Compact binaries and gravitational-wave astronomy

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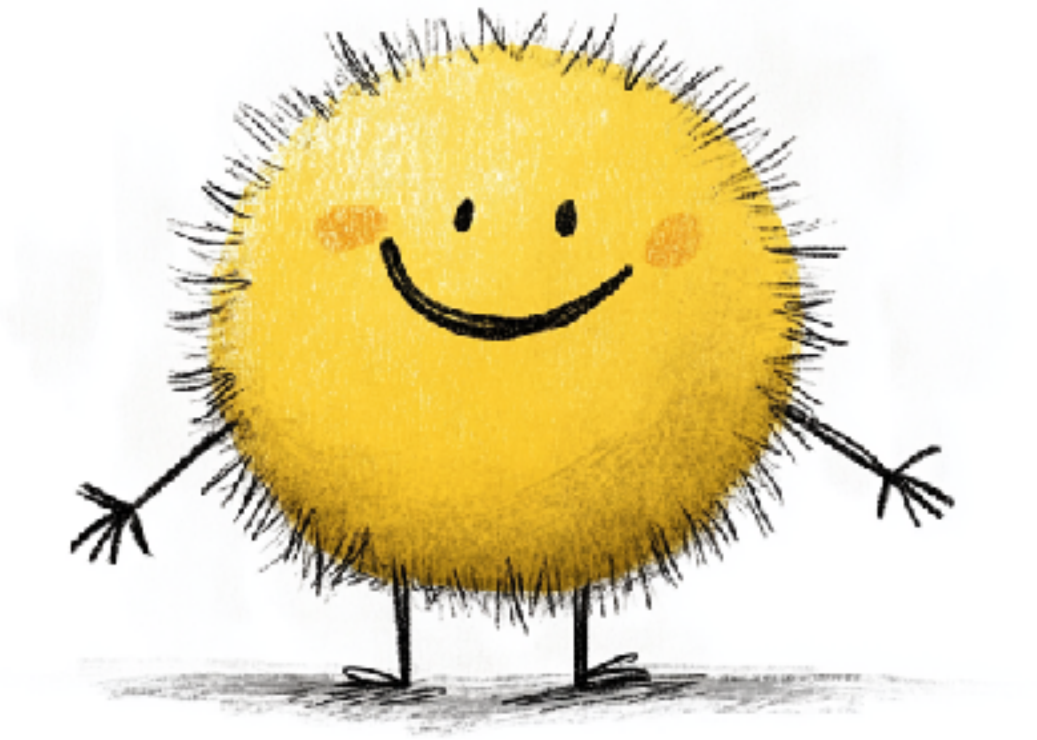
Lecture plan

- Evolution of massive stars and formation of compact objects
- Evolution of binary massive stars and formation of binary compact objects
- Gravitational-wave astronomy
- Gravitational-wave observations of binary compact objects

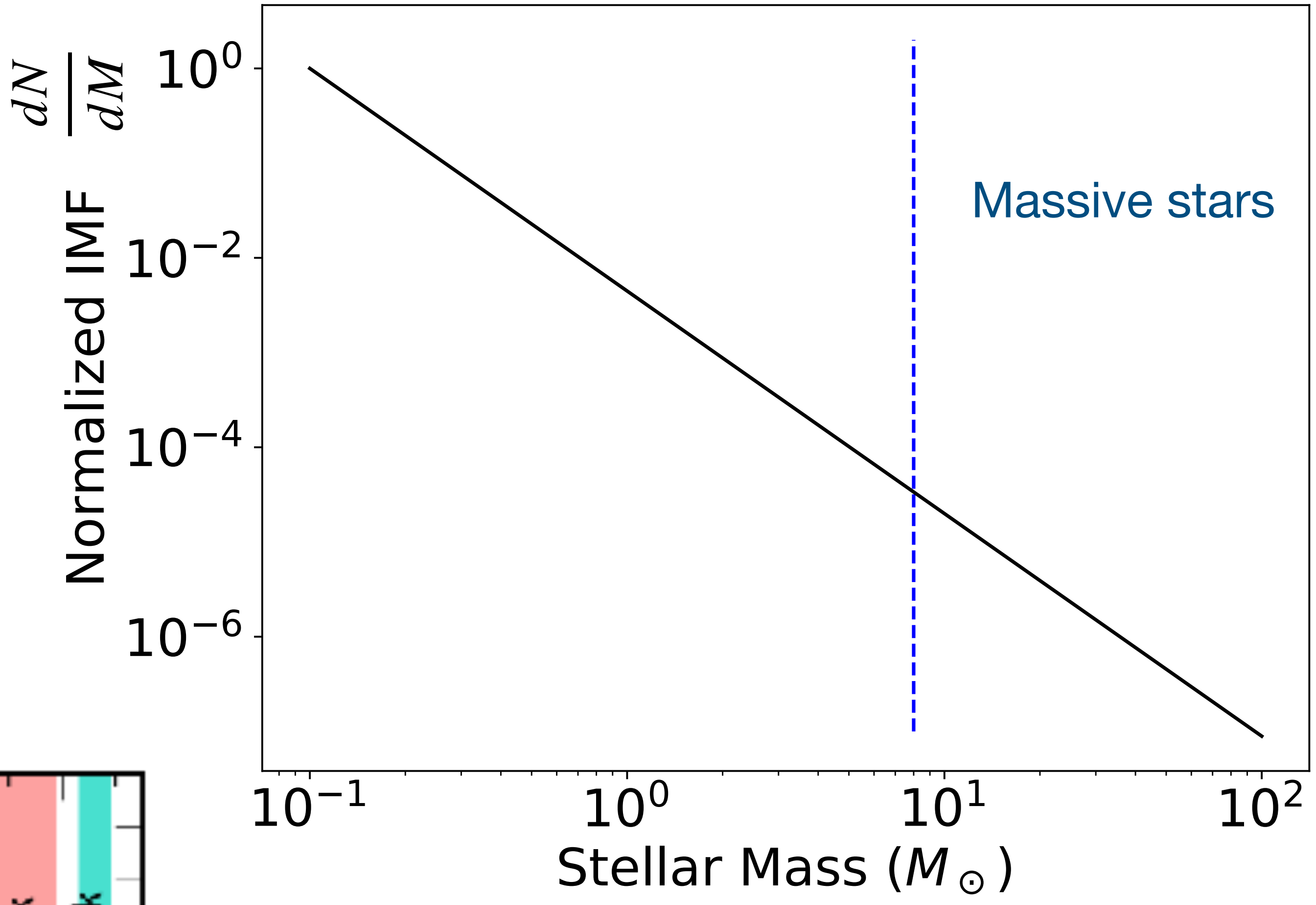
Lecture plan

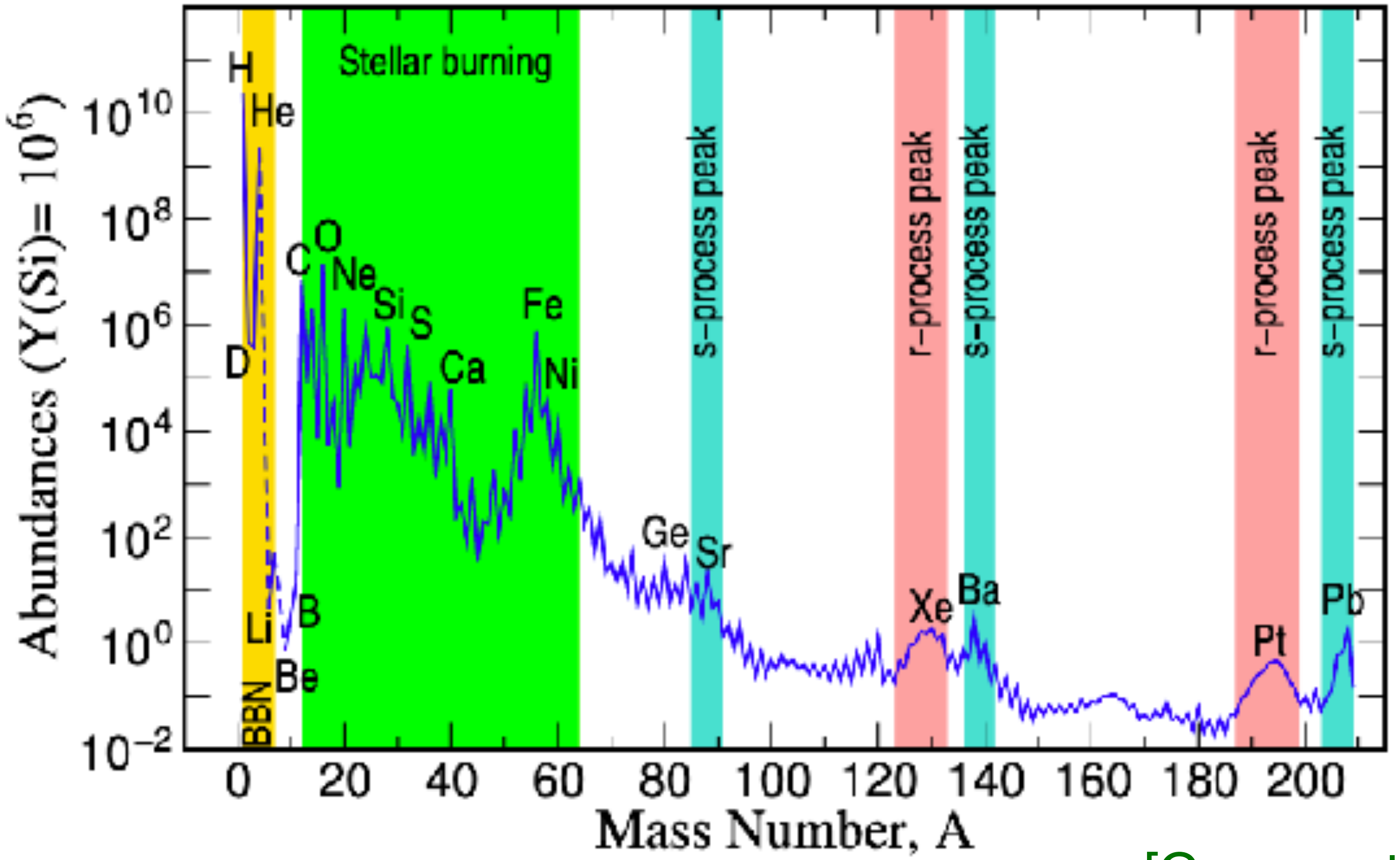
- Evolution of massive stars and formation of compact objects
  - Evolution up to core collapse
  - Core collapse
  - Remnant - neutron star of black hole?



Massive stars

- Masses: $M \gtrsim 8\,M_\odot$
- Radii: up to $\sim 100\,R_\odot$
- Luminosities: up to $\sim 10^6\,L_\odot$
- Fraction by number: $\lesssim 0.3\,\%$



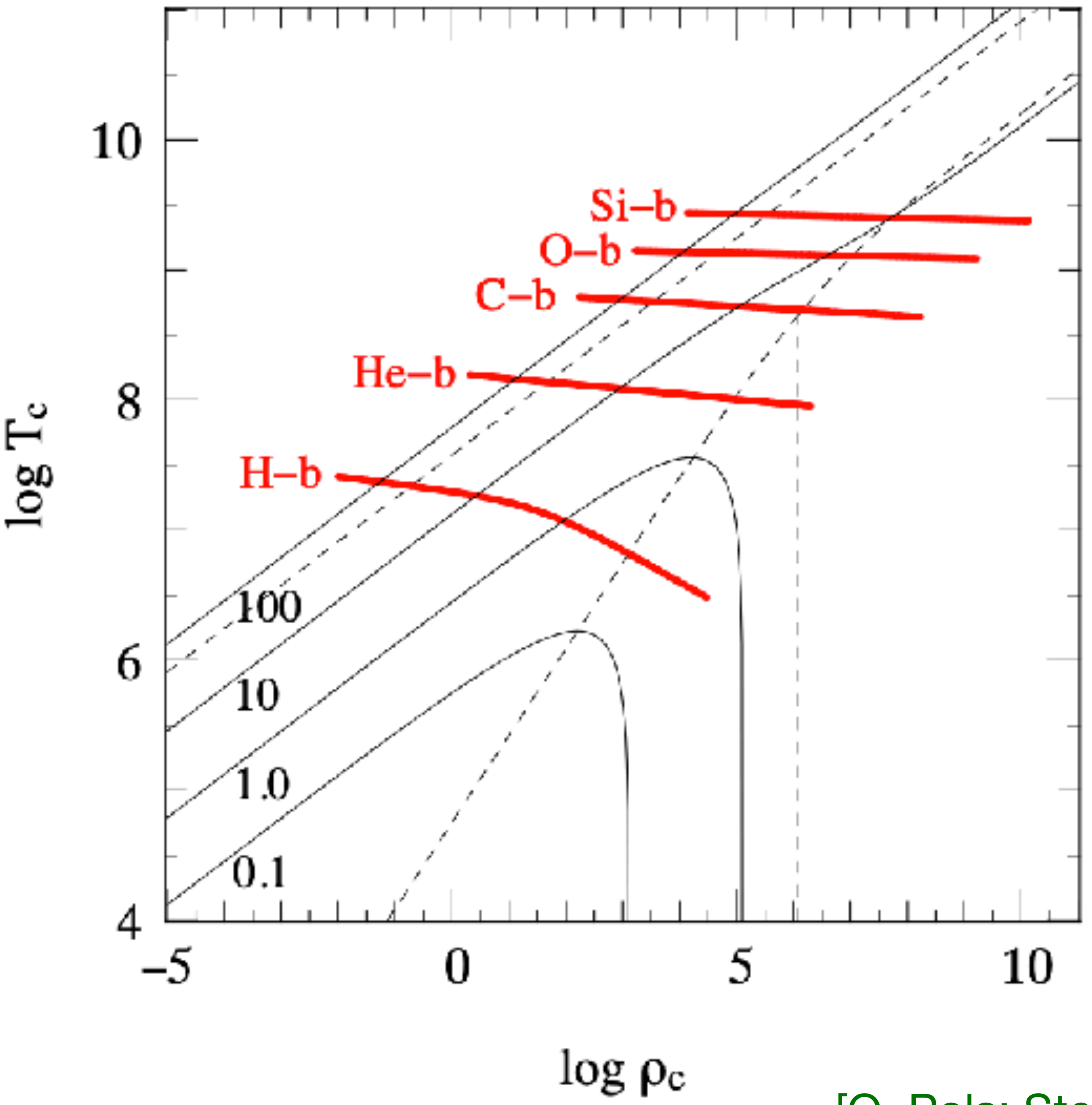


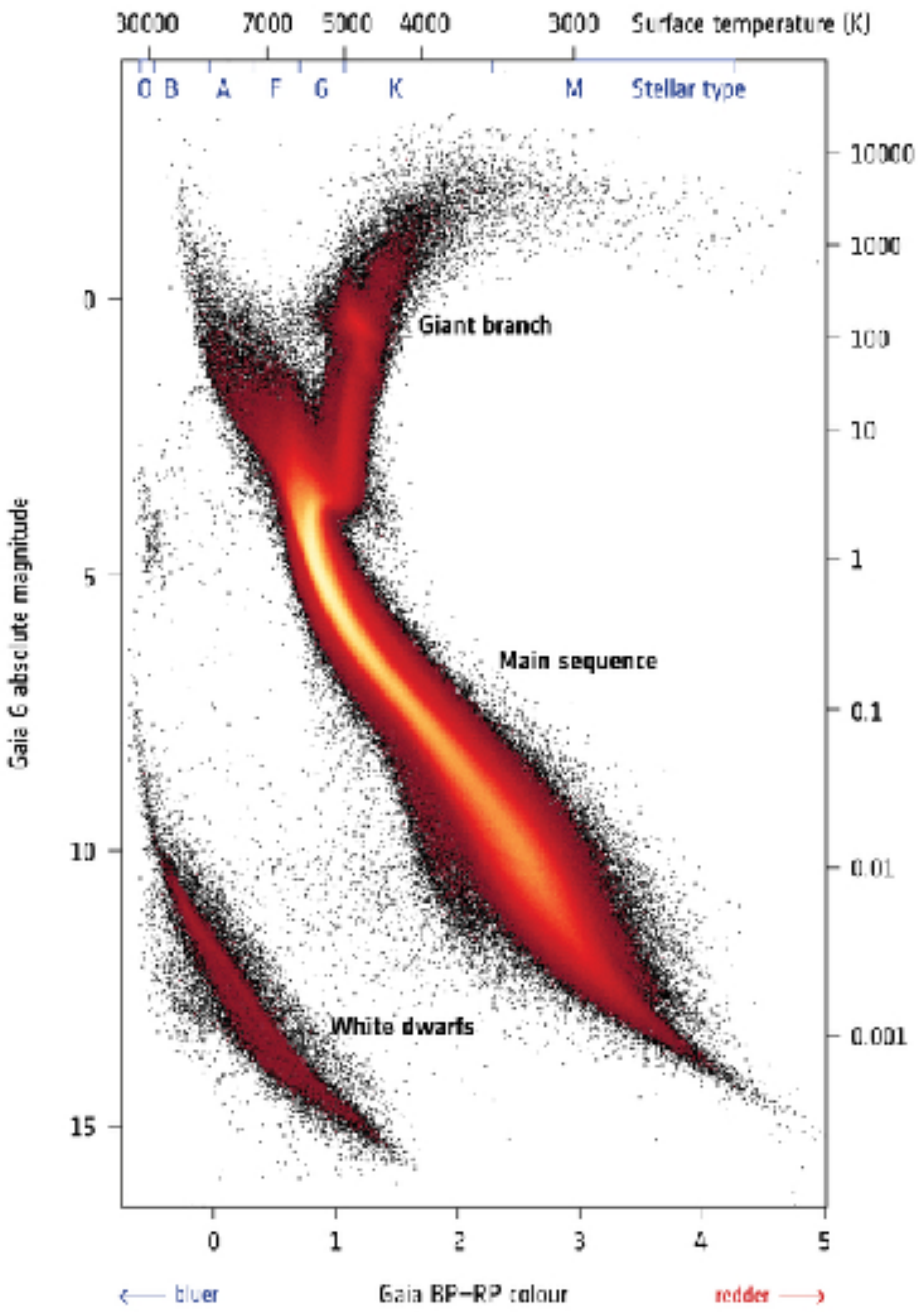
Chemical elements up to the iron group are synthesised in the cores of massive stars

[Cowan et al., arXiv:1901.01410]

Nuclear burning cycles

- Main sequence: hydrogen fusion into helium $4\,^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He}$





[O. Pols: Stellar structure and evolution]

Nuclear burning cycles

- Main sequence: hydrogen fusion into helium

$$4\,^{1}\mathrm{H} \rightarrow {}^{4}\,\mathrm{He}$$

- Triple alpha: helium fusion into carbon

$$3\,^{4}\mathrm{He} \rightarrow {}^{12}\,\mathrm{C}^{*} \rightarrow {}^{12}\,\mathrm{C}$$

- Carbon and oxygen fusion

$$^{12}\mathrm{C} + {}^{4}\,\mathrm{He} \rightarrow {}^{16}\,\mathrm{O}$$

$$^{16}\mathrm{O} + {}^{4}\,\mathrm{He} \rightarrow {}^{20}\,\mathrm{Ne}$$

$$^{16}\mathrm{O} + {}^{16}\,\mathrm{O} \rightarrow {}^{28}\,\mathrm{Si} + {}^{4}\,\mathrm{He}$$

- Neon fusion

$$^{20}\mathrm{Ne} + {}^{4}\,\mathrm{He} \rightarrow {}^{24}\,\mathrm{Mg}$$

- Silicon fusion

$$^{28}\mathrm{Si} + {}^{4}\,\mathrm{He} \rightarrow {}^{32}\,\mathrm{S}$$

- … and so on up to iron

$^{36}\mathrm{Ar}$, $^{40}\mathrm{Ca}$, $^{44}\mathrm{Ti}$, $^{48}\mathrm{Cr}$, $^{52}\mathrm{Fe}$, $^{56}\mathrm{Ni}$, $^{56}\mathrm{Co}$

Nuclear burning cycles

- Nuclear fuel in the current cycle is exhausted.
- Nuclear reaction rate decreases.
- Stellar core contracts.
- Central temperature and density increase.
- A new fusion cycle starts.

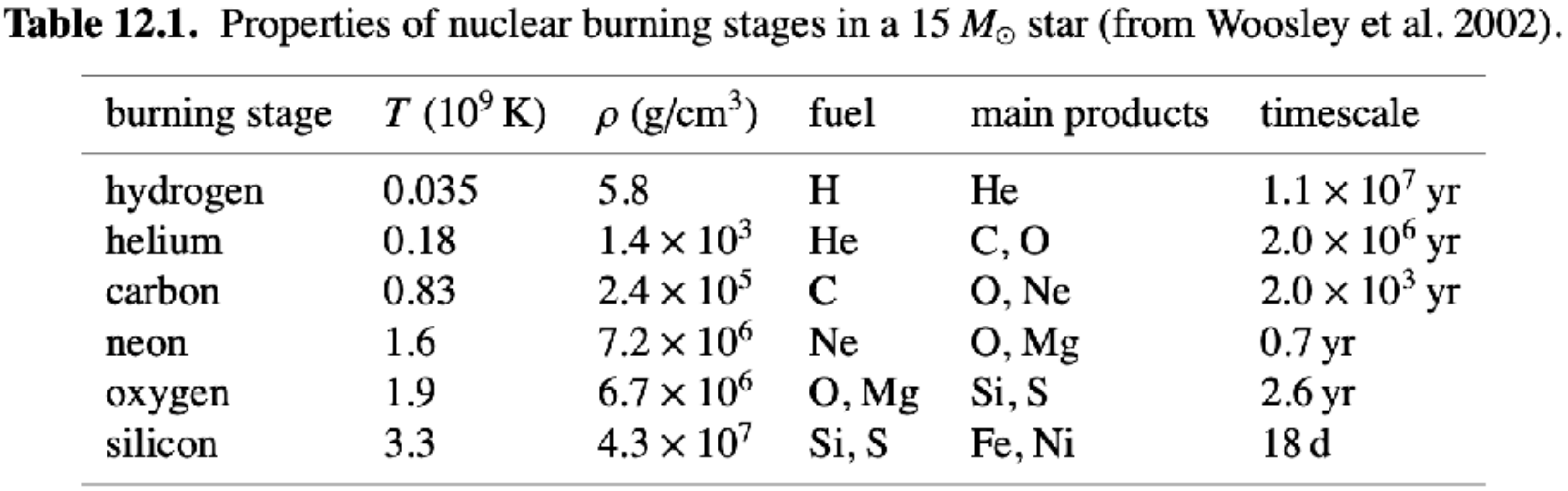
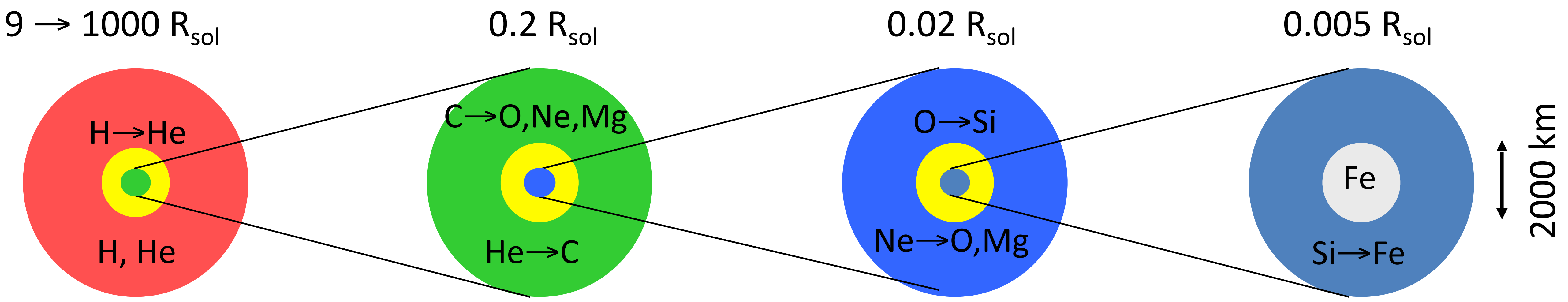


Table 12.1. Properties of nuclear burning stages in a 15 $M_\odot$ star (from Woosley et al. 2002).

burning stage	T (10^9 K)	ρ (g/cm^3)	fuel	main products	timescale
hydrogen	0.035	5.8	H	He	1.1×10^7 yr
helium	0.18	1.4×10^3	He	C, O	2.0×10^6 yr
carbon	0.83	2.4×10^5	C	O, Ne	2.0×10^3 yr
neon	1.6	7.2×10^6	Ne	O, Mg	0.7 yr
oxygen	1.9	6.7×10^6	O, Mg	Si, S	2.6 yr
silicon	3.3	4.3×10^7	Si, S	Fe, Ni	18 d

[O. Pols: Stellar structure and evolution]

Onion shell structure



9 → 1000 R_{sol}

H→He

H, He

0.2 R_{sol}

C→O,Ne,Mg

He→C

0.02 R_{sol}

O→Si

Ne→O,Mg

0.005 R_{sol}

Fe

Si→Fe

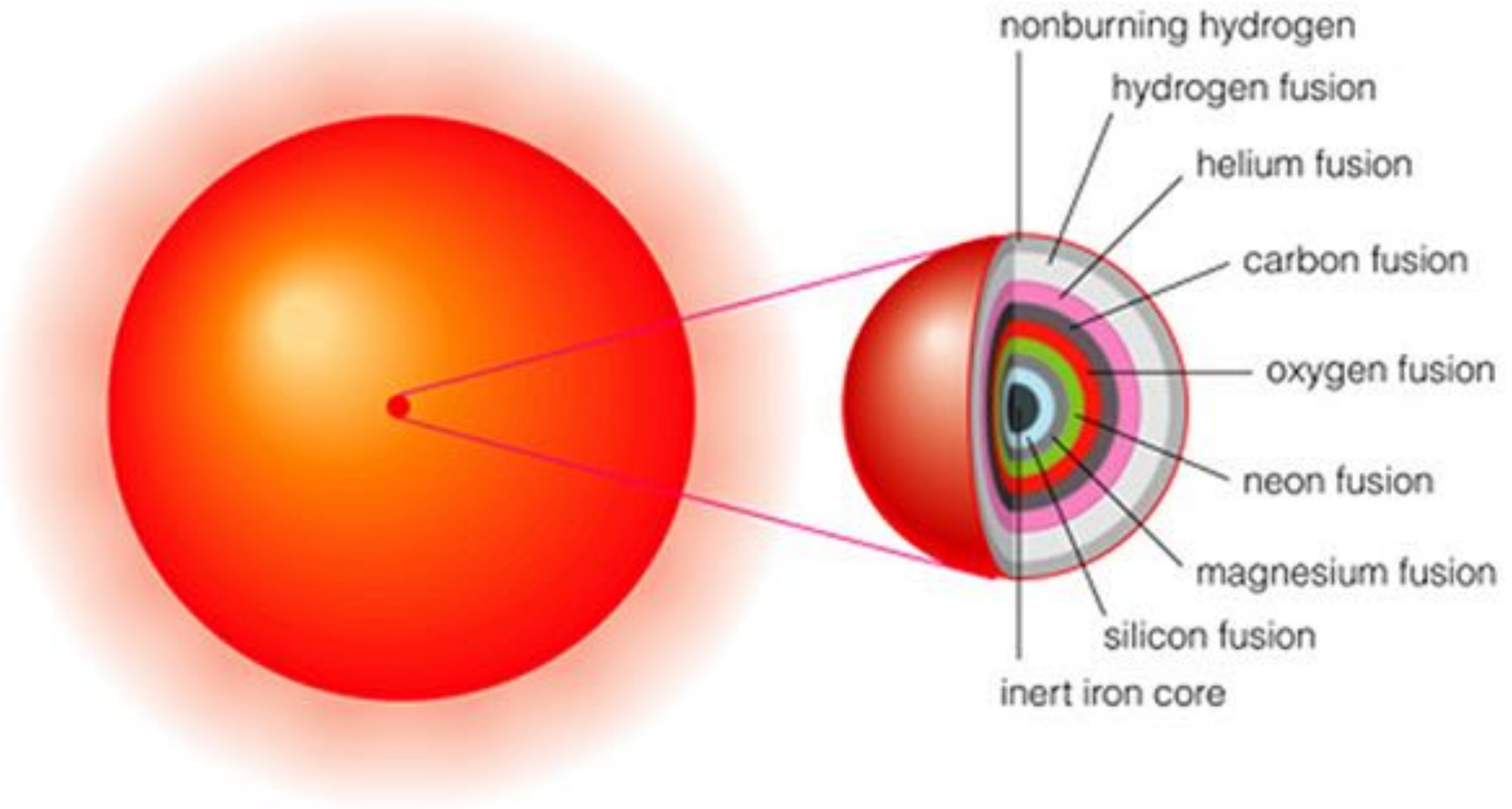
2000 km

H → He : $\tau \approx 10^7$ yrs

He → C : $\tau \approx 5\ 10^5$ yrs
C → O : $\tau \approx 10^3$ yrs

Ne → O, Mg : $\tau \approx 1$ yr
O → Si : $\tau \approx 6$ months

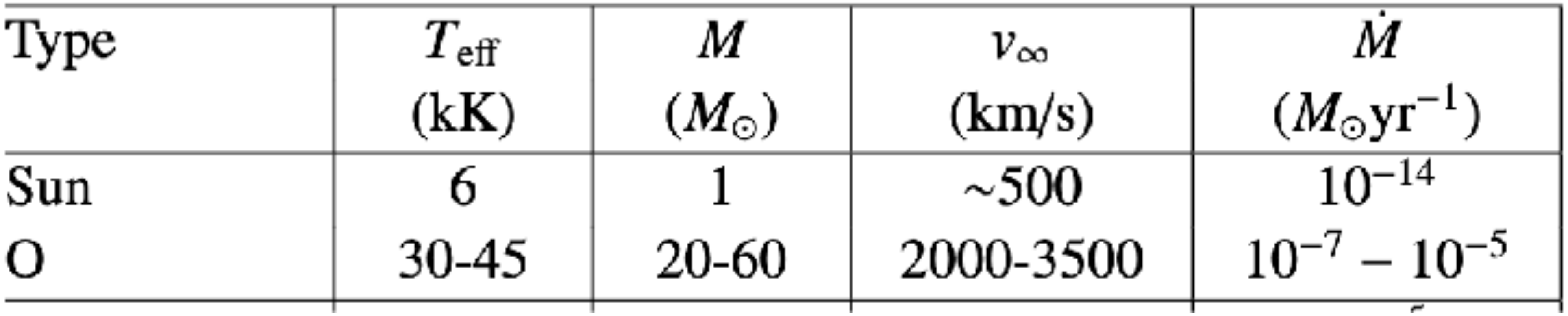
Si → Fe : $\tau \approx 1$ day



Credit: http://astronomy.nmsu.edu/tharriso/ast110/class19.html

Stellar winds

- Massive stars ($> 15M_\odot$) experience rapid mass outflows



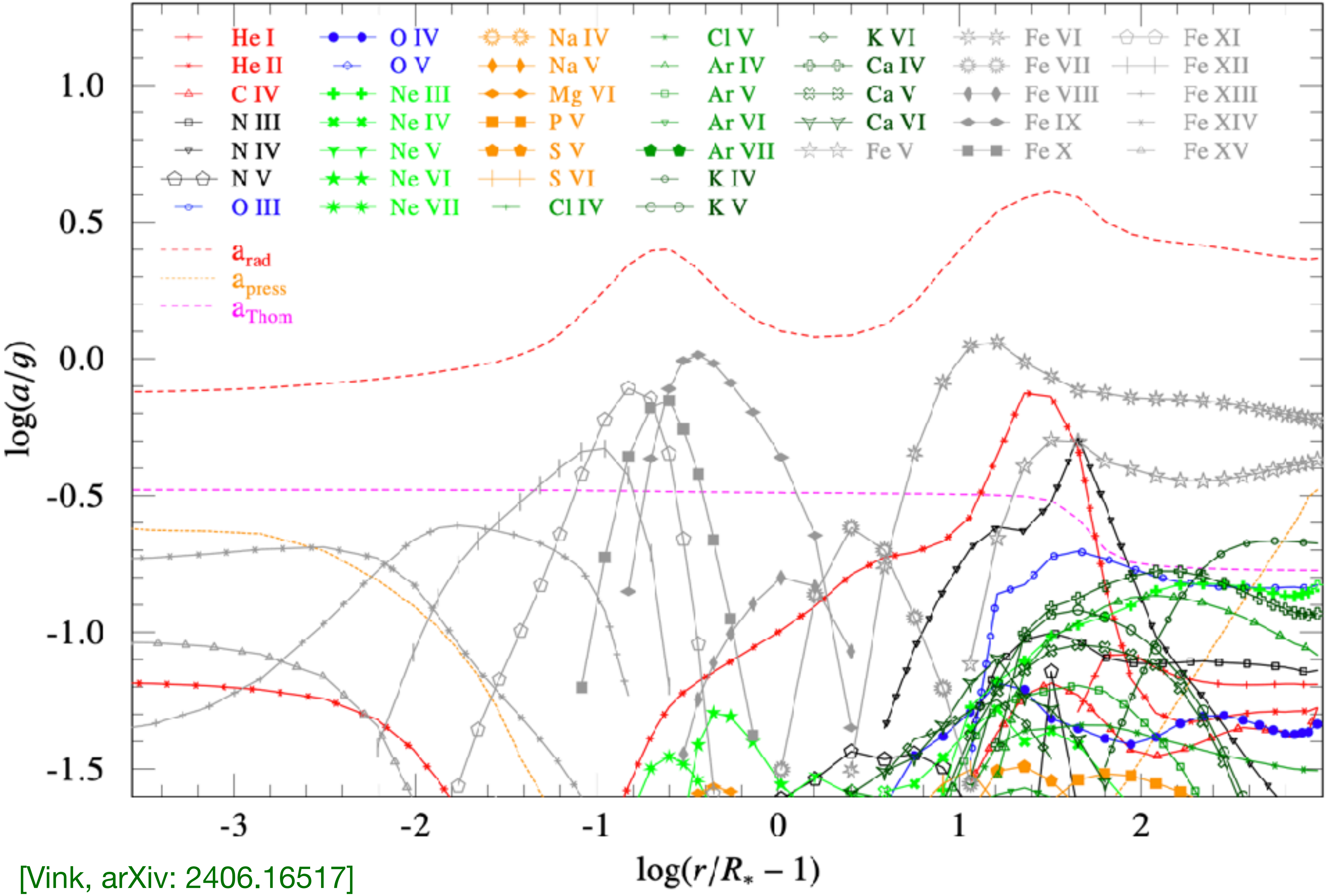
Type	$T_{\rm eff}$ (kK)	M ($M_\odot$)	v_∞ (km/s)	$\dot{M}$ ($M_\odot \mathrm{yr}^{-1}$)
Sun	6	1	~500	10^{-14}
O	30-45	20-60	2000-3500	$10^{-7} - 10^{-5}$

- Modelled as stationary outflow $\dot{M} = 4\pi r^2 \rho(r) v(r)$

- Wind acceleration, with $a(r)$ from radiation (or other sources):

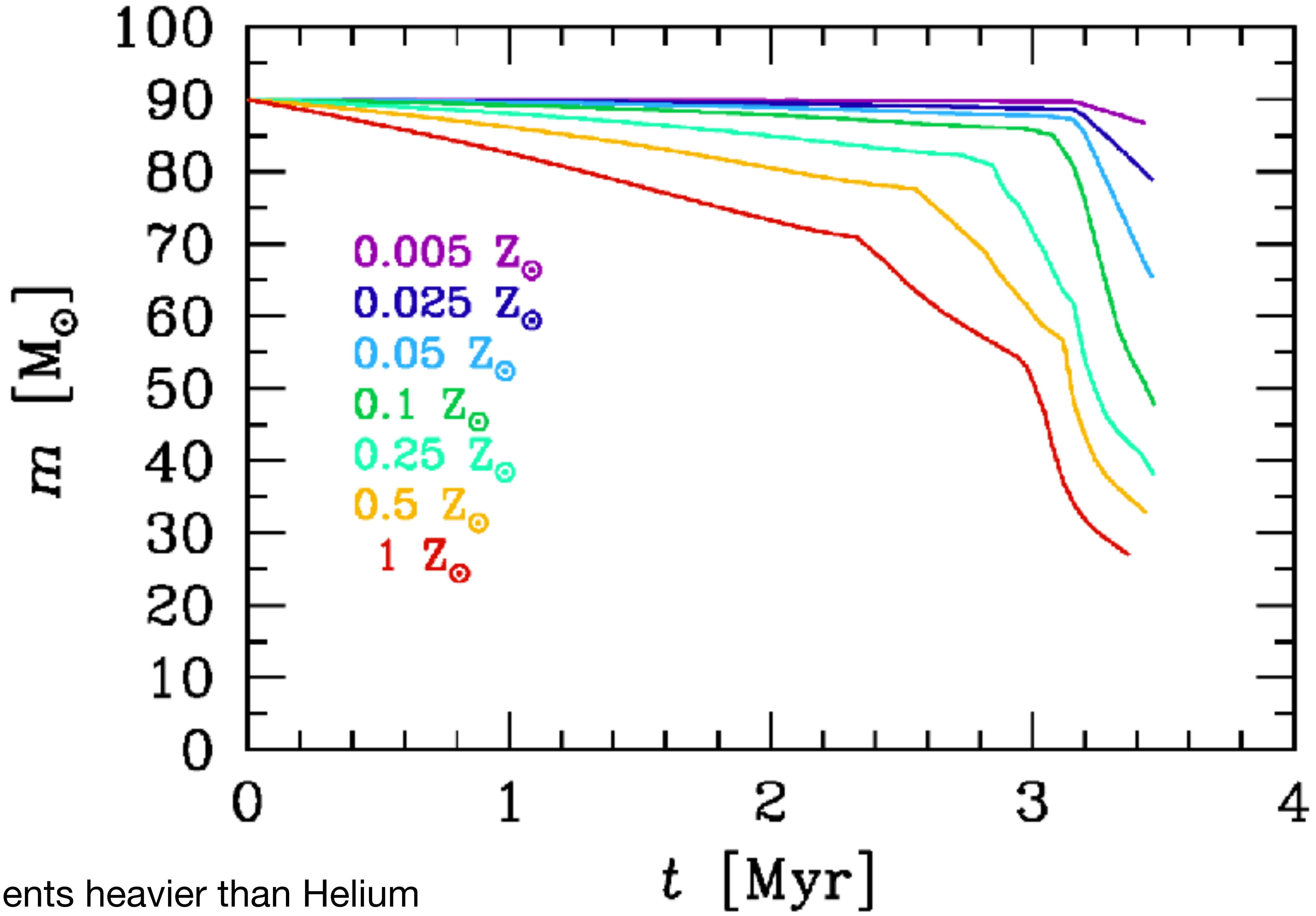
$$v\left(\frac{dv}{dr}\right) = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{dp}{dr} + a(r)$$

Wind acceleration due to radiation pressure



[Vink, arXiv: 2406.16517]

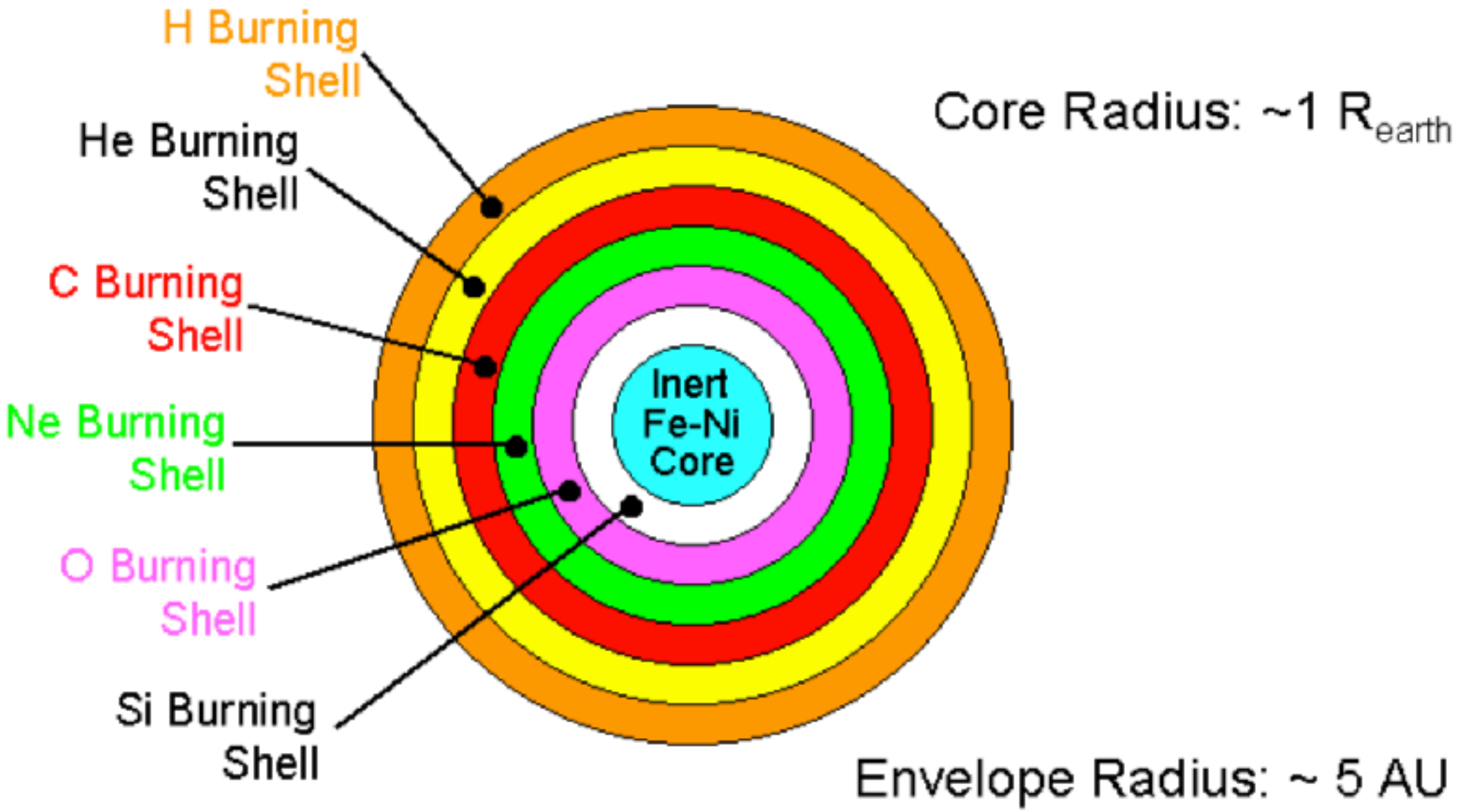
Stellar winds



- *'Metals'*: all elements heavier than Helium
- Metallicity: $Z = \dfrac{M_{*,\mathrm{metals}}}{M_{*,\mathrm{tot}}}$
- Solar metallicity: $Z_\odot = 0.014$

