Large Nc and the QCD Phase Diagram

The $\mu=0$ sector

TDC & L. Ya Glozman *Eur.Phys.J.A* 60 (2024) 9, 171
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An Overview

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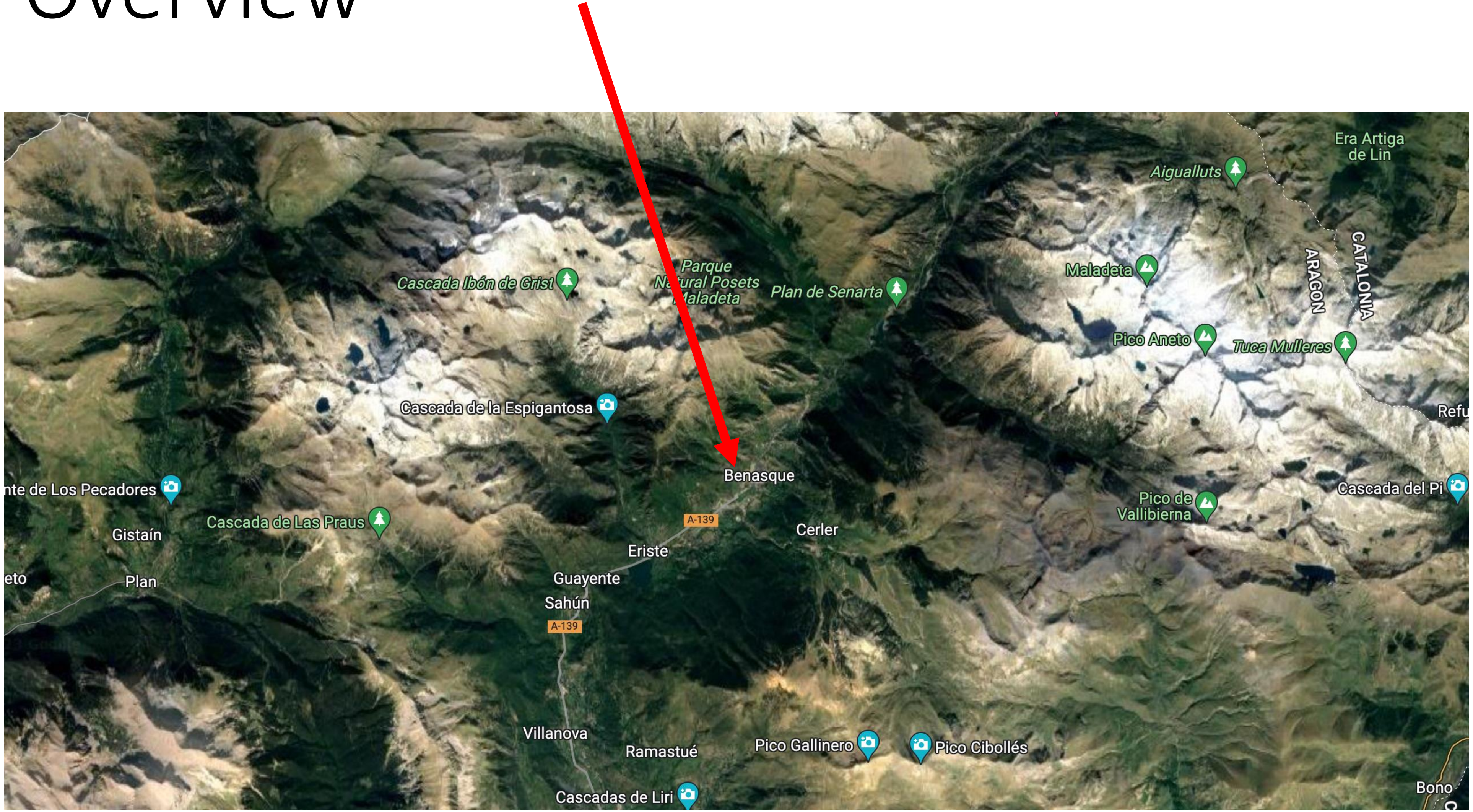
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An Overview

- Introduction
  - Lattice evidence of three regimes of QCD at $\mu=0$; "deconfinement" and chiral crossovers are at qualitatively distinct temperatures.
    - Interpretation in terms of an emergent symmetry ("chiral spin")
      - Dynamical interpretation of intermediate regime as a chirally broken but confined (by electric interaction "stringy fluid"
  - The picture is qualitative; the regimes are not sharp and separated by cross-overs and not phase transitions.
- The picture becomes much sharper theoretically in the combined large Nc and chiral limits of QCD
  - This talk explores what this combined limit can tell us about the three-regime picture

Introduction

- It was long thought that $\mu=0$ QCD has two temperature regimes—a low temperature a hadronic regime where QCD's approximate chiral symmetry is spontaneous broken and a high temperature quark-gluon plasma deconfined regime.
  - These regimes are not separated by phase transitions; it was generally thought the cross-over separating them was near the chiral crossover point $T_c \sim 150$ MeV where the chiral susceptibility ($\chi_{ch} = \frac{\partial_m Z}{V}$ where m is the mass and Z the partition function) maximizes .
- It would have been been heretical to suggest that there were more than two such regimes.

In Spain one needs to be careful of heresy

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- Early lattice studies suggested that the chiral cross-over temperature and the “deconfinement” cross-over temperature (taken to be the region where the the average Polyakov loop rapidly changes from small to large

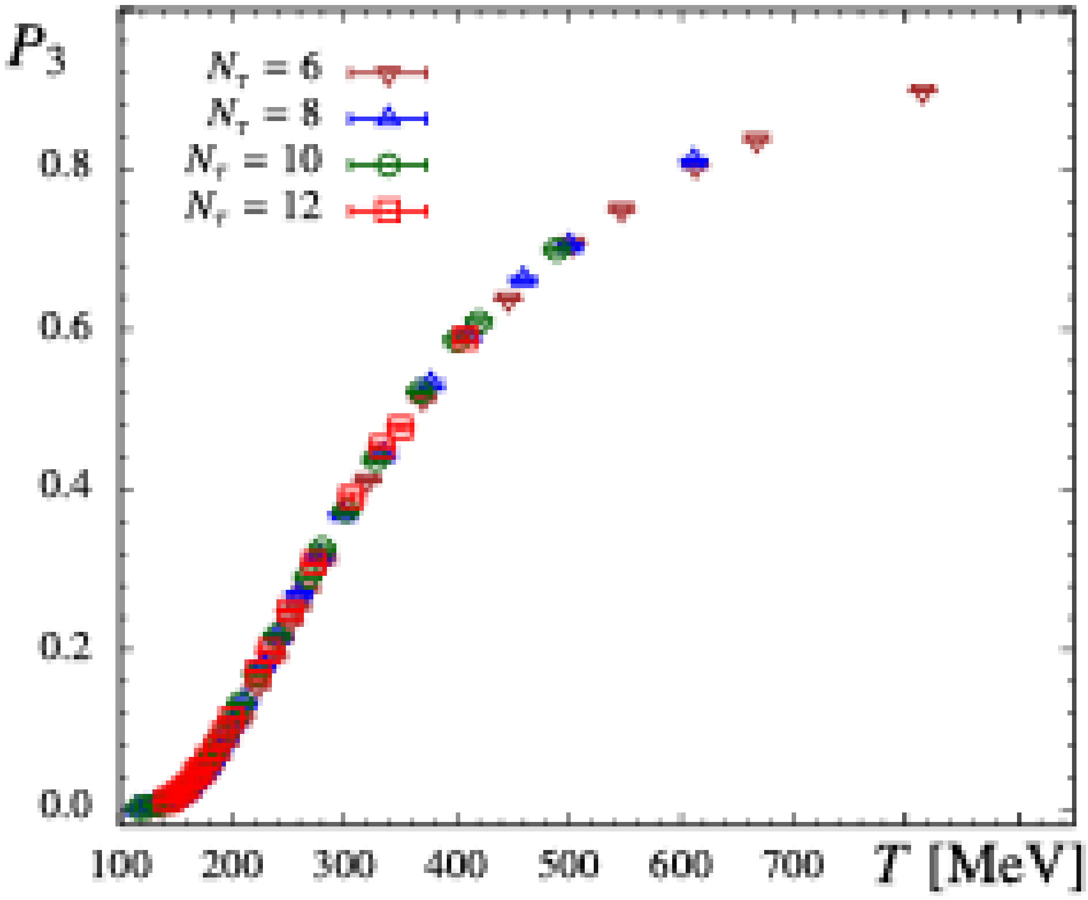
$$\langle l \rangle \text{ where } l \equiv \frac{Tr\left[\mathrm{P}\exp\left(i\int_0^\beta dt\, A_0\right)\right]}{N_c}$$ in Euclidean space.

- l might seems like a natural indicator for a deconfined regime
  - In Yang-Mills theory (pure gauge without quarks) there is a phase transition beween a “confined” phase with a nonzero string tension, and an unbroken Z_{N_c} center symmetry with $\langle l \rangle$=0 and a “deconfined” phase and with no string tension, broken Z_{N_c} center symmetry with $\langle l \rangle \neq 0$.

