Maryland at College Park

Collaborators : Misha Stephanov, Jamie Karthein, Krishna Rajagopal, Yi Yin

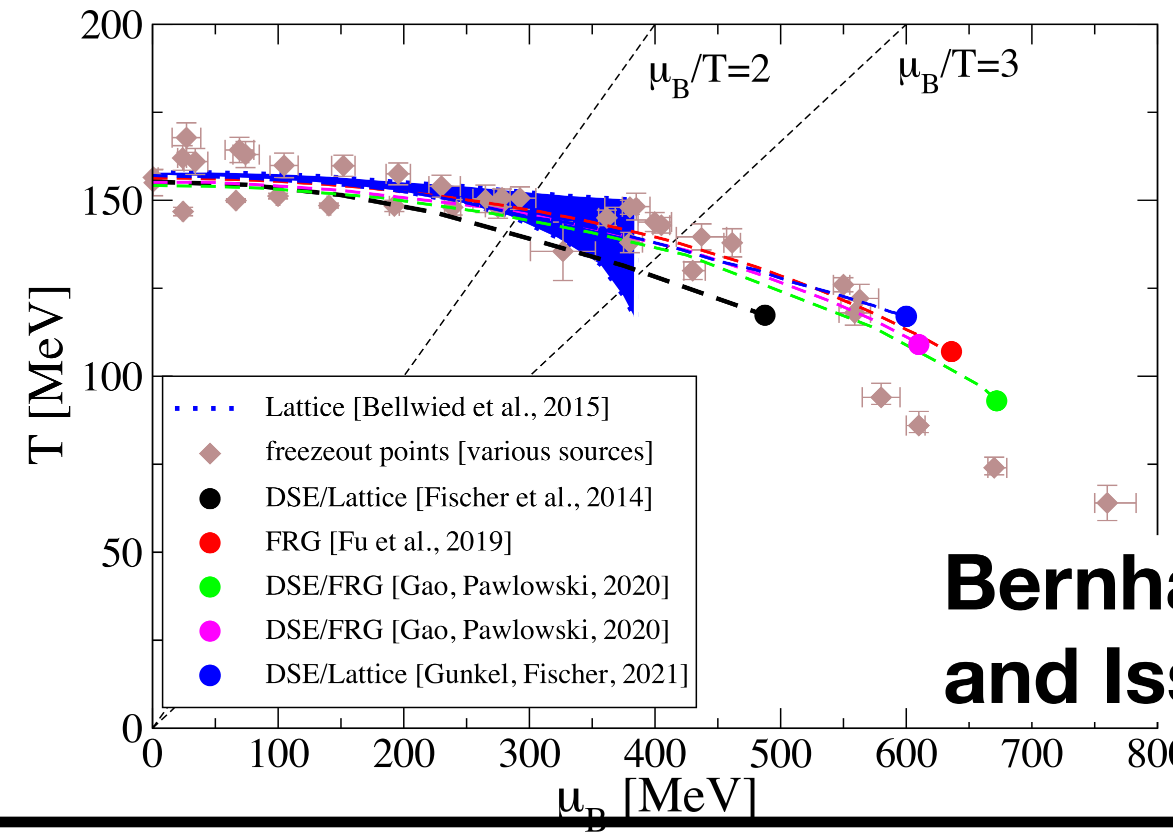
Phys.Rev.Lett. 130 (2023) 16, 16, ***arXiv*** **2409.16249 + Work in progress**

Is there a critical point on the QCD phase diagram?



- At $\mu_B = 0$: It is a cross-over. — Lattice QCD simulations
- At T= 0 : Models predict a first order phase transition — Not known from first principles

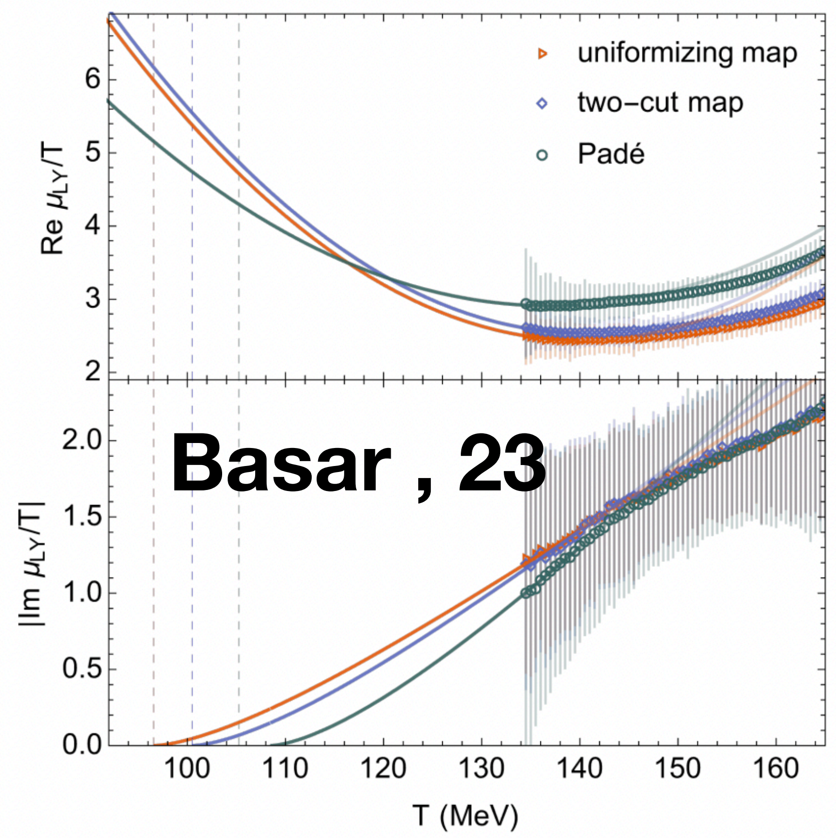
Recent theory guidances for the location of the critical point



FRG/ DSE

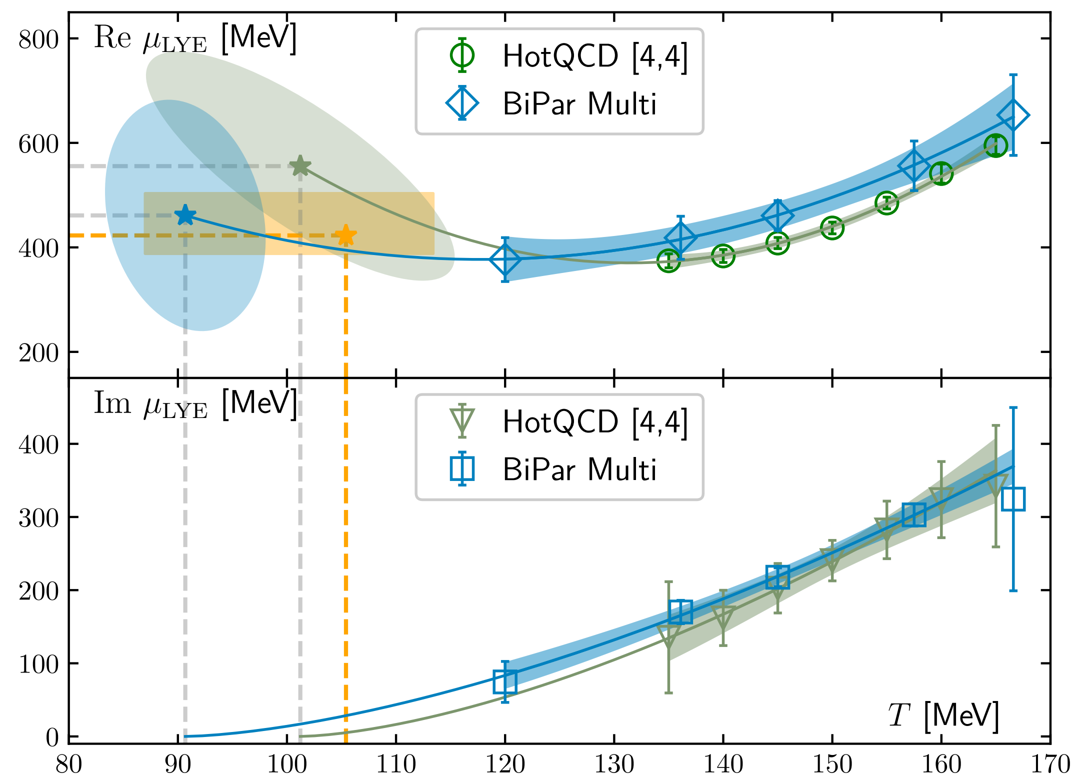
Bernhardt, Fischer and Isserstedt, 23

$$(\mu_{BC}, T_c) = (495 - 654, 108 - 119)\ \text{MeV}$$



Basar , 23

$$(\mu_{BC}, T_c) \approx (580, 100)\ \text{MeV}$$



Clarke et al., 24

$$(\mu_{BC}, T_c) = (422^{+80}_{-35}, 105^{+8}_{-18})\ \text{MeV}$$

Extrapolations of Lee-Yang edge singularities to real axis

Font Color

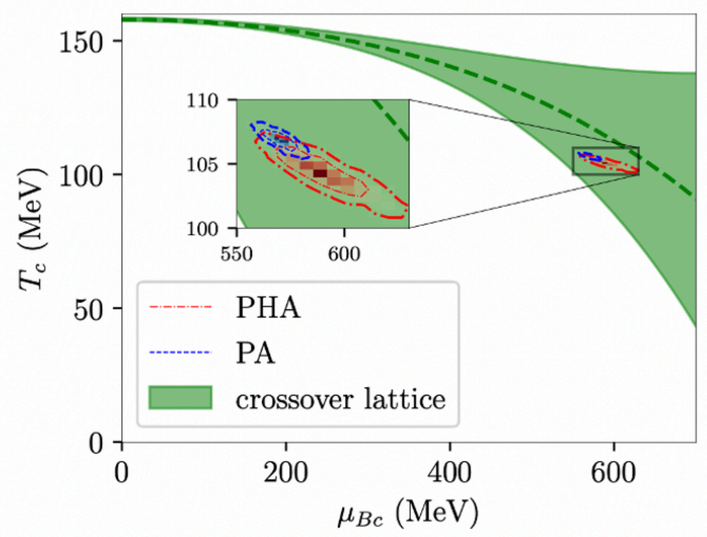
Bayesian Holography + Lattice input at $\mu = 0$

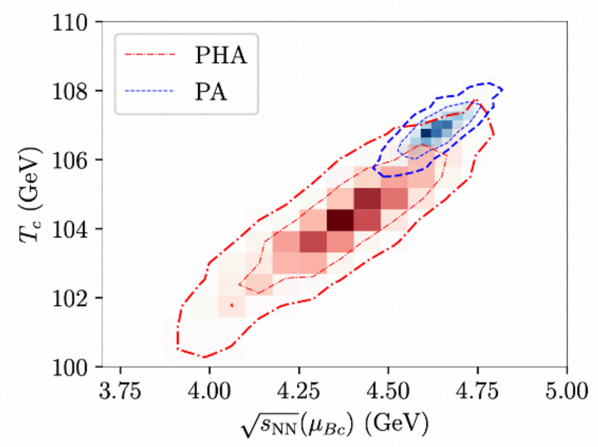
Hippert et al, e-Print: 2309.00579 [nucl-th]

Predict CEP (95% confidence level):

$$T_c = 101 - 108\ \text{MeV} \quad \mu_c = 560 - 625\ \text{MeV}$$

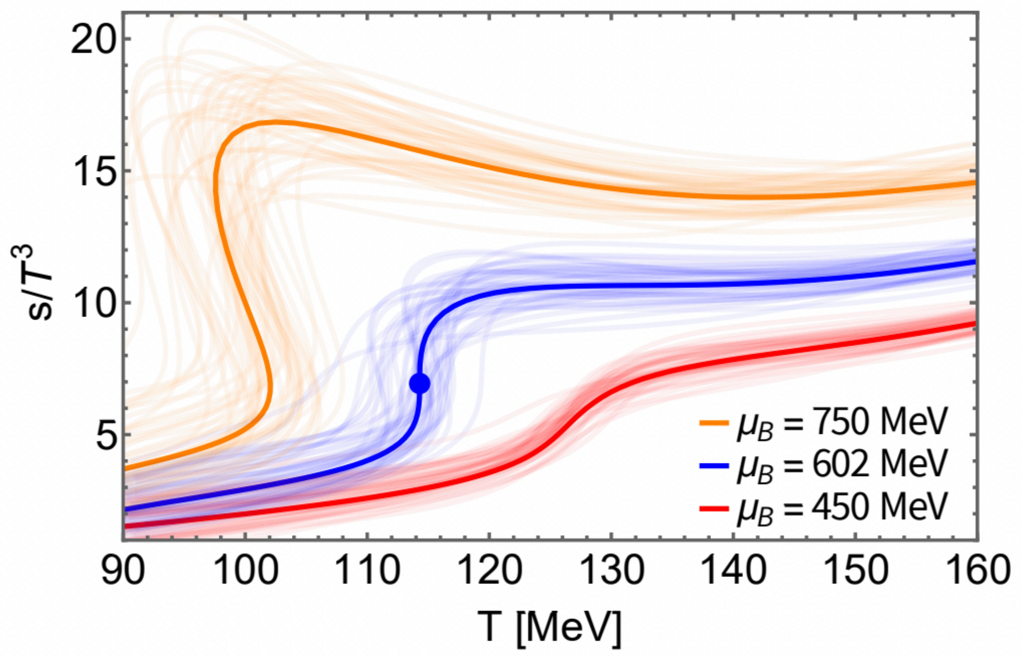
$$\sqrt{s} = 4.0 - 4.8\ \text{GeV}$$





Thermodynamic analysis of a lattice QCD extrapolated EoS

Shah et al., 24

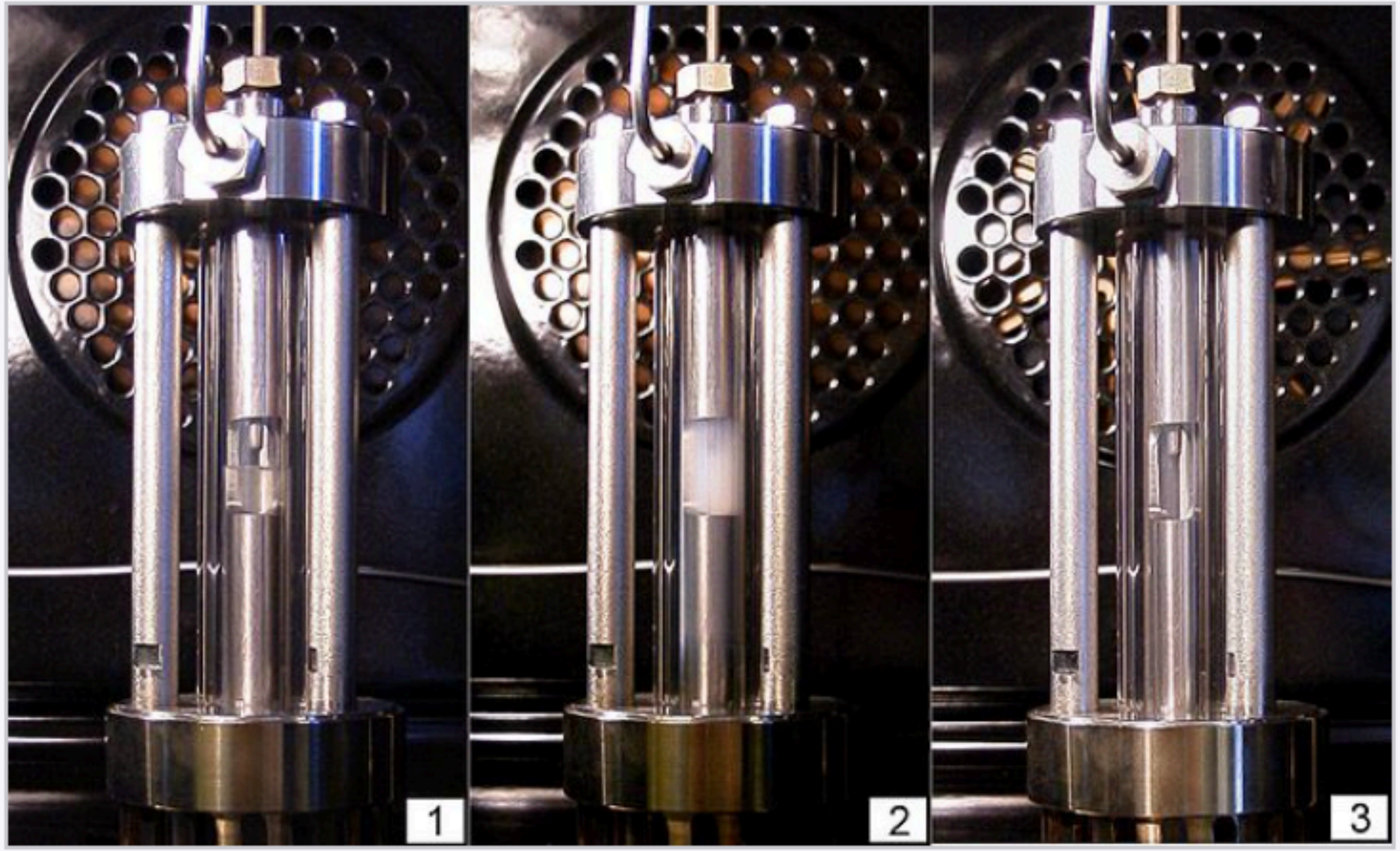


$$(\mu_{BC}, T_c) = (602.1 \pm 62.1\ ,\ 114.3 \pm 6.9)$$

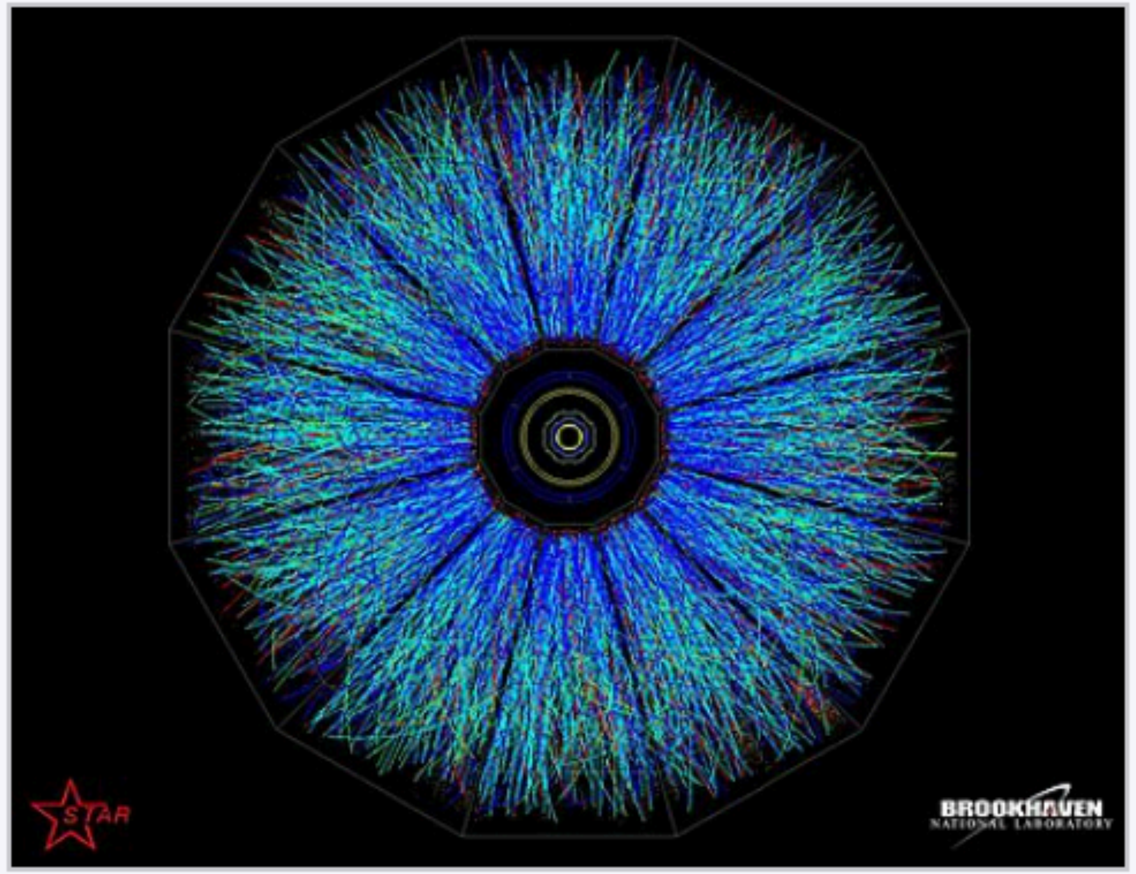
Probing critical points experimentally

Baron Cagniard de la Tour's experiment

Ethane (C_2H_6)	31.17 °C (304.32 K)	48.077 atm (4,871.4 kPa)
Ethanol (C_2H_5OH)	241 °C (514 K)	62.18 atm (6,300 kPa)
Fluorine	−128.85 °C (144.30 K)	51.5 atm (5,220 kPa)
Helium	−267.96 °C (5.19 K)	2.24 atm (227 kPa)
Hydrogen	−239.95 °C (33.20 K)	12.8 atm (1,300 kPa)
Krypton	−63.8 °C (209.3 K)	54.3 atm (5,500 kPa)
Methane (CH_4)	−82.3 °C (190.8 K)	45.79 atm (4,640 kPa)
Neon	−228.75 °C (44.40 K)	27.2 atm (2,760 kPa)
Nitrogen	−146.9 °C (126.2 K)	33.5 atm (3,390 kPa)
Oxygen (O_2)	−118.6 °C (154.6 K)	49.8 atm (5,050 kPa)
Carbon dioxide (CO_2)	31.04 °C (304.19 K)	72.8 atm (7,380 kPa)
Nitrous oxide (N_2O)	36.4 °C (309.5 K)	71.5 atm (7,240 kPa)
Sulfuric acid (H_2SO_4)	654 °C (927 K)	45.4 atm (4,600 kPa)
Xenon	16.6 °C (289.8 K)	57.6 atm (5,840 kPa)
Lithium	2,950 °C (3,220 K)	652 atm (66,100 kPa)
Mercury	1,476.9 °C (1,750.1 K)	1,720 atm (174,000 kPa)
Sulfur	1,040.85 °C (1,314.00 K)	207 atm (21,000 kPa)
Iron	8,227 °C (8,500 K)	
Gold	6,977 °C (7,250 K)	5,000 atm (510,000 kPa)
Aluminium	7,577 °C (7,850 K)	
Water (H_2O)[3][20]	373.946 °C (647.096 K)	217.7 atm (22,060 kPa)