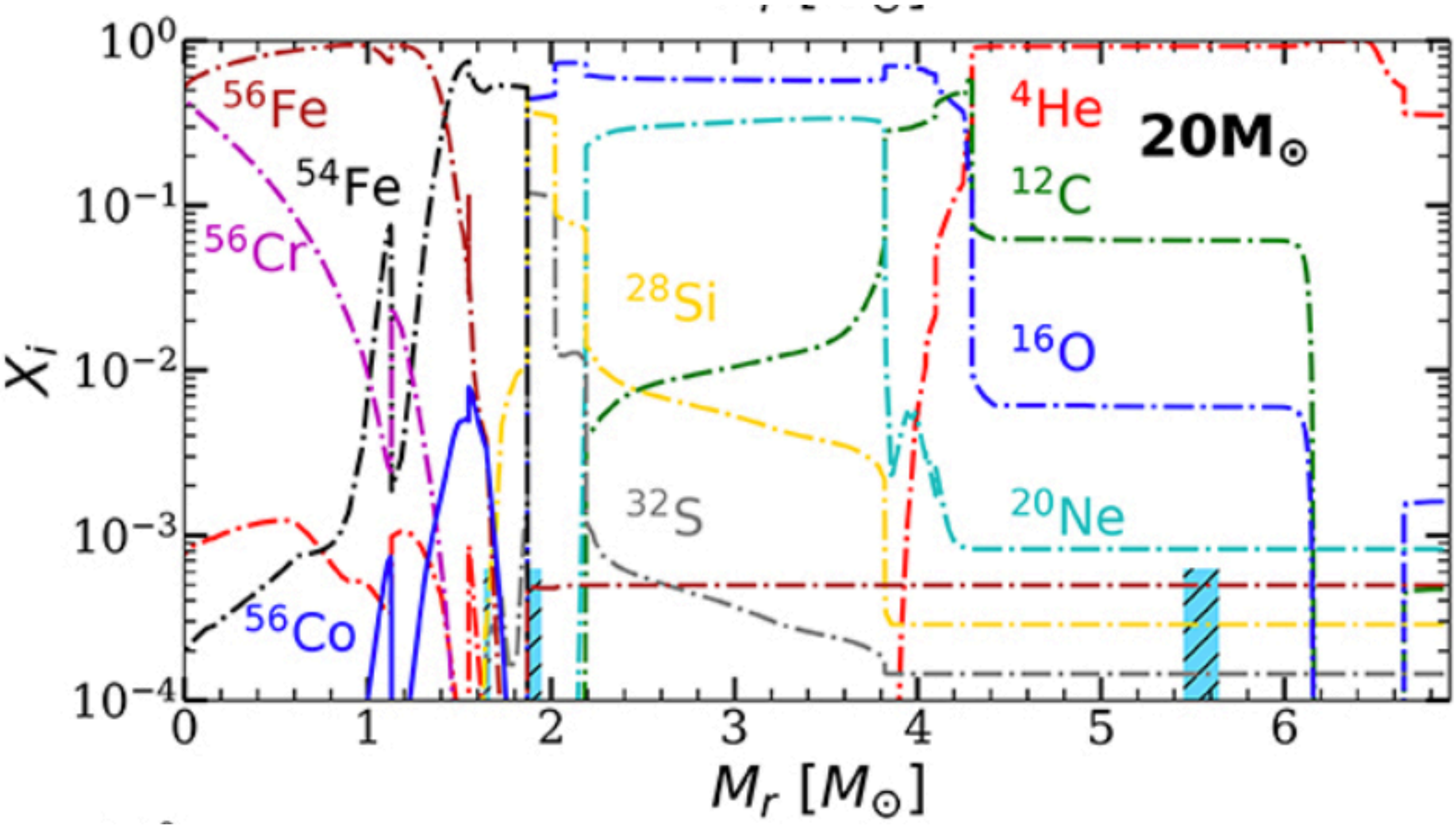
[Mapelli, arXiv:2106.00699]

Massive star right before core collapse



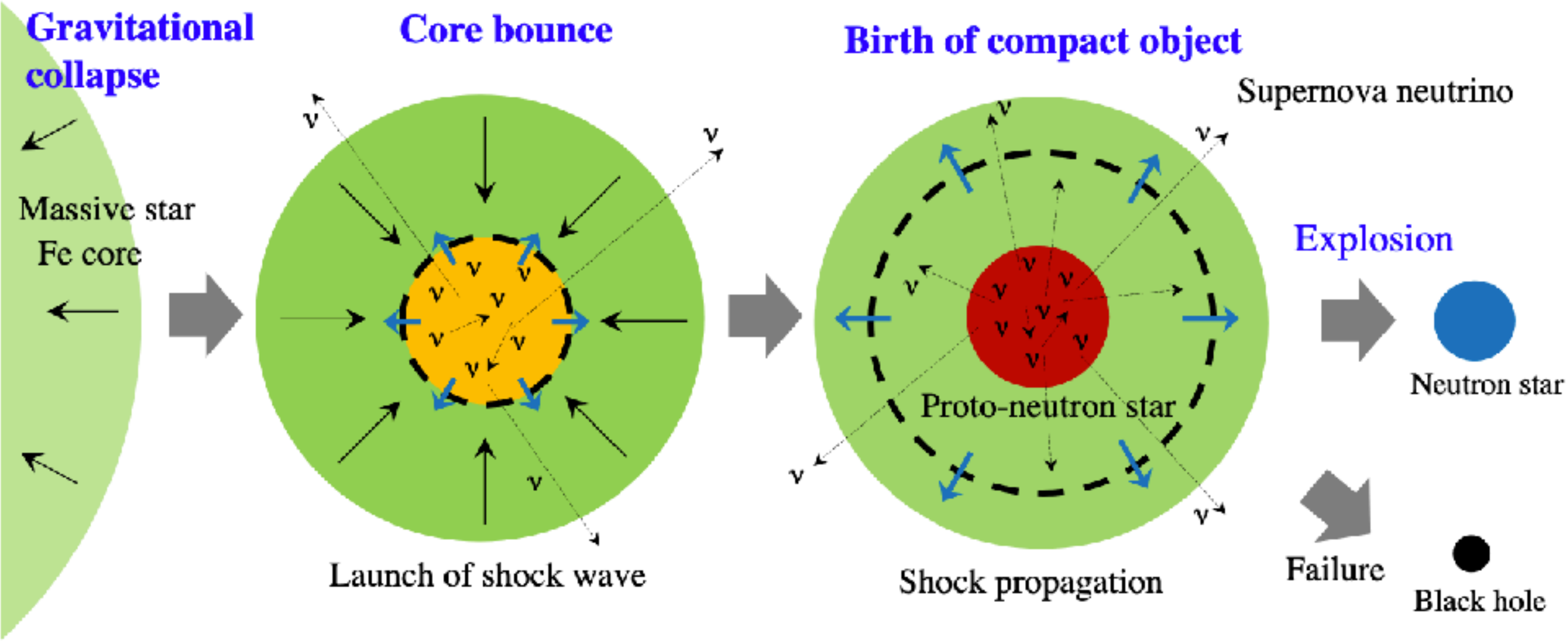
Credit: Richard Pogge

Massive star right before core collapse



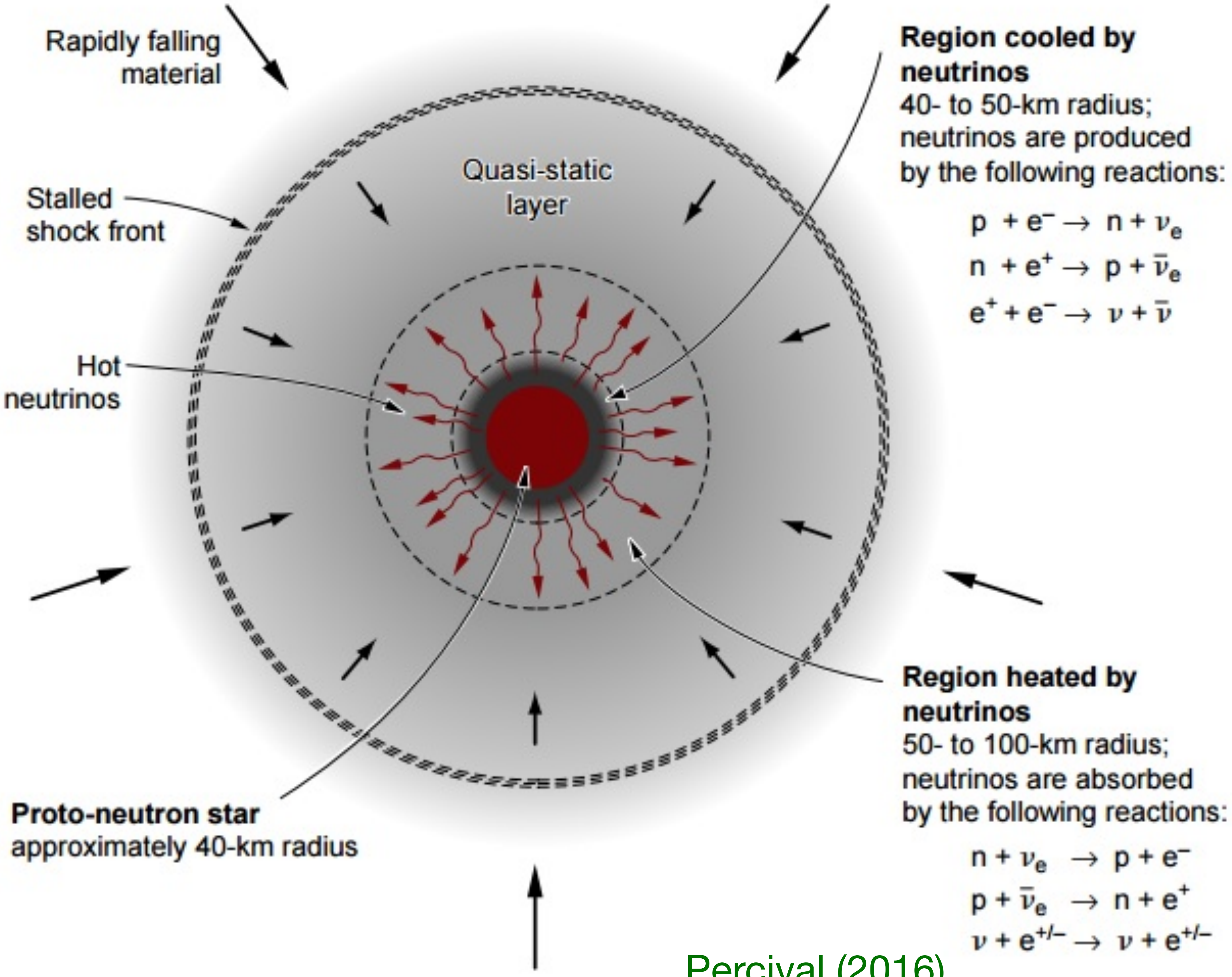
[Griffiths et al., arXiv:2408.03368]

Core collapse



[Sumiyoshi et al., arXiv:2207.00033]

Core collapse



Rapidly falling material

Stalled shock front

Hot neutrinos

Quasi-static layer

Region cooled by neutrinos
40- to 50-km radius;
neutrinos are produced
by the following reactions:

$p + e^- \rightarrow n + \nu_e$

$n + e^+ \rightarrow p + \bar{\nu}_e$

$e^+ + e^- \rightarrow \nu + \bar{\nu}$

Proto-neutron star
approximately 40-km radius

Region heated by neutrinos
50- to 100-km radius;
neutrinos are absorbed
by the following reactions:

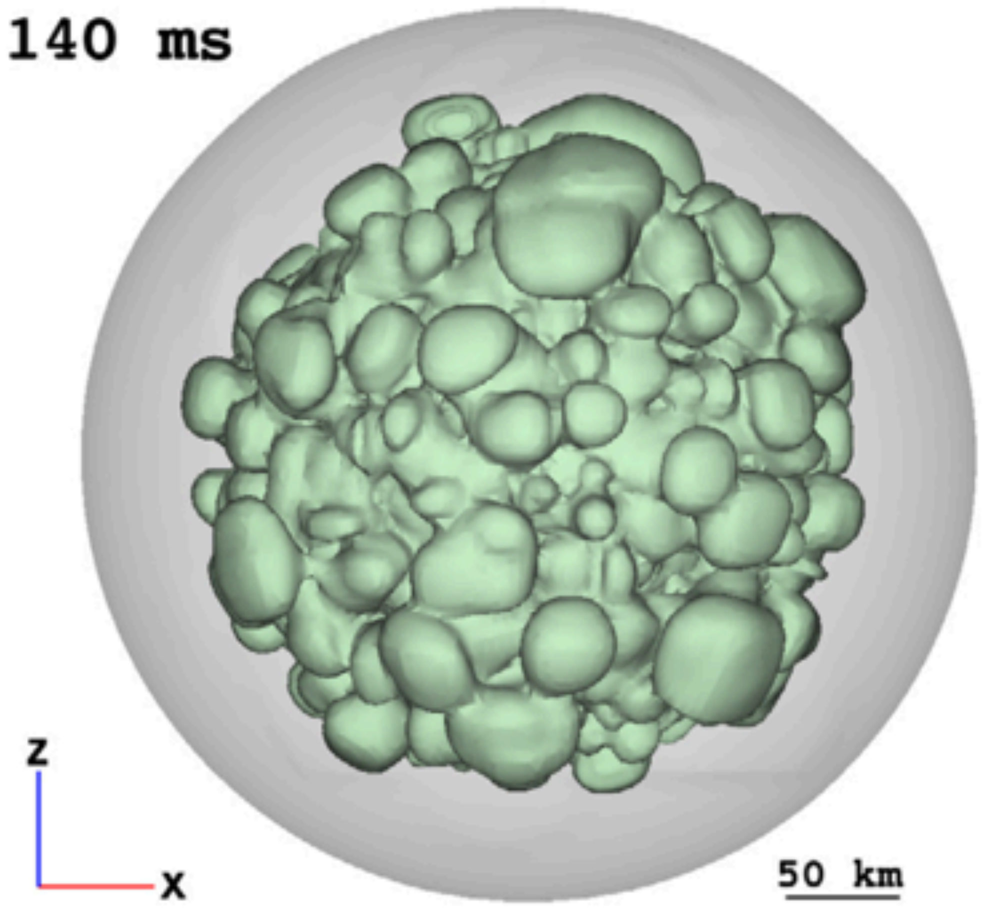
$n + \nu_e \rightarrow p + e^-$

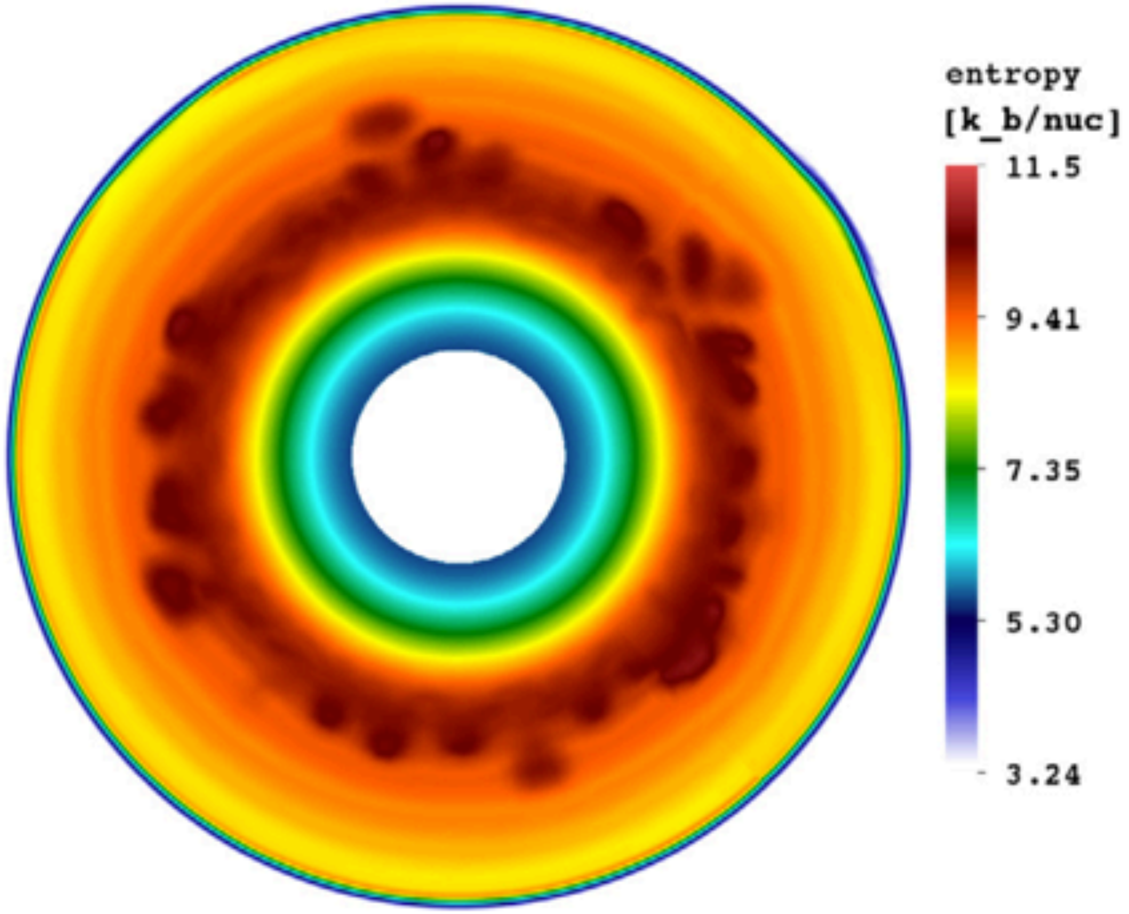
$p + \bar{\nu}_e \rightarrow n + e^+$

$\nu + e^{+/-} \rightarrow \nu + e^{+/-}$

Percival (2016)

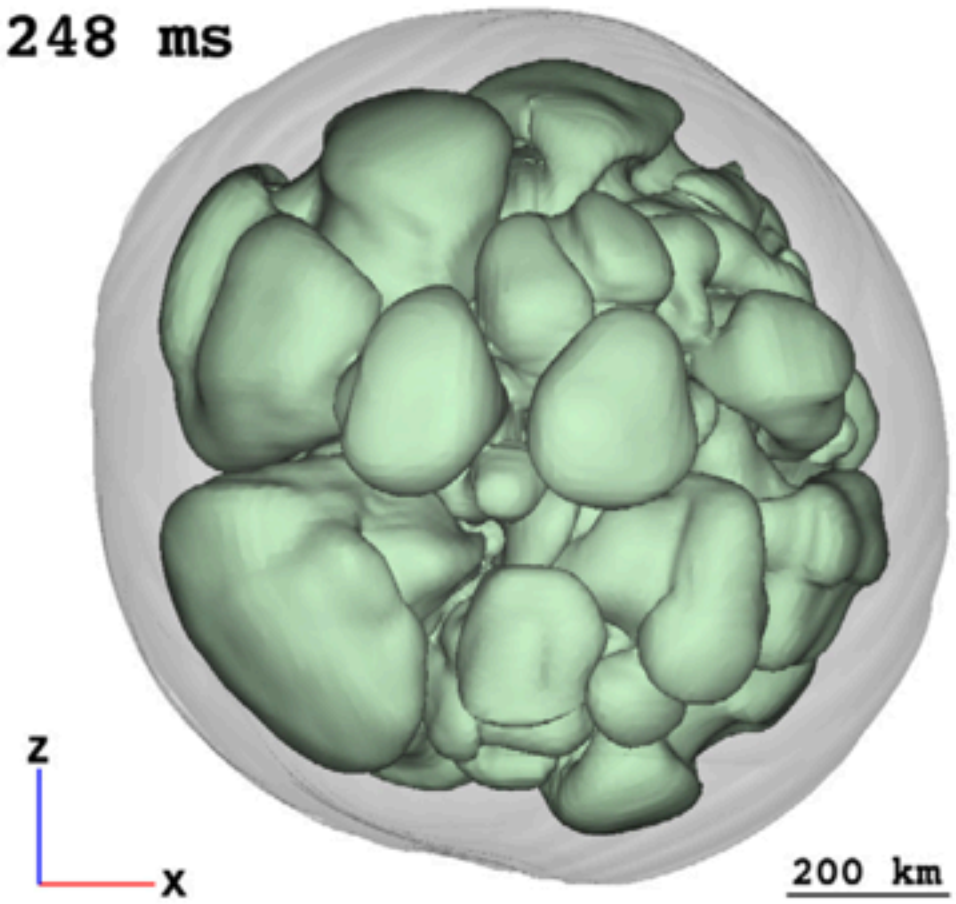
Core collapse: example

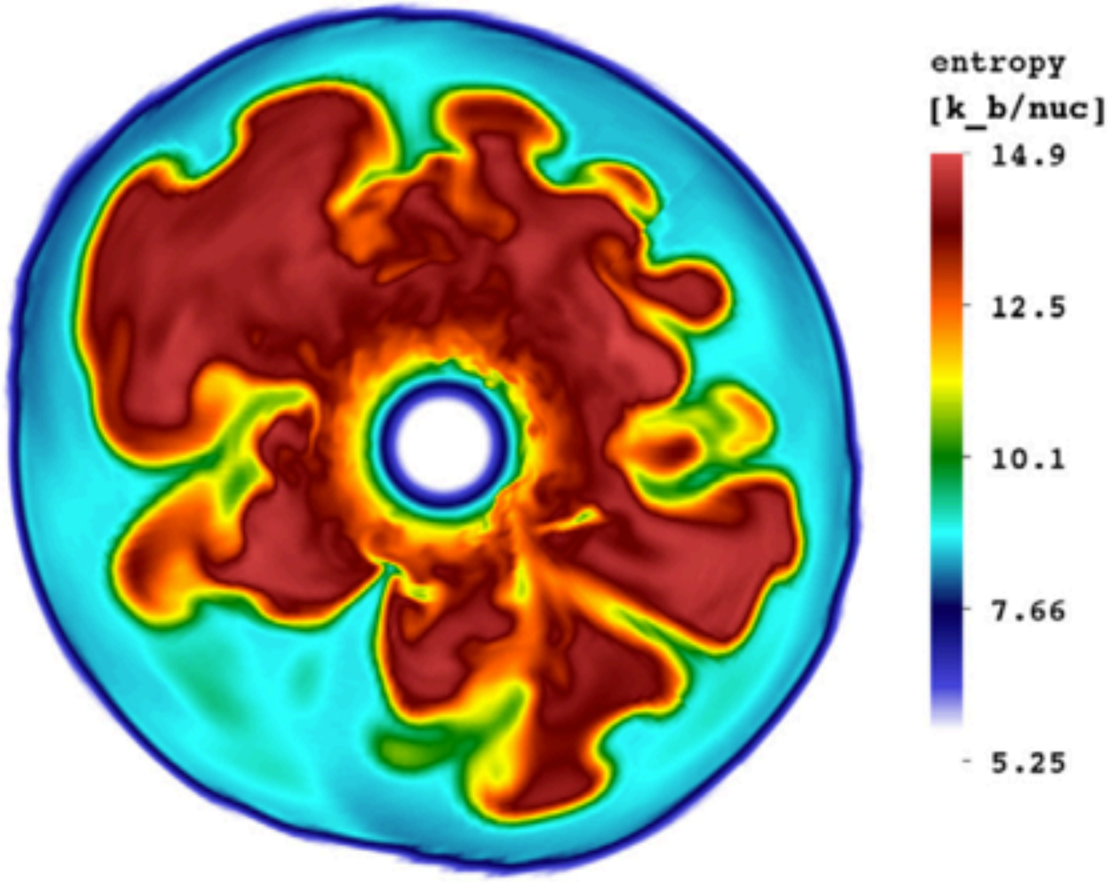




[Wongwathanarat et al., (2013)]

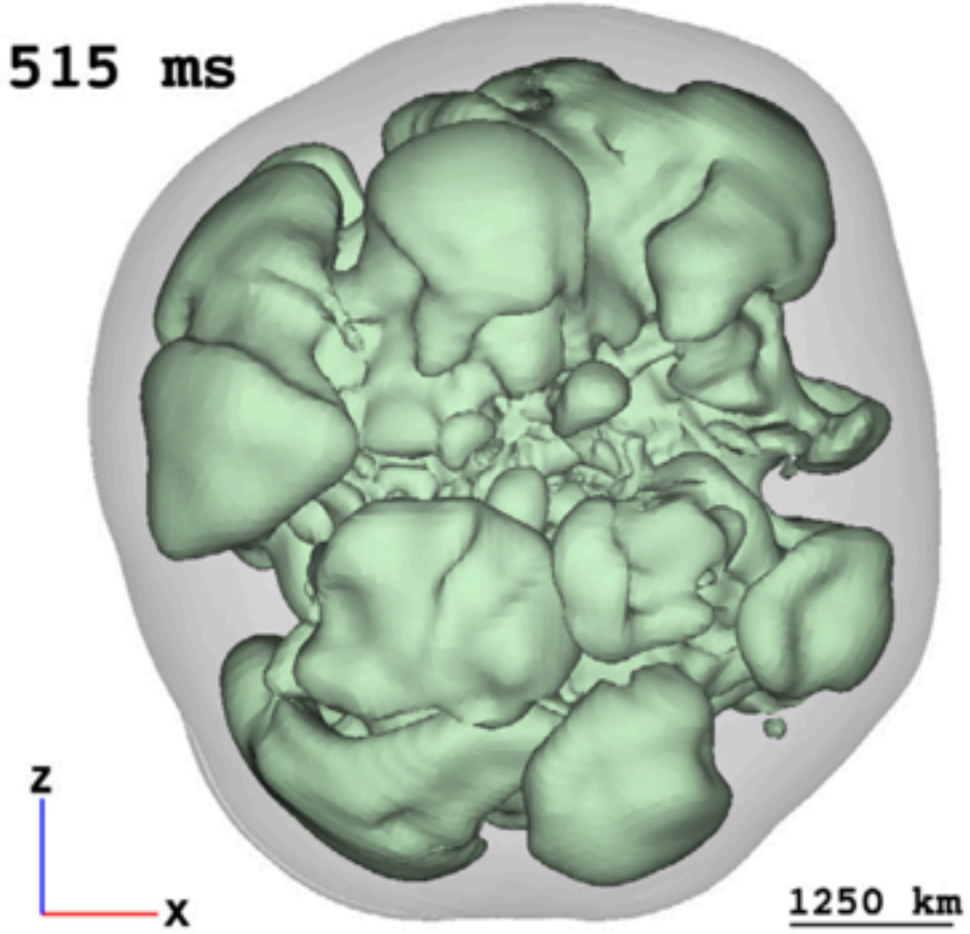
Core collapse: example

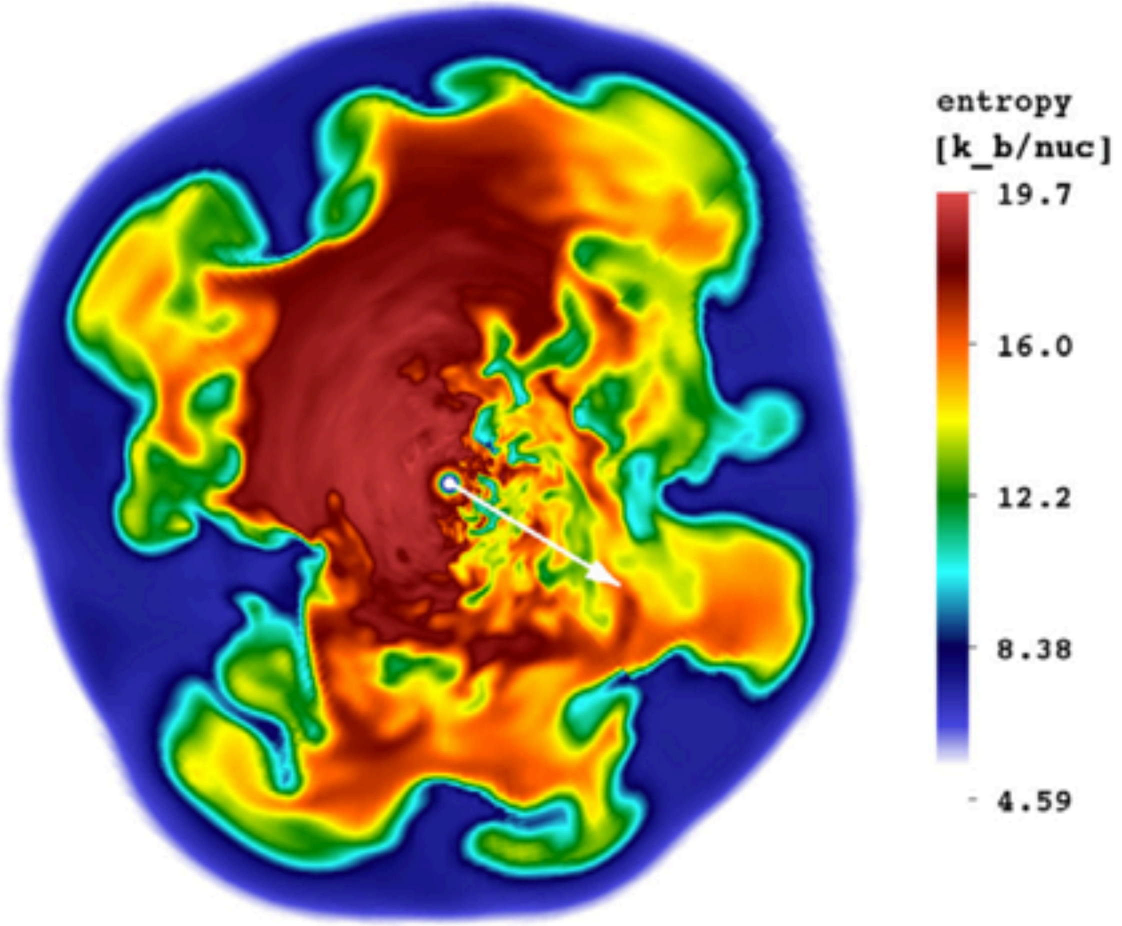




[Wongwathanarat et al., (2013)]

Core collapse: example





[Wongwathanarat et al., (2013)]

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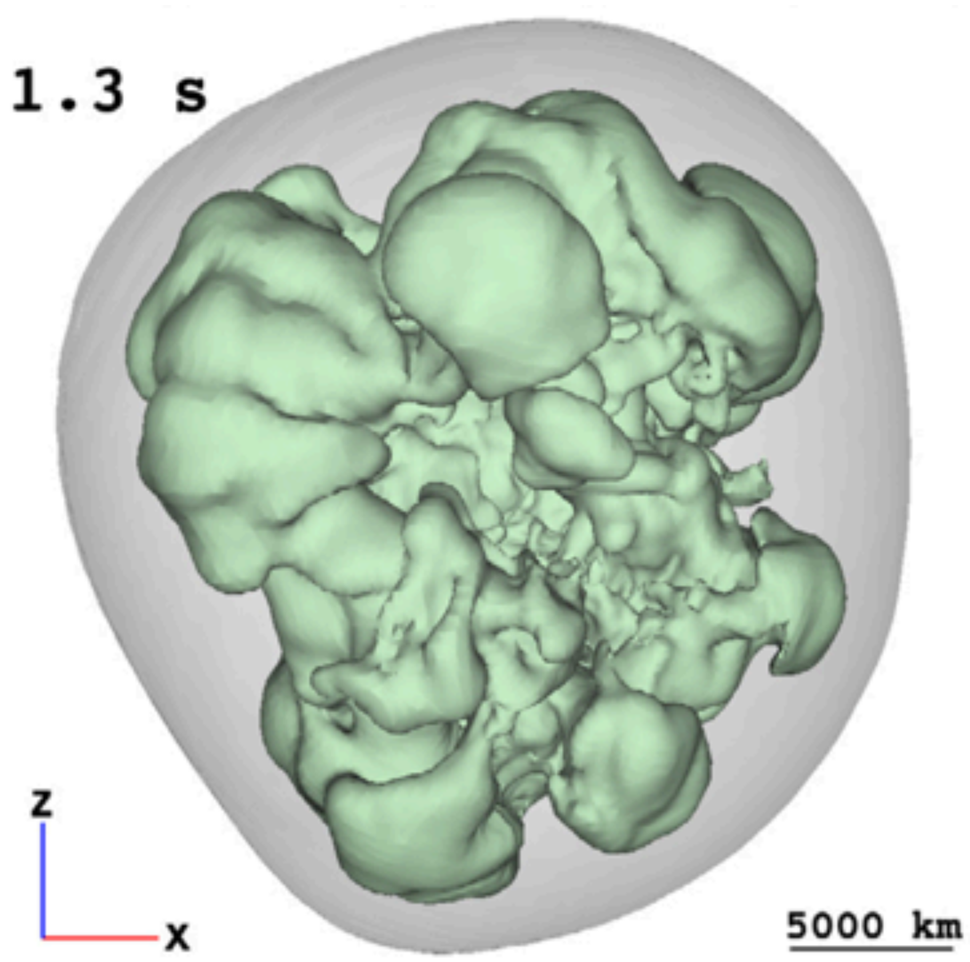
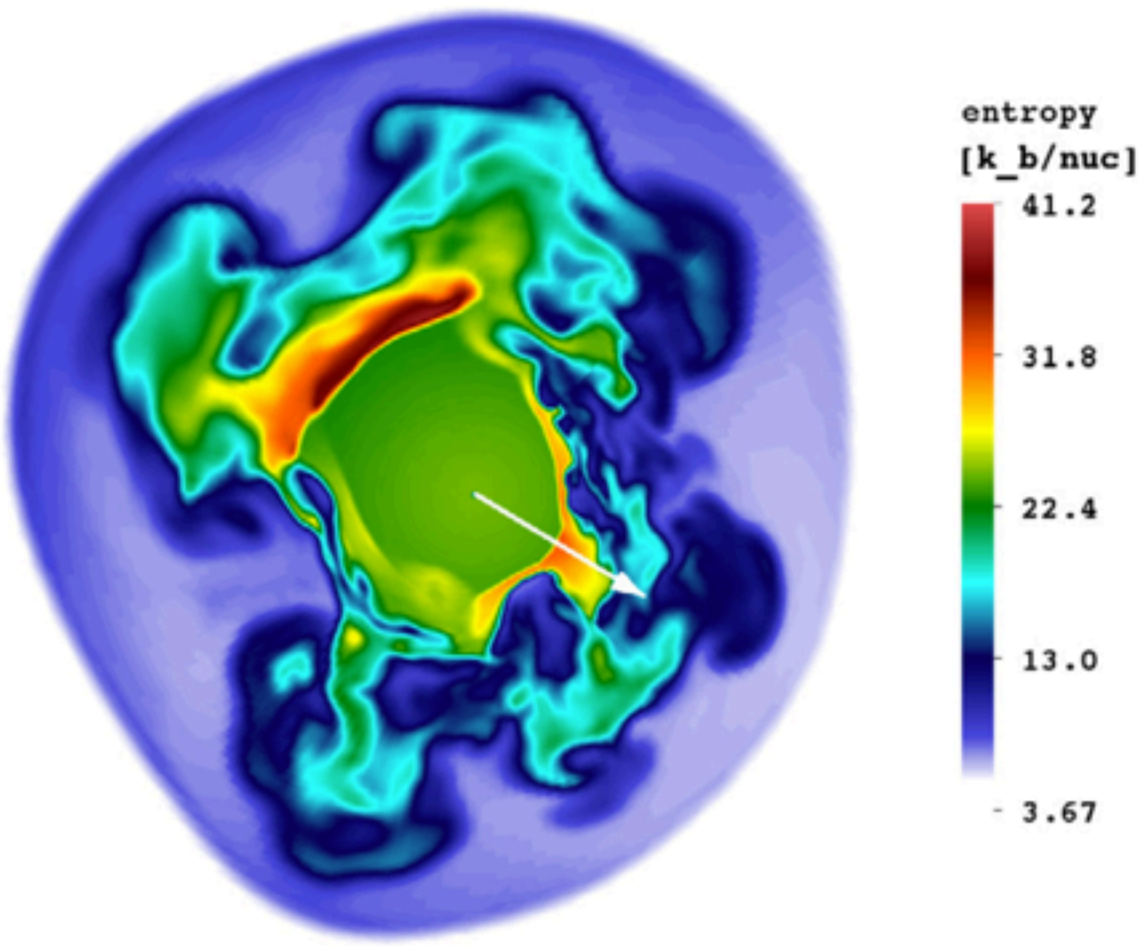
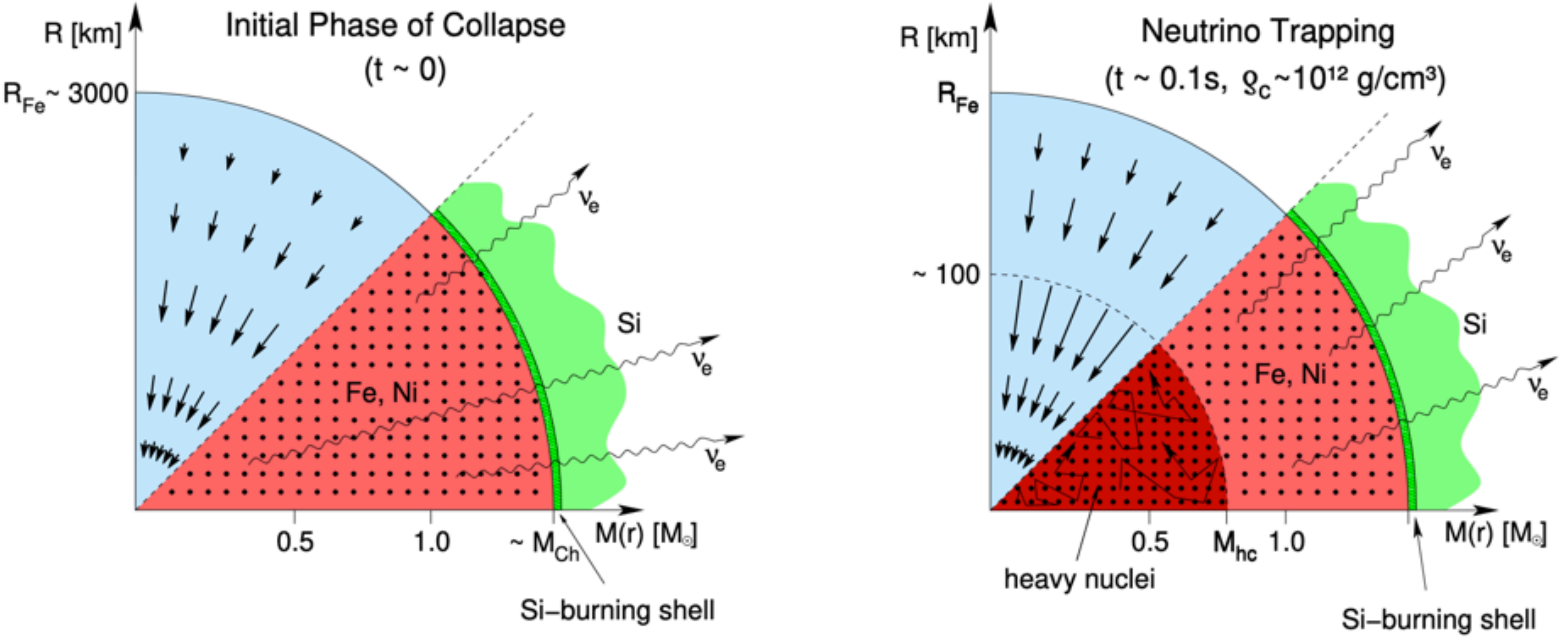




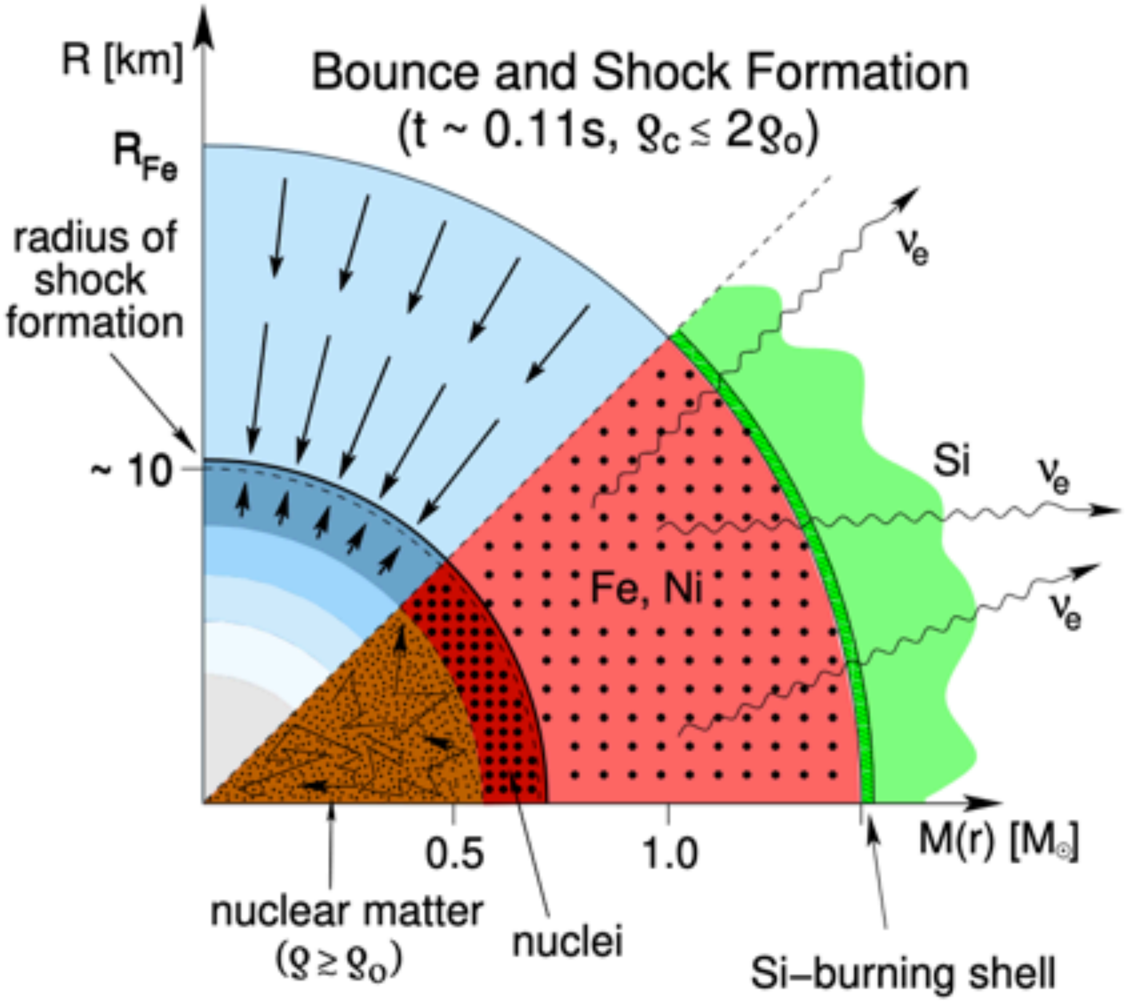
[Wongwathanarat et al., (2013)]

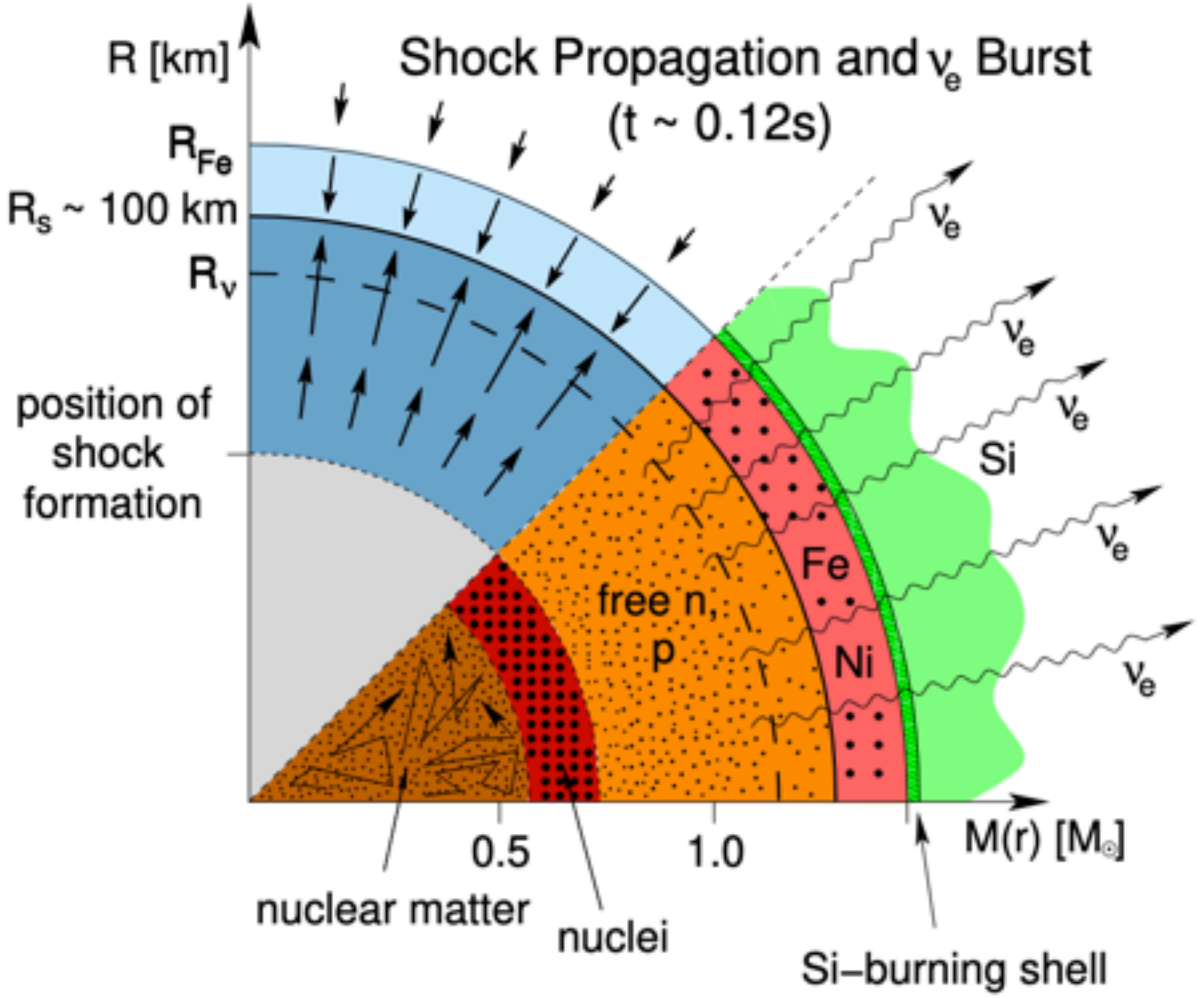
Core collapse and formation of a neutron star