- Modern lattice calculations : it is hard to pin down the deconfinement transition (different schemes proposed typically using quantities connected to $\bar{Q}Q$ states).
- There appears to be reasonably strong evidence that the crossover to "deconfinement" defined via $\langle l \rangle$ is not well connected to the chiral cross-over.

Polyakov Loop Susceptibility and Correlators in the Chiral Limit (David A. Clarke, Olaf Kaczmarek, Frithjof Karsch, Anirban Lahiri, 37th International Symposium on Lattice Field Theory - Lattice2019 16-22 June 2019 Wuhan, China)

"The Polyakov loop does not exhibit any indication of a crossover near the chiral pseudo-critical temperature at lower-than-physical quark mass"



P. Petreczky and H. P. Schadler, Phys. Rev. D 92 (2015) no.9, 094517

The difference between T_{ch} and the regime of large Polyakov loop opens the possibility of an intermediate regime between the hadronic and QGP regimes.

- **An aside**— "deconfinement" in the lattice is generally assumed to be connected with the Polyakov loop. I am very skeptical that this is meaningful—even in pure Yang-Mills where there is a 1st-order phase transition between phases one with $\langle l \rangle$=0 ("confined") and one with $\langle l \rangle \neq 0$("deconfined").
  - In Yang-Mills the Polyakov loop is for the color fundamental indicating confinement of quarks—in a theory that lacks quarks! There is no phase transition for the Polyakov loop for the color adjoint which tells about confinement of gluons.
  - It is a purely Euclidean property that has no obvious physical interpretation in Minkowski space—and physics takes place in Minkowski space.
    - As a result, its value in non-equilibrium situations is obscure at best (and no physical process is ever perfectly equilibrated)

- **An aside**— "deconfinement" is generally assumed to be connected with the Polyakov loop. In Euclidean calculations, it is an order parameter for 1st-order transitionin YM.   Reasons to be skeptical that it is meaningfully connected to confinement—even in Yang-Mills where there is a 1st-order phase transition between phases one with $\langle l \rangle$=0 ("confined") and one with $\langle l \rangle \neq 0$("deconfined").
  - In YM the Polyakov loop is for the color fundamental indicating confinement of quarks—in a theory that lacks quarks! The Polyakov loop for the color adjoint which tells about confinement of gluons is not an order parameter of theory
  - It is Euclidean property; no obvious physical interpretation in Minkowski space—and physics takes place in Minkowski space.
    - As a result, its value in non-equilibrium situations is obscure at best (and no physical process is ever perfectly equilibrated)

Of course, this view is heretical! So, for reasons of clarity I will refer to "deconfinement" based on $\langle l \rangle$, but remain skeptical of physical interpretations.

This heresy does not affect any conclusions in this talk

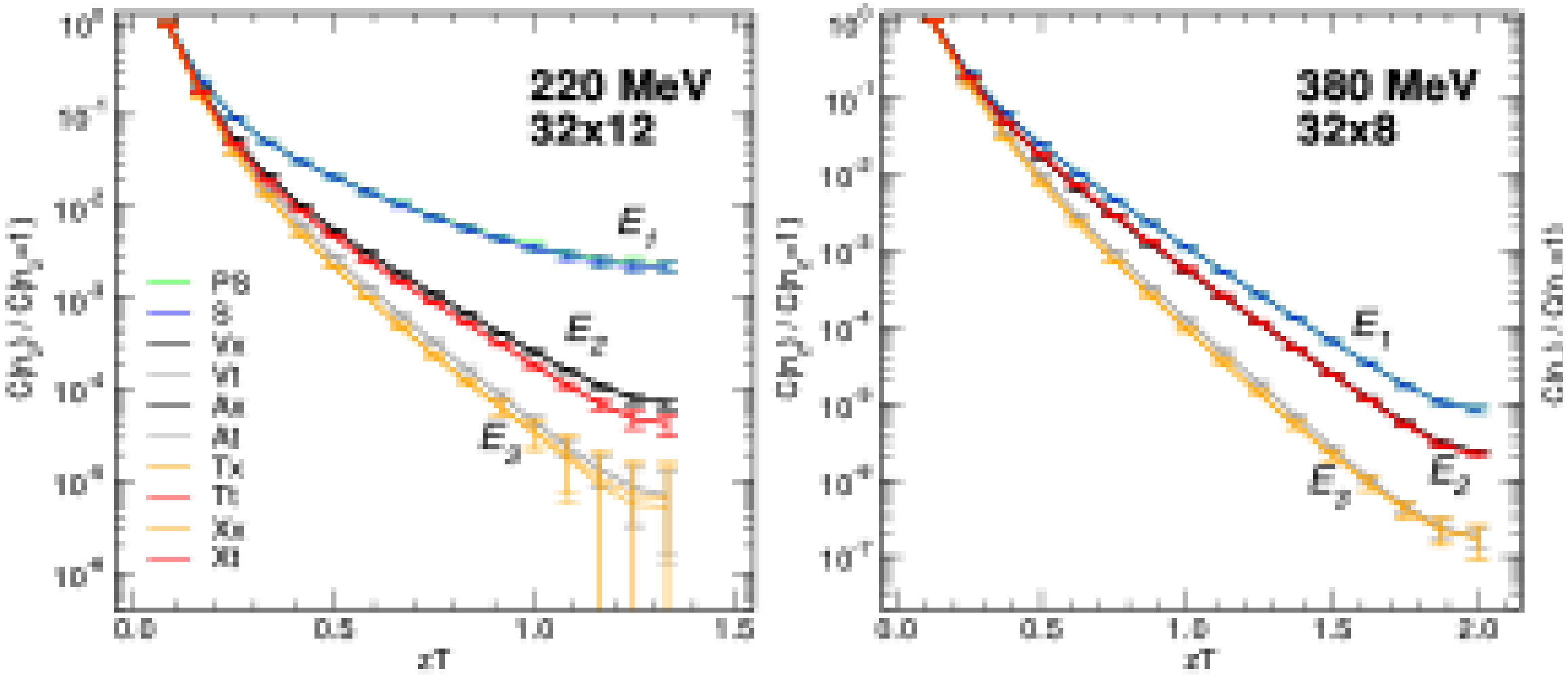
- **The possibility of a gap in temp between a hadronic regime and a QGP regime raise the possibility of the "Graz picture" of 3 regimes (**as discussed in detail in Lenya Glozman's talk).
    - **Evidence underlying this is from lattice studies of spatial correlators of various sources and three regimes were identified on the basis of patterns of near degeneracies of correlators of operators with distinct quantum numbers.**

Low T regime (below chiral cross-over temp T_{ch}) degeneracy pattern as expected from spontaneously broken chiral symmetry.

Intermedia regime (from T_{ch} to around $\sim 3T_{ch}$) correlators of operators connected by approx.chiral sym are nearly degenerated but also additional degeneracies for operators that are not connected by chiral symmetry.

High T regime (above $\sim 3T_{ch}$) correlators of operators connected by approx. chiral sym are nearly degenerate but other degeneracies disappear

- Intermediate regime
    - The lattice data show the existence of a novel pattern of degeneracies



"On chiral spin symmetry and the QCD phase diagram" Lattice 2022, O. Philipsen, L. Ya Glozman, P. Lowdon, R. D. Pisarski

- **Many correlators collapse approximately into 3: E_1 , E_2, & E_3 for intermediate temp**
- **The operators correlators in each band connected by "chiral spin" transformations which includes chiral symmetry but is larger. It is not a QCD symmetry**
- **At low temps the degeneracy pattern returns to that of isospin symmetry only, with spontaneous chiral symmetry breaking**
- **high temps the degeneracy pattern returns to that of ordinary unbroken chiral symmetry**

The Graz interpretation

- **Intermediate regime: “confined” regime, chirally symmetric, with a larger emergent “chiral spin” symmetry.**

What is its nature?

- **In Graz interpretation “chiral spin” is an approximate emergent symmetry of the system that is not a symmetry of QCD. (Assumes the pattern of degeneracies is due to a sym)**
- **Color magnet interactions between quarks (in Hamiltonian picture in Coulomb gauge) violate chiral spin symmetry while electric interactions respect it.**
- **Electric interactions in flux tube picture are responsible for confinement in the sense of linearly growing potential between color fundamental sources in Yang-Mills**
- **Flux tubes look like strings, so regime looks like one of a “stringy fluid”**

- **The lattice data clearly shows nearly identical correlators which is suggestive of an emergent approximate symmetry, but the nature of the intermediate regime should be viewed with some caution.**

One of the principal crocodiles threatening the Graz interpretation of a confined chirally-restored regime is that in QCD there is no order parameter for "confinement".

- Well-defined order parameters for χ-symmetry breaking and confinement exist in limits of QCD with $N_c=3$.
  - χ-symmetry breaking order parameter $\langle \bar{q} q \rangle$ is only a strict order parameter (zero in one phase and nonzero in the other) in the limit $m_q = 0$.
  - center-symmetry breaking order parameter $\langle P \rangle$ generally taken to indicate confinement is only a strict order in the limit $m_q = \infty$.

- These two limits are incompatible for QCD with $N_c=3$! If one pushes QCD toward a regime where one becomes reasonably well defined by altering quark masses the other becomes less well-defined suggesting that is intrinsically ambiguous to ask about both.