1. Subcritical ethane, liquid and gas phase coexist.
2. Critical point (32.17 °C, 48.72 bar), displaying critical opalescence.
3. Supercritical ethane, fluid.[1]

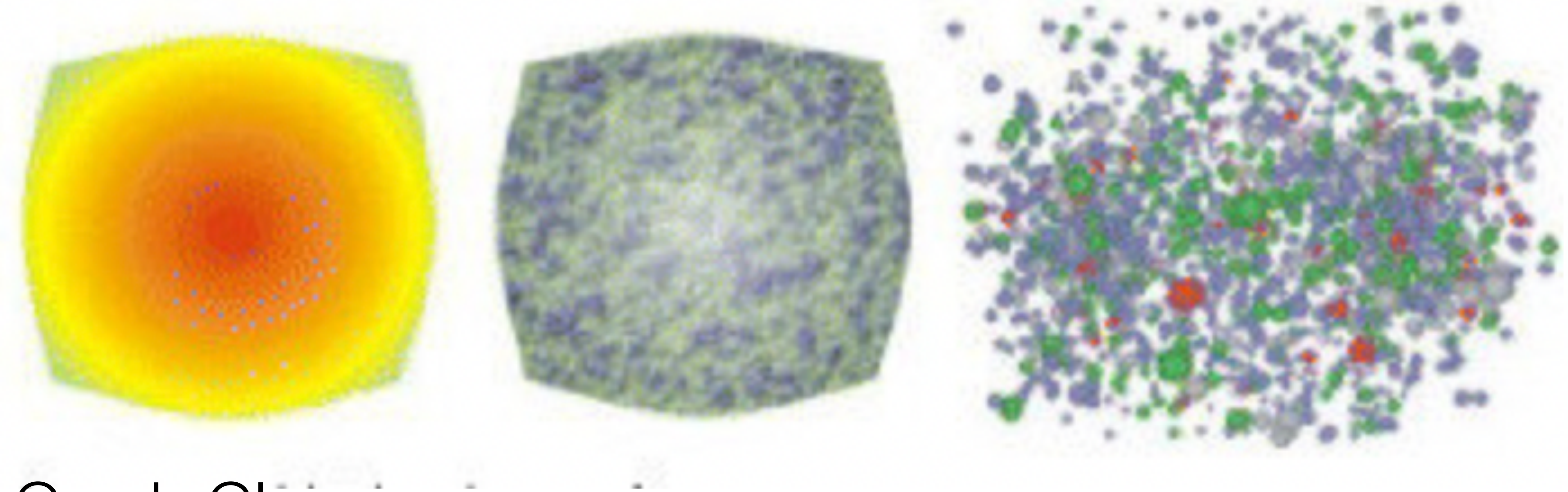


A view of gold ions collisions as captured by the STAR detector.

Colliding heavy-ions at varying center of mass energy

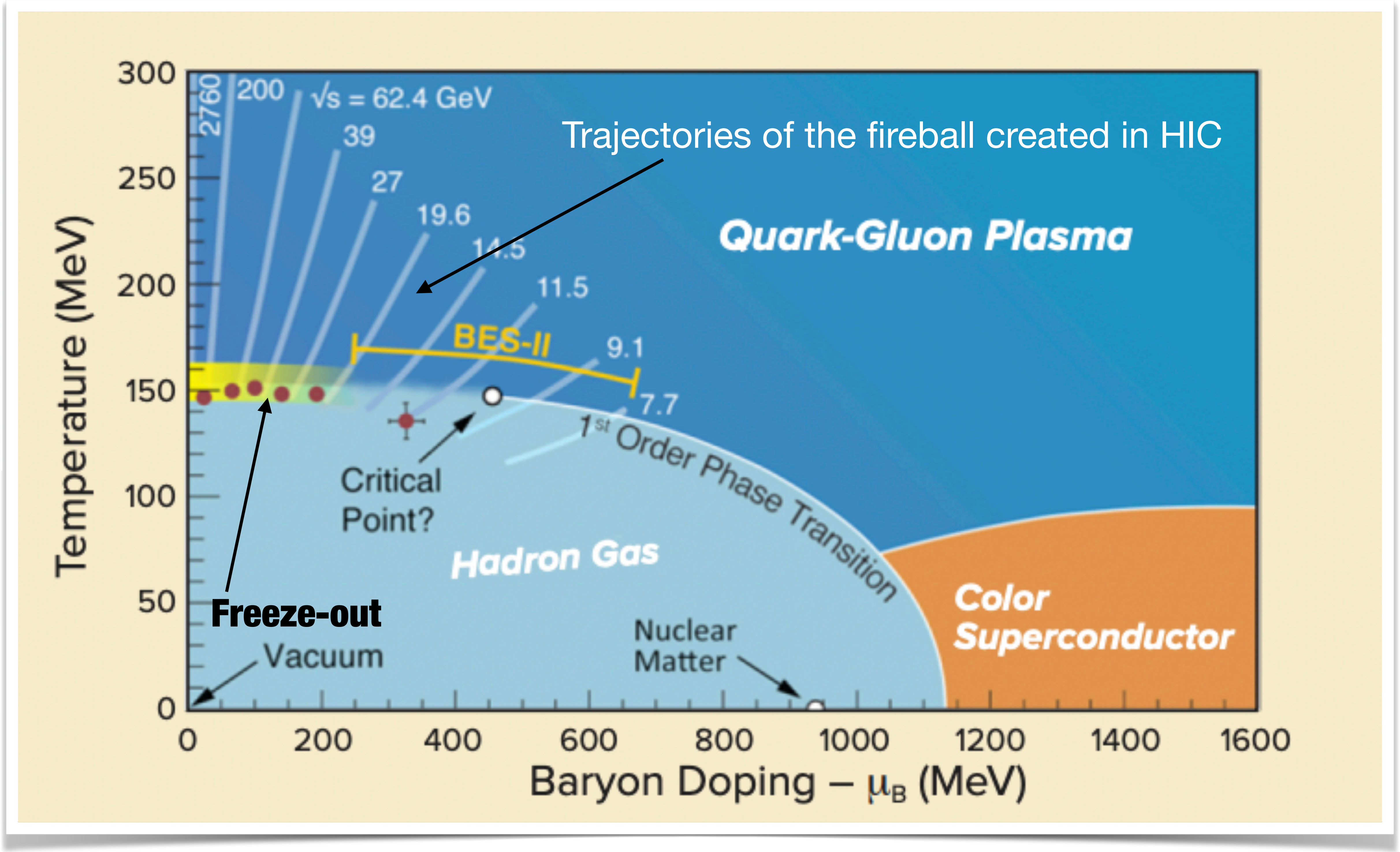
Wikipedia

Probing the EoS via heavy-ion collisions



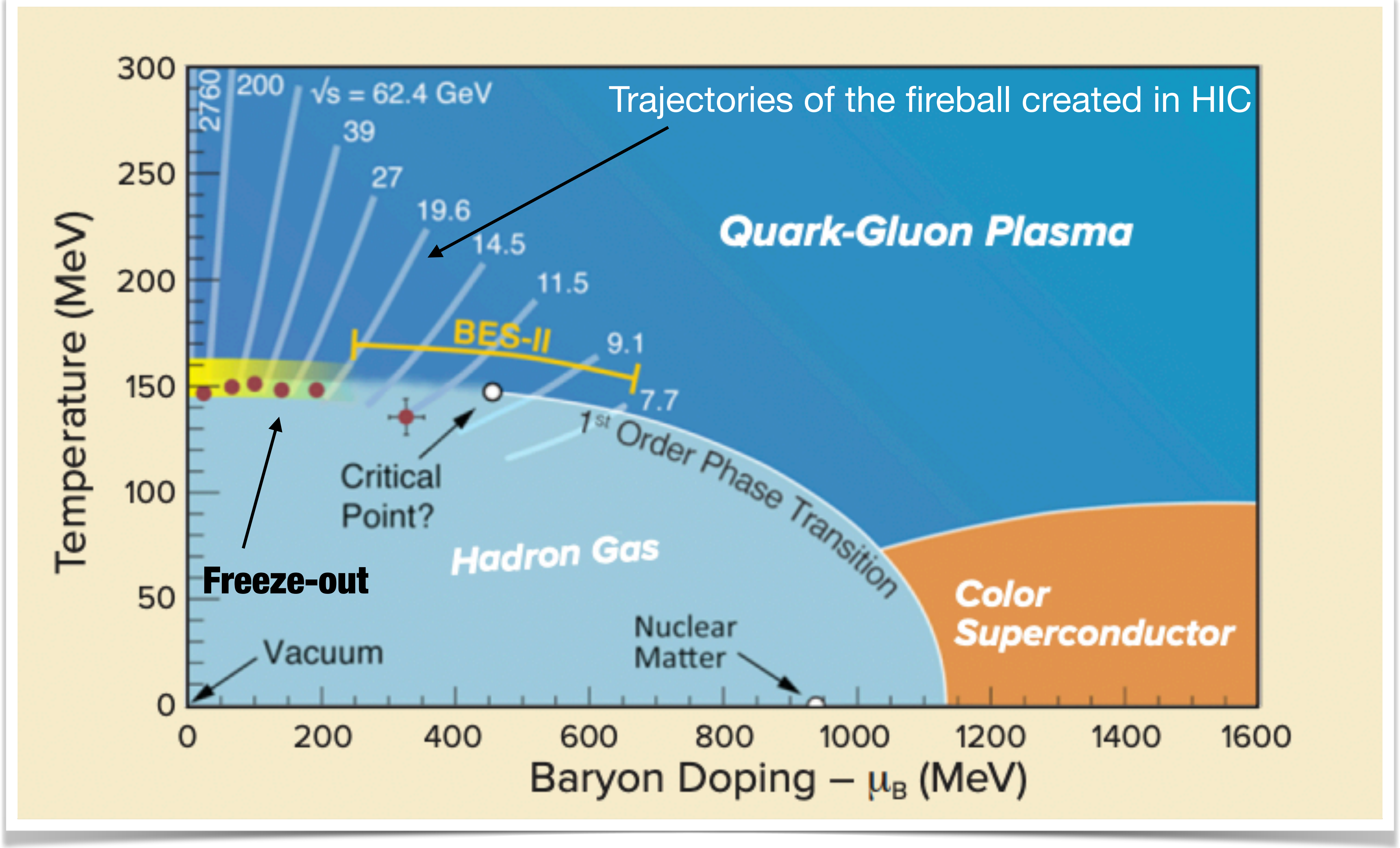
Quark-Gluon Plasma   Critical region   Freeze-out

Figure by S.A.Bass



- QGP thermalizes in ~ 1fm/c
- Freeze-out at about 10 fm/c
- The event by event distribution of the particle multiplicities are measured at the detectors.

Non-monotonic dependence of cumulants as a function of collision energy



Diverges at CP

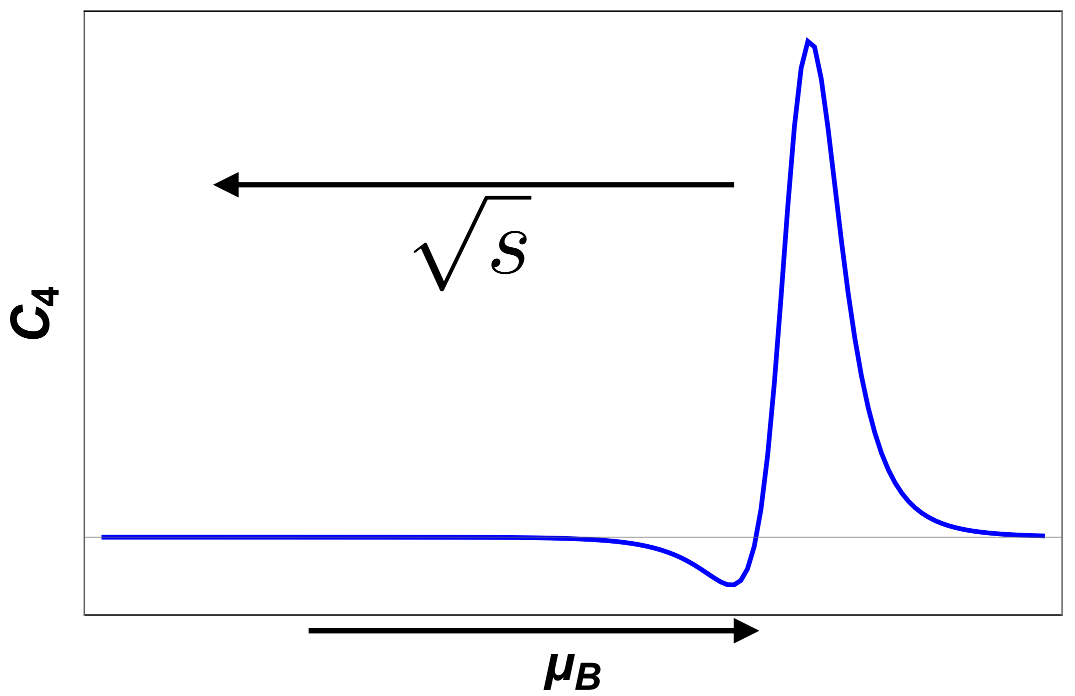
$$C_k \equiv \left\langle \delta N_B^k \right\rangle_c \overset{\text{in eq.}}{=} V T^{k-1} \frac{\partial^k P}{\partial \mu^k}$$

Fluctuations are enhanced near CP

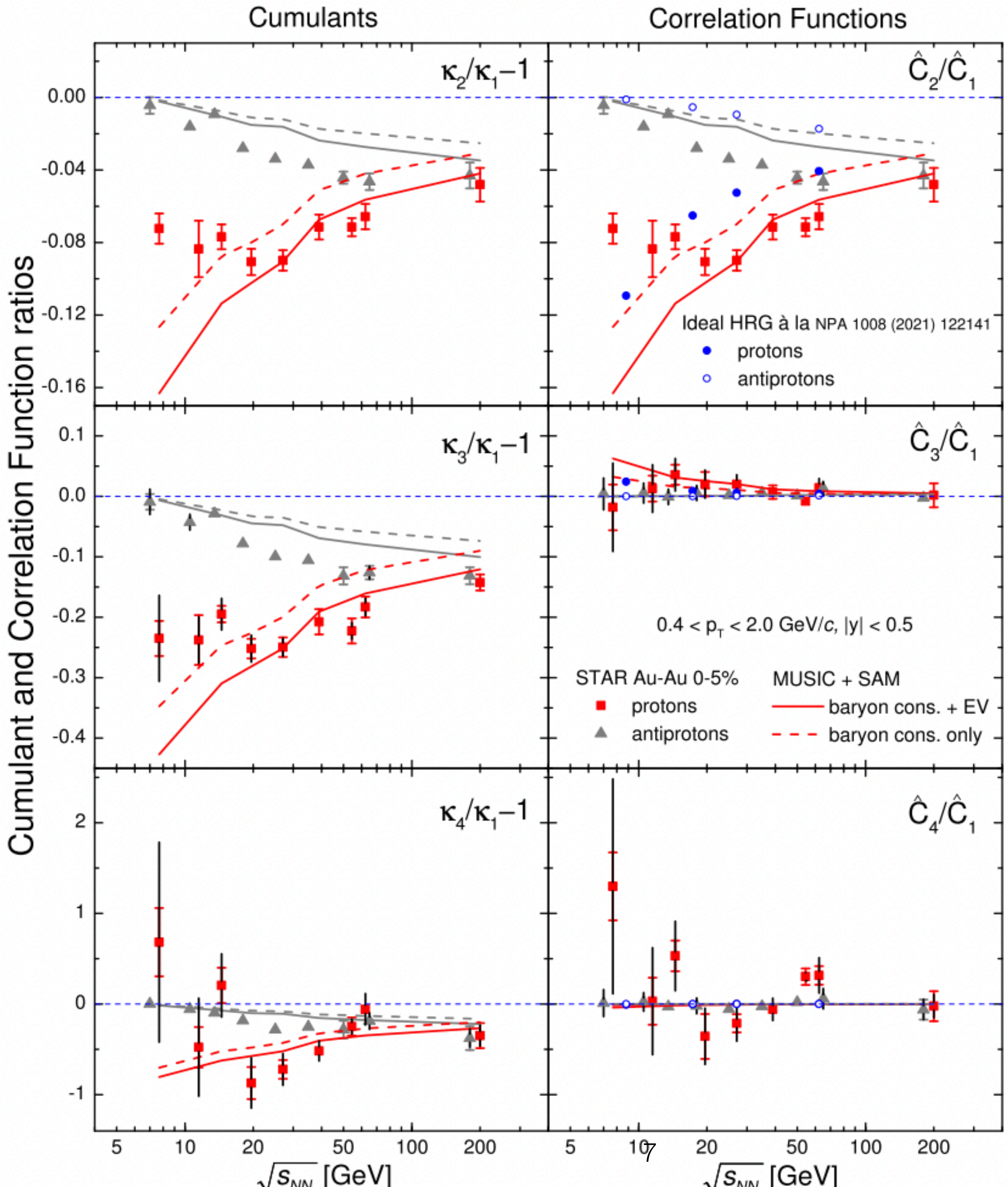
$$\left\langle \delta N_B^2 \right\rangle_c = \left\langle (N_B - \langle N_B \rangle)^2 \right\rangle \, , \left\langle \delta N_B^3 \right\rangle_c = \left\langle (N_B - \langle N_B \rangle)^3 \right\rangle$$

$$\left\langle \delta N_B^4 \right\rangle_c = \left\langle (N_B - \langle N_B \rangle)^4 \right\rangle - 3 \left\langle (N_B - \langle N_B \rangle)^2 \right\rangle$$

