Confined but chirally and chiral spin symmetric hot matter

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Introduction

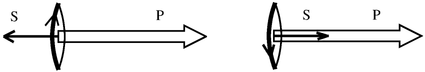
Chiral symmetry

- Free Dirac Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$$

- Left and right fermions

$$\psi_R = \frac{1}{2}(1+\gamma^5)\psi \quad \psi_L = \frac{1}{2}(1-\gamma^5)\psi \quad \psi = \psi_R + \psi_L$$



- Chirally symmetric and nonsymmetric bilinear forms

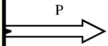
$$\bar{\psi}\gamma^\mu\psi = \bar{\psi}_R\gamma^\mu\psi_R + \bar{\psi}_L\gamma^\mu\psi_L \qquad \bar{\psi}\psi = \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$$

- Strict chiral limit ($m = 0$)
  - ψ_L and ψ_R are decoupled and transform independently
  - Axial current $j^5_\mu = \bar{\psi}\gamma^5\gamma_\mu\psi$ is conserved $\Longrightarrow$ $[Q_5 H] = 0$
  - (Naive) conclusion: Hadrons of opposite parity are degenerate

Chiral symmetry

- Free Dirac Lagran
- Left and right fer

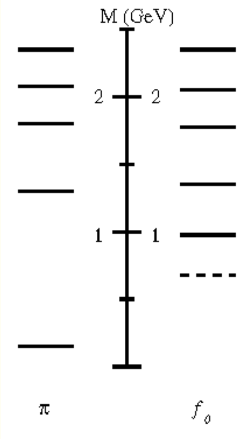
$$\psi_R = \frac{1}{2}(\qquad \psi = \psi_R + \psi_L$$



- Chirally symmetri ... forms

$$\bar{\psi}\gamma^\mu\psi = \bar{\psi} \qquad \bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R$$

- Strict chiral limit



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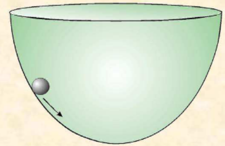
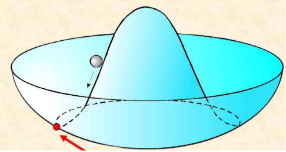
Spontaneous breaking of symmetry





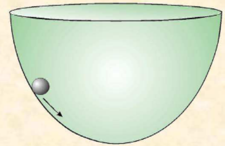
- Symmetric vacuum is stable
- Massive excitations over vacuum

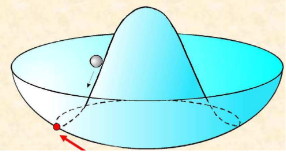
$$V(\phi) = \underbrace{V(0)}_{\text{Energy shift}} + \frac{1}{2}\underbrace{V''(0)}_{m^2>0}\phi^2 + \dots$$

- No tachyon in the spectrum
- Spectrum of excitations inherits symmetry of the vacuum

- Symmetric vacuum is unstable
- Tachyon in the spectrum built over symmetric vacuum: $V''(0) = m^2 < 0$
- True vacuum is selected among many possibilities $\Longrightarrow$ Symmetry is spontaneously broken
- Physical vacuum and excitations over it are not symmetric
- Massless mode — Goldstone boson

Spontaneous breaking of symmetry





At temperatures above some critical temperature $T_{\rm ch}$ chiral symmetry is restored in the QCD vacuum:

What happens to hadrons?

- Symm
- Mass

$V(\phi)$ ... 2

Energy shift $m^2 > 0$

- No tachyon in the spectrum
- Spectrum of excitations inherits symmetry of the vacuum

...ver

...$^2 < 0$

- True vacuum is selected among many possibilities $\Longrightarrow$ Symmetry is spontaneously broken
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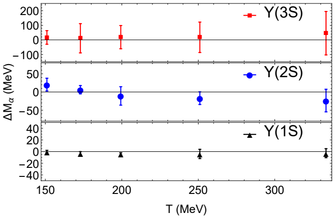
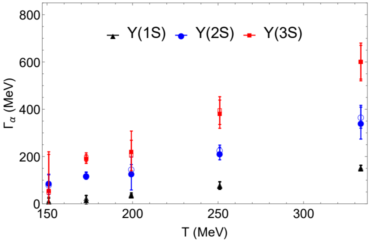
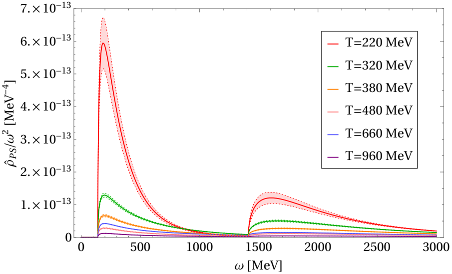
Hadrons @ $T > T_{\rm ch}$ — hints from lattice





(Larsen,Meinel,Mukherjee,Petreczky'2019)



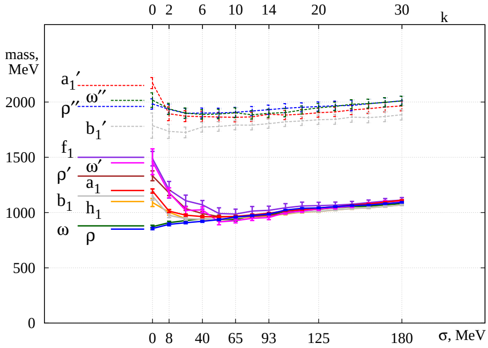
(Lowdon,Philipsen'2022)

Hadrons on "truncated" lattice configurations

$$\langle\bar{\psi}\psi\rangle = -\lim_{m\to 0}\int_0^\infty d\lambda\rho(\lambda,m)\frac{2m}{m^2+\lambda^2} = -\pi\rho(0)$$

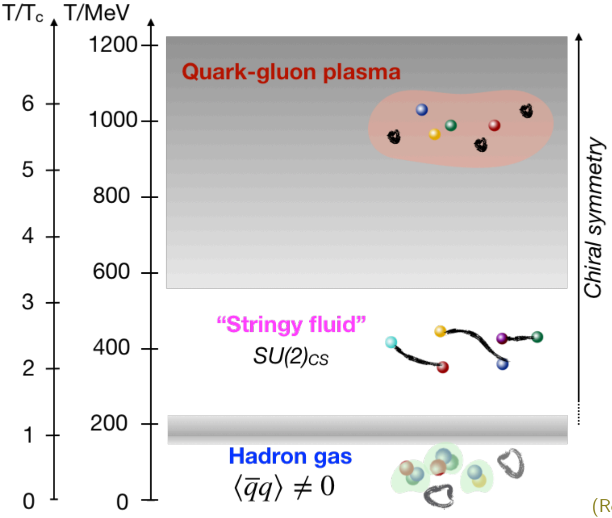
(Banks,Casher'1980)

$$\tilde{S} = S - \sum_{n=1}^{k}\frac{1}{\lambda_n}|\lambda_n\rangle\langle\lambda_n| \qquad i\not{D}\psi_n(x) = \lambda_n\psi_n(x)$$



(Denissenya,Glozman,Lang'2015)

Regimes of QCD at finite temperatures (?)