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  - These two limits are incompatible! If one pushes QCD toward a regime where one becomes reasonably well defined by altering quark masses the other becomes less well-defined suggesting that is intrinsically ambiguous to ask about both.

*The problem exists even if one disregards my heretical suspicion that $\langle P \rangle$ is not a legitimate indicator of confinement.

The large N_c limit

- **The large N_c is a theoretically clean way to evade this intrinsic ambiguity.**
  - **As $N_c \to \infty$ center symmetry → emergent symmetry; $\langle l \rangle \to$ a valid order parameter**
    - **This is a natural consequence of the fact that the number of gluon species $\sim N_c^2$ while the number of quark species $\sim N_c^1$.**
    - **Diagrammatically gluon loops dominate as $N_c \to \infty$ and every quark loop "costs" a factor of $\sim N_c^{-1}$.**
- **Combined $N_c \to \infty$ and $m_q \to 0$ limits allows one to ask whether theory is "confining" without invalidating asking whether χ– symmetry is broken ($\langle \bar{q}q \rangle \neq 0$) .**
  - **Confinement and χ–symmetry simultaneously have well-defined order parameters and are unambiguously present or absent**

- **Not the first time the $N_c \rightarrow \infty$ and $m_q \rightarrow 0$ limits used to probe confinement and χ symmetry simultaneously in clean way.**
  - **McLerran and Pisarski used large N_c arguments in the context of a non-zero μ to suggest a χ restored but confined quaryonic* phase (Nucl.Phys.A796, 83,2007)**
  - **The argument here is that lattice calculations at μ=0 and finite *T*, hint that a χ restored but confined phase could occur.**

*later incarnation of the idea of quarkyonic matter had χ broken in a non-translationally invariant way.

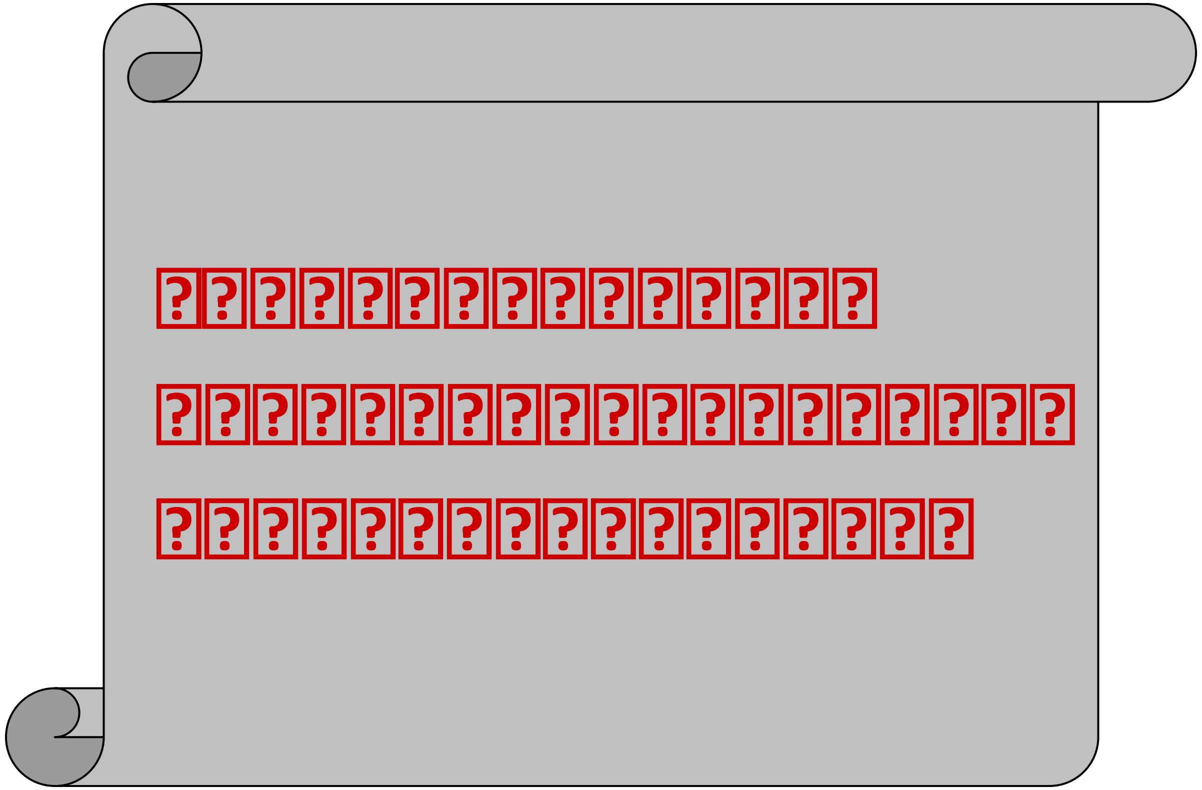
The combined $N_c \to \infty$ & $m_q \to 0$ limits

- **Rather than confining or chirally broken "regimes" one has "phases" in which the system either has $\langle l \rangle \neq 0$ or not and either has $\langle \bar{q}q \rangle \neq 0$ or not.**

- **If the large N_c world is similar to the real world of $N_c = 3$ one expects the existence or nonexistence of 3 distinct phases rather than regimes**

Low T Hadronic Phase with $\langle P \rangle = 0$ & $\langle \bar{q}q \rangle \neq 0$
An Intermedia Phase with $\langle P \rangle = 0$ & $\langle \bar{q}q \rangle = 0$
High T QGP Phase with $\langle P \rangle \neq 0$ & $\langle \bar{q}q \rangle = 0$

Caveat Emptor: the motto for $1/N_c$ practitioners



- The large N_c world may be a useful cartoon version of the physical world--but for any given observable there are no guarantees that $1/N_c$ corrections are small and the cartoon may look nothing like the physical world.
- The cartoon is quite recognizable for many observbles—but not all.
- An example: In $N_c \to \infty$ limit, QCD has an arbitrarily large number of narrow glueballs, unmixed with mesons, distinct phenomenologically. The PDG indicates no unambiguous glueball states; the few candidates are not narrow and substantially mixed.

Properties of mesons* & glueballs large Nc QCD in the hadronic phase (from TDC in Encylopedia of Particle Physics)

Glueball N_c-scaling properties	scaling
Generic n-glueball interaction amplitude	N_c^{2-n}
Mass	N_c^0
Width	N_c^{-2}
2-body scattering cross section	N_c^{-4}

Properties of the glueball spectrum	
Number of glueball states	∞
$N(m)$ (Number of glueballs with mass< m)	Hagedorn spectrum

*Mesons include quantum number exotic hybrids

Meson N_c-scaling properties	**Leading scaling**
Generic *n*-meson interaction amplitude	$N_c^{1-n/2}$
Mass	N_c^{0}
Width	N_c^{-1}
2-body scattering cross section	N_c^{-2}
Properties of the meson spectrum	
Number of meson states	∞
$N(m)$ (Number of mesons with mass$<m$)	Hagedorn spectrum

Mixed glueball-meson N_c-scaling properties	**Leading scaling**
Generic n_g-glueball, n_m-meson interaction amplitude	$N_c^{1-n_g-\frac{1}{2}n_m}$
Glueball-meson mixing amplitude	$N_c^{\frac{1}{2}}$

- **Implications of $N_c \to \infty$ for hadronic phase**
  - Hadrons masses scale as N_c^0
  - Expectation values of connected n-point functions of quark bilinears (such as $\langle \bar{q}q \rangle$) scale as N_c^1
  - Expectation values of of connected n-point functions of single-color-trace gluon operators (such as $\langle F^{\mu\nu} F_{\mu\nu} \rangle$) scale as N_c^2
  - Hadrons do not interact with each other, thus:
    - Energy density, pressure & entropy density are given by standard Boltzmann distributions for free noninteracting hadrons with masses $\sim N_c^0$. (We know that the temp of this phase is order N_c^0)

$$\epsilon_{\rm had} \sim N_c^0 \;,\; P_{\rm had} \sim N_c^0 \;,\; s_{\rm had} \sim N_c^0$$

    - Connected n-point functions of single-color-trace gluon operators & quark operators are independent of T in hadronic phase!
      - This is consistent with χPT where $m_q = 0$ & $F_\pi \sim N_c^{1/2}$

$$\frac{\Sigma(T)}{\Sigma(0)} = 1 - \frac{T^2}{8F_\pi^2} - \frac{T^4}{384 F_\pi^4} - \cdots$$

- Implications of $N_c \to \infty$ for high T QGP phase
  - QCD evolution equations work as $N_c \to \infty$ and imply that Λ_{QCD} scales as N_c^0
    - QCD becomes weakly coupled at a temperature of order N_c^0 and thermal quantities reliably calculated. In this regime:

$$\epsilon_{QGP} \sim N_c^2 \ , \ P_{QGP} \sim N_c^2 \ , \ s_{QGP} \sim N_c^2$$

    - This implies that there must be (at least one) phase transition between the hadronic phase and the QGP phase due to the distinct scaling.
      - The order of the transition is not determined by scaling argument
      - Very strong numerical evidence (Oxford Group) evidence that it the transition to the QGP phase is 1st-order—regardless of whether QCD at large N_c has 2 or 3 phases.