Similar relations between cumulants of entropy density and derivatives of pressure with respect to temperature



Rajagopal, Shuryak, Stephanov, 98, 99, Stephanov, 09



2021, STAR Collaboration

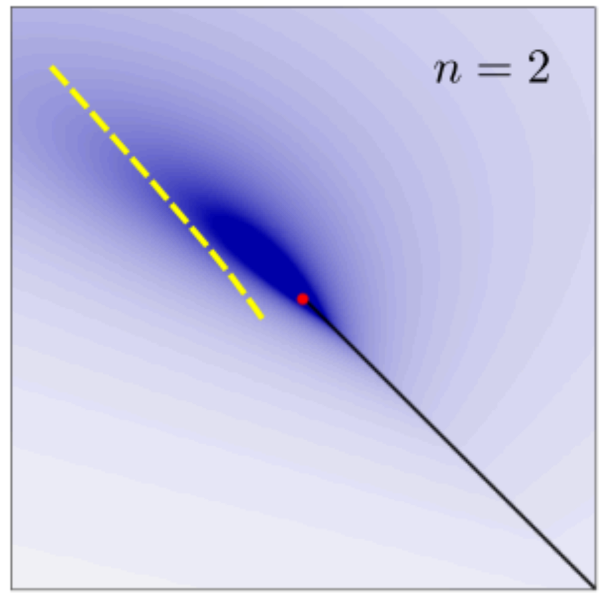
BES-I data for proton multiplicity cumulants

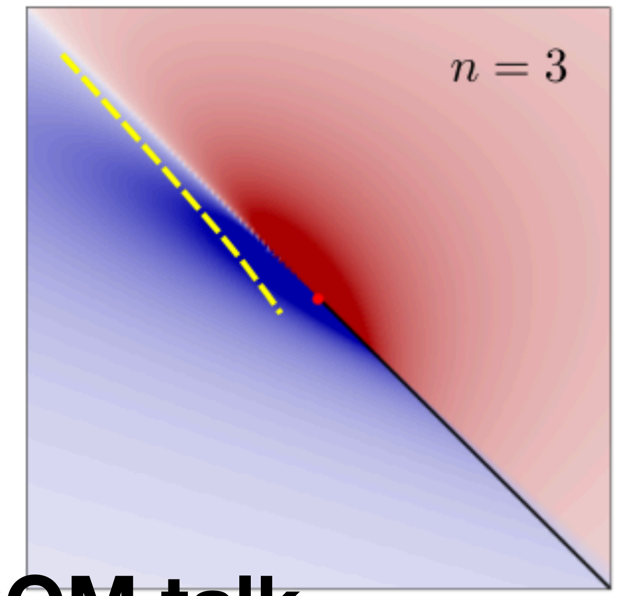
A *clear excess* of scaled proton-number variance from non-critical baseline reported for $\sqrt{s_{NN}} \leq 10\,\mathrm{GeV}$

Vovchenko, Koch, Shen, 22

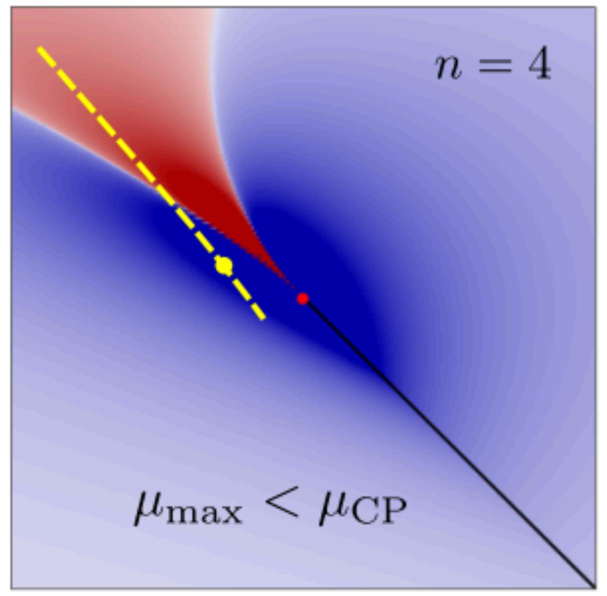
Theory vs BES-II data

(universal EOS) critical χ_n:



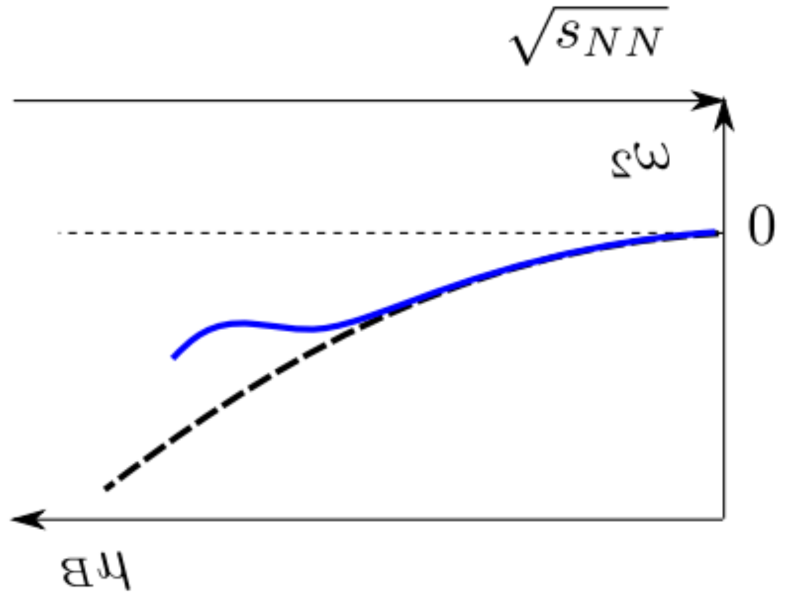


Misha Stephanov's SQM talk

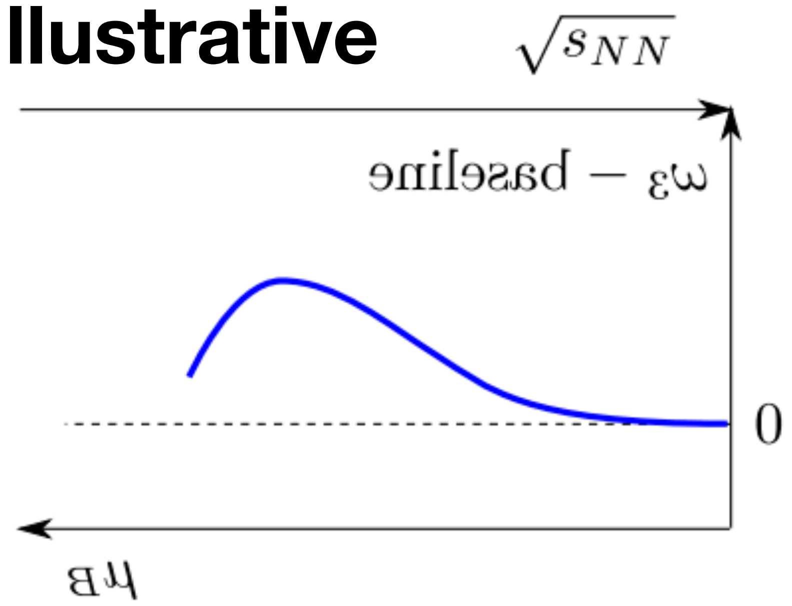


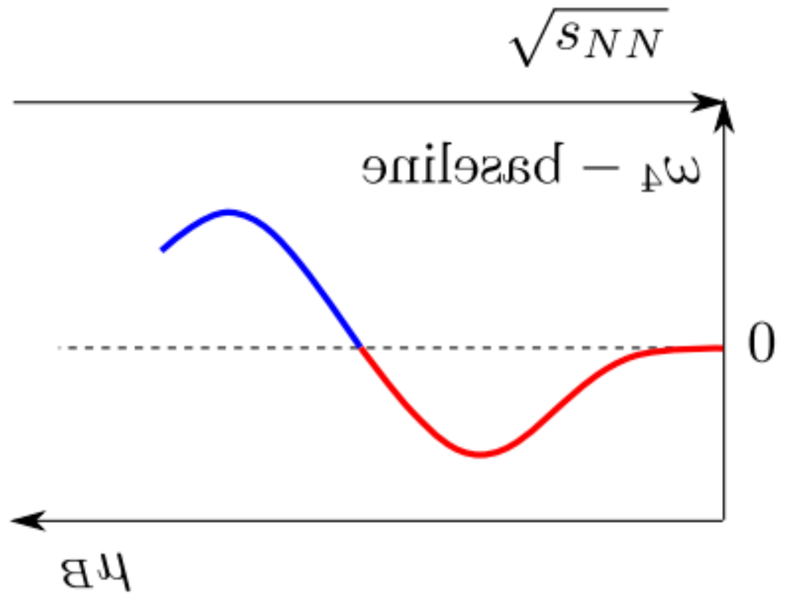
Bzdak et al review 1906.00936

(irreducible correlations) $\text{FC}_n[N_p] \sim \chi_n$ (Pradeep, MS 2211.09142), $\omega_n \equiv \text{FC}_n/\text{FC}_1$

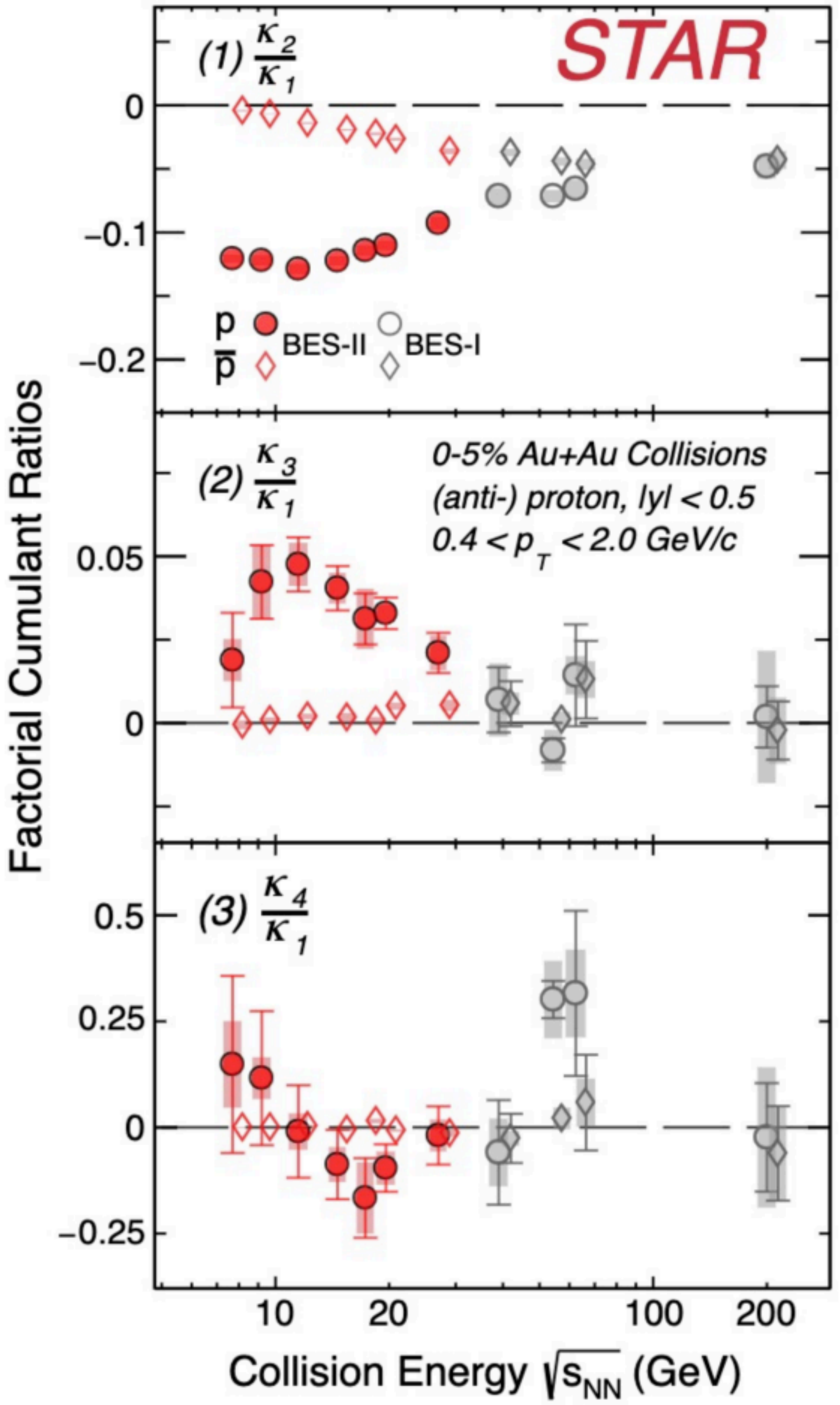


Illustrative





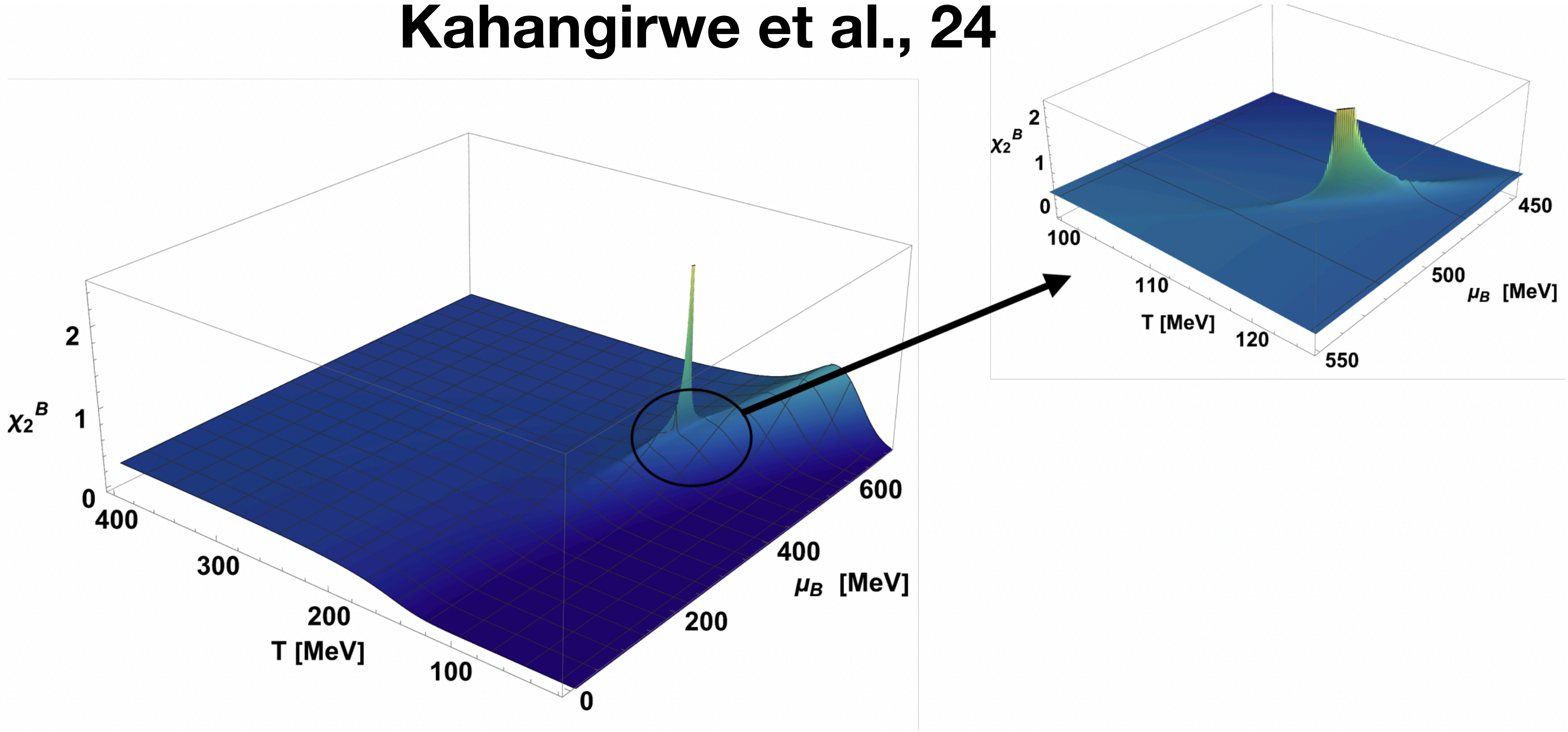
Ashish Pandav's CPOD talk



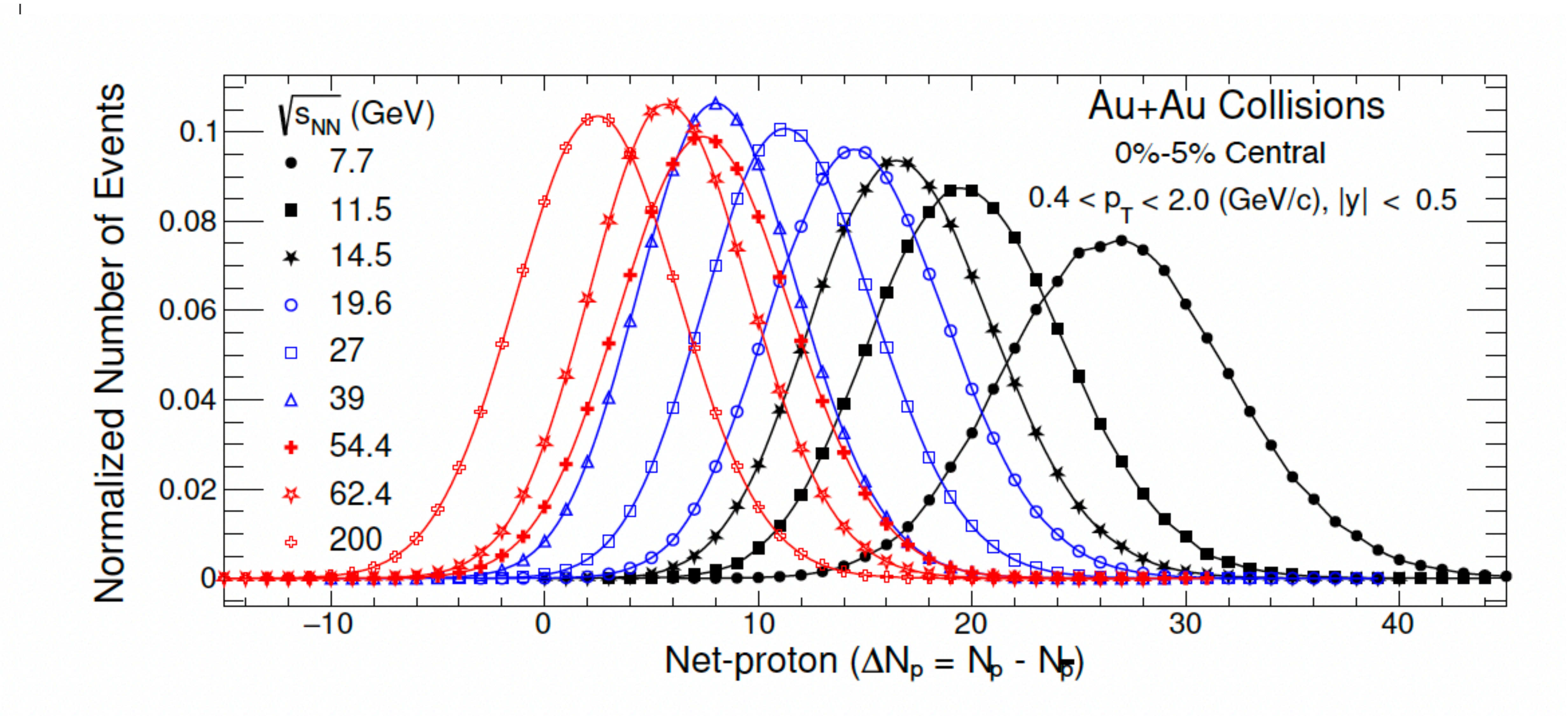
Expected signatures: bump in ω_2 and ω_3, dip then bump in ω_4 for CP at $\mu_B > 420$ MeV

