QCD Thermodynamics Explored with Chiral Fermions

Yasumichi Aoki

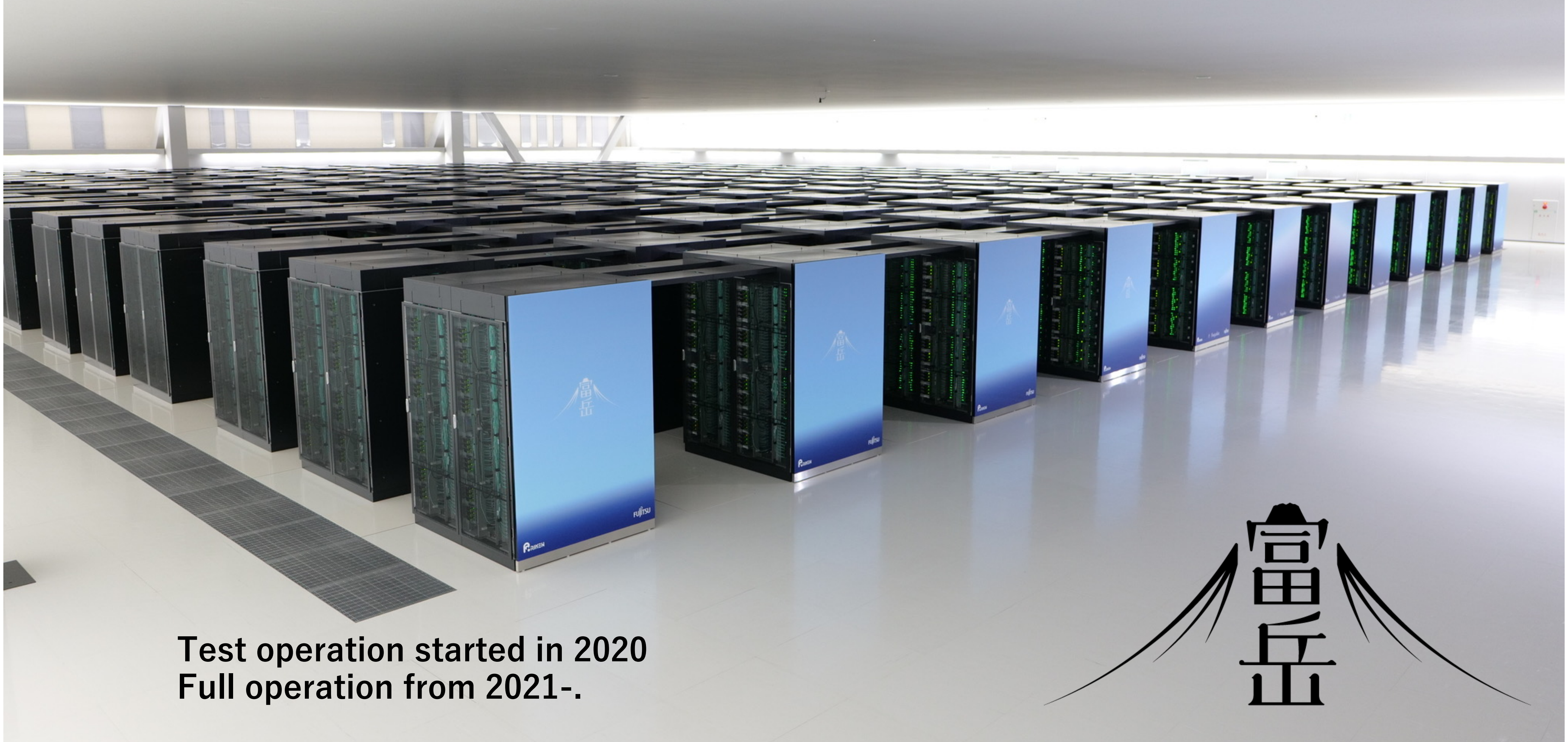
Field Theory Research Team
RIKEN Center for Computational Science (R-CCS)

Confinement and symmetry from vacuum to QCD phase diagram
@ Benasque Science Center, Feb.10, 2025

https://www.r-ccs.riken.jp/labs/ftrt/

R-CCS: hosting Japanese flagship machine:

Supercomputer Fugaku: 富岳



Test operation started in 2020
Full operation from 2021-.

Members of Field Theory Research Team

Regular Members

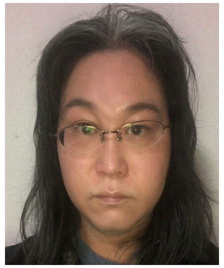

Y. Aoki

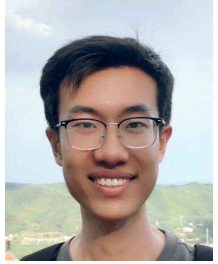

I. Kanamori


J. Goswami


K. Nakayama


Y. Nakamura (concurrent)


K. Nitadori (concurrent)


Xiaoyang Wang (concurrent)

Visiting Affiliates


S. Aoki


S. Hashimoto


D. Lin


M. Fukuma


A. Tomiya

Former Members (since 2018)


E. Shintani


T. Ishikawa


Z. Yu


R. Tsuji


A. Portelli

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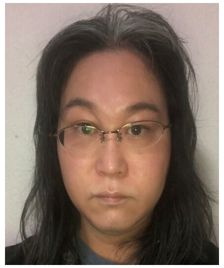
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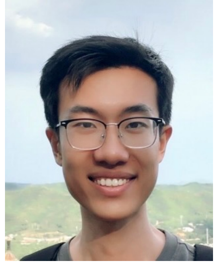
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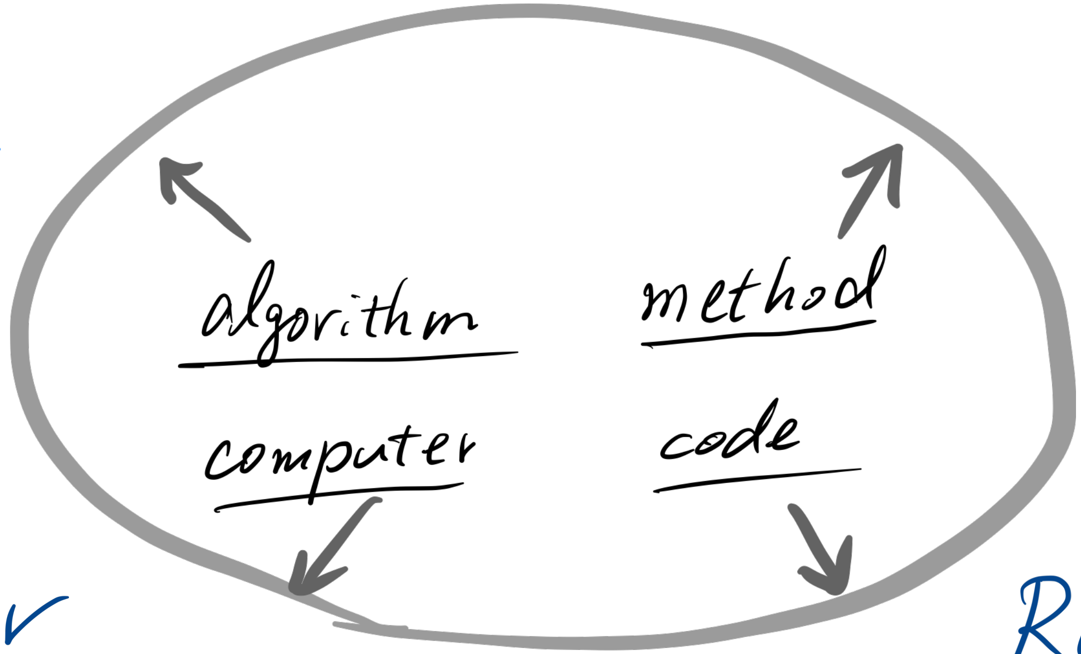
Pushing forward the reach of quantum field theories

nucleon

ex: structure
- radius

meson / CKM

ex:
- unitarity
- inclusive ↕ exclusive



State of matter

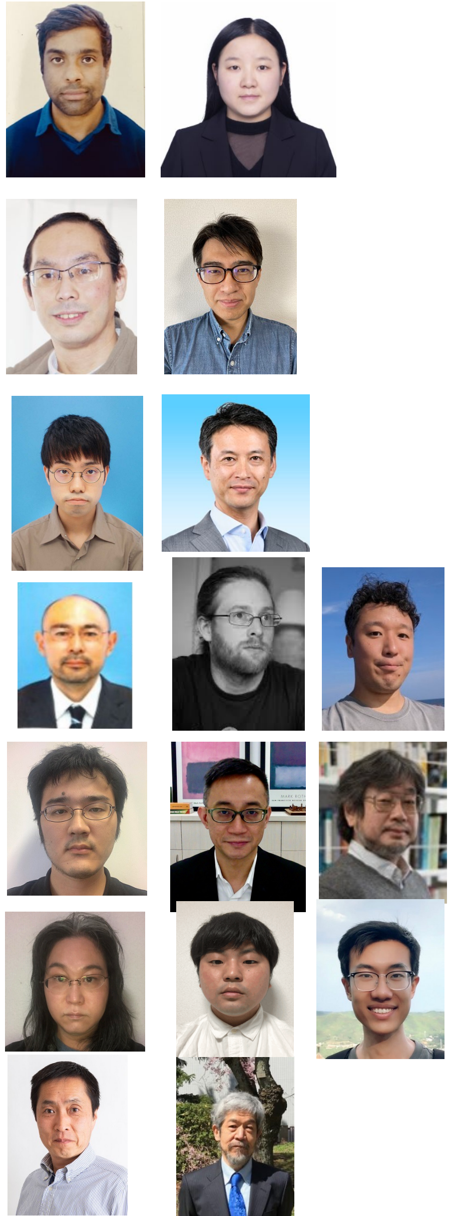
ex: QCD phase transition
- $\mu = 0 \;/\; \neq 0$

Real time dynamics

Scattering etc.

Pushing forward the reach of quantum field theories - current

- Use of chiral fermions for fine lattices
  - **QCD phase transition** — *Goswami, Zhang, Kanamori, Nakamura, Aoki*
  - heavy quark physics (B mesons) — *Aoki, Kanamori, Nakamura*
- Beyond conventional LQCD "measurements"
  - towards nucleon properties on Big lattices - *R. Tsuji (JRA), Aoki, with PACS*
  - coarser lattices, new scheme – proton decay – *Aoki, et al & Aoki, Shintani, Tsuji,,*
  - QED and isospin breaking effects → B meson – *Portelli, Aoki, et al*
- Beyond conventional LQCD "simulations"
  - Tensor networks, higher dim., systematic error – *Nakayama, Nakamura, [Lin]*
  - AI acceleration, new alg. (world volume) *[Tomiya], [Fukuma]*
  - Quantum *Goswami, Nakayama, Wang*
- Beyond Fugaku:
  - FugakuNEXT Feasibility Study - *Aoki, Kanamori, Nakamura, Nitadori*
- Beyond conventional data share: *Nakamura, Aoki*
  - JLDG (Japan Lattice Data Grid) & ILDG (International Lattice Data Grid) → ILDG 2.0
- LQCD frameworks
  - QWS : *Nakamura, Kanamori, Nitadori,,,*
  - Grid/Hadrons: *Portelli;* tuning: *Kanamori, Nakamura*; new measurements: *Zhang,*
  - Bridge++: *Kanamori, Goswami* et al, → DWF meas., R&D for multi-grid etc
  - BQCD: *Nakamura* et al.

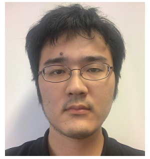


Pushing forward the reach of quantum field theories - current → near term

- Accelerating Simulation / Computation in LQCD
  - *R&D for AI acceleration for LQCD* — *Tomiya w/ couple of Grad students*

- Beyond $\mu = 0 \to \mu \neq 0$
  - *seeking seeds in new methods* - *Fukuma et al*

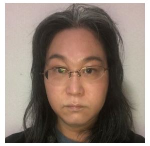
- Target Physics Computation of Tensor Network
  - *seeking good model to study* – *Lin, Nakayama*

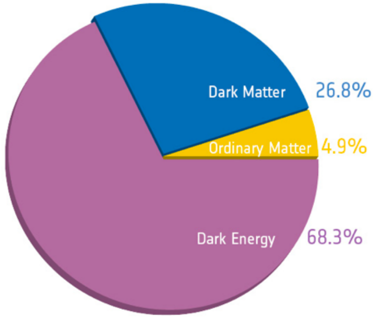
- Quantum Computing
  - *seeking seeds towards particle physics* — *Goswami, Nakayama*
  - *to be discussed* — *involving Wang (concurrent)*


- Beyond Fugaku:
  - *FugakuNEXT codesign (using Bridge++)*- *Kanamori, Nakamura, Nitadori*
  - *→ online ~2030-*



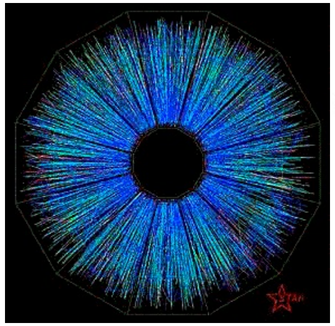


LQCD opportunities

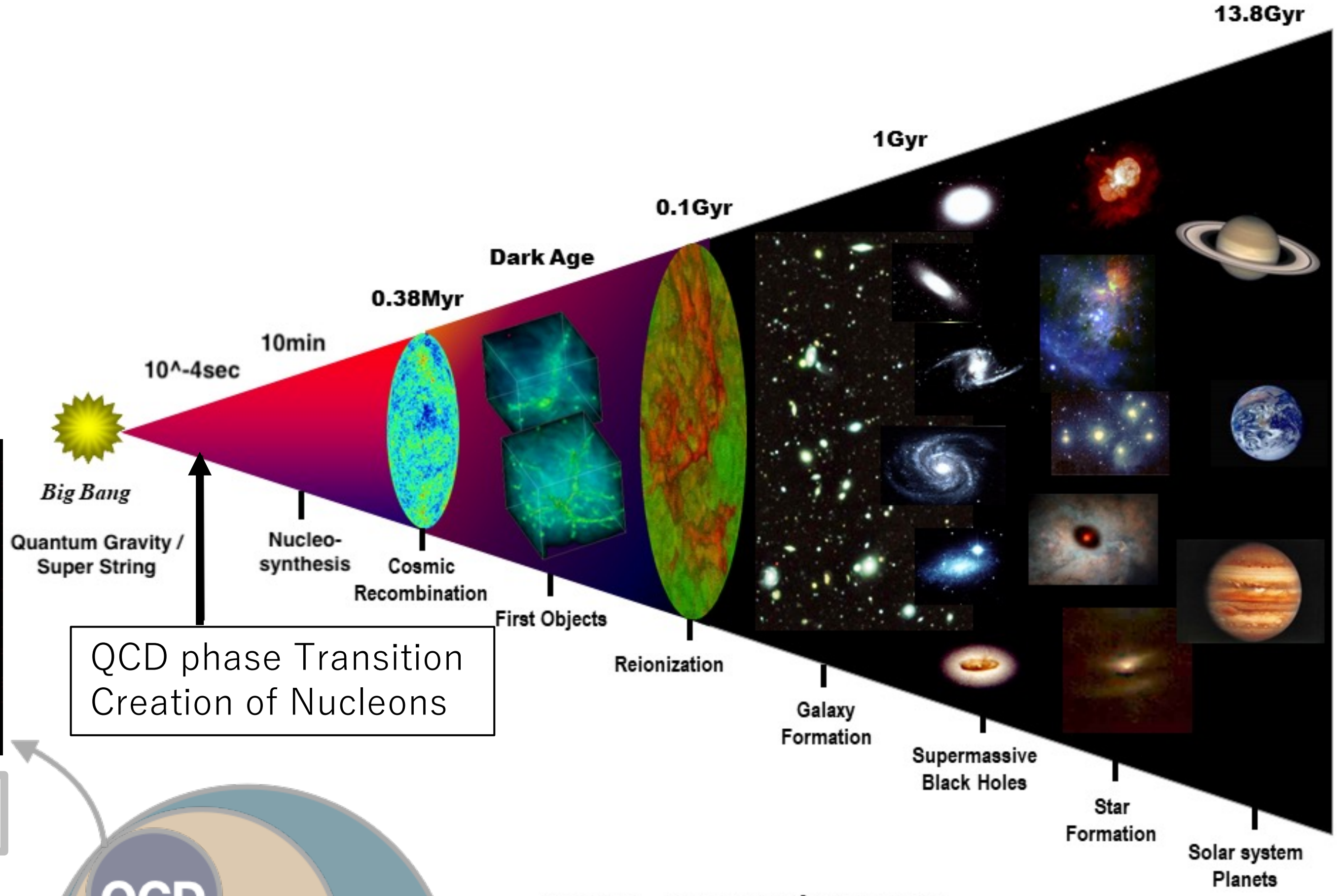


Dark Matter

BNL/CERN

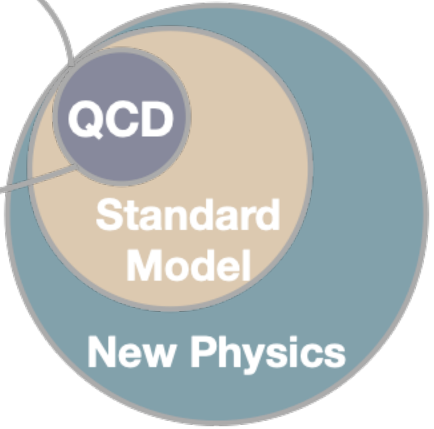


QCD phase



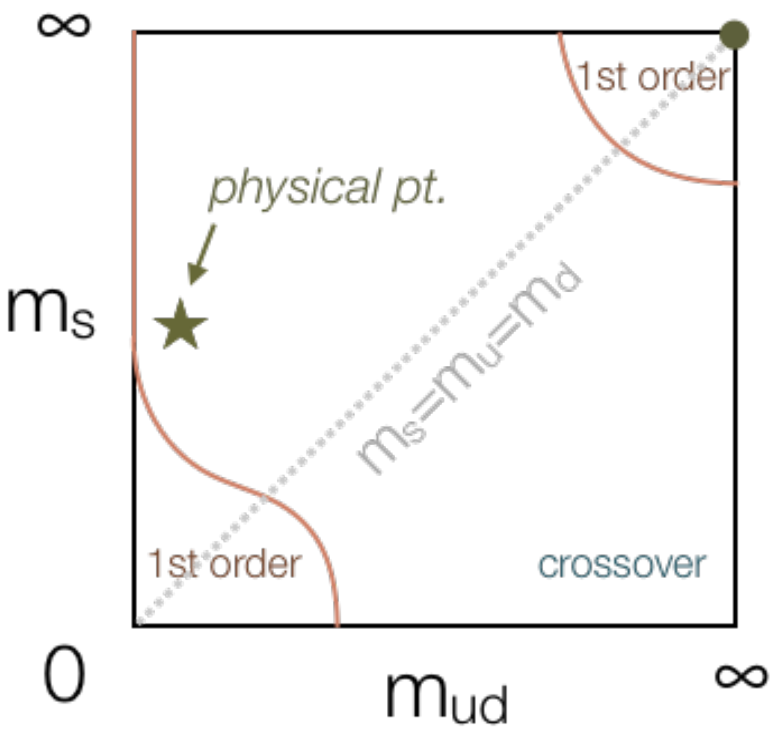
QCD phase Transition
Creation of Nucleons

Based on © Masayuki Umemura

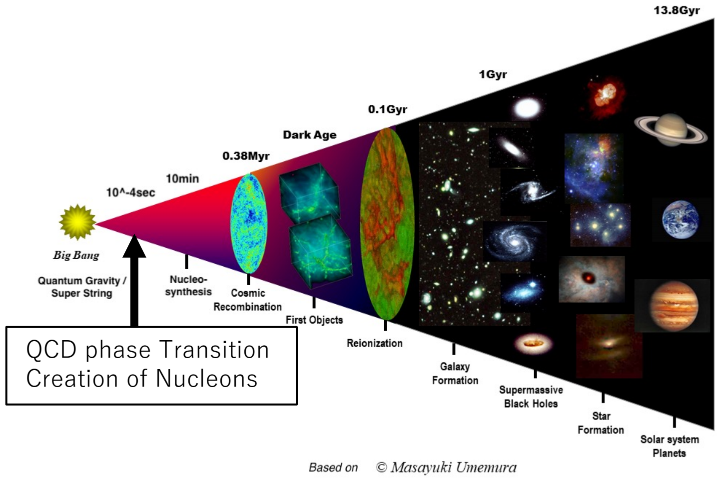


QCD phase transition

“Columbia Plot” is often used to discuss QCD phase
- for arbitrary values of u=d, s quark masses
- phase structure itself is interesting object
- help understand the physical point
- Focus: **chiral** regime: $m \to 0$



- **This is the plot many of us would have drawn a few years ago**



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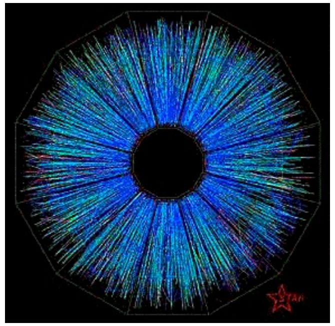

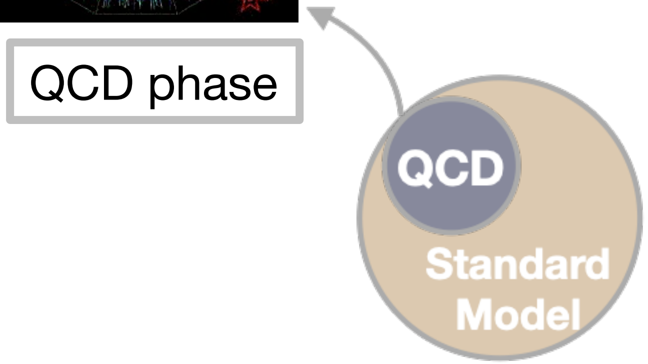

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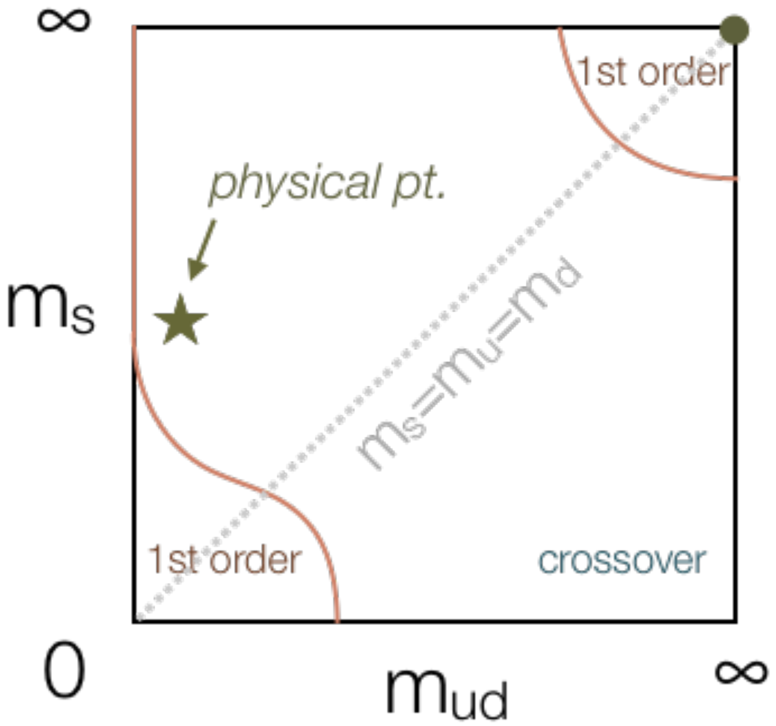

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- **Pisarski and Wilczek PRD 1984**
  - effective model / ε expansion
  - 1st order expected near origin (N_f=3 chiral limit)
- **Columbia group: Brown et al PRL 1990**
  - first lattice QCD computation

N_f=3 chiral limit is a hot topic

- **related non-lattice studies**
  - G. Fejos, PRD 22
  - Kousvos and Stergiou, SciPost Phys. 23
  - Pisarski and Rennecke, PRL 23
  - Fejos and Hatsuda PRD 24