(Rohrhofer et al'2019)

(Cohen & Glozman'2024 for large-N_c picture)

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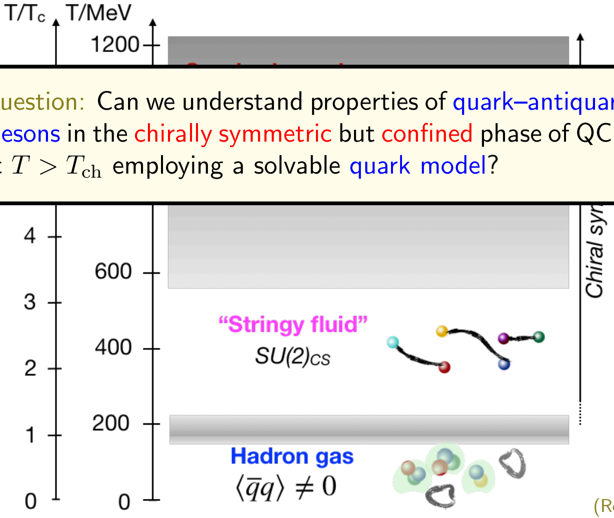

Question: Can we understand properties of quark–antiquark mesons in the chirally symmetric but confined phase of QCD at $T > T_{\rm ch}$ employing a solvable quark model?

(Rohrhofer et al'2019)

(Cohen & Glozman'2024 for large-N_c picture)

NJL model (Nambu & Jona-Lasinio'1961)

Lagrangian of the model ($N_f = 1$)

$$\mathcal{L}_{\text{int}} = \frac{\lambda}{4} \int d^3x \left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi} i \gamma^5 \psi\right)^2 \right] = \lambda \int d^3x \left(\bar{\psi}_R \psi_L\right)\left(\bar{\psi}_L \psi_R\right)$$

Gap (mass-gap equation):

$$m = \Sigma = 2$$

$$= \frac{i}{2}\lambda \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} S(p) = \lambda \int \frac{d^3p}{(2\pi)^3} \frac{m}{\sqrt{\boldsymbol{p}^2 + m^2}}$$

$$m \left(1 - \lambda \int^{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{\boldsymbol{p}^2 + m^2}} \right) = 0$$

- Weak coupling regime $\lambda < \lambda_{\text{crit}} = \left(\frac{2\pi}{\Lambda}\right)^2 \implies m = 0$
- Strong coupling regime $\lambda > \lambda_{\text{crit}}$

$m \neq 0 \implies$ Gap in the spectrum of excitations

$\langle \bar{\psi}\psi \rangle \neq 0 \implies$ Chiral symmetry is broken spontaneously

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Gap (mass-

m … ^{2}

NJL model:

- $+$ Simple and physically transparent
- $+$ Explains SBCS
- $-$ The only mass scale comes from cut-off
- $-$ No confinement

- Weak coupling regime $\lambda < \lambda_{\rm crit} = \left(\frac{2\pi}{\Lambda}\right)^2 \implies m = 0$
- Strong coupling regime $\lambda > \lambda_{\rm crit}$

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Model for QCD in two dimensions

QCD_2 in axial gauge

('t Hooft'1974;Bars,Green'1978,...)

- Lagrangian of QCD_2 ('t Hooft model)

$$L(x) = -\frac{1}{4}F^a_{\mu\nu}(x)F^a_{\mu\nu}(x) + \bar{\psi}(x)\Big[i(\partial_\mu - igA^a_\mu t^a)\gamma_\mu - m\Big]\psi(x)$$

- Interaction Hamiltonian in axial (Coulomb) gauge

$$H_{\text{int}} = -\frac{g^2}{2}\int dxdy\left(\psi^\dagger(t,x)\frac{\lambda^a}{2}\psi(t,x)\right)|x-y|\left(\psi^\dagger(t,y)\frac{\lambda^a}{2}\psi(t,y)\right)$$

- Large-N_c limit

$$\gamma = \frac{g^2 N_c}{4\pi} \underset{N_c\to\infty}{\to} \text{const}$$

- Dressed fermion field

$$\psi(t,x) = \int\frac{dk}{2\pi}e^{ikx}[b(k,t)u(k) + d^\dagger(-k,t)v(-k)]$$

$$u(k) = T(k)\begin{pmatrix}1\\0\end{pmatrix} \quad v(-k) = T(k)\begin{pmatrix}0\\1\end{pmatrix} \quad T(k) = e^{-\frac{1}{2}\theta(k)\gamma_1}$$

Bogoliubov transformation: From "bare" to "dressed" fermions

- Hamiltonian in terms of "bare" particles (fermion+antifermion)

$$H = \epsilon(b_0^\dagger b_0 - d_0 d_0^\dagger) + \Delta(b_0^\dagger d_0^\dagger + d_0 b_0) \qquad E_{\rm vac}^{(0)} = {}_0\langle 0|H|0\rangle_0 = -\epsilon$$

- "Dressed" particles (quasiparticles)

$$b_0 = ub - vd^\dagger \qquad d_0 = ud + vb^\dagger \qquad u^2 + v^2 = 1 \implies \{bb^\dagger\} = \{dd^\dagger\} = 1$$

with a convenient parametrisation: $u = \cos\theta$ and $v = \sin\theta$

- Hamiltonian in terms of dressed operators ($H = H_0 + :H_2:$)

$$H_0 = -(\epsilon\cos\theta + \Delta\sin\theta)$$

$$:H_2: = (\epsilon\cos\theta + \Delta\sin\theta)(b^\dagger b + d^\dagger d) + (\Delta\cos\theta - \epsilon\sin\theta)(b^\dagger d^\dagger + db)$$

- Equation for θ

$$\Delta\cos\theta - \epsilon\sin\theta = 0$$

ensures both E_{vac}=min and $:H_2:$ is diagonal

- Diagonalised Hamiltonian in terms of dressed operators

$$H = \sqrt{\epsilon^2+\Delta^2}(b^\dagger b - dd^\dagger)$$

- Physical versus trivial vacuum

$$|0\rangle = \frac{1}{\sqrt{2}}\left(\cos\theta + d_0^\dagger b_0^\dagger \sin\theta\right)|0\rangle_0 \qquad b\,|0\rangle = d\,|0\rangle = 0$$

- Trivial vacuum is unstable

$$E_{\text{vac}} = \langle 0|H|0\rangle = -\sqrt{\epsilon^2+\Delta^2} < E_{\text{vac}}^{(0)}$$

$$:H_2: = (\epsilon\cos\theta + \Delta\sin\theta)(b^\dagger b + d^\dagger d) + (\Delta\cos\theta - \epsilon\sin\theta)(b^\dagger d^\dagger + db)$$