EoS $\overset{?}{\longleftrightarrow}$ Particle multiplicity distributions

Kahangirwe et al., 24



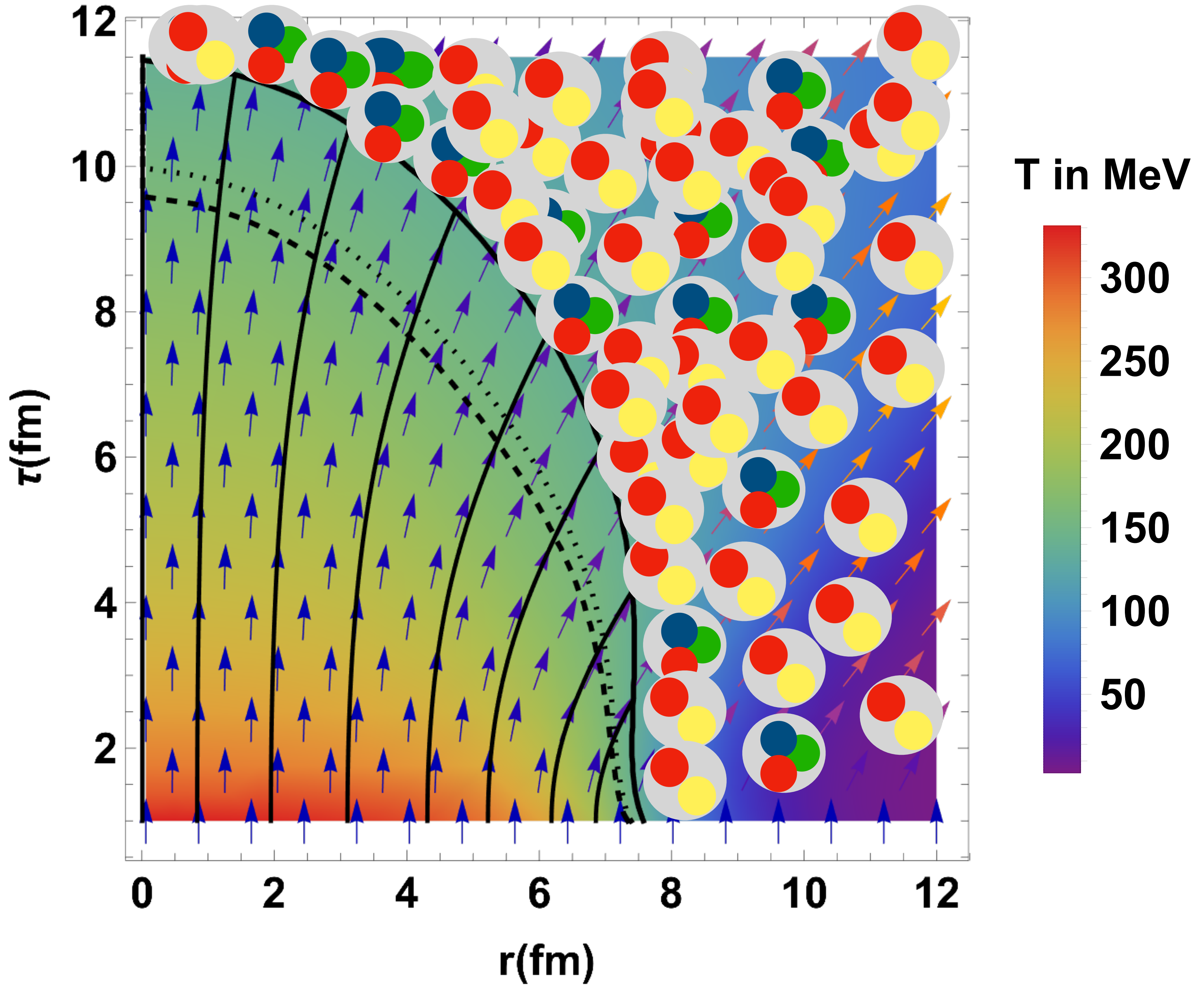
Susceptibilities that diverge at CP



Imprints on Particle Distribution Functions?

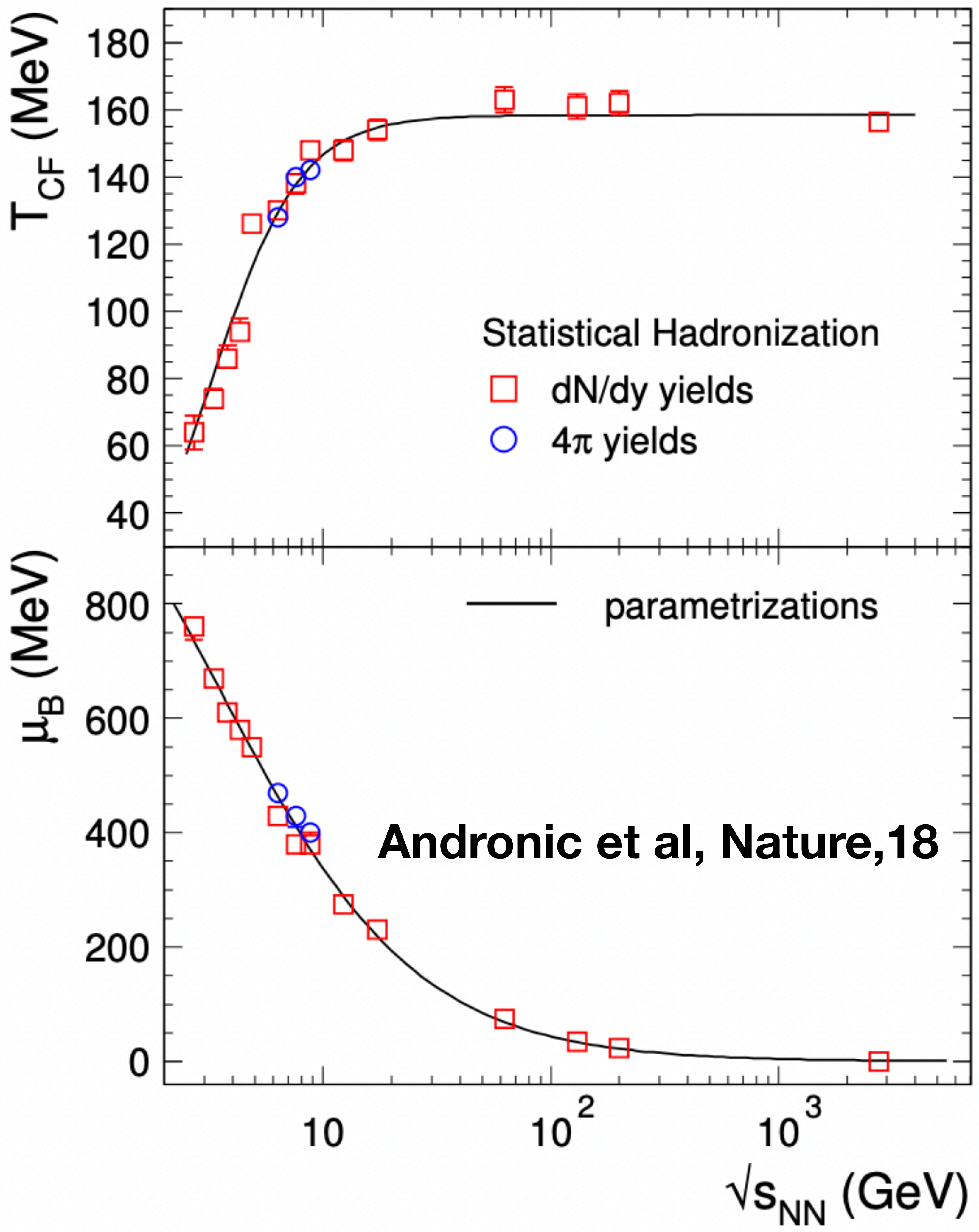
STAR Collaboration, PRL 2014, PRL 2021

Freeze-out in heavy-ion collisions

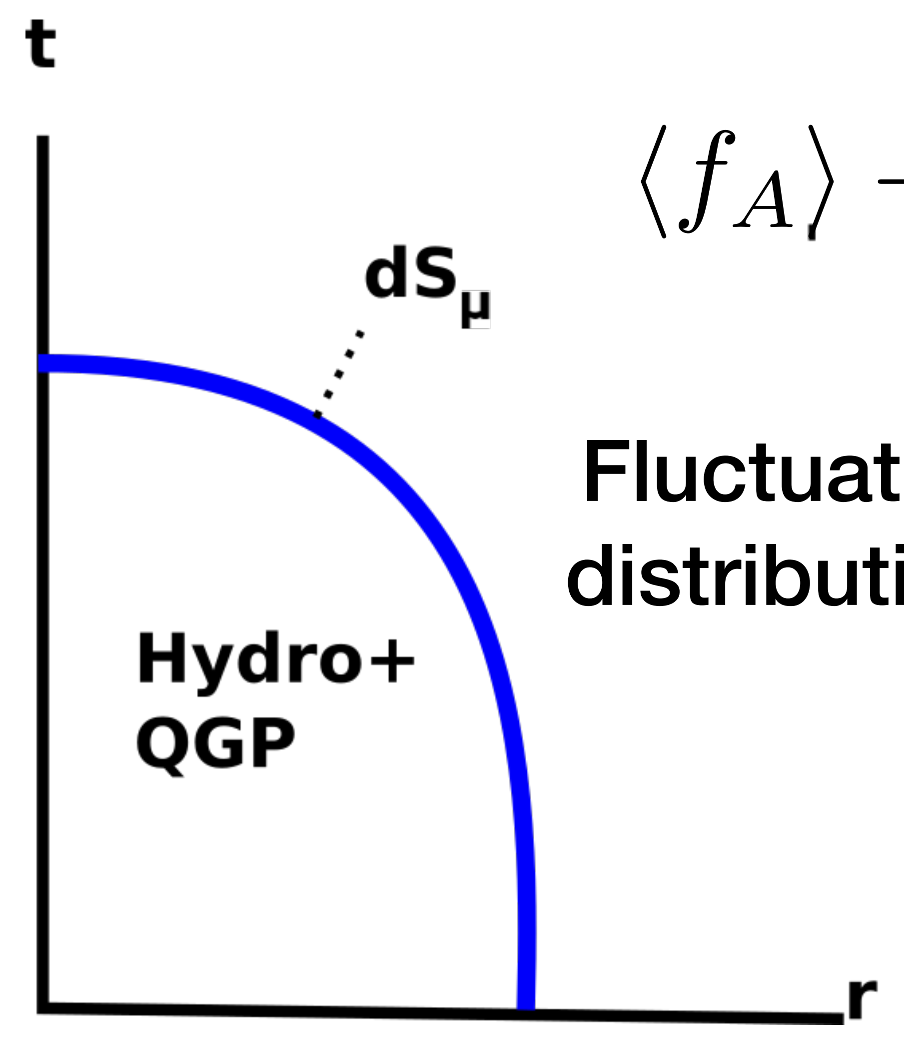


Freeze-out hypersurface : Both the hydrodynamics and kinetic description in terms of a hadron resonance gas apply

Freeze-out fits for mid-rapidity yields of hadrons



Variables at freeze-out



$\langle f_A \rangle + \delta f_A$

Fluctuating particle distribution function

Hydrodynamic mean densities

$$\{\langle \epsilon u^\mu \rangle\,,\, \langle n \rangle\} \equiv \Psi^a$$

Hydrodynamic correlations

$$\Psi^a,\, \left\langle \delta\Psi^a \delta\Psi^b \right\rangle \equiv H^{ab}, \ldots\, H^{abc\ldots}$$

Particle distribution function at freeze-out

$$\langle f_A \rangle = \bar{f}_A\,,\, \langle \delta f_A \delta f_B \rangle = G_{AB}\,,\, \langle \delta f_A \delta f_B \delta f_C \rangle = G_{ABC}\ \ldots$$

MP, Stephanov, PRL, 23

Matching conditions at freeze-out

$$\langle \epsilon\, u^\mu \rangle = \sum_A \int_{p_A} \bar{f}_A\, p_A^\mu\,, \quad \langle n \rangle = \sum_A q_A \int_{p_A} \bar{f}_A$$

$$\Psi^a = \sum_A \int_{p_A} \bar{f}_A P_A^a$$

$$P_A = \begin{bmatrix} p_A^\mu \\ q_A \end{bmatrix}$$

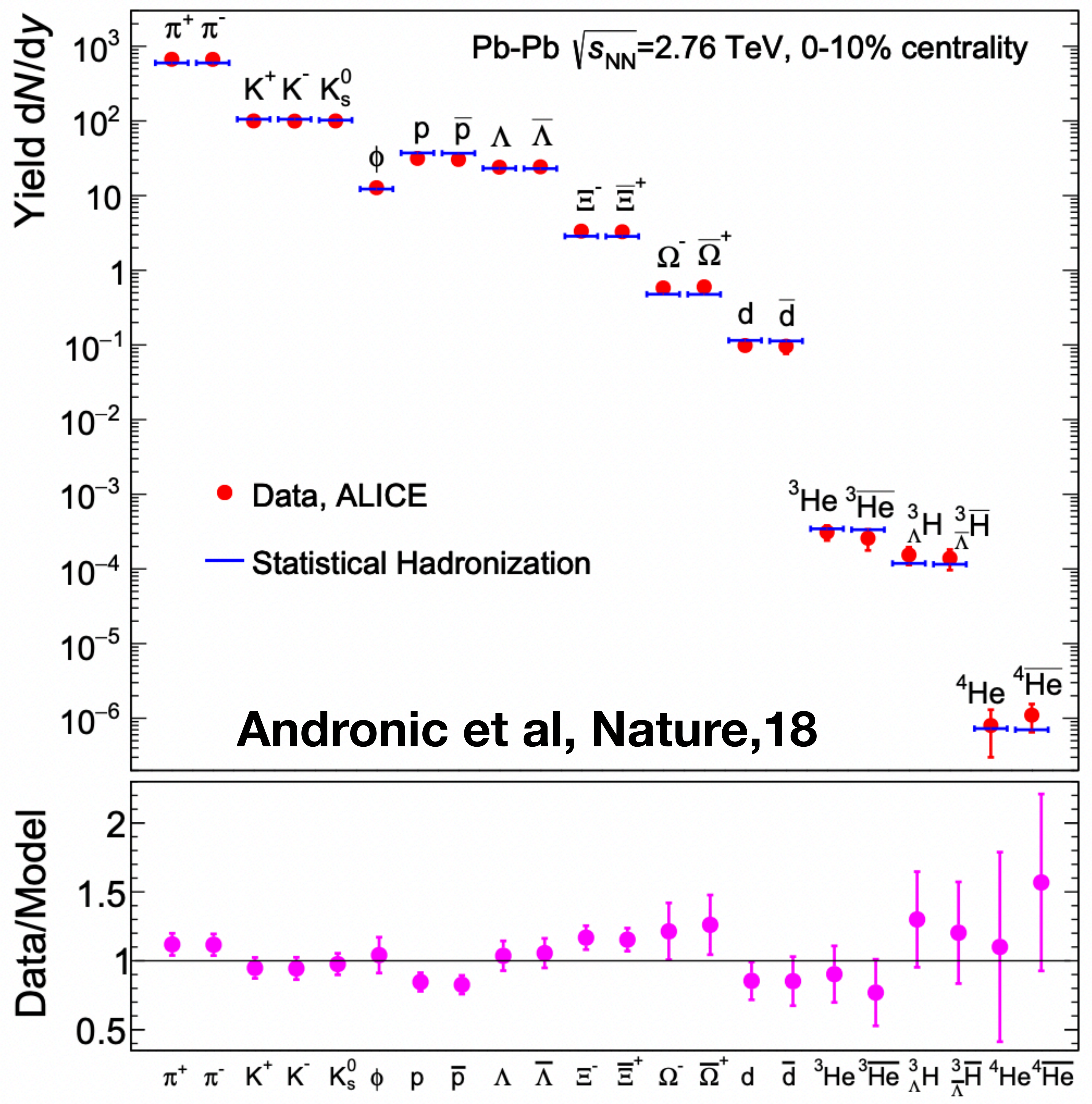
$$H^{abc\dots} = \sum_{A,B,C,\dots} \int_{p_A p_B p_C \dots} G_{ABC\dots} P_A^a P_B^b P_C^c \dots$$

- Matching conditions for averages of conserved densities
- Infinitely many sets of distribution functions that satisfy these matching conditions
- Freeze-out prescription corresponds to choosing one of these sets - **How to choose**?

MP, Stephanov, PRL, 23

Maximum entropy freeze-out without fluctuations

- Results in a thermal gas of hadrons specified by the local temperature and chemical potential. Coincides with Cooper-Frye, 74
- Recent work on extension to viscous hydrodynamics : Everett-Heinz-Chattopadhyay, 21



Maximum entropy freeze-out

Given all the information about the hydrodynamic densities on the freeze-out hypersurface, what is the ***least biased*** ensemble of free streaming particles after freeze-out that obeys the matching conditions?

It is the one which maximizes the entropy of the fluctuating particle distribution function:

$$S[\bar{f}, G_2, G_3, \ldots]$$

subject to the constraints of the matching conditions.

MP, Stephanov, PRL, 23

Equilibrium

$$S[\bar{f}]$$

Which entropy to maximize?