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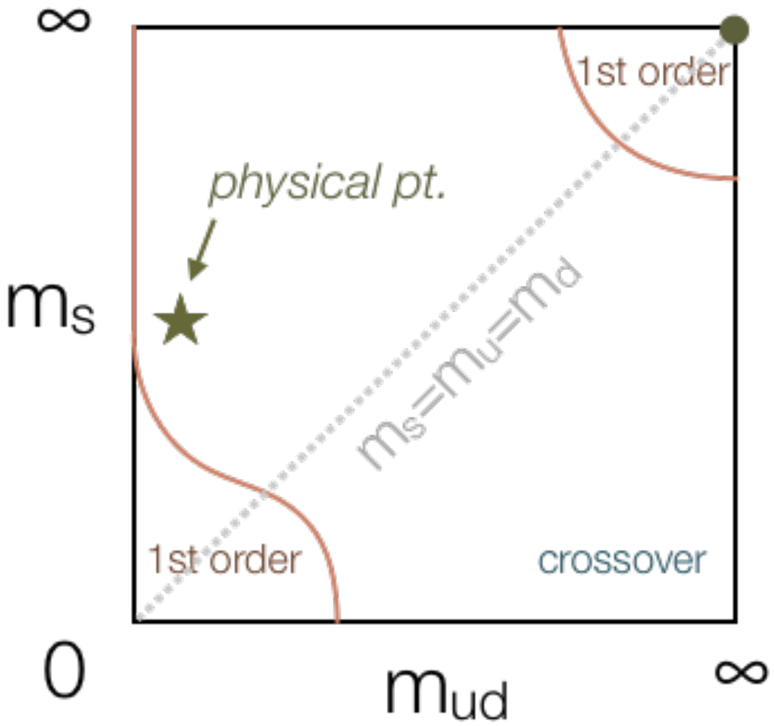

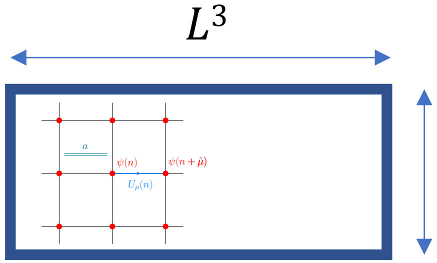

$$\frac{1}{T} = aN_t$$

Diagonal search: Fate of the chiral 1st order region

- **Wilson Fermion: Kuramashi et al (2020)**
  - 1st order observed for $a > 0$
  - region shrinks towards $a \to 0$

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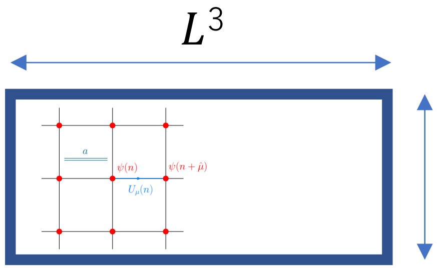

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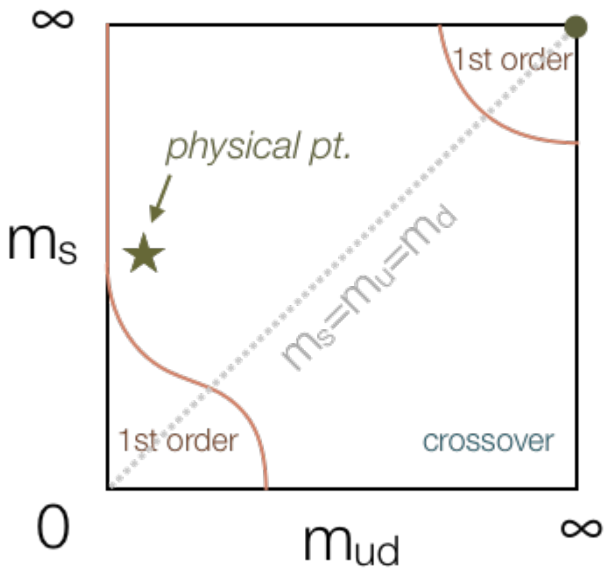

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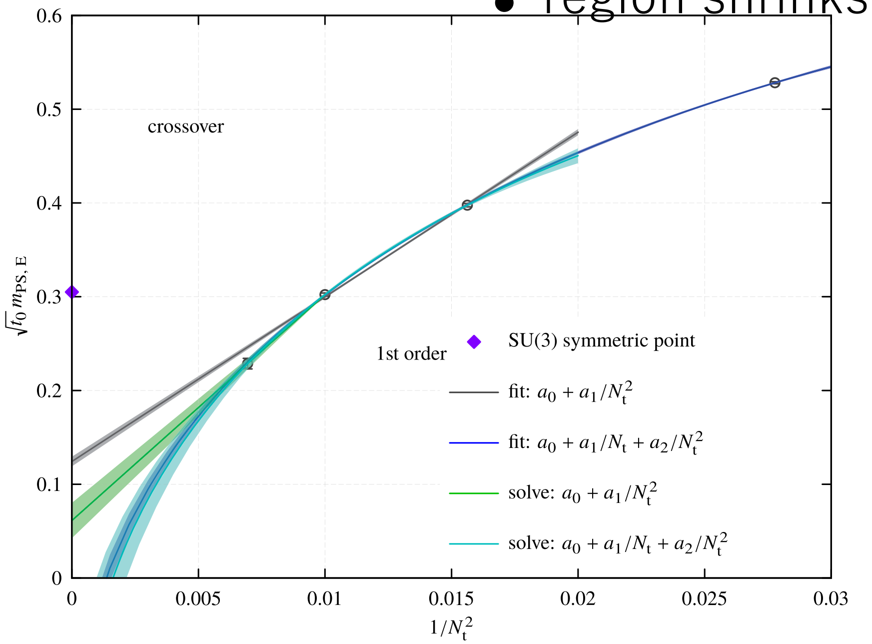

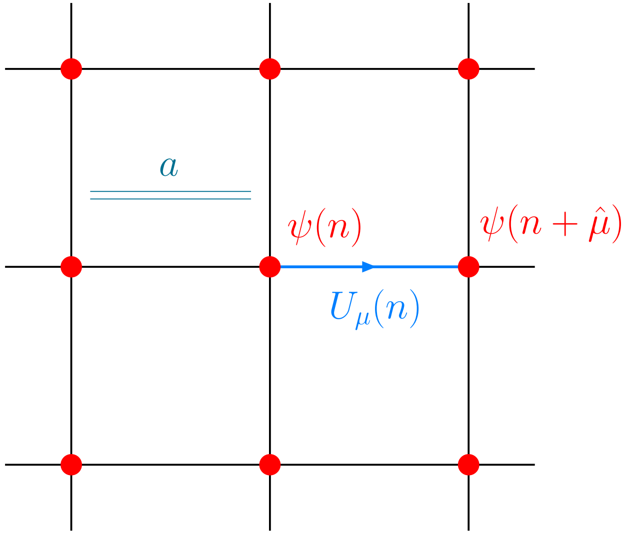

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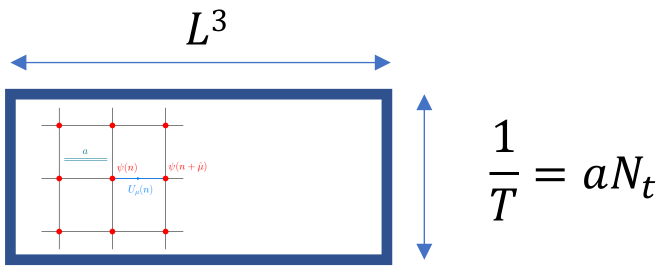

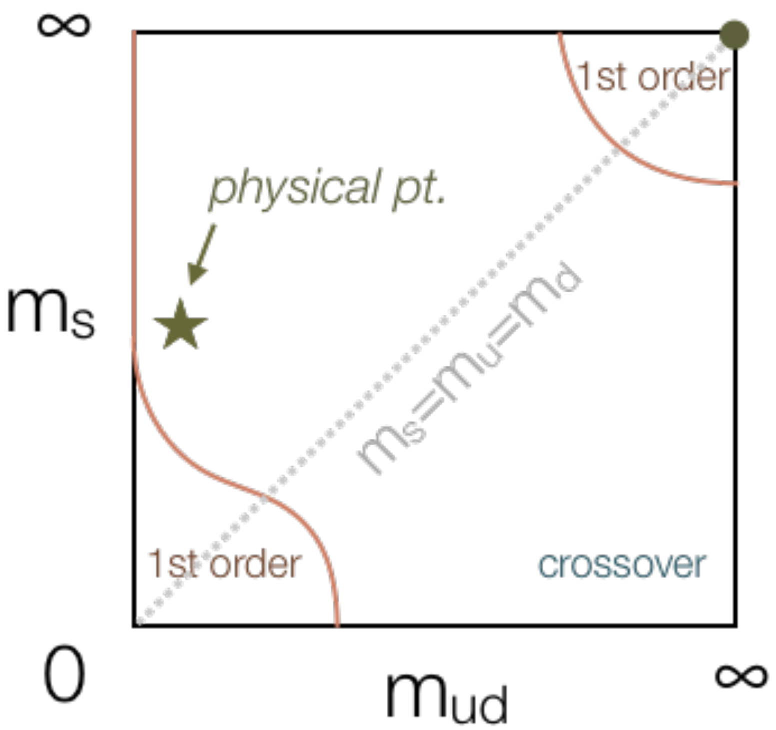

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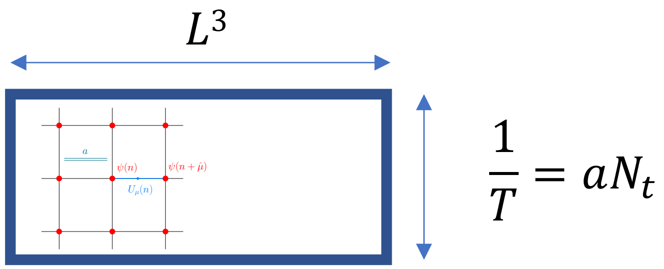
Diagonal search: Fate of the chiral 1st order region

- **Wilson Fermion: Kuramashi et al (2020)**
  - 1st order observed for $a > 0$
  - region shrinks towards $a \to 0$
- **Naive Staggered Fermion: Cuteri et al (2021)**
  - no 1st order region for $N_f = 3$
- **HISQ (improved staggered): Dini et al (2022)**
  - consistent with no 1st order (critical scaling)
- **We tackle this problem with "Chiral Fermion"**

LQCD fermions

Each fermion has its pros and cons

- chiral: domain wall fermion (DWF), overlap fermion
- non-chiral: Wilson fermion, staggered fermion

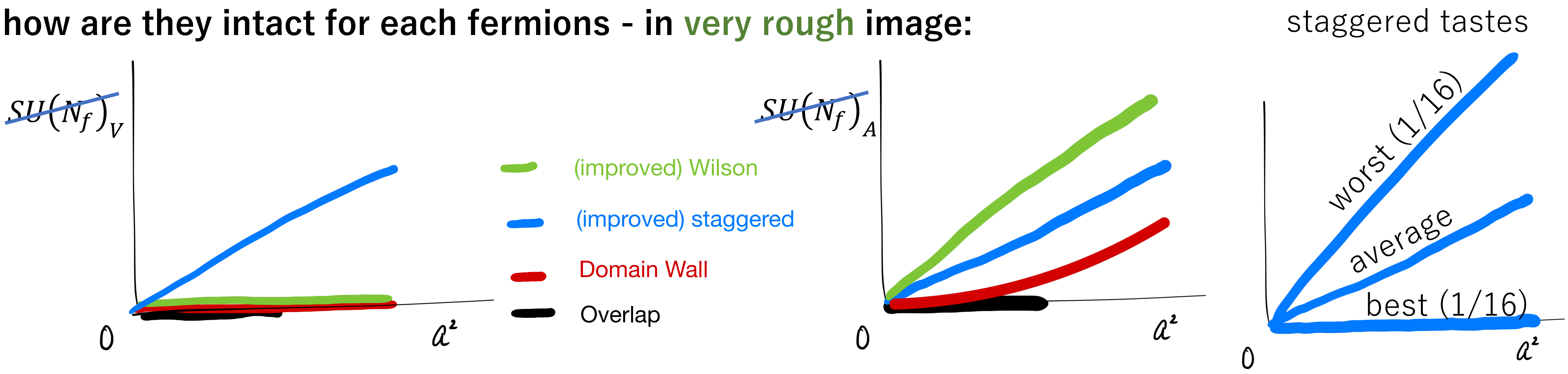


Flavor, chiral symmetries for N_f number of flavors: 2 for u,d, 3 for u,d,s

$$SU(N_f)_L \times SU(N_f)_R = SU(N_f)_V \times SU(N_f)_A$$

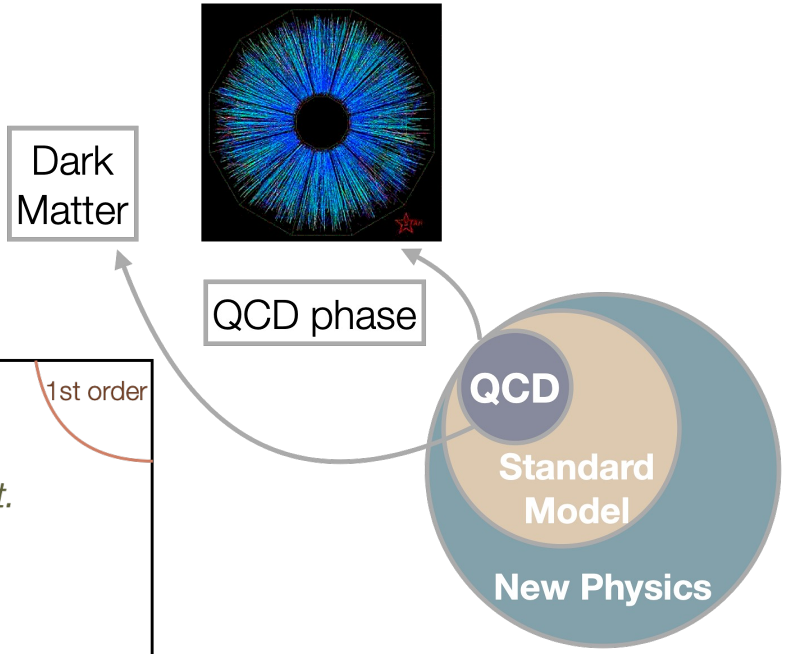
$SU(N_f)_A$ is broken at low T by QCD dynamics, but formulation should have the symmetry

how are they intact for each fermions - in very rough image:



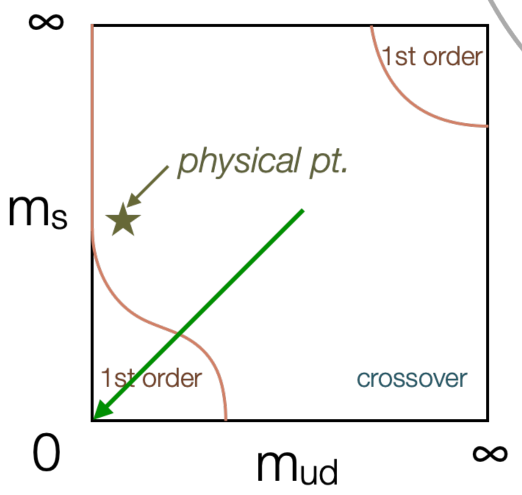
We use domain wall fermions (best practical); cons = computationally demanding

QCD phase and thermodynamics

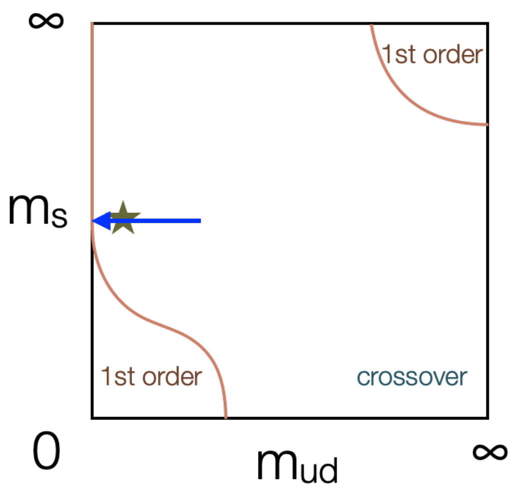


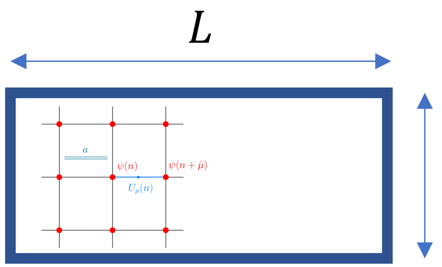
QCD phase
- fundamental understanding of QCD phase
- Simulation on (pseudo) critical temperature
  - with Domain-Wall fermions

Diagonal search
- Wilson fermion shows large discretization error
- Staggered fermion consistent with no 1st order
- ➢ use of chiral (Domain-wall) fermion would help



On/near physical point
- High temperature ($T > T_c$) overlap fermions
- ➢ $T = T_c$ using DWF with Fugaku
  - QCD thermodynamics





$$\frac{1}{T} = aN_t$$

Current authors of projects / under JLQCD

- **N_f=3 phase hunting**

YA[1], S. Hashimoto[2][3], I. Kanamori[1], T. Kaneko[2][3][4], Y. Nakamura[1], Y. Zhang[5]

- **N_f=2+1 thermodynamics with line of constant physics**

YA[1], H. Fukaya[6], J. Goswami[1],

S. Hashimoto [2][3], I. Kanamori[1], T. Kaneko [2][3][4], Y. Nakamura[1], Y. Zhang[5]

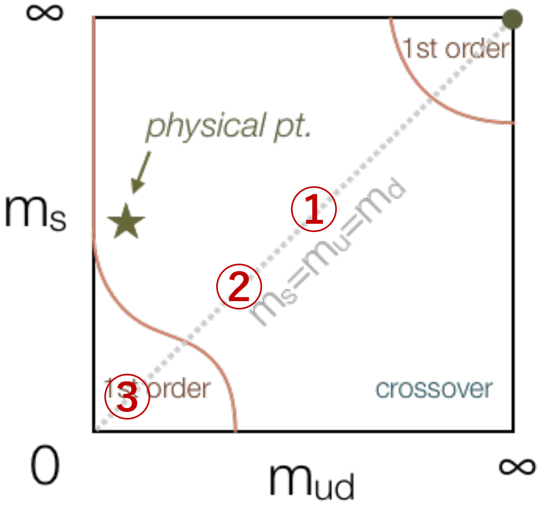
(1): RIKEN Center for Computational Science
(2): KEK
(3): SOKENDAI
(4): Kobayashi-Maskawa Institute, Nagoya Univ.
(5): Bielefeld University
(6): Osaka University

QCD phase hunting

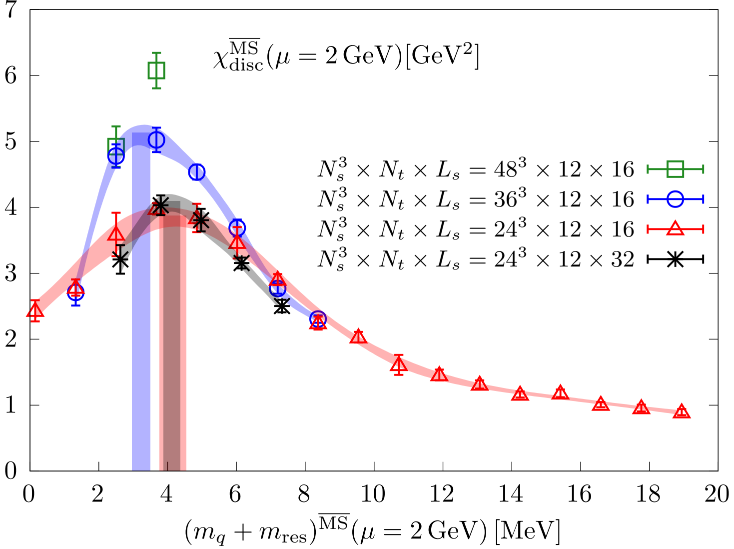
- T>0 QCD using fine lattice chiral fermions: domain-wall (DWF) [JLQCD]
  - N_f=3 - hunting QCD phase boundary
    - Columbia plot: a **long-standing problem** in Lattice QCD
    - $m \simeq 220\ MeV$: crossover !
    - $m \simeq 40\ MeV$: crossover !
    - $m \simeq 4\ MeV$: crossover likely
    - [PoS Lattice 2021, 22, 23, 24]



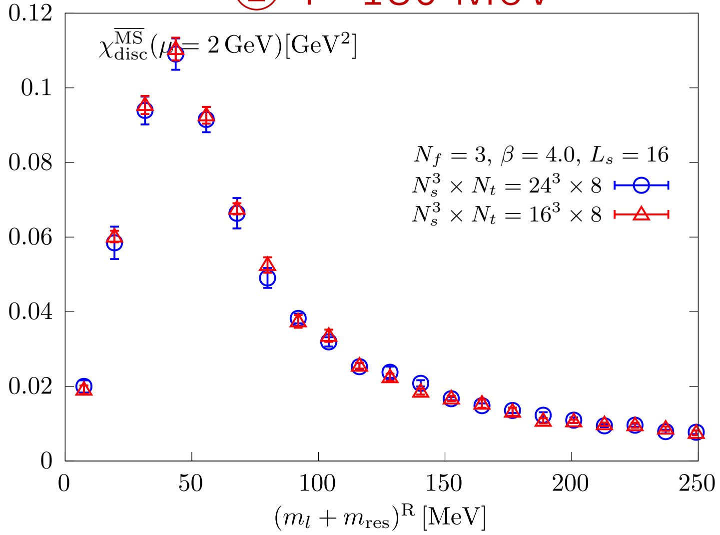


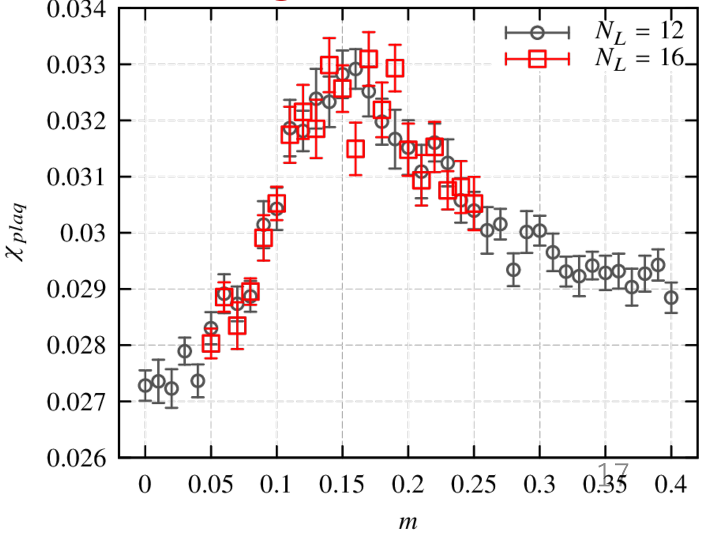
③ T=120 MeV



② T=180 MeV



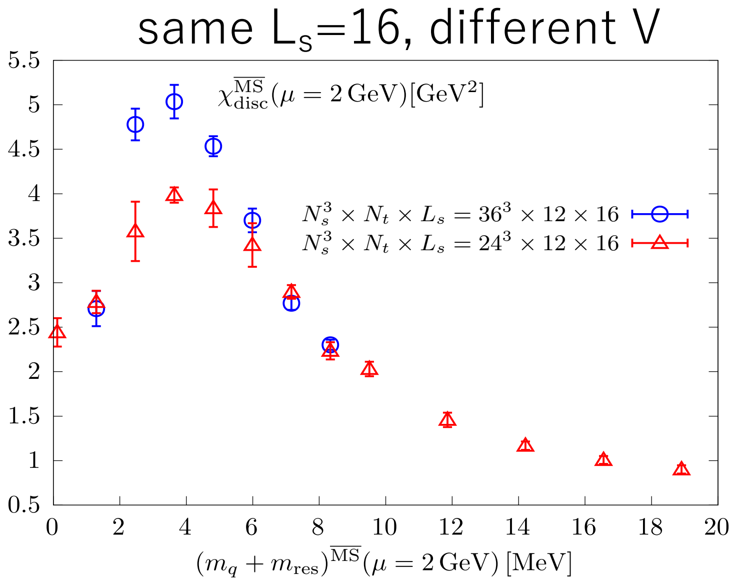
① T=240 MeV

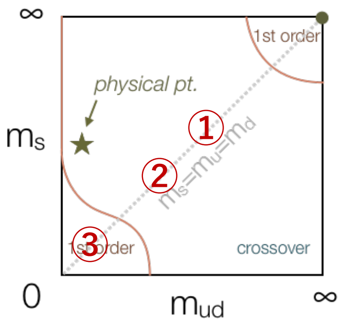


QCD phase hunting: ③ T=120 MeV

DWF and Overlap Fermion

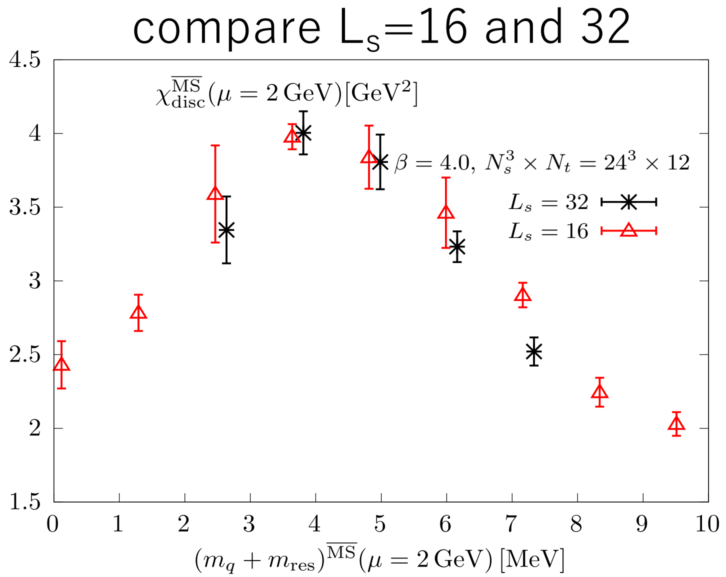
- **DWF: 4D + extra 1D formulation**
- **DWF($L_s \to \infty$) → Overlap**
- **DWF: small residual breaking m_{res}**
  - $m_{res} \sim 1/L_s$
- **Overlap: exact symmetry: $m_{res} = 0$**