- In fact, there is some remarkable and surprising thermal behavior at large N_c

An amusing aside

Before turning to the central question of a putative intermediate phase, it worth looking at the remarkable consequence of having a first-order transition to a QGP phase.

- **In the $N_c \to \infty$ world, there is a first-order transiton so the QGP phase can super cool.**

- **Remarkably the metastable super-cooled phase will must include temperatures at which the pressure is negative in an absolute sense, i.e. the pressure is less than the vacuum.** (T .D. Cohen S. Lawrence and Y. Yamauchi *Phys.Rev.C* 102 (2020) 6, 065206)

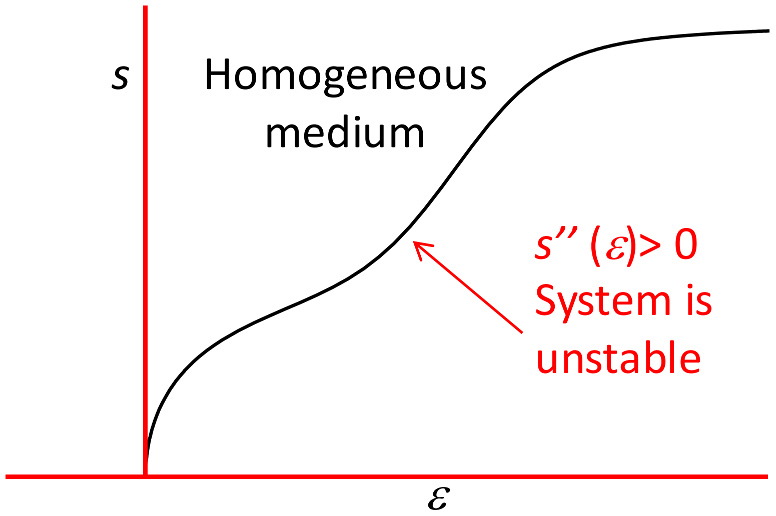
  - **The basic argument holds whether there are 2 or 3 phases—so for simplicity lets look at 2.**
  - **Minimally, the argument illustrates just how different the $N_c \to \infty$ world can differ from ours**

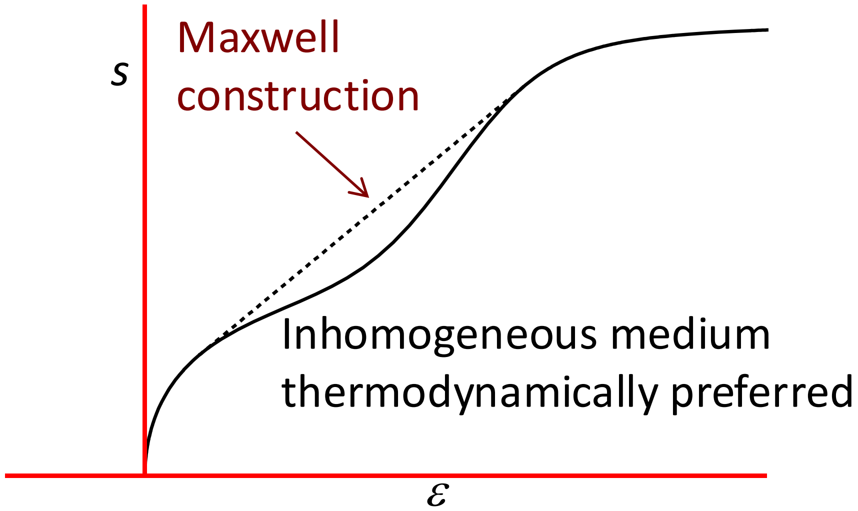
- **Use the microcanonical ensemble as this provides the most insight for this issue.**

  - **Key quantity $S(E)$ where $S(E)$ is the log of the number of accessible states at E.**
  - **$S'(E)=1/T$**
  - **In thermodynamic limit of large volumes relevant quantities are entropy density, s, and energy density ε:**

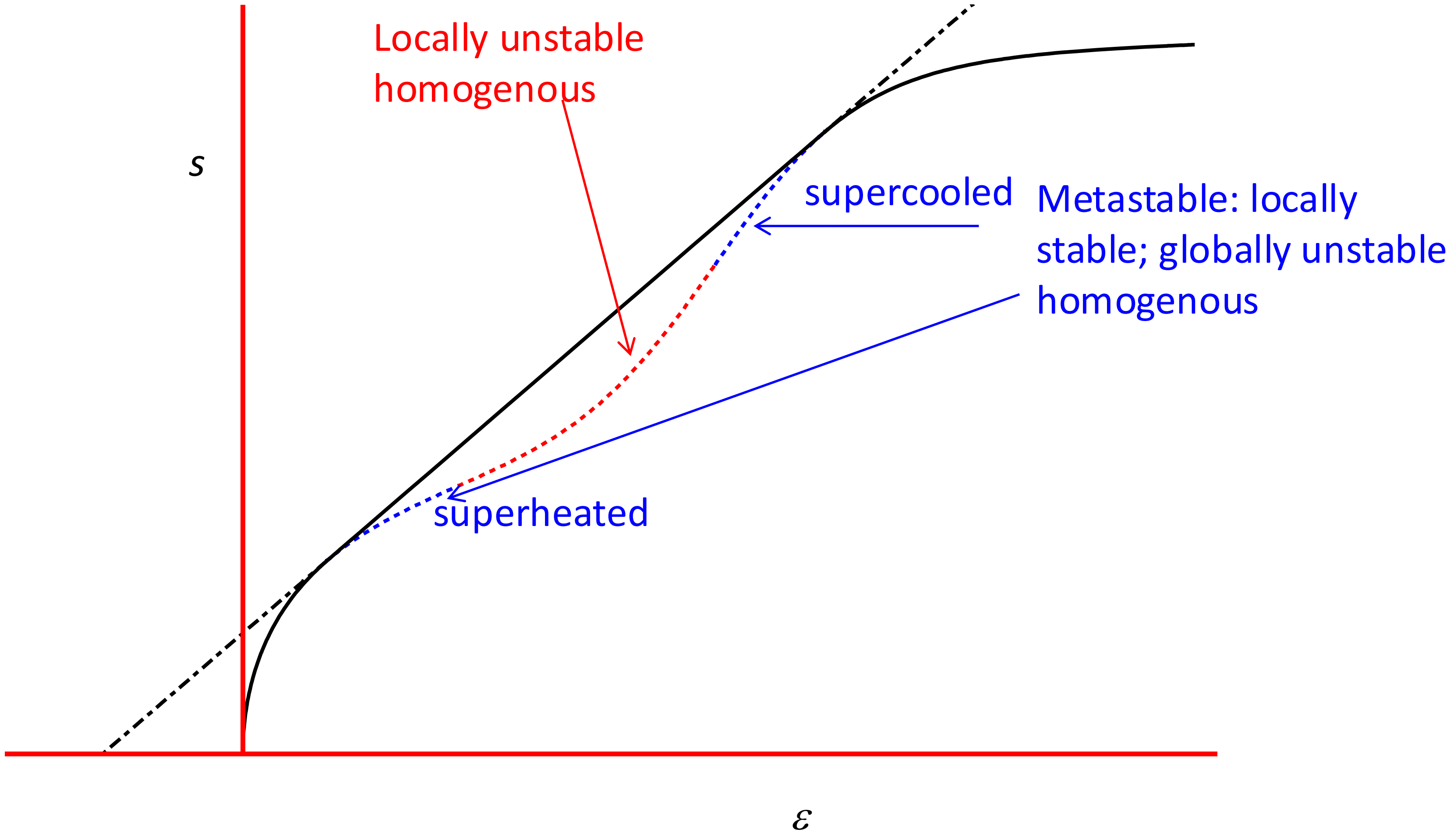
  $$s(\varepsilon) = \mathrm{Lim}_{V \to \infty} \, S(\varepsilon V)/V$$