Generalized

$$S[P(f)]$$

$$S[P(f)] = S_0\left[\bar{f}\right] + \int_f P(f)\log\frac{P_{\text{eq}}(f)}{P(f)}$$

$$S[\bar{f}, G_2, G_3, \ldots]$$

G s are the correlation functions in the Hadron Gas description

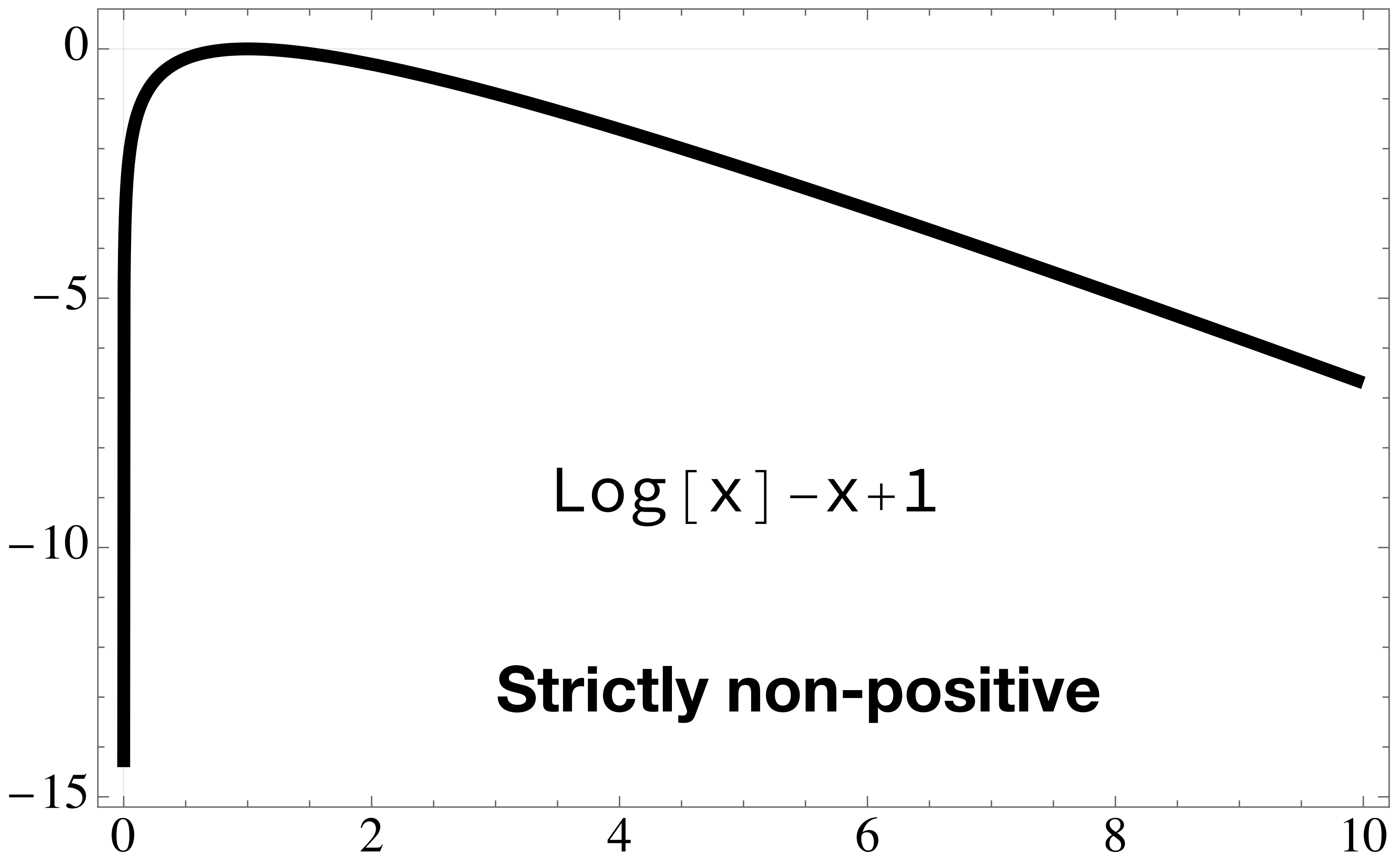
- Maximize the relative entropy when correlations are out of equilibrium
- Constraints from matching conditions

MP, Stephanov, PRL, 23

Entropy to describe out-of equilibrium two-point correlations in ideal HRG

$$S_2 = S_0 + \frac{1}{2}\mathrm{Tr}\left[\log G\bar{G}^{-1} - G\bar{G}^{-1} + 1\right] \,,$$

Equilibrium



Similar to 2-PI action

Berges, 04, Stephanov, Yin, 17…

2-PI entropy

MP, Stephanov, PRL, 23

Two-point correlation function of proton multiplicities in phase space,

$$\Delta G_{pp}(k,k^{'}) = \#_{pp}^{\epsilon\epsilon}(k,k^{'})\Delta H_{\epsilon\epsilon} + \#_{pp}^{n\epsilon}(k,k^{'})\Delta H_{n\epsilon} + \#_{pp}^{nn}(k,k^{'})\Delta H_{nn} + \dots$$

Contributions from baryon and energy density correlations

Δ denotes deviation of the hydrodynamic correlation function from its value in Ideal Hadron Resonance Gas

MP, Stephanov, PRL, 23

Organizational Scheme for Non-Gaussian Correlations

$$G_{nnn}(p,p^{'},p^{''}) - \bar{G}_{nnn}(p,p^{'},p^{''}) = \sum_{a,b,c=\epsilon,n} \#^{abc}_{pp'p''}\,\Delta H_{abc} + \sum_{a,b=\epsilon,n} \#^{ab}_{pp'p''}\,\Delta H_{ab} + \ldots$$

IRC

IRC

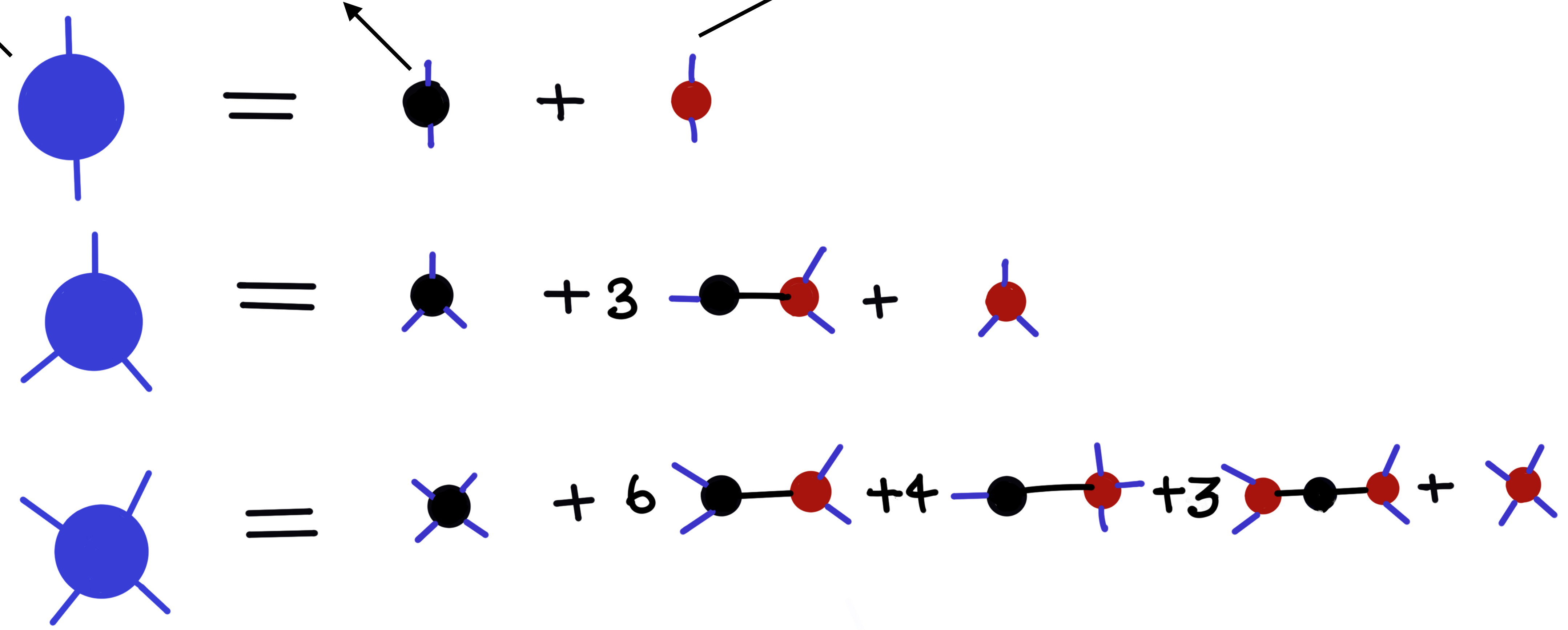
Baseline distribution/EoS

$$\hat{\Delta} G_{ABC} = \mathcal{F}^{abc}_{ABC}(\bar{H},\bar{G})\,\hat{\Delta} H_{abc}$$

Total

IRC

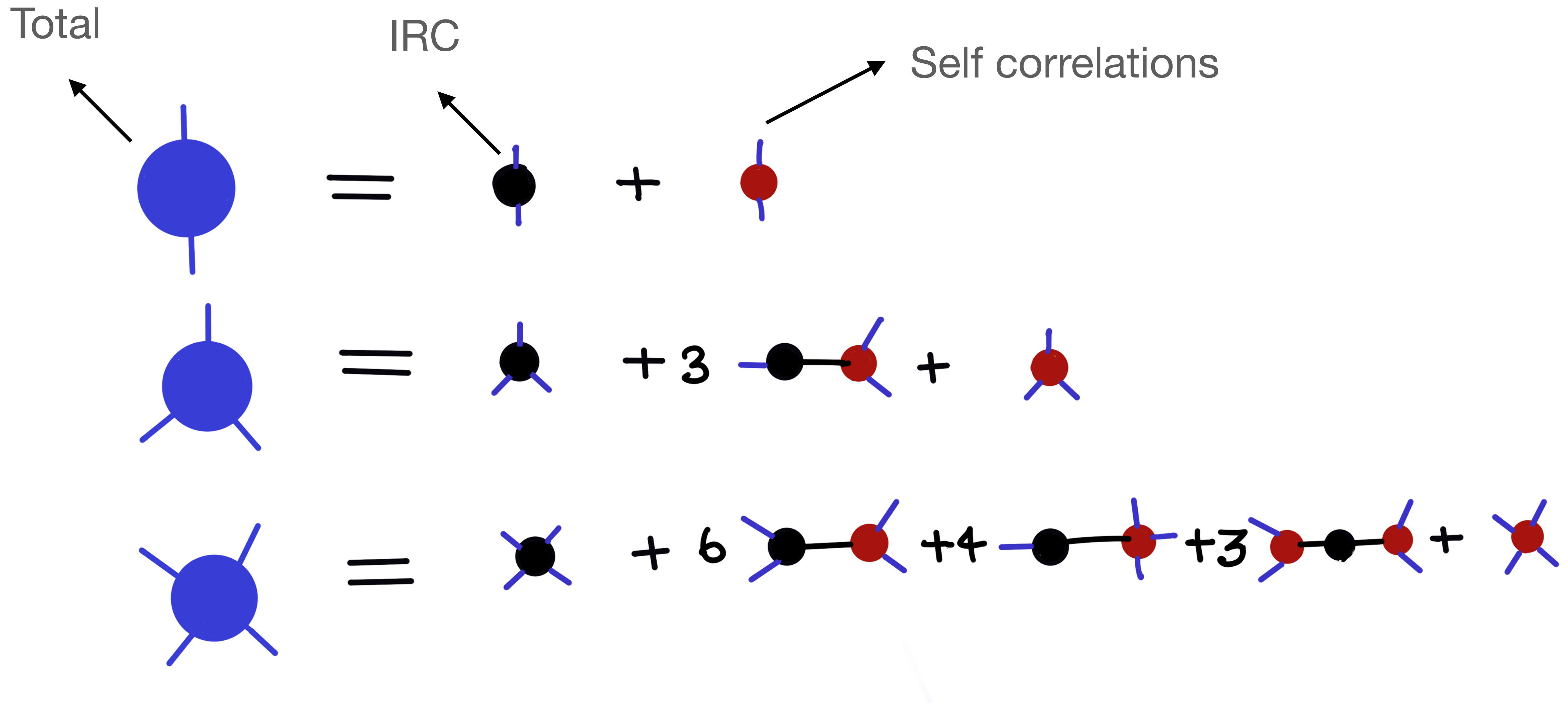
Self correlations in HRG



MP, Stephanov, PRL, 23

Irreducible relative cumulants

For classical gas, irreducible relative cumulants (IRCs) reduce to so called “factorial cumulants”.



- For gases obeying different statistics, IRCs quantify the non-trivial correlations
- Non-trivial correlations relative to any specified baseline distribution

MP, Stephanov, PRL, 23

Maximum entropy freeze-out procedure

$$\hat{\Delta} \left\langle \delta N^k_{A_1 \ldots A_k} \right\rangle = X^{b_1}_{A_1} X^{b_2}_{A_2} \ldots X^{b_k}_{A_k} \hat{\Delta} H_{b_1 \ldots b_k}$$

- Integrating over phase space bins, we get factorial cumulants of particle multiplicities
- Factorial cumulants remove various sources of trivial correlations
- They contain crucial information about criticality in the Equation of State / Hydro

MP, Stephanov, PRL, 23

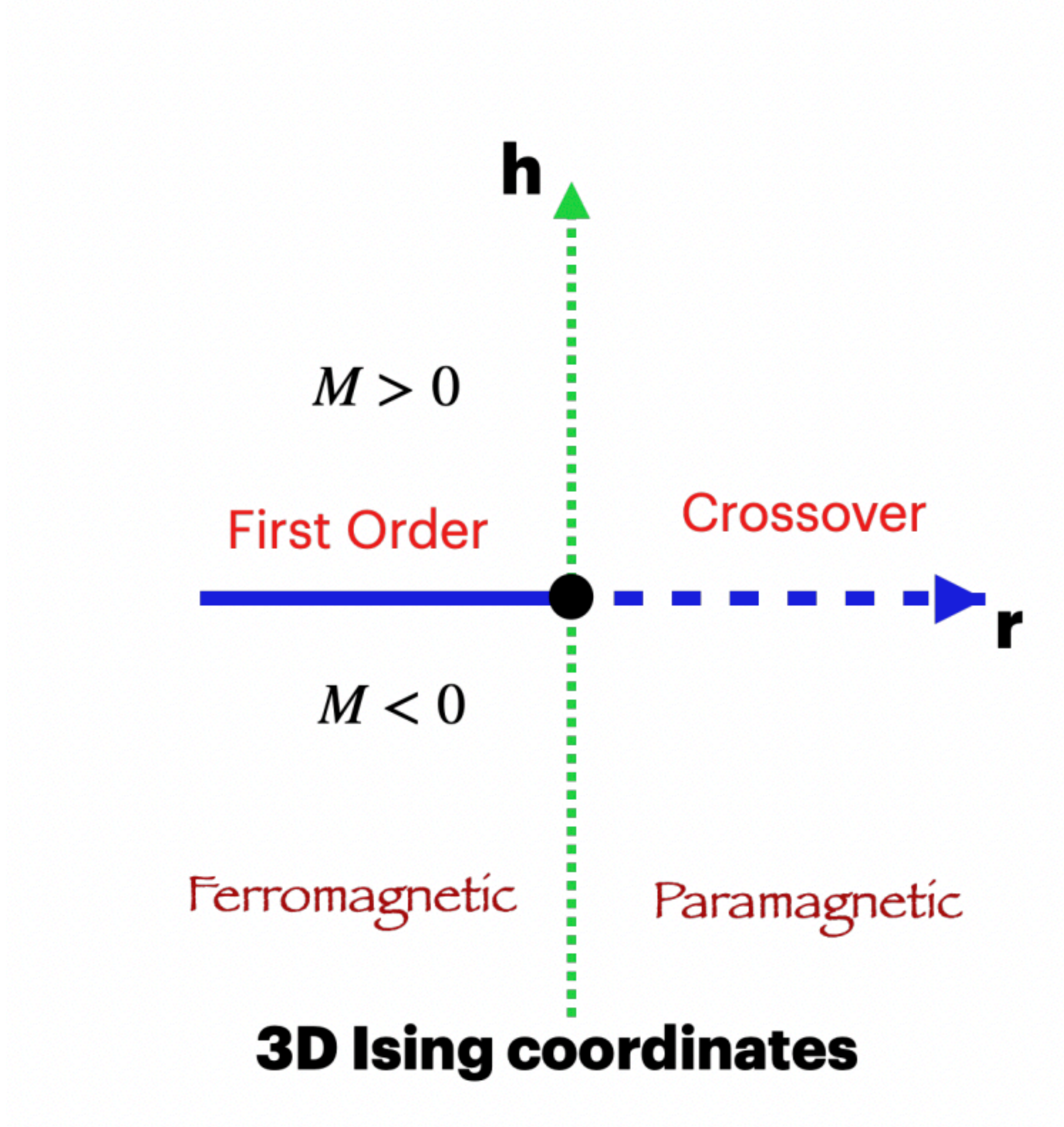
But, the QCD EoS near the critical point is not known from first-principles?!

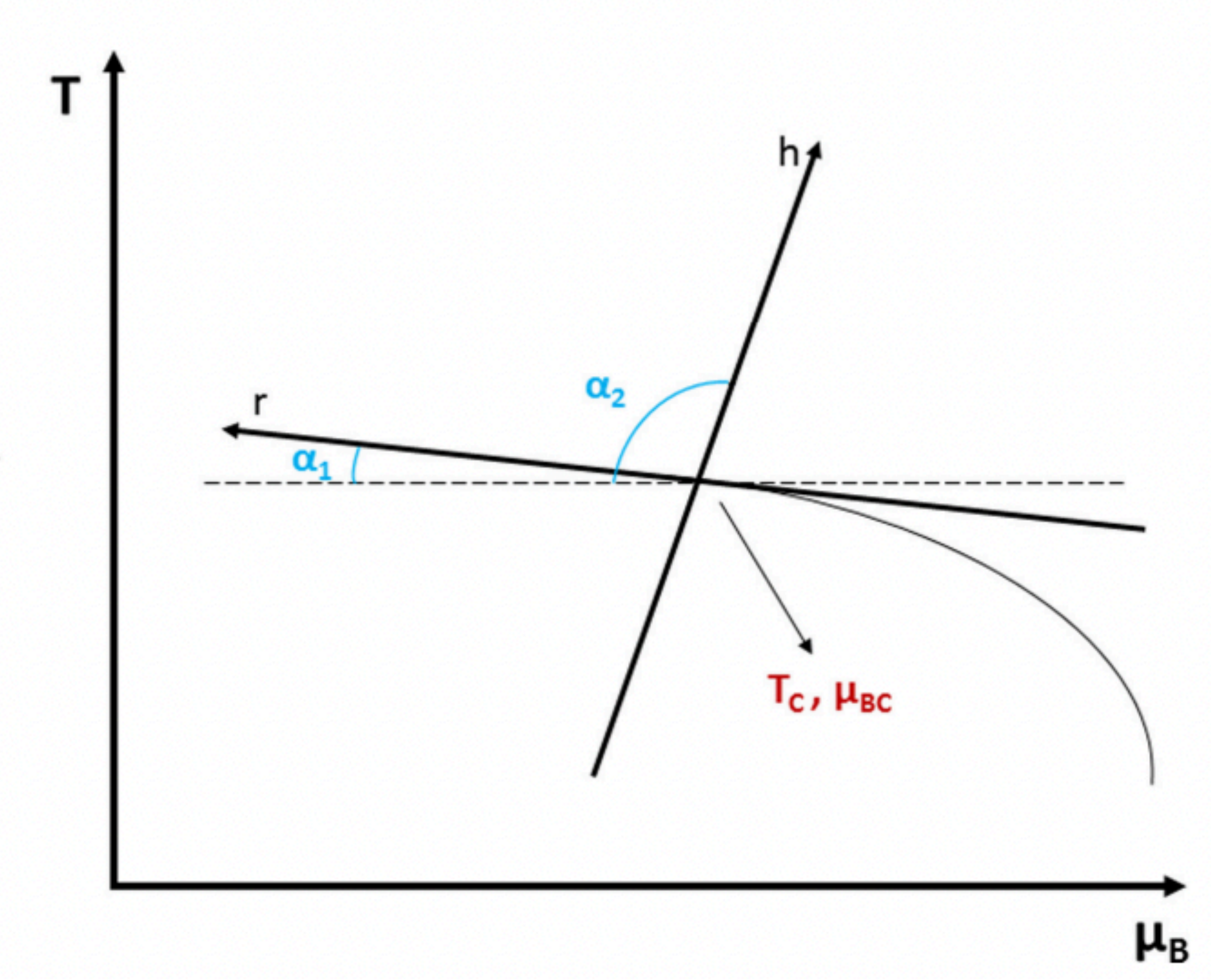
Equation of States with a QCD critical point