Hamiltonian approach to 't Hooft model

- Normally ordered Hamiltonian ($\psi \sim b + d^\dagger$)

$$H = H_0 + :H_2: + :H_4:$$

- The vacuum energy is a minimum

$$E_{\text{vac}} = \langle 0|H|0\rangle = H_0 = \text{min}$$

- Quadratic part $:H_2:$ (describes dressing of quarks) is diagonal
- Quartic part $:H_4:$ (describes interaction of dressed quarks) is suppressed by N_c
- Mass-gap equation

$$p\cos\theta(p) - m\sin\theta(p) = \frac{\gamma}{2}\int\frac{dk}{(p-k)^2}\sin[\theta(p)-\theta(k)]$$

- Dressed quark dispersion law

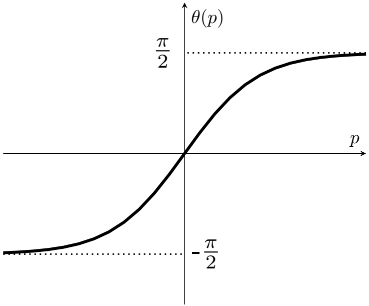
$$E_p = m\cos\theta(p) + p\sin\theta(p) + \frac{\gamma}{2}\int\frac{dk}{(p-k)^2}\cos[\theta(p)-\theta(k)]$$

Solutions of mass-gap equation in 't Hooft model

- Free solution

$$\theta = \arctan \frac{p}{m} \qquad E_p = \sqrt{p^2 + m^2}$$

- Physical chirally nonsymmetric solution



$$\langle \bar{\psi}\psi \rangle = -\frac{N_c}{\pi} \int_0^\infty dp \cos\theta(p) \neq 0$$

$$\langle \bar{\psi}\psi \rangle_{m=0} = -\frac{1}{\sqrt{6}} N_c \sqrt{\gamma}$$

From quarks towards quark-antiquark mesons

Operators creating and annihilating quark-antiquark pairs

$$M^\dagger(p,p') = \frac{1}{\sqrt{N_c}}\sum_\alpha b^\dagger_\alpha(p')d^\dagger_\alpha(-p) \qquad M(p,p') = \frac{1}{\sqrt{N_c}}\sum_\alpha d_\alpha(-p)b_\alpha(p')$$

Hamiltonian (including $:H_4:$ part) in terms of such compound operators

$$H \sim H_0 + M^\dagger M + \frac{1}{2}\left(M^\dagger M^\dagger + MM\right)$$

(Kalashnikova,AN'2000)

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(Kalashnikova,AN'2000)

Hamiltonian H is subject to a second (bosonic) Bogoliubov transformation from compound $q\bar{q}$ operators M to mesonic operators m

$$m^\dagger = M^\dagger \varphi^+ + M\varphi^- \quad m = M\varphi^+ + M^\dagger\varphi^-$$

$$(\varphi^+)^2 - (\varphi^-)^2 = 1$$

Bound state equation in 't Hooft model

Meson creation/annihilation operators

$$m_n^\dagger(Q) = \int \frac{dq}{2\pi} \left\{ M^\dagger(q-Q,q)\varphi_n^+(q,Q) + M(q,q-Q)\varphi_n^-(q,Q) \right\}$$

$$m_n(Q) = \int \frac{dq}{2\pi} \left\{ M(q-Q,q)\varphi_n^+(q,Q) + M^\dagger(q,q-Q)\varphi_n^-(q,Q) \right\}$$

Orthogonality & completeness

$$\int \frac{dp}{2\pi} \left(\varphi_n^+(p,Q)\varphi_{n'}^+(p,Q) - \varphi_n^-(p,Q)\varphi_{n'}^-(p,Q) \right) = \delta_{nn'}$$

$$\sum_{n=0}^{\infty} \left(\varphi_n^+(p,Q)\varphi_n^+(k,Q) - \varphi_n^-(p,Q)\varphi_n^-(k,Q) \right) = 2\pi\delta(p-k)$$

Bound state equation

$$\begin{cases} [E_p + E_{Q-p} - Q_0]\varphi^+(p,Q) = \gamma \displaystyle\int \frac{dk}{(p-k)^2} \left[C(p,k,Q)\varphi^+(k,Q) - S(p,k,Q)\varphi^-(k,Q) \right] \\ [E_p + E_{Q-p} + Q_0]\varphi^-(p,Q) = \gamma \displaystyle\int \frac{dk}{(p-k)^2} \left[C(p,k,Q)\varphi^-(k,Q) - S(p,k,Q)\varphi^+(k,Q) \right] \end{cases}$$

The chiral pion

Solution of bound state equation for the chiral pion

$$\varphi_\pi^\pm(p,Q) = \sqrt{\frac{\pi}{2Q}}\left(\cos\frac{\theta(Q-p)-\theta(p)}{2} \pm \sin\frac{\theta(Q-p)+\theta(p)}{2}\right)$$

The pion decay constant f_π

$$\langle\Omega\,|\,J_\mu^5(x)\,|\,\pi(Q)\,\rangle = f_\pi Q_\mu \frac{e^{-iQx}}{\sqrt{2Q_0}} \qquad f_\pi = \sqrt{\frac{N_c}{\pi}}$$

Pion mass

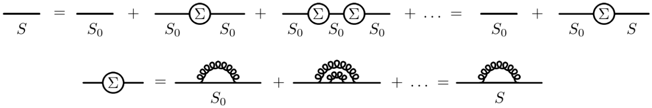
$$M_\pi^2 = 2m\int_0^\infty dp\cos\theta(p)$$

Gell-Mann-Oakes-Renner relation

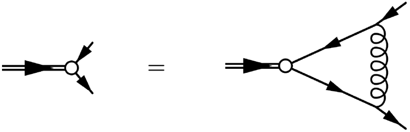
$$f_\pi^2 M_\pi^2 = -2m\langle\bar\psi\psi\rangle$$

Diagrammatic approach to 't Hooft model

- Dyson series (in rainbow approximation) for dressed quark propagator



- Bethe-Salpeter equation (in ladder approximation) for quark-antiquark meson



- Mass-gap equation and bound-state equation derived using diagrams and Hamiltonian approach coincide

Infrared divergent and finite quantities: An instructive lesson

- Interquark potential in principal value prescription

$$V(x) = -P \int_{-\infty}^{+\infty} \frac{dp}{2\pi} \frac{e^{ipx}}{p^2} = -|x| \int_0^{+\infty} \frac{d\xi}{\pi} \frac{\cos\xi - 1}{\xi^2} = \frac{1}{2}|x|$$

- Interquark potential with finite infrared regulator

$$V(x) = -\int_{-\infty}^{+\infty} \frac{dp}{2\pi} \frac{e^{ipx}}{p^2 + \mu_{\rm IR}^2} = -\frac{1}{2\mu_{\rm IR}} e^{-\mu_{\rm IR}|x|} \underset{\mu_{\rm IR}\to 0}{=} -\frac{1}{2\mu_{\rm IR}} + \frac{1}{2}|x| + \dots$$

- Infrared divergent piece appears in not observable quantities (potential, E_p, etc) but cancels in physical ones (chiral angle, bound state equation, etc)
- Infrared divergence shows up in not gauge invariant objects (e.g. single quark) indicating that they are not observable
- Only gauge invariant objects are Poincare invariant (Bars & Green'1978)