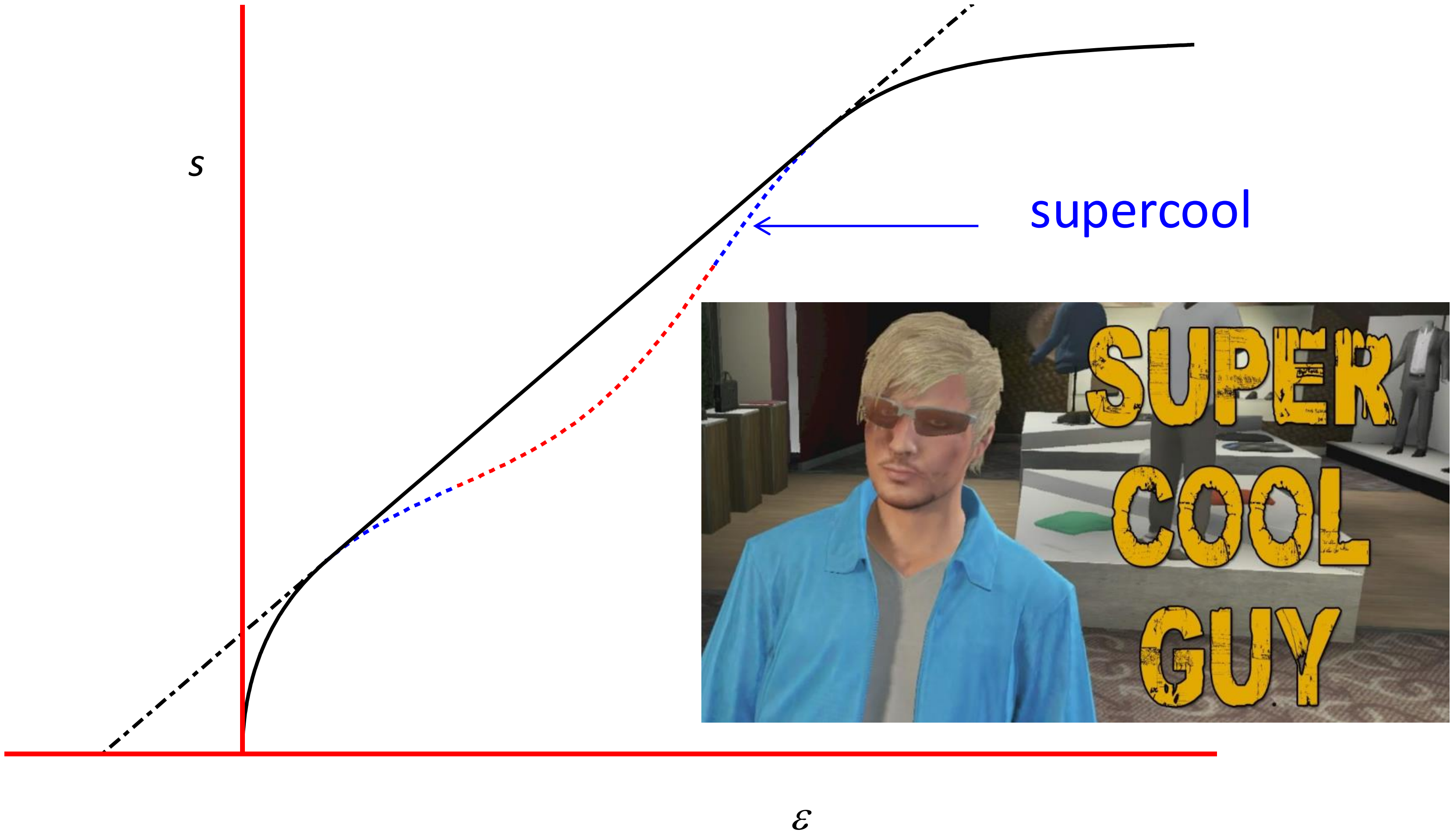
  - **Thermodynamic stability implies $s''(\varepsilon) \leq 0$.**