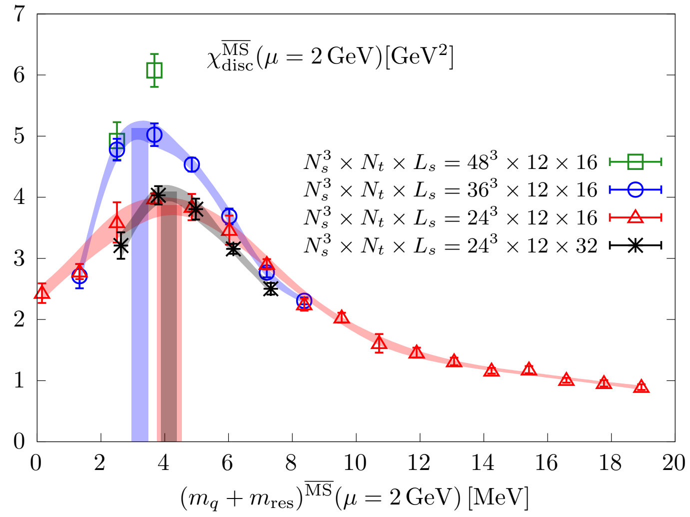


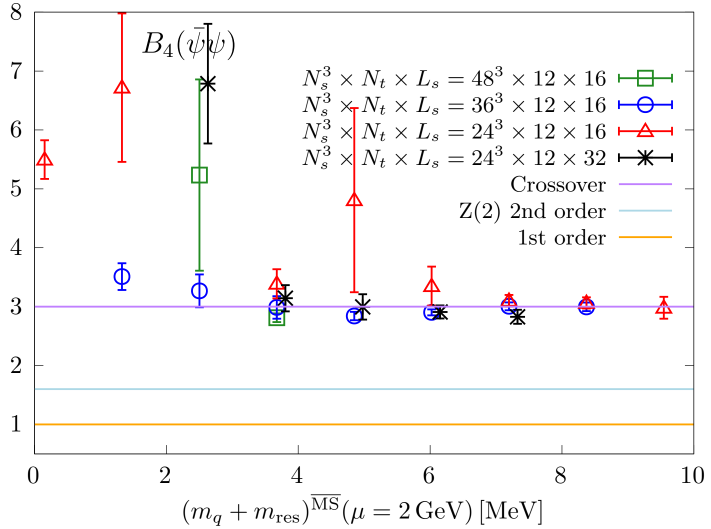
testing the effect of finite L_s

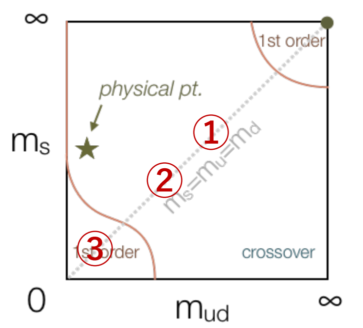
- **L_s=16 (main)**
  - $am_{res} = 0.00613(9)$
  - $m_{res}^{MS} = 7.2(1)$ MeV
- **L_s=32**
  - $m_{res}^{MS} \simeq 4$ MeV
- **Susceptibility: consistent**
  - finite L_s effect properly captured

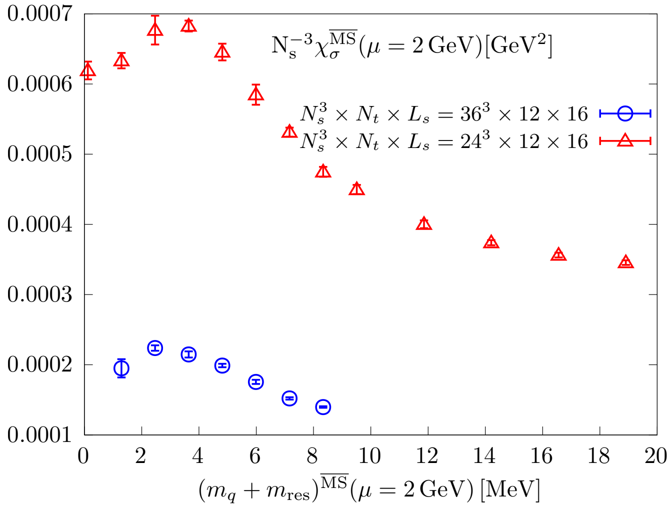


QCD phase hunting: ③ T=120 MeV









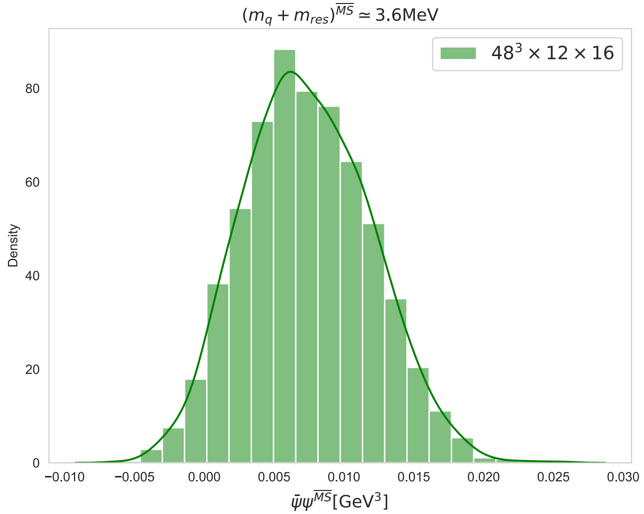
volume scaling for 1st order phase transition is too bad

Binder cumulant

$$B_4(\bar\psi\psi) = \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2},$$

$$\delta\bar\psi\psi = \bar\psi\psi - \langle\bar\psi\psi\rangle$$

consistent with crossover

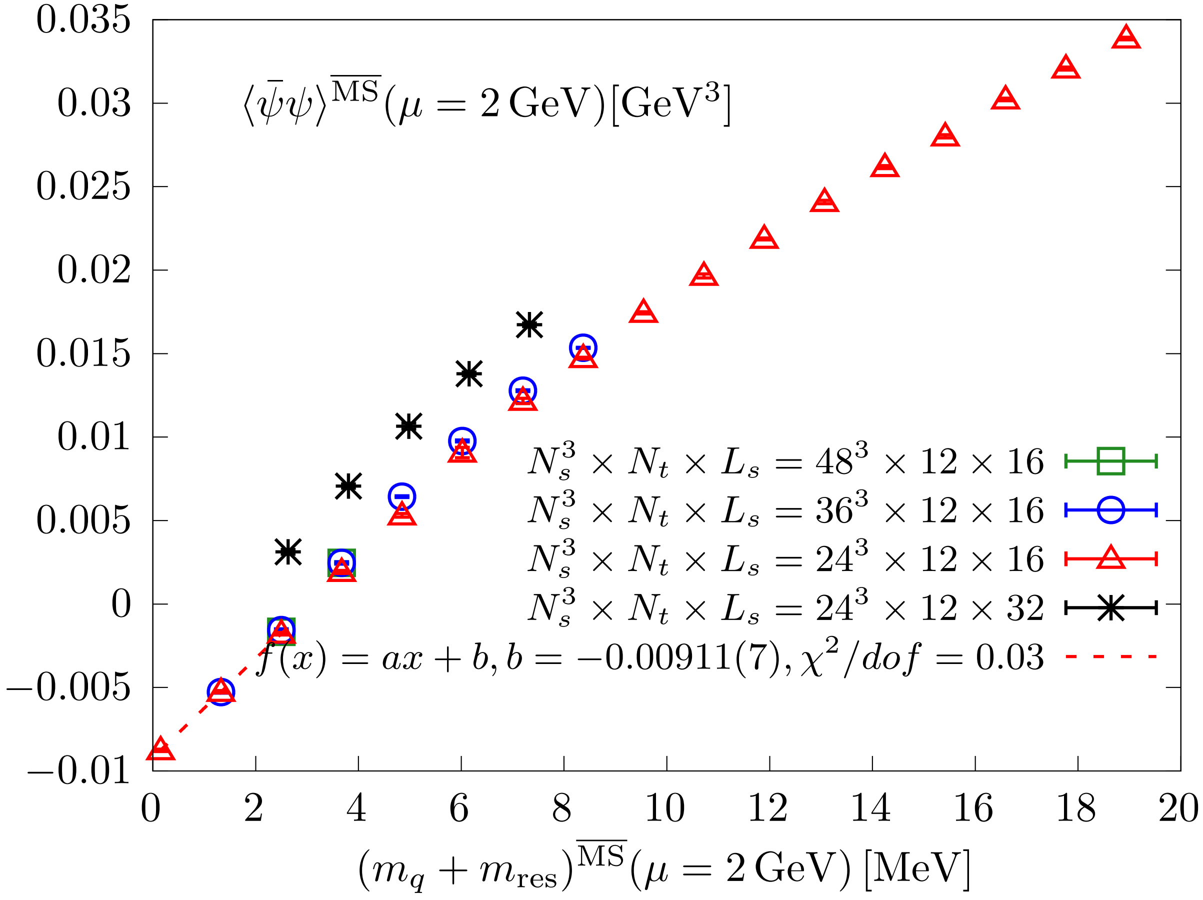


unlikely to be true phase transition

Light quark $\Sigma = -\langle\overline{\psi}\psi\rangle$: no power div. in disconnected susceptibility

- $\chi_{disc} = \langle\overline{u}u \cdot \overline{d}d\rangle - \langle\overline{u}u\rangle\,\langle\overline{d}d\rangle$
  - power divergence in $\langle\overline{\psi}\psi\rangle$ cancels out
  - no new divergence over Σ because no new contact terms
  - needs multiplicative renormalization for logarithmic divergence
  - $Z_S(\beta) = 1/Z_m(\beta)$
  - we stick for now on this quantity → See next talk (Kanamori)
- $\chi_{total} = \langle\overline{\psi}\psi \cdot \overline{\psi}\psi\rangle - \langle\overline{\psi}\psi\rangle\,\langle\overline{\psi}\psi\rangle$
  - has power divergence everywhere
  - needs to understand the power divergence of $\Sigma = -\langle\overline{\psi}\psi\rangle$ first

N_f=3, N_t=12 chiral condensate



- only multiplicative renormalization applied
- quark mass: m_{res} shift applied to x axis
- @T=0: $m_\pi \to 0$, $(m_q + m_{res}) \to 0$
- L_s=16
  - three volumes: 24^3, 36^3, 48^3
- L_s=32
  - smaller m_{res}, 24^3
- Intercept = $C_D \dfrac{-(1-x)m_{res}}{a^2} < 0$
  - need to be subtracted

Light quark $\Sigma = -\langle\bar{\psi}\psi\rangle$: residual power divergence

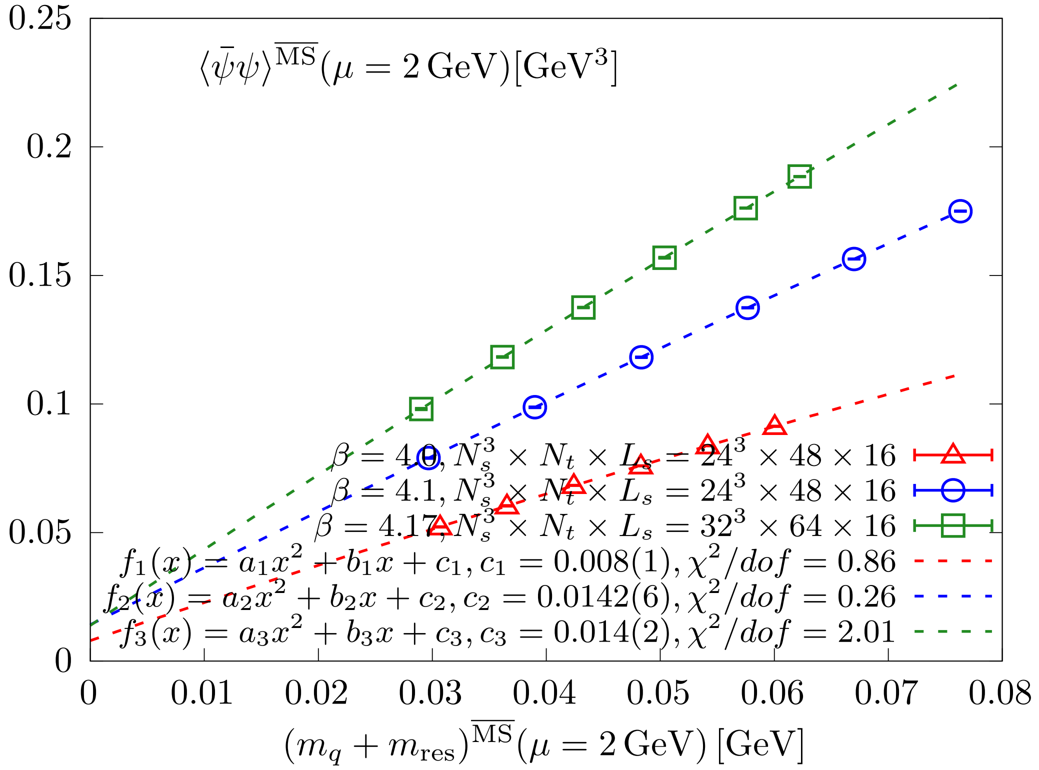
- $\Sigma|_{DWF} = C_D \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$ S. Sharpe (arXiv: 0706.0218)

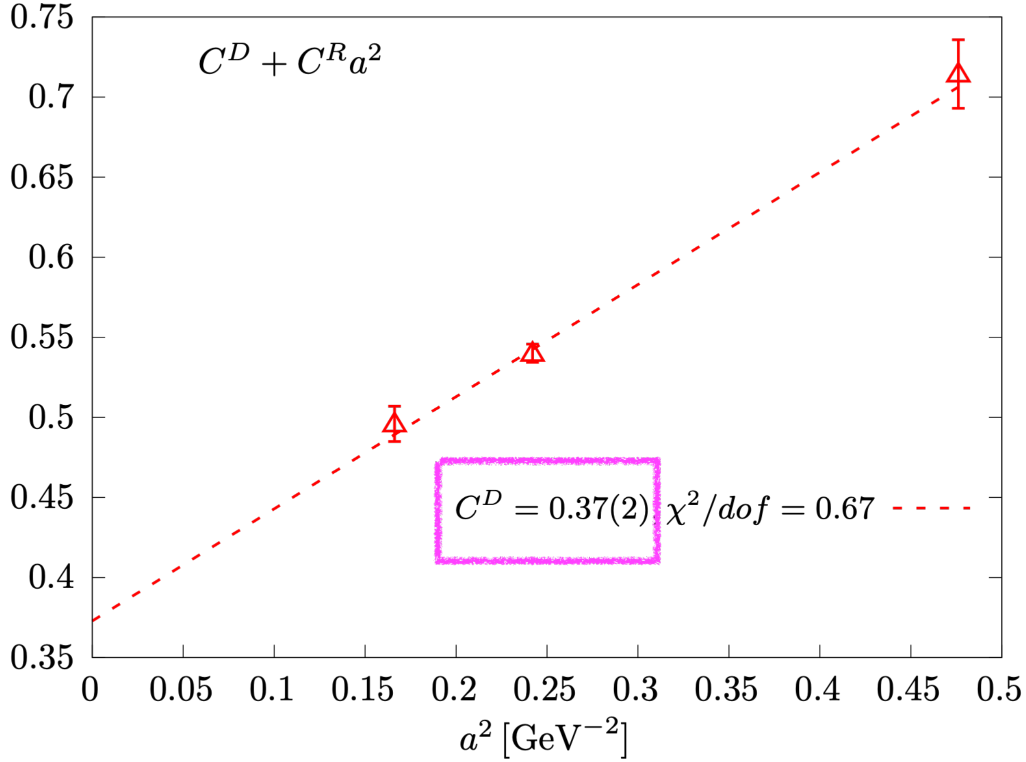
  $m_{res} \neq x m_{res}; \quad x = O(1) \neq 1$

  - "Since x is not known, this term gives an uncontrolled error in the condensate. It can be studied and reduced only by increasing L_s - a very expensive proposition." – S. Sharpe.

- $\boldsymbol{N_f = 3}$ case
  - T>0 problem @ β =4.0
  - T=0 exercise @ β =4.0, 4.1, 4.17
  - T>0 div free Σ

Nf=3, T=0 chiral condensate

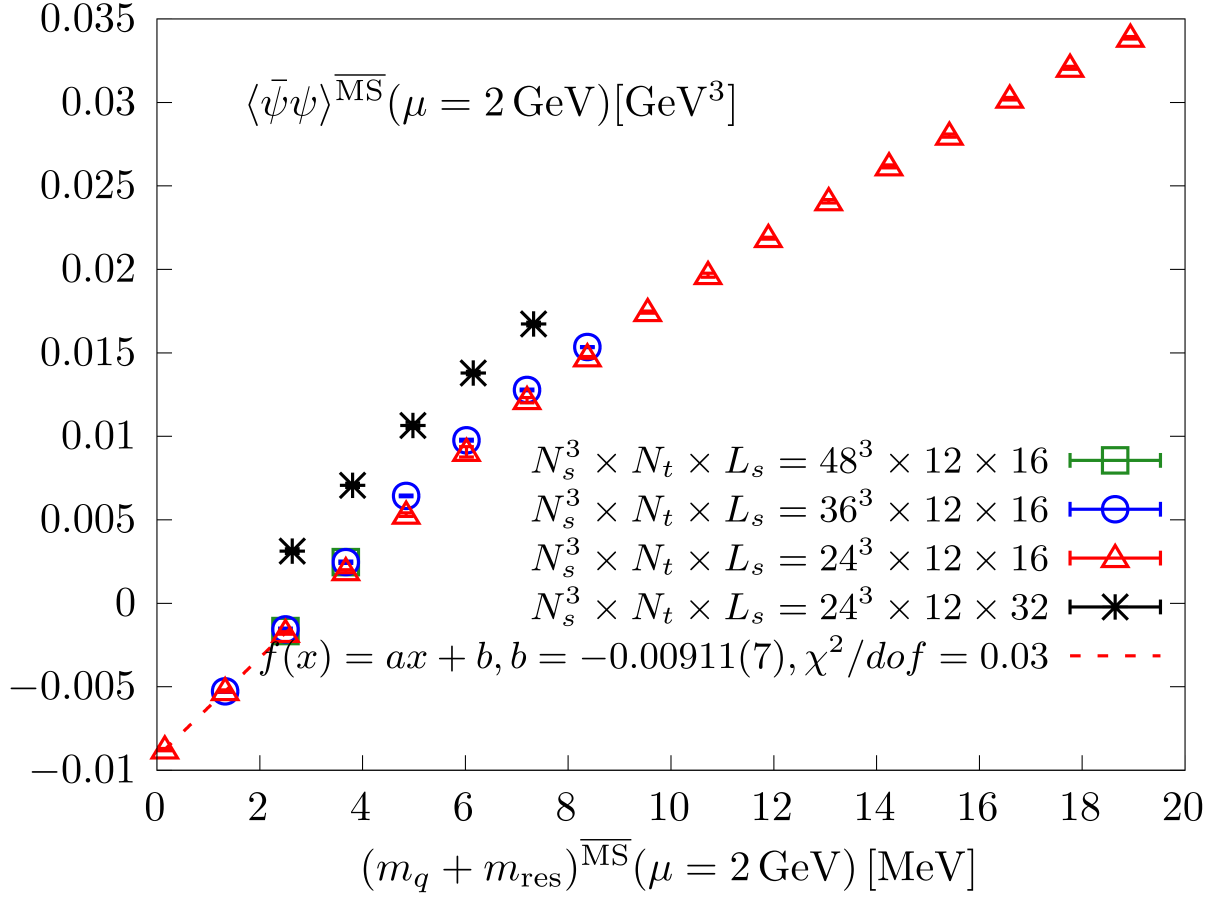


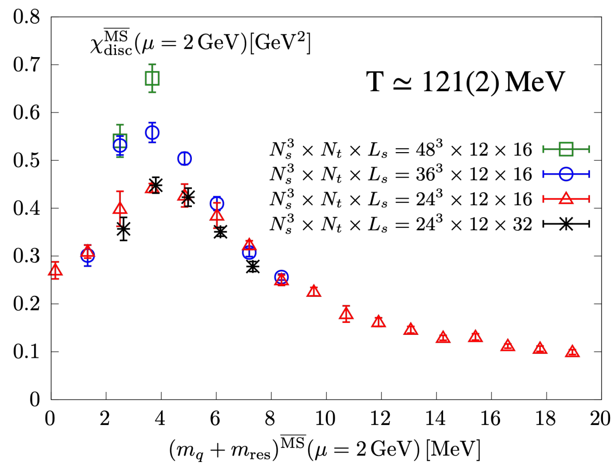


- $\Sigma(m) = C_0 + C_1\, m + C_2\, m^2$ fit
- $C_1 = \frac{C_D}{a^2} + C_R$
  - C_D/a^2: divergent, C_R: regular

- $\Sigma|_{DWF} = C_D \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$
- $C_D + C_R a^2$
  - $C_D = 0.37(2)$ from linear fit

N_f=3, N_t=12 chiral condensate

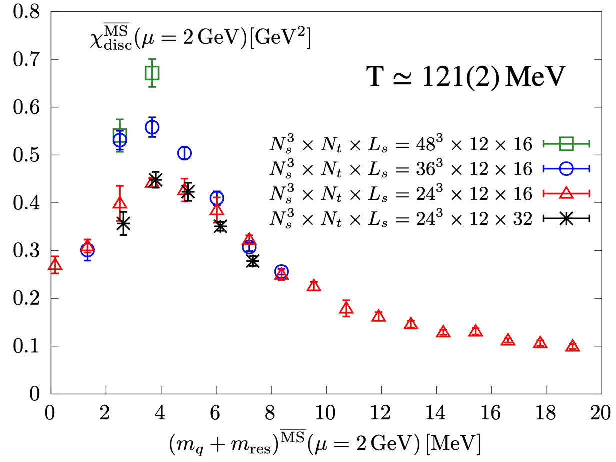


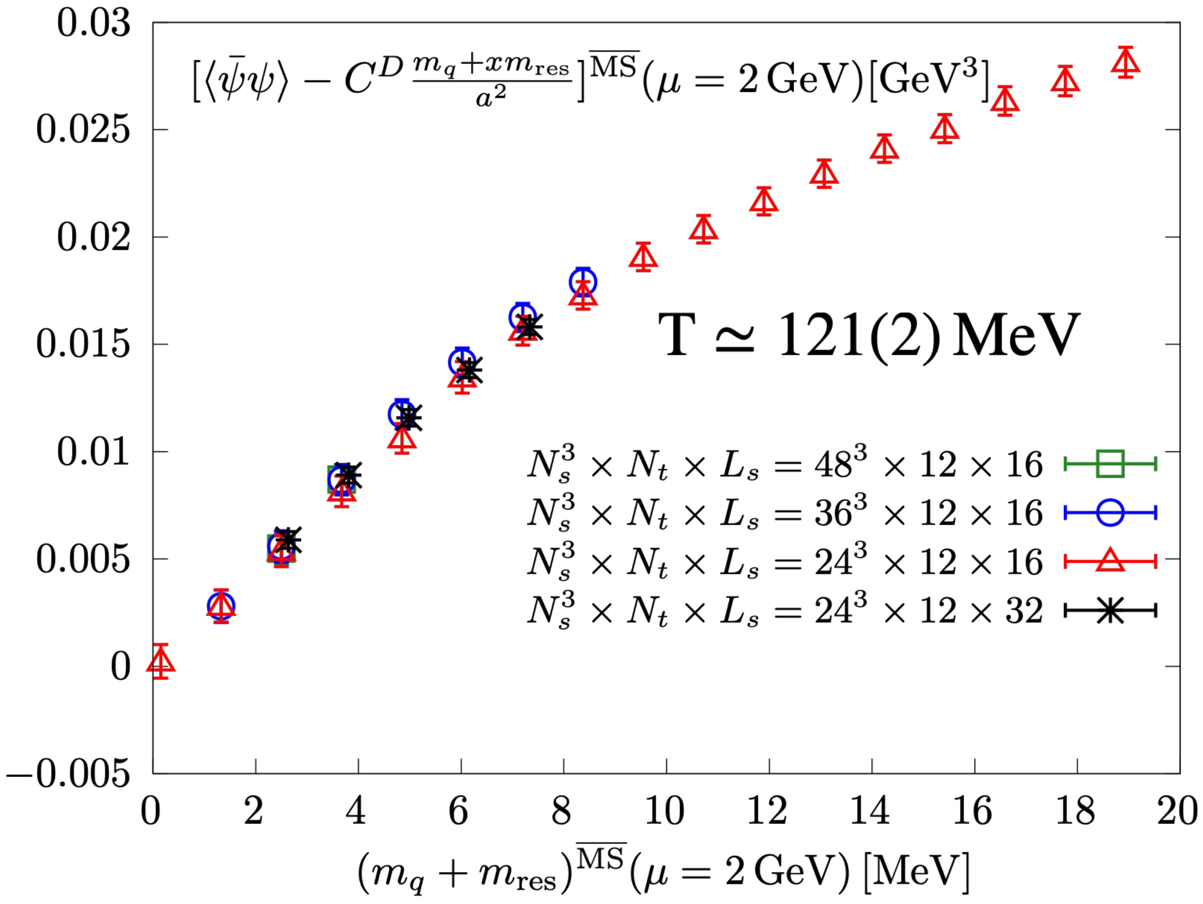


- $m_{pc} \simeq 4$ MeV
- $m < m_{pc}$: high T "phase"
- $\Sigma|_{DWF} \to C_D \frac{-(1-x)m_{res}}{a^2} + \Sigma|_{cont.}$; $(m_f \to -m_{res})$