Confining chiral quark model for QCD in four dimensions

Confining chiral quark model for QCD in 3+1

(Orsay group'1980s;Adler,Devis'1984;Bicudo,Ribeiro'1990s)

- Interacting colour charge densities

$$H_{\rm int} = \frac{1}{2}\int d^3x d^3y \left(\psi^\dagger(t,\boldsymbol{x})\frac{\lambda^a}{2}\psi(t,\boldsymbol{x})\right) V(|\boldsymbol{x}-\boldsymbol{y}|) \left(\psi^\dagger(t,\boldsymbol{y})\frac{\lambda^a}{2}\psi(t,\boldsymbol{y})\right)$$

$$\psi(t,\boldsymbol{x}) = \sum_{s=\uparrow,\downarrow}\int \frac{d^3p}{(2\pi)^3} e^{i\boldsymbol{p}\boldsymbol{x}}\left(b_{\boldsymbol{p}s}u_{\boldsymbol{p}s}[\varphi_p] + d^\dagger_{-\boldsymbol{p}-s}v_{-\boldsymbol{p}-s}[\varphi_p]\right)$$

- Normally ordered Hamiltonian

$$H = E_{\rm vac}[\varphi_p] + :H_2: + :H_4:$$

- The energy of the vacuum is a minimum $\Longrightarrow$ mass-gap equation for φ_p
- Quadratic part $:H_2:$ describes dressed quarks
- Quartic part $:H_4:$ describes mesons
- Employ large-N_c logic $\Longrightarrow$ nonplanar diagrams neglected & only leading-order contributions in N_c retained

Mass-gap equation

- Define auxiliary functions

$$A_p = m + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}-\boldsymbol{k}) \sin\varphi_k$$

$$B_p = p + \frac{1}{2}\int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}-\boldsymbol{k})(\hat{\boldsymbol{p}}\hat{\boldsymbol{k}}) \cos\varphi_k$$

- Vacuum energy

$$E_{\rm vac}[\varphi_p] = -N_c V \int \frac{d^3p}{(2\pi)^3}\left(A_p \sin\varphi_p + B_p \cos\varphi_p\right)$$

- Dressed quark dispersion law

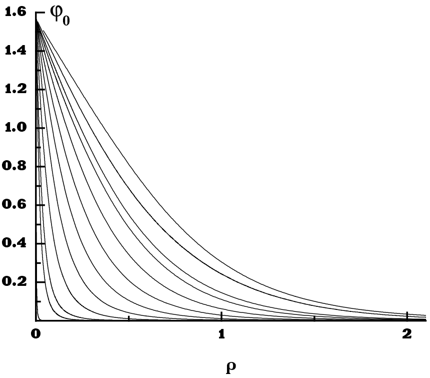
$$E_p = A_p \sin\varphi_p + B_p \cos\varphi_p$$

- Mass-gap equation for the chiral angle

$$A_p \cos\varphi_p - B_p \sin\varphi_p = 0$$

Power-like confining potential

(Orsay group'1980s, Bicudo, AN'2003)



$$V(r) = K_0^{1+\alpha} r^{\alpha}$$

$$0 \leqslant \alpha \leqslant 2$$

$$(2\pi)^3 K_0 \delta^{(3)}(\boldsymbol{p}) < V(\boldsymbol{p}) \leqslant (2\pi)^3 K_0^3 \Delta\delta^{(3)}(\boldsymbol{p})$$

Properties of the chiral angle

(Glozman,AN,Ribeiro'2005)

Mass-gap as "loop" equation ($V(r) = \sigma r$)

$$pc\sin\varphi_p - mc^2\cos\varphi_p = \frac{\hbar}{2}\int\frac{d^3k}{(2\pi)^3}V(\boldsymbol{p}-\boldsymbol{k})\left[\cos\varphi_k\sin\varphi_p - (\hat{\boldsymbol{p}}\hat{\boldsymbol{k}})\sin\varphi_k\cos\varphi_p\right]$$

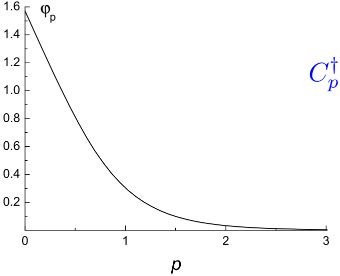
"Perturbative" regime (heavy quarks with $m \gg \sqrt{\sigma}$)

$$\varphi_p = \sum_{n=0}^{\infty}\left(\frac{\sigma\hbar c}{(mc^2)^2}\right)^n \tilde{f}_n\left(\frac{p}{mc}\right) = \arctan\frac{mc}{p} + \sum_{n=1}^{\infty}\left(\frac{\hbar}{\mathcal{S}}\right)^n \tilde{f}_n\left(\frac{p}{mc}\right)$$

"Nonperturbative" regime (light quarks with $m \ll \sqrt{\sigma}$)

$$\varphi_p = \sum_{n=0}^{\infty}\left(\frac{mc^2}{\sqrt{\sigma\hbar c}}\right)^n f_n\left(\frac{pc}{\sqrt{\sigma\hbar c}}\right) \underset{p\to 0}{\approx} \frac{\pi}{2} - \text{const}\frac{pc}{\sqrt{\sigma\hbar c}} + \dots$$

Chirally broken vacuum



(Bicudo,Ribeiro'1990s)

$$C_p^\dagger = \sum_{\alpha=1}^{N_c} \sum_{s,s'=\uparrow,\downarrow} b^\dagger_{\alpha s}(\boldsymbol{p}) \underbrace{[(\boldsymbol{\sigma}\hat{\boldsymbol{p}})i\sigma_2]_{ss'}}_{^3P_0 \text{ operator}} d^\dagger_{\alpha s'}(\boldsymbol{p})$$

$$Q^\dagger = \frac{1}{2}\sum_{\boldsymbol{p}} \varphi_p C_p^\dagger$$

Broken vacuum

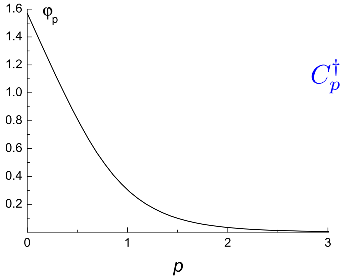
$$|0\rangle = e^{Q-Q^\dagger}|0\rangle_0 = \prod_p \left[\cos^2\frac{\varphi_p}{2} + \sin\frac{\varphi_p}{2}\cos\frac{\varphi_p}{2}C_p^\dagger + \frac{1}{2}\sin^2\frac{\varphi_p}{2}C_p^{\dagger 2}\right]|0\rangle_0$$

Chiral condensate

$$\langle\bar\psi\psi\rangle = -\frac{N_C}{\pi^2}\int_0^\infty dp\, p^2 \sin\varphi_p$$

