Confinement and symmetry from vacuum to QCD phase diagram

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Center symmetry and confinement in Yang-Mills

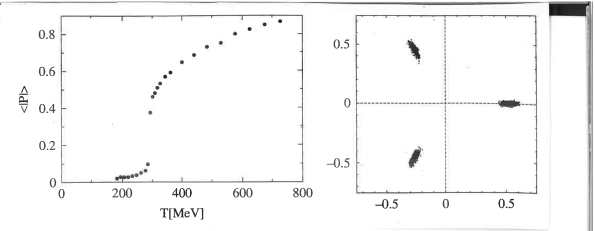
At T=0: Wilson loop and its area law: Wilson, 1974.
Observation of a linear confining potential between static quarks on the lattice - Creutz, 1977. Formation of a flux tube.

Confinement can be connected to the center Z_{N_c} symmetry of the pure glue action; a Polyakov loop is an order parameter: at $T < T_d$ the center symmetry is unbroken; at $T > T_d$ the center symmetry is spontaneously broken. Polyakov, 1978

In the deconfined phase deconfined gluon degrees of freedom should be relevant and thermodynamical quantities scale as $N_c^2 - 1$. An observable that is sensitive to $N_c^2 - 1$ gluons is the Polyakov loop:

$$P(\mathbf{x}) = \frac{1}{N_c} Tr \left[T \exp \left(i \int_0^\beta d\tau A_0(\mathbf{x}, \tau) \right) \right].$$

The first order deconfinement phase transition around $T_d \sim 300$ MeV (Fig. from Gattringer, Lang book "Quantum chromodynamics on the lattice"):



Symmetries of electric charge and electric interaction in QED and QCD

Do we have a symmetry with light quarks that is sensitive to confining electric interaction? Yes!

$$
\begin{aligned}
div\mathbf{E} &= 4\pi\rho \\
rot\mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} &= \frac{4\pi}{c}\mathbf{j} \\
rot\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} &= 0 \\
div\mathbf{B} &= 0
\end{aligned}
$$

How do we define **E** and **B** in a given Lorentz frame?

$$\mathbf{F} = q\mathbf{E} + q\frac{\mathbf{v}}{c} \times \mathbf{B}$$

Possible to measure directly **F** in electrodynamics but not possible in quantum chromodynamics. Is there another method to distinguish **E** and **B**? Yes!

Consider charges to be particles with $s = 1/2$. They are characterized by helicities (chiralities for massless particles):

$$\begin{pmatrix} R \\ L \end{pmatrix}$$

Consider a $SU(2)_{CS}$ chiral spin transformation that mixes R and L (L.Ya.G., 2015):

$$\begin{pmatrix} R \\ L \end{pmatrix} \to \begin{pmatrix} R' \\ L' \end{pmatrix} = \exp\left(i\frac{\varepsilon^n \sigma^n}{2}\right)\begin{pmatrix} R \\ L \end{pmatrix}$$

Dirac eq. prohibits such transformation.

What happens with the charge density ρ?

$$R'^{\dagger}R' + L'^{\dagger}L' = R^{\dagger}R + L^{\dagger}L$$

i.e.

$$\rho' = \rho$$

Charge density is invariant under the chiral spin transformation.

What happens with the current density $\mathbf{j} = \rho\mathbf{v}$? Upon the chiral spin transformation $\mathbf{v}$ and $\mathbf{j}$ change.

$$\mathbf{F}_E = q\mathbf{E}$$

$$\mathbf{F}_B = \sim \mathbf{j} \times \mathbf{B}$$

The interaction of a charge with the electric field is invariant under $SU(2)_{CS}$, while the interaction of a current with the magnetic field is not.

We can distinguish the electric and magnetic fields by the chiral spin symmetry. The electric part of the EM theory is more symmetric than the magnetic part.

IT IS A GAUGE-INVARIANT STATEMENT.

$$\mathcal{L} = \mathcal{L}(\mathbf{E},\mathbf{B}) - \rho\phi + \mathbf{j}\cdot\mathbf{A} + \quad Dirac \quad Lagrangian$$

The Dirac Lagrangian is not invariant under the $SU(2)_{CS}$.

The chromoelectric field of QCD is defined via interaction with the color charge

$$\mathbf{F} = Q^a\mathbf{E}^a; \qquad Q^a = \int d^3x\ q^\dagger(x)T^a q(x), \qquad a = 1,...,8$$

It is invariant under $SU(2)_{CS}$:

$$q \to q' = \exp\left(i\frac{\varepsilon^n\Sigma^n}{2}\right)q, \qquad \Sigma = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}$$

The transformation can be local.

$$\overline{q}(x)\gamma^\mu T^a q(x)\ A^a_\mu = q^\dagger(x)T^a q(x)\ A^a_0 + \overline{q}(x)\gamma^i T^a q(x)\ A^a_i$$

In a given Lorentz frame interaction of quarks with the electric part of the gluonic field is chiral spin invariant like in electrodynamics.