[Janka et al., arXiv:0612072]

Core collapse and formation of a neutron star





[Janka et al., arXiv:0612072]

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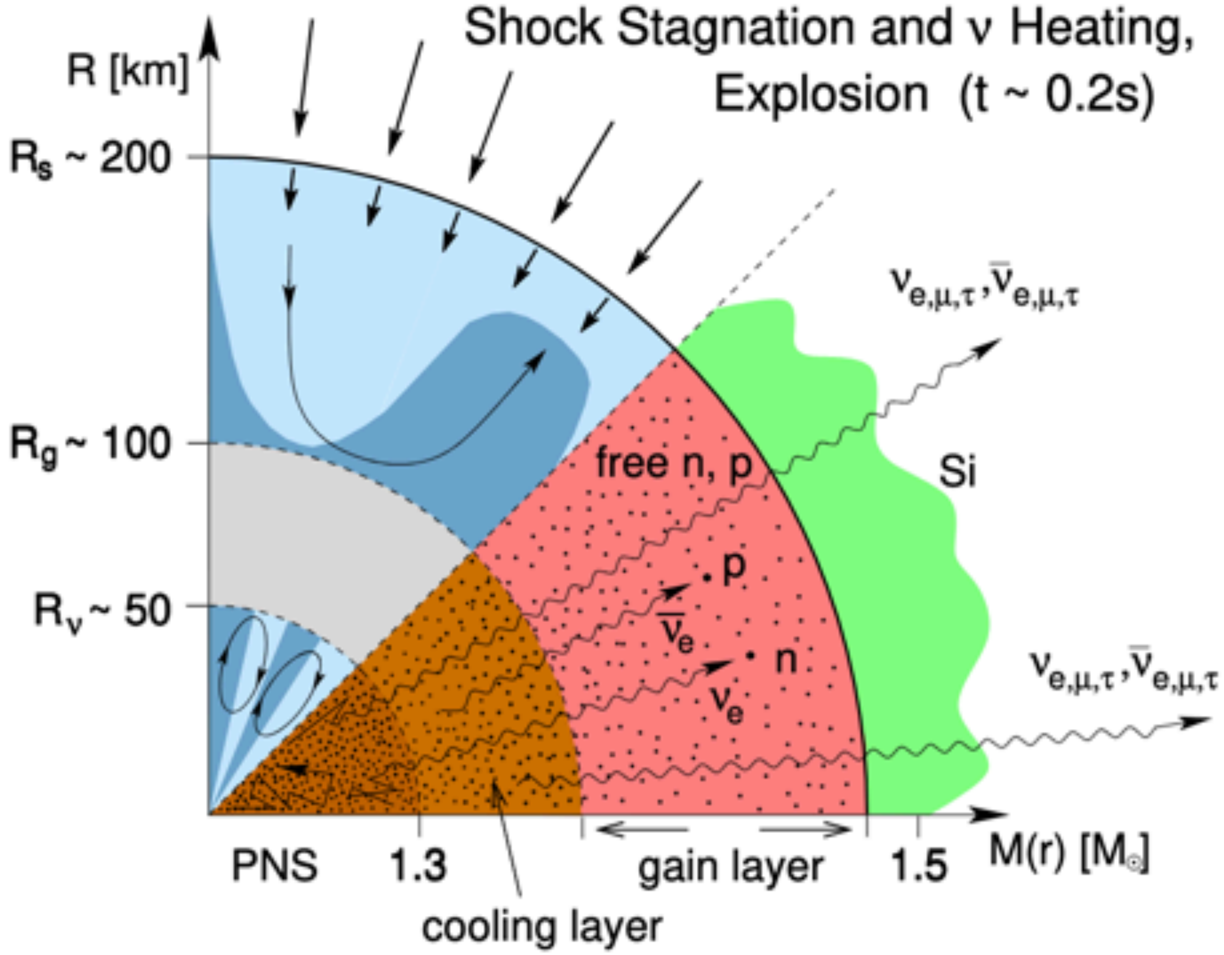
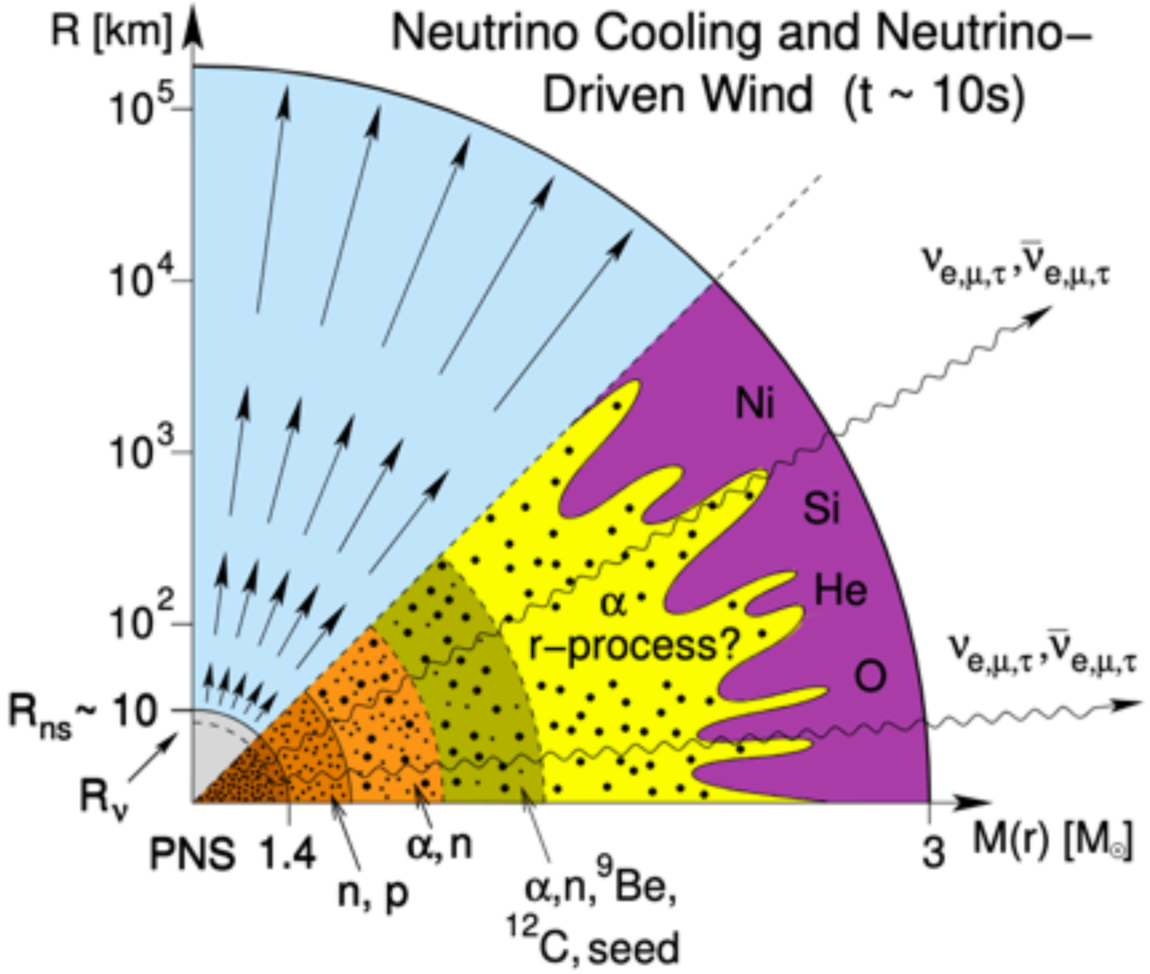
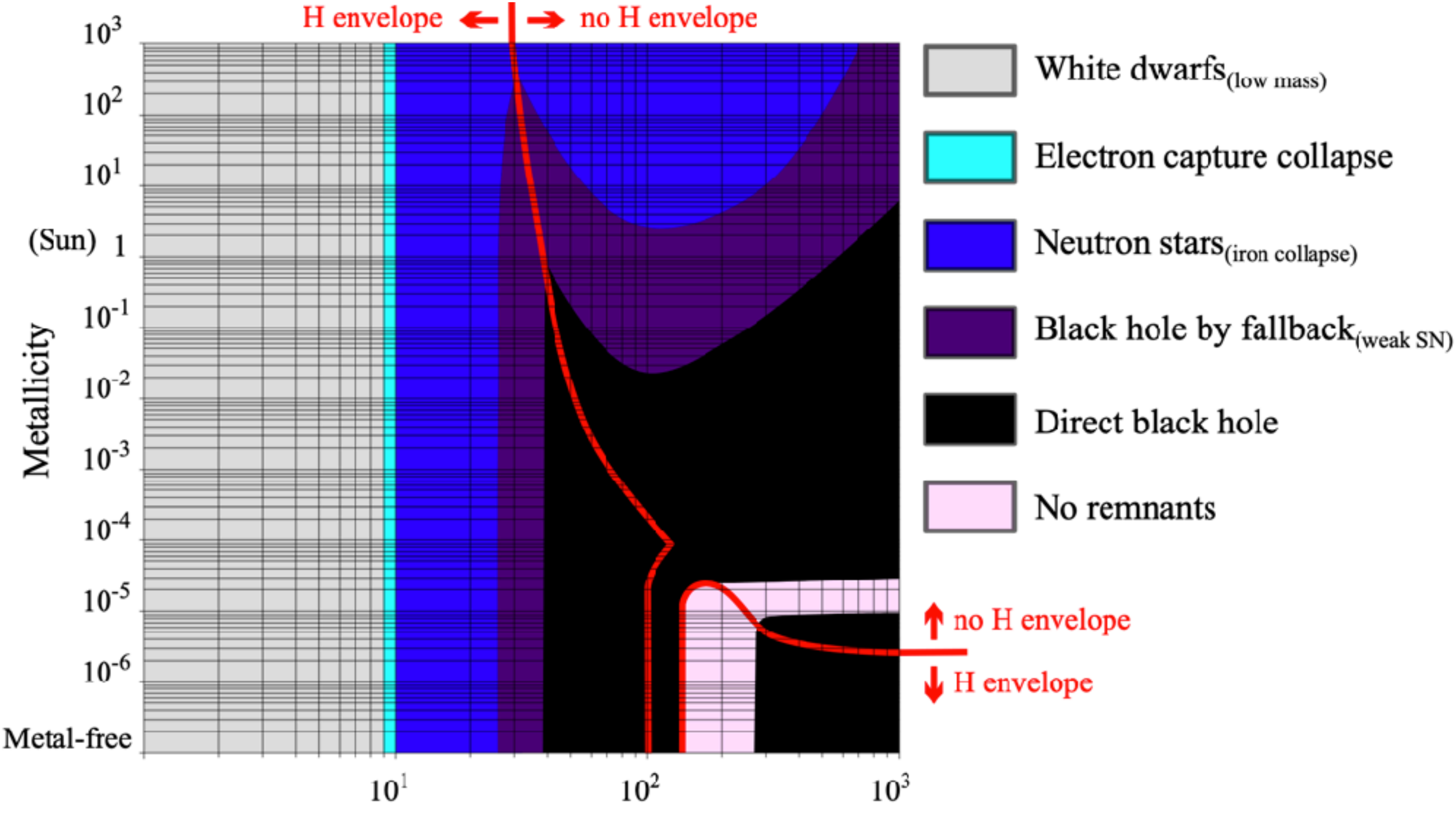




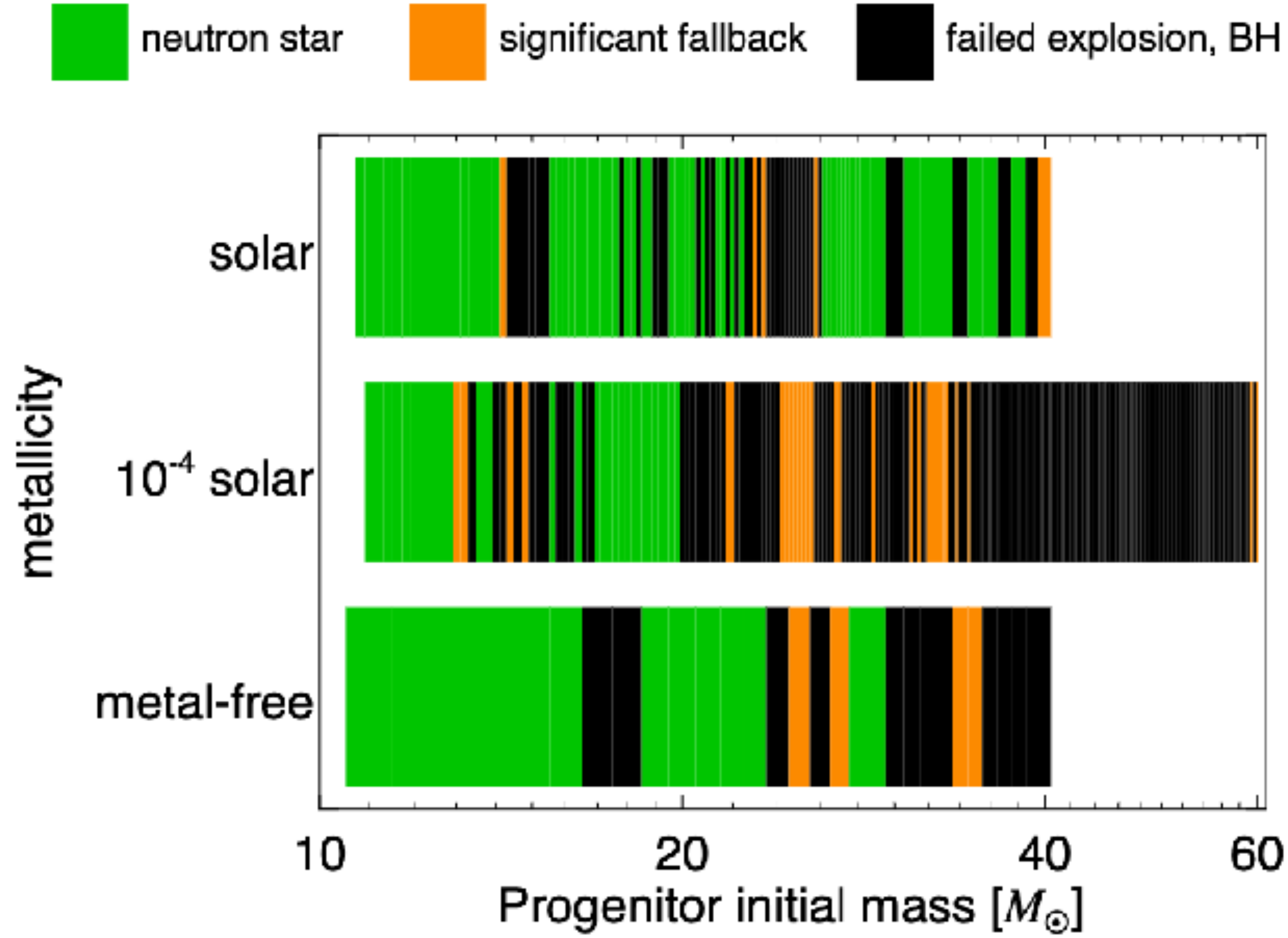
[Janka et al., arXiv:0612072]

The fate of massive stars



Zero-age Main Sequence mass (solar masses)

The fate of massive stars: islands of explodability



[Pejcha & Thompson, arXiv:1409.0540]

Explodability criteria

- Criteria based on pre-supernova structure
  - Criteria for zero-age main sequence stars depend also on stellar evolution

- Compactness

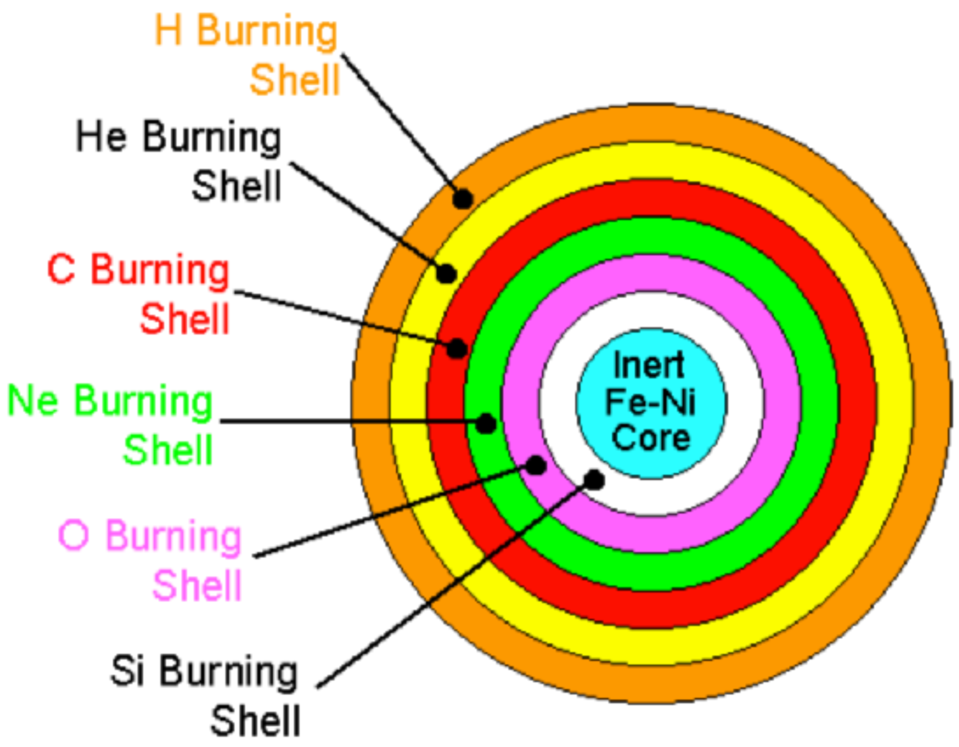
$$\xi_M = \frac{M/M_\odot}{R(M_{bary} = M)/1000\ km}$$

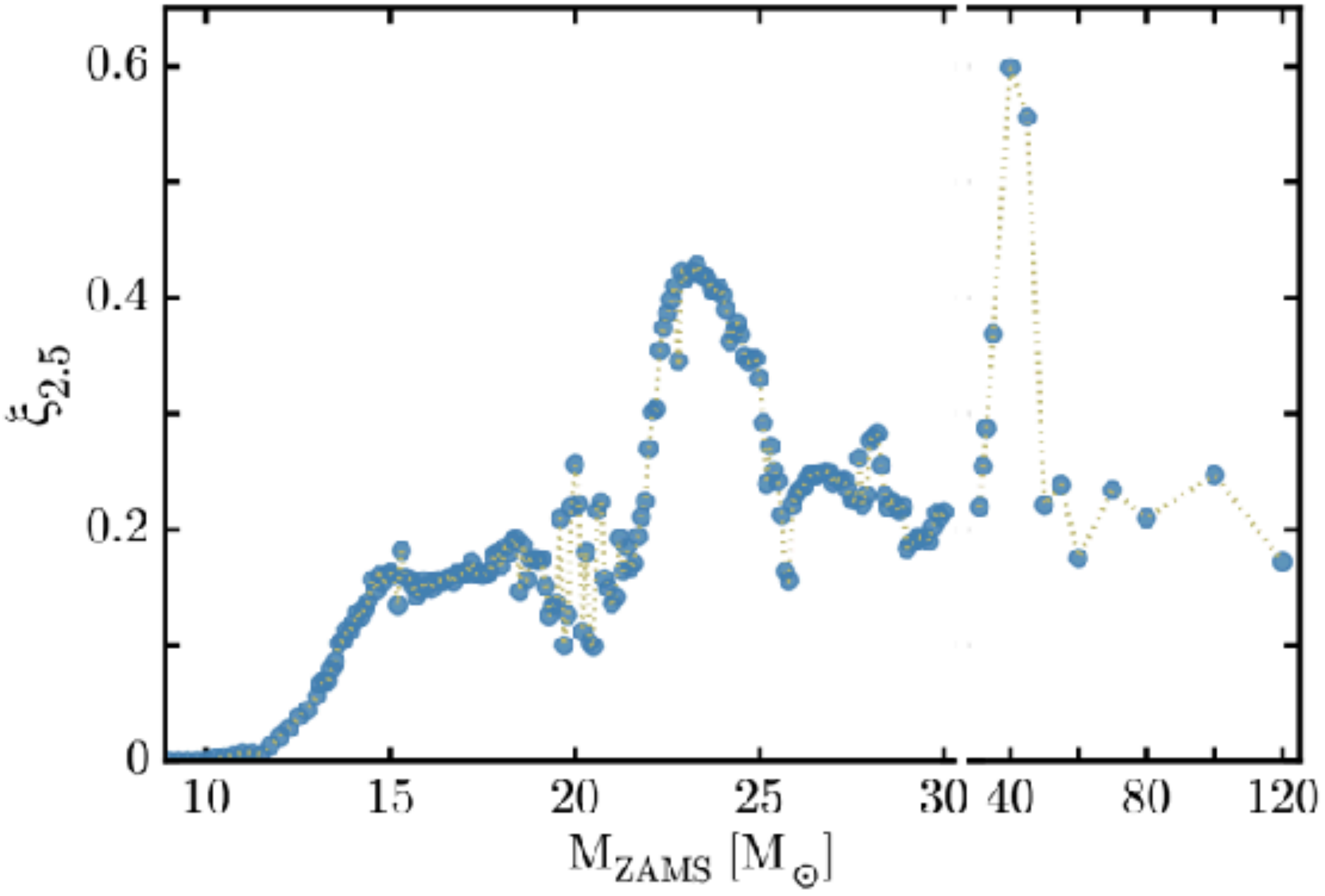
  - For example: if $\xi_2 < 0.5$ star explodes

Explodability of various stellar models

- Compactness seems to be non-monotonic with (initial) stellar mass!

- Episodes of convective burning in shells surrounding the core (carbon, oxygen, silicon) have a large effect on compactness





Credit: T. Sukhbold (PhD thesis)

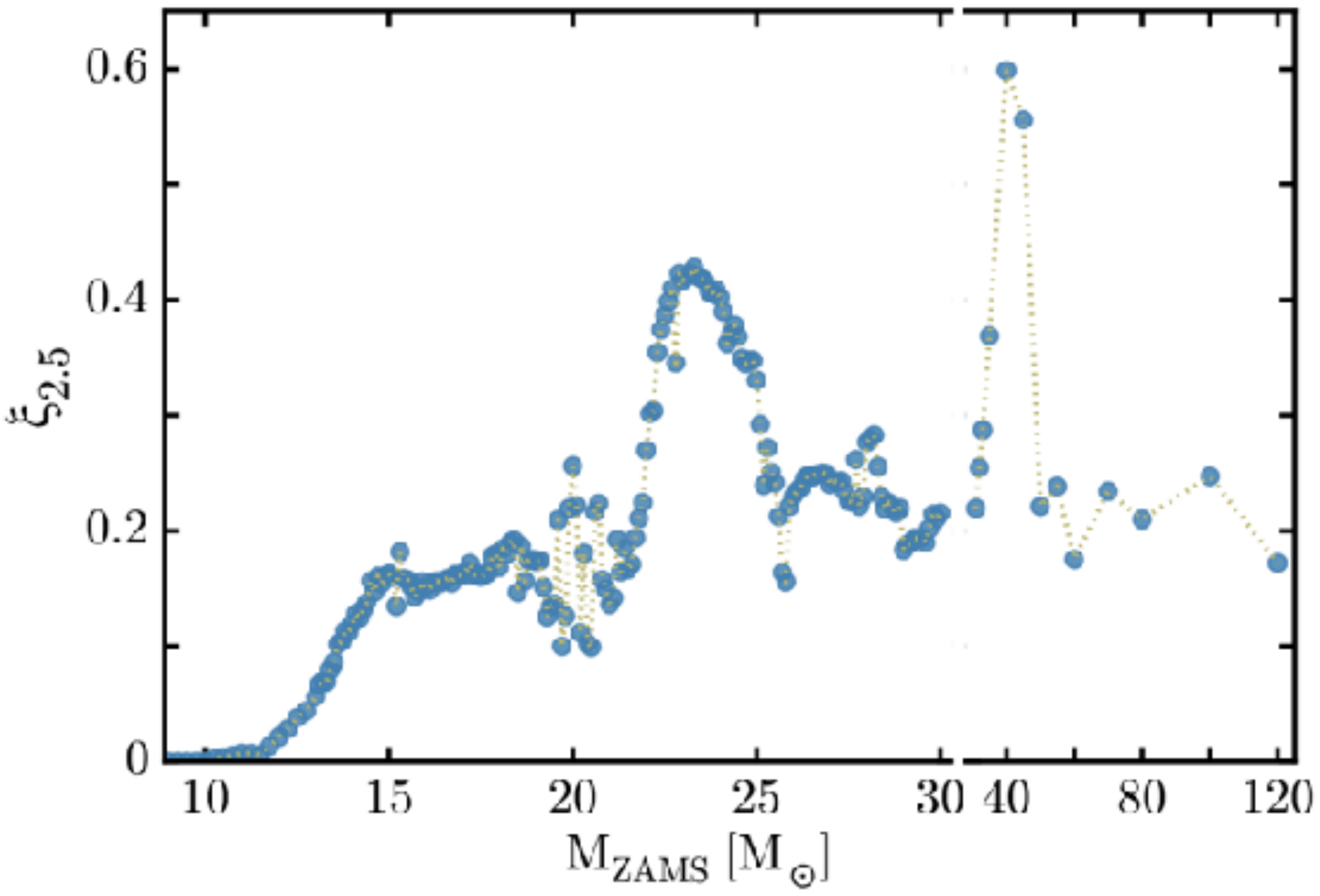
Explodability of various stellar models

- Compactness seems to be non-monotonic with (initial) stellar mass!

- Episodes of convective burning in shells surrounding the core (carbon, oxygen, silicon) have a large effect on compactness

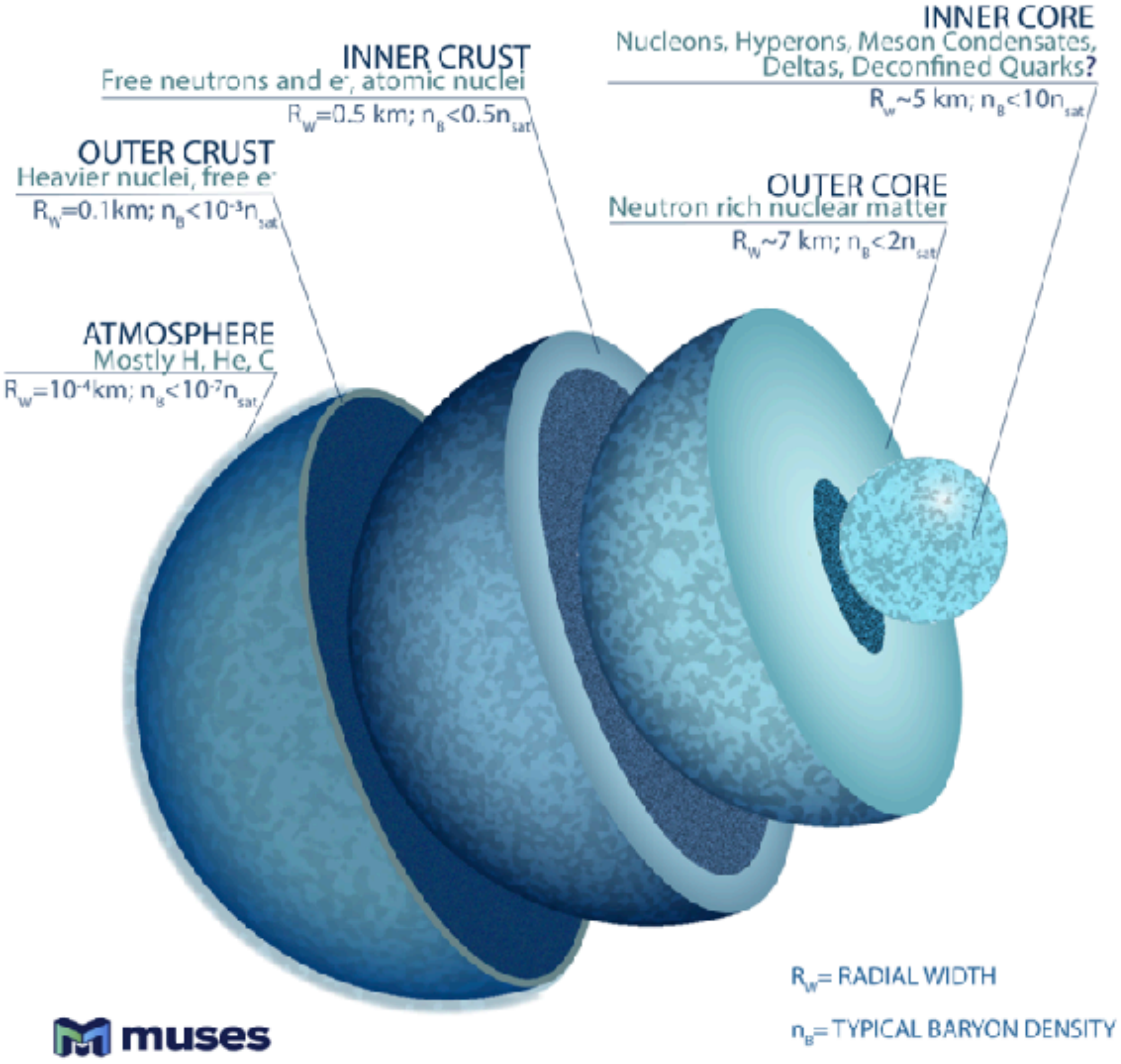
- Modelling uncertainties: convection regions; rates of some nuclear reactions

$$^{12}\text{C} + ^{4}\text{He} \rightarrow ^{16}\text{O}$$



Credit: T. Sukhbold (PhD thesis)

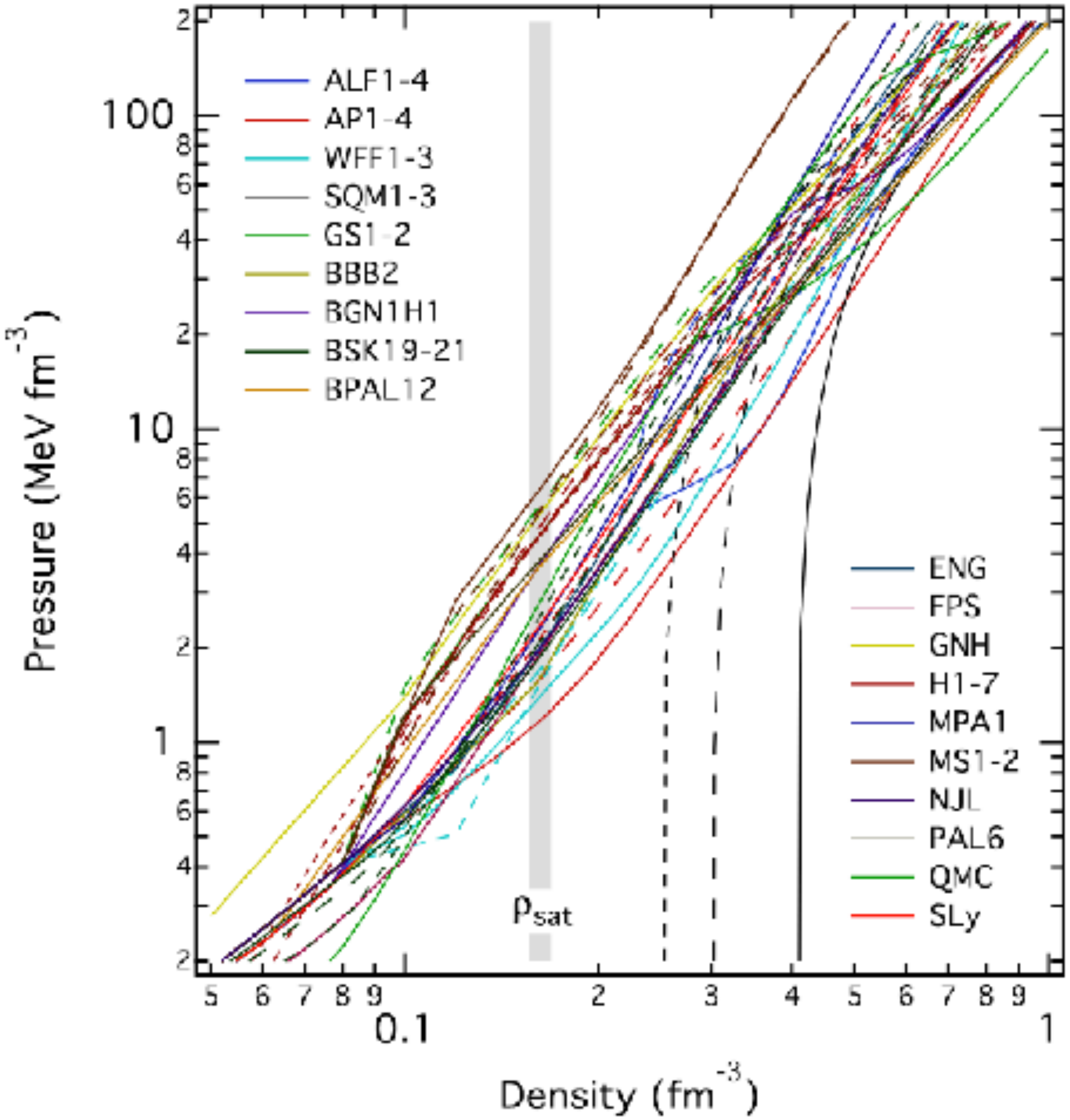
Neutron star structure



Neutron star equation of state

- Pressure vs. density $P(\rho)$ at and above the nuclear saturation density

- Tolman-Oppenheimer-Volkov (TOV) equations



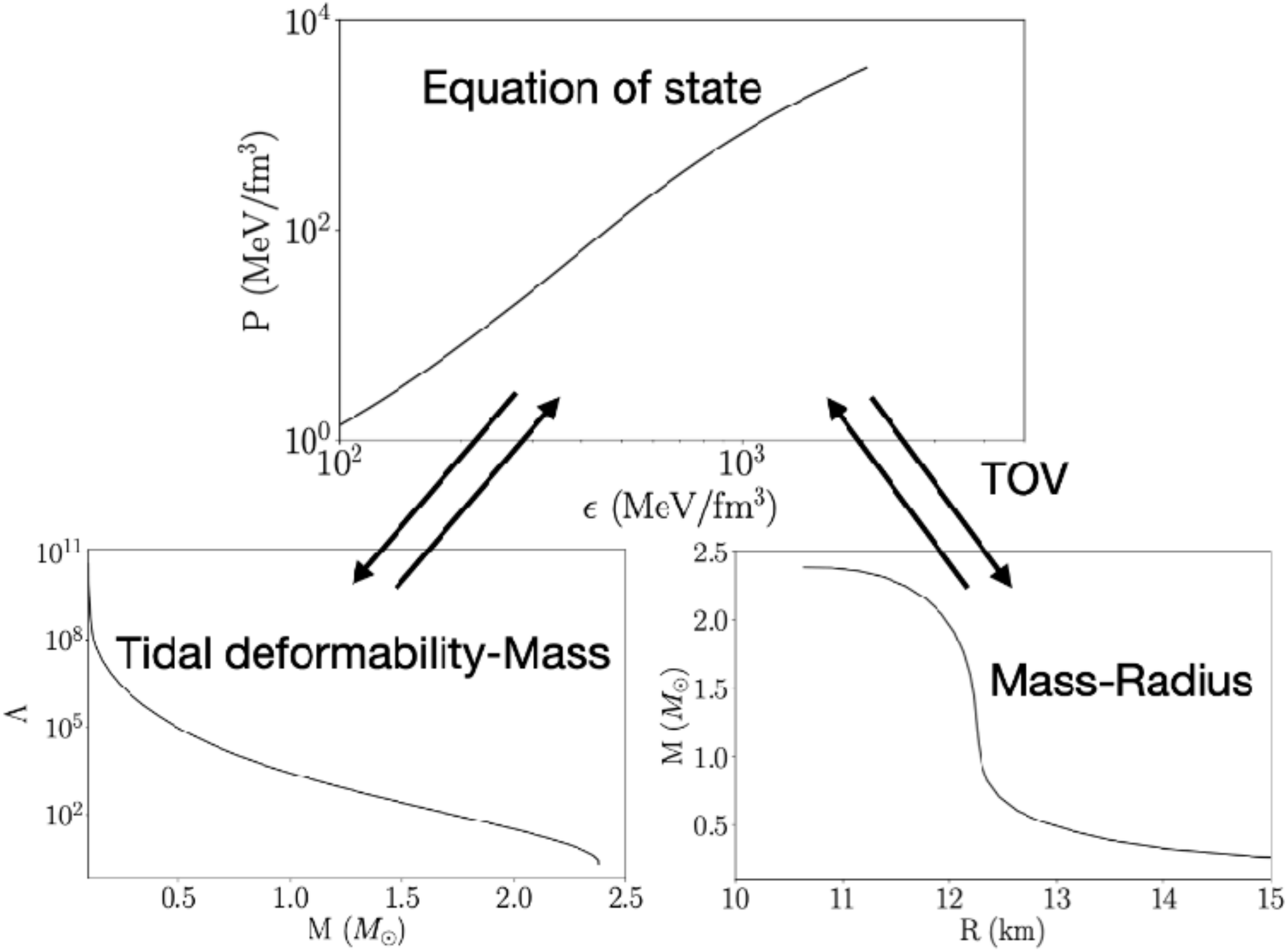
$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}\left(1+\frac{P}{\rho}\right)\left(1+\frac{4\pi r^3 P}{M(r)}\right)\left(1-\frac{2GM}{r}\right)^{-1}$$

Tidal deformability

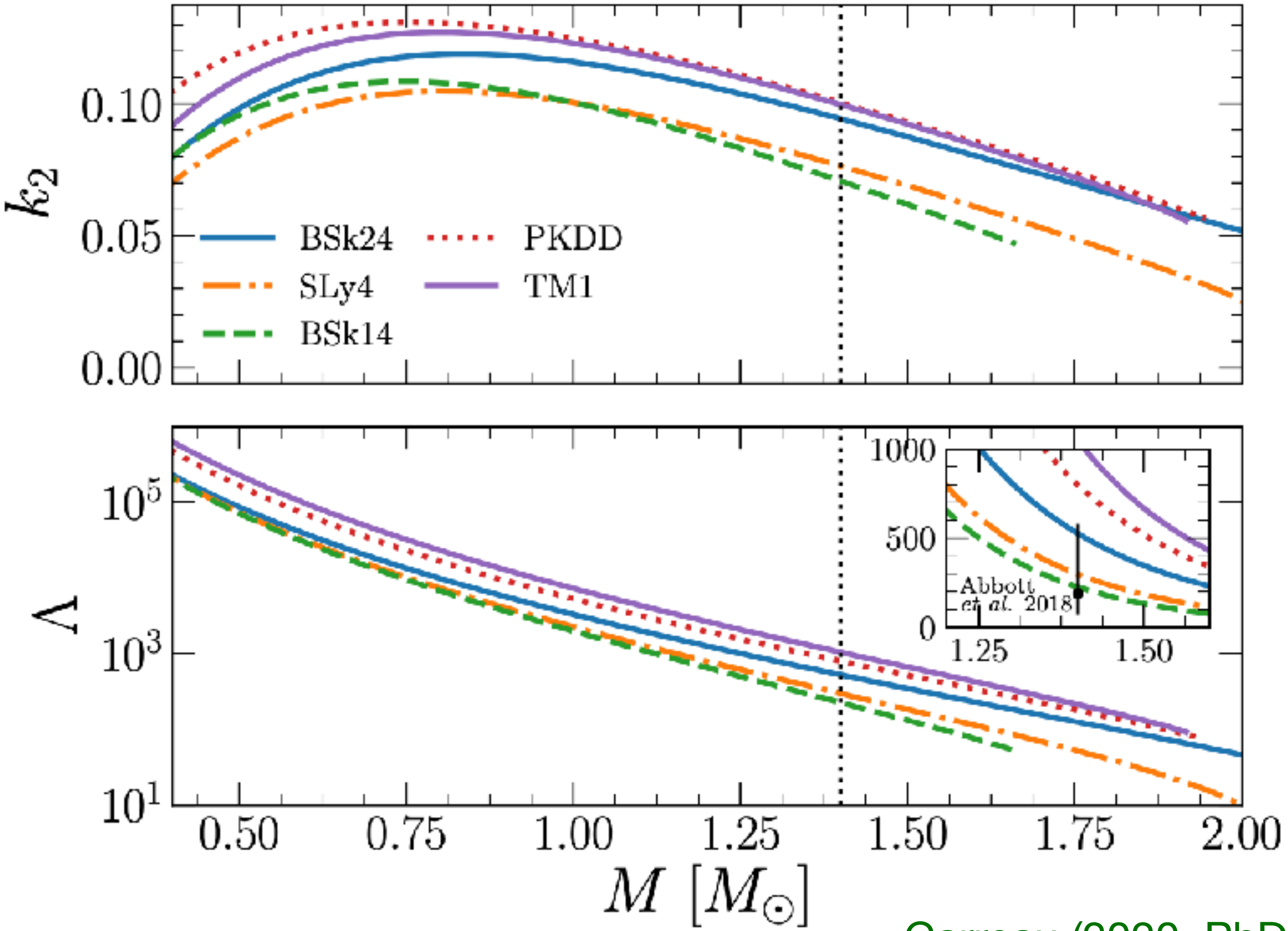
- A spherically symmetric neutron star is placed in a static external quadrupolar **tidal field** ε
- The induced quadrupolar moment is: $Q = -\lambda\varepsilon$
- **Tidal deformability** λ
- Tidal Love number k_2 (obtained by solving the TOV eqs): $\lambda = \frac{2}{3}k_2\left(\frac{Rc^2}{G}\right)^5$
- **Dimensionless tidal deformability**: $\Lambda = \frac{\lambda}{M^5}$

Neutron stars: from theory to observed quantities



See [Chatziioannou et al., arXiv:2407.11153]

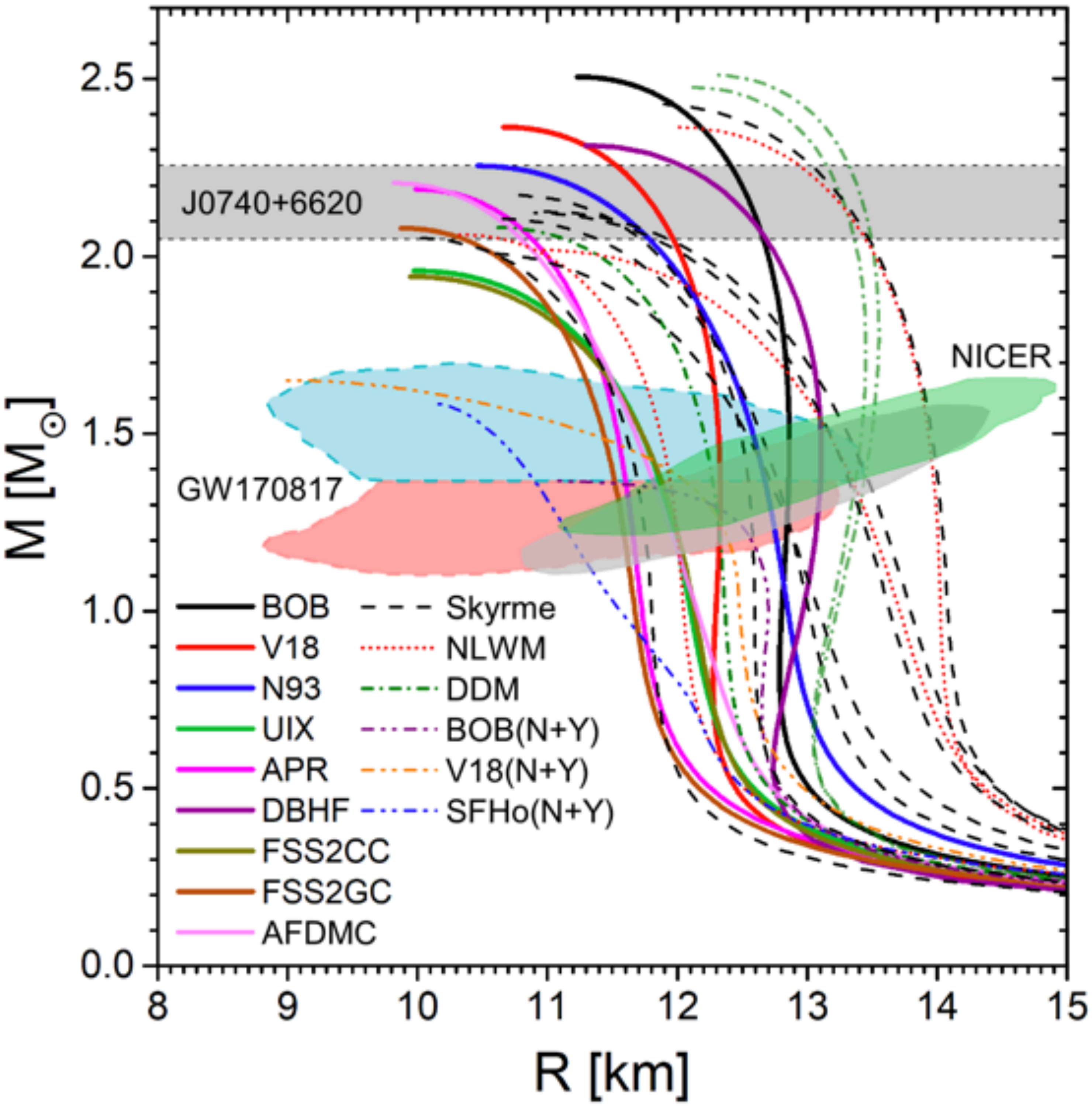
Tidal deformability



Carreau (2020, PhD thesis)