- $\Sigma|_{cont.}$=0 : renormalization cond.
  - applied to determine x
  - x=-0.6(1) from 24^3x12x16

N_f=3, N_t=12 chiral condensate

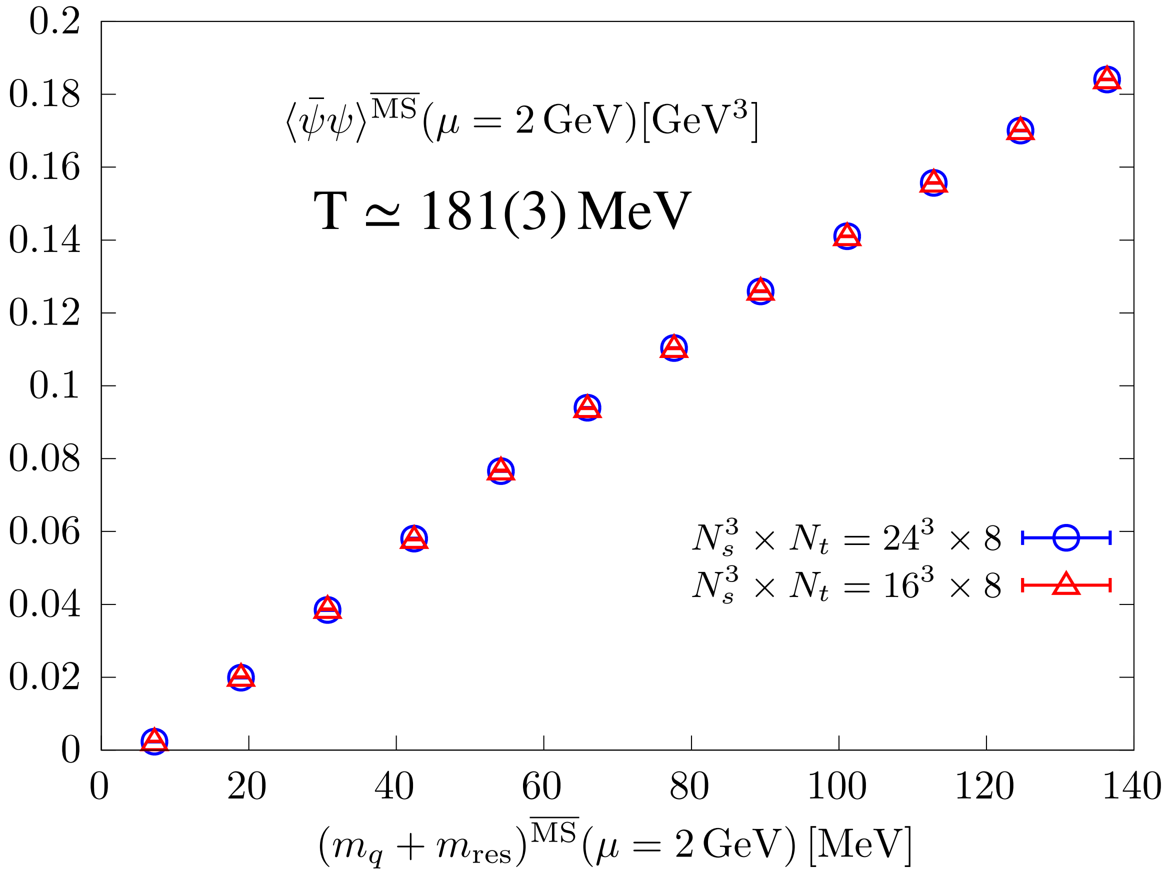




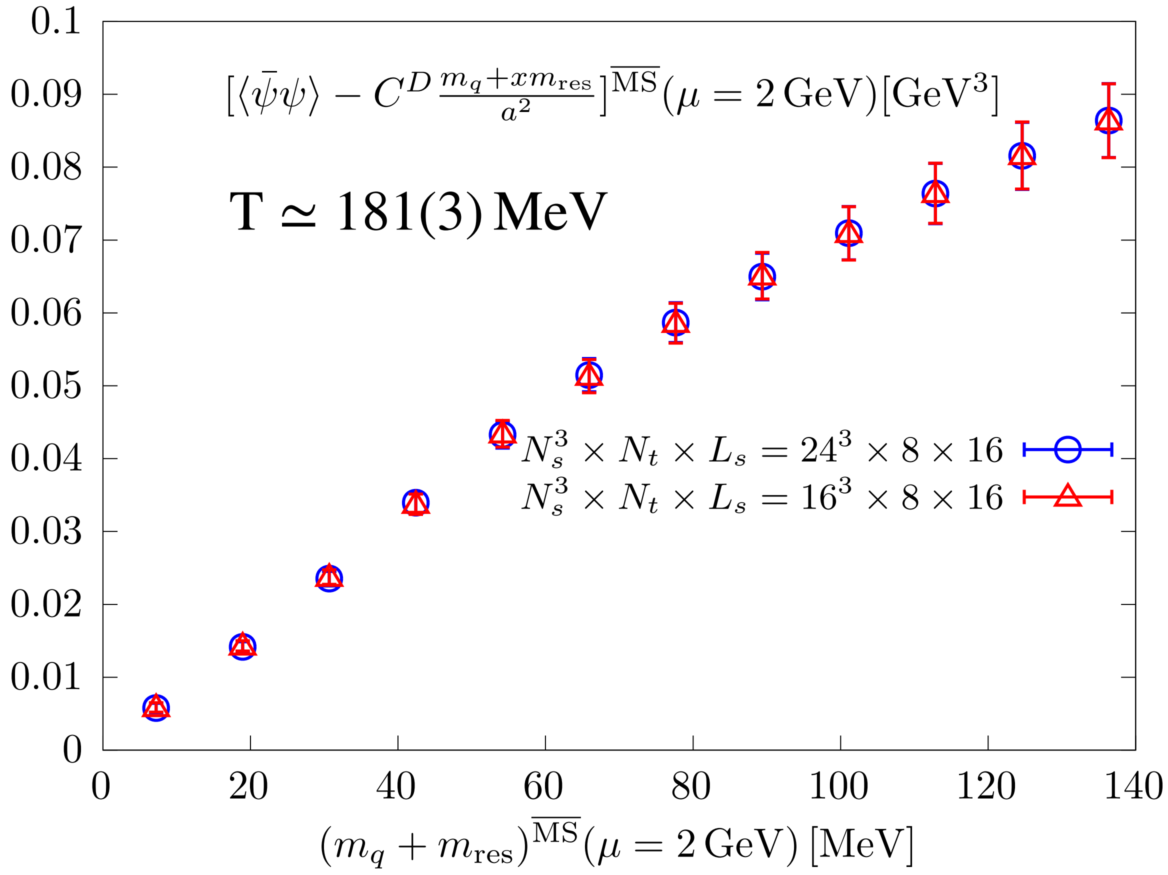
- $m_{pc} \simeq 4$ MeV
- $m < m_{pc}$: high T "phase"
- $\Sigma|_{DWF} \to C_D \frac{-(1-x)m_{res}}{a^2} + \Sigma|_{cont.}$; $(m_f \to -m_{res})$
- $\Sigma|_{cont.}$=0 : renormalization cond.
  - applied to determine x
  - x=-0.6(1) from 24^3x12x16
- **subtraction using these to all sets**
  - **note: consistency L_s=16 <-> 32**

Renormalized chiral condensate

Multiplicatively renormalized chiral condensate



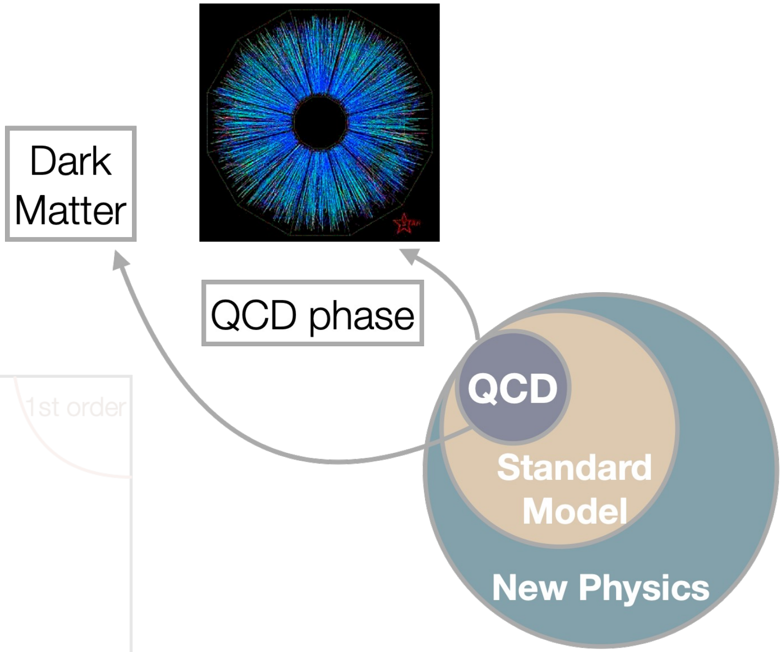
Additive and multiplicatively renormalized chiral condensate



Subtracted chiral condensate vanishes in the chiral limit as expected since T> T_c

X

QCD phase and thermodynamics



QCD phase

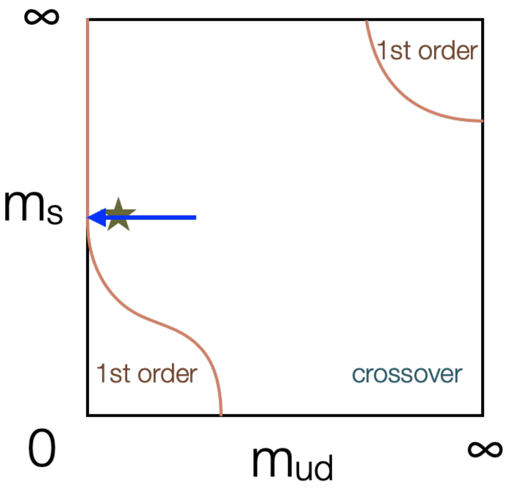
- fundamental understanding of QCD phase
- Simulation on (pseudo) critical temperature
  - with Domain-Wall fermions

Diagonal search

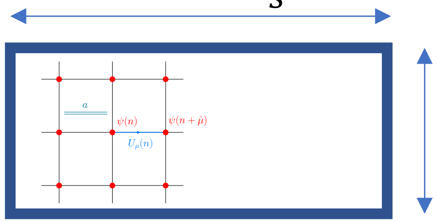
- Wilson fermion shows large discretization error
- Staggered fermion consistent with no 1st order
- ➢ use of chiral (Domain-wall) fermion would help

On/near physical point

- High temperature ($T > T_c$) overlap fermions
- ➢ $T = T_c$ using DWF with Fugaku
  - QCD thermodynamics



$L = aN_s$



$$\frac{1}{T} = aN_t$$

Modes of Simulations

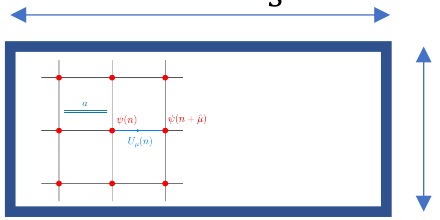
to locate phase transition

- tune parameters near transition

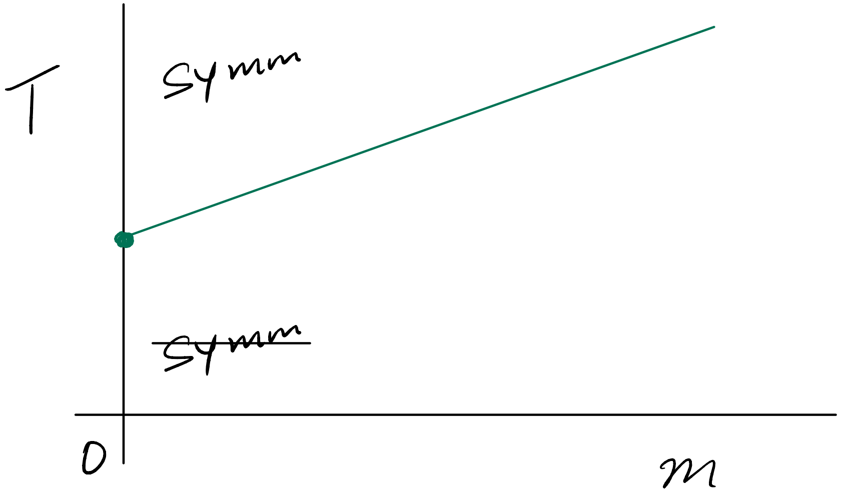
➢ T: fixed, change m

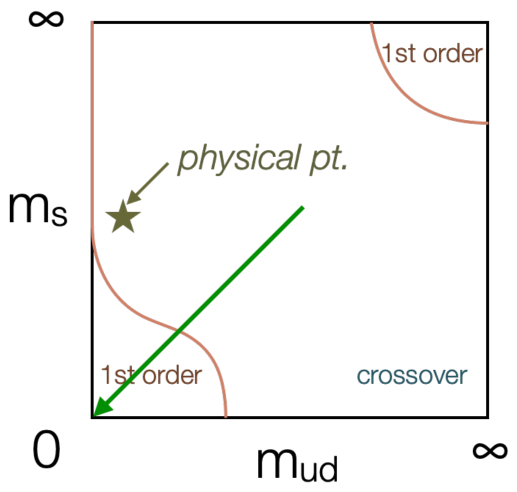
➢ m: fixed, change T

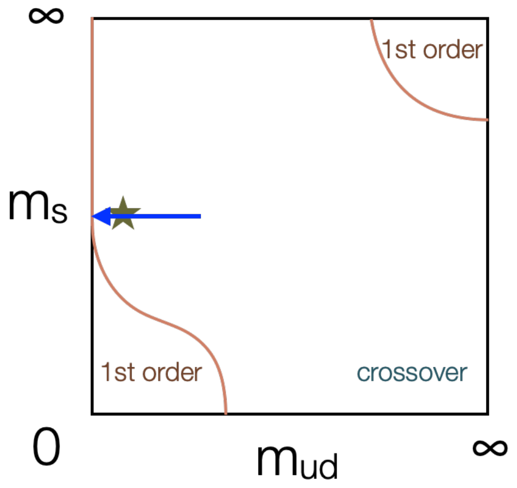
$L = aN_s$



$\frac{1}{T} = aN_t$



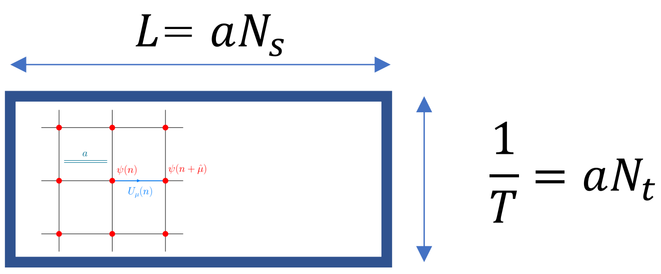


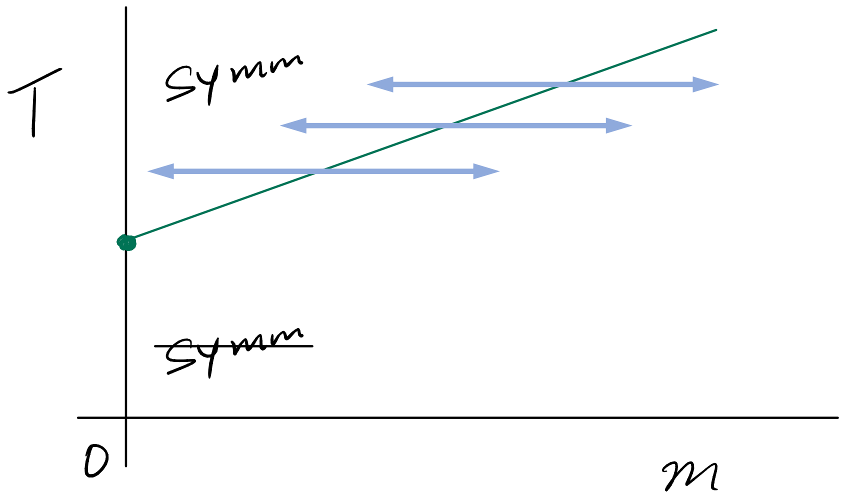


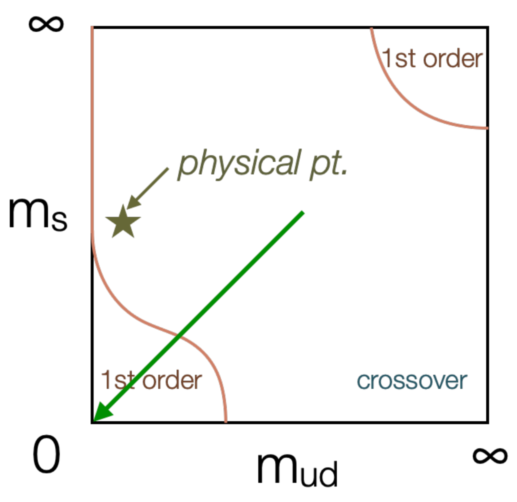
Modes of Simulations

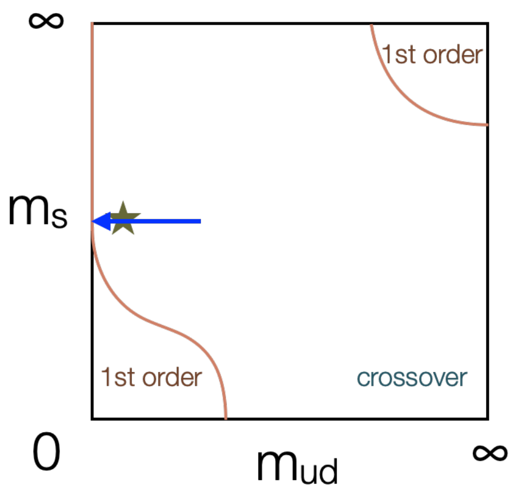
to locate phase transition

- tune parameters near transition
- ➢ T: fixed, change m
- ➢ m: fixed, change T







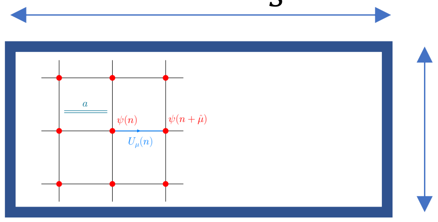


Modes of Simulations

to locate phase transition

- tune parameters near transition
  - T: fixed, change m
  - m: fixed, change T

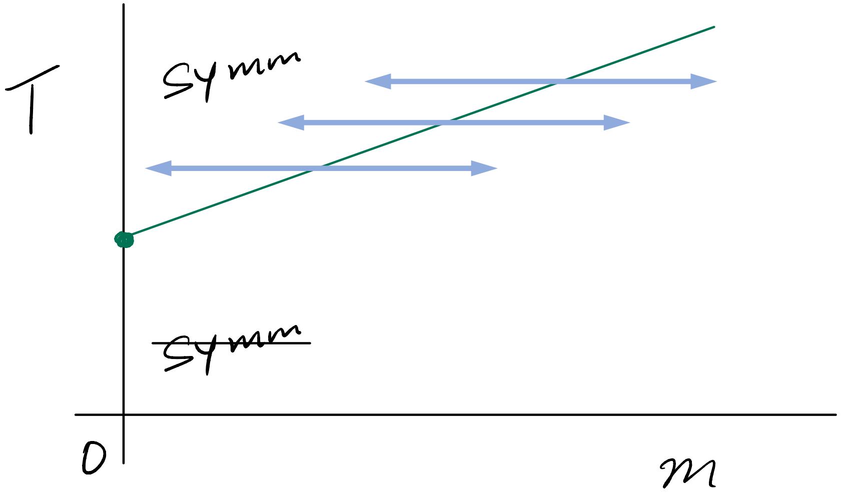
$L = aN_s$

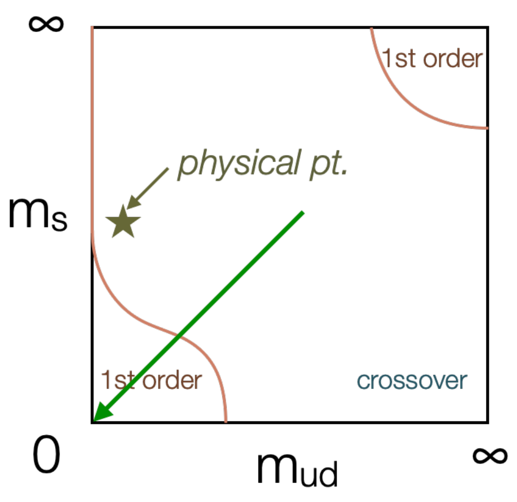


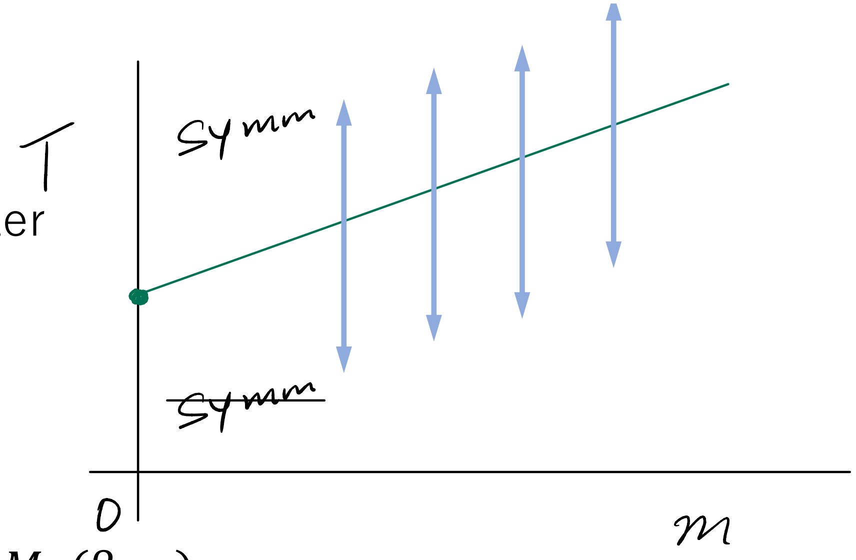
$$\frac{1}{T} = aN_t$$

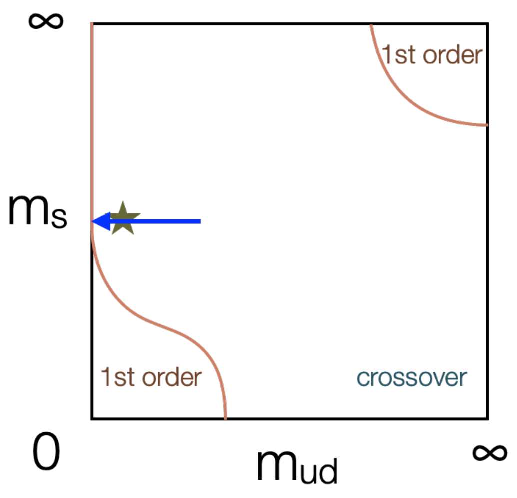
Fixing / changing the controlling parameter

- T: controled by
  - $a(\beta)$: controlled by β
  - N_t : discrete
- m: controlled by
  - input quark mass
  - $m(\beta) \leftarrow$ matching with hadronic scale: $M_H(\beta, m)$









N_f=2+1 Möbius DWF LCP for **2023-**

For the **L**ine of **C**onstant **P**hysics: $\boldsymbol{am_s(\beta)}$ with $\boldsymbol{a(\beta)}$

- Step 1: determine $\boldsymbol{a(\beta)}$ [fm] with t_0 (BMW) input
  - at $\beta = \mathbf{4.0}, \sout{4.1}^*, 4.17, 4.35, 4.47$

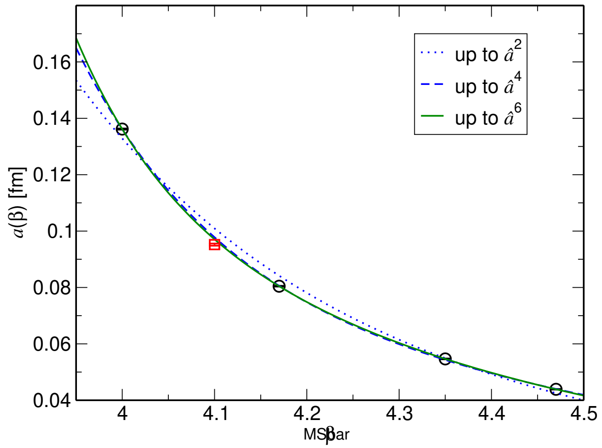
    * β=4.0 new data, to add support at small β

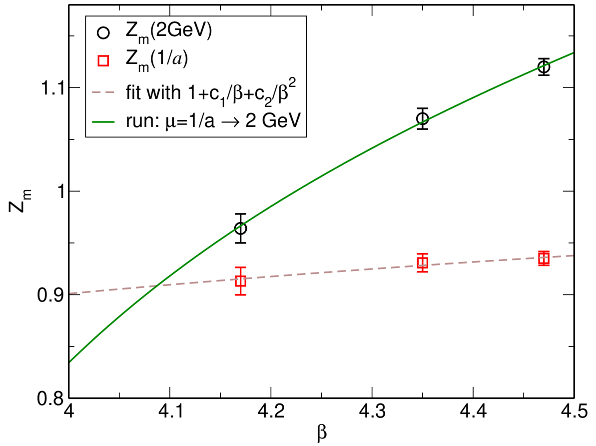
    * β=4.1 old pilot study data, removed - small volume and statistics
- Step 2: determine $Z_m(\beta)$ using Non-Perturbative Renormalization results
  - at $\beta =$ $4.17, 4.35, 4.47$; Z_m with $\overline{MS}$ 2 GeV are available
  - NNNLO running: $\mu = 2\,GeV \rightarrow 1/a$ & β polynomial fit & running back
  - use $Z_m(\beta)$ so obtained for $\beta \geq 4.0$: $\beta < 4.17$ region is extrapolation
  - $1/Z_m(\beta)$ will be used to renormalize scalar operator, **chiral condensate**
- Step 3: solve $\boldsymbol{am_s(\beta)}$ with input (*quark mass input*):
  - $m_s^R = Z_m \cdot am_s^{latt} \cdot a^{-1} = 92$ MeV
  - $\frac{m_s}{m_{ud}} = 27.4$ (See for example FLAG 2019)
- See for details in Lattice 2021 proc by S.Aoki et al.