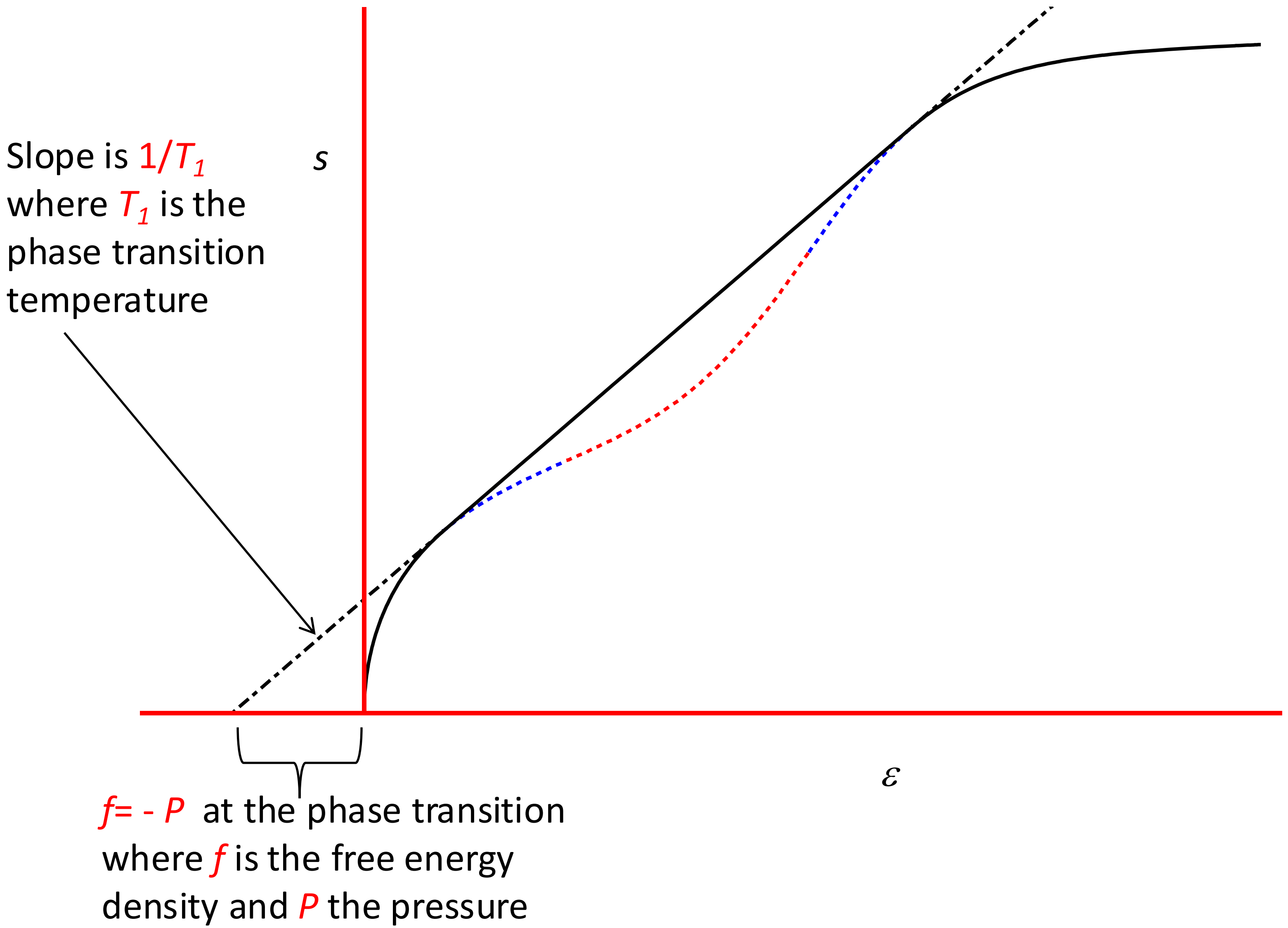




Generic first order transition



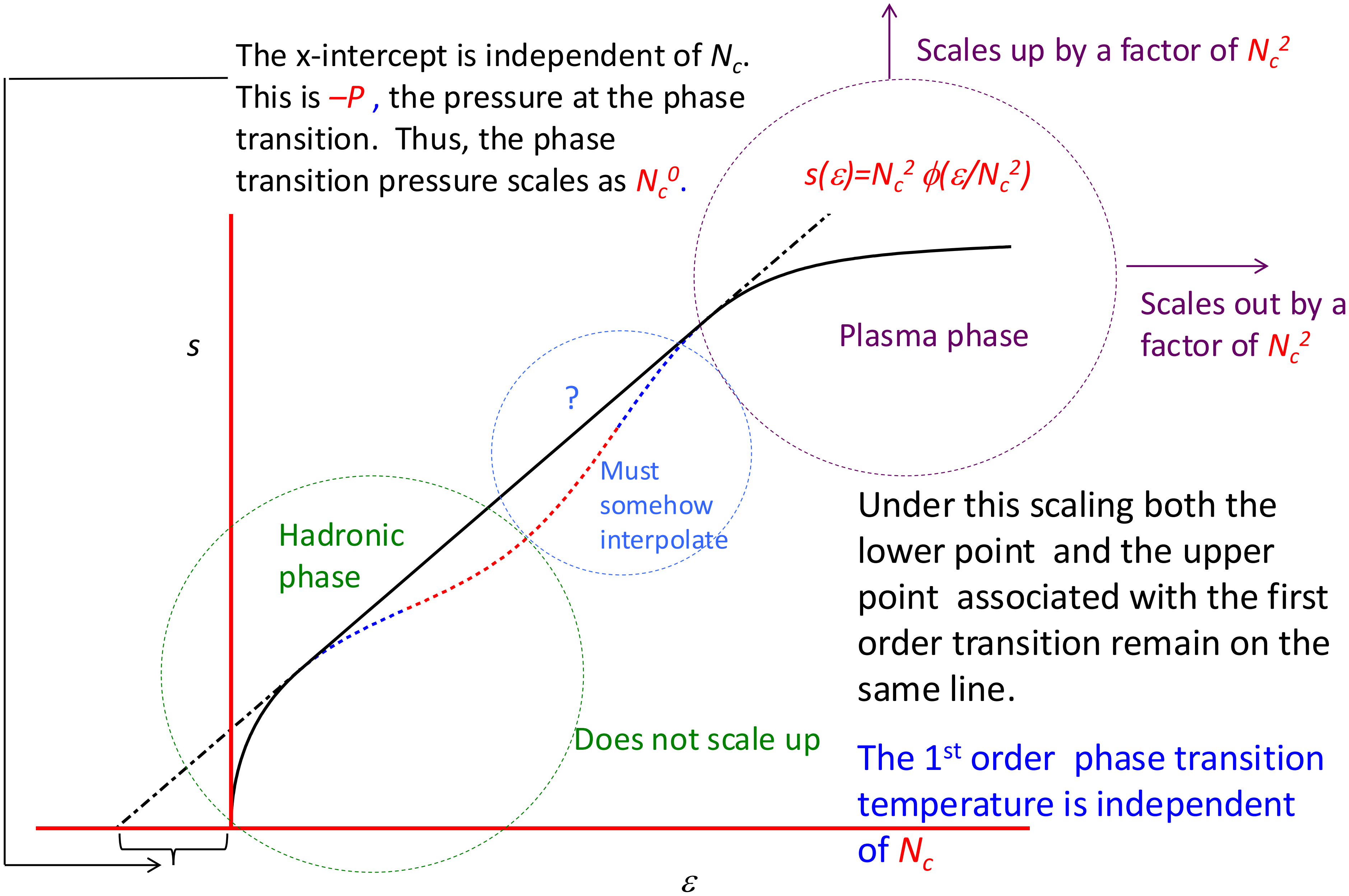

s
supercool
SUPER
COOL
GUY
ε



Slope is 1/T_1
where T_1 is the
phase transition
temperature
s
ε
f= - P at the phase transition
where f is the free energy
density and P the pressure

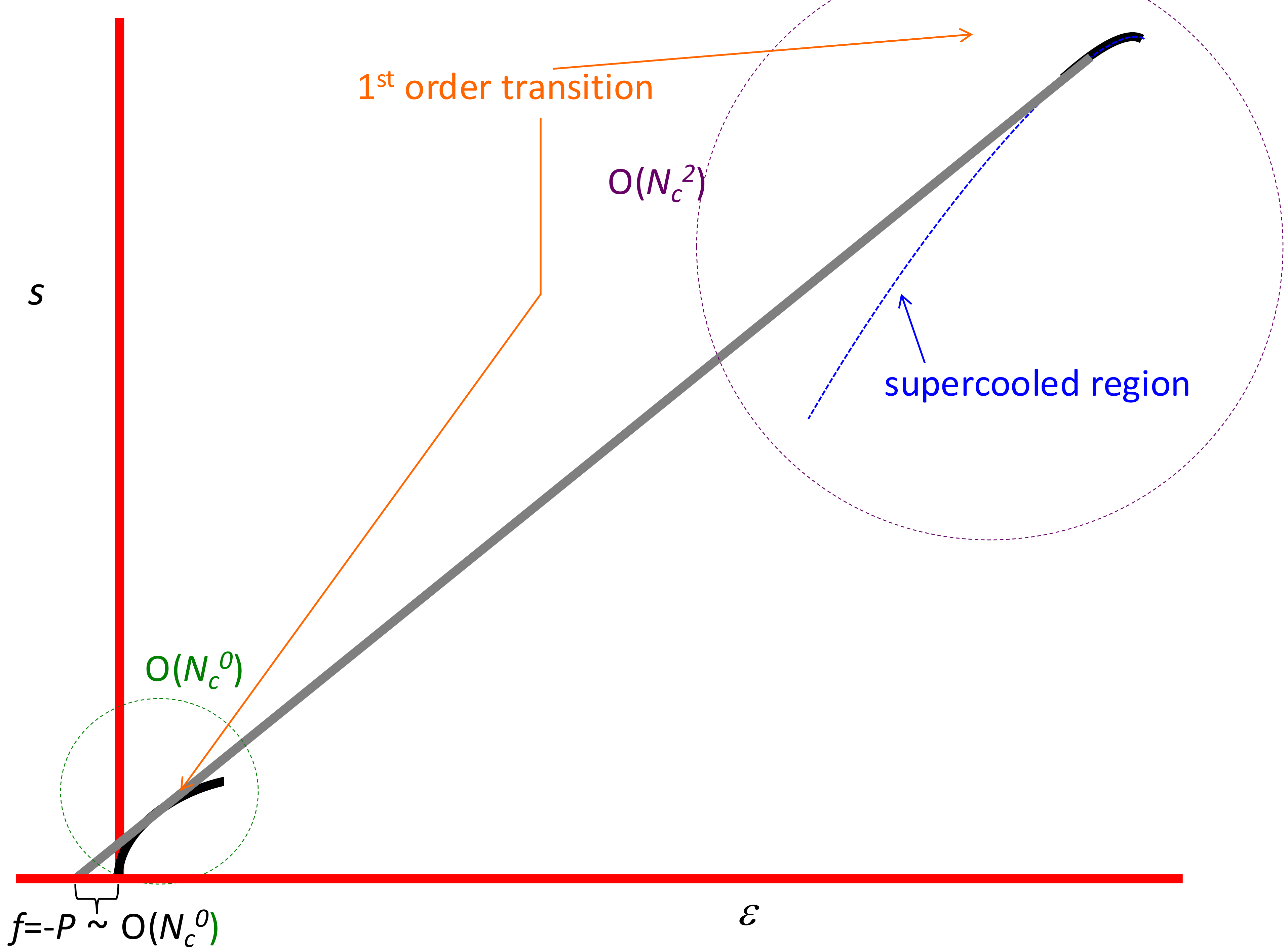
What happens at large N_c ?

The x-intercept is independent of N_c. This is $-P$, the pressure at the phase transition. Thus, the phase transition pressure scales as N_c^0.

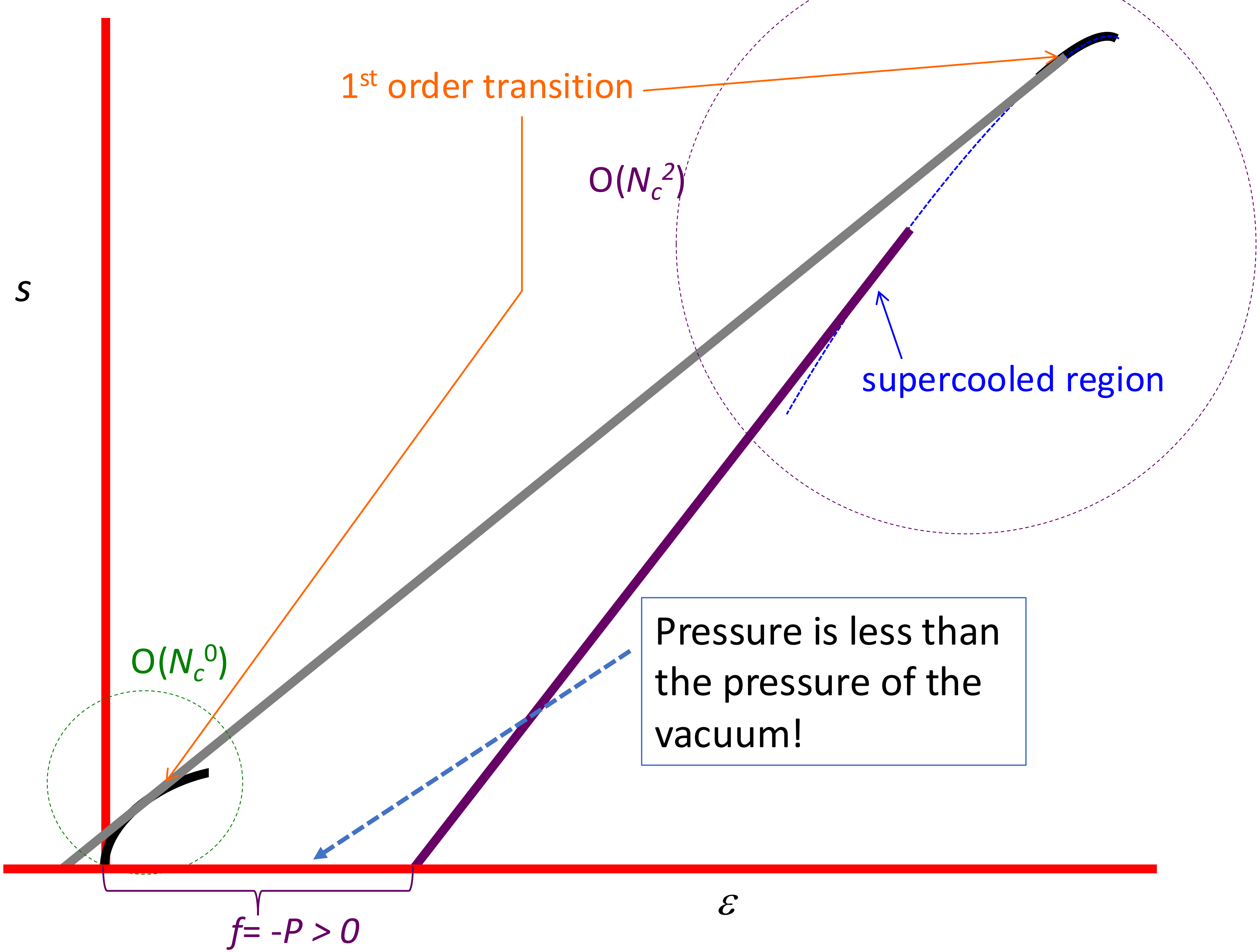


Under this scaling both the lower point and the upper point associated with the first order transition remain on the same line.