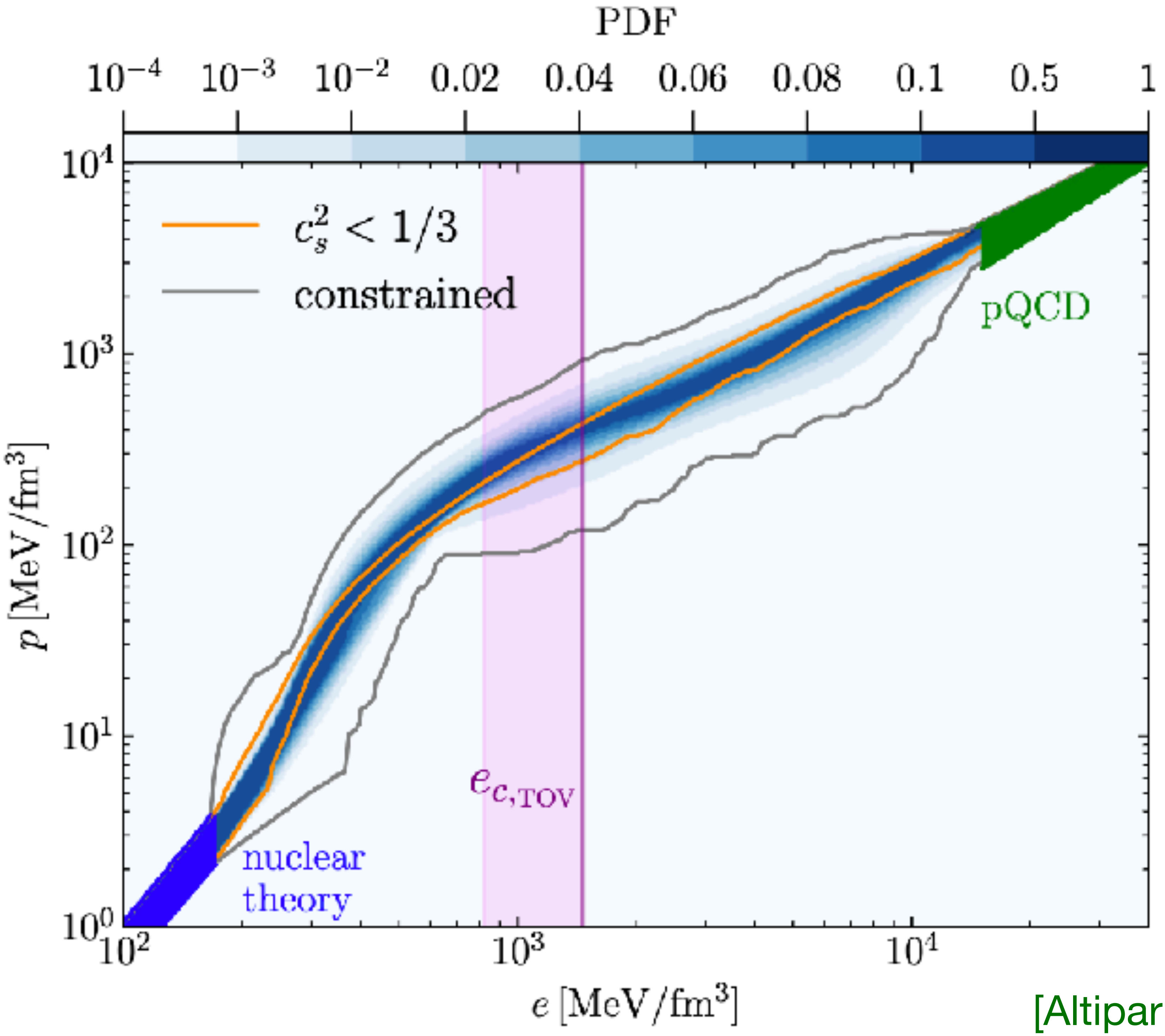
Neutron star mass-radius relation

- Stiff EOS: high maximum mass, large radii
- Soft EOS: low maximum mass, small radii



Burgio et al., arXiv:2105.03747

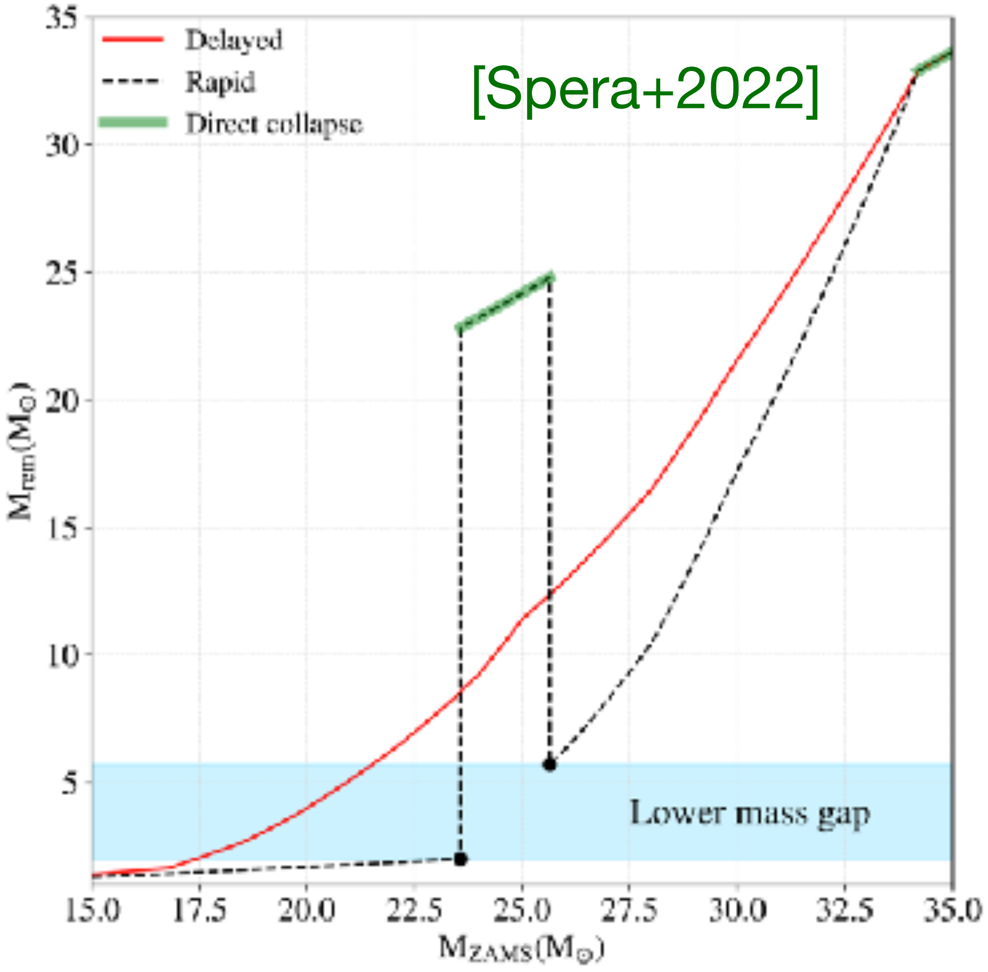
Neutron star equation of state: observational constraints

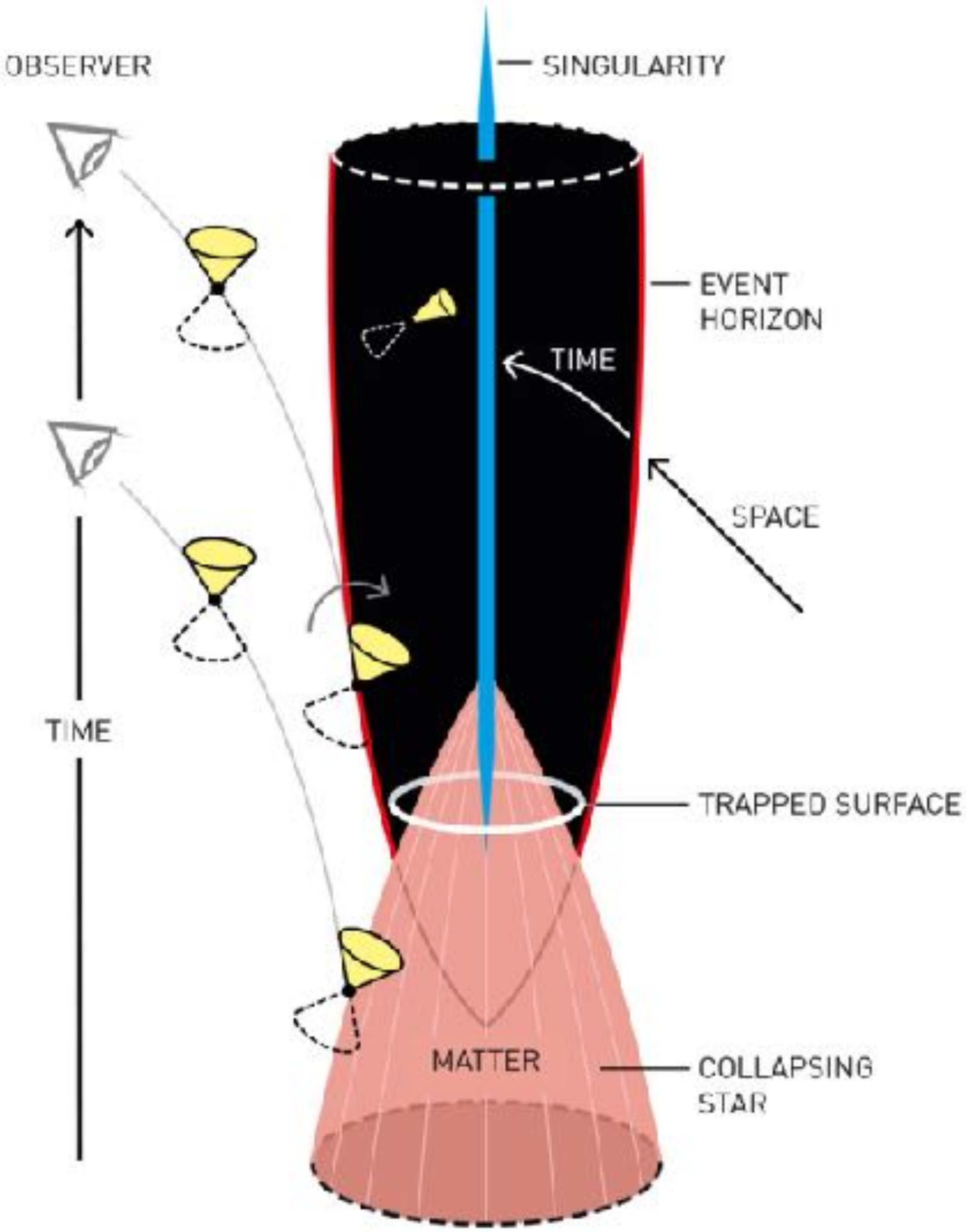


[Altiparmak+2017]

Black hole formation

- If the mass of the proto-neutron star is above the maximally allowed limit: further collapse to a black hole
- Until recent years: no black holes heavier than $\sim 5M_\odot$. Is there a mass gap between neutron stars and black holes?



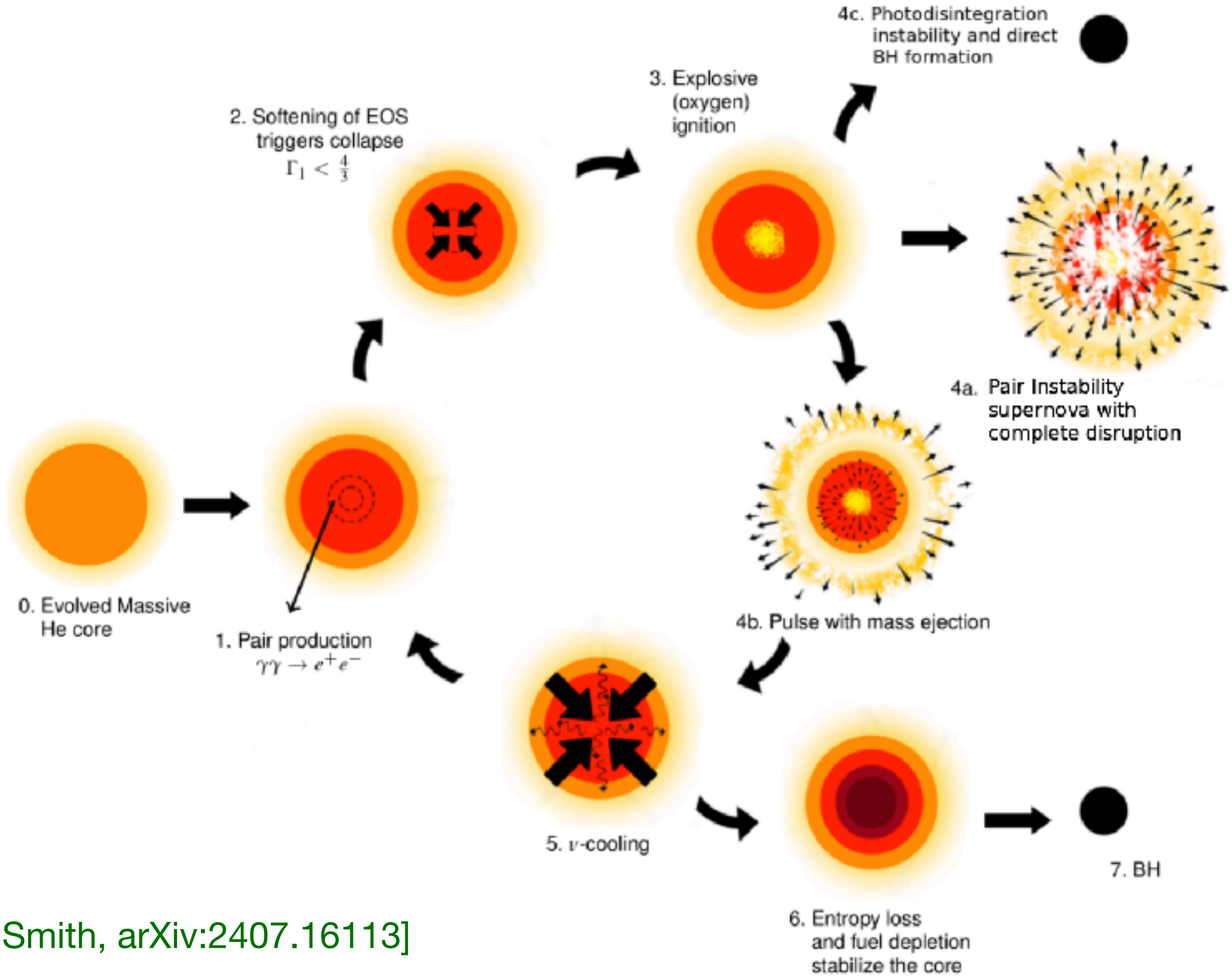


Credit: Johan Jarnestad

Pair instability in massive stars

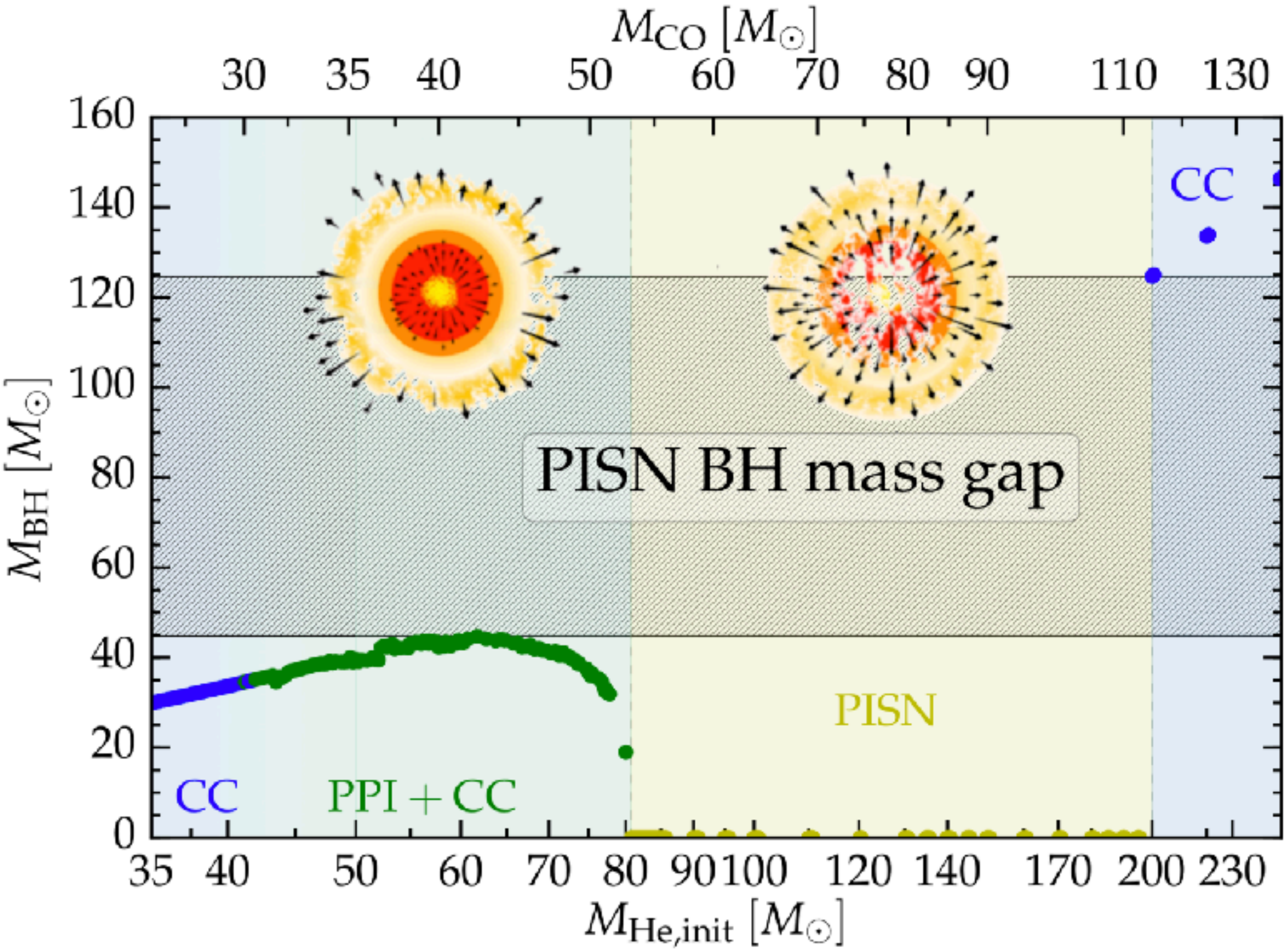
- Electron-positron pairs remove pressure from the star $\gamma + \gamma \rightarrow e^+ + e^-$
- If pressure is radiation-dominated, this leads to local thermal instability
- Contraction, increase in temperature, increase in pair production rate
- Runaway oxygen/silicon burning
- Disruption of the star

Pair instability (PI) and pulsational pair instability (PPI)



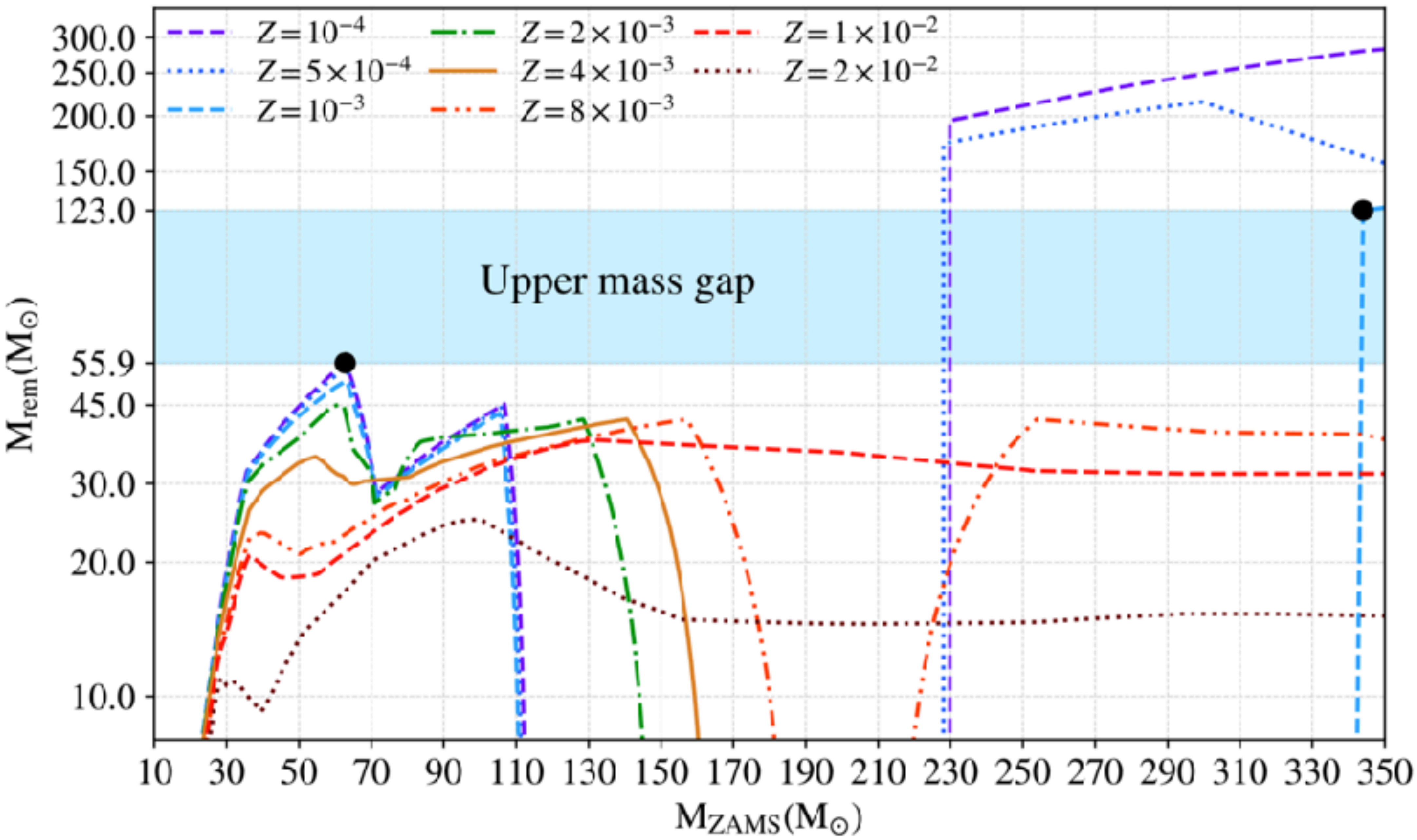
[Renzo and Smith, arXiv:2407.16113]

Pair instability and the upper mass gap



[Renzo and Smith, arXiv:2407.16113]

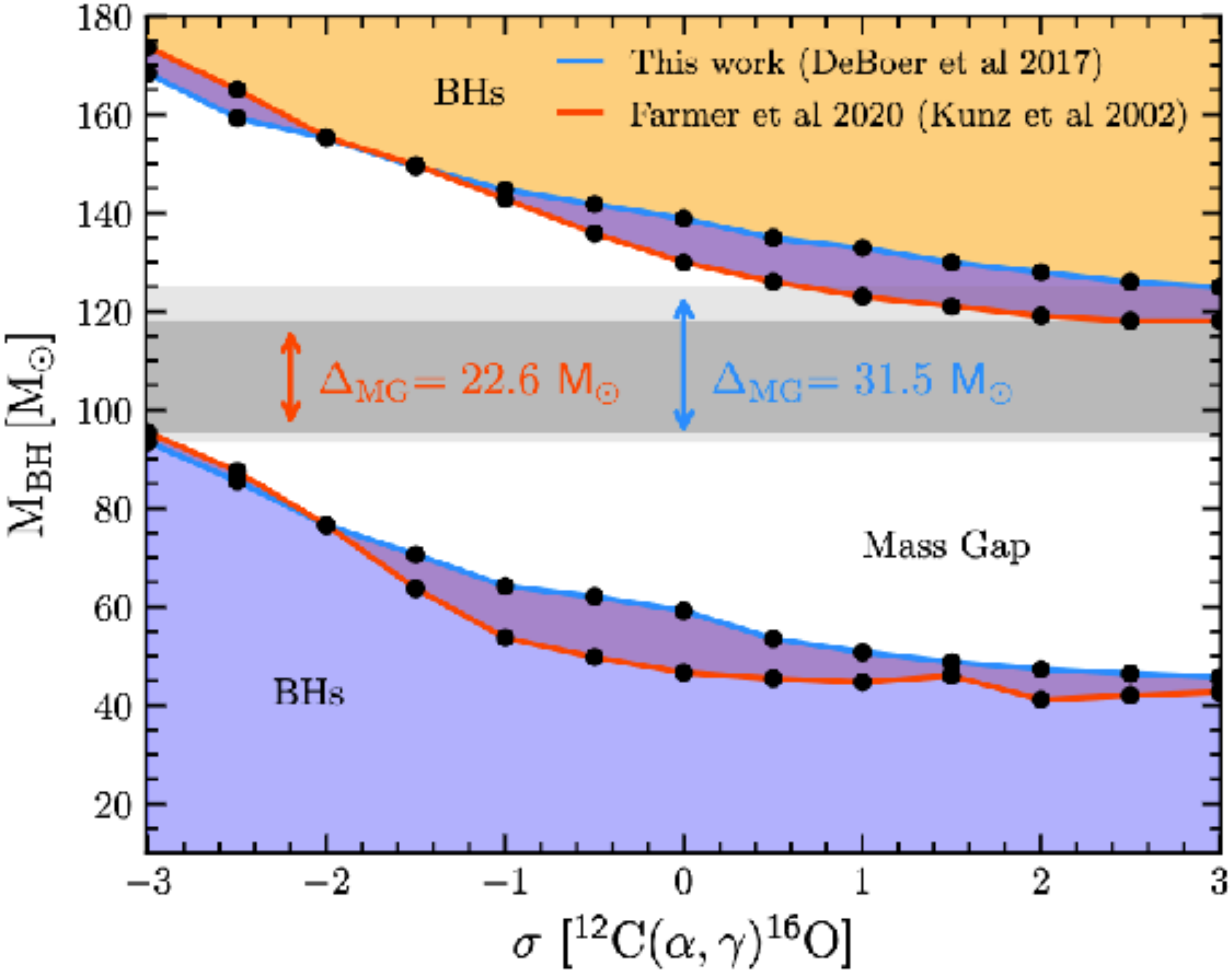
Pair instability: dependence on metallicity



[Spera et al., arXiv:2206.15392]

Uncertainty in nuclear reaction rates

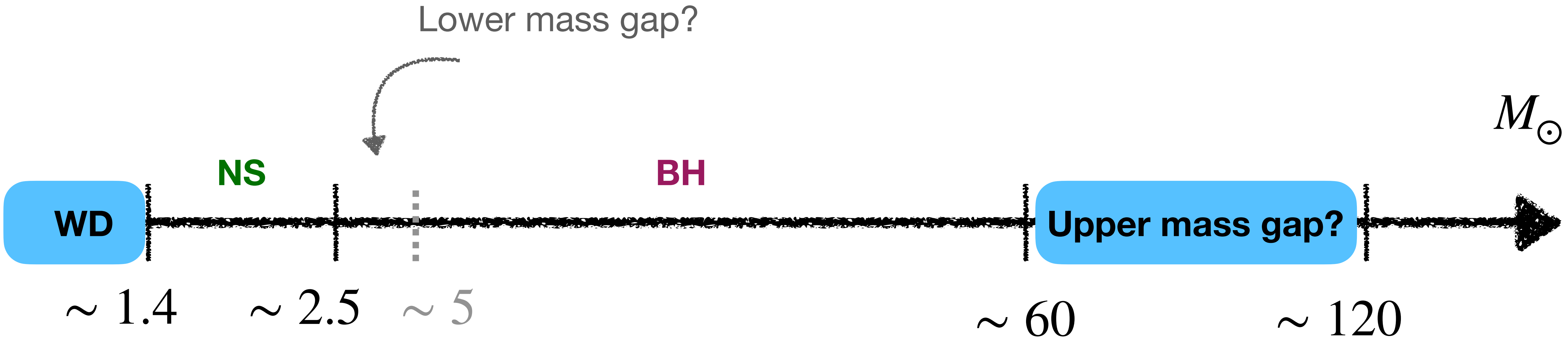
$^{12}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O}$ determines the amount of oxygen available for explosive burning



[Renzo & Smith, arXiv:2407.16113]

Neutron star and (stellar) black hole masses

- Lower limit on **NS**: Chandrasekhar mass
- Upper limit on **NS**/lower limit on **BH**: equation of state
- Mass gap between **NS** and **BH** ???
- Upper limit on **BH**: pair-instability (initial stellar mass?)

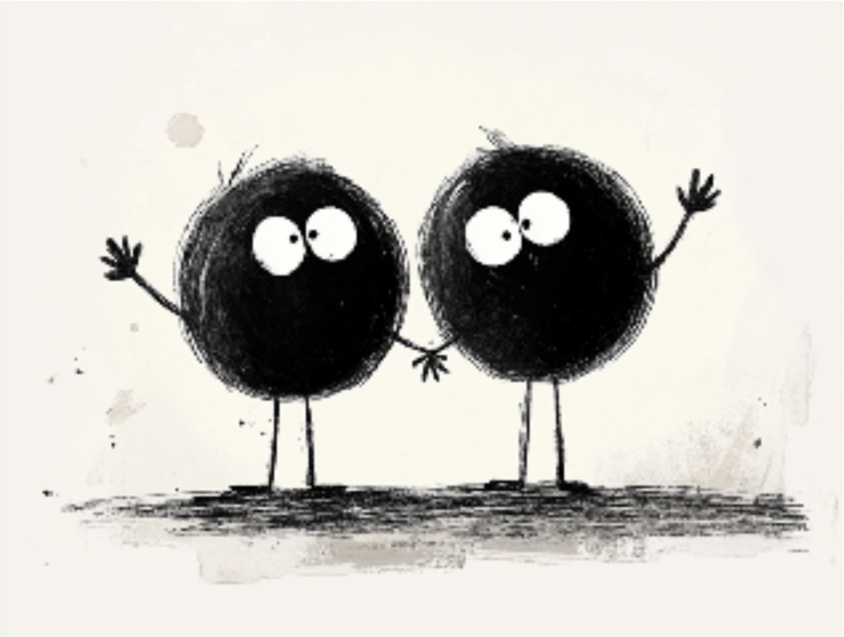


Lecture plan

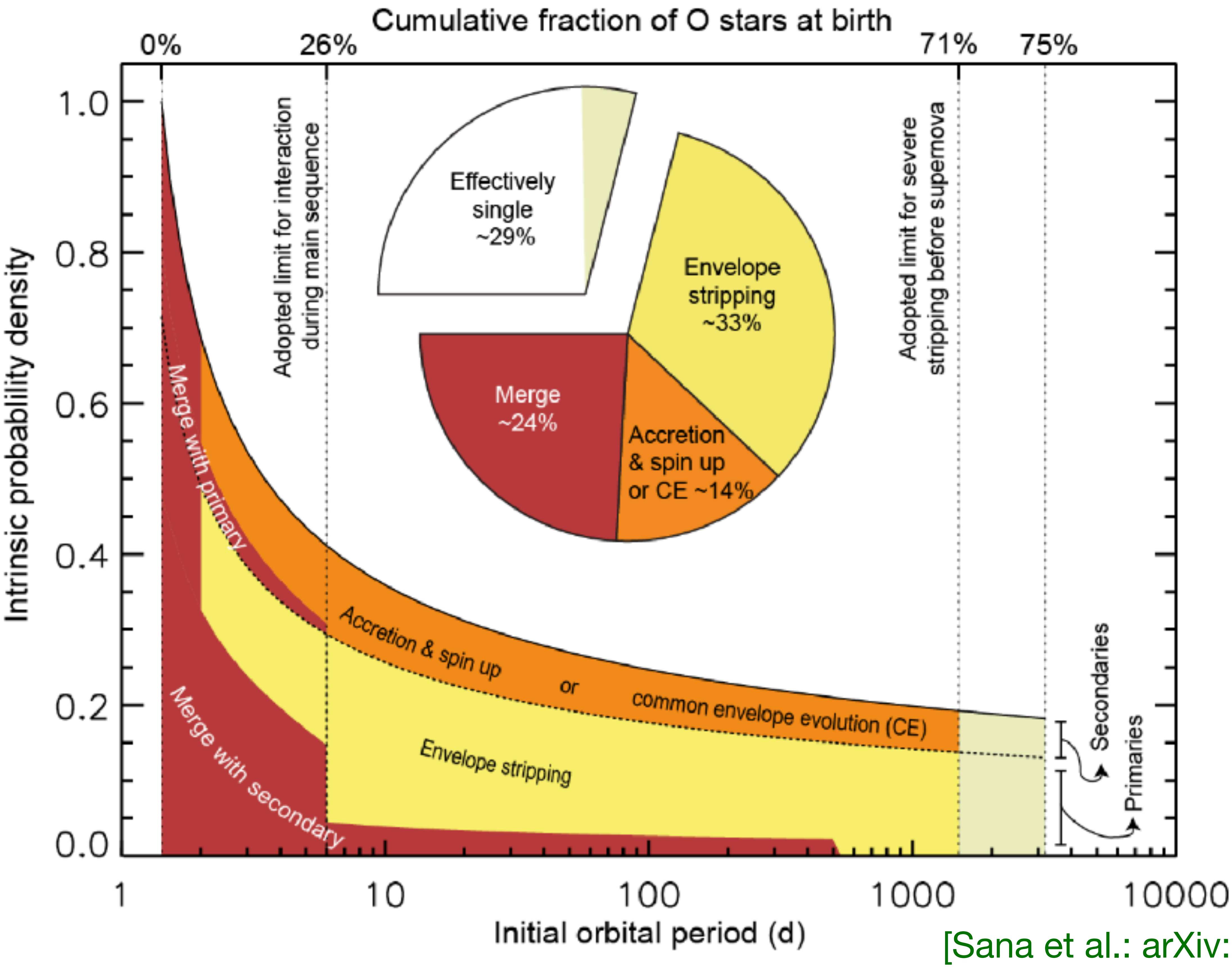
- Evolution of massive stars and formation of compact objects
- Evolution of binary massive stars and formation of binary compact objects
- Gravitational-wave astronomy
- Gravitational-wave observations of binary compact objects

Lecture plan

- Evolution of binary massive stars and formation of binary compact objects
  - Physical processes in binary stars
  - Modelling methods and challenges
  - Observables



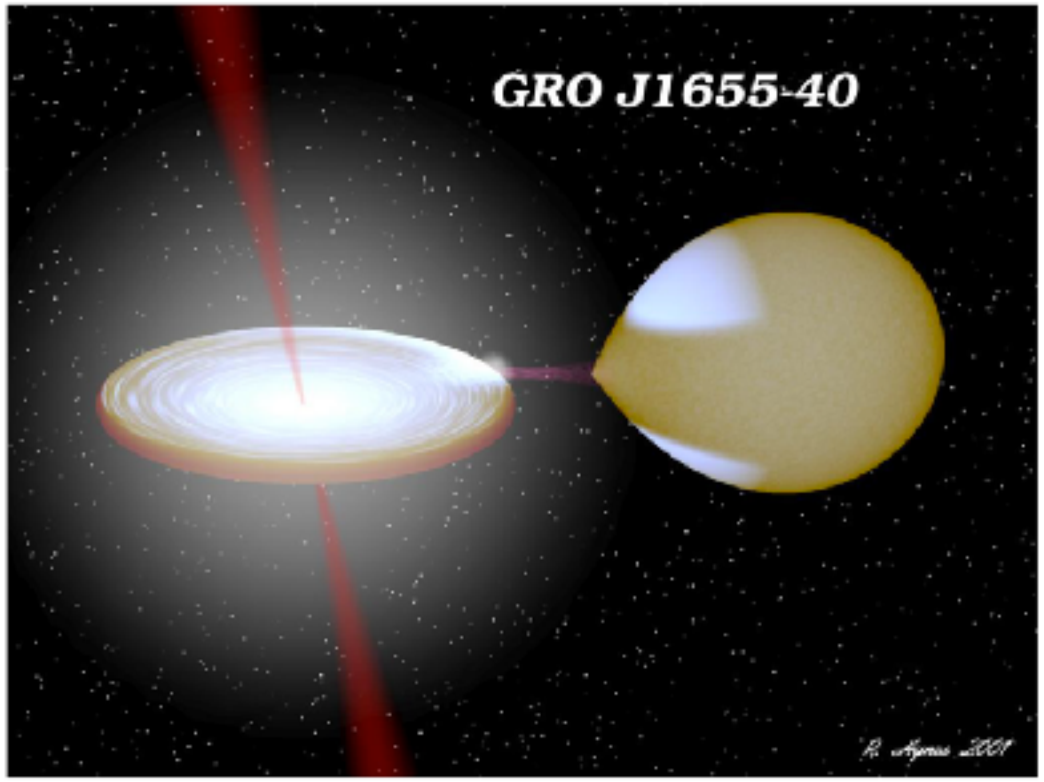
Most stars live in binaries



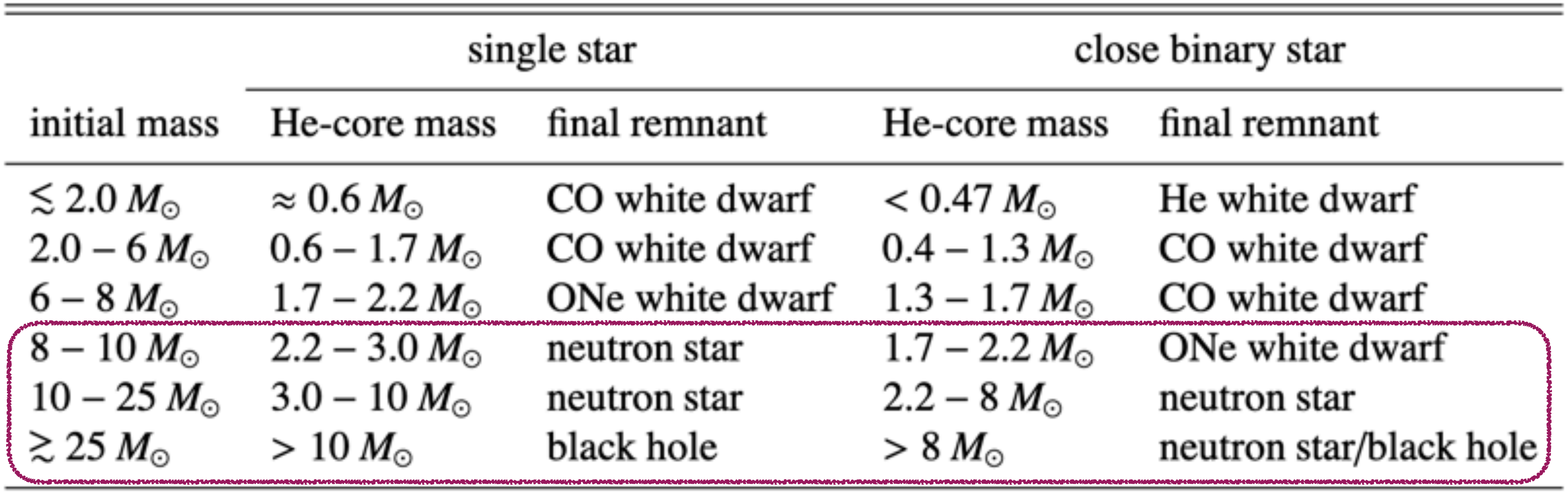
[Sana et al.: arXiv:1207.6397]

Key processes in binary stars

- Stable mass transfer
- Unstable mass transfer
- Supernova kicks and binary disruption



[Credit: Robert Hynes]

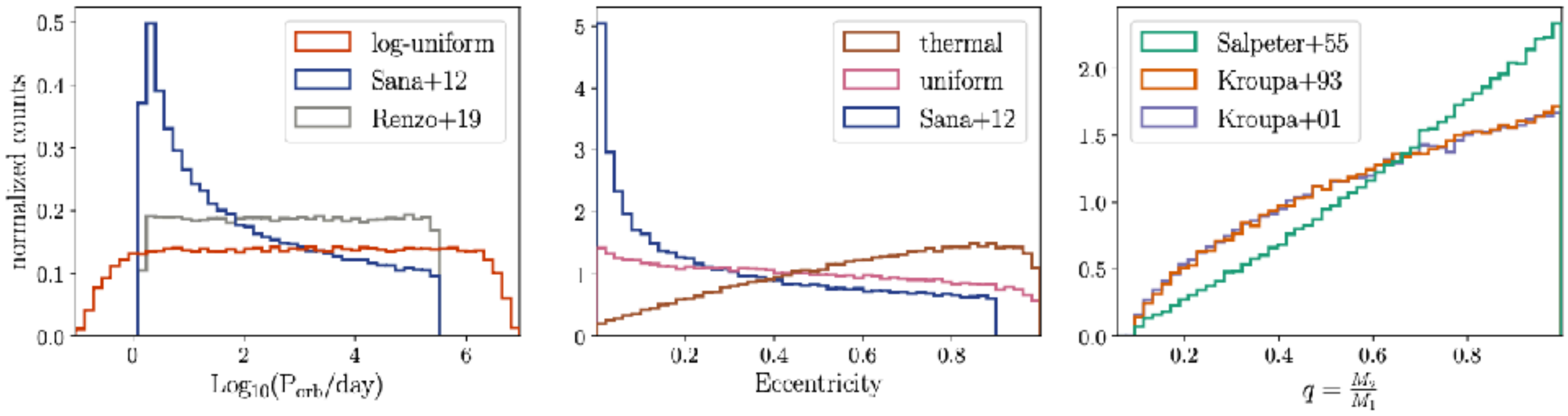


	single star		close binary star	
initial mass	He-core mass	final remnant	He-core mass	final remnant
$\lesssim 2.0\, M_\odot$	$\approx 0.6\, M_\odot$	CO white dwarf	$< 0.47\, M_\odot$	He white dwarf
$2.0 - 6\, M_\odot$	$0.6 - 1.7\, M_\odot$	CO white dwarf	$0.4 - 1.3\, M_\odot$	CO white dwarf
$6 - 8\, M_\odot$	$1.7 - 2.2\, M_\odot$	ONe white dwarf	$1.3 - 1.7\, M_\odot$	CO white dwarf
$8 - 10\, M_\odot$	$2.2 - 3.0\, M_\odot$	neutron star	$1.7 - 2.2\, M_\odot$	ONe white dwarf
$10 - 25\, M_\odot$	$3.0 - 10\, M_\odot$	neutron star	$2.2 - 8\, M_\odot$	neutron star
$\gtrsim 25\, M_\odot$	$> 10\, M_\odot$	black hole	$> 8\, M_\odot$	neutron star/black hole

[O. Pols: Stellar structure and evolution]