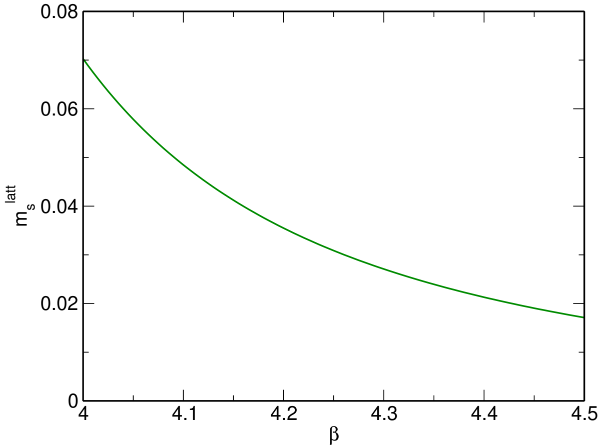
Do simulation

- Step 4: proper tuning of input mass: correct m_{res}

Do simulation 2nd round / correction with reweighting + valence meas.



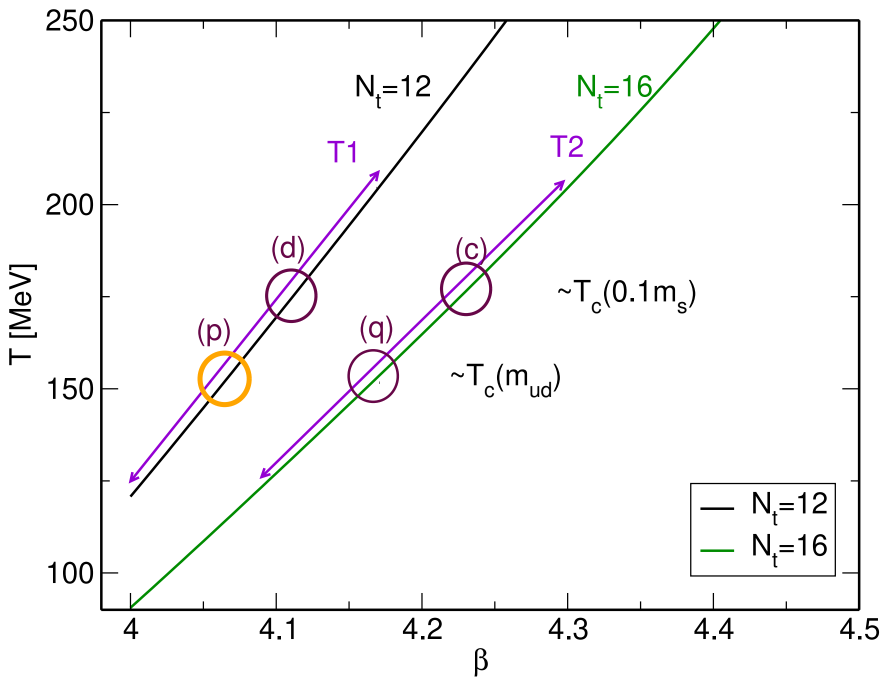


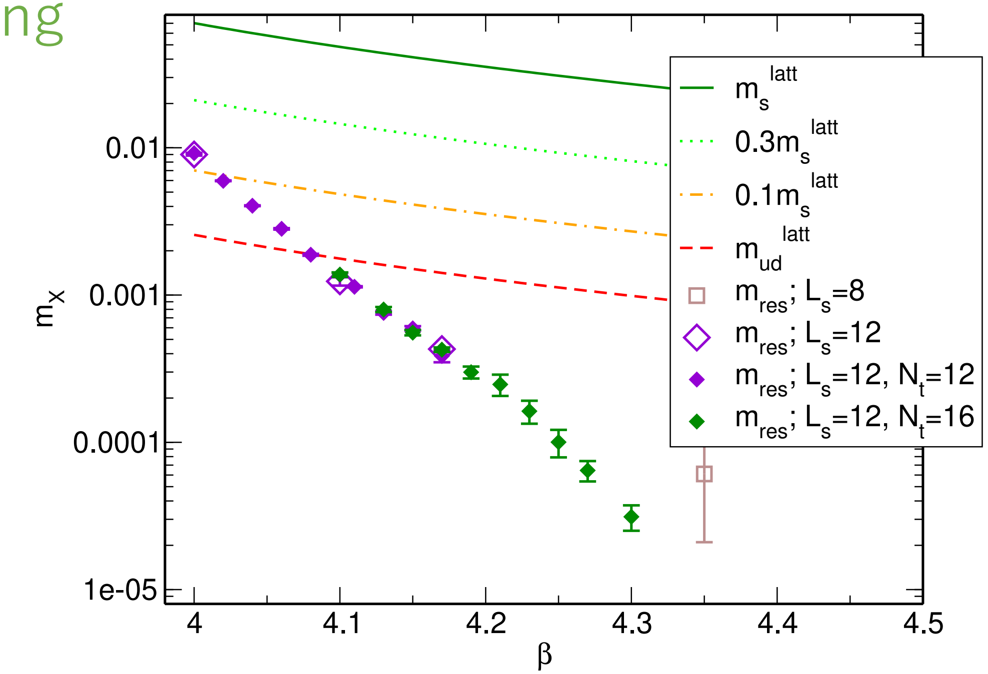


Simulation plan: 2nd round w/ treatment of m_{res} effect

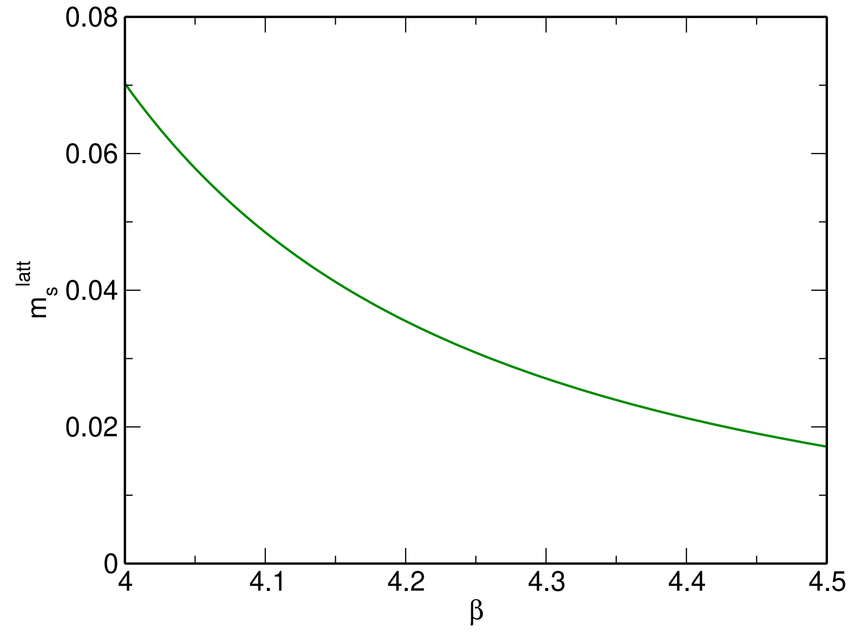
$L_s = 12$ fixed throughout this study

- T1-(d)
  - $N_t = 12$
  - $m_l = 0.1 m_s$
  - $m_q^{input} = m_q^{LCP} - m_{res}$
  - $V_s = 24^3, 36^3$
- T2-(c)
  - $N_t = 16$
  - $m_l = 0.1 m_s$
  - m_{res} shift by reweighting
  - $V_s = 32^3$
- T1-(p)
  - $N_t = 12$
  - $m_l = m_{ud}$
  - $m_q^{input} = m_q^{LCP} - m_{res}$
  - $V_s = 36^3, 48^3$
- T2-(q)
  - $N_t = 16$
  - $m_l = m_{ud}$
  - $m_q^{input} = m_q^{LCP} - m_{res}$
  - $V_s = 48^3$





LCP remarks for **FT2023-**

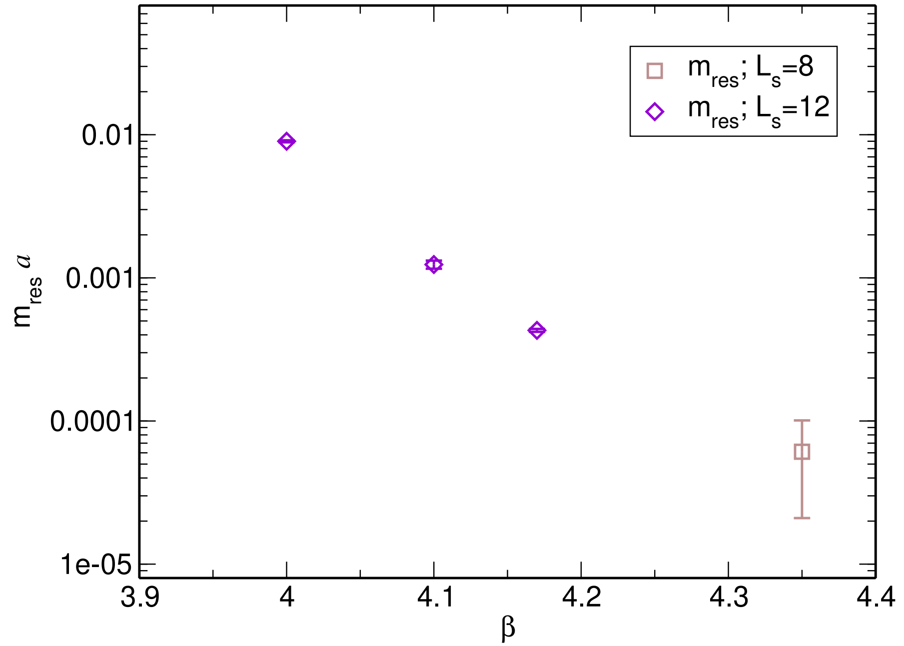


Features

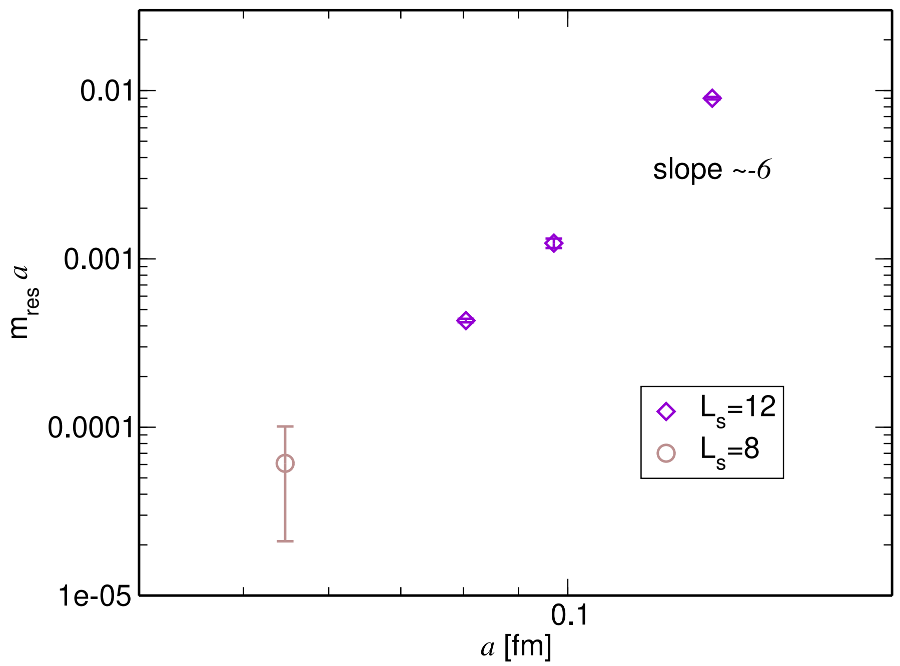
- Fine lattice: use of existing results ($\mathbf{0.04} \leq a \leq \mathbf{0.08}$ fm)
  - Granted preciseness towards continuum limit
- Coarse lattice parametrization is an extrapolation
  - Preciseness might be deteriorated
  - Newly computing Z_m e.g. at $\beta = 4.0$ (lower edge) might improve, but not done so far
    - NPR of Z_m at $a^{-1} \simeq$1.4 GeV may have sizable error (window problem) anyway
- Smooth connection from fine to coarse should not alter leading $O(a^2)$
  - Difference should be higher order
- Error estimated from Kaon mass (at physical point)
  - $\Delta m_K \sim$ ~~10 %~~ at $\beta = 4.0$ ($a \simeq 0.14$ fm) → $\Delta m_K \sim$ a few %
  - $\Delta m_K \sim$ a few % at $\beta = 4.17$ ($a \simeq 0.08$ fm)

Domain wall fermions

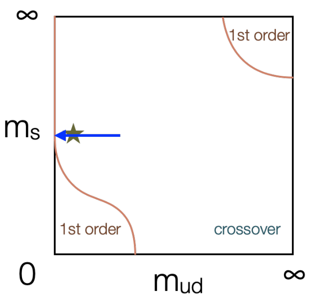
- Möbius DWF → OVF by reweighting
  - Successful (w/ error growth) at $\beta = 4.17$ ($a \simeq 0.08$ fm)
    - See Lattice 2021 JLQCD (presenter: K.Suzuki)
  - Questionable for
    - Coarser lattice: rough gauge, DWF chiral symmetry breaking
    - Finer lattice: larger V (# sites)
- Chiral fermion with continuum limit
  - A practical choice is to stick on DWF
- Controlling chiral symmetry breaking with DWF
  - WTI residual mass m_{res}: $m_\pi^2 \propto (m_f + m_{res})(1 + h.o.)$
  - Understanding $m_{res}(\beta)$ with fixed L_s (5-th dim size)
- $m_{res}[MeV] \sim a^X$, where $X \sim 5$
  - Vanishes quickly as $a \to 0$
  - 1st (dumb) approximation: forget about m_{res}
  - Better : $m_f^{cont} \leftrightarrow (m_f + m_{res})$ but, this is not always enough



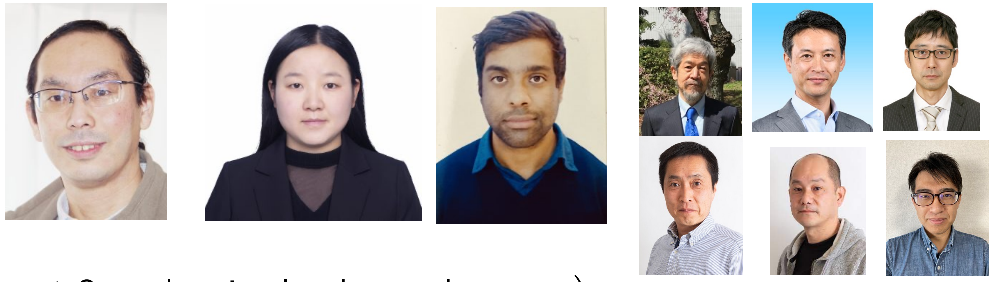
MDWF N_f=2+1, L_s=12



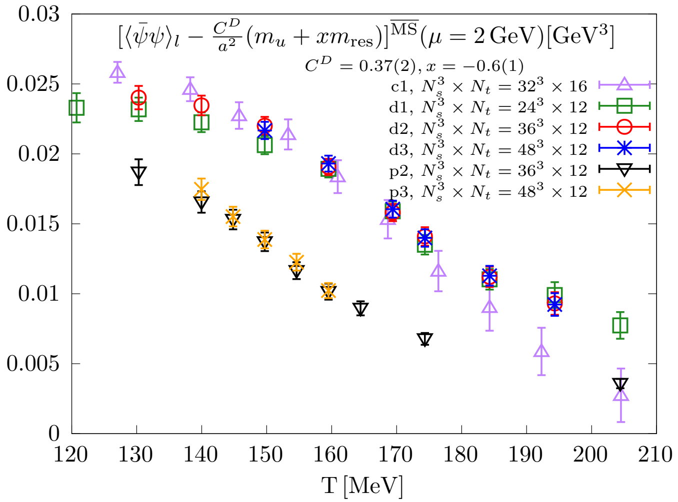
QCD phase transition near and on the physical point

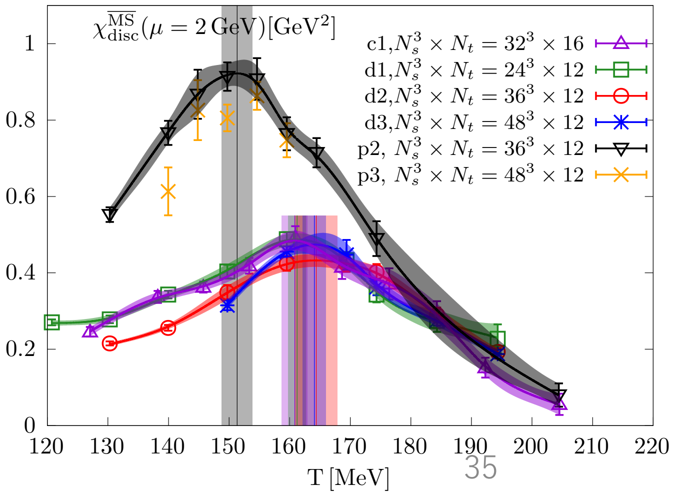


- N_f=2+1, 2 fine lattice DWF simulation and reweighting to overlap [PRD(2021), PTEP(2022)]
  - Profound relation among: chiral symmetry, axial anomaly and topological susceptibility
- R & D for the N_f=2+1 thermodynamics with Line of Constant Physics (LCP)
  - Codes: Grid, Hadrons, Bridge++
  - LCP / Reweighting
  - Chiral order parameter and renormalization
  - Quark number susceptibility



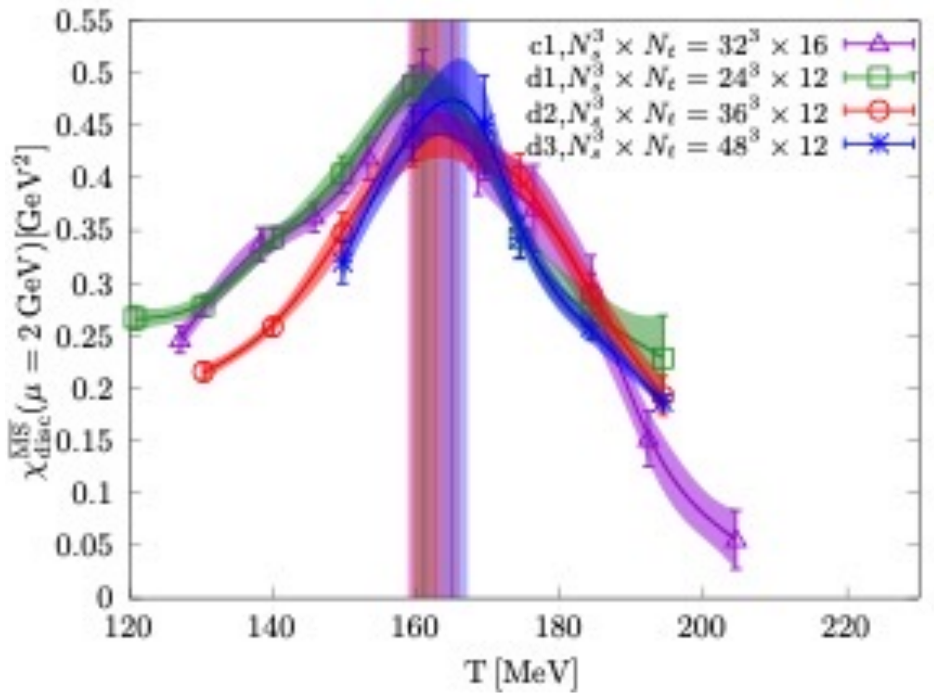
- N_f=2+1 - thermodynamics with LCP (mass = ms/10 = about 3 x physical ud quark mass)
  - 2 step renormalization for chiral condensate (power and log divergence) with an xm_{res} correction
  - 2 lattice spacings N_t=12, 16
  - 3 volumes N_s/N_t=2, 3, 4
  - *No phase transition !*
  - *T_{pc} determined $T_{pc} = 165(2)$ MeV*
  - PPR-Fugaku FY2020-2022
  - [PoS Lattice 2021, 2022]
- Physical point study
  - PPR-Fugaku 2023- preliminary results →

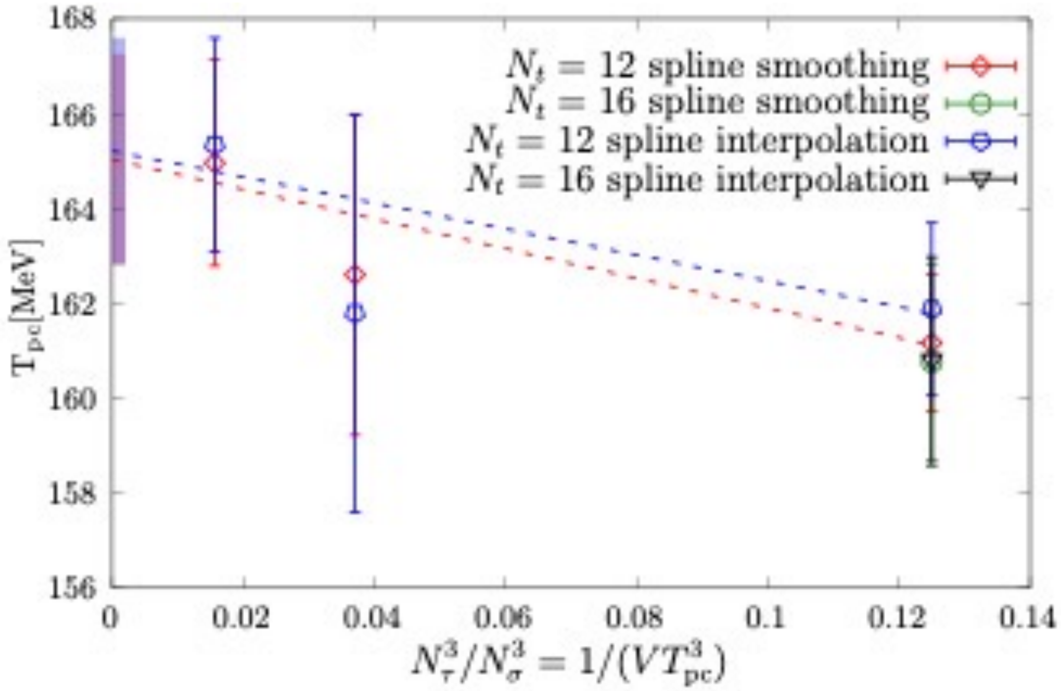




Chiral susceptibility (disconnected)