Chirally broken vacuum

(Bicudo,Ribeiro'1990s)



$$C_p^\dagger = \sum_{\alpha=1}^{N_c} \sum_{s,s'=\uparrow,\downarrow} b_{\alpha s}^\dagger(\boldsymbol{p}) \underbrace{[(\boldsymbol{\sigma}\hat{\boldsymbol{p}})i\sigma_2]_{ss'}}_{{}^3P_0 \text{ operator}} d_{\alpha s'}^\dagger(\boldsymbol{p})$$

$$Q^\dagger = \frac{1}{2}\sum_{\boldsymbol{p}} \varphi_p C_p^\dagger$$

Broken vacuum

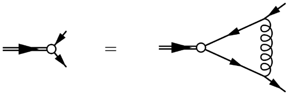
$|0$ ... $\rangle_0$

Conclusion: small momenta are most crucial for SBCS

Chiral condensate

$$\langle\bar{\psi}\psi\rangle = -\frac{N_C}{\pi^2}\int_0^\infty dp\, p^2 \sin\varphi_p$$

Bound state equation



$$\chi(\boldsymbol{p}; M) = i \int \frac{d^4q}{(2\pi)^4} V(\boldsymbol{p}-\boldsymbol{q})\gamma_0 S(q_0 + M/2, \boldsymbol{q})\chi(\boldsymbol{q}; M)S(q_0 - M/2, \boldsymbol{q})\gamma_0$$

$$\begin{cases} [2E_p - M]\phi^+(\boldsymbol{p}; M) = \int \frac{d^3q}{(2\pi)^3} V(\boldsymbol{p}-\boldsymbol{q})\Big[\mathcal{P}_{++}\phi^+(\boldsymbol{q}; M)\mathcal{P}_{--} + \mathcal{P}_{+-}\phi^-(\boldsymbol{q}; M)\mathcal{P}_{+-}\Big] \\ [2E_p + M]\phi^-(\boldsymbol{p}; M) = \int \frac{d^3q}{(2\pi)^3} V(\boldsymbol{p}-\boldsymbol{q})\Big[\mathcal{P}_{-+}\phi^+(\boldsymbol{q}; M)\mathcal{P}_{-+} + \mathcal{P}_{--}\phi^-(\boldsymbol{q}; M)\mathcal{P}_{++}\Big] \end{cases}$$

$$\phi^\pm(\boldsymbol{p}; M) = P_\pm T_p \frac{\chi(\boldsymbol{q}; M)}{2E_p \mp M} T_p P_\mp$$

$$T_p = \exp\left[\frac{1}{2}(\boldsymbol{\gamma}\hat{\boldsymbol{p}})\left(\frac{\pi}{2} - \varphi_p\right)\right] \quad P_\pm = \frac{1}{2}(1 \pm \gamma_0) \quad \mathcal{P}_{\lambda_1\lambda_2} = P_{\lambda_1} T_p T_q^\dagger P_{\lambda_2} \quad \lambda_{1,2} = \pm$$

The chiral pion

Matrix wave functions for the pion

$$\phi_\pi^\pm(\boldsymbol{p}; M) = \frac{i}{\sqrt{2}}\sigma_2 Y_{00}(\hat{\boldsymbol{p}})\varphi_\pi^\pm(p)$$

Bound state equation for the pion in centre-of-mass frame

$$\begin{cases} [2E_p - M_\pi]\varphi_\pi^+(p) = \int \frac{q^2 dq}{(2\pi)^3}\left[T_\pi^{++}(p,q)\varphi_\pi^+(q) + T_\pi^{+-}(p,q)\varphi_\pi^-(q)\right] \\ [2E_p + M_\pi]\varphi_\pi^-(p) = \int \frac{q^2 dq}{(2\pi)^3}\left[T_\pi^{-+}(p,q)\varphi_\pi^+(q) + T_\pi^{--}(p,q)\varphi_\pi^-(q)\right] \end{cases}$$

possesses solution (near the chiral limit $M_\pi \to 0$)

$$\varphi_\pi^\pm(p) = \mathcal{N}_\pi\left(\sin\varphi_p \pm O\left(M_\pi\right)\right)$$

With this w.f., the pion bound-state equation is equivalent to the mass-gap equation for the chiral angle φ_p

Chiral symmetry in heavy-light mesons

(Kalashnikova,AN,Ribeiro'2005)

- Bound state equation for opposite-parity heavy-light mesons ($\varphi^+=\psi$, $\varphi^-=0$)

$$\psi'(\boldsymbol{p})=(\boldsymbol{\sigma}\hat{\boldsymbol{p}})\psi(\boldsymbol{p})$$

$$E_p\psi(\boldsymbol{p})+\int\frac{d^3k}{(2\pi)^3}V(\boldsymbol{p}-\boldsymbol{k})\left[C_pC_k+(\boldsymbol{\sigma}\hat{\boldsymbol{p}})(\boldsymbol{\sigma}\hat{\boldsymbol{k}})S_pS_k\right]\psi(\boldsymbol{k})=E\psi(\boldsymbol{p})$$

$$E_p\psi'(\boldsymbol{p})+\int\frac{d^3k}{(2\pi)^3}V(\boldsymbol{p}-\boldsymbol{k})\left[S_pS_k+(\boldsymbol{\sigma}\hat{\boldsymbol{p}})(\boldsymbol{\sigma}\hat{\boldsymbol{k}})C_pC_k\right]\psi'(\boldsymbol{k})=E'\psi'(\boldsymbol{p})$$

$$C_p=\sqrt{\frac{1+\sin\varphi_p}{2}}\qquad S_p=\sqrt{\frac{1-\sin\varphi_p}{2}}$$

- If $\varphi_p\to 0$ mesons with opposite parity become degenerate

$$C_p^2-S_p^2=\sin\varphi_p$$

- GNJL provides a microscopic picture for phenomena related to chiral symmetry

Chiral symmetry in heavy-light mesons

(Kalashnikova,AN,Ribeiro'2005)

- Bound state equation for opposite-parity heavy-light mesons ($\varphi^+ = \psi$, $\varphi^- = 0$)

- Chiral quark model:

$$\Delta M_\pm \ll \Delta M_+, \Delta M_-$$

- Naive approach (no chiral symmetry, Salpeter equation):

$$\Delta M_\pm \simeq \Delta M_+, \Delta M_-$$

- If $\varphi_p \to 0$ mesons with opposite parity become degenerate

$$C_p^2 - S_p^2 = \sin\varphi_p$$

- GNJL provides a microscopic picture for phenomena related to chiral symmetry

Infrared-finite quark energy

- Linear confining potential

$$V(r) = -\int \frac{d^3p}{(2\pi)^3} \frac{8\pi\sigma}{(p^2+\mu_{\rm IR}^2)^2} e^{i\boldsymbol{pr}} = \frac{\sigma}{\mu_{\rm IR}} e^{-\mu_{\rm IR} r} \underset{\mu_{\rm IR}\to 0}{=} -\frac{\sigma}{\mu_{\rm IR}} + \sigma r + \dots$$

- Auxiliary functions A_p and B_p

$$A_p = \frac{\sigma}{2\mu_{\rm IR}} \sin\varphi_p + A_p^{\rm fin} \qquad B_p = \frac{\sigma}{2\mu_{\rm IR}} \cos\varphi_p + B_p^{\rm fin}$$