Initial distribution of binary parameters

- Orbital period
- Eccentricity
- Mass ratio



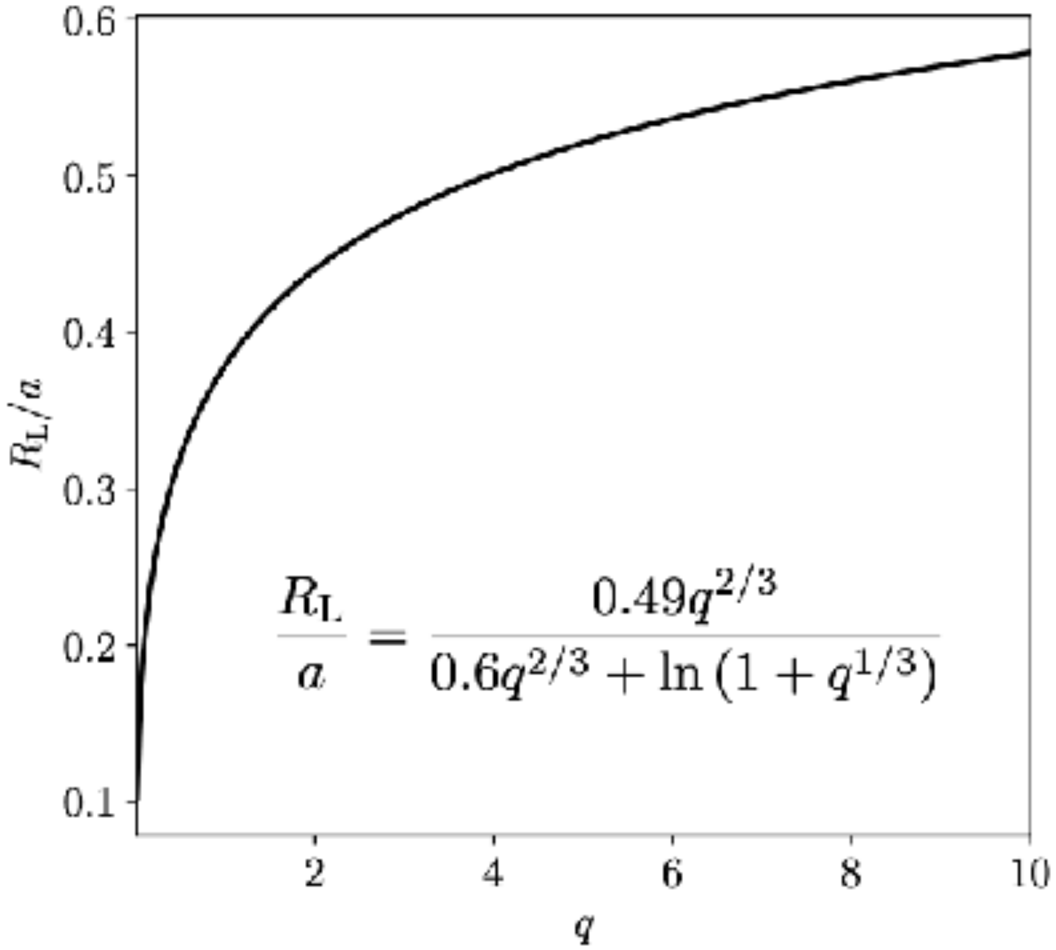
[Credit: Clément Pellouin]

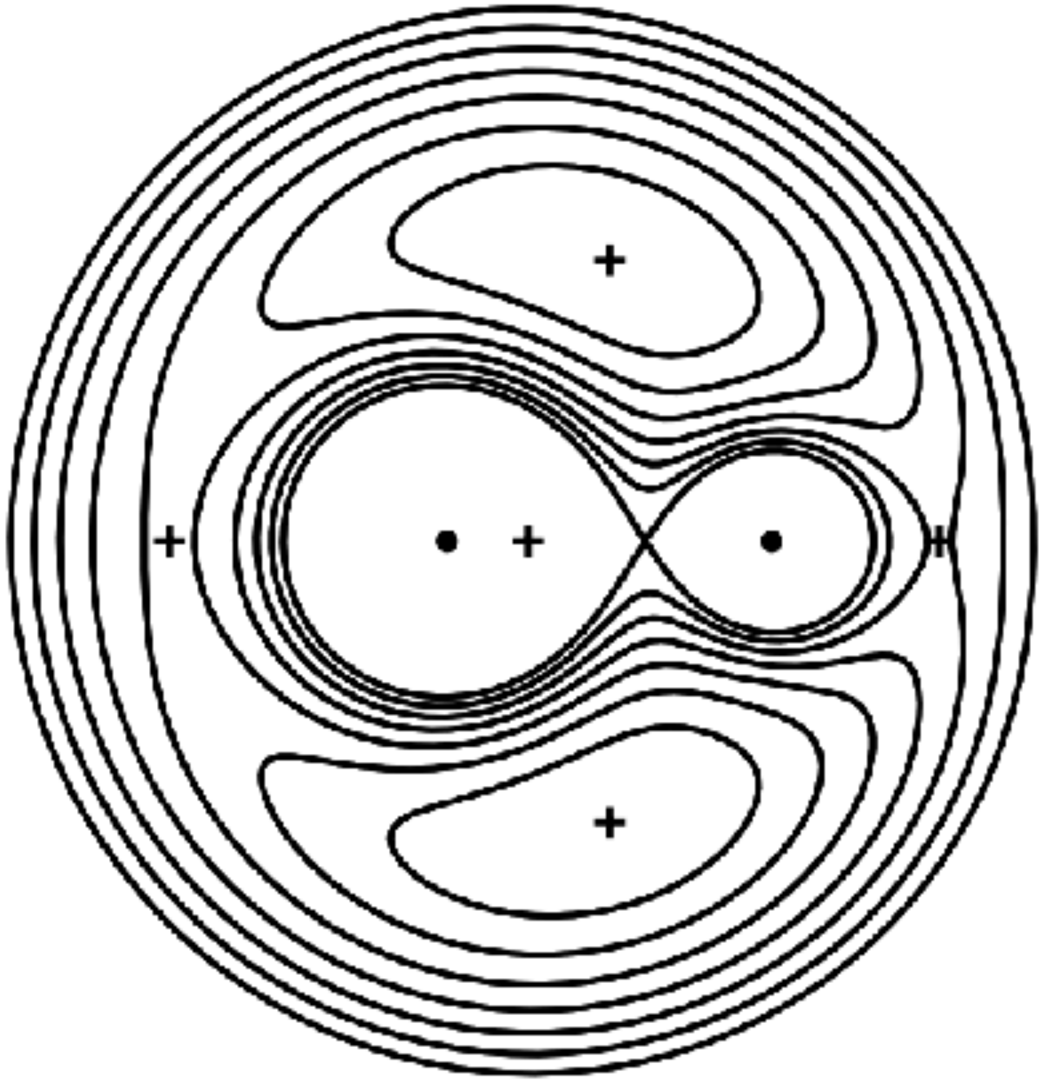
Roche lobe

Effective Roche lobe radius

Equipotential surfaces

$R_{L,1}/a$

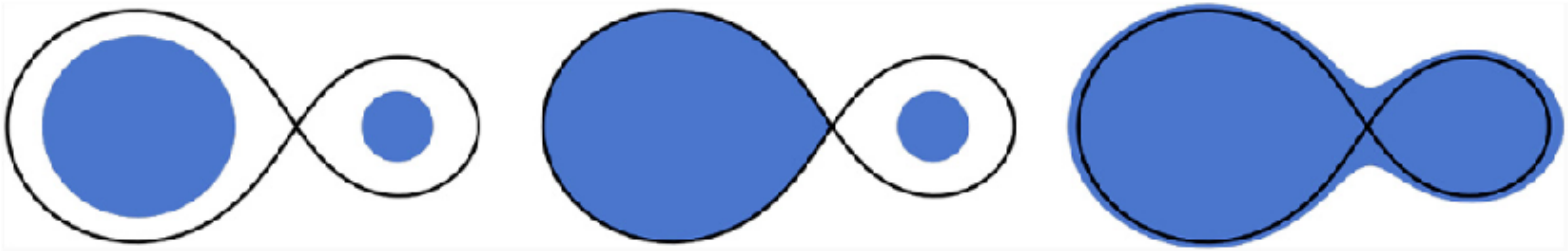




a : separation between the stars

$q = M_1/M_2$: mass ratio

Binary stars and their Roche lobes



Detached

Stars may be deformed but no mass exchange

Semi-detached

Stable mass transfer: Roche lobe overflow (RLO)

Possible formation of an accretion disk

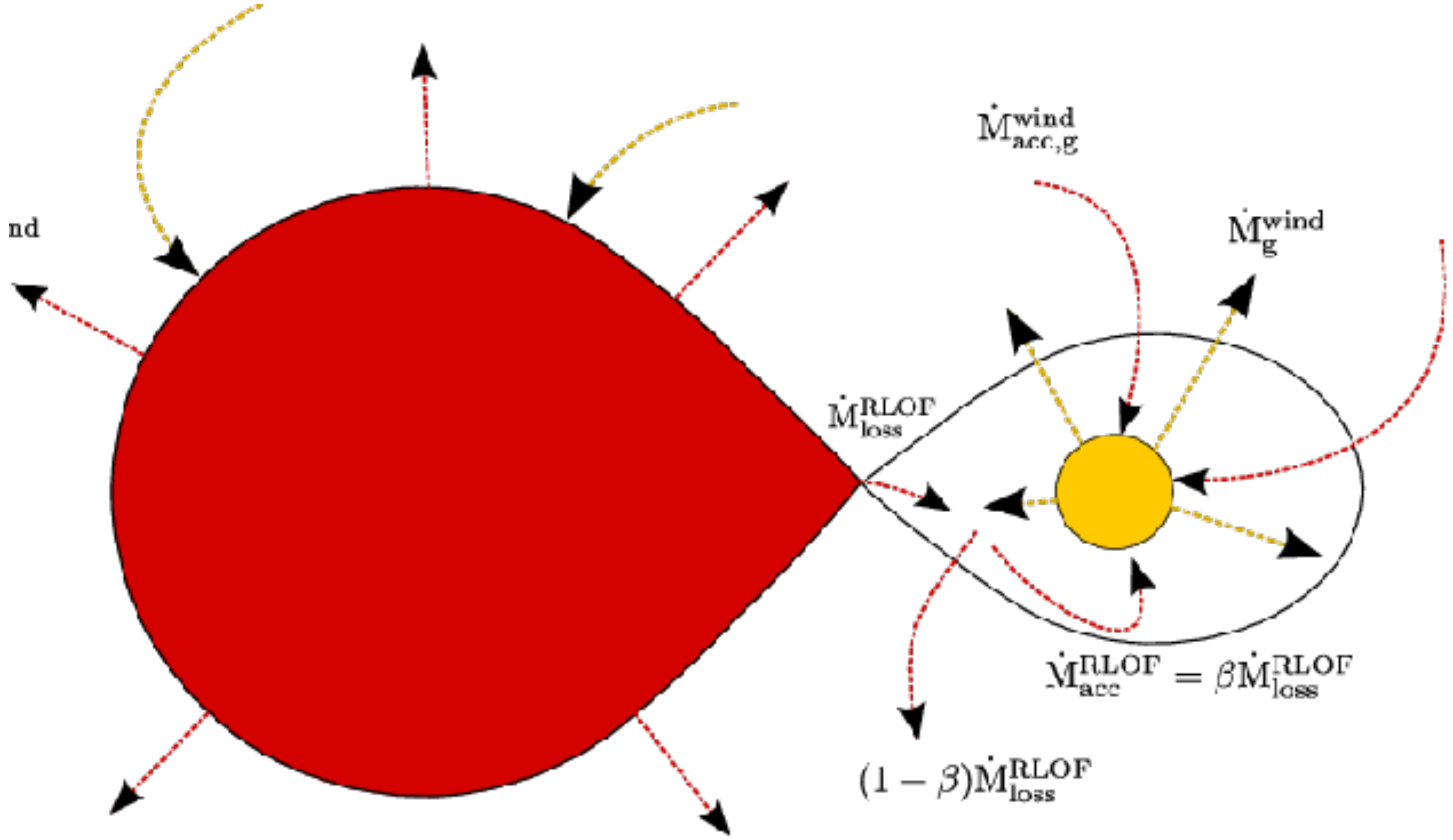
Contact binary

Unstable mass transfer?

Merger?

Stable mass transfer

- Accretion of (some fraction) of wind from the companion
- Roche Lobe Overflow:
    - Binary orbit shrinks
    - Radius of one of the star increases



Siess et al. (2013)

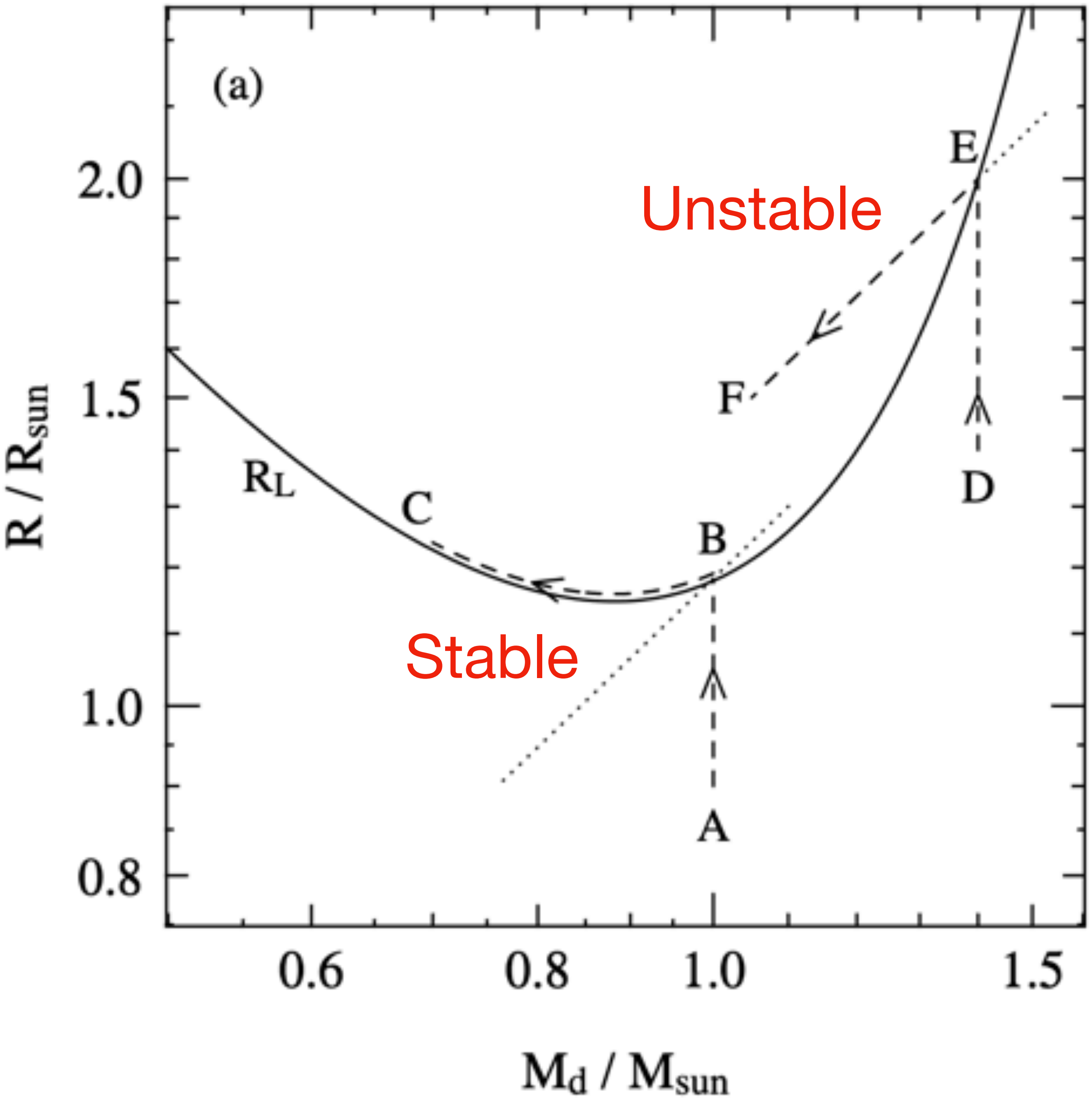
Stability of mass transfer

- Derivative of donor radius to its mass: $\zeta_* = \frac{dR}{dM}$
- Derivative of donor Roche Lobe radius to its mass: $\zeta_L = \frac{dR_L}{dM}$
- If $\zeta_* > \zeta_L$: mass transfer is stable (Roche Lobe radius increases faster than stellar radius)

Stability of mass transfer

$M_{\rm tot} = 2\,M_\odot$

Total mass and angular momentum are conserved



Donor star fills its Roche lobe:

Example 1: A—> B

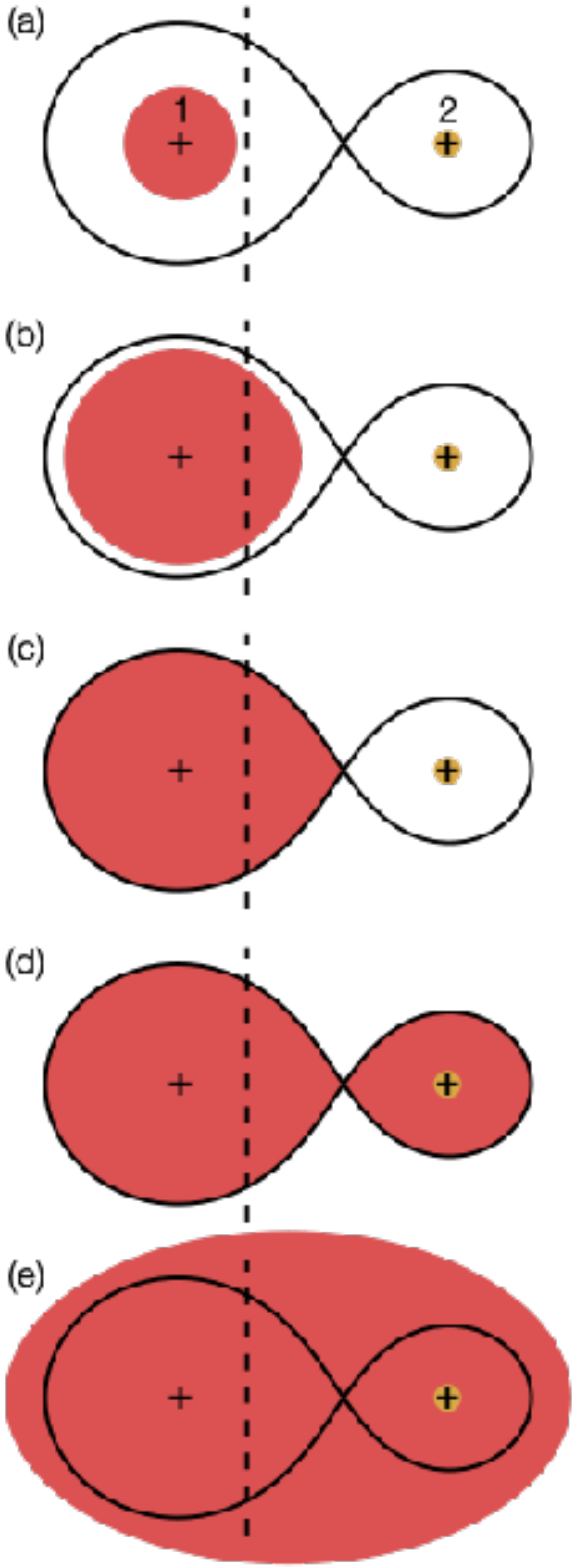
Example 2: D —> E

Stellar radius reacts to mass loss

Example 1: $R < R_L$

Example 2: $R > R_L$

Common envelope



- Necessary to explain the existence of compact binaries!
- Merger time due to emission of gravitational waves:

$$t_{\rm merg} = \frac{5}{256}\frac{c^5}{G^3}\frac{a_{\rm i}^4}{M_1 M_2 (M_1+M_2)}$$

- Initial separation needed to merge within a Hubble time:

$$a_{\rm i} \simeq 4.8 R_\odot \left(\frac{M}{1.4\ M_\odot}\right)^{3/4} \left(\frac{t_{\rm merg}}{13.8\ {\rm Gyr}}\right)^{1/4}$$

- But radius of a red supergiant: $10^2\,R_\odot - 10^3\,R_\odot$

Simple common envelope model

- Envelope binding energy

$$E_{\text{bind}} = -\int_{M_c}^{M} \frac{Gm}{R(m)} dm = -\frac{GM_{\text{don}}M_{\text{env}}}{\boxed{\lambda} R_{\text{don}}}$$

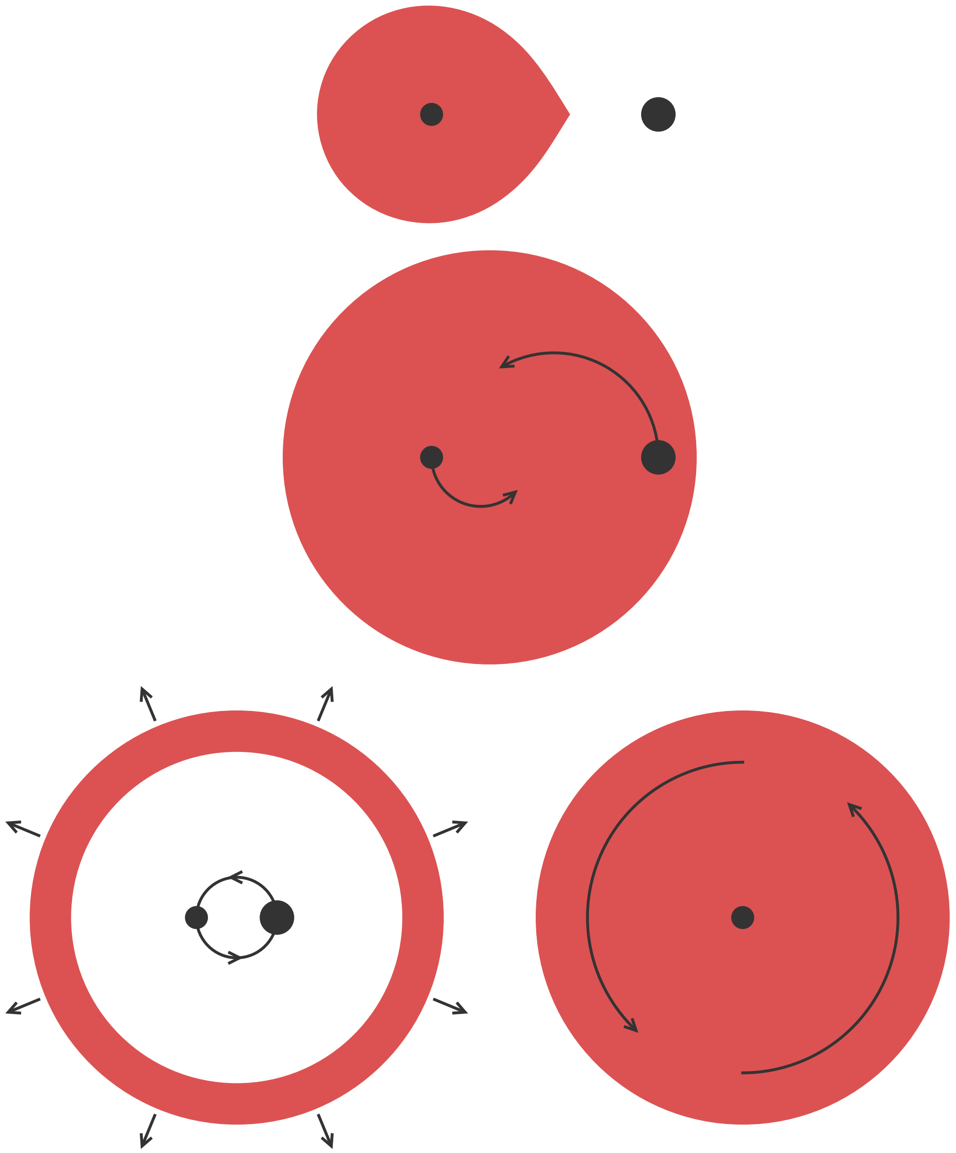
- Orbital energy: $E_{\text{orb}} = -\frac{1}{2}\frac{GM_{\text{don}}M_{\text{comp}}}{a}$

- Some fraction is used to unbind the envelope

$$\Delta E_{\text{bind}} = \boxed{\alpha_{\text{CE}}} \Delta E_{\text{orb}}$$

- Assuming envelope is ejected:

$$E_{\text{orb,f}} = E_{\text{orb,i}} + \frac{E_{\text{bind,i}}}{\alpha_{\text{CE}}}$$

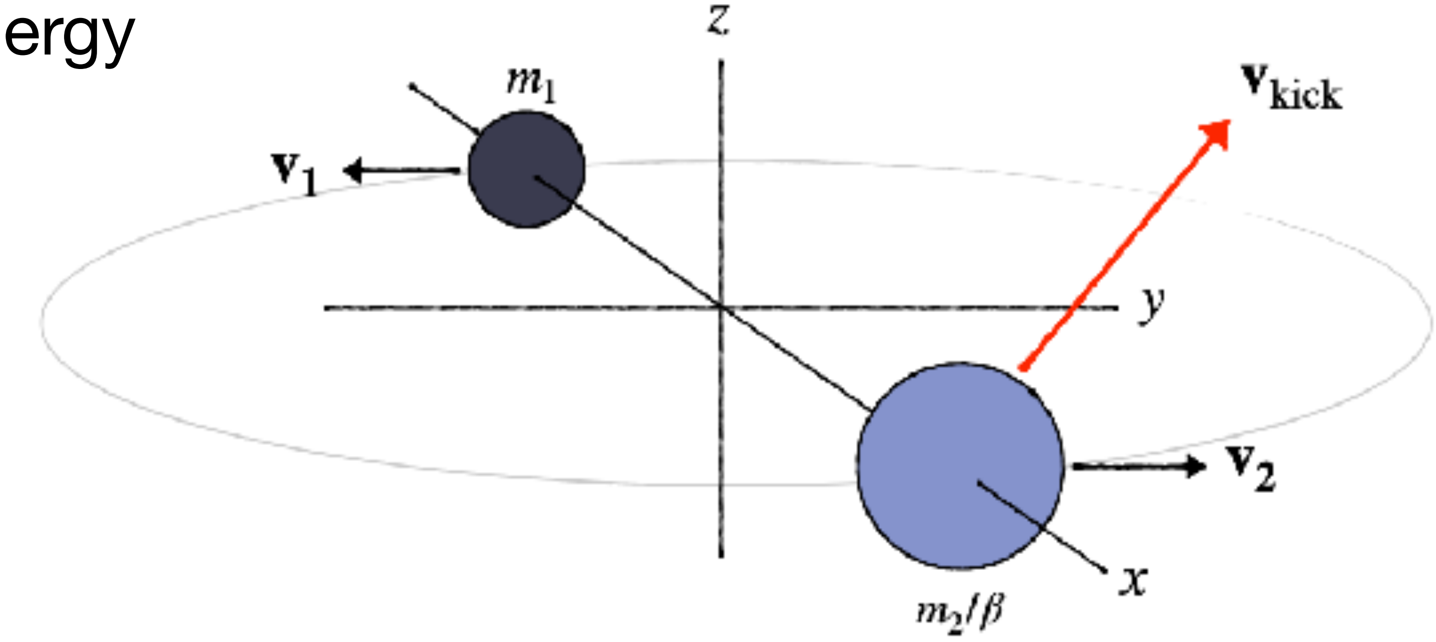


Supernova natal kicks

- Assume Maxwellian distribution of kick velocity after each supernova

$$P(v_{\rm k}) = \sqrt{\frac{2}{\pi}}\frac{v_{\rm k}^2}{\sigma_{\rm k}^3}\exp\left(-\frac{v_{\rm k}^2}{2\sigma_{\rm k}^2}\right)$$

- Velocity dispersion: $\sigma_k \sim \mathcal{O}(100)$
- Reduced kicks due to fallback
- Reduced kicks if low explosion energy
- Random kick direction

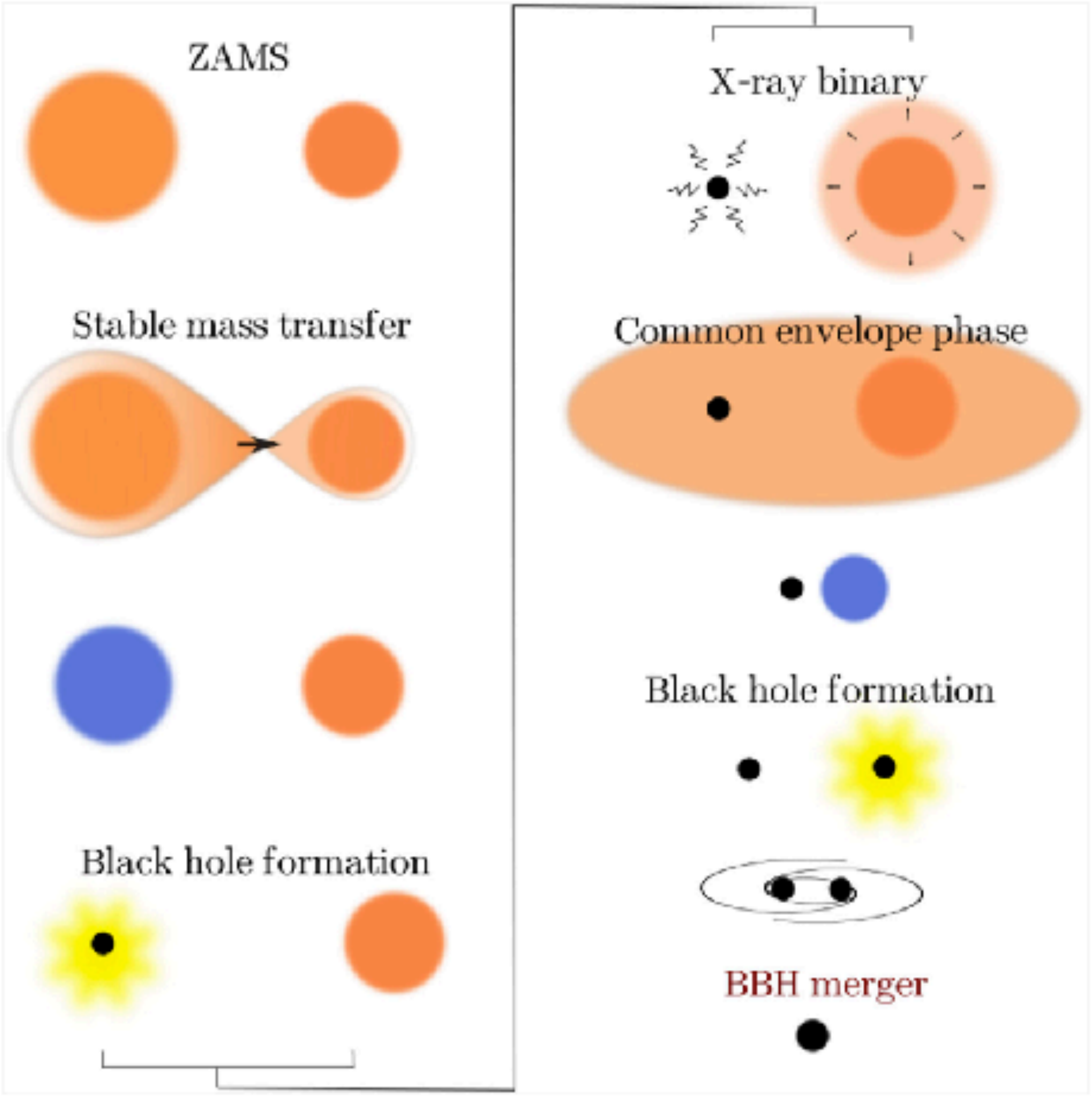


[Callister et al., arXiv:2011.09570]

Formation of compact binaries

'Standard' scenario:

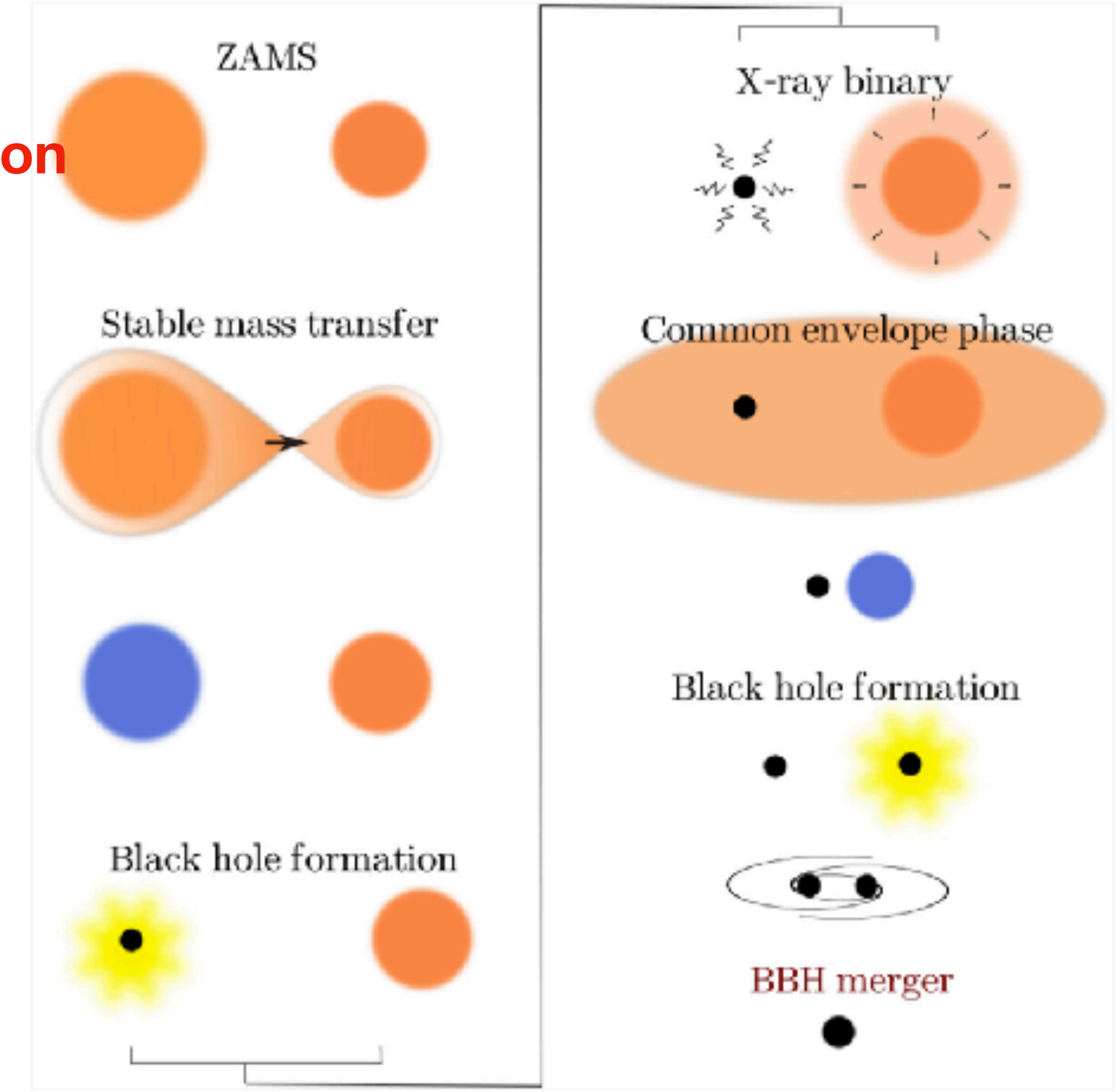
- Massive stellar binary
- First supernova (possible birth kick)
- NS/BH + massive star
- Massive star expands, Roche lobe overflow
- Common envelope
- NS/BH + He star
  - Possible: second Roche lobe overflow
- Second supernova (possible birth kick)
- NS/BH + NS/BH
- Merger



Binary neutron stars: formation

‘Standard' scenario:

- Massive stellar binary **+single stellar evolution**
- First supernova (possible birth **kick**)
- NS/BH + massive star
- Massive star expands, **Roche lobe overflow**
- **Common envelope**
- NS/BH + He star
  - Possible: second Roche lobe overflow
- Second supernova (possible birth **kick**)
- NS/BH + NS/BH
- Merger



Modelling binary stars

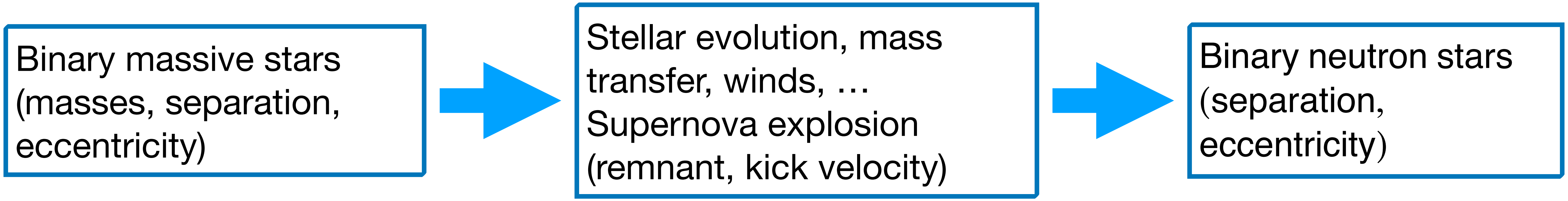
- Full hydrodynamical simulations (1D/2D/3D)

Modules for Experiments in Stellar Astrophysics

MESA

mesastar.org

- Semi-analytic population synthesis codes:

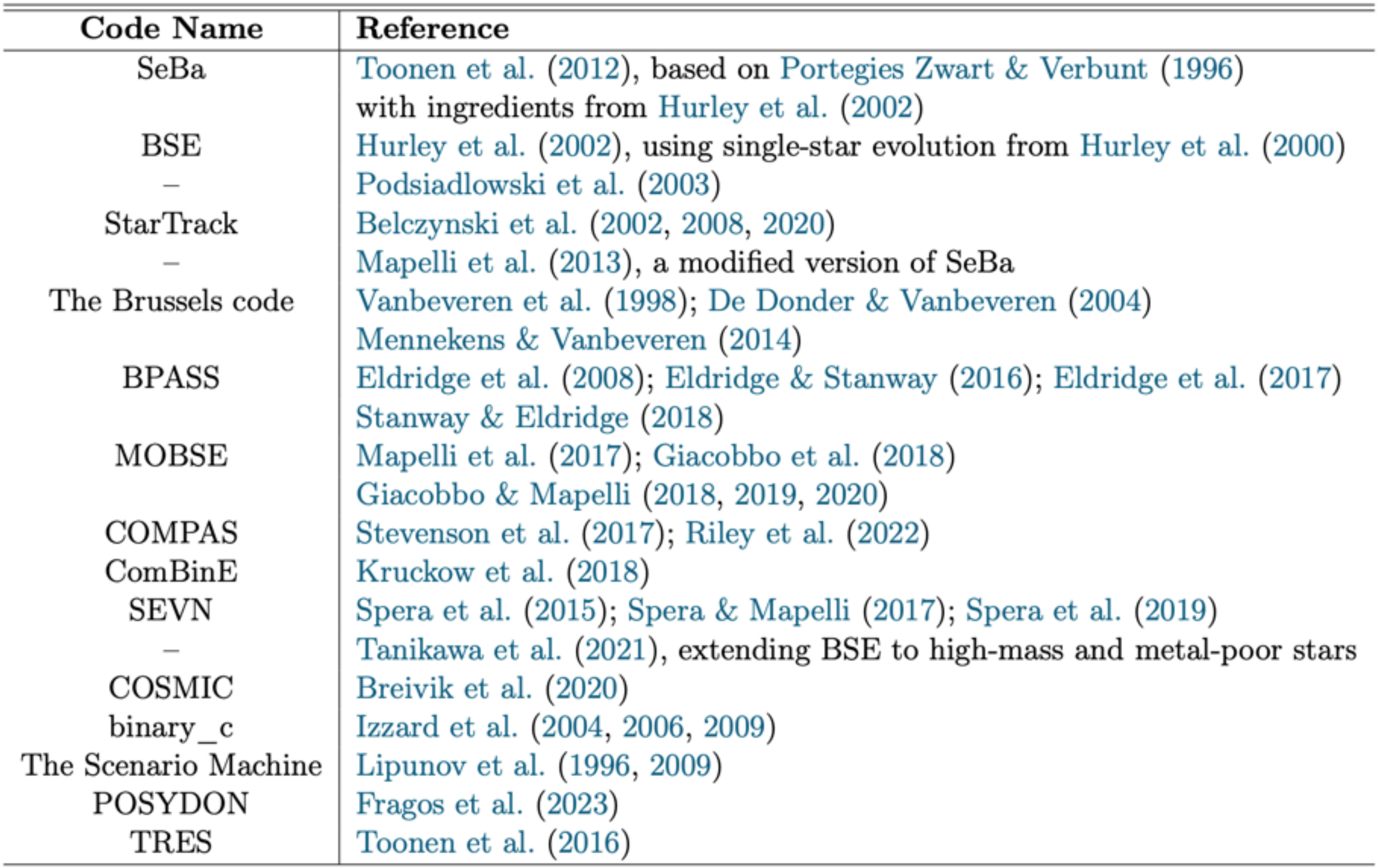


Binary massive stars (masses, separation, eccentricity)

Stellar evolution, mass transfer, winds, …
Supernova explosion (remnant, kick velocity)

Binary neutron stars (separation, eccentricity)

Stellar population synthesis codes



Code Name	Reference
SeBa	Toonen et al. (2012), based on Portegies Zwart & Verbunt (1996) with ingredients from Hurley et al. (2002)
BSE	Hurley et al. (2002), using single-star evolution from Hurley et al. (2000)
–	Podsiadlowski et al. (2003)
StarTrack	Belczynski et al. (2002, 2008, 2020)
–	Mapelli et al. (2013), a modified version of SeBa
The Brussels code	Vanbeveren et al. (1998); De Donder & Vanbeveren (2004) Mennekens & Vanbeveren (2014)
BPASS	Eldridge et al. (2008); Eldridge & Stanway (2016); Eldridge et al. (2017) Stanway & Eldridge (2018)
MOBSE	Mapelli et al. (2017); Giacobbo et al. (2018) Giacobbo & Mapelli (2018, 2019, 2020)
COMPAS	Stevenson et al. (2017); Riley et al. (2022)
ComBinE	Kruckow et al. (2018)
SEVN	Spera et al. (2015); Spera & Mapelli (2017); Spera et al. (2019)
–	Tanikawa et al. (2021), extending BSE to high-mass and metal-poor stars
COSMIC	Breivik et al. (2020)
binary_c	Izzard et al. (2004, 2006, 2009)
The Scenario Machine	Lipunov et al. (1996, 2009)
POSYDON	Fragos et al. (2023)
TRES	Toonen et al. (2016)

[Credit for compilation: Clément Pellouin]

Stellar population synthesis codes

Challenges:

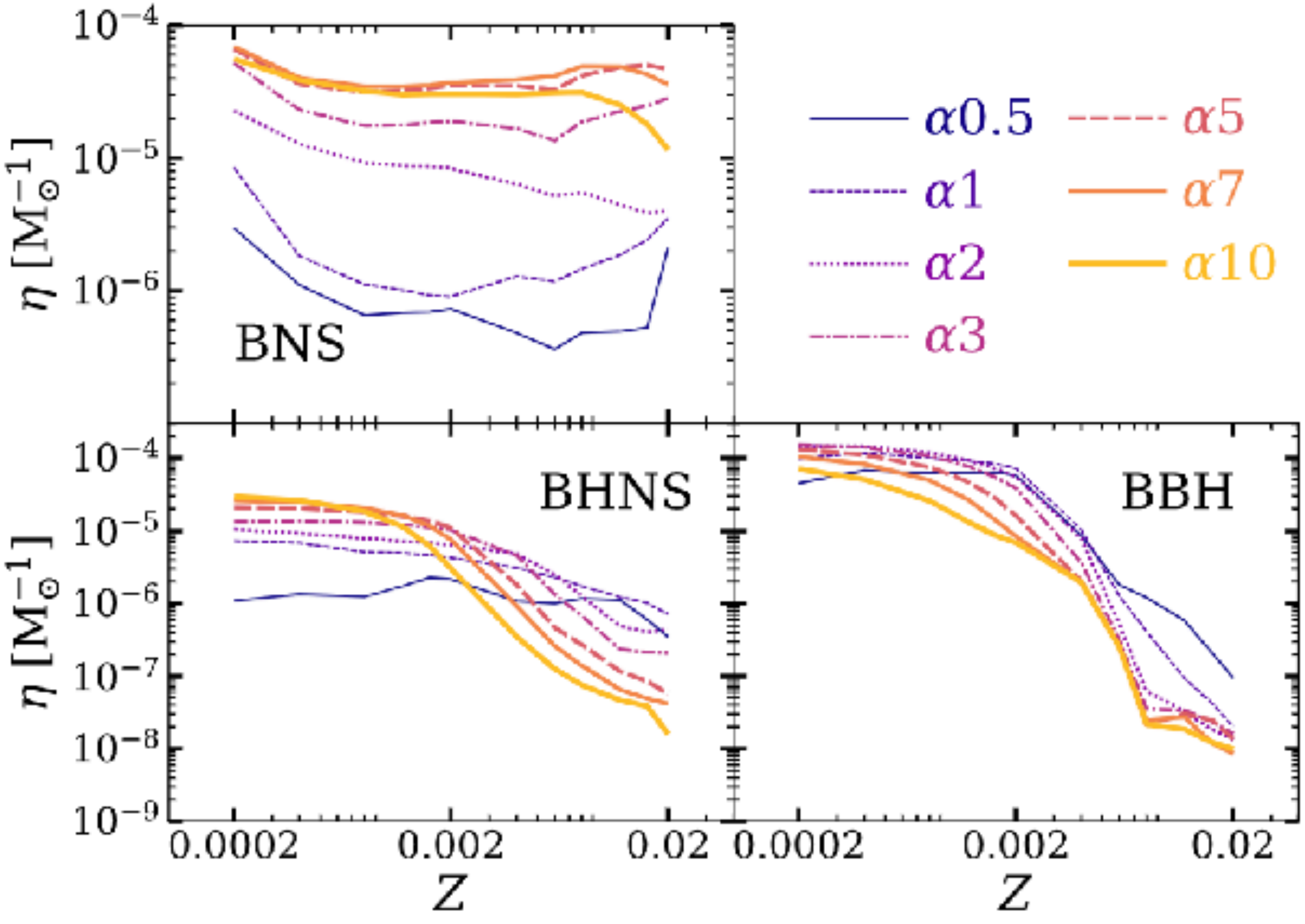
- Uncertainties in single stellar evolution (ex: evolution of stellar radius)
- Large uncertainties in some key processes (ex: supernova explosion)
- Over-simplification of some complex processes (ex: common envelope)
- Very large number of parameters