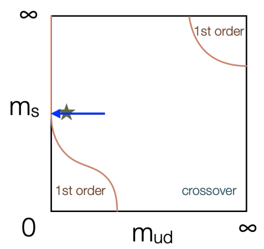
$m_l = 0.1 m_s$ (about 3 time larger than physics u,d mass)

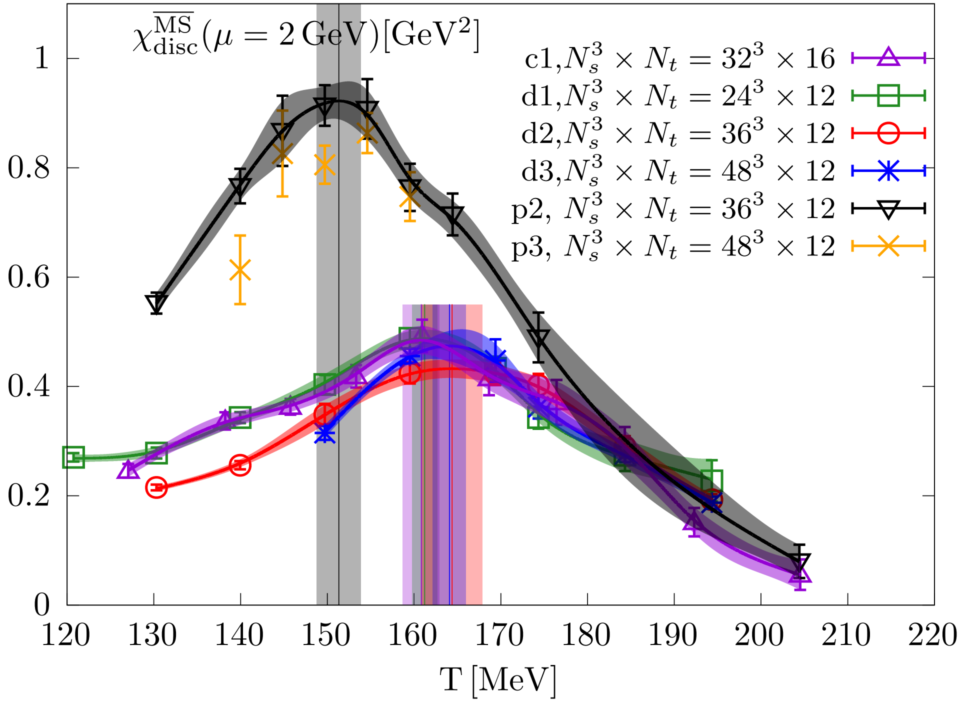




- no subtraction needed in addition to vacuum subtraction
- peak position : mild volume dependence → infinite volume limit
- observing no dependence for N_t=12 and 16 (LT=2)
- $\boldsymbol{T_{pc} = 165\ (2)}$ MeV from the disconnected chiral condensate

Disconnected chiral susceptibility at average **physical** u and d quark mass





$m_l = m_s/10$

- d1,d2,d3 : $N_t = 12$, LT=2,3,4
  - almost no volume dependence → **cross over**
- c1 : $N_t = 16$, LT=2
  - good scaling $N_t = 12$ -16 observed for LT=2

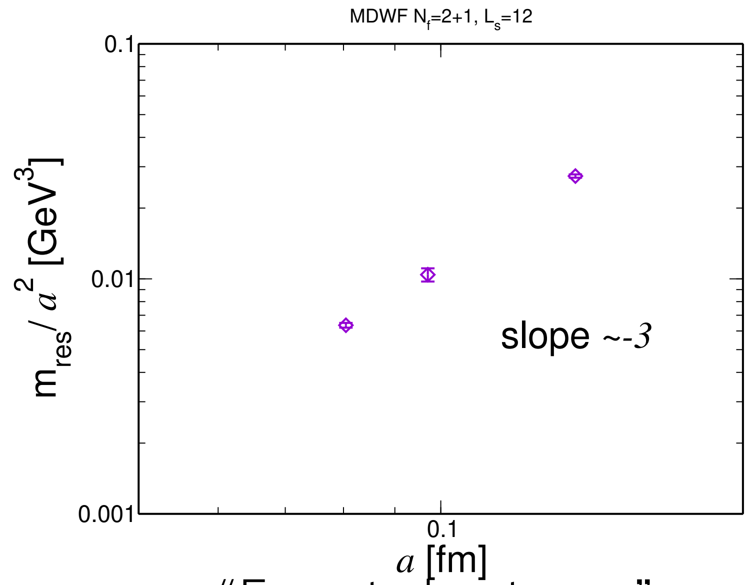
$m_l = m_{ud}$

- p2,p3: N_t=12, aspect ratio LT = 3, 4
  - Statistics is ~20,000 MDTU for LT=3, sampled every 10 MDTU
  - LT=4 very preliminary, currently running to get to planned satat.
- $\boldsymbol{T_{pc} = 151\,(3)}$ MeV (preliminary) on $\boldsymbol{36^3 \times 12}$, compared with
  - $\boldsymbol{T_{pc} = 155\,(1)(8)}$ w/ DWF (**N_t=8**) by HotQCD (2014)
  - $\boldsymbol{T_{pc} = 156.5\,(1.5)}$ w/ HISQ by HotQCD (2019) (≃disconnected)
  - $\boldsymbol{T_{pc} = 158.0\,(0.6)}$ w/ stout staggered, Budapest-Wuppertal (2020)

Likely **NO phase transition** at **physical point** with chiral fermions.
No surprise happened so far..

Light quark $\Sigma = -\langle\bar{\psi}\psi\rangle$: conventional and residual power divergence

- $\Sigma|_{DWF} \sim C_D \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$ S. Sharpe (arXiv: 0706.0218)
  - $m_{res} \neq x m_{res}; \quad x = O(1) \neq 1$
    - "Since x is not known, this term gives an uncontrolled error in the condensate. It can be studied and reduced only by increasing L_s - a very expensive proposition." – S. Sharpe.
- cf: $m_\pi^2 \propto (m_f + m_{res})$ [1+h.o.]
- $\Sigma|_{DWF} \rightarrow \quad C_D \frac{x m_{res}}{a^2} \quad + \Sigma|_{cont.} + \cdots; (m_f \rightarrow 0)$
- $\Sigma|_{DWF} \rightarrow C_D \frac{-(1-x) m_{res}}{a^2} + \Sigma|_{cont.}; \quad (m_f \rightarrow -m_{res})$



"Forget about m_{res}" is dumber for Σ, but⋯

Light quark $\Sigma = -\langle\overline{\psi}\psi\rangle$: residual power divergence

- $\Sigma|_{DWF} = C_D \frac{m_f + x m_{res}}{a^2} + \Sigma|_{cont.} + \cdots$ S. Sharpe (arXiv: 0706.0218)

  $m_{res} \neq x m_{res}$; $x = O(1) \neq 1$

  - "Since x is not known, this term gives an uncontrolled error in the condensate. It can be studied and reduced only by increasing L_s - a very expensive proposition." – S. Sharpe.

- (we proposed another way to utilize m'_{res}, which end up mixing T=0 C_R into high T)
- Yet another way of subtraction including $x m_{res}$ using $\boldsymbol{N_f = 3, T = 0}$ & $\boldsymbol{T > T_c}$ information

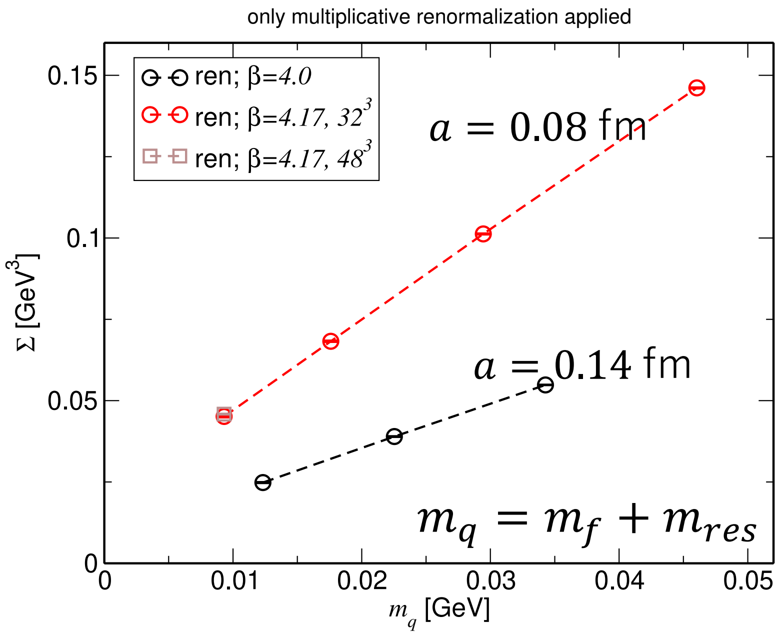
  →see the talk by Yu Zhang

1. Prepare several different lattice spacing for $\boldsymbol{T = 0}$
2. Compute coefficient linear in m_f : $\Sigma|_{DWF} \sim const. + (\frac{C_D}{a^2} + C_R) m_f + \cdots$
3. Separate divergent term : *linear fit in* a^2 *of*: $C_D + a^2 C_R \rightarrow \boldsymbol{C_D = 0.37(2)}$
4. Estimate x using $\boldsymbol{T > T_c}$ through $\Sigma|_{DWF} \rightarrow \frac{-C_D(1-x) m_{res}}{a^2} = 0$ $(m_f \rightarrow -m_{res})$ [ren.cond. $\Sigma|_{cont.} = 0$]

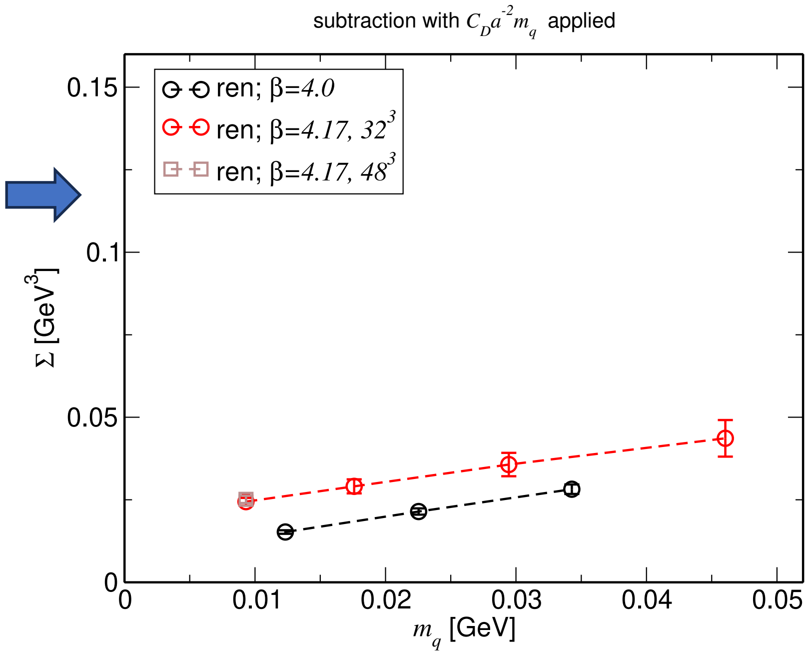
→ $\boldsymbol{N_f = 3; \beta = 4.0}$ estimate: $\boldsymbol{x = -0.6(1)}$

- In general, $\boldsymbol{x}$ may depend on $\boldsymbol{\beta}$, for now use this value as a reference for all $\boldsymbol{\beta}$
- We also use C_D (single flavor normalization) of $N_f = 3$ for $N_f = 2 + 1$

test on $N_f = 2+1, T = 0$ measurements

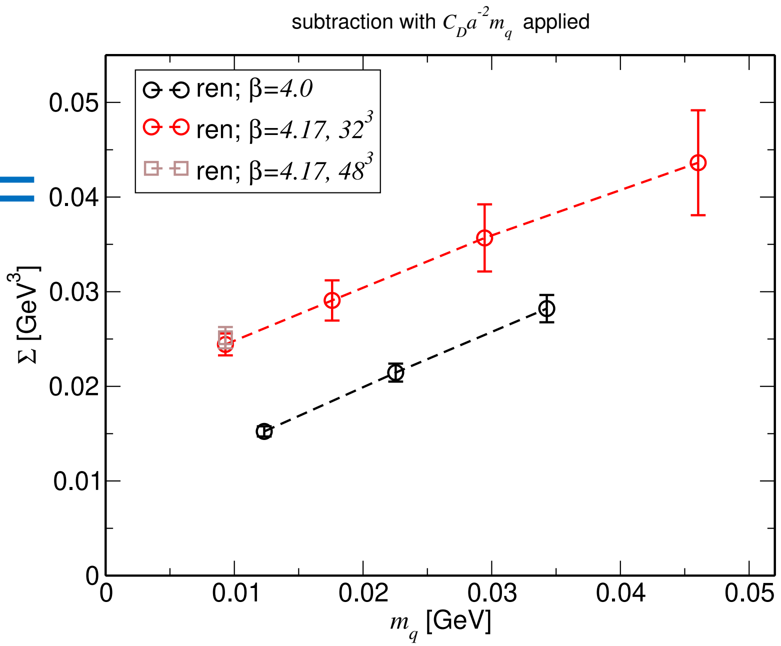


only multiplicative renormalizations applied



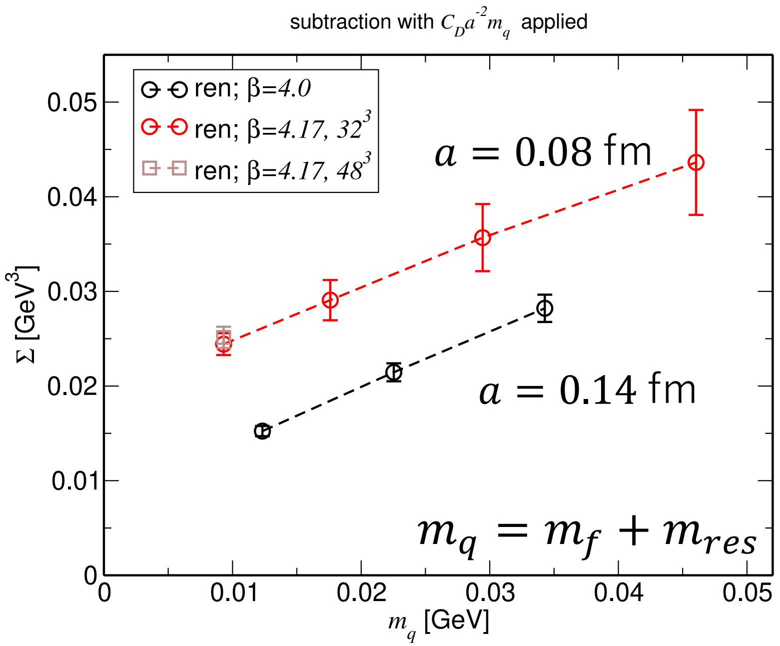
$C_D a^{-2} m_q$ subtraction applied

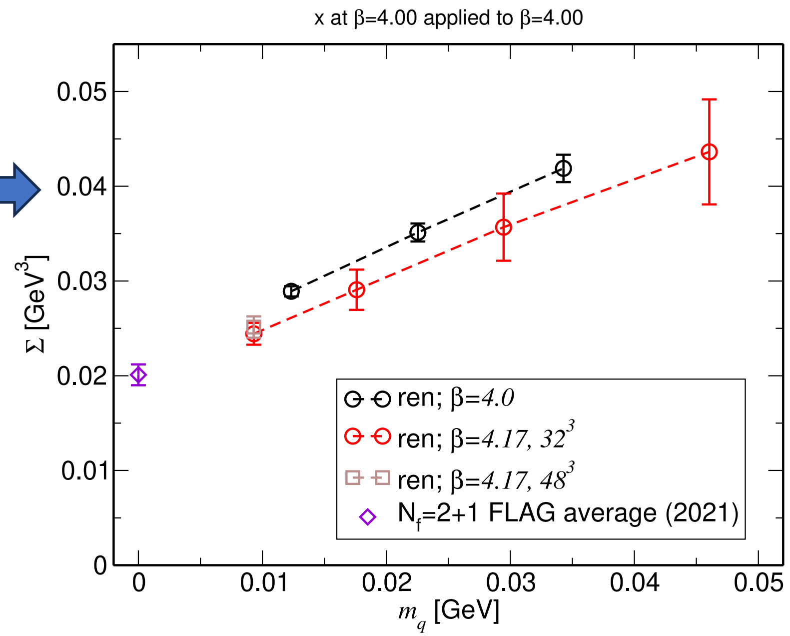
=

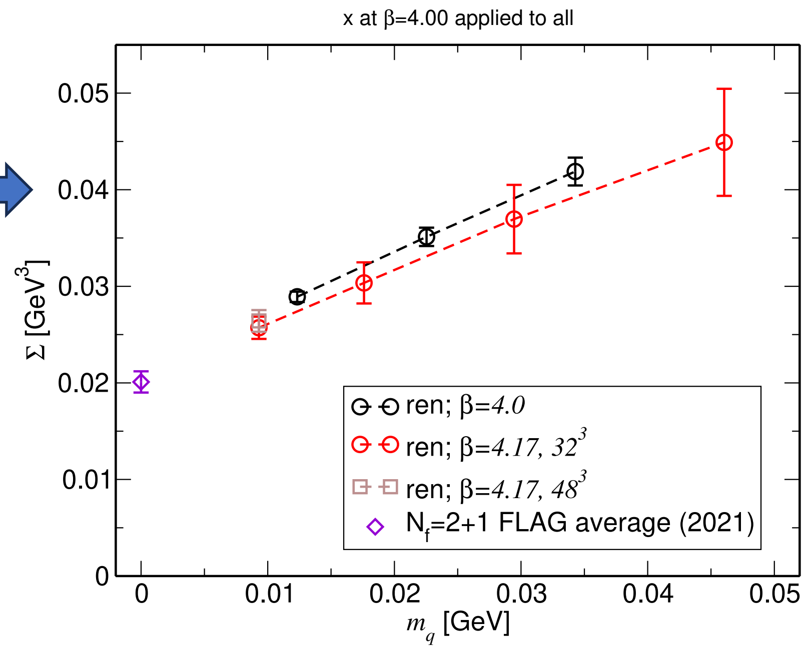


changing y-axis range

test on $N_f = 2 + 1, T = 0$ measurements







$C_D a^{-2} m_q$ subtraction applied

$C_D a^{-2}(1-x) m_{res}$ subtraction applied only to β =4.0

$C_D a^{-2}(1-x) m_{res}$ subtraction applied to all assuming x is universal

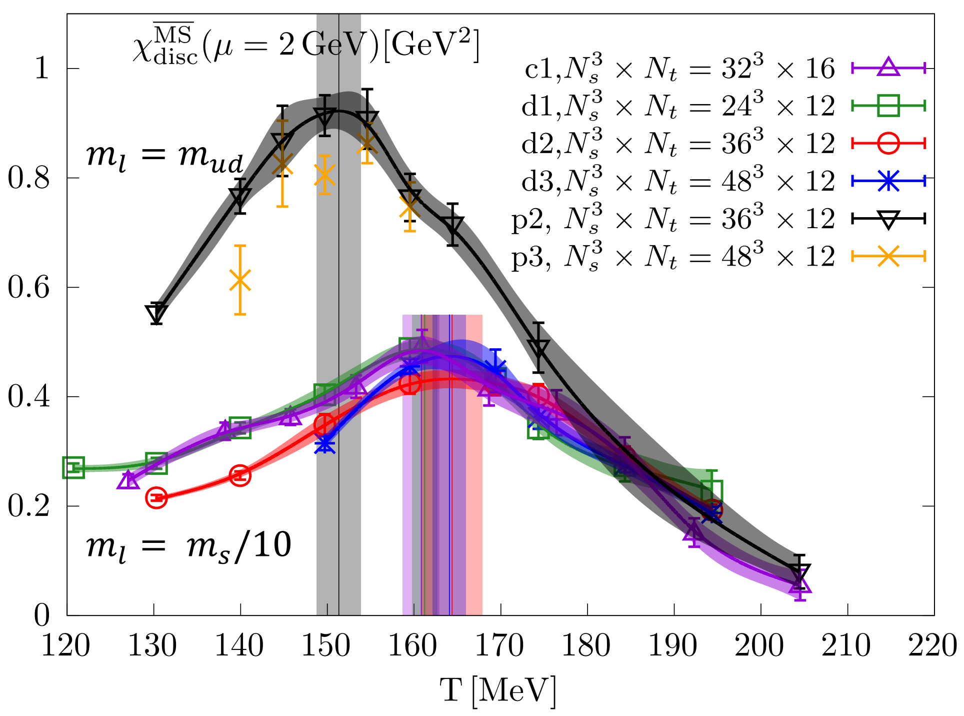
Seemingly, both conventional and residual divergence are controlled, but

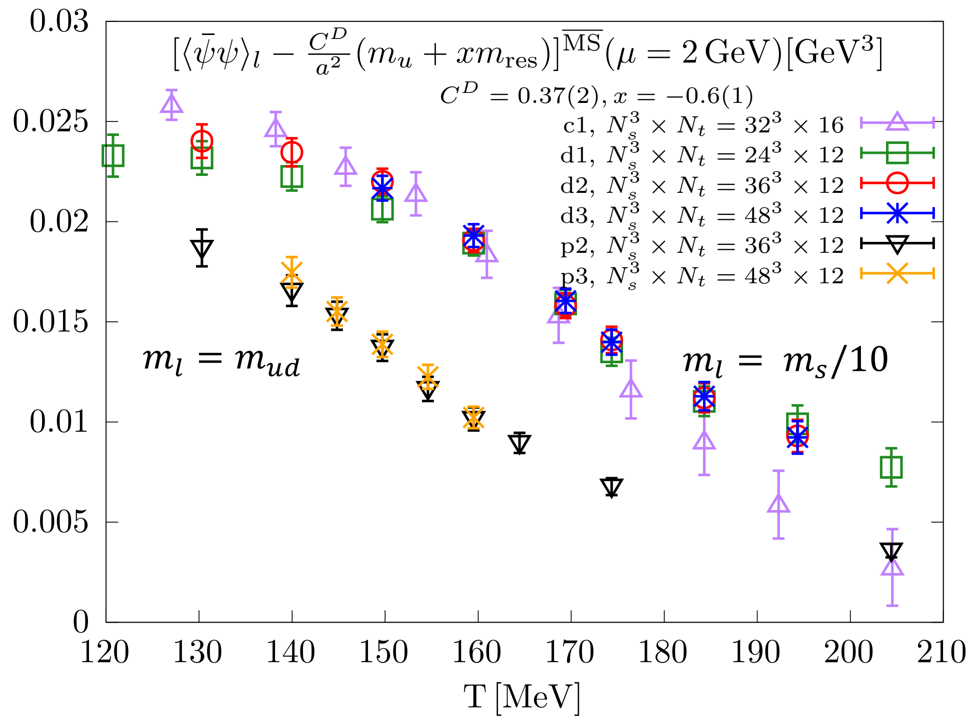
- need to check if x does not depend much on β
- refinement of precision and check applicability range of C_D necessary

Disconnected chiral susceptibility and chiral condensate

$m_l = m_{ud}$

all divergences subtracted assuming x is universal



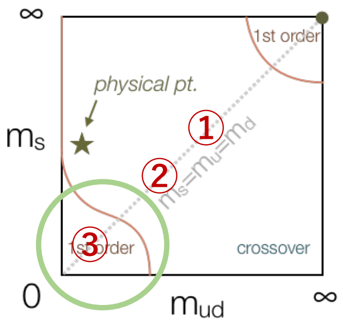


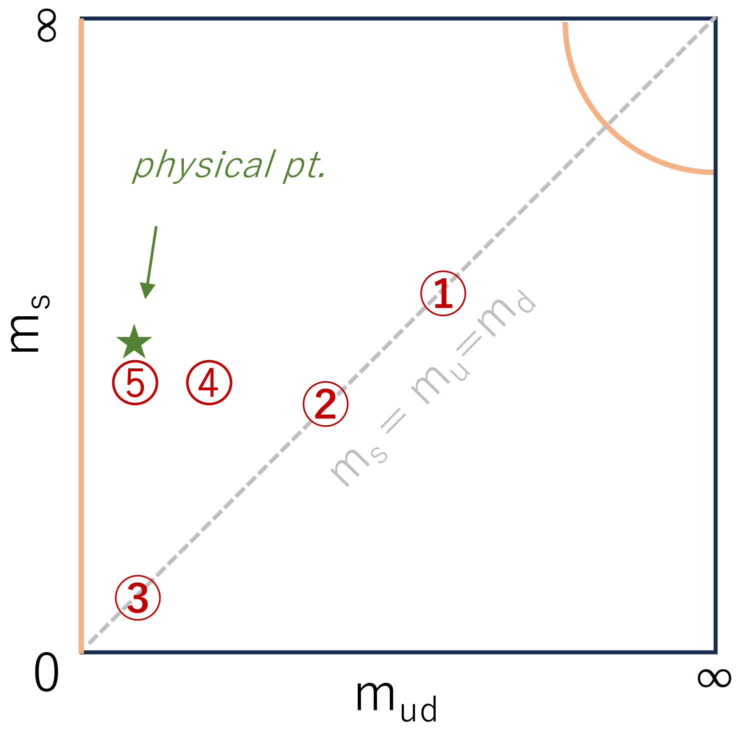
Likely NO phase transition at physical point with chiral fermions.
No surprise happened so far..