- Mass-gap equation

$$A_p^{\rm fin}\cos\varphi_p - B_p^{\rm fin}\sin\varphi_p = 0$$

- Dispersion law

$$E_p = \frac{\sigma}{2\mu_{\rm IR}} + \dots$$

- Infrared-finite dynamical quark mass

$$\omega_p = \left(p^2 + \underbrace{(p\tan\varphi_p)}_{M_p}{}^2\right)^{1/2}$$

Mass-gap equation at finite temperatures

- Fermion propagator at $T > 0$

$$\Delta S(p_0, \boldsymbol{p}; T) = 2\pi i \Big[n_p \Lambda_+(\boldsymbol{p}) \delta(p_0 - E_p) - \bar{n}_p \Lambda_-(\boldsymbol{p}) \delta(p_0 + E_p) \Big] \gamma_0$$

$$\Lambda_\pm(\boldsymbol{p}) = \frac{1}{2}[1 \pm \gamma_0 \sin\varphi_p \pm (\boldsymbol{\alpha}\hat{\boldsymbol{p}}) \cos\varphi_p]$$

- Fermi-Dirac distributions at $T \neq 0$

$$\langle b^\dagger_{\boldsymbol{p}s} b_{\boldsymbol{p}s} \rangle = n_p = \left(1 + e^{(\sqrt{p^2+M_p^2}-\mu)/T}\right)^{-1} \underset{T\to\infty}{\to} \frac{1}{2}$$

$$\langle d^\dagger_{\boldsymbol{p}s} d_{\boldsymbol{p}s} \rangle = \bar{n}_p = \left(1 + e^{(\sqrt{p^2+M_p^2}+\mu)/T}\right)^{-1} \underset{T\to\infty}{\to} \frac{1}{2}$$

- Modified auxiliary functions at finite T

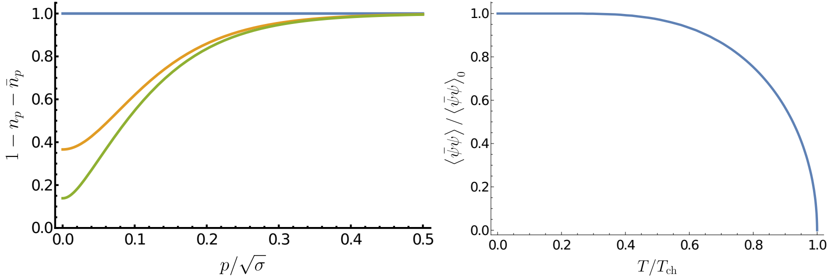
$$\tilde{A}_p = m + \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} (1 - n_k - \bar{n}_k) V(\boldsymbol{p} - \boldsymbol{k}) \sin\varphi_k$$

$$\tilde{B}_p = p + \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} (1 - n_k - \bar{n}_k) V(\boldsymbol{p} - \boldsymbol{k}) (\hat{\boldsymbol{p}}\hat{\boldsymbol{k}}) \cos\varphi_k$$

(for derivation in imaginary time formalism: Kocic'1986)

Critical temperature and chiral restoration

(Glozman,AN,Wagenbrunn'2024)



Prediction of the model with linear confinement

$$|\langle\bar{\psi}\psi\rangle_0|^{1/3} \approx 2.75\ T_{\rm ch}$$

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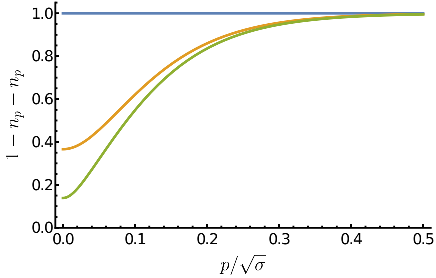

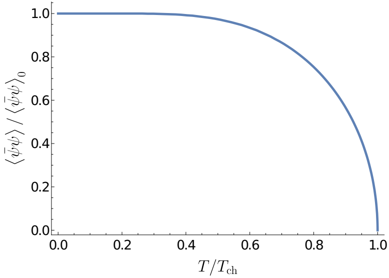

Prediction of the model with linear confinement

$$|\langle\bar{\psi}\psi\rangle_0|^{1/3} \approx 2.75\, T_{\rm ch}$$

Numerical estimate for $\langle\bar{\psi}\psi\rangle_0 = -(250 \text{ MeV})^3$

$$T_{\rm ch} \approx 90 \text{ MeV}$$

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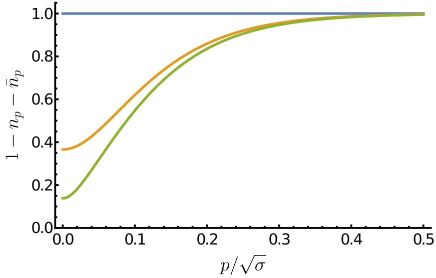

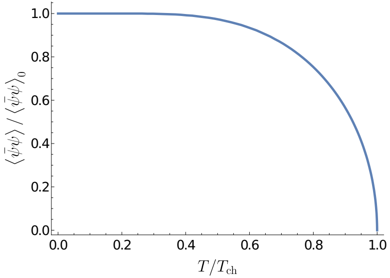

Prediction of the model with linear confinement

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$$T_{\rm ch} \approx 90\text{ MeV}$$

To confront with

- $T_{\rm ch} \approx 100$ MeV (Quandt et al.'2018)
- $T_{\rm ch}^{\rm lat} \approx 130$ MeV (HotQCD'2019)

Bound state equation at finite temperature

$$\chi(\boldsymbol{p}; M) = i \int \frac{d^4q}{(2\pi)^4} V(\boldsymbol{p}, \boldsymbol{q}; T)\gamma_0 S(q_0 + M/2, \boldsymbol{q}; T)\chi(\boldsymbol{q}; M) S(q_0 - M/2, \boldsymbol{q}; T)\gamma_0$$

$$\underbrace{V(\boldsymbol{p} - \boldsymbol{q})}_{T=0} \quad \Longrightarrow \quad \underbrace{V(\boldsymbol{p}, \boldsymbol{q}; T)}_{T>0} = (1 - n_q - \bar{n}_q) V(\boldsymbol{p} - \boldsymbol{q})$$

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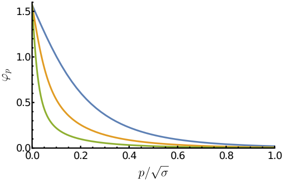
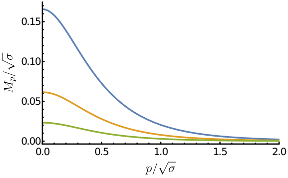
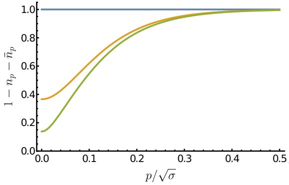
- Temperature dumps chiral angle and dynamical quark mass