$SU(2)_{CS} \times SU(N_F) \subset SU(2N_F)$; $SU(2N_F)$ is also a symmetry of the color charge.

The color charge and electric part of the theory have a $SU(2N_F)$ symmetry that is larger than the chiral symmetry of QCD as a whole.

The fundamental vector of $SU(2N_F)$ at $N_F = 2$

$$\Psi = \begin{pmatrix} u_{\mathrm{R}} \\ u_{\mathrm{L}} \\ d_{\mathrm{R}} \\ d_{\mathrm{L}} \end{pmatrix} .$$

L.Ya.G., EPJA, 2015
L.Ya.G., M.Pak,PRD, 2015
L.Ya.G., Prog. Part. Nucl. Phys., 131 (2023) 104049

The $SU(2)_{CS}$ and $SU(2N_F)$ are explicitly broken by magnetic interaction, by quark kinetic term and by the quark condensate. They can be seen if and only if the breaking effect is small.

Since the confining interaction is related to formation of the electric flux tube, we can consider the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries as symmetries of the confining interaction with ultrarelativistic light quarks.

Compare with the center symmetry.

CS and $SU(4)$ multiplets

$(0,0)$	$f_1(0,1^{++})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^i)\Psi$		$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^i)\Psi$
$(1/2,1/2)_a$	$b_1(1,1^{+-})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^4\gamma^i)\Psi$	$\xleftrightarrow{SU(2)_A}$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^4\gamma^i)\Psi$
	$U(1)_A$		$U(1)_A$
$(1/2,1/2)_b$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^4\gamma^i)\Psi$	$\xleftrightarrow{SU(2)_A}$	$h_1(0,1^{+-})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^4\gamma^i)\Psi$
$(1,0)\oplus(0,1)$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^i)\Psi$	$\xleftrightarrow{SU(2)_A}$	$a_1(1,1^{++})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^i)\Psi$

$(0,0)$	$f_1(0,1^{++})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^i)\Psi$	$SU(4)$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^i)\Psi$	$SU(2)_{CS}$
$(1/2,1/2)_a$	$b_1(1,1^{+-})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^4\gamma^i)\Psi$	$\longleftrightarrow$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^4\gamma^i)\Psi$	
$SU(2)_{CS}$				
$(1/2,1/2)_b$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^4\gamma^i)\Psi$	$\longleftrightarrow$	$h_1(0,1^{+-})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^4\gamma^i)\Psi$	
$(1,0)\oplus(0,1)$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^i)\Psi$	$\longleftrightarrow$	$a_1(1,1^{++})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^i)\Psi$	

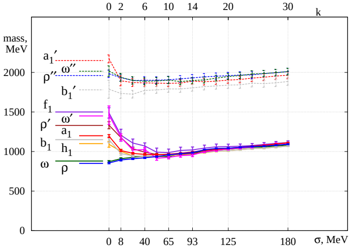
Observation of the chiral spin symmetry and its implications for hadrons

Banks-Casher:

$$i\gamma_\mu D_\mu \psi_n(x) = \lambda_n \psi_n(x), \qquad <\bar{q}q> = -\pi\rho(0).$$

Low mode truncation, M.Denissenya, L.Ya.G., C.B.Lang, 2014-2015:

$$S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$



$SU(2)_{CS}$ and $SU(4)$ symmetries.

The magnetic interaction is located predominantly in the near zero modes while the confining electric interaction is distributed among all modes. Confinement and chiral symmetry breaking are not directly related.

A clear implication for the genesis of hadron spectra: the $SU(4)$ degeneracy of the electric string is lifted by chiral symmetry breaking.

Minkowski QCD Hamiltonian in Coulomb gauge:

$$H_{QCD} = H_E + H_B + \int d^3x \Psi^\dagger(\mathbf{x})[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla}]\Psi(\mathbf{x}) + H_T + H_C,$$

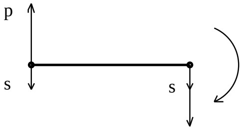
with the transverse and instantaneous "Coulombic" interactions to be:

$$H_T = -g \int d^3x \, \Psi^\dagger(\mathbf{x})\boldsymbol{\alpha} \cdot t^a \mathbf{A}^a(\mathbf{x}) \, \Psi(\mathbf{x}) \ ,$$

$$H_C = \frac{g^2}{2} \int d^3x \, d^3y \, J^{-1} \ \rho^a(\mathbf{x}) F^{ab}(\mathbf{x}, \mathbf{y}) \, J \, \rho^b(\mathbf{y}) \ .$$