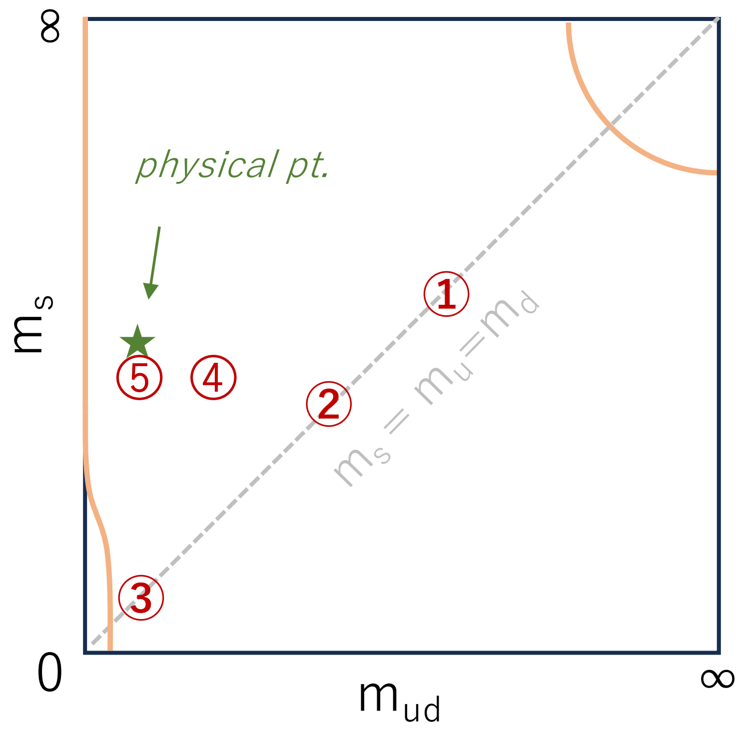
$m_l = m_{ud}$

- p2,p3: N_t=12, aspect ratio LT = 3, 4
  - Statistics is ~20,000 MDTU for LT=3, sampled every 10 MDTU
  - LT=4 very preliminary, currently running to get to planned satat.
- $\boldsymbol{T_{pc} = 151\,(3)}$ MeV (preliminary) on $\boldsymbol{36^3 \times 12}$

QCD phase hunting: outcome so far

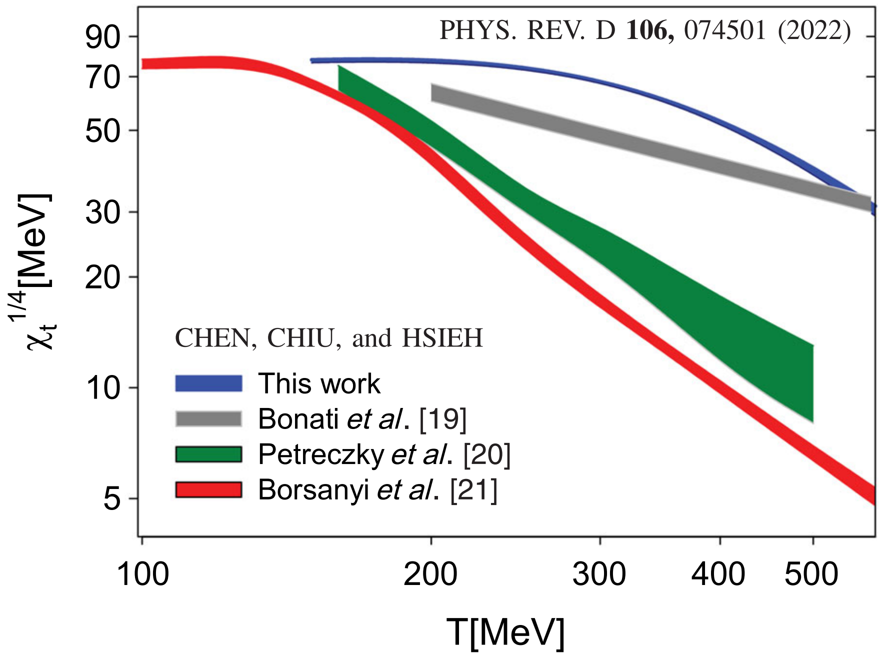






topological susceptibility @ physical point

Recent Summary by Chen et al (TWQCD)



Chen et al: optimal DWF N_f=2+1+1
(yet another DWF)

- Crucial input to axion dark matter scenarios
- Large difference: blue (latest) <-> Red / Green

topological susceptibility

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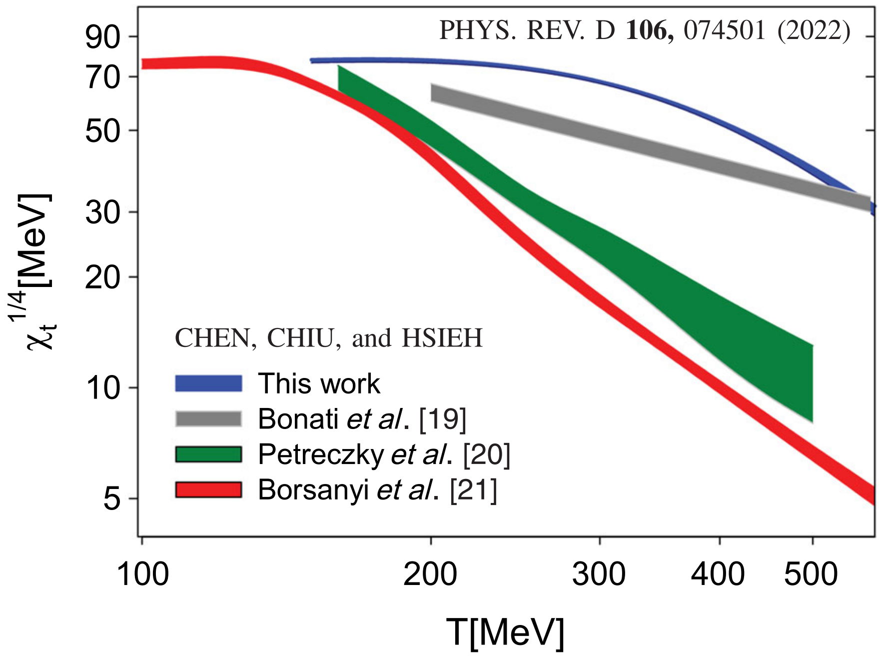

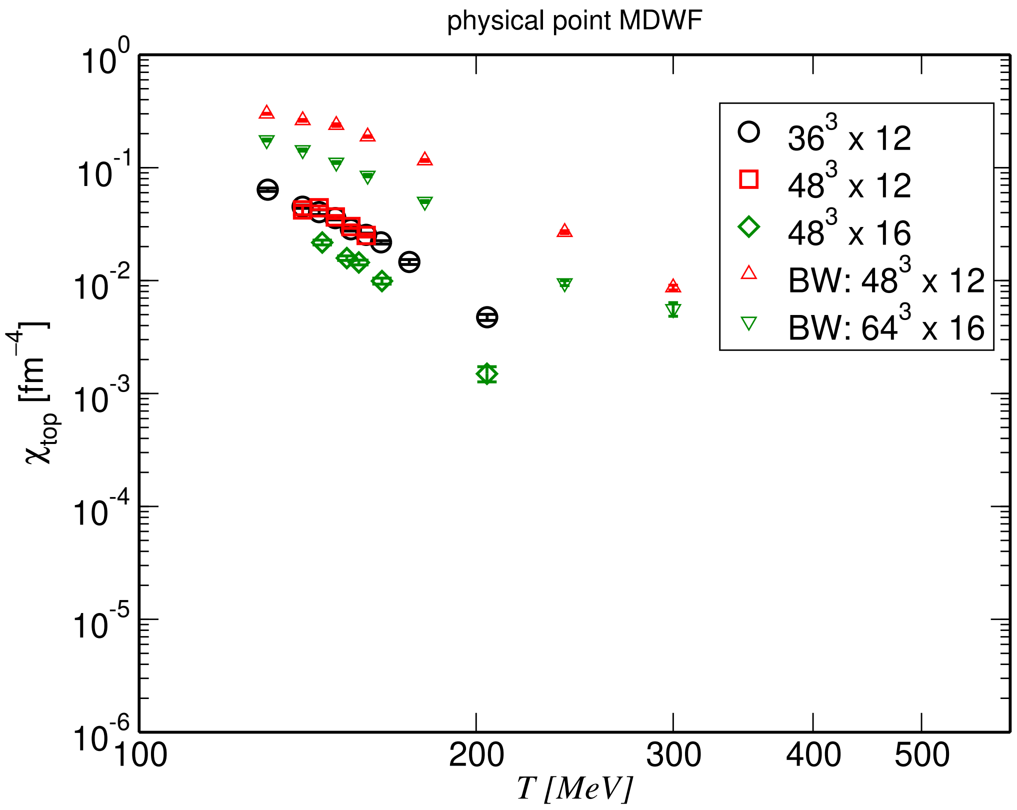

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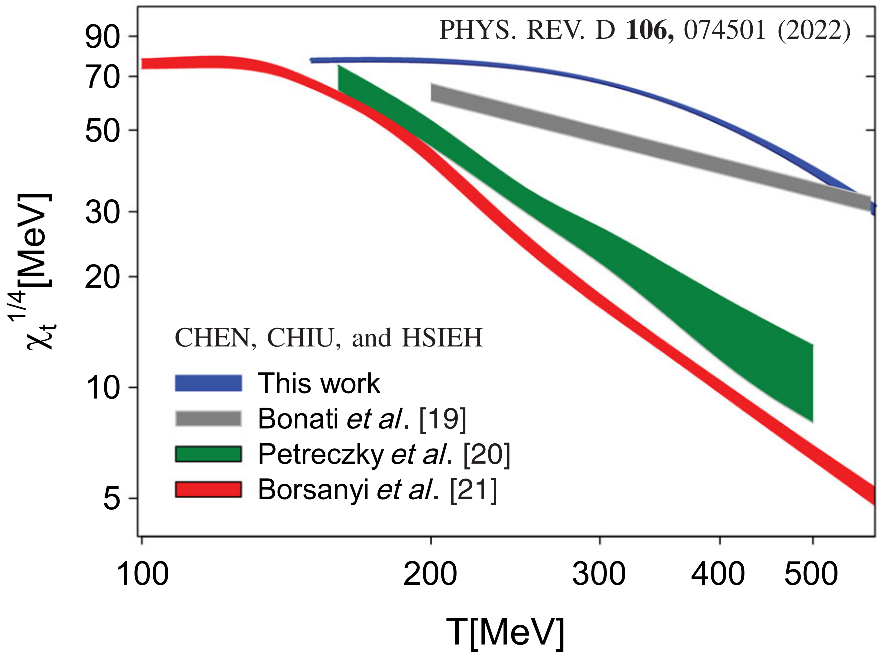

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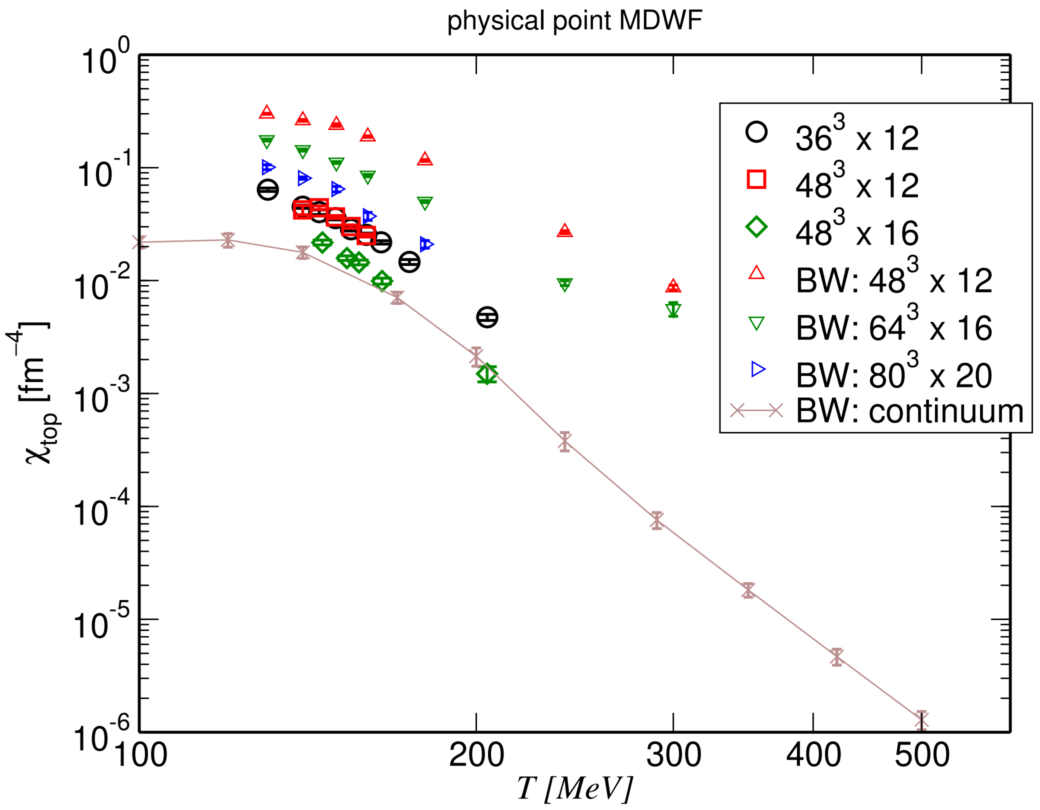

MDWF(JLQCD) χ_t at physical point

- inconsistent with Chen et al (optimal DWF)
- getting closer to BW[continuum] for $a \to 0$
- N_t=16 already ~continuum or even undershoot ?

• more detailed study needed

Summary and Outlook

Summary

- **Chiral fermion simulation using domain wall fermions on Fugaku underway**
- **Finite temperature QCD studies are on-going**
  - N_f=2+1 near physical point using LCP
  - N_f=3 phase hunting
  - N_f=2, 2+1: fate of $U(1)_A$ and relation with topology with DWF→Overlap Fermion
    - led by Hidenori Fukaya (Osaka)
  - N_f=2, 2+1: hadron correlation and extended symmetry at high T
    - Hidenori Fukaya (Osaka), David Ward (Osaka / R-CCS), et al
- **These activities are done mostly with JLQCD collaboration**
- **controlling the residual symmetry violation – promising recent progress**
- **With deep chiral simulations, no signal of 1st order transition so far**

Outlook

- **N_f=2+1 physical point → continuum limit project underway → FY2025**
- **thermodynamics observables : charge fluctuations are to be investigated (J. Goswami)**
  - direct relevance to Heavy Ion Collision experiments
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  - topology → dark matter → may eventually shed light on the current confusing status

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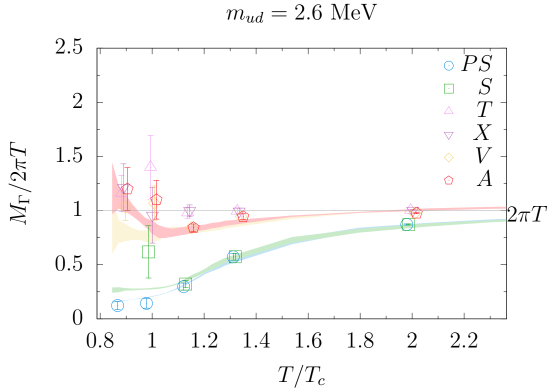
David Ward



arXiv: 2501.12675

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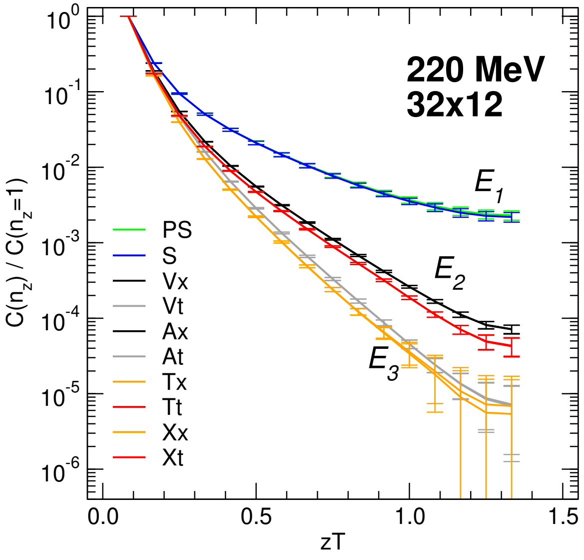
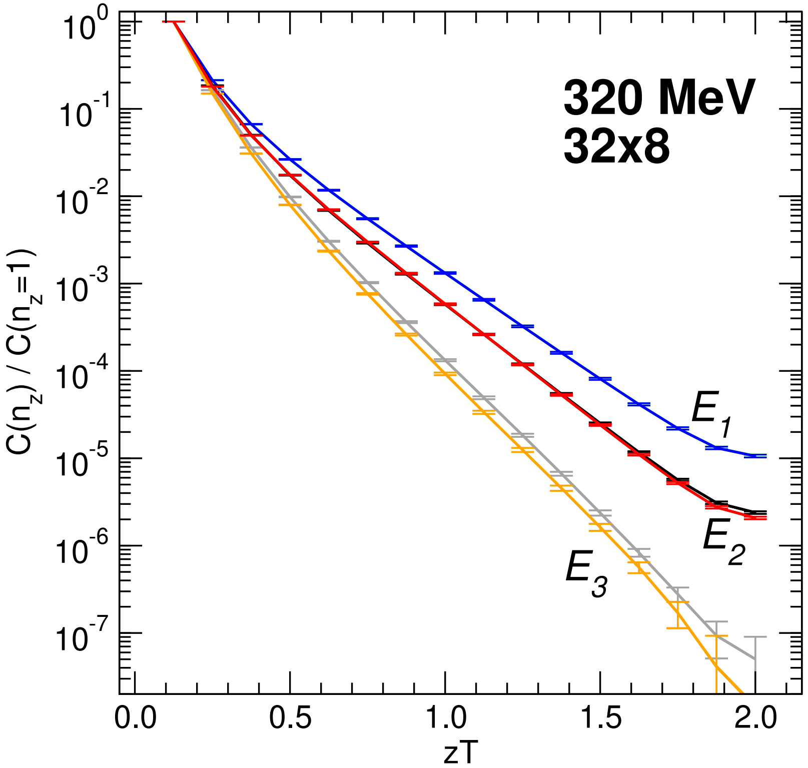
$N_f = 2$ screening mass



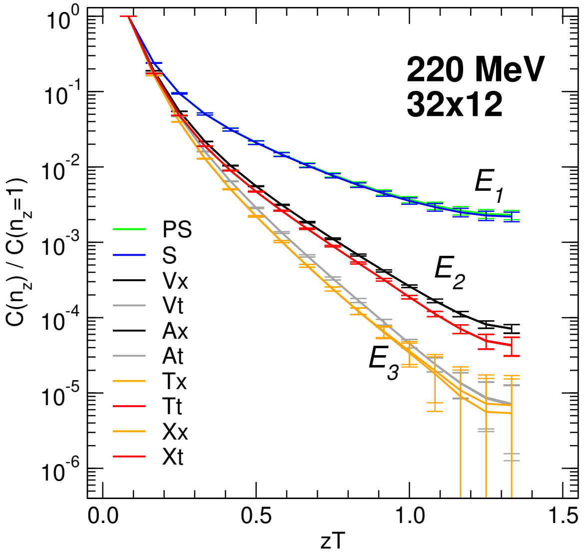


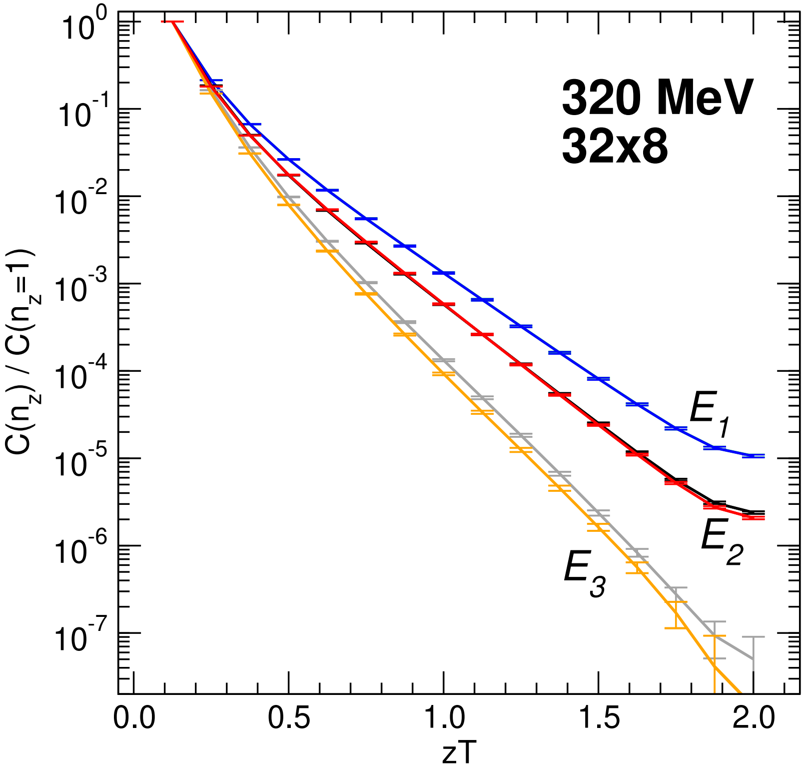
Rohrhofer et al PRD (2019)