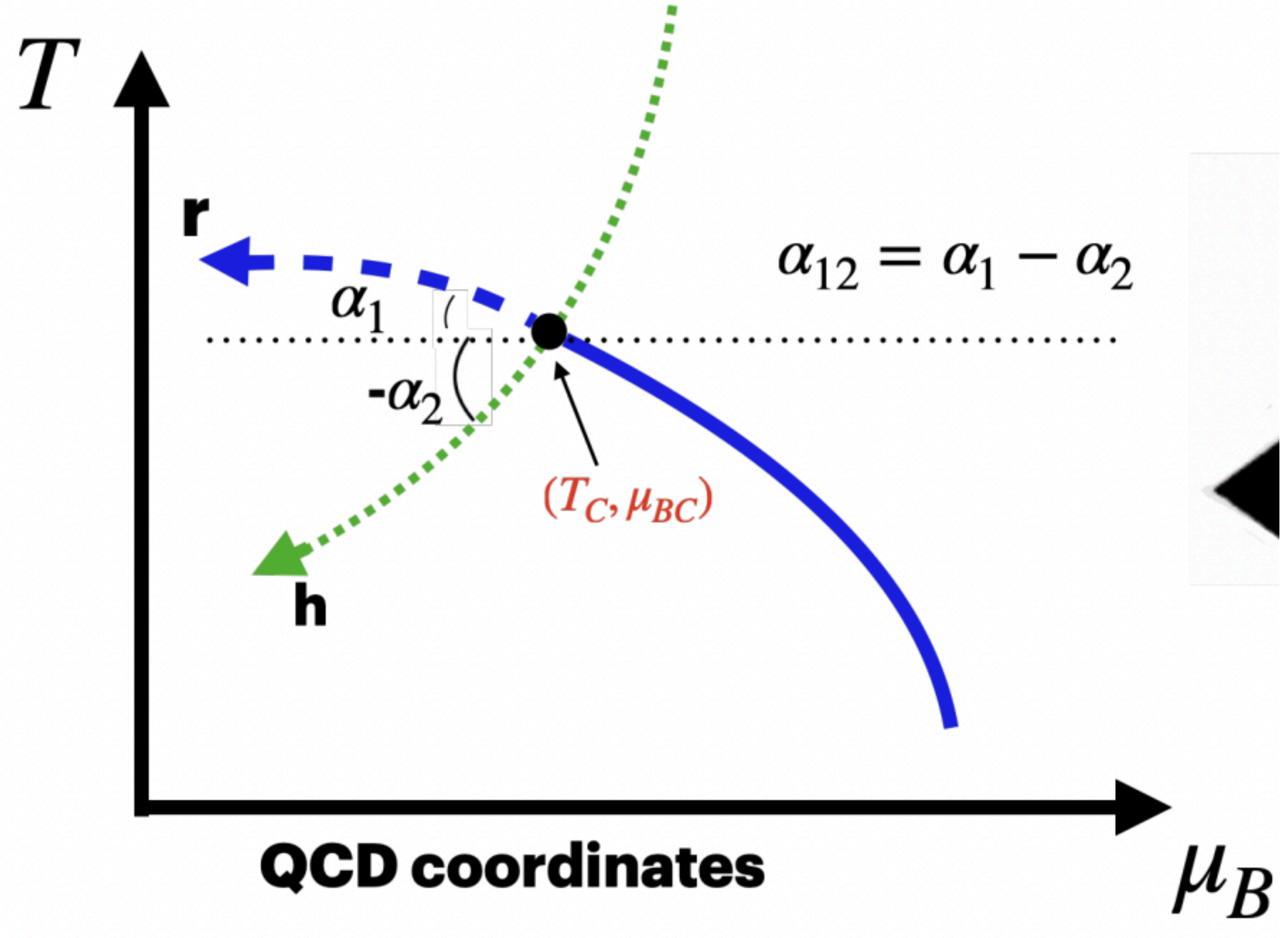
- Must agree with the Taylor Expanded EoS from lattice
- Compatible with other limits : PQCD, HRG
- Critical point in the 3D Ising universality class

Examples of recently developed EoSs that have a CP in the Ising universality class but differ in their implementation:
Parotto et al, 19, Karthein et al.,21, Grefa et al., 21,Kapusta&Welle,22, Kahangirwe et al.,24

QCD EoS near the Critical Point







Parotto et al., 18, Karthein et al., 21

Kahangirwe et al., 24

Summation scheme by WB collaboration

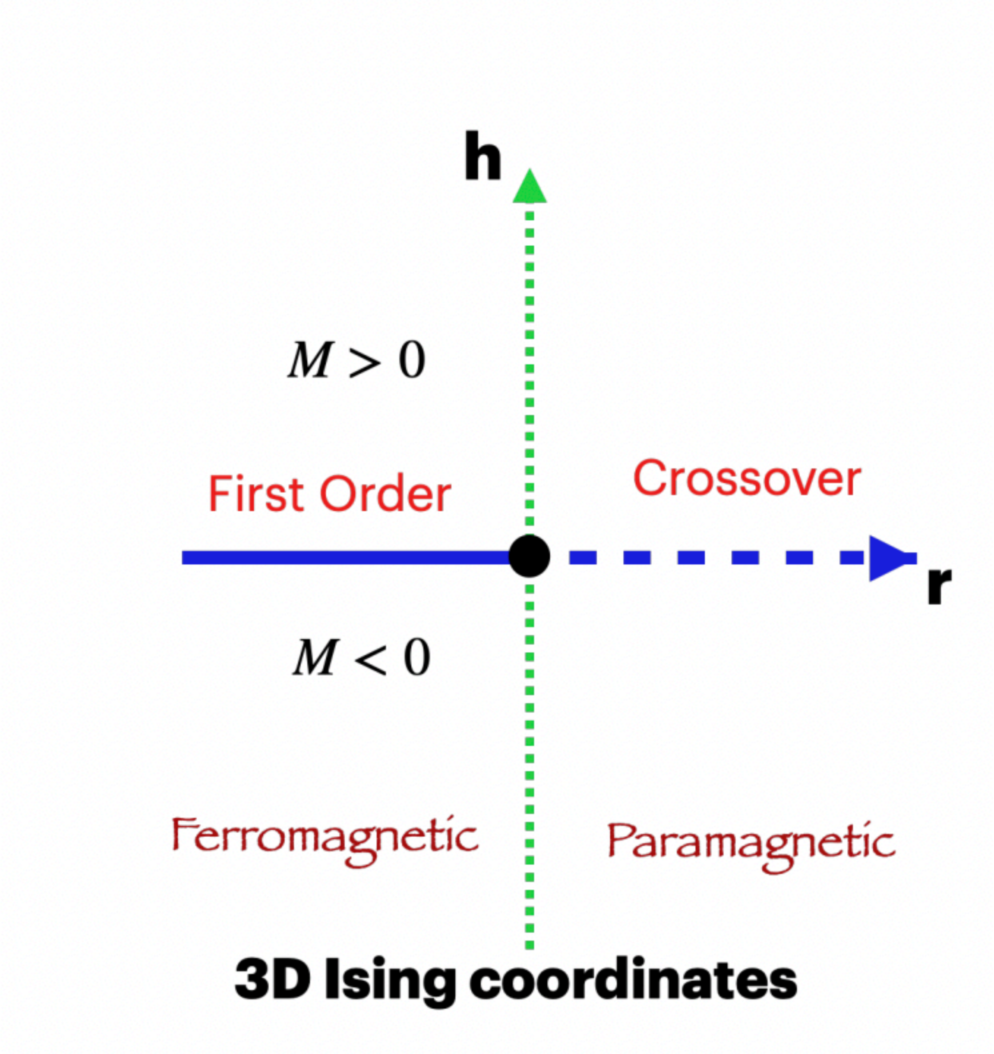
Borsanyi et al,21

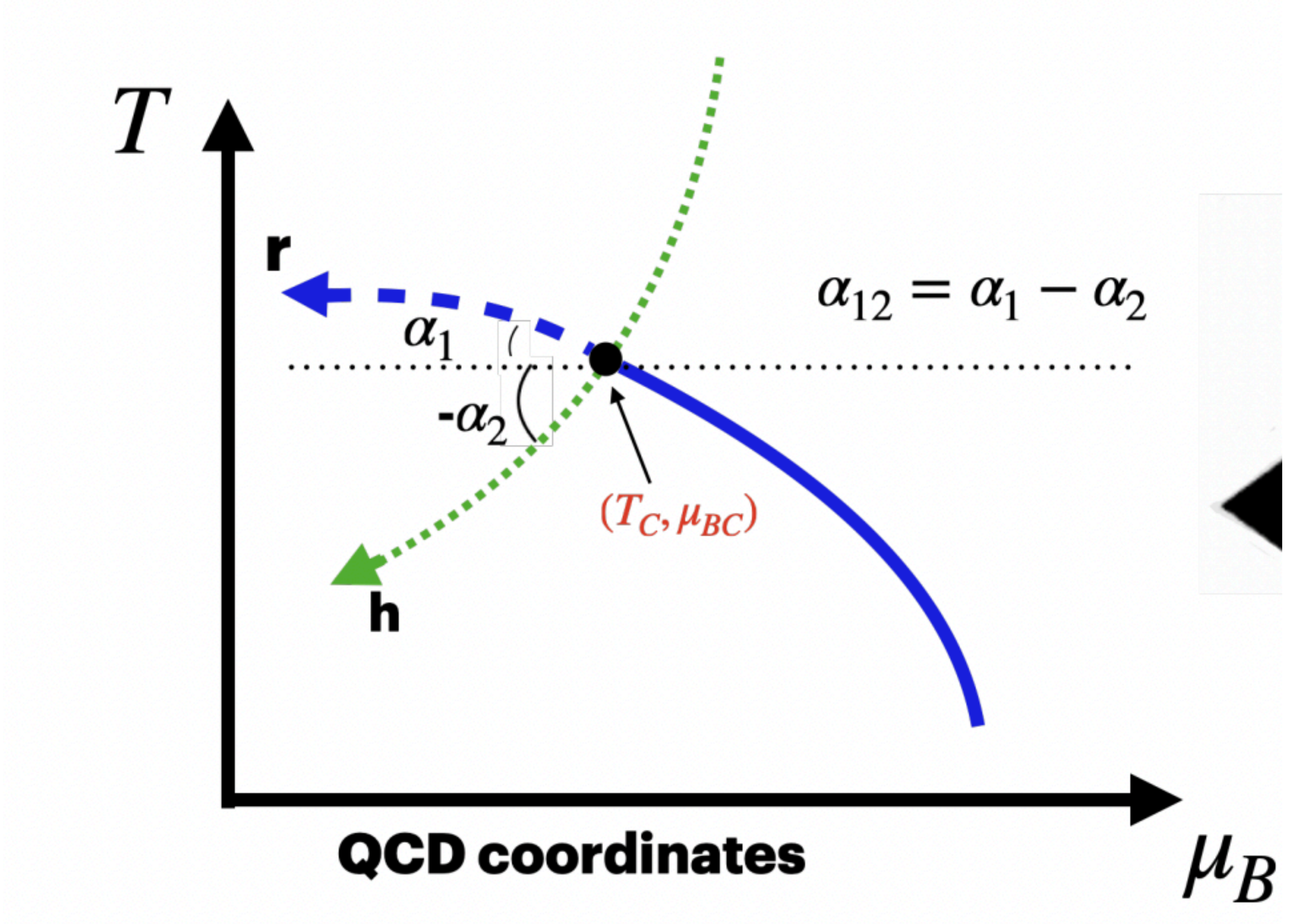
Non-universal map from QCD to Ising variables

$$P_{\mathrm{QCD}}(\mu, T) = P_{\mathrm{BG}}(\mu, T) + A\,G(r(\mu, T), h(\mu, T))$$

A general class of candidate EoSs

$$P_{\rm QCD}(\mu,T) \;=\; P_{\rm BG}(\mu,T) \,+\, A\,G(r(\mu,T),h(\mu,T))$$





23

Kahangirwe et al., 24

Independent & non-universal parameters

$$\mu_c\,,\;\alpha_{12}\,,\;\rho\,,\;w$$

Weakly constrained in the chiral limit

MP, Stephanov, 19

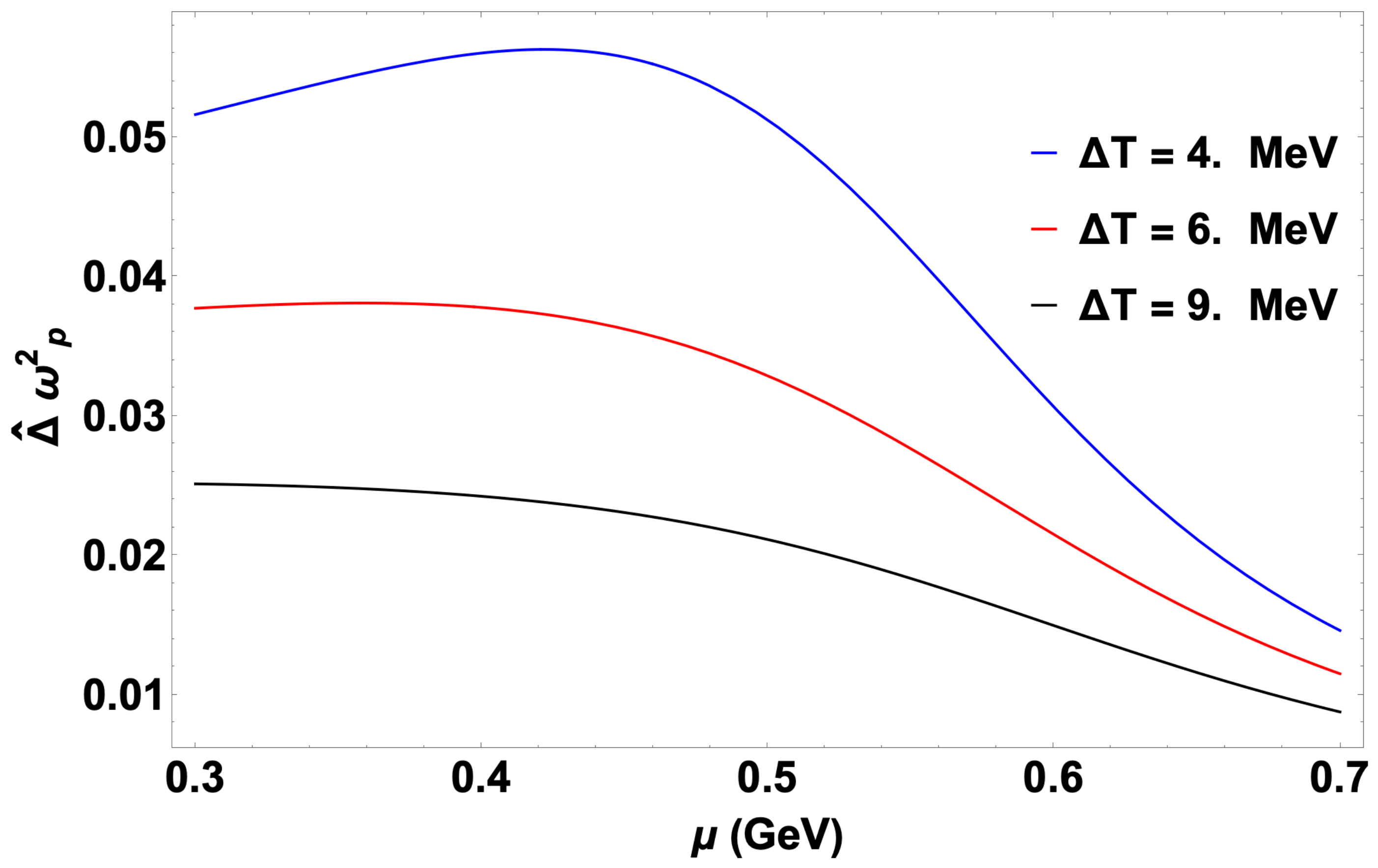
Range of Validity improved

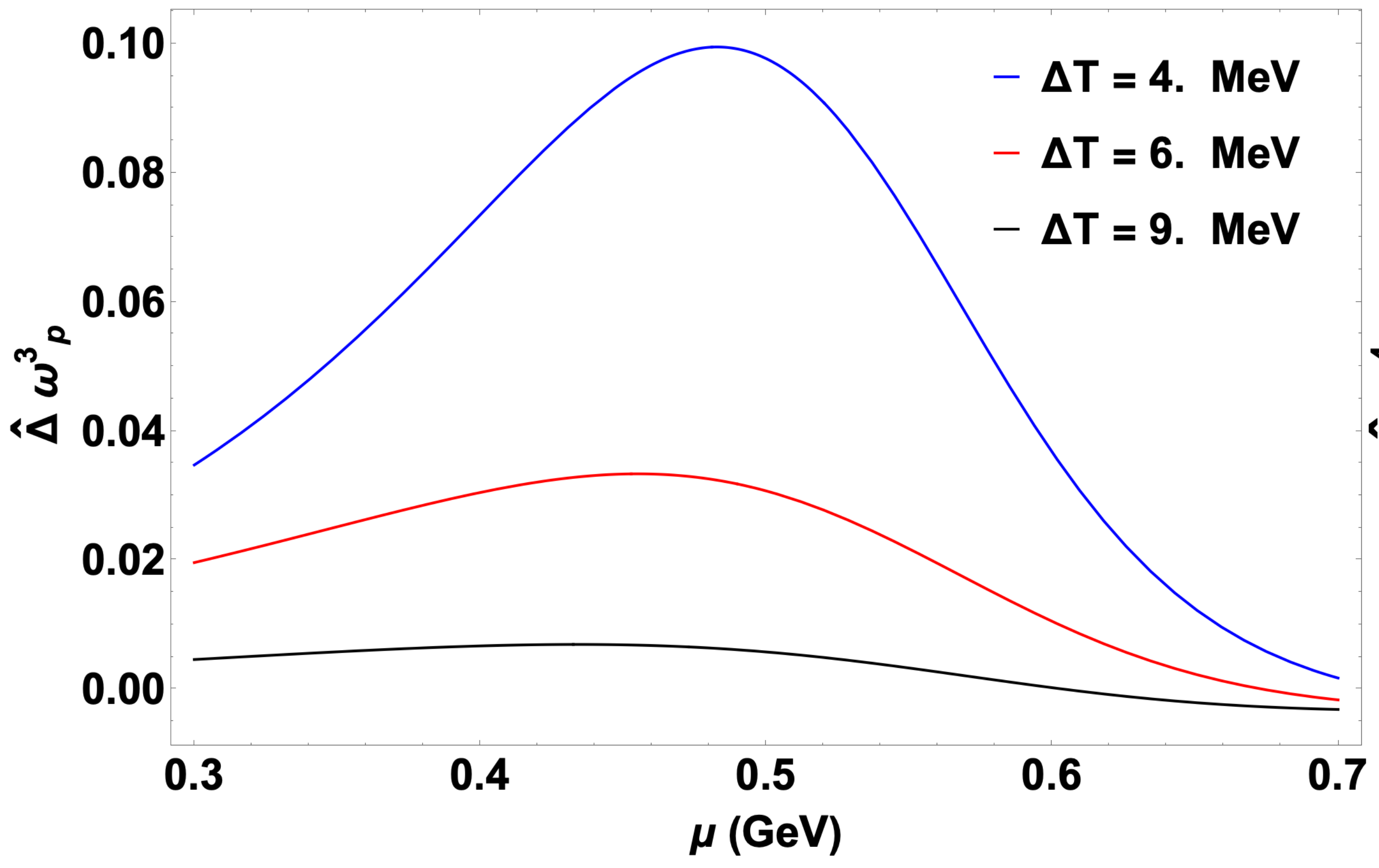
$$0\leq\mu_B\leq700\,\text{MeV}\,,\;25\,\text{MeV}\leq T\leq800\,\text{MeV}$$