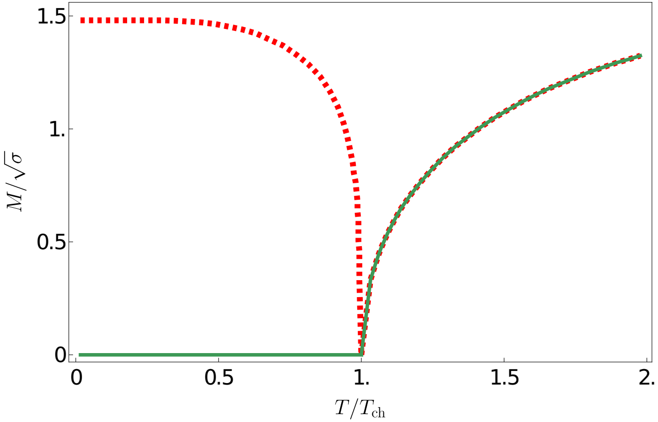


- Temperature dumps interaction potential



Chiral symmetry in spectrum of mesons

(Glozman,AN,Wagenbrunn'2024)



- 1^1S_0 meson (chiral pion) mass (green solid line)
- 1^3P_0 ("σ") meson mass (red dotted line)

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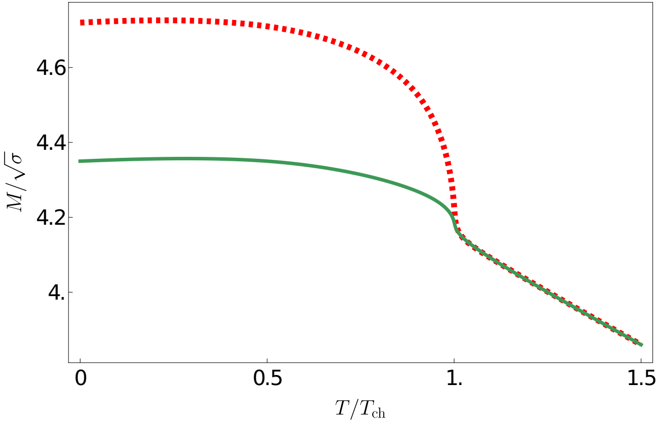
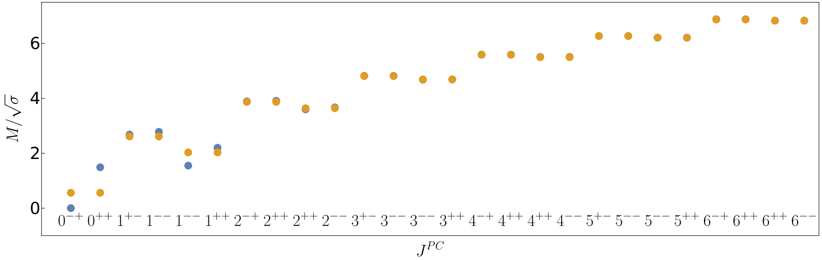


- 3^1S_0 meson mass (green solid line)
- 3^3P_0 meson mass (red dotted line)

Symmetries of spectrum of light-light mesons

(Glozman,AN,Wagenbrunn'2024)

$T = 0$ (blue dots) and $T = 1.1T_{\rm ch}$ (yellow dots)



Above $T_{\rm ch}$:

- Confinement persists $\Longrightarrow$ hadrons survive as bound states of quarks
- Chiral symmetry is restored $\Longrightarrow$ opposite-parity states become degenerate
- Spectrum of $\bar{q}q$ mesons demonstrates higher emergent symmetry

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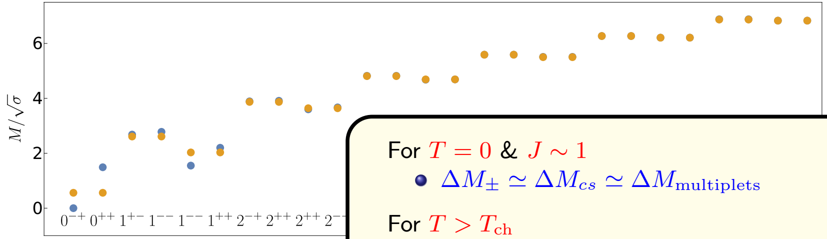

For $T=0$ & $J\sim 1$

- $\Delta M_{\pm} \simeq \Delta M_{cs} \simeq \Delta M_{\text{multiplets}}$

For $T>T_{\text{ch}}$

- $\Delta M_{\pm} = 0$
- $\Delta M_{cs} \ll \Delta M_{\text{multiplets}}$

Above T_{ch}:

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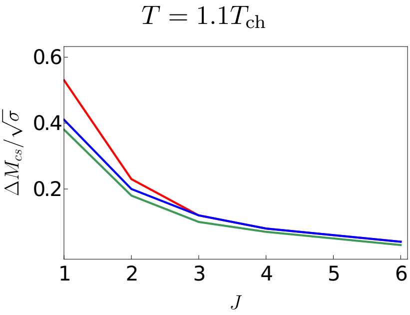
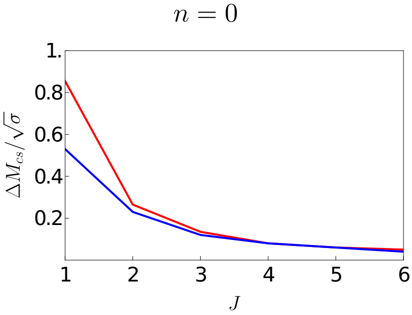
Symmetries of spectrum of light-light mesons



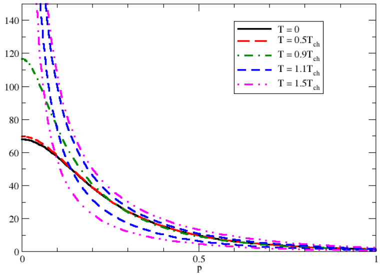


- $n = 0$ (red line)
- $n = 1$ (blue line)
- $n = 2$ (green line)

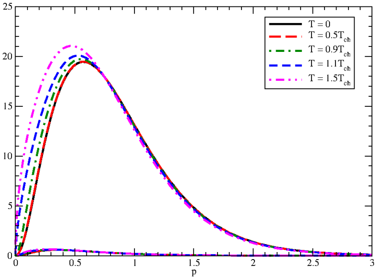
- $T = 0$ (red line)
- $T = 1.5T_{\rm ch}$ (blue line)

Wave functions of low-lying mesons @ $T > T_{\rm ch}$

$J^{PC} = 0^{-+}\ (n = 0)$

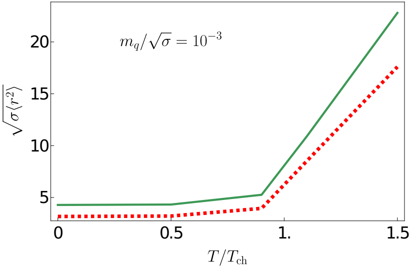


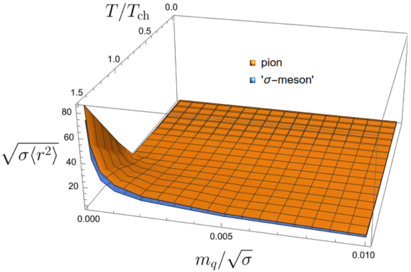
$J^{PC} = 2^{-+}\ (n = 0)$



- Above $T_{\rm ch}$, radial wave functions of low-lying mesons grow at $p \to 0$
- No problem with w.f. normalisation due to cancellations in $\varphi_+^2 - \varphi_-^2$
- Expect consequences for observables!