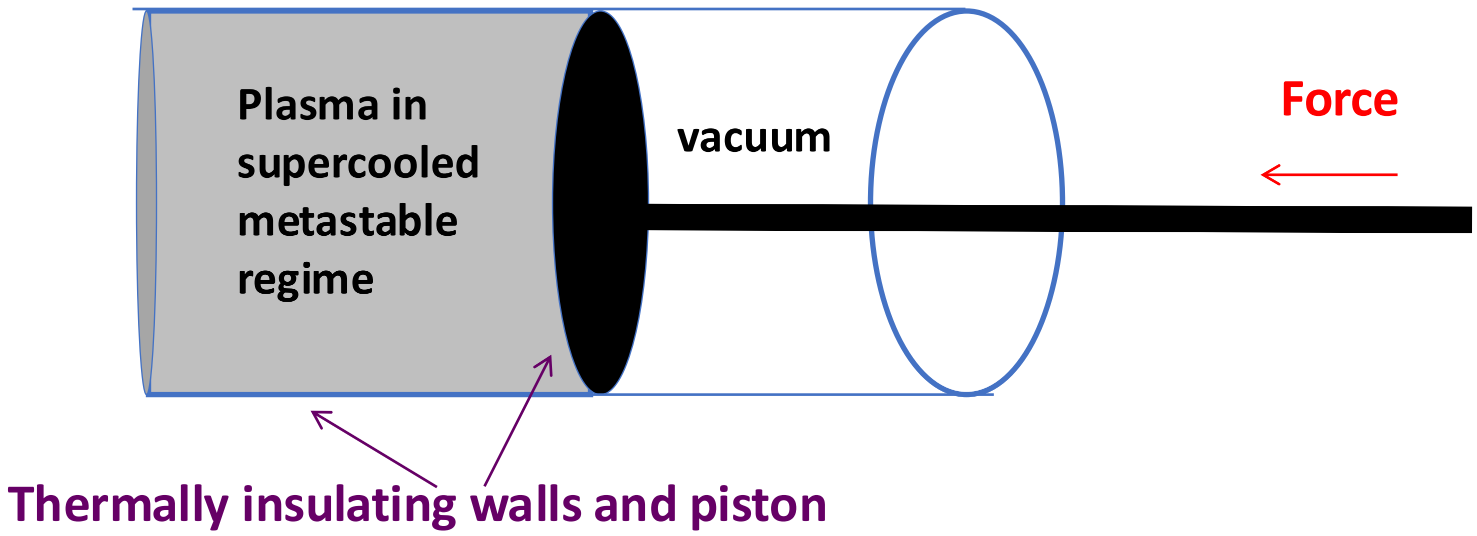
The 1st order phase transition temperature is independent of N_c



1st order transition
O(Nc2)
s
supercooled region
O(Nc0)
f=-P ~ O(Nc0)
ε

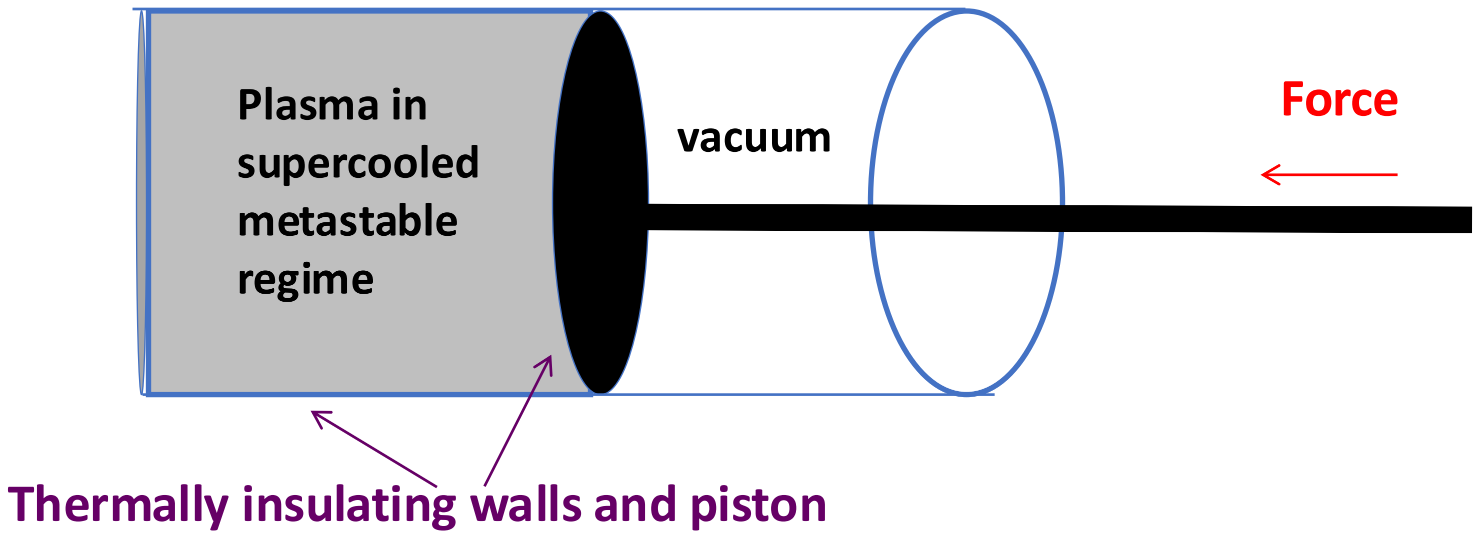

1st order transition
$O(N_c^2)$
s
supercooled region
$O(N_c^0)$
Pressure is less than the pressure of the vacuum!
ε
$f = -P > 0$

Negative absolute pressure is remarkable



The medium is not just weird—it sucks!

Negative absolute pressure is remarkable



The medium is not just weird—it sucks!

Indicates a breakdown of intuition based on kinetic theory: whatever this medium is, the pressure is not describable in terms of particles or quasiparticles that strike the wall, transferring momentum and imparting an outward pressure. **This requires a strongly coupled theory where quasiparticle motion is not dominant.**

Return to the main issue: the possibility of an intermediate phase in $\boldsymbol{N_c \to \infty \,,\, m_q \to 0}$ limits

Caveat Emptor: the motto for $1/N_c$ practitioners

No guarantee that for the issue of interest here the $N_c \to \infty$ world is similar to $N_c=3$.

Possible scenarios in $N_c \to \infty$ limit:

1. **Intermediate regime disappears.**
2. **Two phases with a 1st -order transition. High T phase has 2 qualitative regimes with a cross-over from one with "hadron-like" structures to a QGP regime.**
3. **Three phases with a 2nd -order transition from hadronic to intermediate and a 1st -order transition to QGP phase**
4. **Three phases with two 1st -order transitions separating them**

3 & 4 are the Interesting possibilities

N_c scaling of thermodynamic properties for the 3 phases

$\epsilon_{\rm had} \sim N_c^0$, $P_{\rm had} \sim N_c^0$, $s_{\rm had} \sim N_c^0$ — Hadronic Phase

$\epsilon_{\rm int} \sim N_c^1$, $P_{\rm int} \sim N_c^1$, $s_{\rm int} \sim N_c^1$ — Intermediate Phase

$\epsilon_{\rm QGP} \sim N_c^2$, $P_{\rm QGP} \sim N_c^2$, $s_{\rm QGP} \sim N_c^2$ — QGP Phase

$\epsilon_{\rm int} \sim N_c^1$, $P_{\rm int} \sim N_c^1$, $s_{\rm int} \sim N_c^1$ **Intermediate Phase**

Why this scaling of putative intermediate phase?

As $N_c \to \infty$ thermodynamics at the scale of N_c^2 comes solely from gluons.
At $O(N_c^2)$ thermodynamic quantities in QCD and YM are Indistinguishable

- **YM: 1st –order transition to "deconfined" phase, below which all gluonic operators are unchanged from vacuum values → thermodynamic quantities differ from hadronic phase by at most $O(N_c^1)$**

In hadronic phase in

- **Expectation values of connected n-point functions of quark bilinears (such as $\langle \bar{q} q \rangle$) scale as N_c^1.**
- **Expectation values are independent of *T***
- **By hypothesis, intermediate phase is chirally restored: $\langle \bar{q} q \rangle = 0$ → so quark bilinears must shift by $O(N_c^1)$ → so thermpdynamics quantities (which depend on quark bilinears, e g. $\bar{q} \not{D} q$) shift by $O(N_c^1)$**

- An open question about the the intermediate phase at large N_c:

- Is chiral spin an exact or approximate symmetry in the intermediate phase of the combined $N_c \to \infty$, $m_q \to 0$ limits?
    - Previous argument focused on the change in the chiral condensate.
    - It may seem plausible that if a chirally restored intermediate phase exists that the chiral spin symmetry becomes exact in combined limit, but there are also reasons to doubt it
        - In either case if $N_c \to \infty$ world is reasonable cartoon of N_c=3 connected correlators of quark-binear operators (which are generically $O(N_c^1)$) are very different from in the hadronic phase.
        - Reinforces argument that operators shift by $O(N_c^1)$ in the intermediate phase.