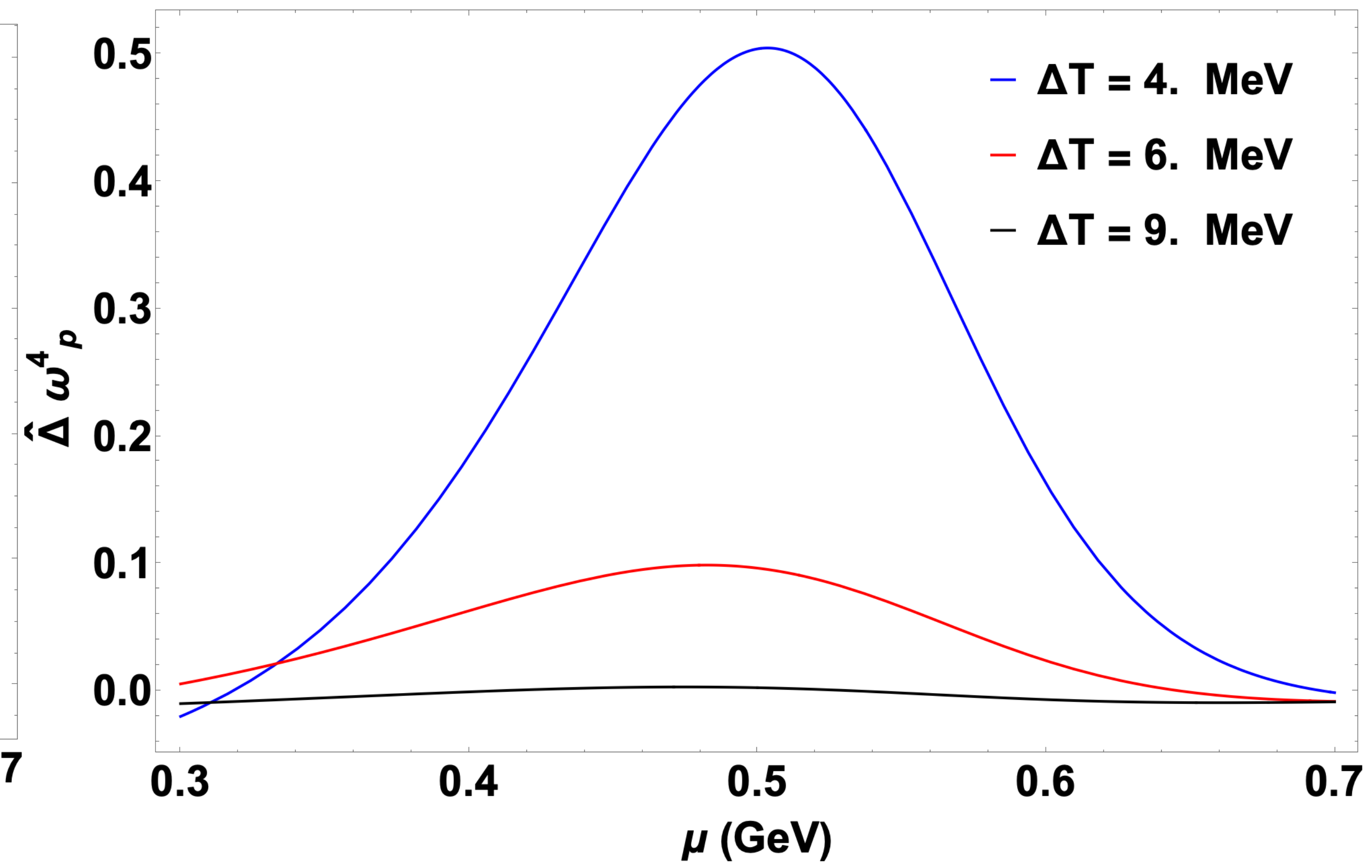
The new construction is causal and stable for a larger range of ρ and w

Equilibrium estimates for critical contribution to factorial cumulants of proton multiplicities

Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)





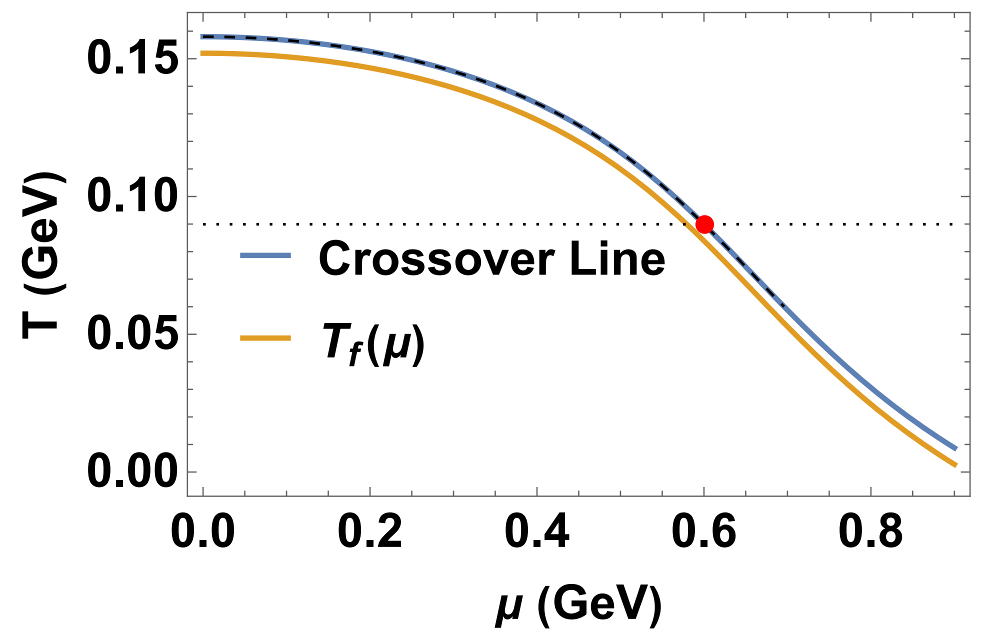


$$T_f(\mu) = T_{\rm co}(\mu) - \Delta T$$

$$\mu_c = 600\,\text{MeV}\,,\ \alpha_2 = 0^\circ\,,\ \rho = 1\,,\ w = 20$$

Example **choice of Mapping Parameters**

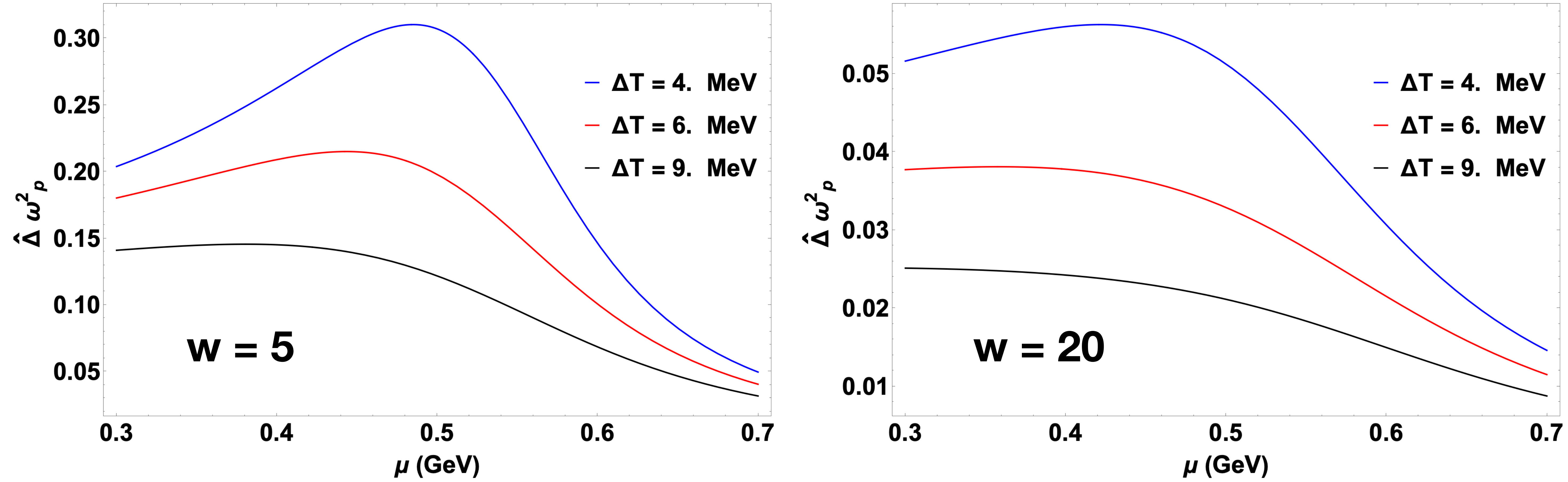
$$\hat{\Delta}\omega_p^k = \frac{\hat{\Delta}\,\langle \delta N_p^k \rangle}{\langle N_p \rangle}$$



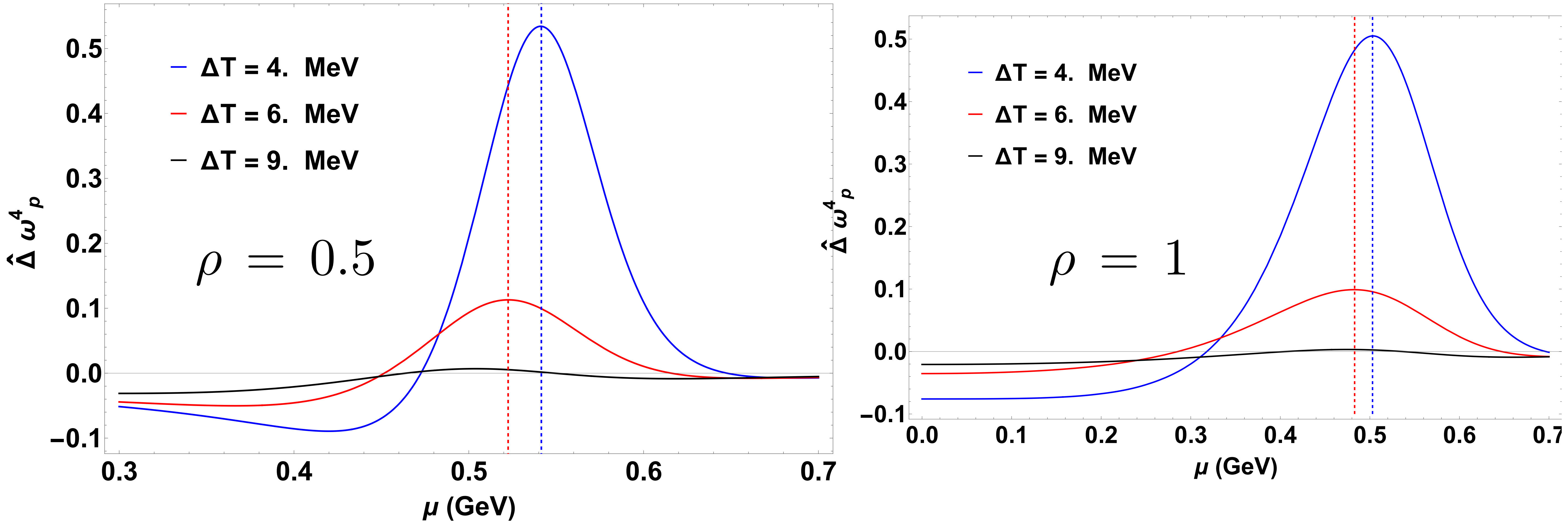
Freeze-out curve

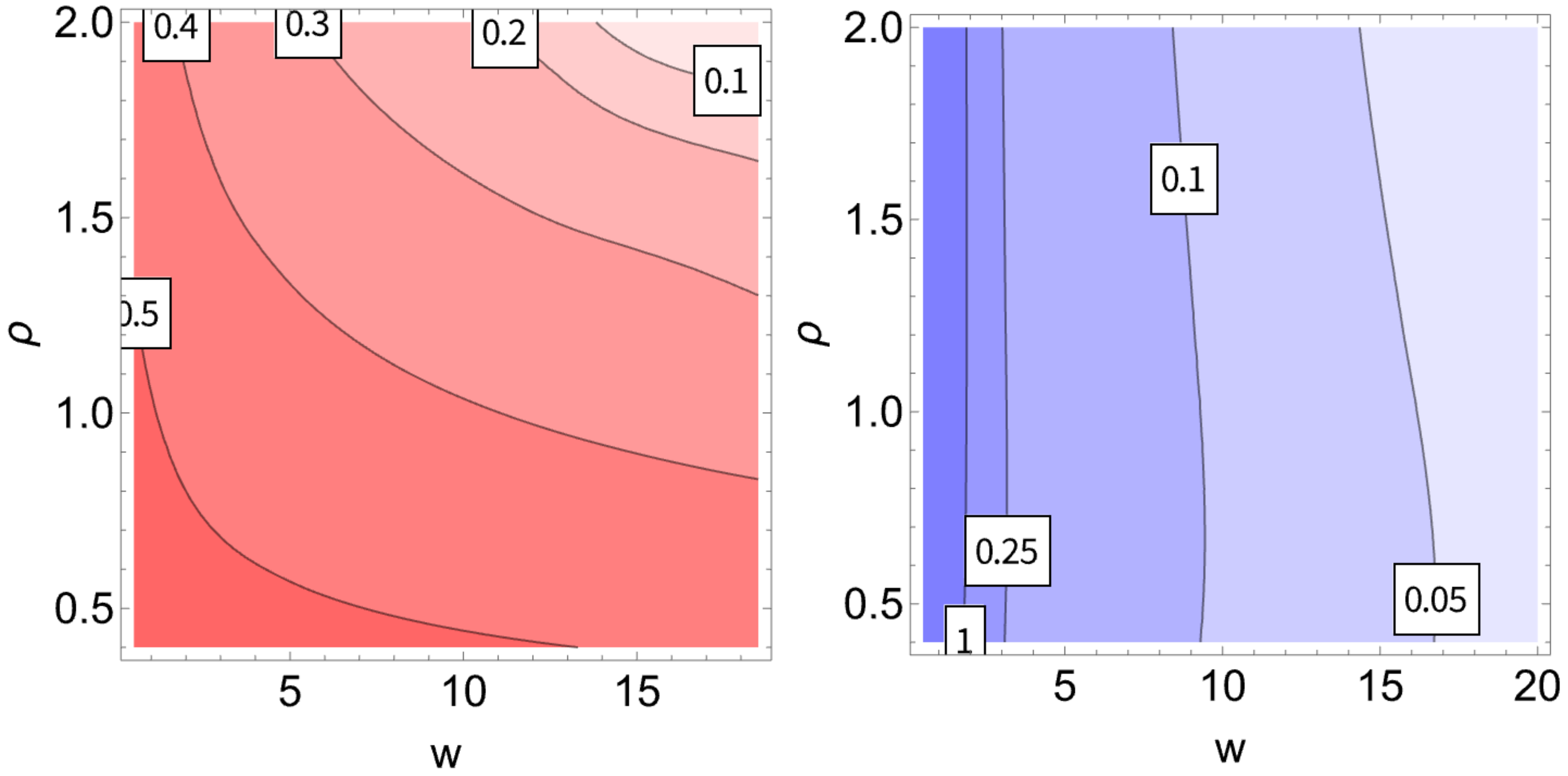
Karthein, MP, Rajagopal, Stephanov, Yin (in preparation)

Role of scale factors



- Peak value of cumulants along the freeze-out curve is fixed by w
- Location of the maxima on the freeze-out curve fixed by ρw





Fluctuation dynamics

Stochastic Approach

- Stochastic differential equations for hydro fields
- Implementation using Metropolis algorithm developed and applied to various models, including Model H

Schaefer and Skokov, 22, Chattopadhyay et al, 23, Florio et al, 21, Basar et al, 24,Chattopadhyay et al, 24…

Deterministic Approach

- Deterministic evolution for hydro correlation functions
- Semi-realistic estimates for two-point correlation functions computed and connected to observables

Teaney, Akamatsu, Mazeliauskas, 16 along with Yan,Yin , 19, Stephanov, Yin,17, An, Basar, Stephanov,Yee, 19,20,22,24…

An, Basar, Stephanov,Yee, 22,24

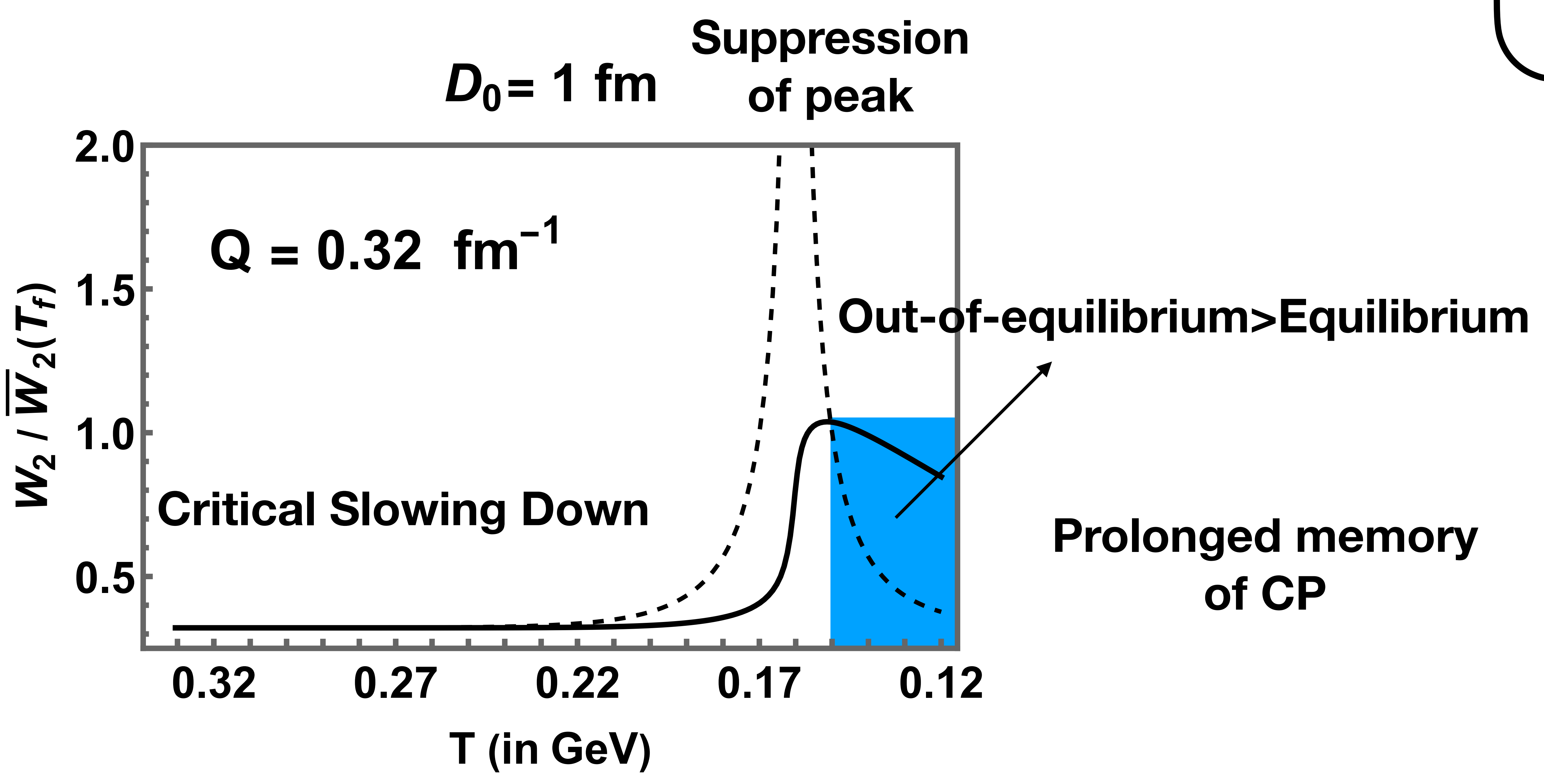
Rajagopal,Ridgway,Weller,Yin,19,Du,Heinz,Rajagopal, Yin,20
MP, Rajagopal, Stephanov, Yin, 22,Mukherjee, Venugpalan, Yin 15

Refer to Johannes Roth's talk

Out-of-equilibrium effects of fluctuations near the CP

$$\langle \delta\hat{s}(x_+)\delta\hat{s}(x_-)\rangle = \int e^{i\mathbf{Q}\cdot\mathbf{\Delta x}}\, W_2(\mathbf{Q})\,,\ \mathbf{\Delta x} = x_+ - x_-$$