Advantages:

- Large populations
- Observables: masses, redshifts, merger rates, …
- Can combine with galaxy models

Stellar population synthesis codes: some results

- Efficiency of compact binary formation is very small
- Difficult to form black hole binaries at high metallicity

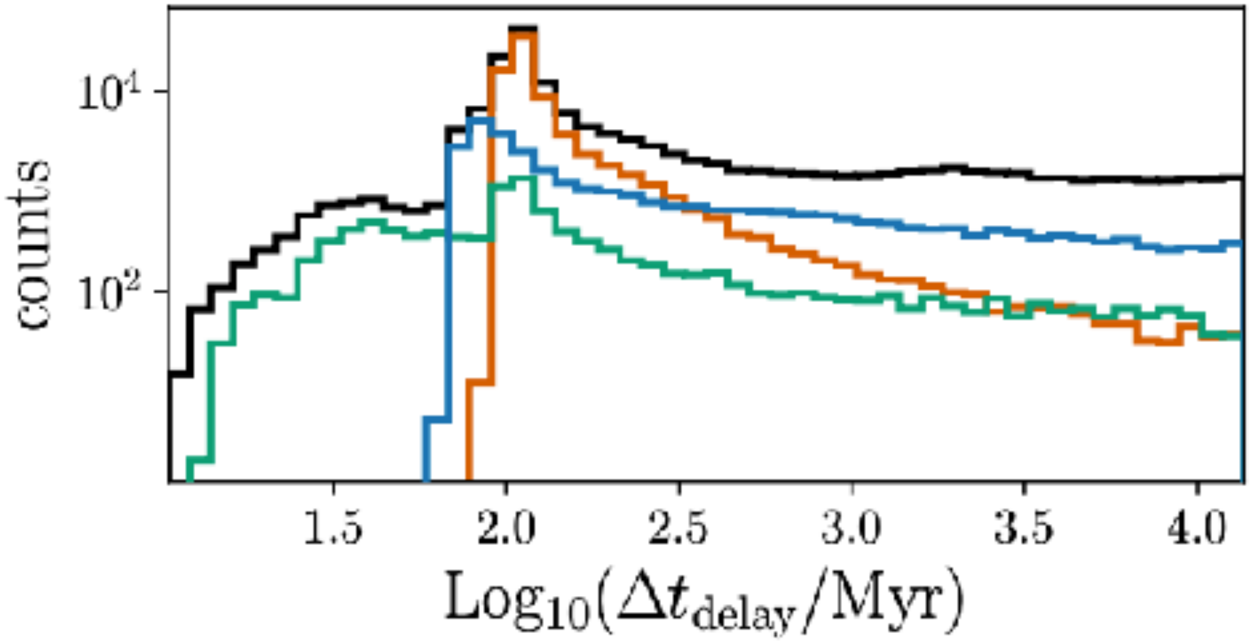


[Santoliquido et al., arXiv:2009.03911]

Stellar population synthesis codes: some results

- Long time delays between the birth of the binary star and the merger of the compact binary
- These time delays depend on the details of the formation channel

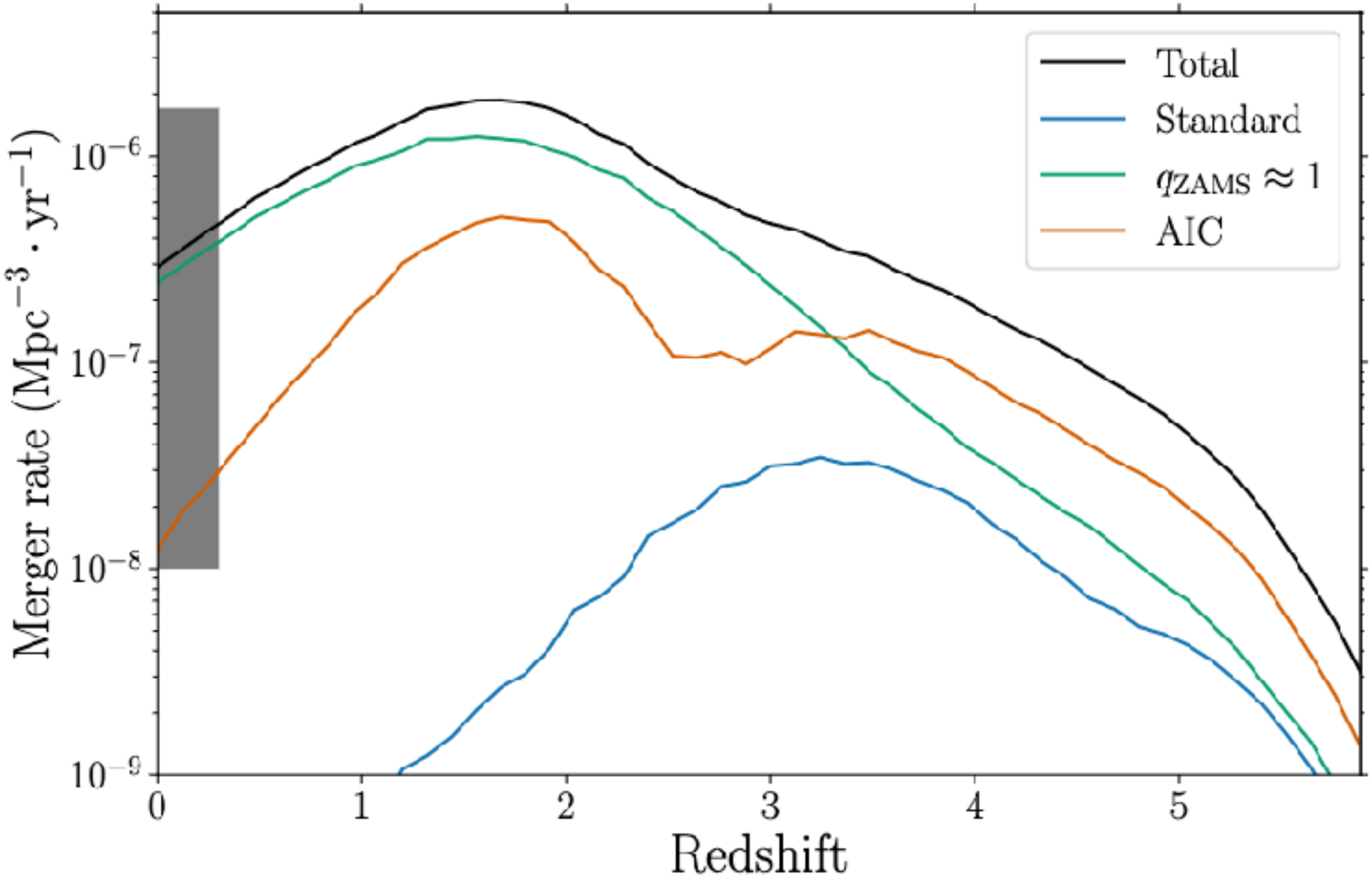
BNS, low-metallicity progenitors



[Pellouin et al., arXiv:2411.04563]

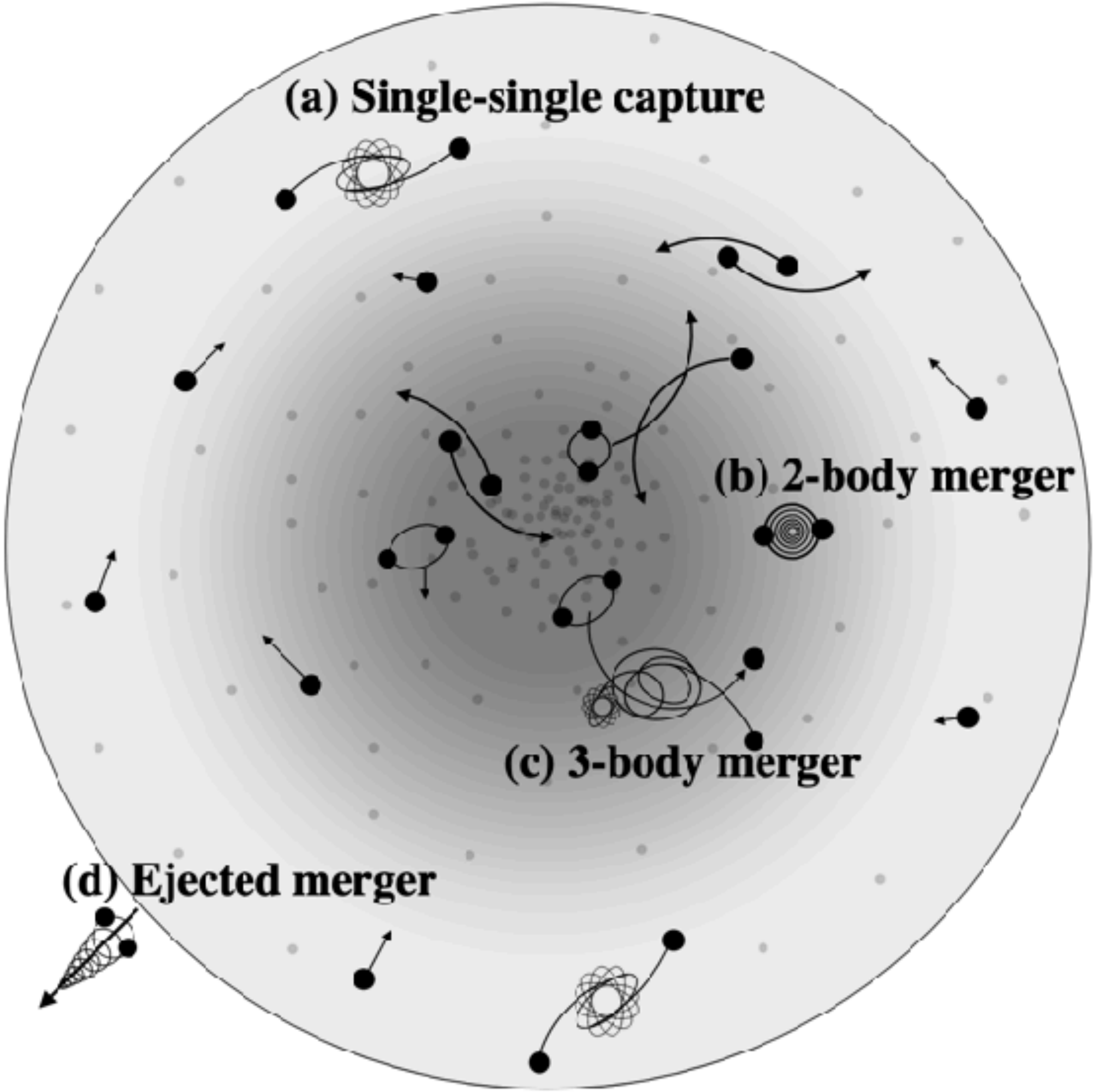
Stellar population synthesis codes: some results

- Models fit the observed merger rate… but there are a lot of parameters!



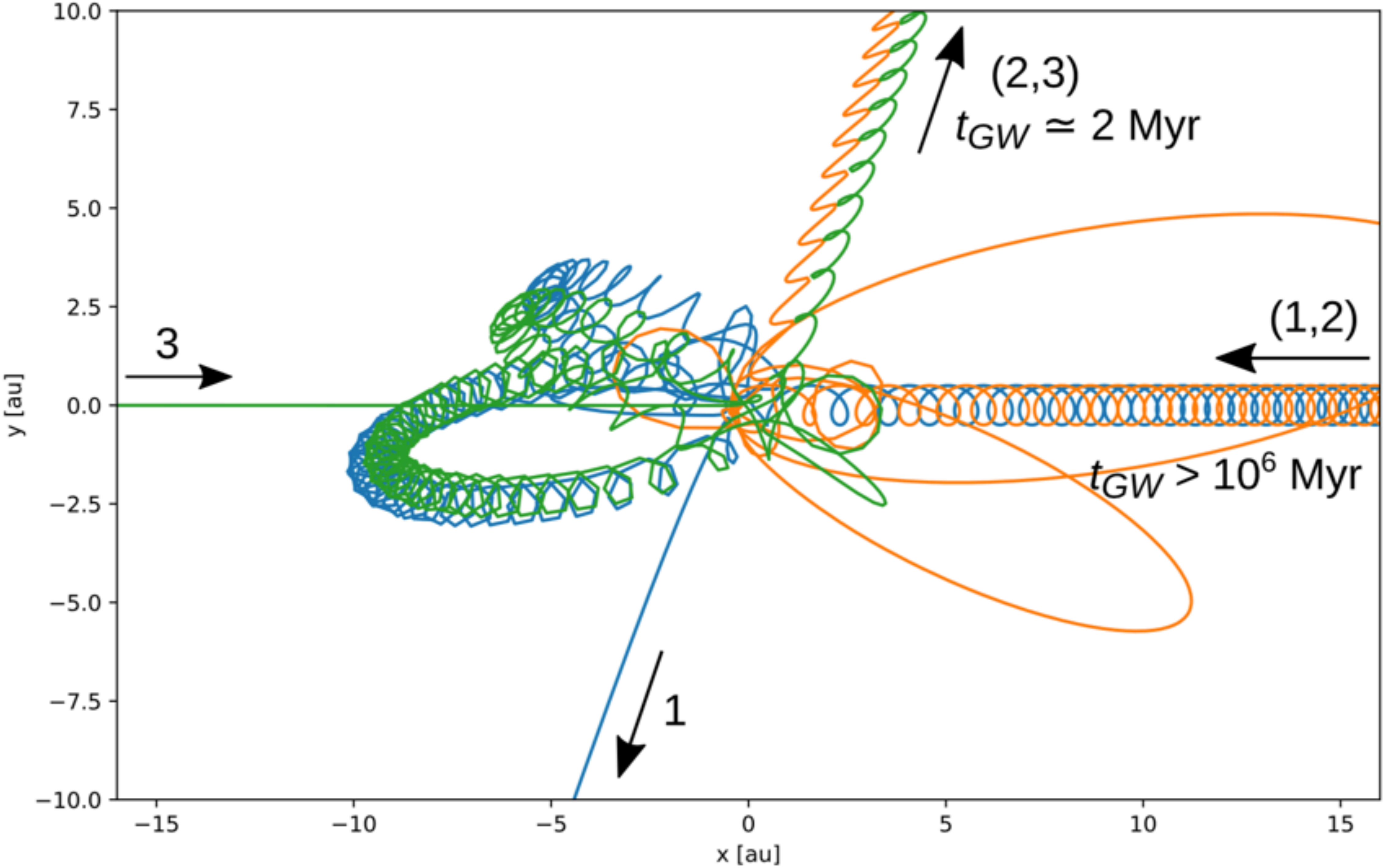
[Pellouin et al., arXiv:2411.04563]

Dynamical formation channel



[Samsing+2020]

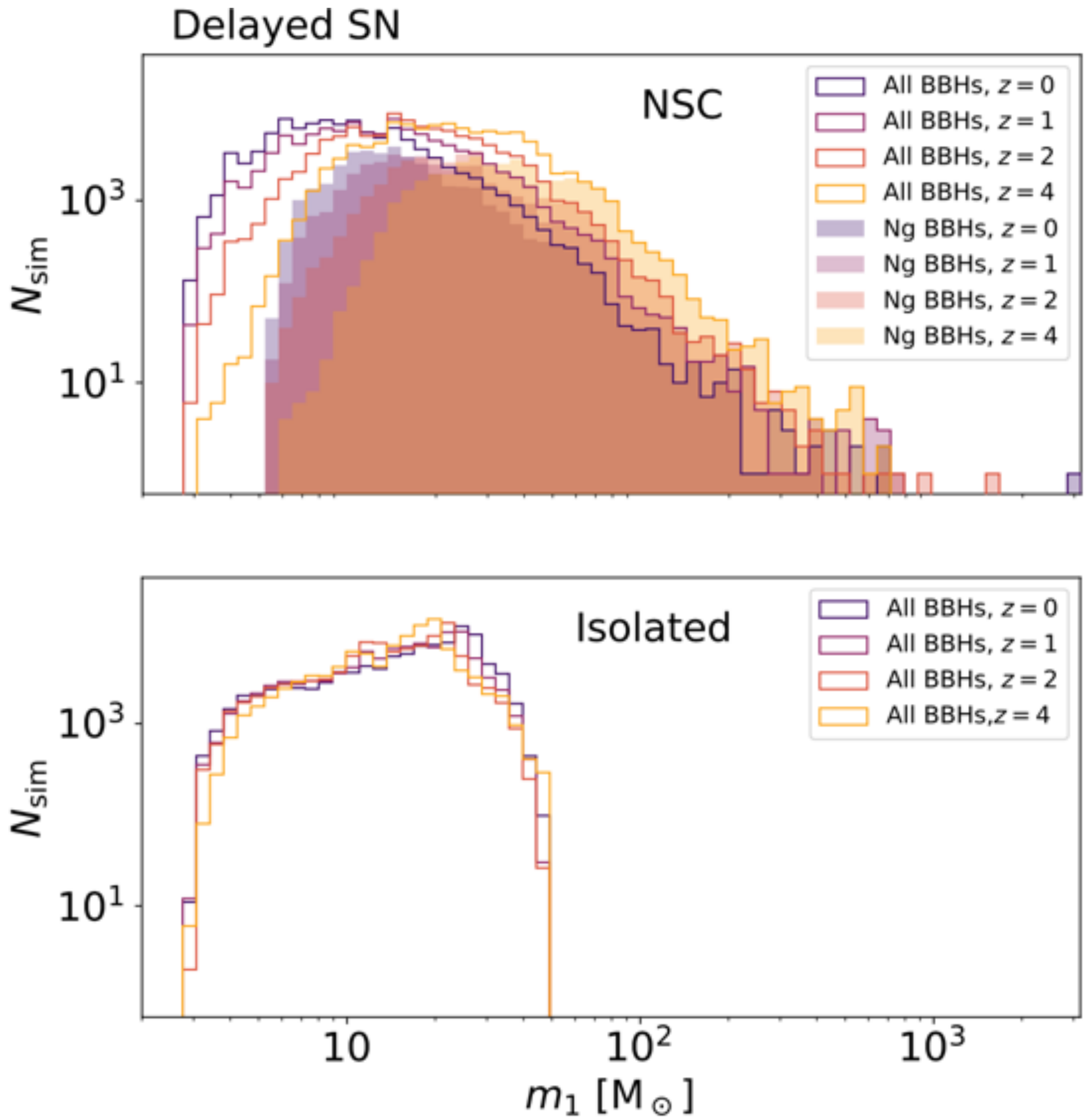
Dynamical formation channel



[Spera et al., arXiv:2206.15392]

Dynamical formation channel

- Allows to obtain black holes in the upper mass gap

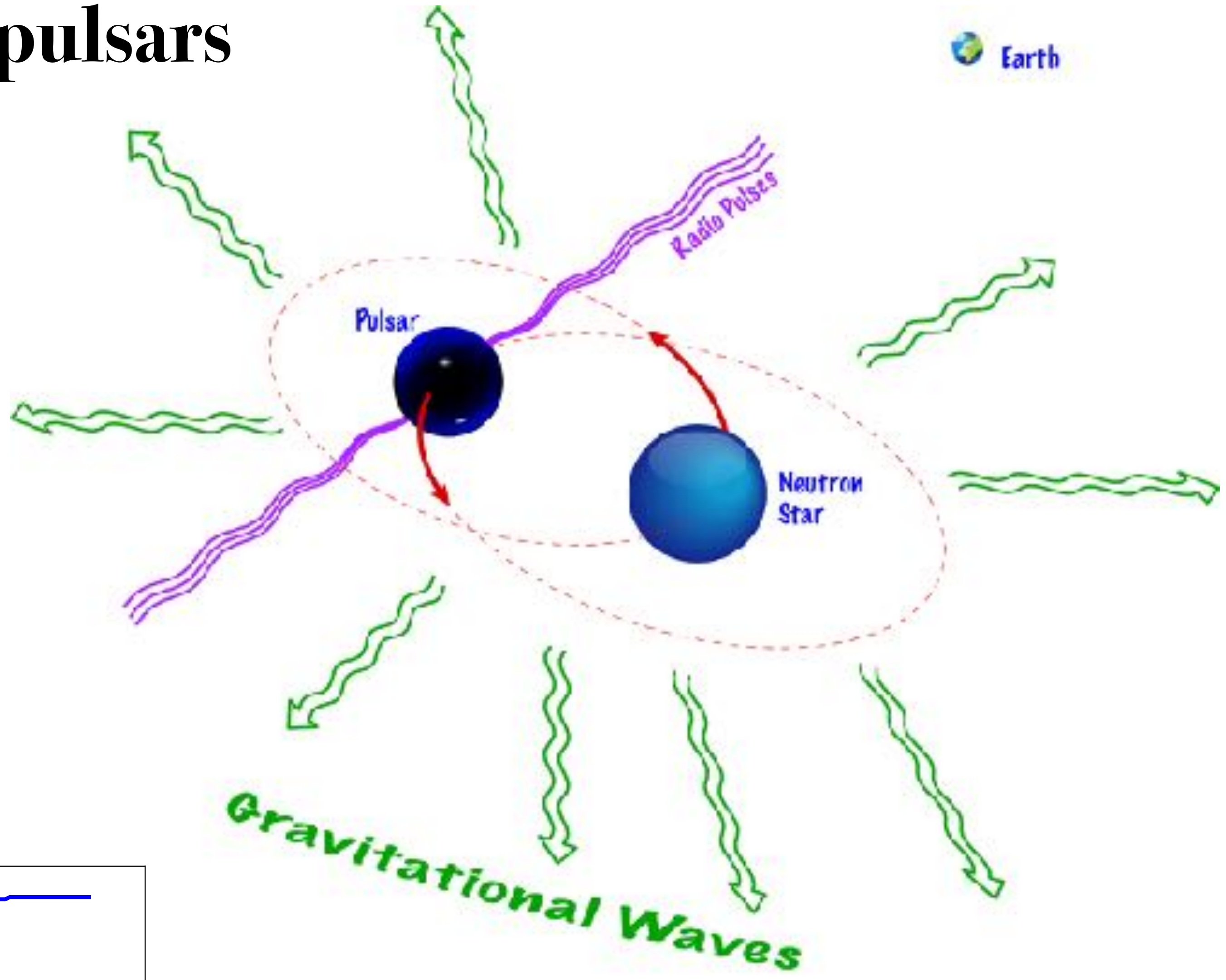


[Mapelli et al., arXiv:2109.06222]

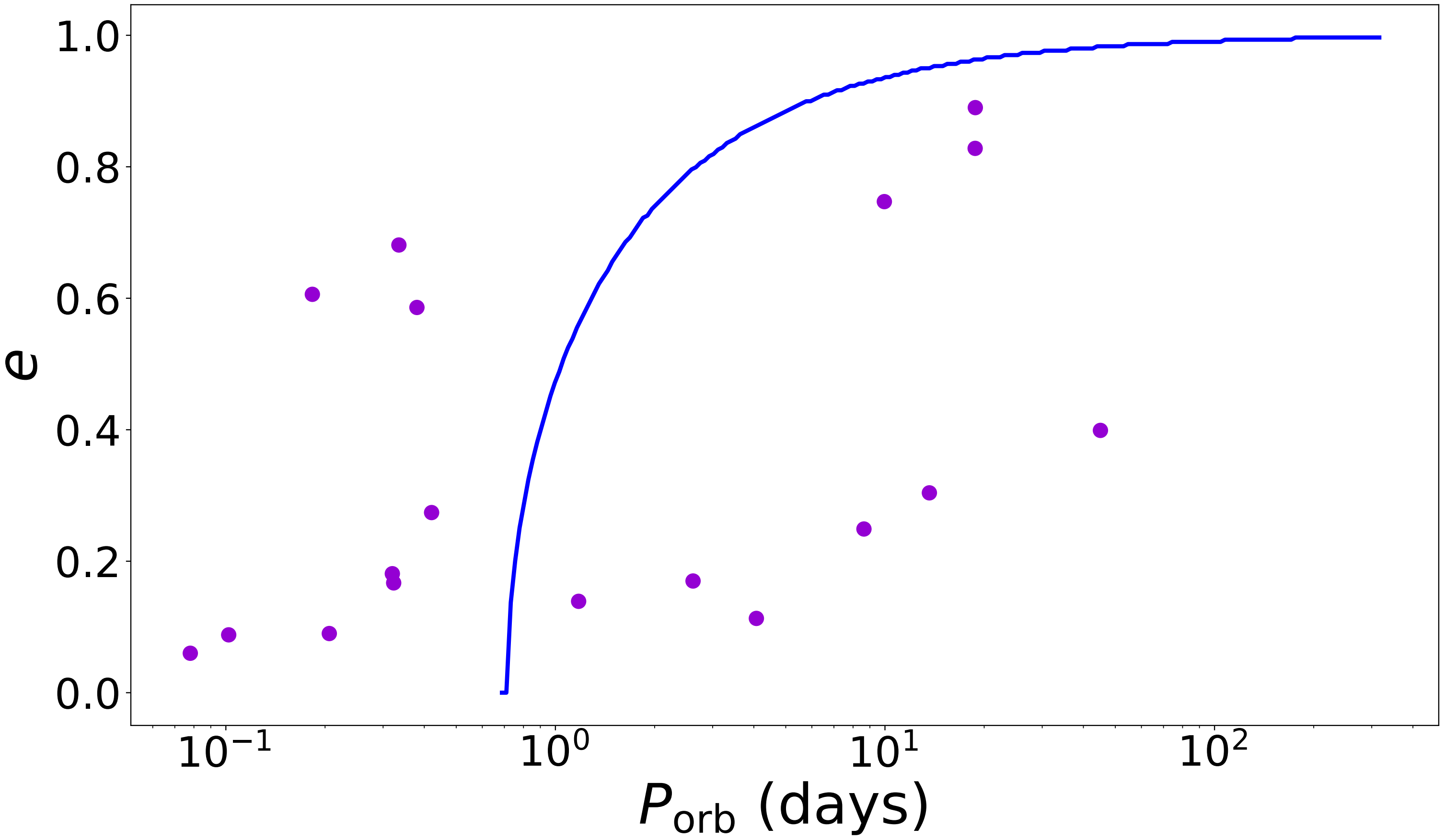
Binary massive stars and binary compact objects: observables

- Binary pulsars in the Galaxy (masses, orbital parameters)
- X-ray binaries (masses, orbital parameters)
- Gamma-ray bursts (rates, redshifts, location in host galaxies)
- Compact binary mergers: gravitational waves (masses, redshifts, merger rates…)

Binary pulsars



- About 3000 pulsars known in the Galaxy
- About 300 are in binary systems
- 18 confirmed binary neutron stars (BNS)



Time to coalescence due to emission of gravitational waves:

$$t_{GW} \propto a^4$$

Merger times: 10 Myr to 10^5 Gyr

Lecture plan

- Evolution of massive stars and formation of compact objects
- Evolution of binary massive stars and formation of binary compact objects
- Gravitational-wave astronomy
- Gravitational-wave observations of binary compact objects

Lecture plan

- **Gravitational-wave astronomy**
    - **Gravitational waves: theory**
    - **Binary compact object mergers**
    - **Observing gravitational waves**

Linearized theory

Solve Einstein's equations assuming a small perturbation of the flat spacetime

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

with

$$|h_{\mu\nu}| \ll 1$$

Linearized theory

Solve Einstein's equations assuming a small perturbation of the flat spacetime

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$ with $$|h_{\mu\nu}| \ll 1$$

Linearized equation in Lorenz gauge $\partial^\nu \bar{h}_{\mu\nu} = 0$. with $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$

$$\boxed{\left(-\frac{\partial^2}{c^2 \partial t^2} + \nabla^2\right) \bar{h}_{\alpha\beta} = -\frac{16\pi G}{c^4} T_{\alpha\beta}}$$

Wave equation

Outside the source

$$\left(-\frac{\partial^2}{c^2\partial t^2}+\nabla^2\right)\bar{h}_{\alpha\beta}=0$$

The perturbation travels with the speed of light

Plane wave solution

$$\boxed{\bar{h}_{\alpha\beta}=A_{\alpha\beta}\exp(ik_\gamma x^\gamma)}$$

Lorenz gauge

$$\bar{h}_{\mu\nu}{}^{,\nu}=0\rightarrow A_{\mu\nu}k^\nu=0$$

orthogonal to propagation vector

Any solution is a superposition of plane waves

How many degrees of freedom?

The metric perturbation can be decomposed into 4 scalars, 2 transverse vectors, and a transverse trace-free tensor

Take wavevector in the z direction

$$h_{\mu\nu} = \begin{pmatrix} h_{00} & h_{01} & h_{02} & h_{03} \\ h_{10} & h_{11} & h_{12} & h_{13} \\ h_{20} & h_{21} & h_{22} & h_{23} \\ h_{30} & h_{31} & h_{32} & h_{33} \end{pmatrix}$$

h_{00} is a scalar under spatial rotations

1 d.o.f = scalar

h_{0i} is a 3-vector

$$\vec{h}_{0i} = \vec{\nabla}\Phi + \vec{\nabla} \times \vec{V}$$

3 d.o.f = divergence + transverse vector

h_{ij} contains trace + scalar $\partial^i \partial^j h_{ij}$ + transverse vector + traceless transverse tensor

6 d.o.f = divergence + trace + transverse vector + TT tensor

Polarizations (most general case!)

The metric perturbation can be decomposed into 4 scalars, 2 transverse vectors, and a transverse trace-free tensor

But only 2 scalar, 1 transverse vector and the TT tensor are invariant to coordinate transformations -> **6 d.o.f.**

$$h_+, h_\times, h_b, h_l, h_x, h_y$$

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_b + h_+ & h_\times & h_x \\ 0 & h_\times & h_b - h_+ & h_y \\ 0 & h_x & h_y & h_l \end{pmatrix}$$

Polarizations (most general case!)

The metric perturbation can be decomposed into 4 scalars, 2 transverse vectors, and a transverse trace-free tensor

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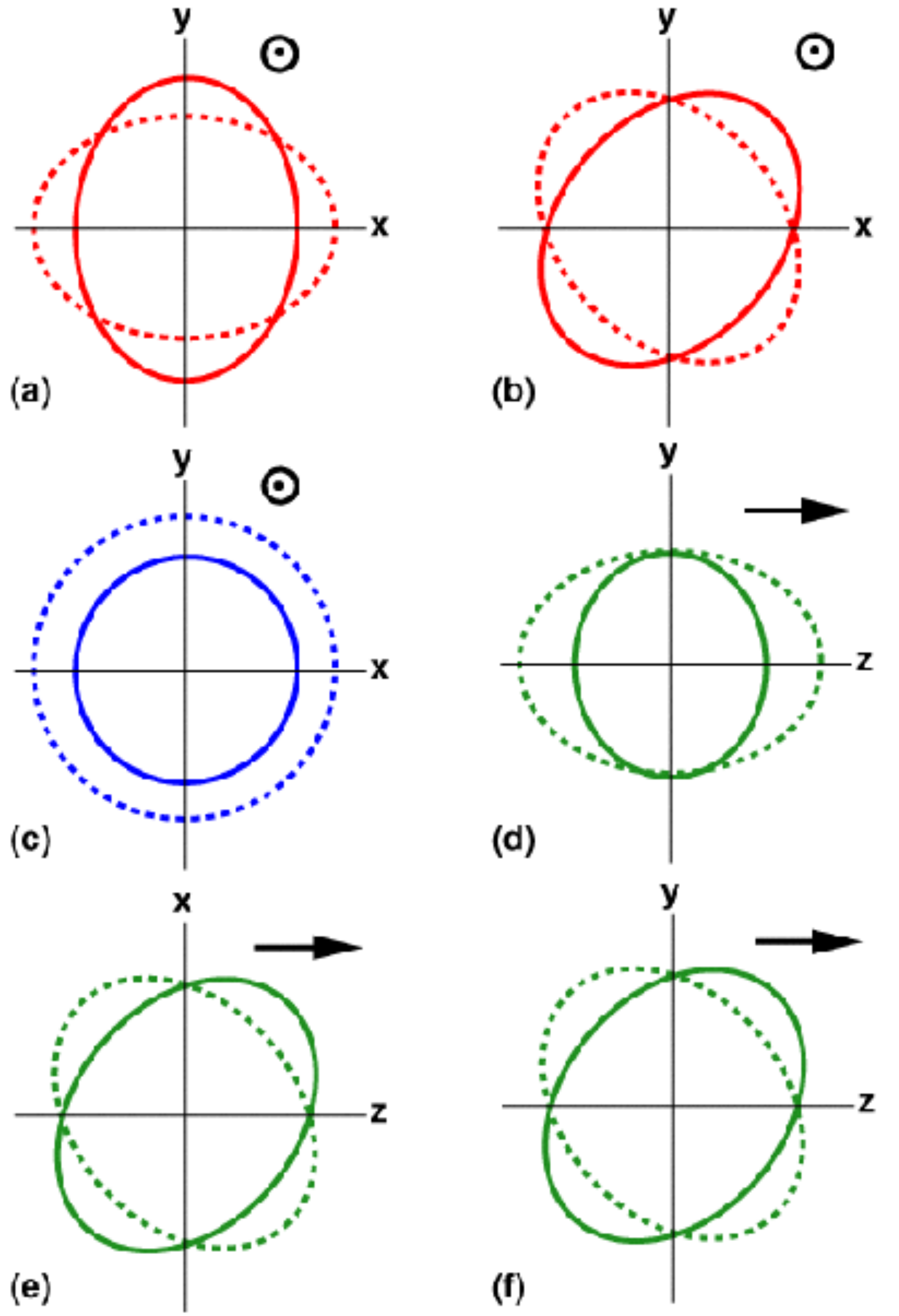
Gravitational–Wave Polarization



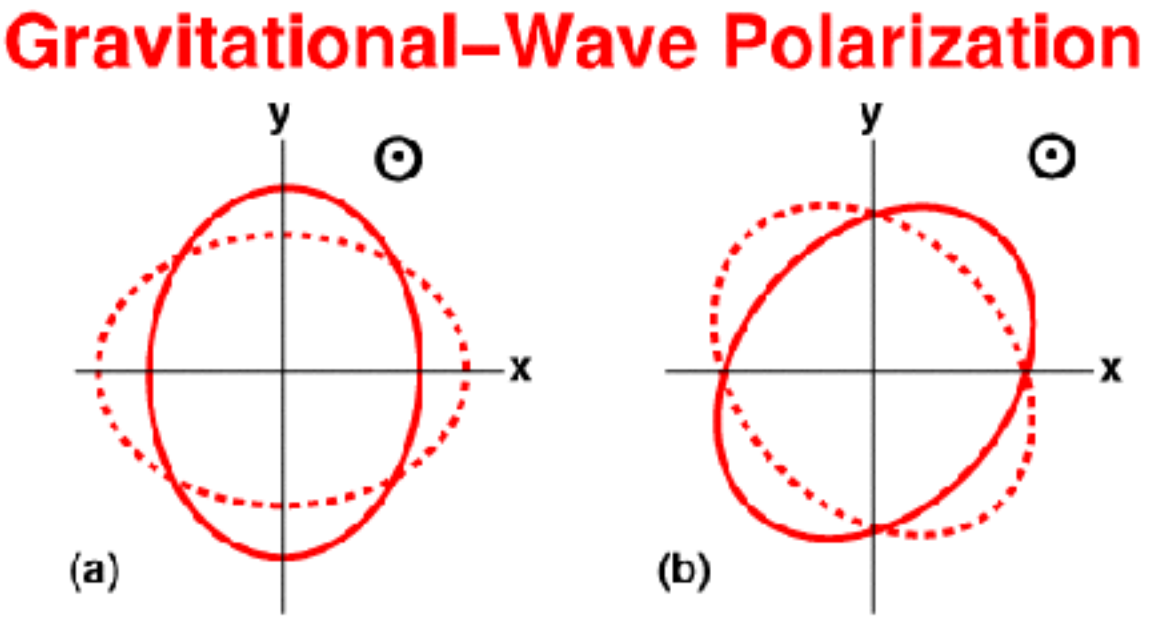
[C. Will, in Living Reviews in Relativity]

Polarizations in GR

In GR, out of the 6 remaining Einstein equations, 4 are constraint equations (no second-order time derivatives)

Only 2 equations are evolution equations -> 2 d.o.f. $h_+, h_\times$

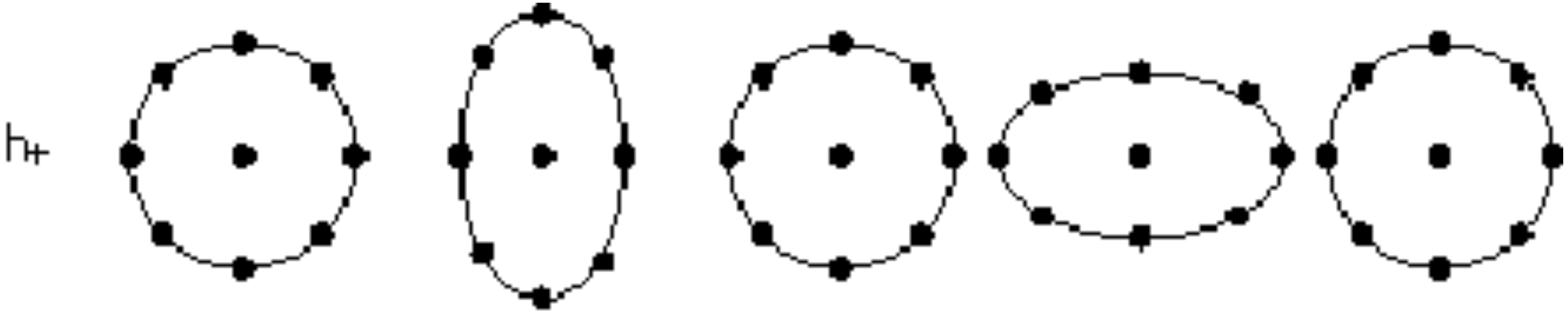
$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



Plus polarization

$$h_{\mu\nu}(t - z/c) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & 0 & 0 \\ 0 & 0 & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \cos\left(\omega(t - z/c)\right)$$

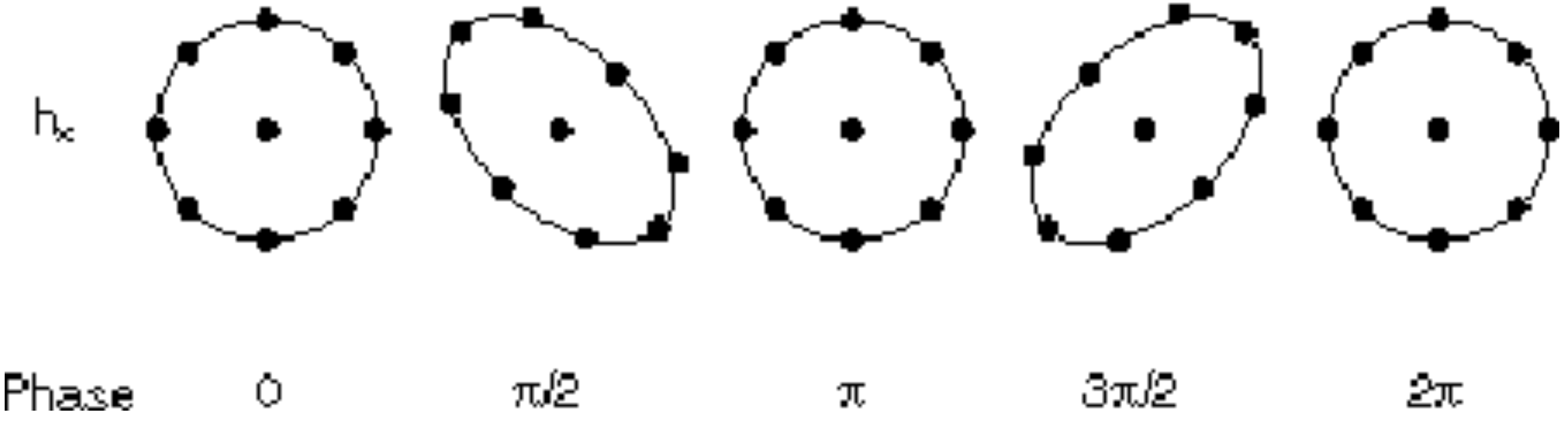
$$ds^2 = -c^2dt^2 + dz^2 + \left(1 + h_+ \cos[\omega(t - z/c)]\right) dx^2 + \left(1 - h_+ \cos[\omega(t - z/c)]\right) dy^2$$

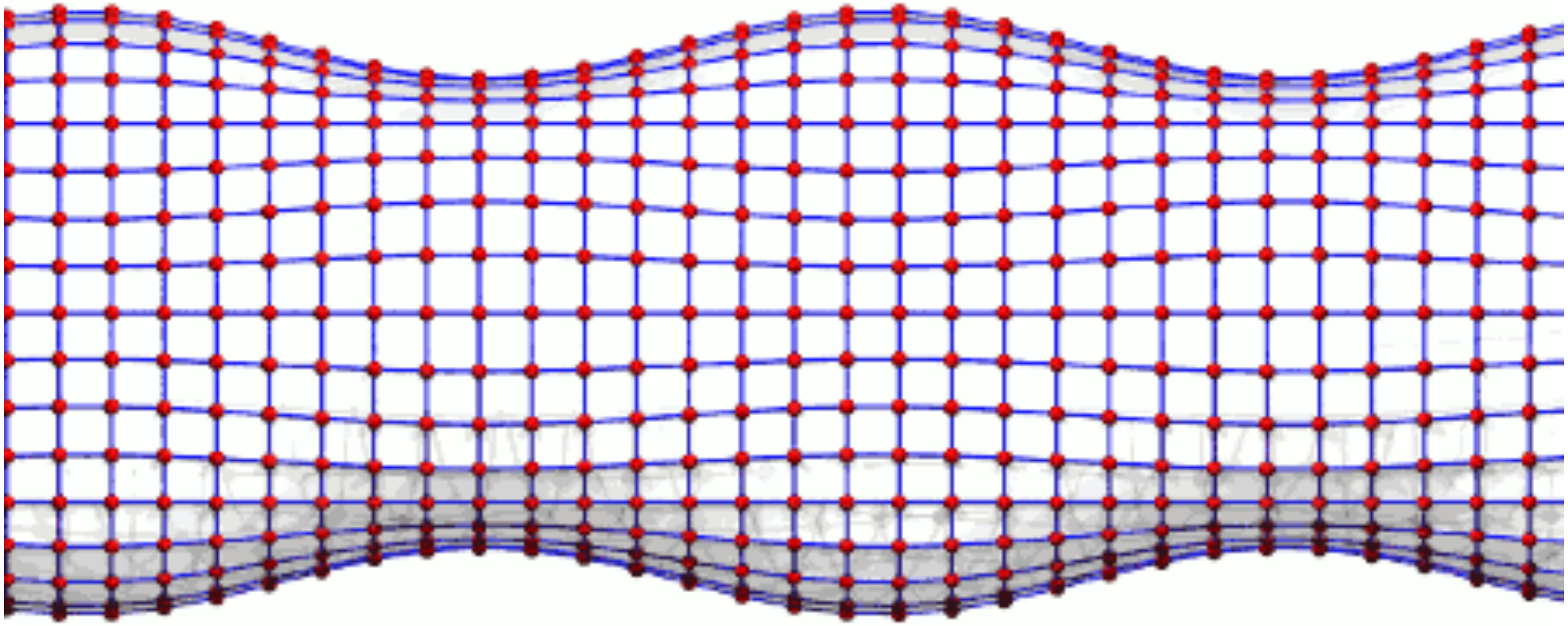


Cross polarization

$$h_{\mu\nu}(t - z/c) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & h_\times & 0 \\ 0 & h_\times & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \cos\left(\omega(t - z/c)\right)$$