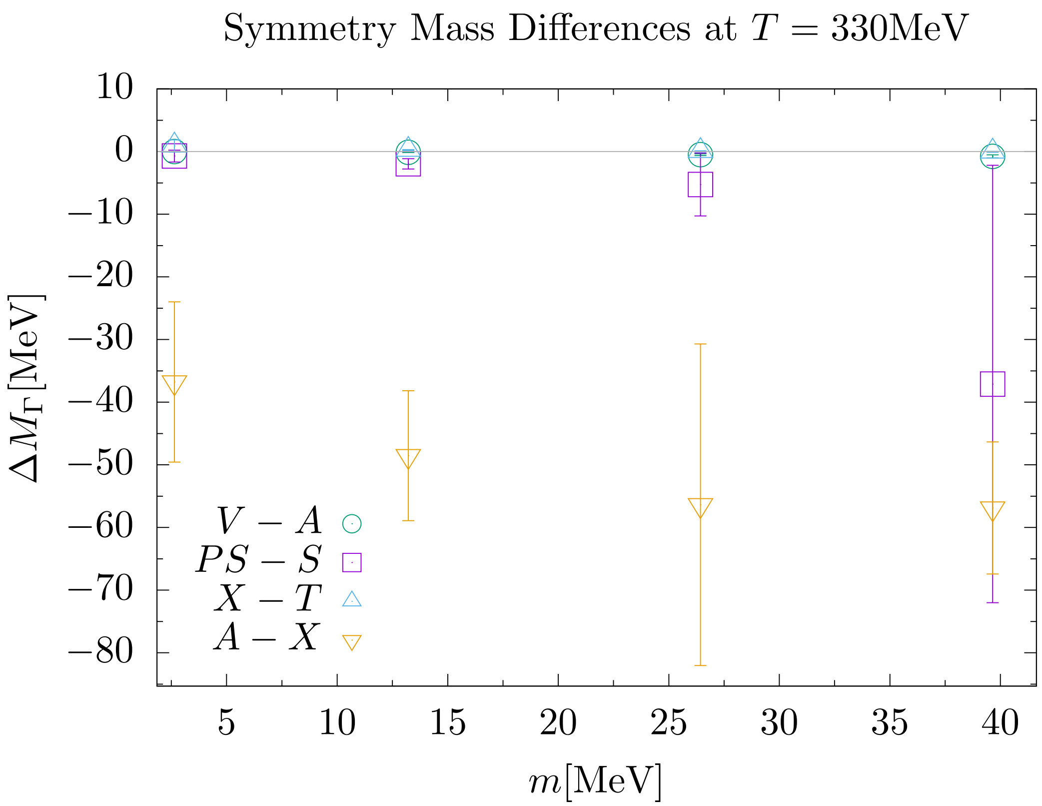
$N_f = 2$ screening mass

Hidenori Fukaya

David Ward







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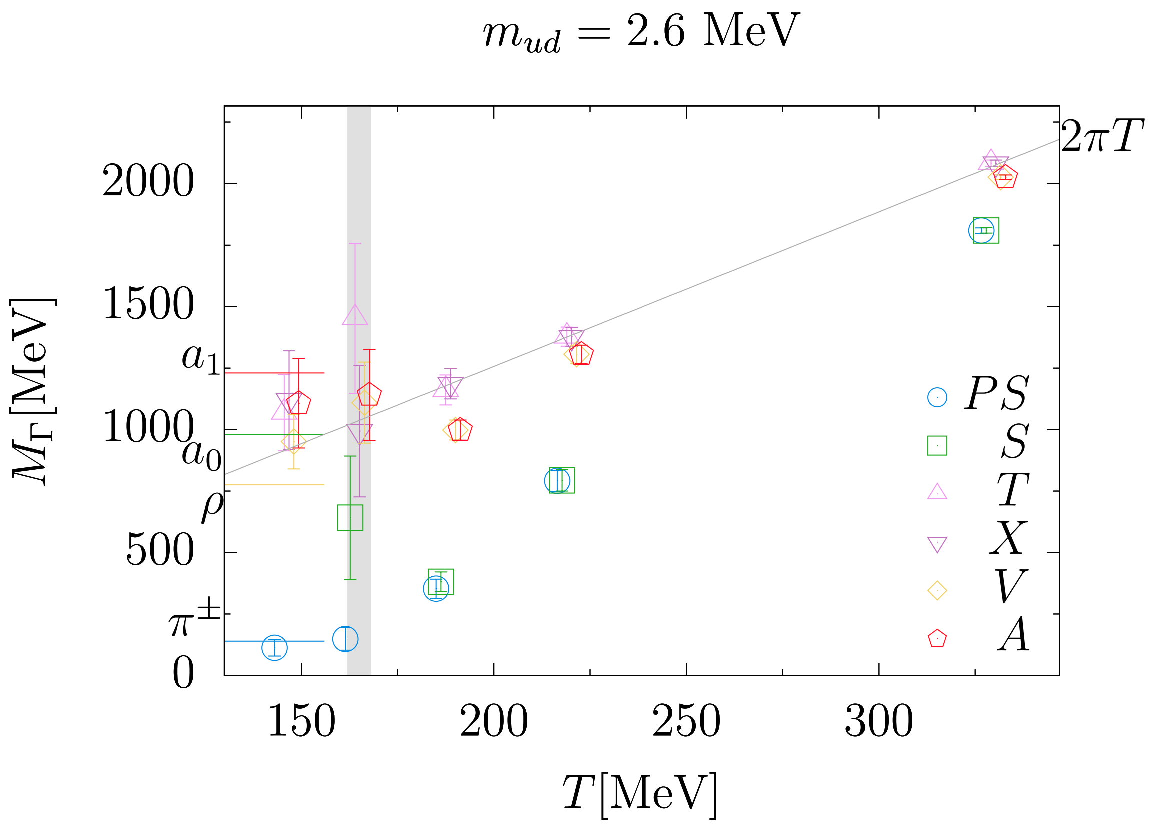
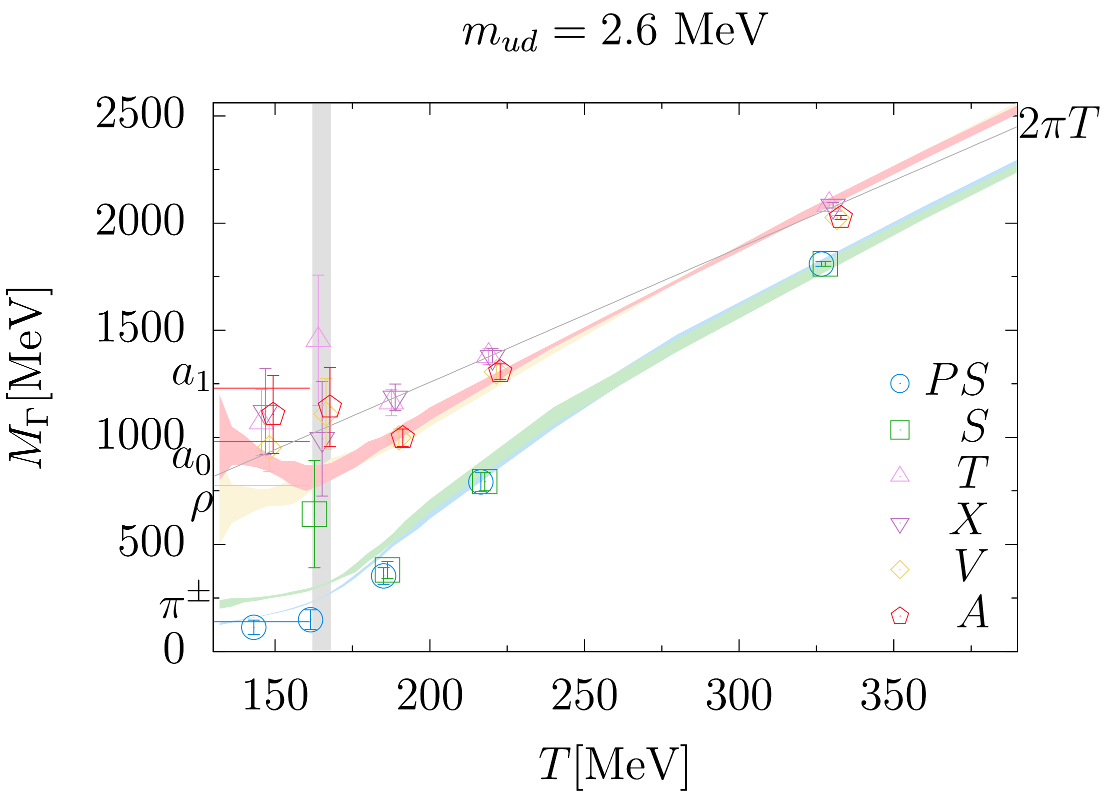
Hidenori Fukaya　David Ward





shaded bands: HISQ 2+1 HotQCD (2019)

Outlook:

- N_f=2+1 similar computation
- N_f=2+1 physical point investigation near T_{pc}

arXiv/2501.12675

Acknowledgements

Codes used:

- Grid (HMC)
- BQCD (Measurements)
- Bridge++ (Measurements)
- Hadrons (Measurements)

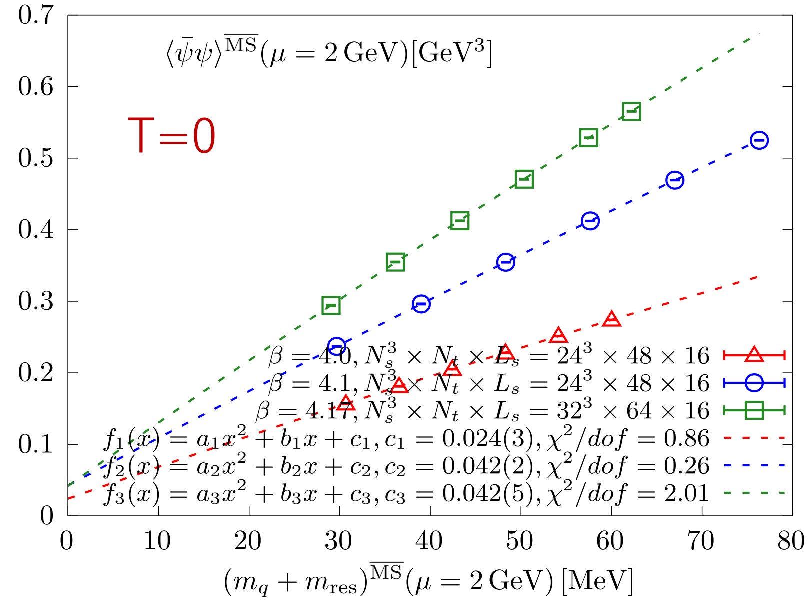
Grants:

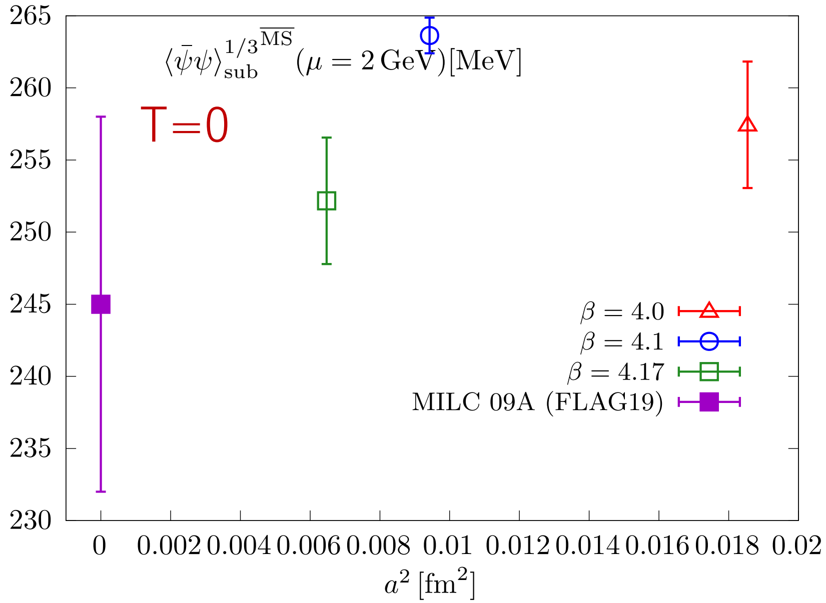
- KAKANHI – (FY2020-2024) - QCD phase diagram explored by chiral fermions – 20H01907
- MEXT Program for Promoting Researches on the Supercomputer Fugaku
  - (FY2020-2022) - Simulation for basic science: from fundamental laws of particles to creation of nuclei - JPMXP1020200105
  - (FY2023-2025) - Simulation for basic science: approaching the new quantum era - JPMXP1020230411

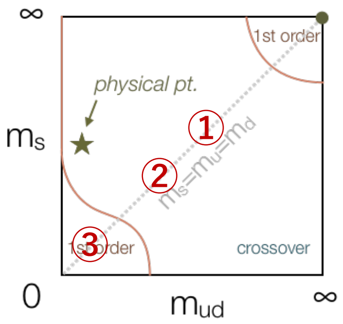
Computers:

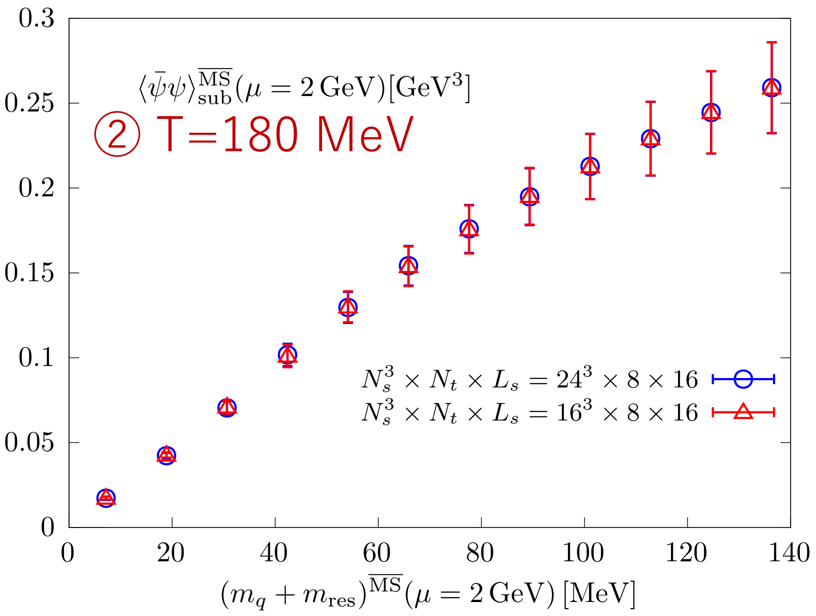
- RIKEN Hokusai BW
- Ito at Kyushu University (hp190124, hp200050)
- Polaire and Grand Chariot at Hokkaido University (hp200130)
- supercomputer Fugaku at R-CCS (hp210032,hp220108,hp220233; hp200130, hp230207)

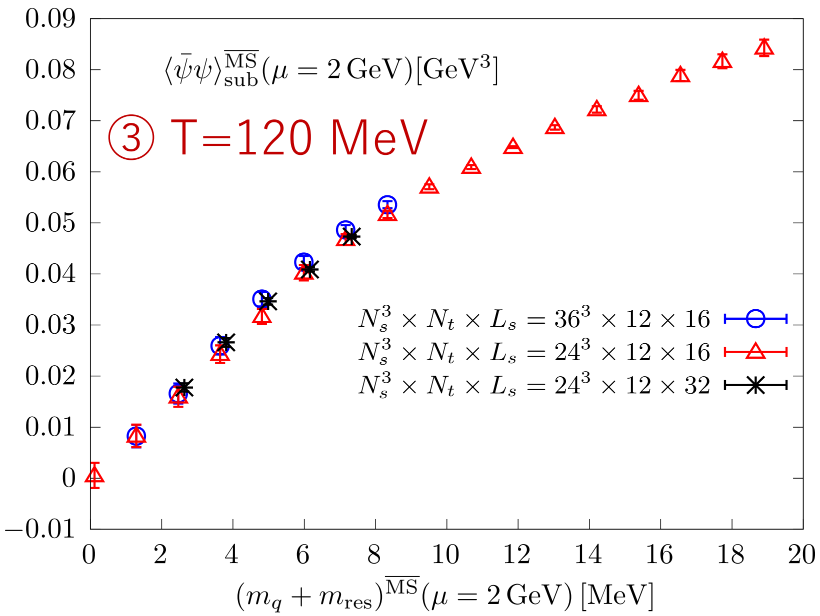
Chiral condensate of DWF



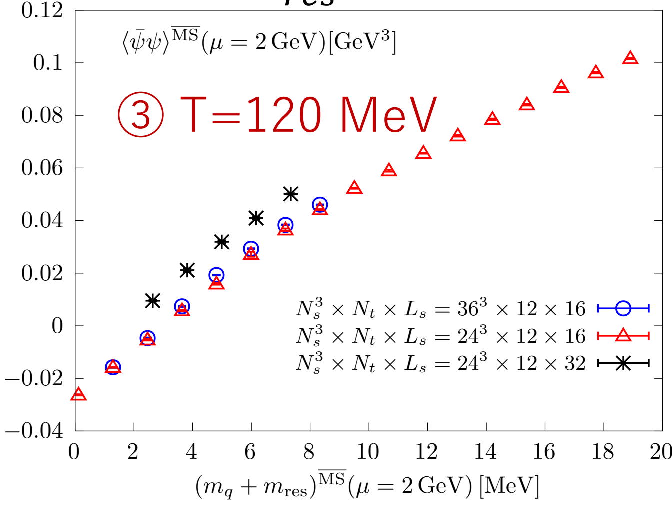






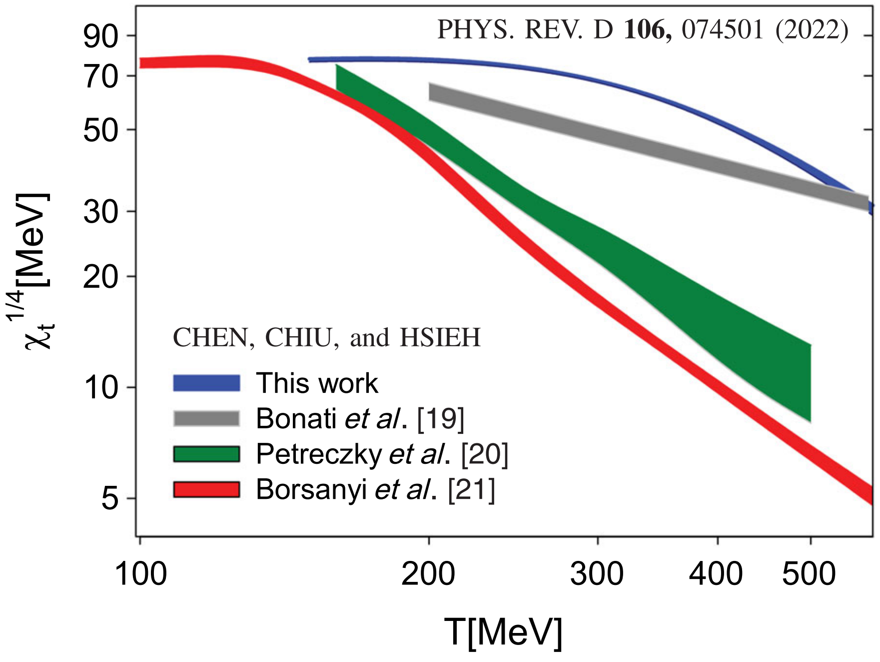


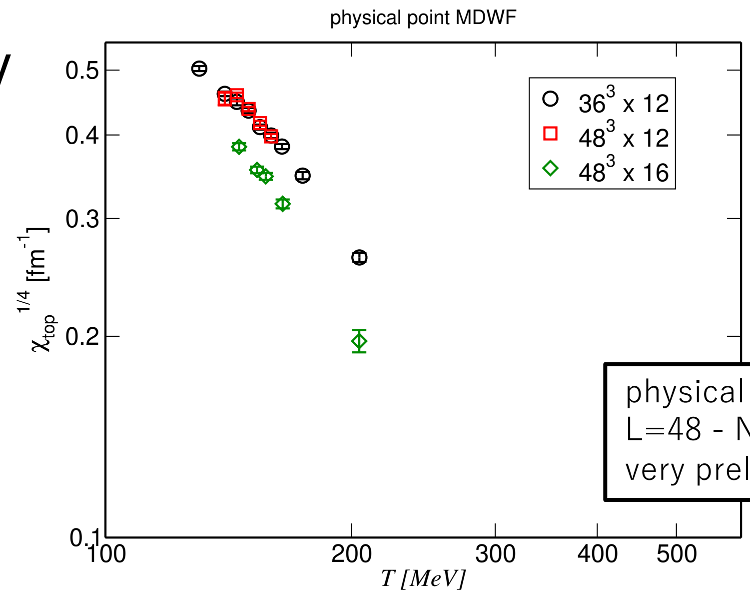
before xm_{res} subtraction



topological susceptibility

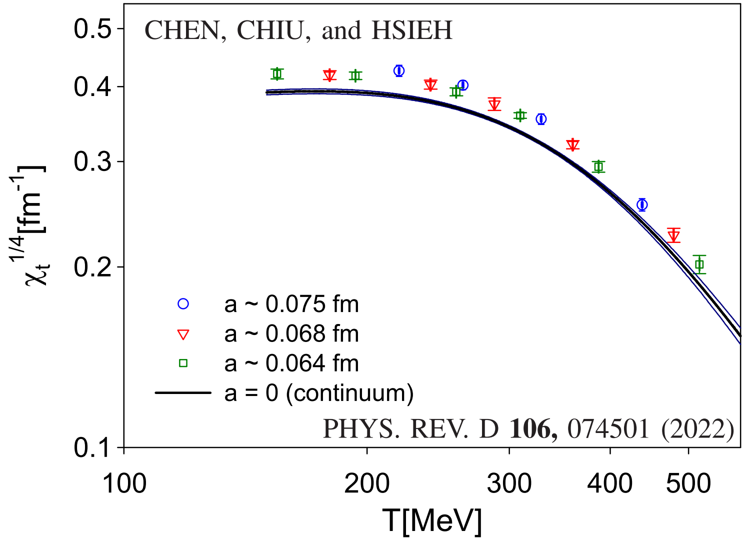
Summary by Chen et al (TWQCD)





MDWF
N_f=2+1

physical point
L=48 - Nt=12 and 16 are
very preliminary (low statistics)



optimal DWF
N_f=2+1+1

Keywords (title: Chiral Fermion Simulation and QCD Thermodynamics)

Chiral Fermion

- **lattice** fermion formulation which preserves “chiral symmetry”

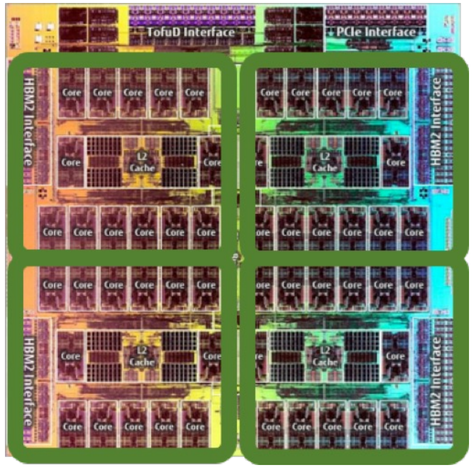
Fugaku

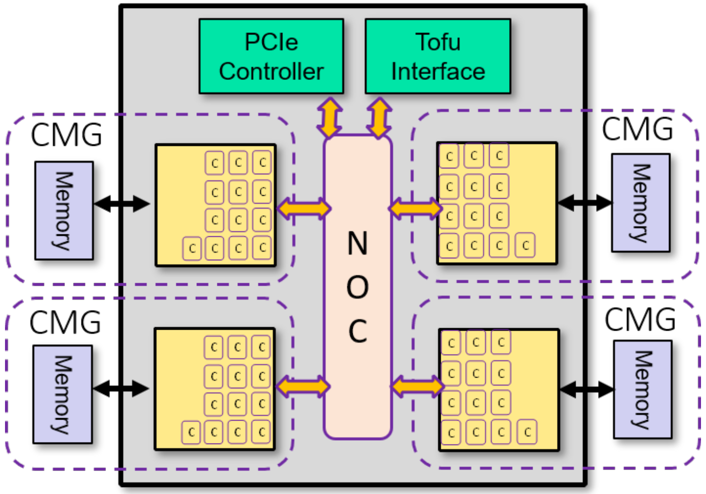
- Japanese flagship supercomputer with A64FX (Arm-based) CPU and Tofu network

Fugaku Tour @ CCP2023

Fugaku – A64FX

https://www.r-ccs.riken.jp/en/fugaku/

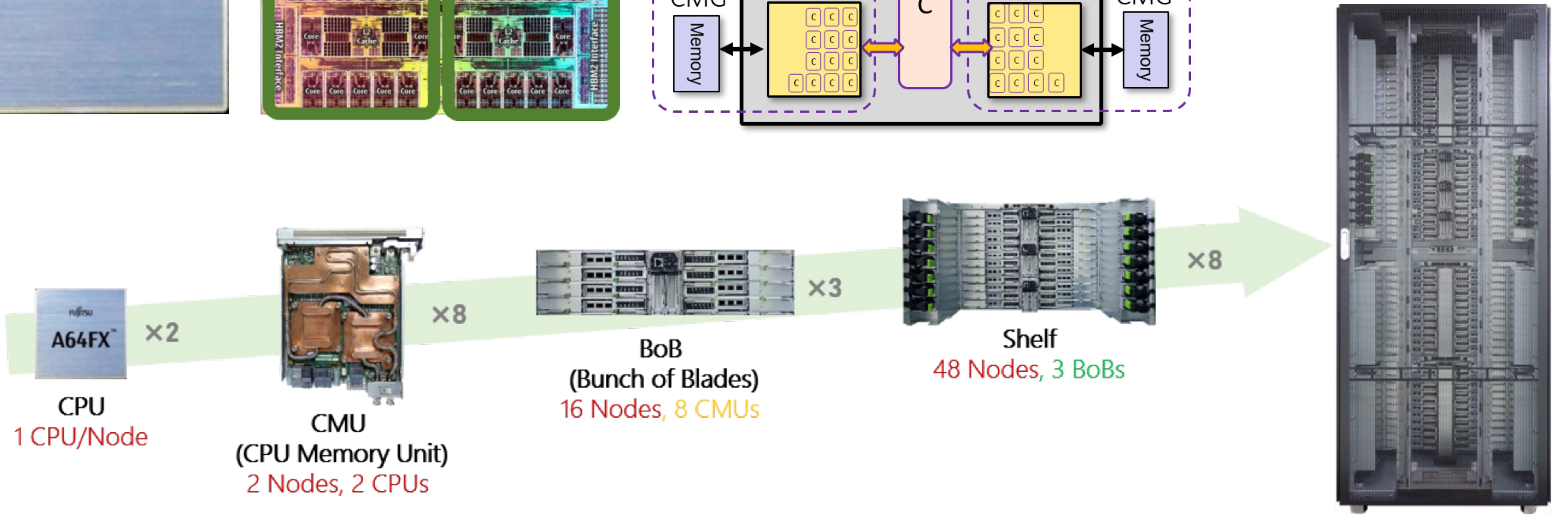




cpu 7nm technology by TSMC

48 cores / cpu

512 bit SIMD register



total 158,976 nodes

Courtesy of FUJITSU LIMITED