- A peculiarity of the intermediate phase at large N_c:
  - Ordinary mesons disappear and are replace by some other dynamics.
  - Glueballs remain; properties the same as in the hadronic phase (with relative order N_c^{-1} corrections)
    - At leading order YM and QCD are identical for purely gluonic observables, *eg*. correlators of glueball operators.
    - In Yang-Mills at large N_c these observables independent of T until the "deconfinement" transition; remains true for QCD.
    - This reflects the different scaling rules for glueballs and mesons.

  - This is an issue which is irrelevant, at least directly, for $N_c=3$, since glueballs play very little role in the hadronic spectra.
    - But is a reminder that the large N_c limit is not necessarily a useful guide.

- **Order of transition between hadronic and intermediate phases :**
  - **Arguments so far do not distinguish whether scenario 3 or 4 (i.e. 2nd- vs 1st –order transition) is more plausible.**
  - **There is however a strong reason to believe the transition is 1st-order.**
    - **Recall that in hadronic phase the hadrons do not interact. So naïve non-interacting hadron resonace gas describes thermodynamics.**

$$\epsilon_{\rm had}(T) = \sum_k (2S_k+1)(2I_k+1)\int \frac{d^3p}{(2\pi)^3}\,\frac{\sqrt{p^2+m_k^2}}{e^{\sqrt{p^2+m_k^2}/T}-1}$$

- In a naïve non-interacting hadron resonance gas describes Implies that a 2nd-order transition cannot take place– nonalayticity in thermodynamic quantities are absent

unless the spectrum itself causes noanalyticity which requires the number of hadrons to grows exponentially (Hagedorn spectrum)

  - Asymptotically $N(m)$ number of hadrons with mass less than m is given by $N(m) = P(m)\exp\left(\frac{m}{T_H}\right)$ where $P(m)$ is subexponential and T_H is a parameter (the Hagedorn temp)

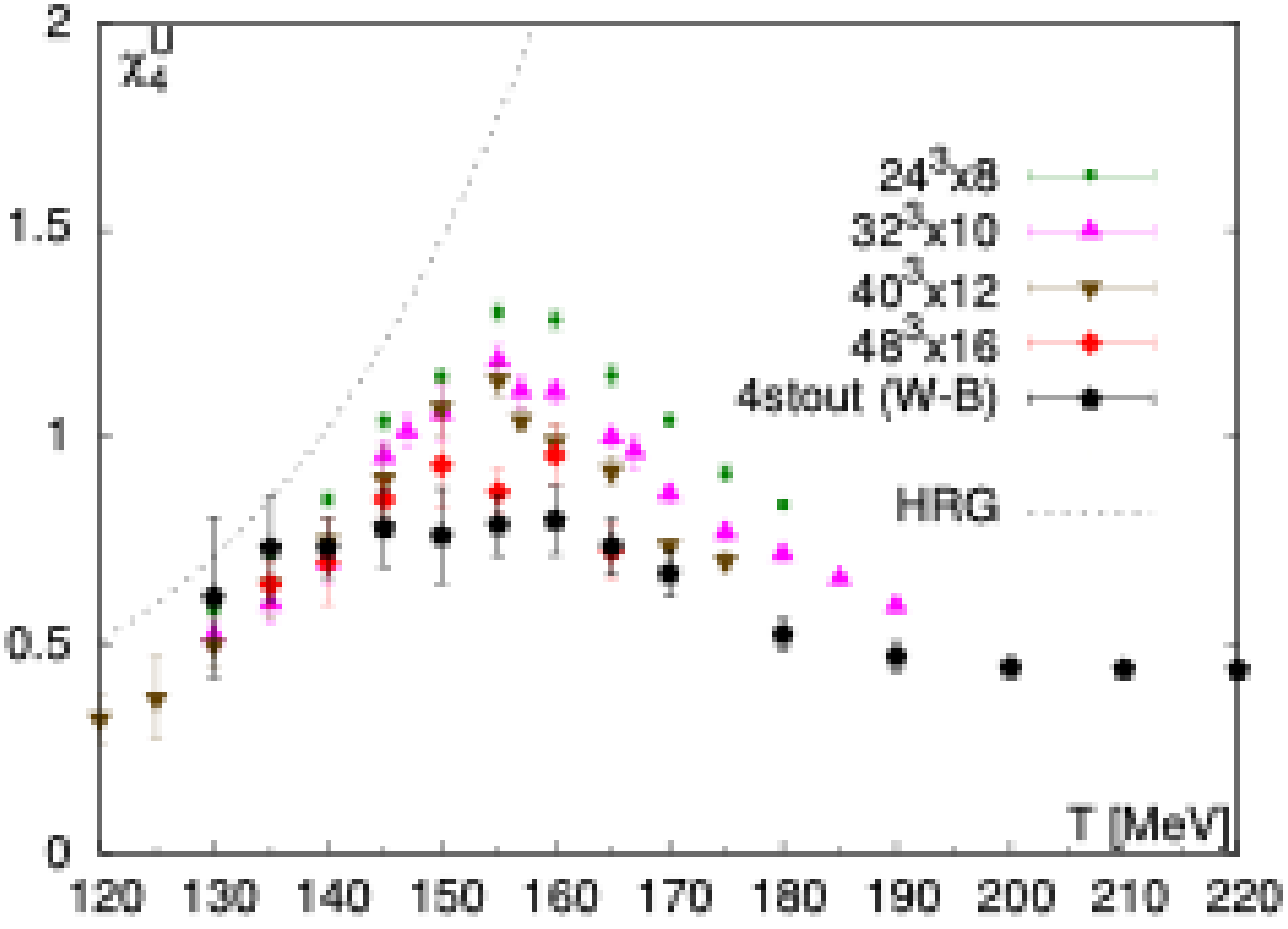
- Good model-independent reasons to suspect that QCD as $N_c \to \infty$
(TDC *JHEP* 06 (2010) 098)

- But Hagedorn spectrum does not *require* a 2nd –order transition.
  - It implies that either a 2nd –order transition takes place (at $T=T_H$) or a 1st –order transition happens at ($T<T_H$) .

- Expect 1st-order. Natural picture for a Hagedorn spectrum as $N_c \to \infty$ is a string picture.
  - Flux tubes don't break at large Nc; highly excited states:long fluxtubes; act "stringy".
    - Closed strings→glueballs ; open strings→ mesons
    - Highest lying states should be described by a string theory.
  - String theories have Hagedorn spectra.
    - In string theory T_H is the same for open and closed strings
  - Phenomenologically we know that as $N_c \to \infty$ there is a 1st –order transition to a QGP phase at T_{QGP} ; $T_{QGP} < T_H$
    - By hypothesis $T_{Inter} < T_{QGP}$ & $T_{QGP} < T_H$ So $T_{Inter} < T_H$
    - 2nd –order is ruled out if high-lying spectra given by string theory as $N_c \to \infty$

Fluctuations of conserved charges

- Taken as signature of deconfinement
- Occurs at or near χ transition
- But observable is $O(N_c)$ not $O(N_c^2)$
- If intermediate phase exists it is naturally associated with transition to intermediate phase.



$$\chi^{u,d,s}_{i,j,k} = \frac{T\partial^{i+j+k}\left(P/T^4\right)}{(\partial\mu_u)^i(\partial\mu_d)^j(\partial\mu_s)^k}.$$

Caveats

- As noted previously, there is no guarantee that the qualitative phenomenological features seen in the regimes at $N_c = 3$ persist as phase when $N_c \to \infty$.
- Formal issue: Is an intermediate phase is possible $N_c \to \infty$?
  - General arguments 4-volume independence (E-K) of $N_c \to \infty$ QCD (which implies T independence) holds when center symmetry is unbroken—in confining phases. (P. Kovtun, M) Unsal, L. Yaffe, JHEP 0706:019,2007)
    - Appears to rule out the intermediate phase (or quarkyonic matter)
    - But while formal argument is “straightforward” for gluonic operators, it requires subtle ordering of limits when fundamental matter is included
      - No careful analysis has been conducted as to whether the argument breaks down if a phase transition were to occur for dynamics at $O\left(N_c^1\right)$ and there are strong reasons to believe it will

Summary

- There is strong numerical evidence for three distinct regimes for QCD at μ=0 including an intermediate region
  - The intermediate region appears to be both "confining" ($\langle l \rangle$=0) and chirally restored (with an approximate "chiral spin-symmetry")
  - But, the notion of "confinement" in the sense of $\langle l \rangle$=0 and chiral restoration are in tension; they correspond to incompatible limits of QCD at N_c=3.

- The combined $N_c \to \infty$, $m_q \to 0$ limit is a theoretically clean way to have both simultaneous.
  - To the extent that the large N_c is a good cartoon of ours it provides a way to make sense of the intermediate regime
  - The interesting possibility is that the regimes become phases and there are strong reasons to believe at there are 1st-order transitions between them

Summary

- **Asides**
  - **In the $N_c \to \infty$ world there exists a supercolled QGP regime with with neative absolute pres**

  - **There are reasons to doubt whether the Polyakov loop is really a useful indicator of confinement.**