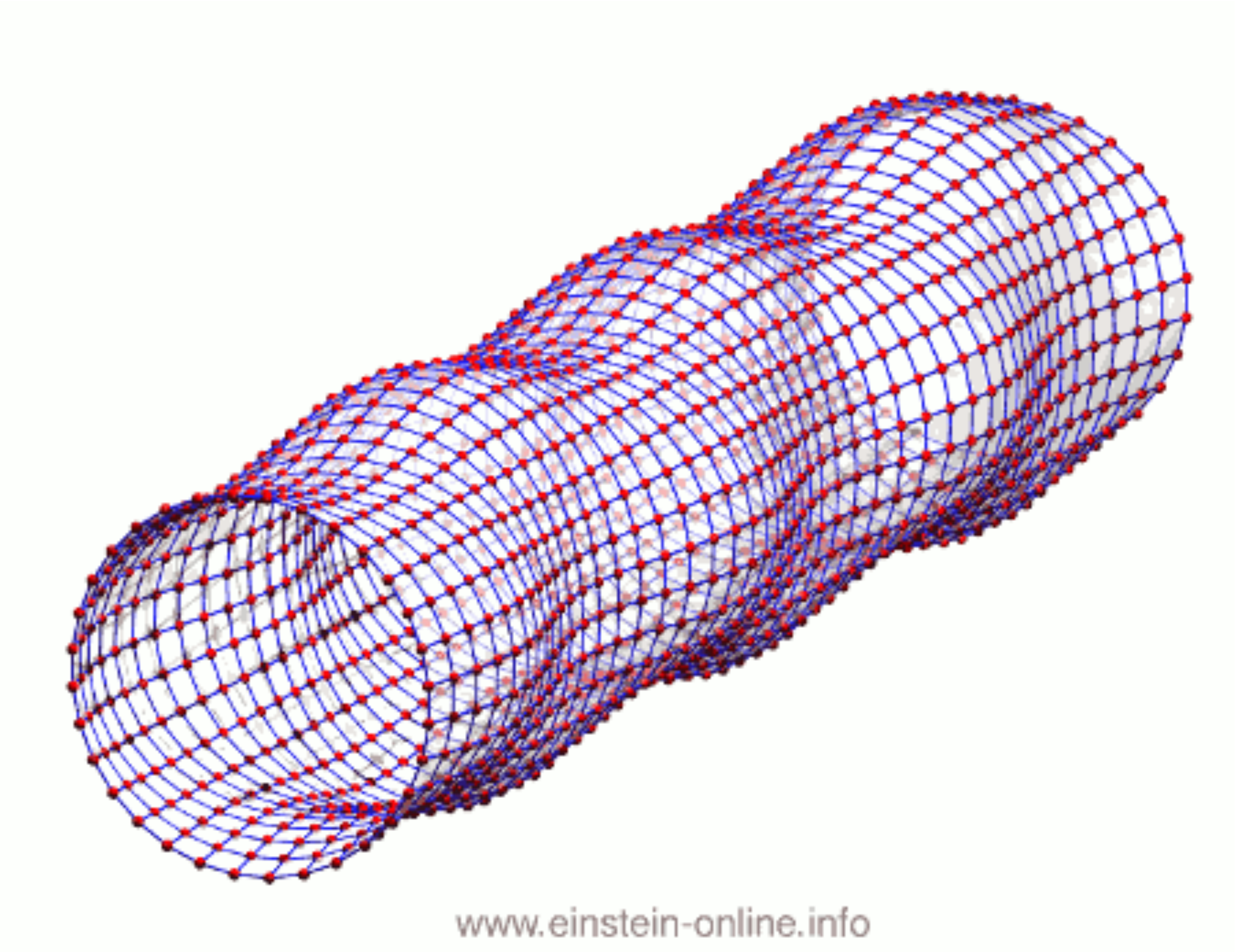
$$ds^2 = -\,c^2dt^2 + dz^2 + dx^2 + dy^2 + 2\left(1 + h_\times \cos[\omega(t - z/c)]\right) dxdy$$





www.einstein-online.info

Linear polarization



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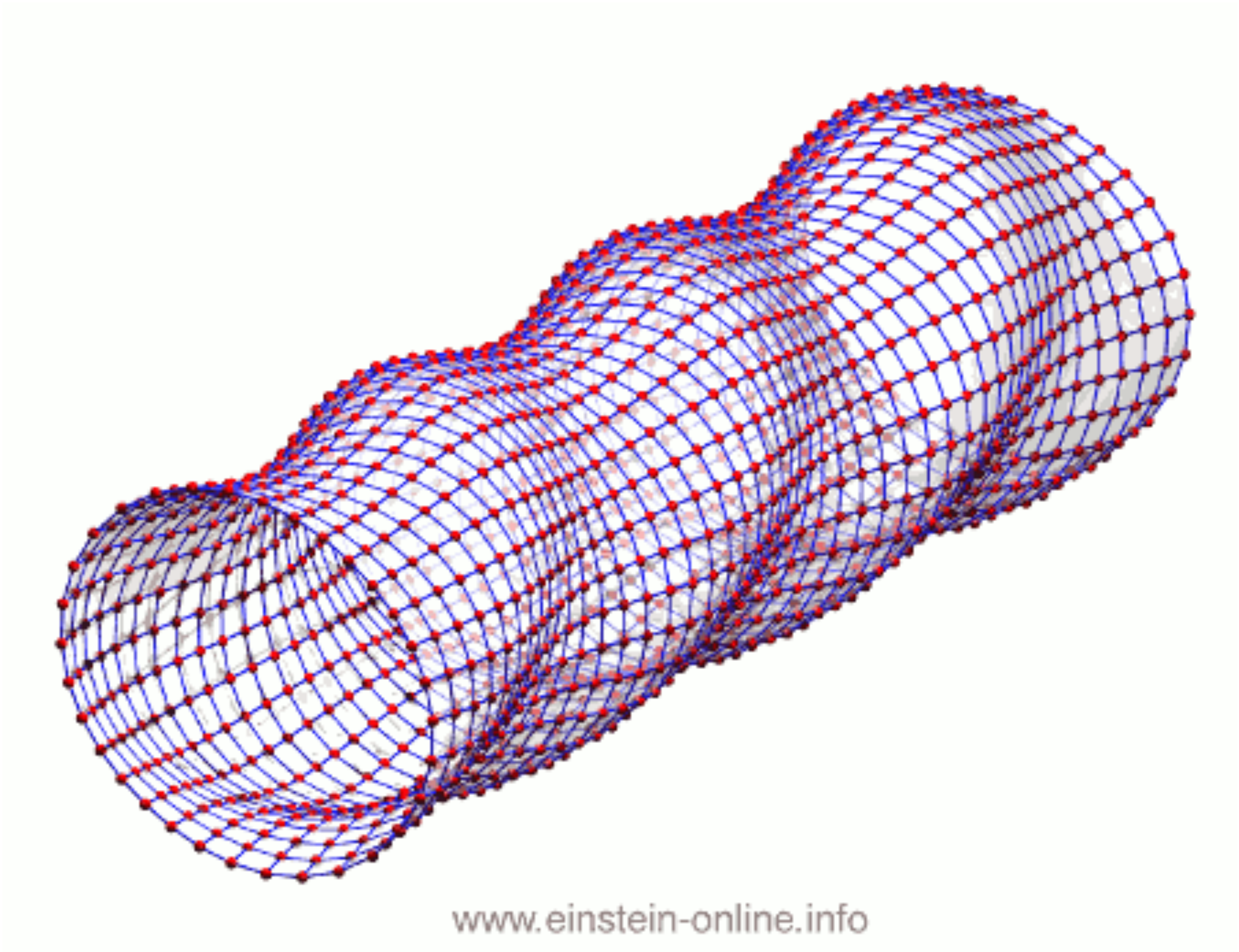
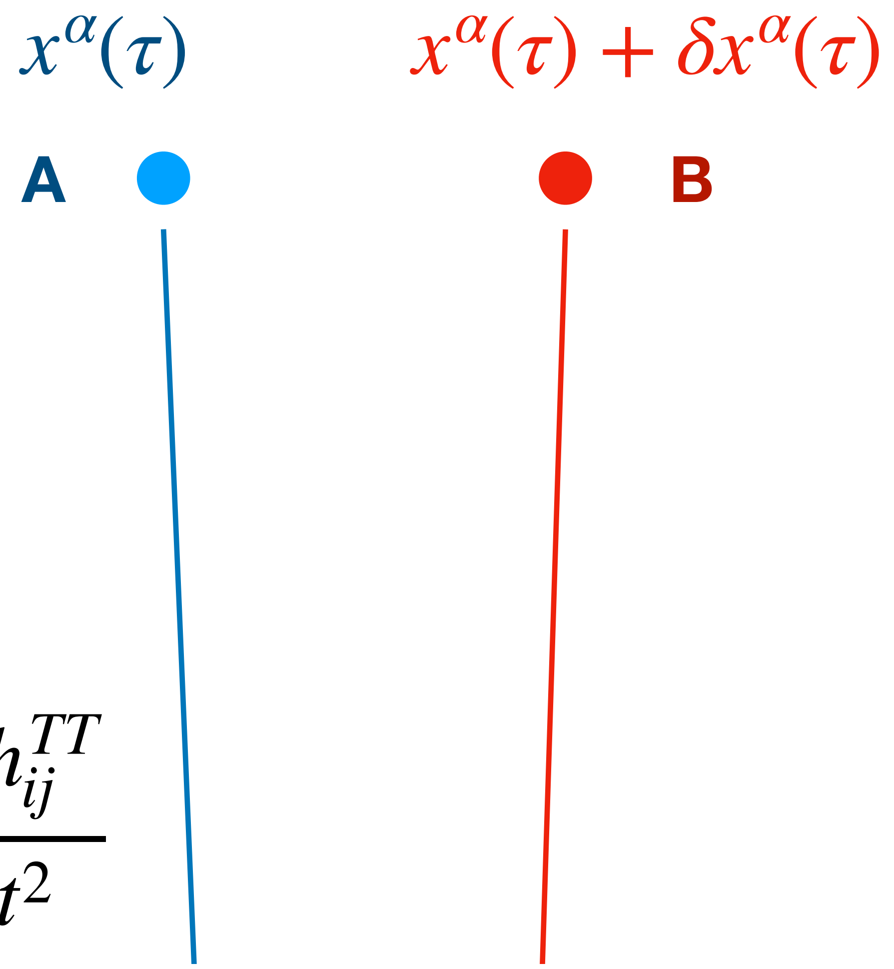
Circular polarization


www.einstein-online.info

GW effect on test masses



Geodesic deviation:

$$\frac{D^2 \delta x^\alpha}{D\tau^2} = R^\alpha{}_{\mu\nu\gamma}\, \dot{x}^\mu\, \dot{x}^\nu\, \delta x^\gamma$$

Take the curve parameters as the time coordinate:

$$\frac{D^2 \delta x^i}{Dt^2} = R^i{}_{00\gamma}\, \delta x^\gamma \qquad \text{with} \qquad R^i{}_{00\gamma} = \frac{1}{2}\frac{\partial^2 h_{ij}^{TT}}{\partial t^2}$$

To linear order:

$$\boxed{\delta x^i(t) = \delta x^i(0) + \frac{1}{2}\left(h_{ij}^{TT}(t) - h_{ij}^{TT}(0)\right)\delta x^j(0)}$$

Strain h :

$$\boxed{\frac{\Delta x}{x} = \frac{1}{2}h}$$

Sources of GW: dimensional analysis

$$\left(-\frac{\partial^2}{c^2\partial t^2}+\nabla^2\right)\bar{h}_{\alpha\beta}=-\frac{16\pi G}{c^4}T_{\alpha\beta}$$

Matter field is characterized by its multipole moments:

$$M=\int \rho d^3x$$ Mass monopole $$\cancel{h\propto\frac{GM}{rc^2}}$$ Mass conservation - cannot vary dynamically

$$D^i=\int \rho x^i d^3x$$ Mass dipole $$\cancel{h\propto\frac{G\dot{D}}{rc^3}}$$ Linear momentum conservation

$$Q^{ij}=\int \rho x^i x^j d^3x$$ Mass quadrupole $$\boxed{h\propto\frac{G\ddot{Q}}{rc^4}}$$

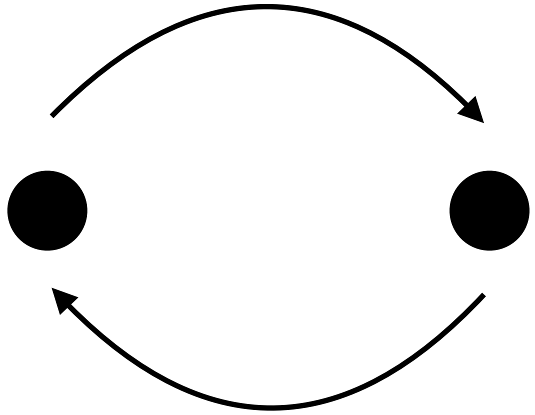
Angular momentum conservation

$$L^i=\int \rho e^i_{jk} x^j v^k d^3x$$ Angular momentum (first moment of mass current) $$\cancel{h\propto\frac{G\dot{L}}{rc^4}}$$

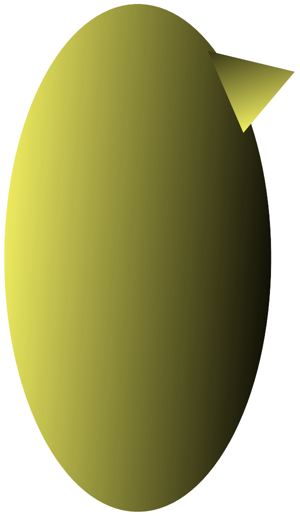
The quadrupole formula (TT gauge)

$$\boxed{\bar{h}_{ij}^{TT}(t,\vec{x}) \simeq \frac{2G}{c^4 r}\ddot{Q}^{TT}(t-r/c)}$$

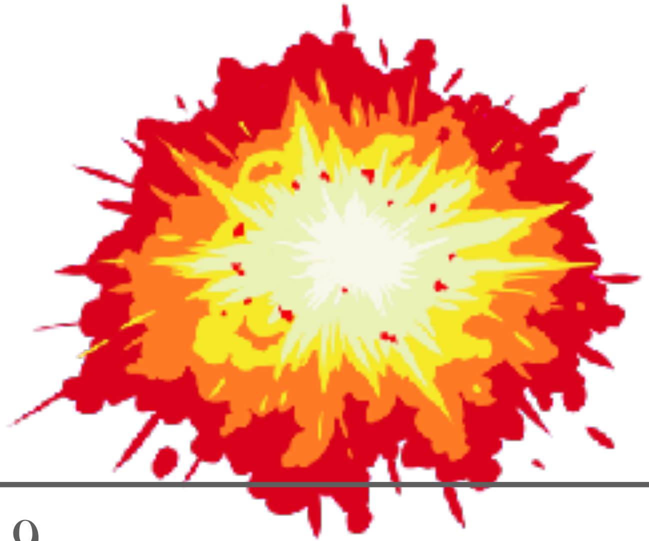
Compact binaries (binary black holes, neutron stars…)



Non-spherical rotating stars



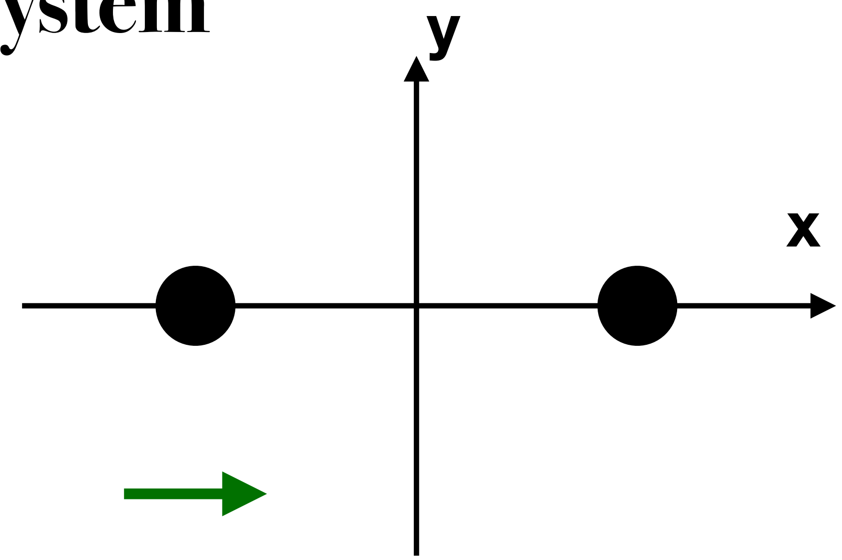
Stellar explosions



The quadrupole formula: binary system

Equal-mass circular binary in the x-y plane,

orbital frequency ω and initial separation a_0



$$Q_{xx} = \frac{1}{4} M a_0^2 \cos(2\omega t)$$

$$Q_{yy} = -\frac{1}{4} M a_0^2 \cos(2\omega t)$$

$$Q_{xy} = \frac{1}{4} M a_0^2 \sin(2\omega t)$$

$$x_1 = \frac{a_0}{2} \cos(\omega t)$$

$$y_1 = \frac{a_0}{2} \sin(\omega t)$$

Radiation in the x direction $$\bar{h}^{TT}_{yy} = -\bar{h}^{TT}_{zz} = \frac{G M a_0^2 \omega^2}{2 c^2 r} \cos(2\omega(t - r/c))$$

Linear polarization aligned with the orbital plane

Binary system: unequal masses

Kepler law:

$$\omega^2 = \frac{G(M_1 + M_2)}{a_0^3}$$

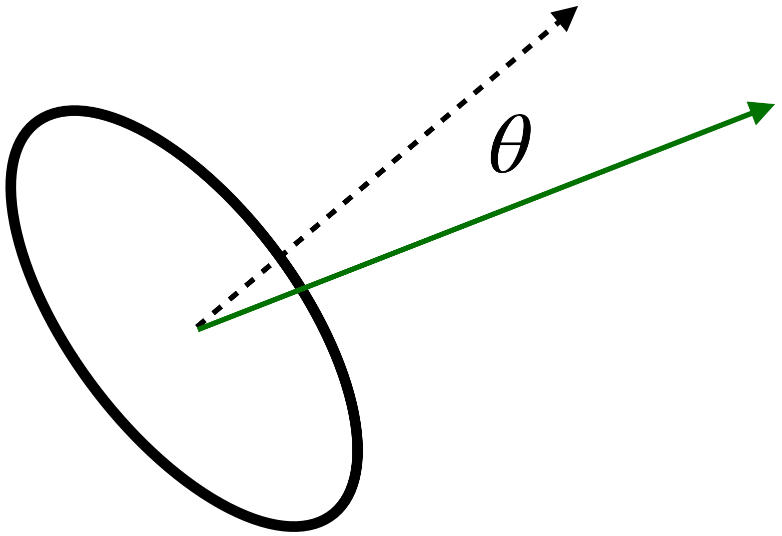
GW frequency:

$$f_{GW} = \frac{\omega}{\pi}$$

Orbital inclination: θ

$$h_+ = \frac{4}{r} \left(\frac{GM_c}{c^2} \right)^{5/3} \left(\frac{\pi f_{GW}}{c} \right)^{2/3} \frac{1 + \cos^2\theta}{2}$$

$$h_\times = \frac{4}{r} \left(\frac{GM_c}{c^2} \right)^{5/3} \left(\frac{\pi f_{GW}}{c} \right)^{2/3} \cos\theta$$



Chirp mass: $M_c = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}$

GW energy flux and luminosity

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Far from the source: the energy is due to GW

Weak field, developing to lowest order

$$t^{TT}_{\mu\nu} = \frac{c^2}{8\pi G} \left(R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \eta^{\delta\rho} R_{\lambda\rho} \right)$$

second order in *h*

GW energy flux and luminosity

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

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Weak field, developing to lowest order

$$t_{\mu\nu}^{TT} = \frac{c^2}{8\pi G}\left(R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\delta\rho}R_{\lambda\rho}\right)$$

second order in *h*

Flux:

$$\frac{dE}{dtdA} = t_{03}^{TT} = -\frac{c^3}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

$$\frac{dE}{dtdA} = -\frac{G}{8\pi c^5 r^2}\left\langle \dddot{Q}_{ij}\dddot{Q}^{ij} \right\rangle$$

GW energy flux and luminosity

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Far from the source: the energy is due to GW

Weak field, developing to lowest order

$$t_{\mu\nu}^{TT} = \frac{c^2}{8\pi G}\left(R_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\delta\rho}R_{\lambda\rho}\right)$$

second order in *h*

Flux:

$$\frac{dE}{dtdA} = t_{03}^{TT} = -\frac{c^3}{16\pi G}\left\langle \dot{h}_+^2 + \dot{h}_\times^2 \right\rangle$$

$$\frac{dE}{dtdA} = -\frac{G}{8\pi c^5 r^2}\left\langle \dddot{Q}_{ij}\dddot{Q}^{ij} \right\rangle$$

Luminosity:

$$\boxed{L_{GW} = -\frac{dE}{dt} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}^{ij} \right\rangle}$$

Luminosity of GW (quadrupole approximation)

$$\boxed{L_{GW} = -\frac{dE}{dt} = \frac{G}{5c^5}\left\langle \dddot{Q}_{ij}\dddot{Q}^{ij} \right\rangle}$$

But $\dfrac{G}{c^5}$ is extremely small…

Assume a compact object $\dfrac{GM}{Rc^2} \sim 1$

Mass quadrupoles and its derivatives:

$$Q \sim MR^2 \qquad \ddot{Q} \sim Mv^2 \sim E_{kin} \qquad \boxed{\dddot{Q} \sim \frac{E_{kin}}{\tau} \sim \frac{E_{kin}}{R/v} \sim \frac{Mv^2}{R/v}}$$

$$\boxed{L_{GW} \sim \frac{G}{c^5}\dddot{Q}^2 \sim \frac{c^5}{G}\left(\frac{GM}{Rc^2}\right)^2\left(\frac{v}{c}\right)^6}$$

Thankfully $\dfrac{c^5}{G}$ is extremely large!

Orbital evolution of a compact binary system

Energy is lost to GW, the orbit shrinks:

$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3 \mu m^2}{c^5 a^3}$$

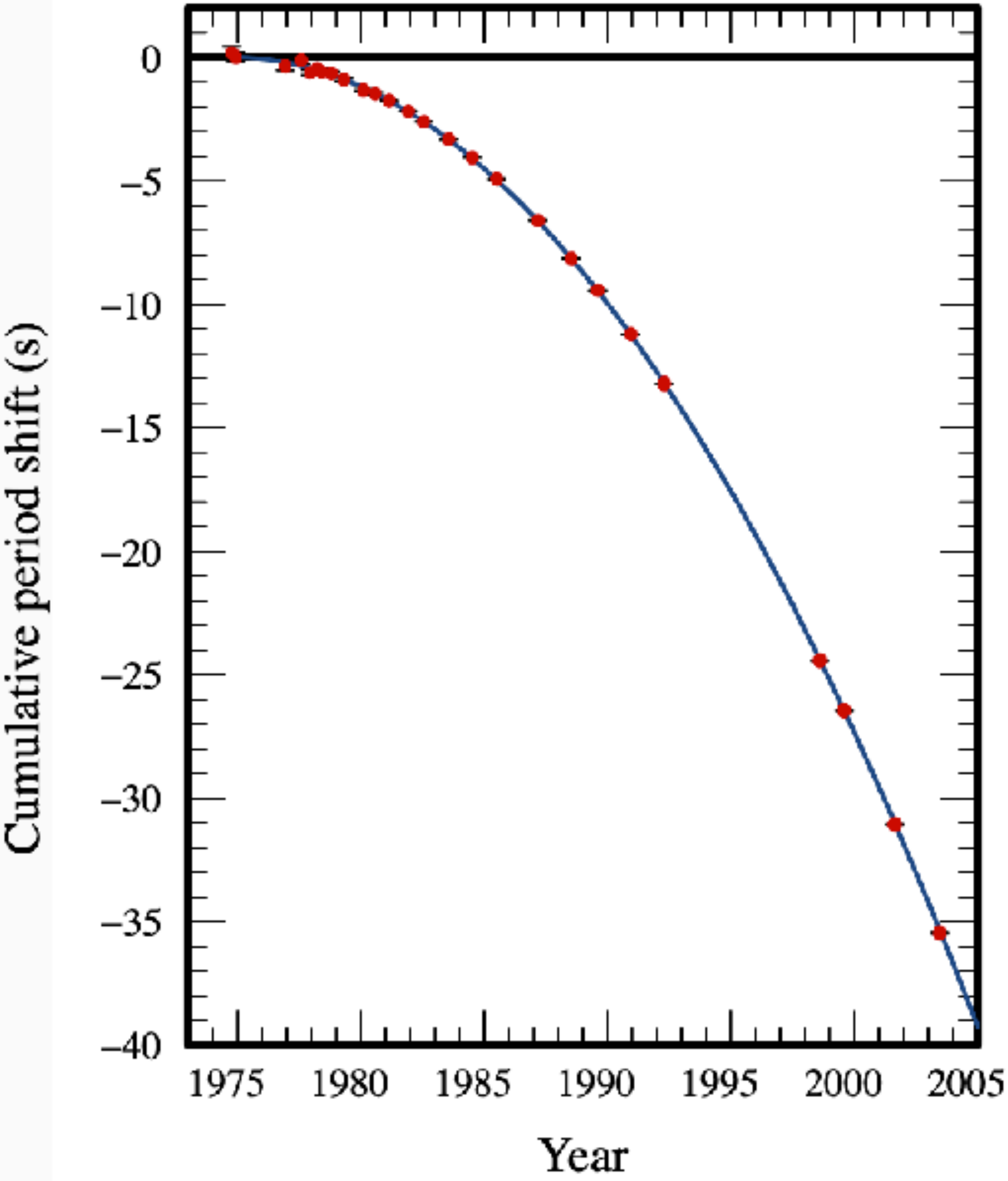
Coalescence time for a circular binary

$$t_{coal} = a_0^4 \cdot \frac{5}{256}\frac{c^5}{G^3}\frac{1}{\mu m^2}$$

Reduced mass $\mu = \frac{M_1 M_2}{M_1 + M_2}$

Total mass $m = M_1 + M_2$

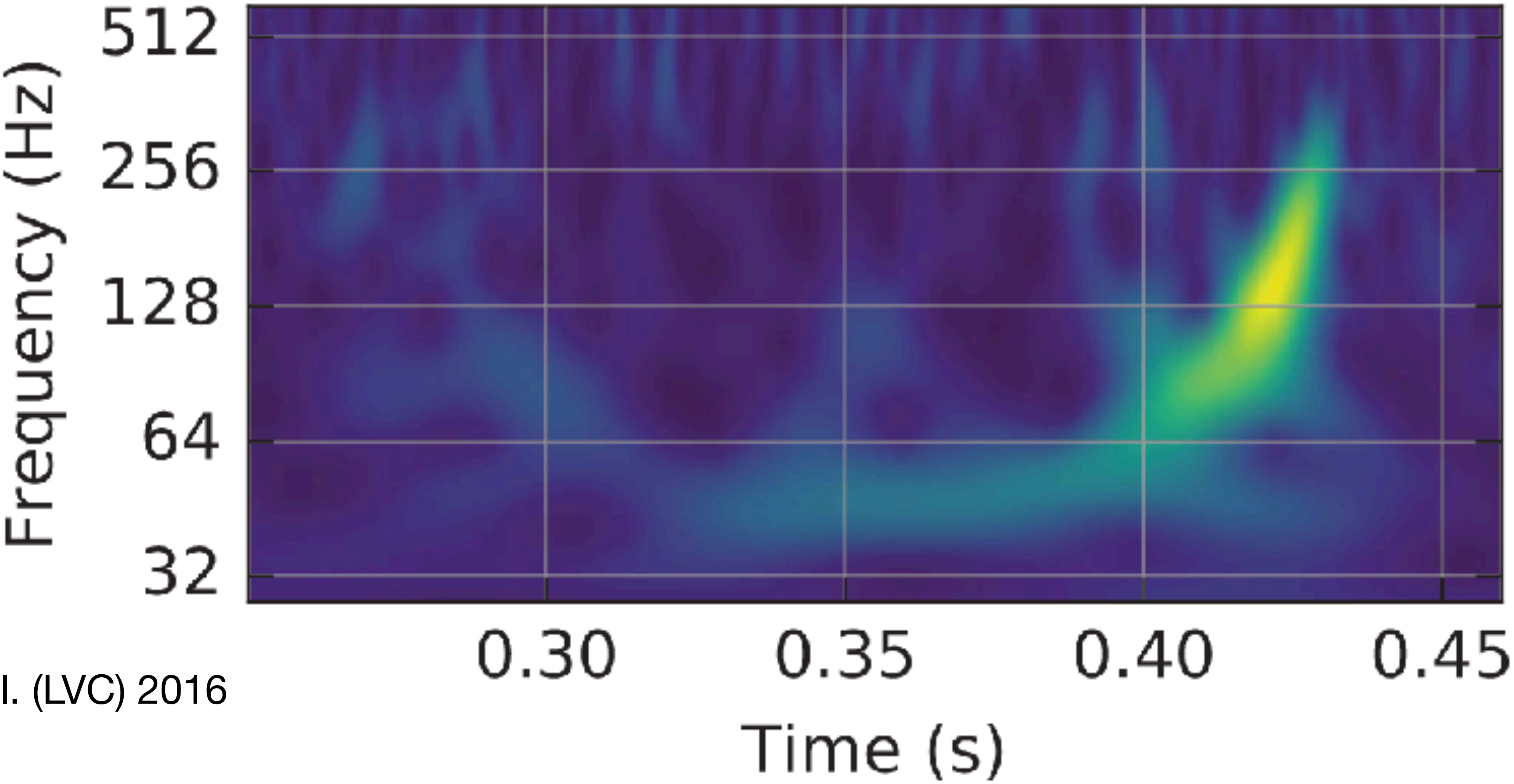
Hulse-Taylor pulsar



GW150914: frequency chirp

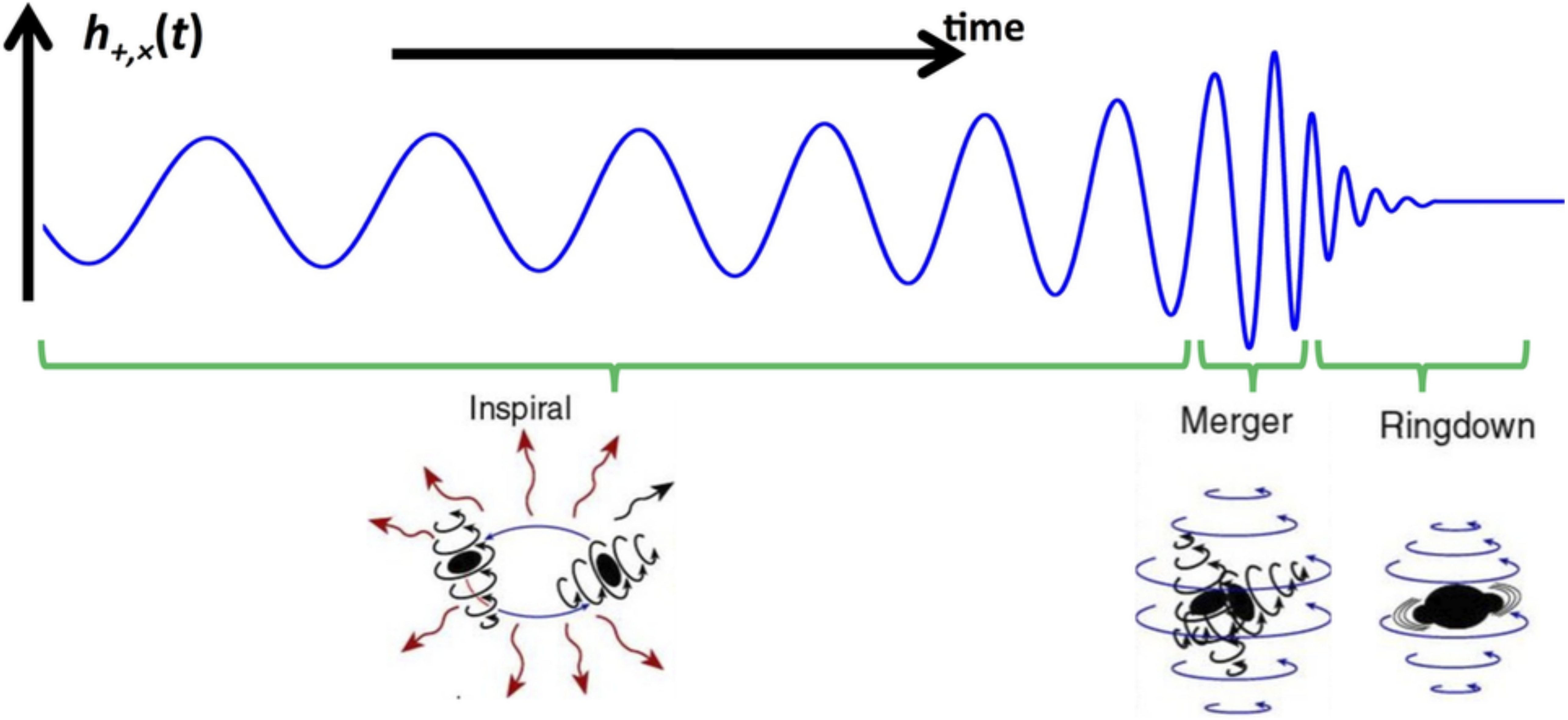
From orbital evolution: $$f_{GW}(t) = \frac{1}{\pi}\left(\frac{GM_c}{c^3}\right)^{-5/8}\left(\frac{5}{256}\frac{1}{(\tau_{coal} - t)}\right)$$

Frequency chirp: $$\dot{f}_{GW}(t) = \frac{96}{5}\pi^{8/3}\left(\frac{GM_c}{c^3}\right)^{5/3} f_{GW}^{11/3}$$



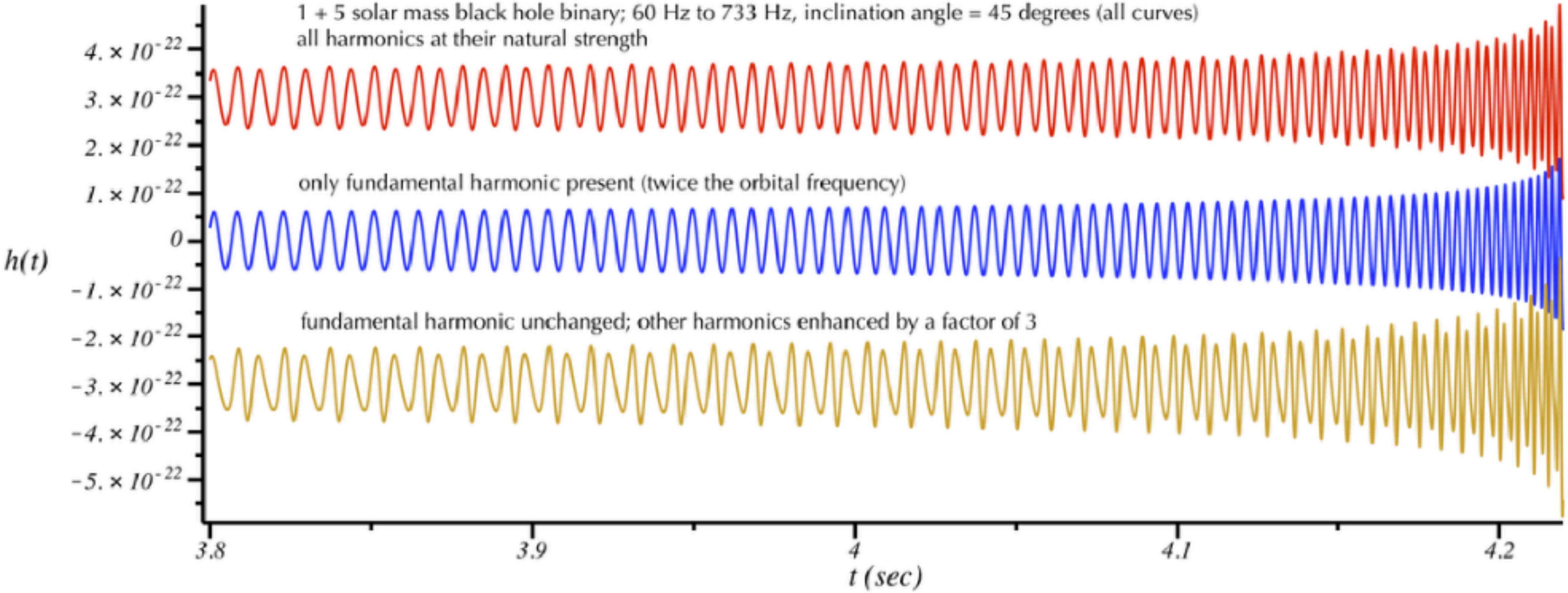
Abbott et al. (LVC) 2016

Compact binary merger



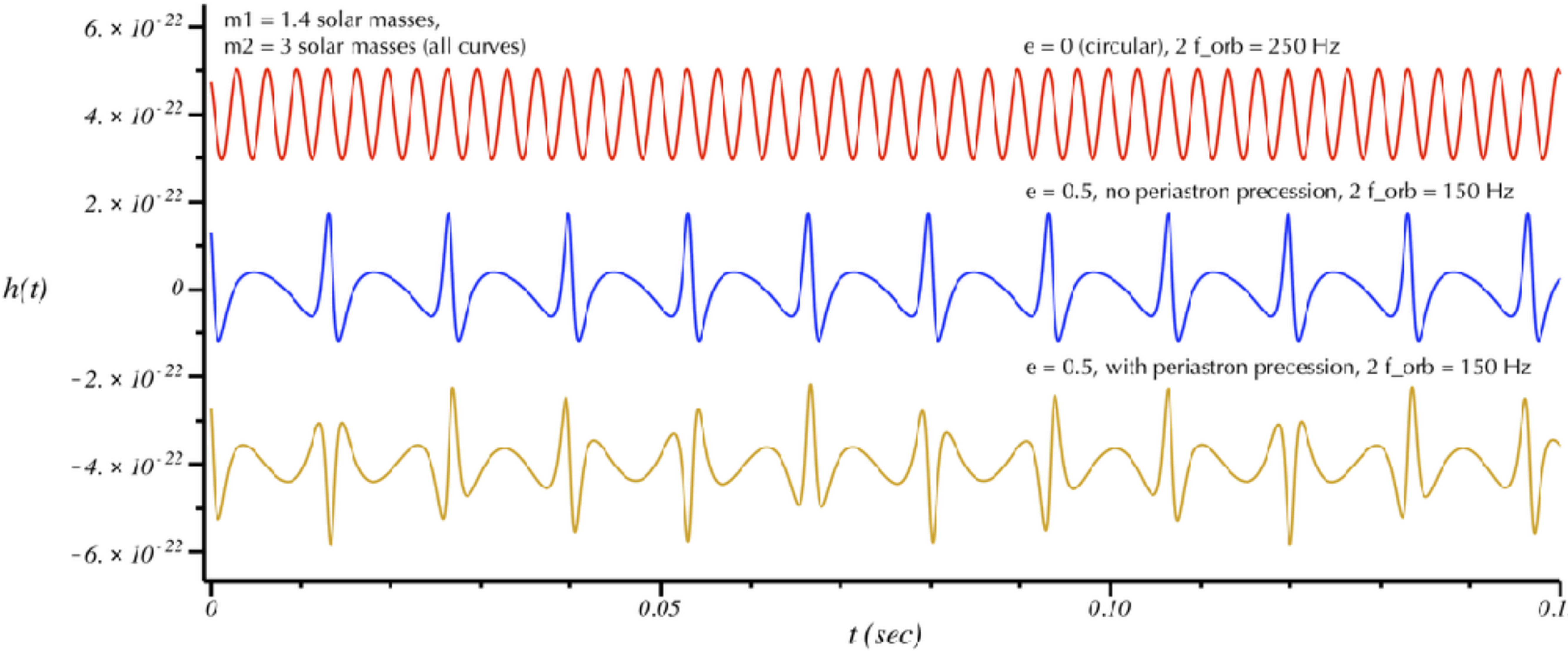
Beyond the quadrupole

- Quadrupole $f_{GW} = 2 \cdot f_{orbit}$
- Octupole $3 \cdot f_{orbit}$
- … $n \cdot f_{orbit}$