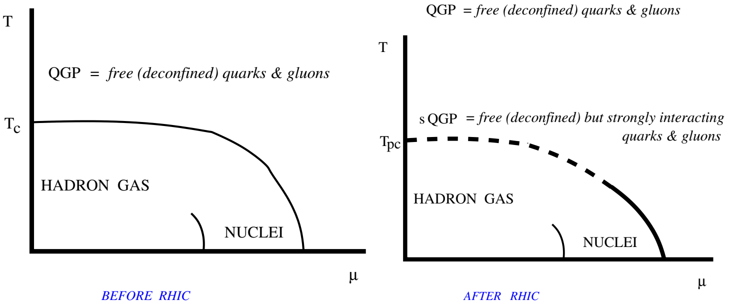
The confining "Coulombic" part is $SU(2N_F) \times SU(2N_F)$-symmetric. A symmetry of confinement in QCD with light quarks is $SU(2N_F) \times SU(2N_F)$

L.Ya.G., Prog. Part. Nucl. Phys., 131 (2023) 104049



Hot QCD. Before and after RHIC

What happens with hadrons in a medium upon increasing T?



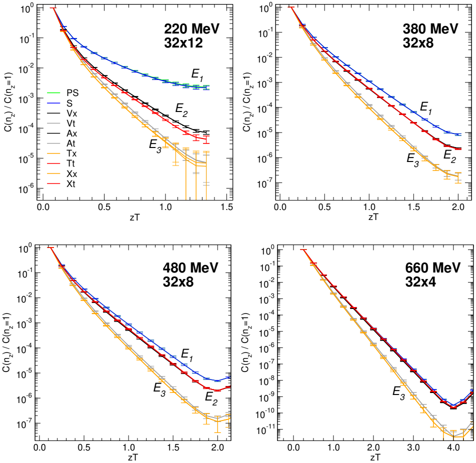
2006-2010 Budapest-Wuppertal and later HotQCD (Bielefeld-BNL) lattice collaborations: The chiral restoration crossover is observed at $T = 120 - 180$ MeV with the pseudocritical temperature at $T_{ch} \sim 155$ MeV. The deconfinement as observed from the Polyakov loop is approximately at the same temperature. The latter statement turned out to be erroneous.

It has been argued and predicted that above $T_{ch} \sim 155$ MeV there should appear approximate chiral spin symmetry and hence QCD should still be in the confining regime. L.Ya.G., 1610.00275

Spatial correlators above T_{ch}

C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, C. Gattringer, L.Ya.G., S. Hashimoto, C.B. Lang, S. Prelovsek , 2017 - 2019

$N_f = 2$ QCD with the chirally symmetric Dirac operator.



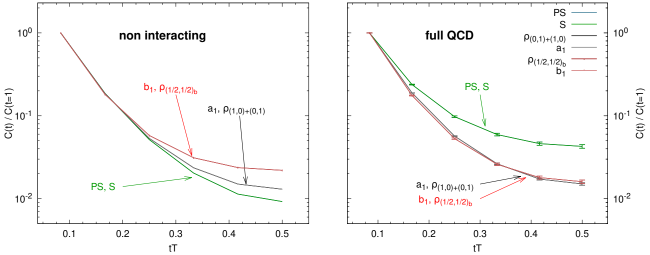
$E1$ - $U(1)_A$ symmetry; $E2$ - chiral spin and $SU(4)$ symmetries; $E3$ consistent with both chiral symmetry and chiral spin ($SU(4)$) symmetry.

The $E1$ and $E2$ multiplet structure persists up to $T \sim 500$ MeV.

Temporal correlators above T_{ch}

C. Rohrhofer, Y. Aoki, L.Ya.G., S. Hashimoto, 2020

$N_F = 2$ QCD at $T = 220$ MeV

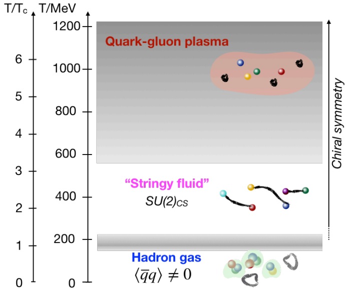


Free quarks: $SU(2)_L \times SU(2)_R$ and $U(1)_A$ multiplets.

Full QCD at $T = 220$ MeV: $U(1)_A$, $SU(2)_L \times SU(2)_R$, $SU(2)_{CS}$ and $SU(2N_F)$ multiplets.