Persistence of critical imprints in the fluctuation observables until freeze-out



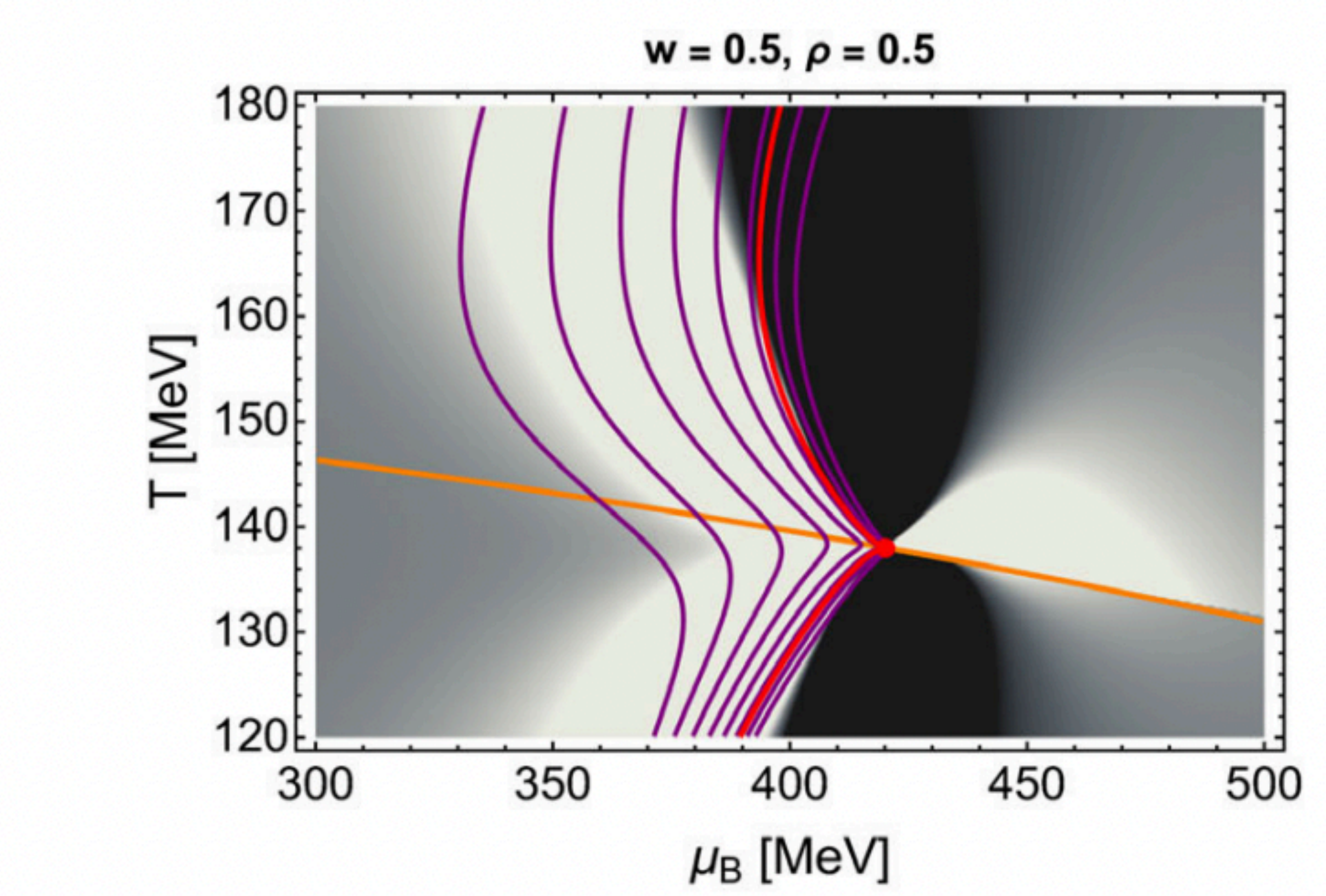
Rajagopal,Ridgway,Weller,Yin,19
Du,Heinz,Rajagopal, Yin,20
MP, Rajagopal, Stephanov, Yin, 22
Mukherjee, Venugopalan, Yin 15

CP fluctuations in MD simulations : V. Kuznietsov et al

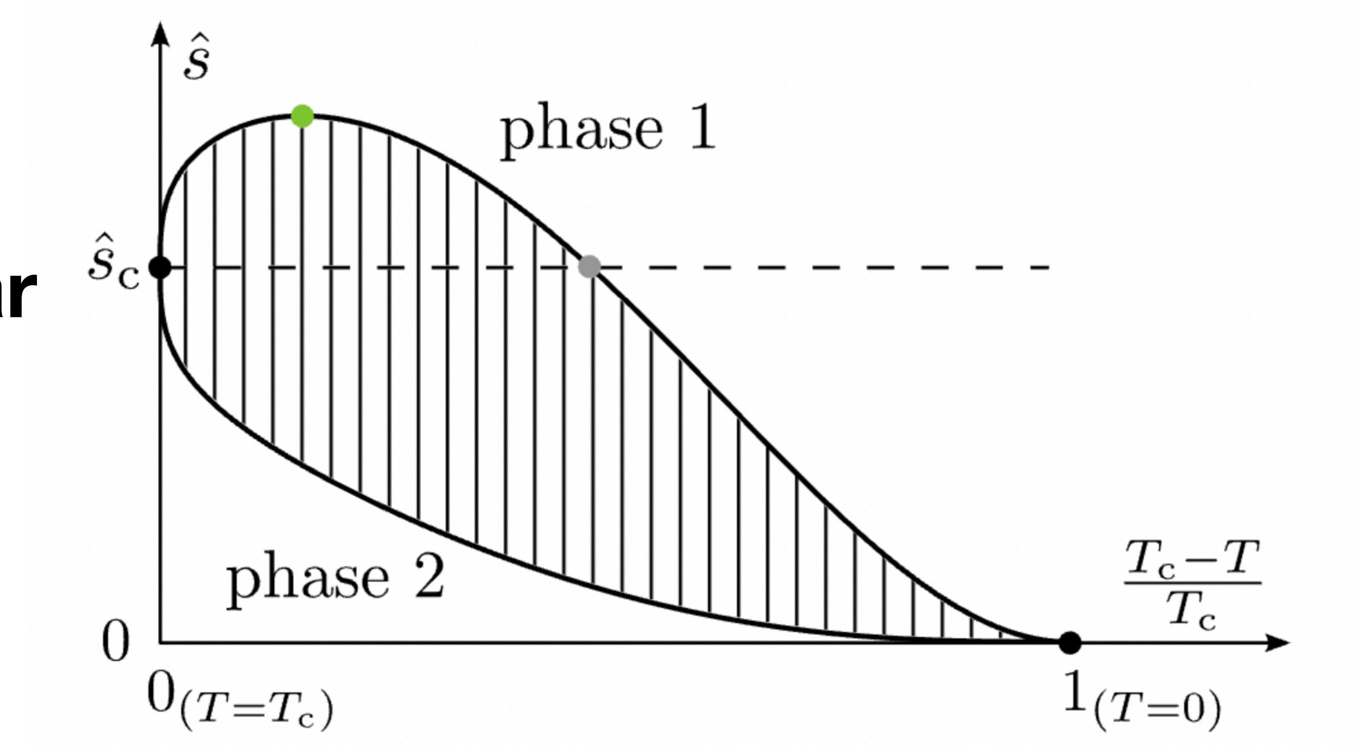
Deformation of hydrodynamic trajectories near CP

MP, Sogabe, Stephanov, Yee,24

Deformations can be broadly classified based on the value of the mapping parameter α_2



Universal scaling near CP

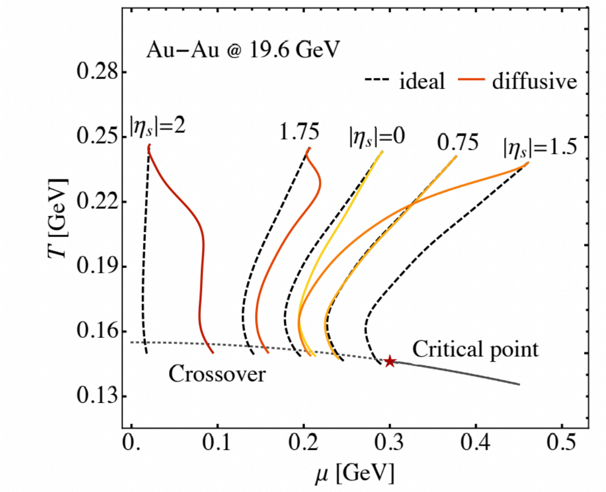


Third Law

- Specific entropy is non-monotonic along one of the branches on the first-order curve

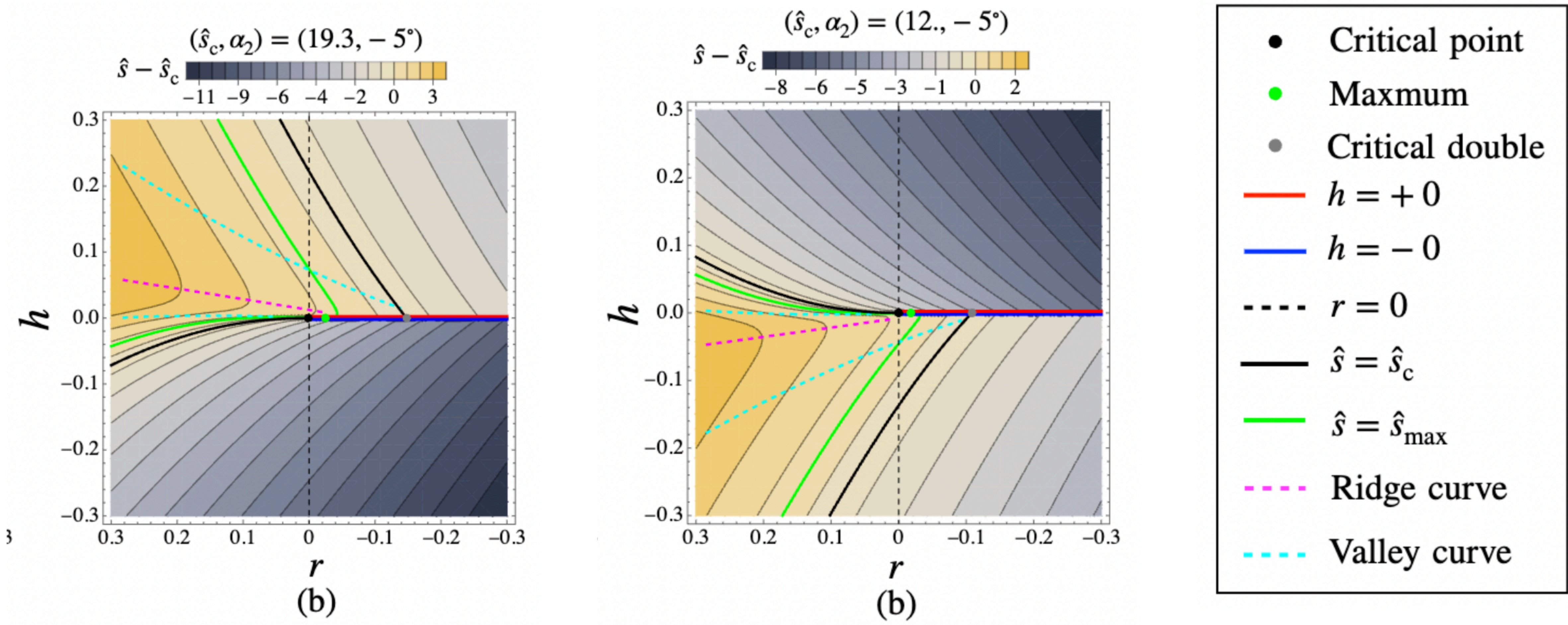
Critical lensing~Dore et al,22, Nonaka&Asakawa, 05

- Consequence of universal ridge-like structure of the isentropes near CP

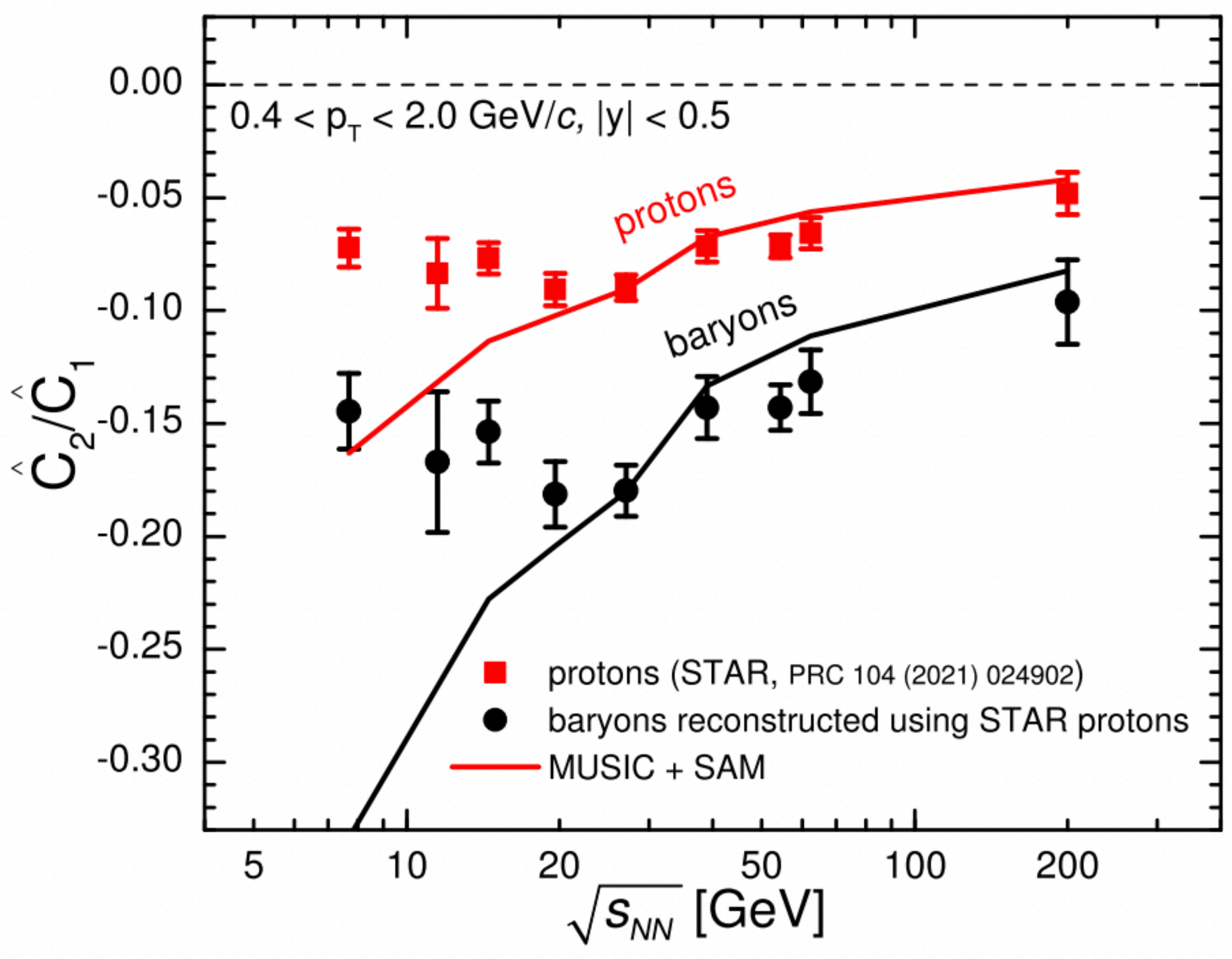


Smearing effect - Du et al, 22

- Phenomenological implications



Summarizing & Looking forward



Vovchenko,Koch,Shen,22

EoS ⟷ Particle multiplicity distributions

$$\chi_j(\mu, T; \mu_c, \alpha_{12}, w, \rho) \xrightarrow{\mathrm{ME}} C_A^k(\mu_F(T_F); \mu_c, \alpha_{12}, w, \rho, \Gamma)$$

Dynamics

A ***family of candidate EoSs with a CP*** that match with the lattice have been developed

Quantitative estimates for ***out-of equilibrium*** corrections to higher order cumulants - needed

Bayesian Analysis of experimental data pertaining to ***multiple observables*** with the theoretical framework may possibly help us learn about QCD EoS near CP, if it exists in the regime scanned by HICs

Thank you!

Back up

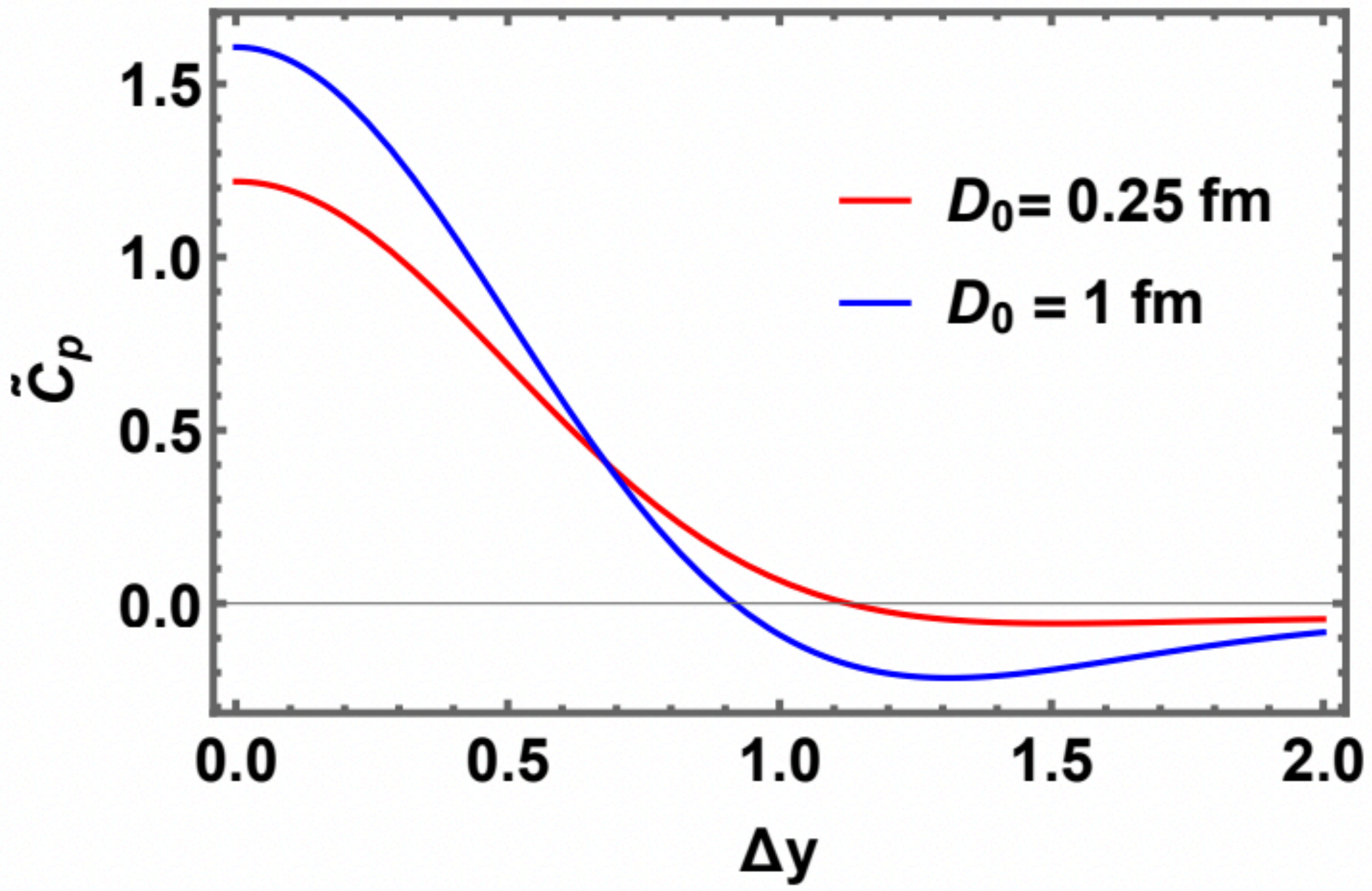


FIG. 6: Normalized proton multiplicity correlator $\tilde{C}(\Delta y)$ for protons from Eq. (60) as a function of the rapidity gap Δy in the Bjorken scenario for two choices of the diffusion parameter D_0.

MP, Rajagopal, Setphanov, Yin, 22