Size of light hadrons above $T_{\rm ch}$





$$\langle h(p')|J_\mu(0)|h(p)\rangle = i(p+p')_\mu F_h(q^2) \qquad \langle r_h^2\rangle = 6\frac{\partial F_h(q^2)}{\partial q^2}\Big|_{q^2=0}$$

- Low-lying mesons made of light quarks "swell" above $T_{\rm ch}$ $(r_{\rm rms} \sim 1/\sqrt{m_q})$
- Size of pion and "σ-meson" at $T = 1.5T_{\rm ch}$ is 5 times their size at $T < T_{\rm ch}$

Conclusions

- Many phenomena inherent in QCD can be studied and understood with the help of quark models
- The employed chiral confining quark model predicts that
  - Chiral symmetry is restored at $T_{\rm ch} \sim 100$ MeV
  - Quark-antiquark mesons survive as confined states above $T_{\rm ch}$
  - Spectrum of mesons above $T_{\rm ch}$ demonstrates higher degeneracy
  - Mesons with light quarks increase their size above $T_{\rm ch}$
  - The underlying mechanism is Pauli blocking of low-lying quark levels with T
- Hadron gas at $T < T_{\rm ch}$ turns to dense system of overlapping “strings” at $T > T_{\rm ch}$

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Work in progress...

Stay tuned!

Bogoliubov transformation: From "bare" to "dressed" bosons

- Hamiltonian in terms of "bare" particles (bosons)

$$H = h_1 M^\dagger M + \frac{1}{2} h_2 (M^\dagger M^\dagger + MM)$$

- "Dressed" particles (quasiparticles)

$$M = um + vm^\dagger \quad M^\dagger = um^\dagger + vm \qquad [mm^\dagger] = [MM^\dagger] = 1 \implies u^2 - v^2 = 1$$

with a convenient parametrisation: $u = \cosh\theta$ and $v = \sinh\theta$

- Hamiltonian in terms of dressed operators ($H = H_0 + :H_2:$)

$$H_0 = -\frac{1}{2} h_1 + \frac{1}{2}(h_1 \cosh 2\theta + h_2 \sinh 2\theta)$$

$$:H_2: = (h_1 \cosh 2\theta + h_2 \sinh 2\theta) m^\dagger m + \frac{1}{2}(h_1 \sinh 2\theta + h_2 \cosh 2\theta)(m^\dagger m^\dagger + mm)$$

Bogoliubov transformation: From “bare” to “dressed” bosons

- Hamilt
- “Dress

$M =$ … $^2 - v^2 = 1$

with a

- Hamilt

$: H_2 :=$ … $m^\dagger + mm)$ … 2

- Condition (equation for θ)

$$h_1 \sinh 2\theta + h_2 \cosh 2\theta = 0$$

ensures both E_{vac}=min and $: H_2 :$ is diagonal

- Physical versus trivial vacuum

$$M\,|0\rangle = 0 \qquad m\,|\Omega\rangle = 0 \qquad |\Omega\rangle \neq |0\rangle$$

- Diagonalised Hamiltonian in terms of dressed operators

$$H = -\frac{1}{2}\left(h_1 - \sqrt{h_1^2 - h_2^2}\right) + \sqrt{h_1^2 - h_2^2}\, m^\dagger m$$

- Vacuum with $E^{(0)}_{\text{vac}} = 0$ is unstable

$$E_{\text{vac}} = \langle\Omega|H|\Omega\rangle < 0$$

Planar and non-planar diagrams

$$\langle (A_\mu)^\alpha_\beta (A_\nu)^\gamma_\delta \rangle \quad \propto \quad \left(\frac{\lambda^a}{2}\right)^\alpha_\beta \left(\frac{\lambda^a}{2}\right)^\gamma_\delta \quad = \quad \frac{1}{2}\left(\delta^\alpha_\delta \delta^\gamma_\beta - \frac{1}{N_c}\delta^\alpha_\beta \delta^\gamma_\delta\right) \quad \xrightarrow[N_c\to\infty]{} \quad \frac{1}{2}\delta^\alpha_\delta \delta^\gamma_\beta$$

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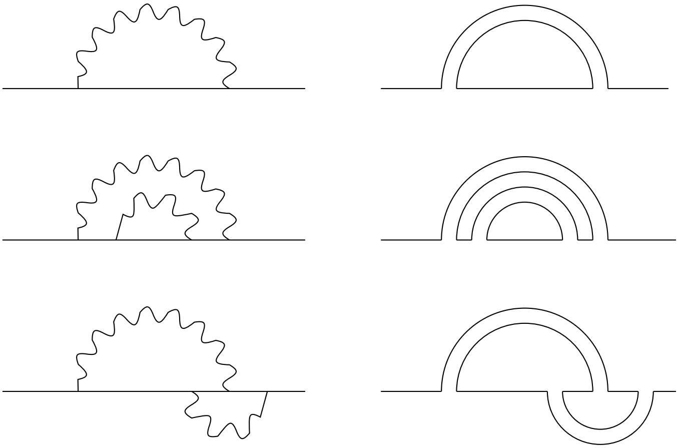

$SU(2)_{CS}$ and chiral multiplets

J	$(0,0)$	$(1/2,1/2)_a$	$(1/2,1/2)_b$	$(0,1)\oplus(1,0)$
0	—	$1,0^{-+} \longleftrightarrow 0,0^{++}$	$1,0^{++} \longleftrightarrow 0,0^{-+}$	—
$2k$	$0,J^{--}\ ;\ 0,J^{++}$	$1,J^{-+} \longleftrightarrow 0,J^{++}$	$1,J^{++} \longleftrightarrow 0,J^{-+}$	$1,J^{++} \longleftrightarrow 1,J^{--}$
$2k-1$	$0,J^{++}\ ;\ 0,J^{--}$	$1,J^{+-} \longleftrightarrow 0,J^{--}$	$1,J^{--} \longleftrightarrow 0,J^{+-}$	$1,J^{--} \longleftrightarrow 1,J^{++}$

$$\Psi(\boldsymbol{x}) \to \Psi'(\boldsymbol{x}) = U\Psi(\boldsymbol{x}), \quad UU^\dagger = U^\dagger U = 1$$

$$U = \exp(i\boldsymbol{\epsilon}\boldsymbol{\Sigma}/2) = \cos\frac{|\boldsymbol{\epsilon}|}{2} + \frac{i(\boldsymbol{\epsilon}\boldsymbol{\Sigma})}{|\boldsymbol{\epsilon}|}\sin\frac{|\boldsymbol{\epsilon}|}{2}$$

$$\boldsymbol{\Sigma} = (\gamma_0, i\gamma_5\gamma_0, -\gamma_5)$$

$$[(\Sigma_i/2),(\Sigma_j/2)] = i\varepsilon_{ijk}(\Sigma_k/2)$$

Spectrum of mesons with $J = 1$