Above T_{ch} QCD is approximately $SU(2)_{CS}$ and $SU(4)$ symmetric.

Three regimes of QCD. C. Rohrhofer et al, PRD 100 (2019) 014502



$0 - T_{ch}$ - Hadron Gas (broken chiral symmetry);

$T_{ch} - 3T_{ch}$ - Stringy Fluid (chiral, $SU(2)_{CS}$ and $SU(4)$ symmetries; electric confinement)

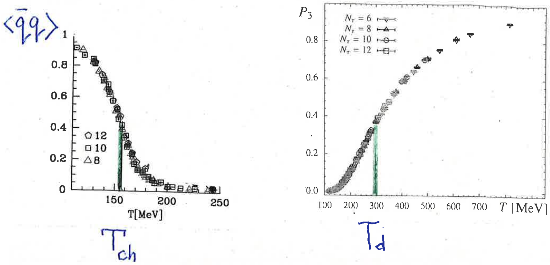
Stringy fluid is mostly populated with $J = 0, 1$ states. It is a densely packed system of the color-singlet clusters that interact strongly. Quark interchanges between the clusters are significant.

$T > 3T_{ch}$ - a QGP (chiral symmetry)
Since the confinement - deconfinement crossover is VERY smooth, one needs a definition of the deconfinement point.

The same is seen with the Polyakov loop: In QCD with light quarks the center symmetry is explicitly broken by quark loops and the first order phase transition is replaced by a VERY smooth crossover around $T_d \sim 300$ MeV.

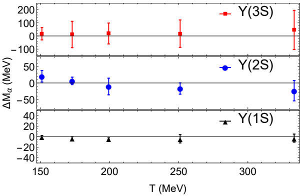
Condensate in 2+1 QCD from: Y. Aoki et al, JHEP 06 (2009) 088

Polyakov loop in 2+1 QCD from: P. Petreczky, H.P. Schadler, PRD 92 (2015) 094517

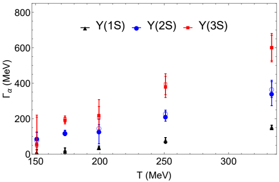


Bottomonium above T_{ch}. R. Larsen et al, PLB 800 (2020) 135119

Mass shifts of the heavy quarkonia in the medium with respect to zero temperature mass:



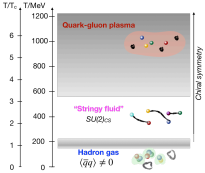
Increasing with T widths:



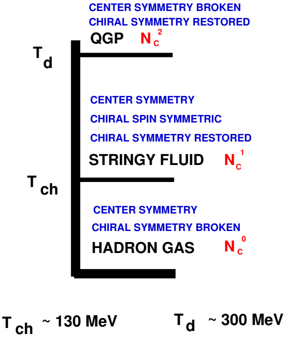
The same is observed with the 1P,2P,3P states.
A Debye-screened potential $\sim 1/re^{-mr}$ does not support any bound state. A Coulomb potential $\sim 1/r$ supports only a 1S state.
An unambiguous evidence for confinement.

Trailer of T.Cohen's talk.

T.D. Cohen, L.Ya.G., EPJA 60 (2024) 171



In the combined large N_c and chiral limit QCD with light quarks is manifestly center symmetric and chirally symmetric. The smooth crossovers may become true phase transitions.



Trailer of A. Nefediev's talk

L.Ya.G., A. Nefediev, R. Wagenbrunn, PLB 854 (2024) 138707; arXiv: 2410.13297

Coulomb gauge QCD at large N_c; only a confining linear potential is retained.
A parameter free connection: $|\langle\bar{\psi}\psi\rangle|^{1/3} \simeq 2.75\ T_{ch}$
$\langle\bar{\psi}\psi\rangle = -(250\ \mathrm{MeV})^3 \quad \longrightarrow \quad T_{ch} \approx 90\ \mathrm{MeV}\ (T_{ch}^{lattice} \approx 130\ \mathrm{MeV})$

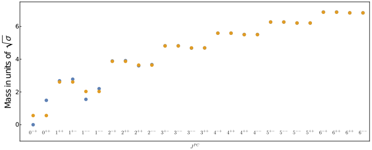


Figure: Symmetries of bound quark-antiquark states $T = 0$ (blue points) and $T = 1.1\,T_{ch}$ (yellow points).

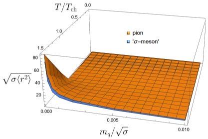


Figure: R.m.s radii of pion and "sigma-meson" depending on T and m_q

A very natural explanation of a collectivity and of a small mean free path of the effective constituents above T_{ch}.