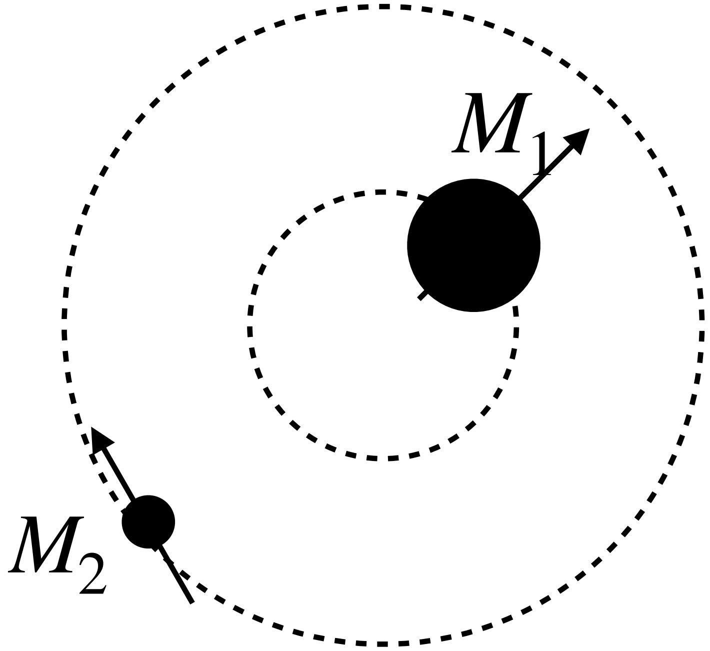
www.soundsofspacetime.org

Eccentric orbits

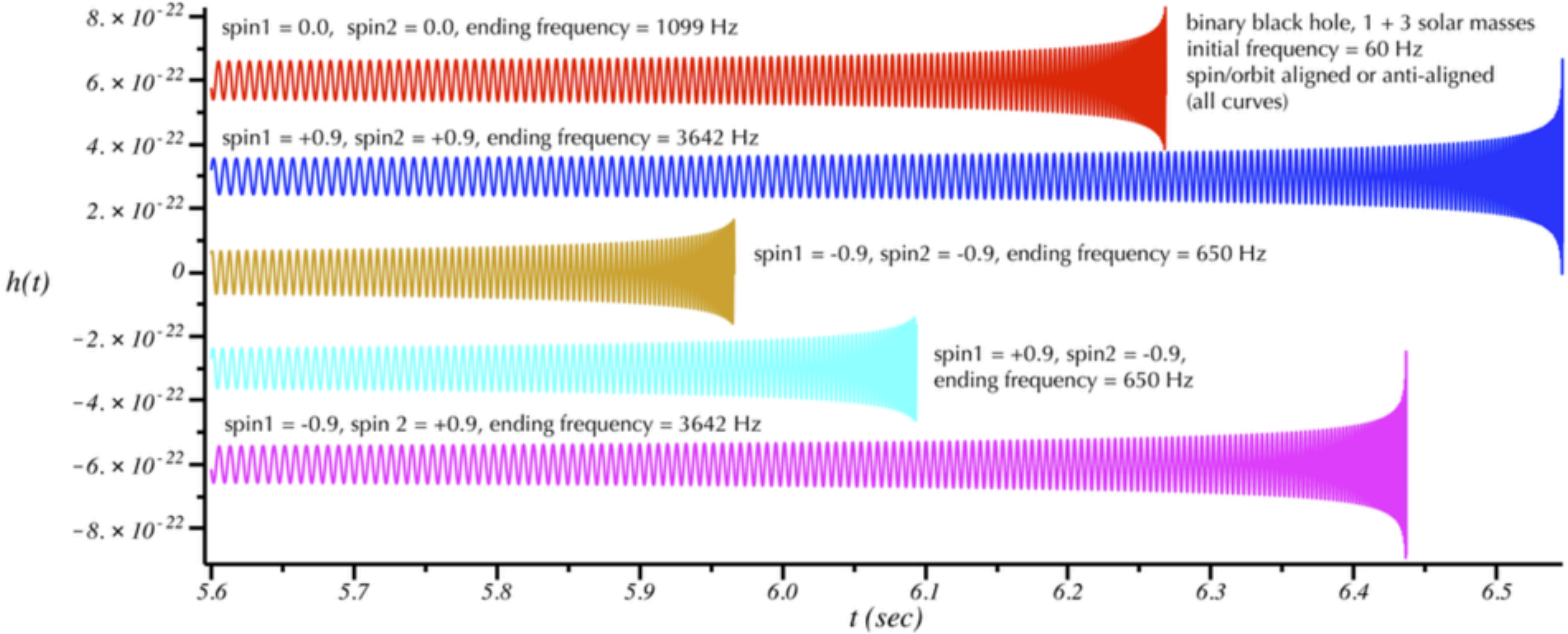


www.soundsofspacetime.org

Spinning black holes



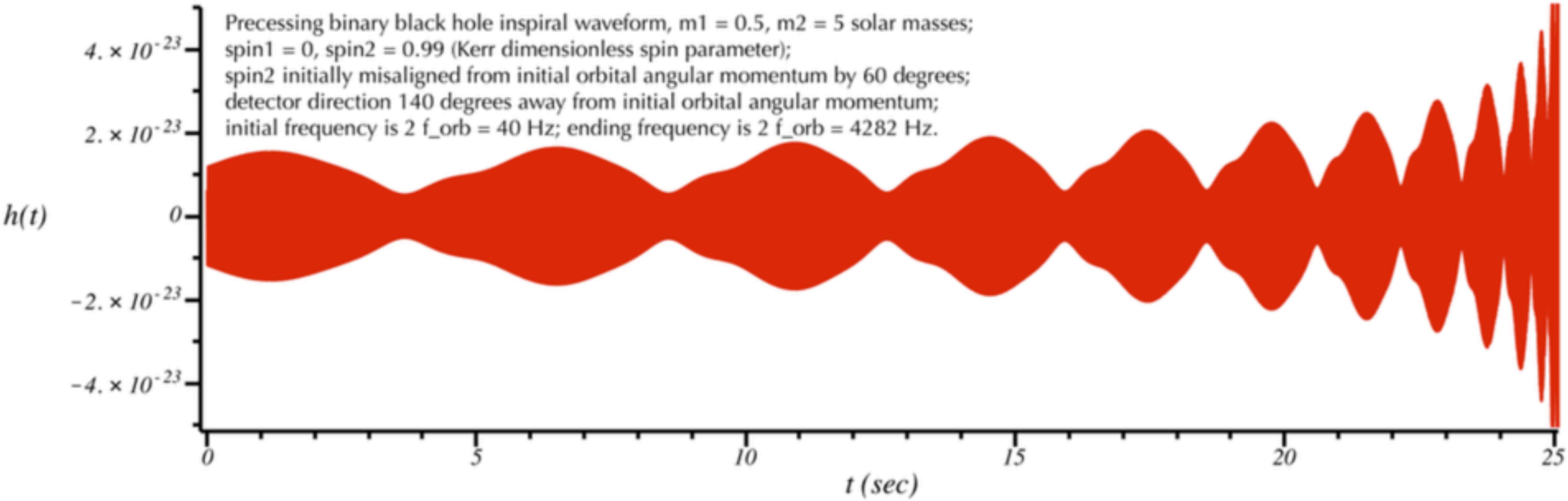
$1M_\odot + 3M_\odot$



www.soundsofspacetime.org

Precessing black holes

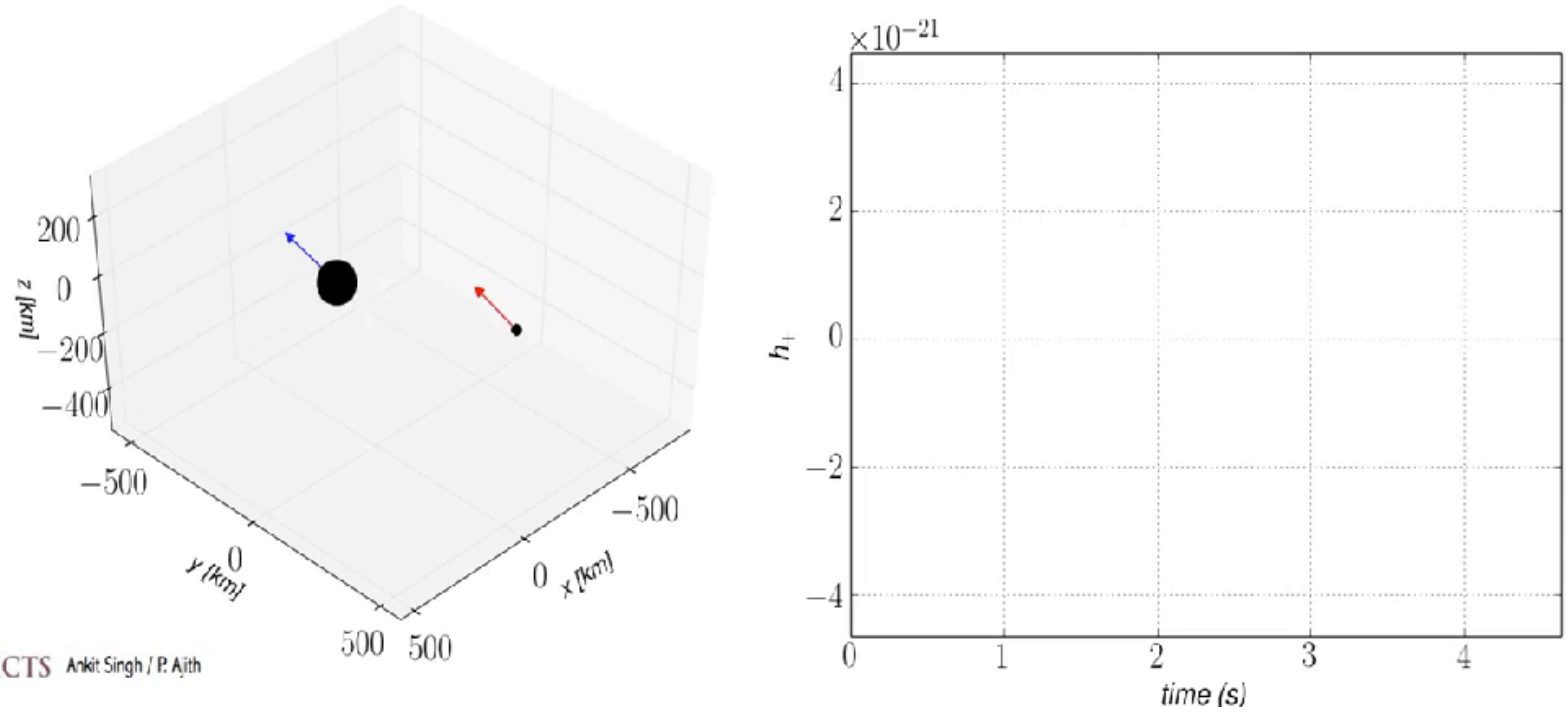
$0.5M_\odot + 5M_\odot$



Precessing binary black hole inspiral waveform, m1 = 0.5, m2 = 5 solar masses;
spin1 = 0, spin2 = 0.99 (Kerr dimensionless spin parameter);
spin2 initially misaligned from initial orbital angular momentum by 60 degrees;
detector direction 140 degrees away from initial orbital angular momentum;
initial frequency is 2 f_orb = 40 Hz; ending frequency is 2 f_orb = 4282 Hz.

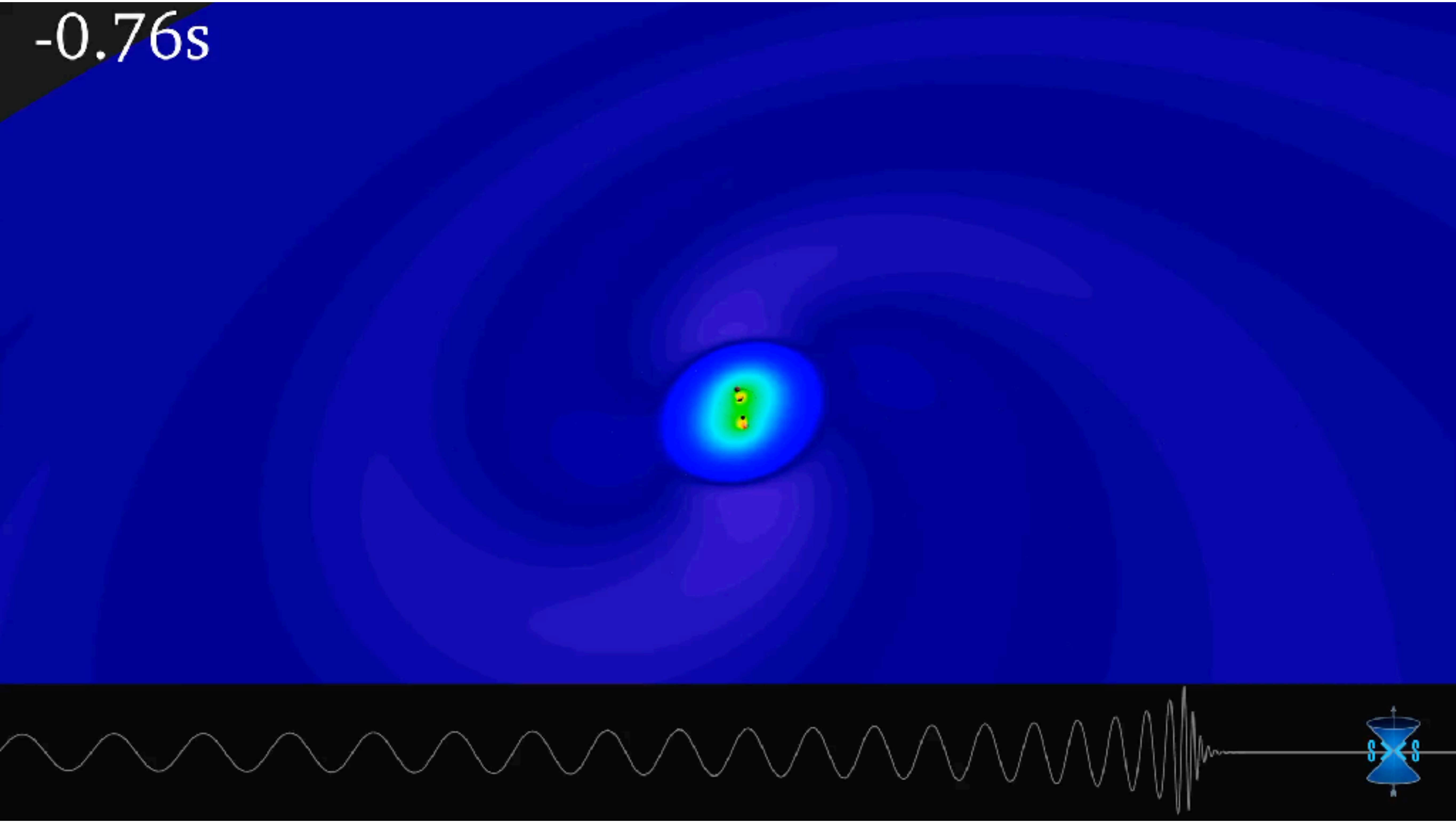
www.soundsofspacetime.org

Precessing black holes



ICTS Ankit Singh / P. Ajith

GW150914 : simulation of the signal

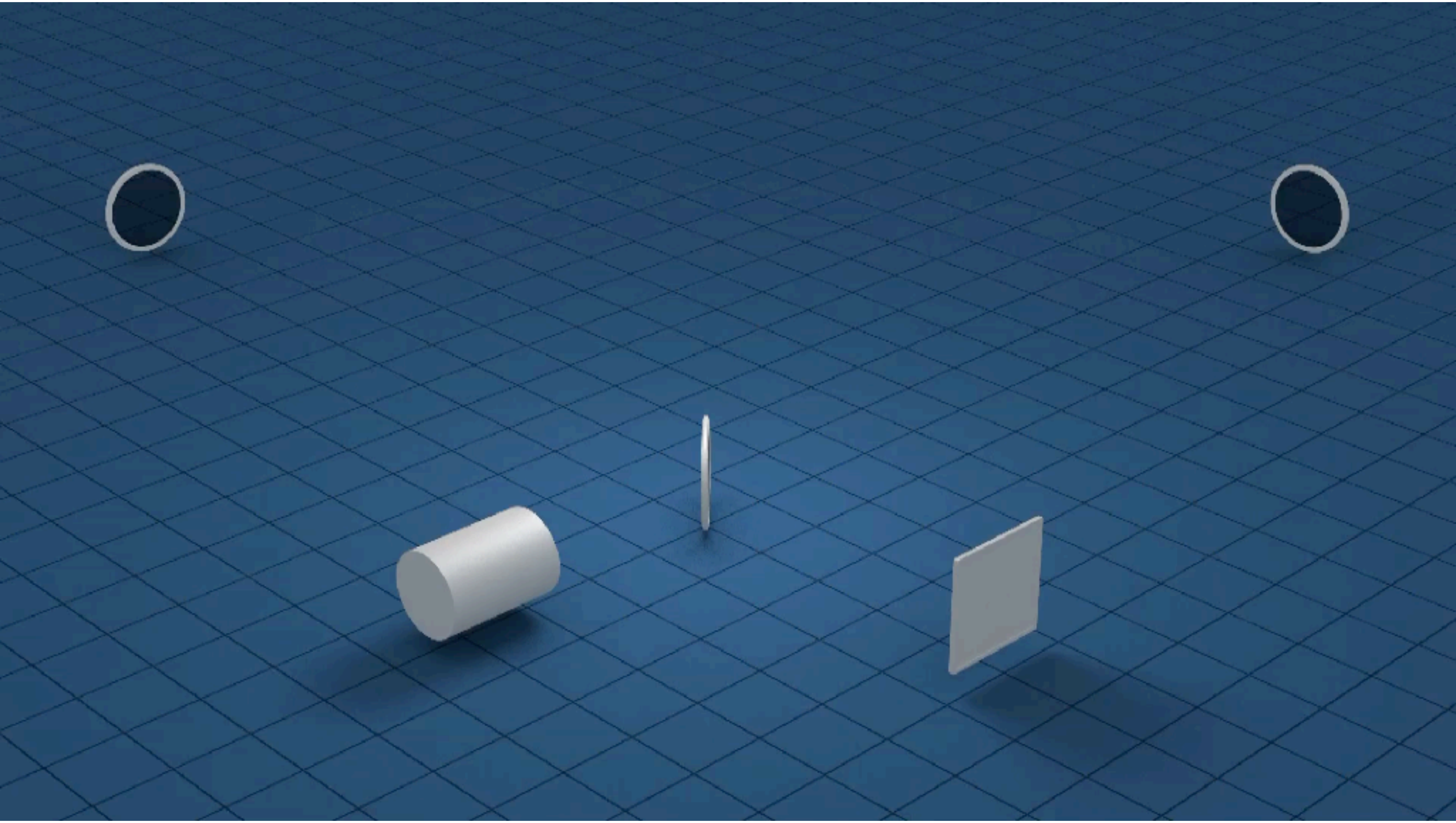


Credit: SXS

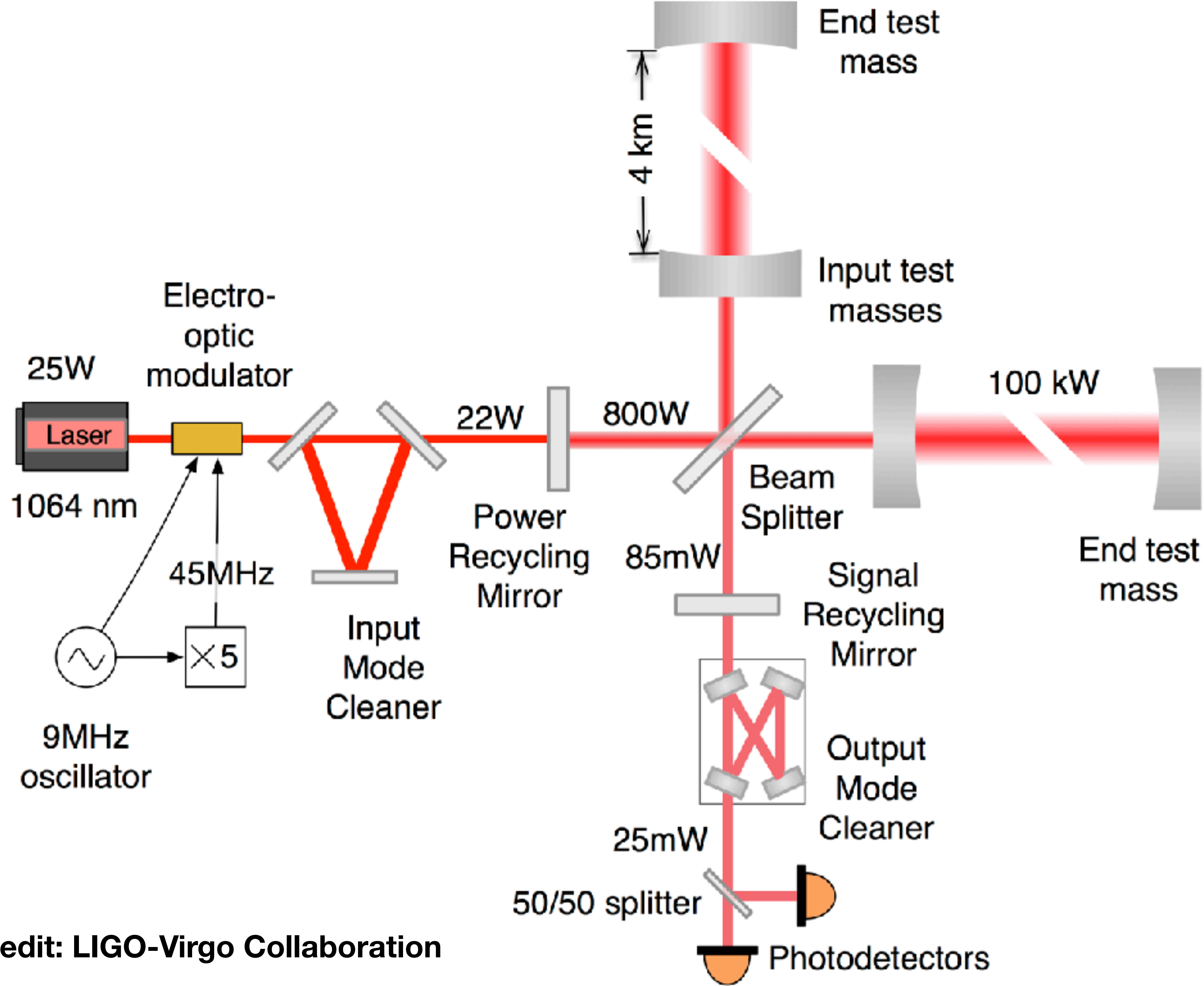
Gravitational-wave observatories

- LIGO (Hanford+Livingston, USA)
- Virgo (Italy)
- Kagra (Japan)

Gravitational-wave observatories: interferometry

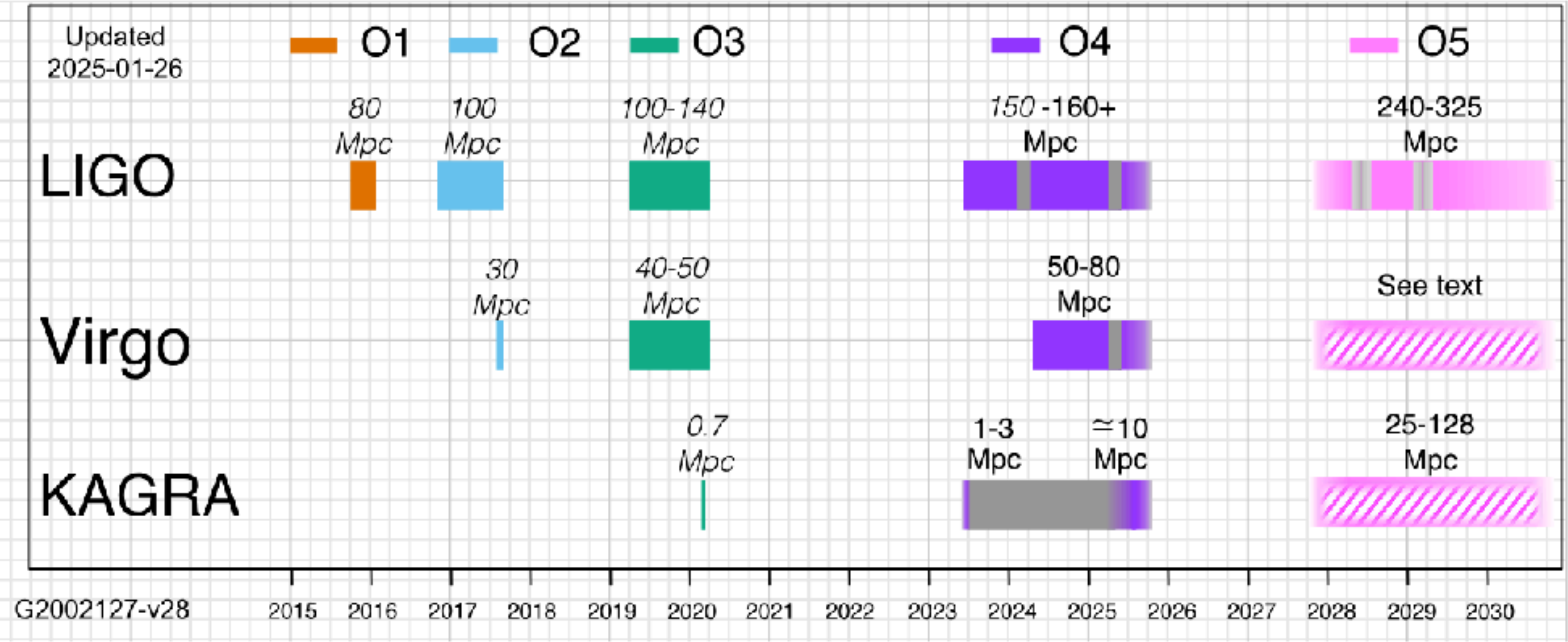


Animation created by T. Pyle, Caltech/MIT/LIGO Lab



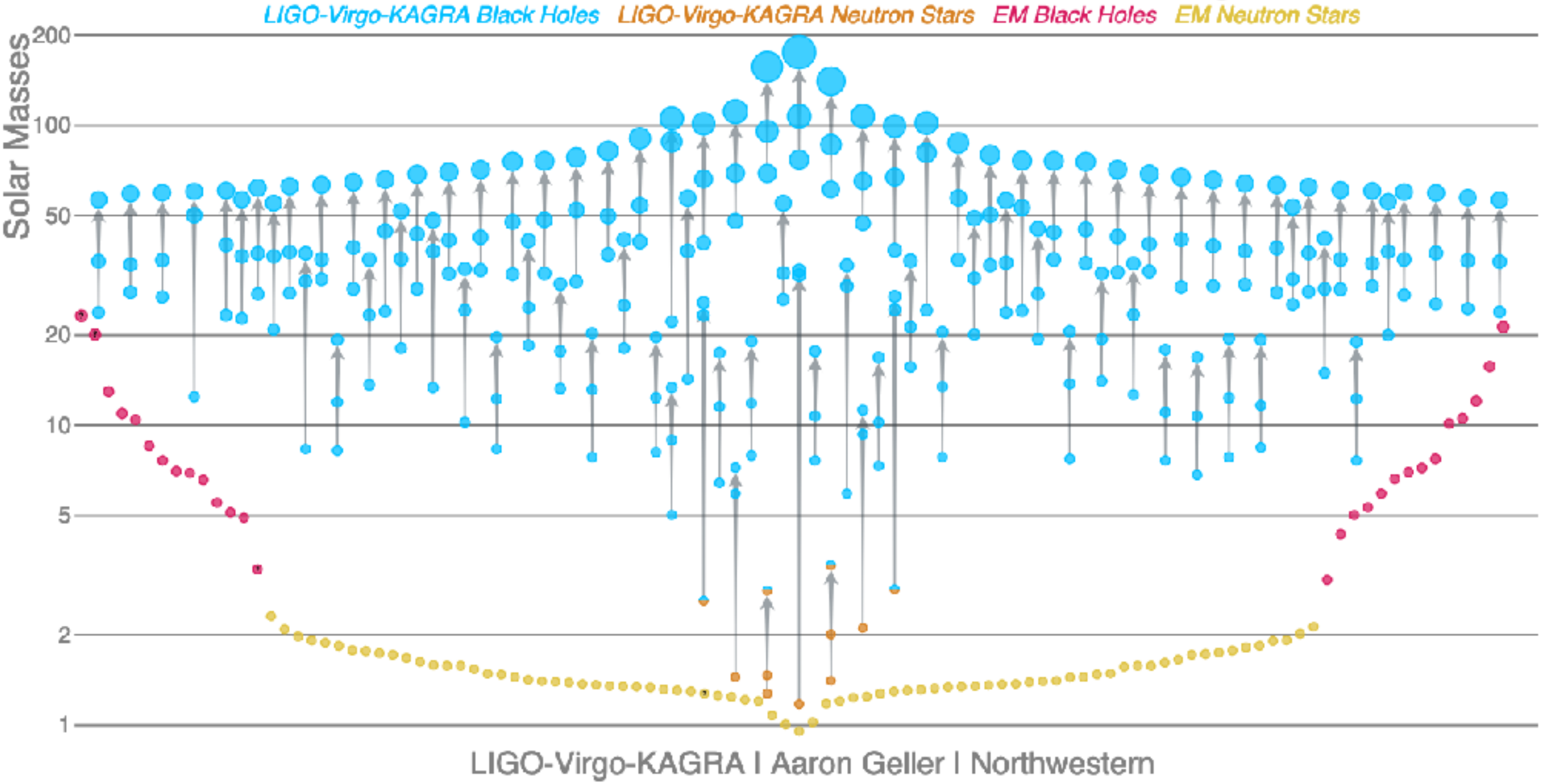
Credit: LIGO-Virgo Collaboration

Prospects for O4/O5 runs



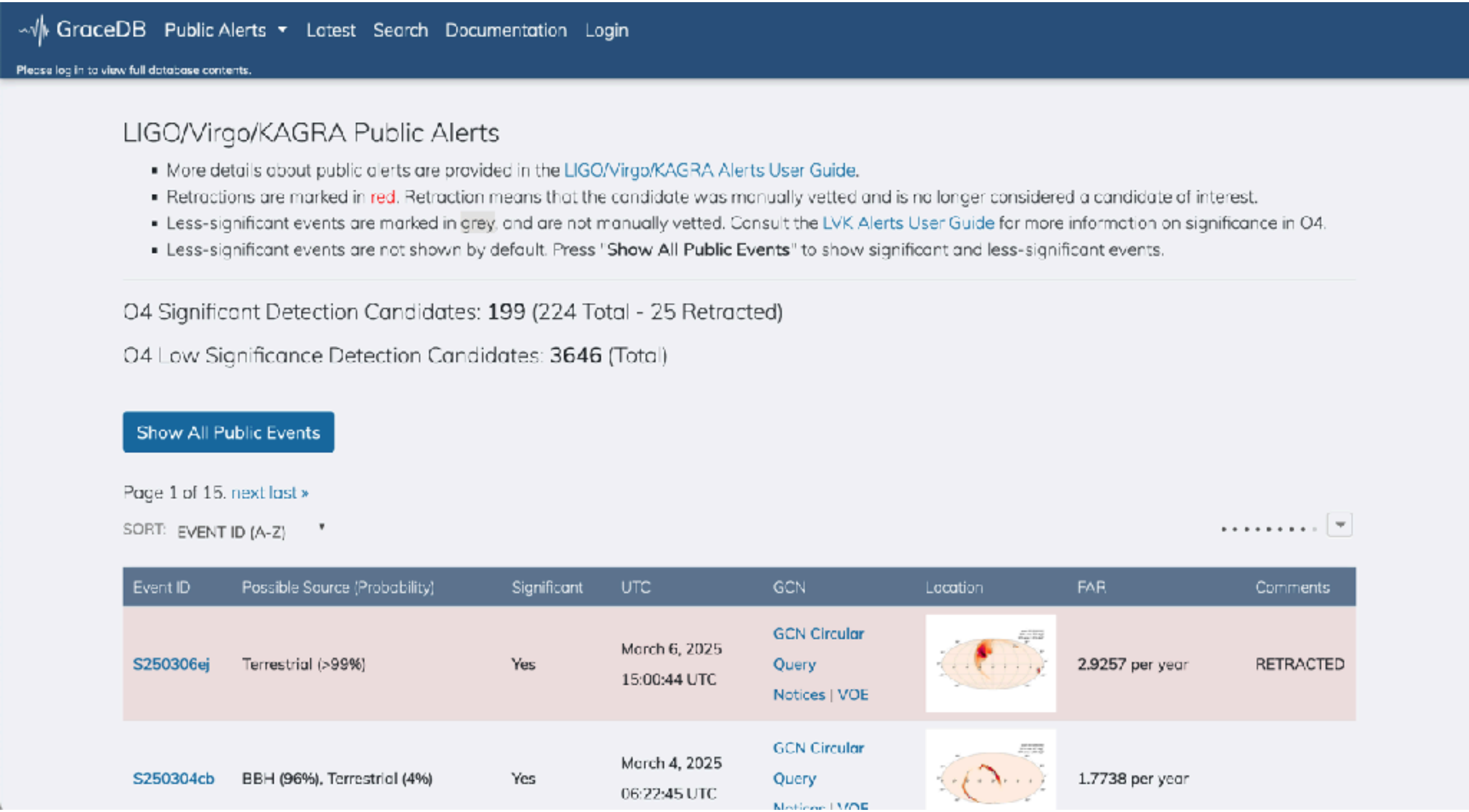
[LIGO Public User Guide: https://emfollow.docs.ligo.org/userguide/capabilities.html]

Masses in the Stellar Graveyard



Abbott et al. 2019, PRX, 9, 031040; Abbott et al. 2021, PRX, 11, 021053;
Abbott et al. 2021, arXiv:2111.03606; Abbott et al. 2021, arXiv:2108.01045

https://gracedb.ligo.org/superevents/public/O4/



Please log in to view full database contents.

LIGO/Virgo/KAGRA Public Alerts

- More details about public alerts are provided in the LIGO/Virgo/KAGRA Alerts User Guide.
- Retractions are marked in red. Retraction means that the candidate was manually vetted and is no longer considered a candidate of interest.
- Less-significant events are marked in grey, and are not manually vetted. Consult the LVK Alerts User Guide for more information on significance in O4.
- Less-significant events are not shown by default. Press '**Show All Public Events**' to show significant and less-significant events.

O4 Significant Detection Candidates: **199** (224 Total - 25 Retracted)

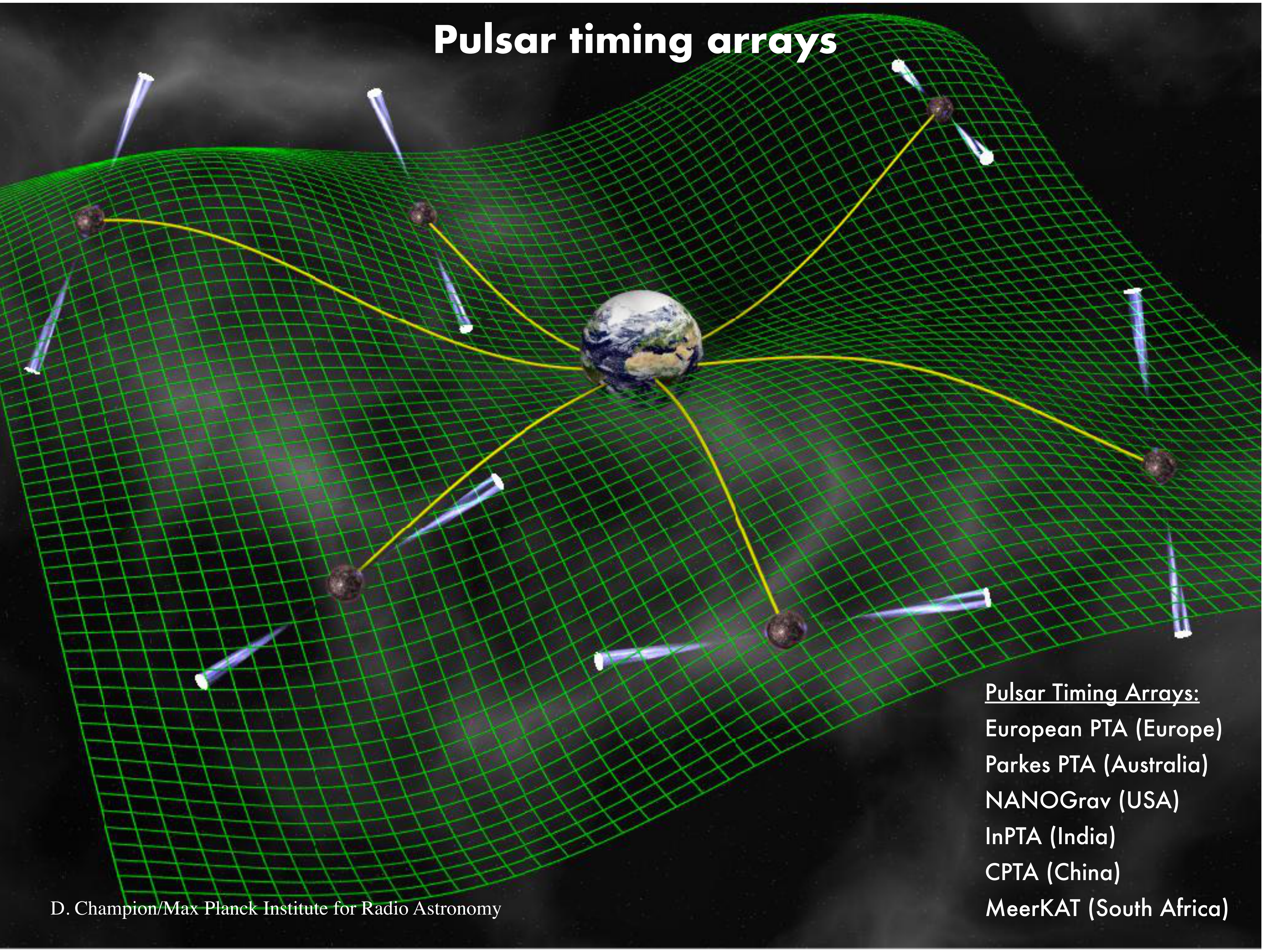
O4 Low Significance Detection Candidates: **3646** (Total)

Page 1 of 15. next last »

SORT: EVENT ID (A-Z)

Event ID	Possible Source (Probability)	Significant	UTC	GCN	Location	FAR	Comments
S250306ej	Terrestrial (>99%)	Yes	March 6, 2025 15:00:44 UTC	GCN Circular Query Notices | VOE		2.9257 per year	RETRACTED
S250304cb	BBH (96%), Terrestrial (4%)	Yes	March 4, 2025 06:22:45 UTC	GCN Circular Query Notices | VOE		1.7738 per year	

Pulsar timing arrays



D. Champion/Max Planck Institute for Radio Astronomy

Pulsar Timing Arrays:

European PTA (Europe)

Parkes PTA (Australia)

NANOGrav (USA)

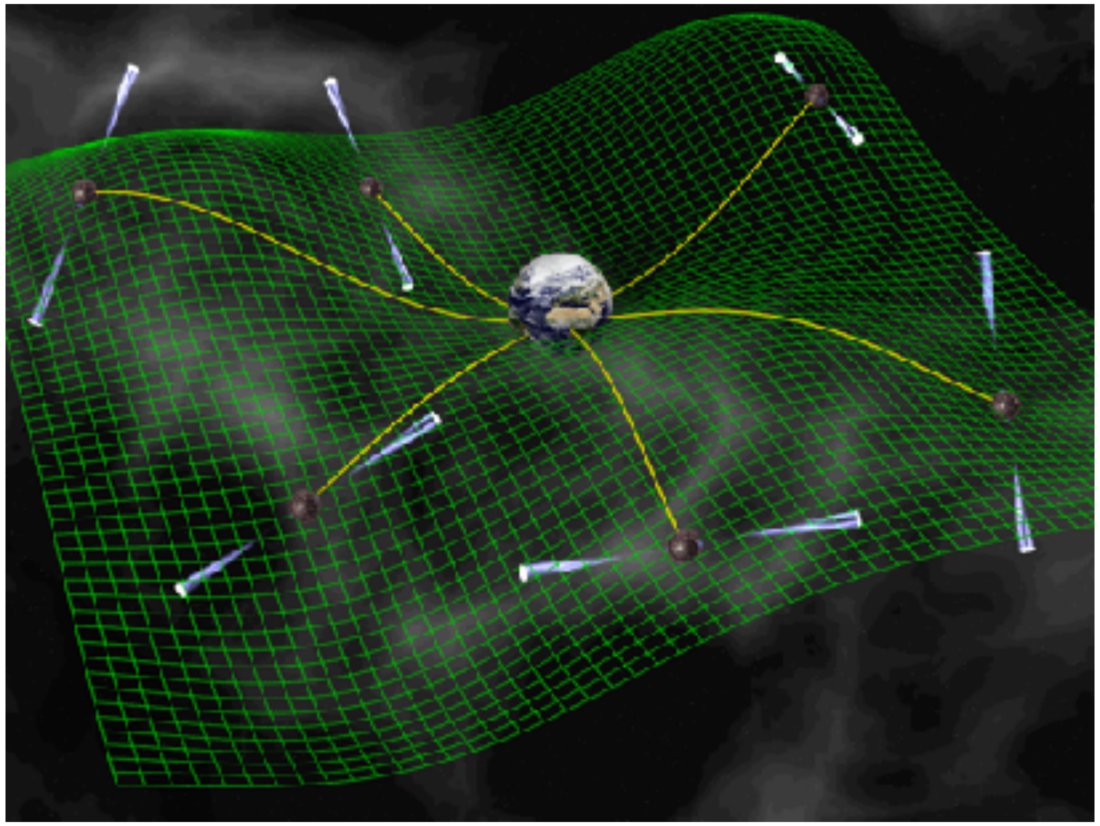
InPTA (India)

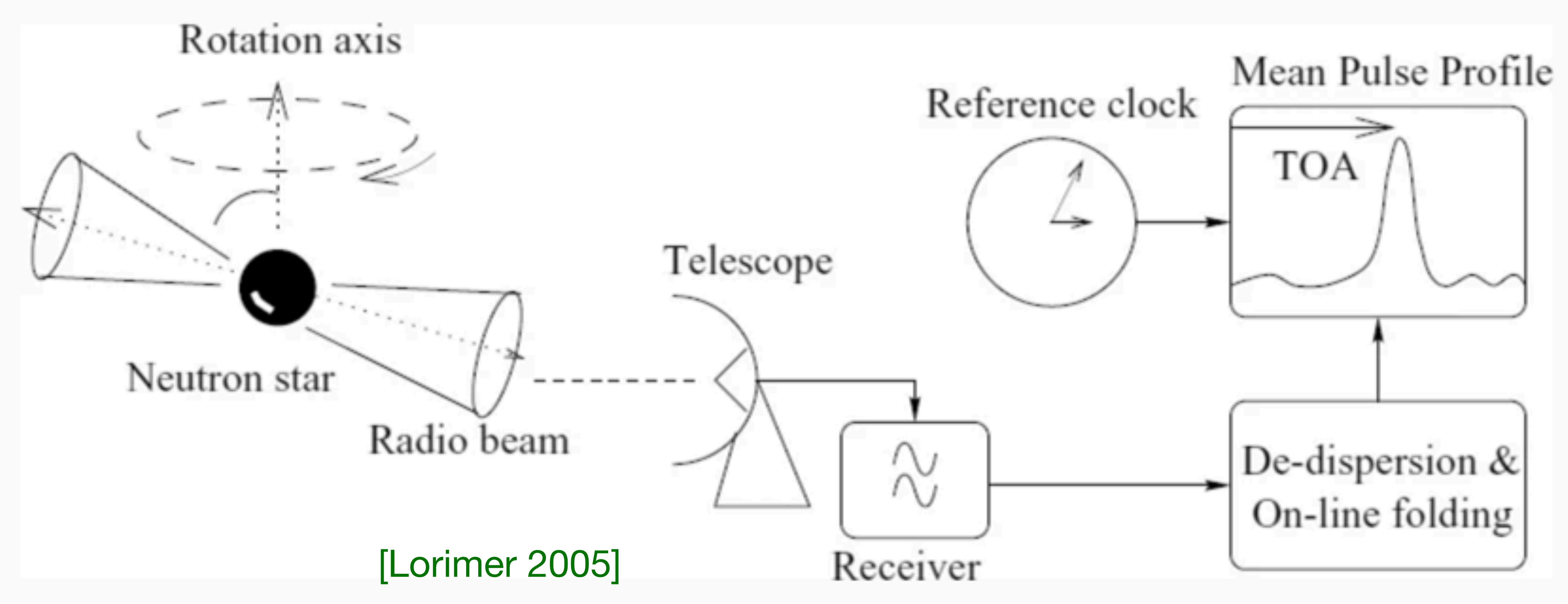
CPTA (China)

MeerKAT (South Africa)

Pulsar Timing Arrays

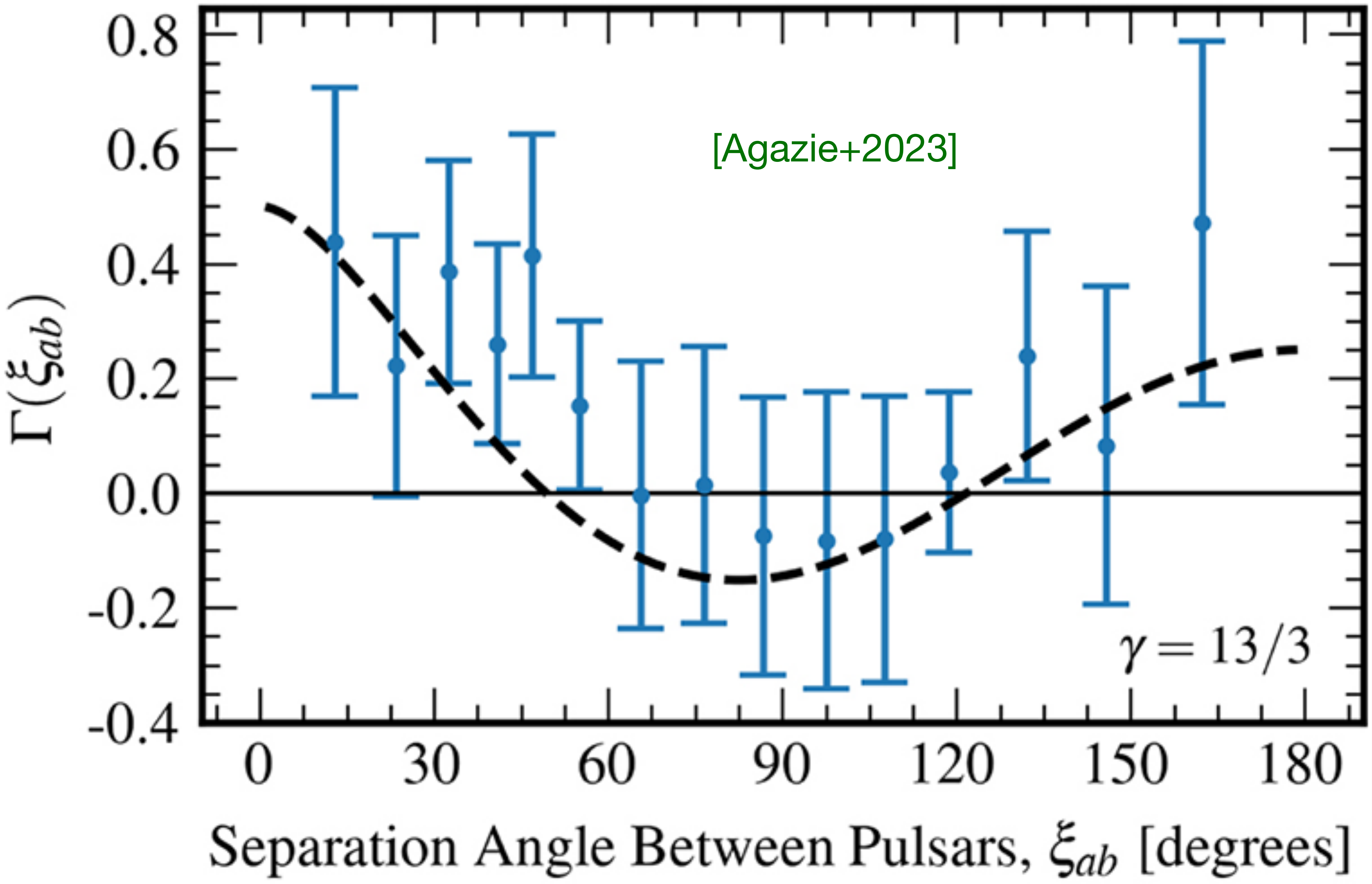
- Compare times of arrival of pulses from a network of pulsars





[Lorimer 2005]

Pulsar Timing Arrays: was the background detected?



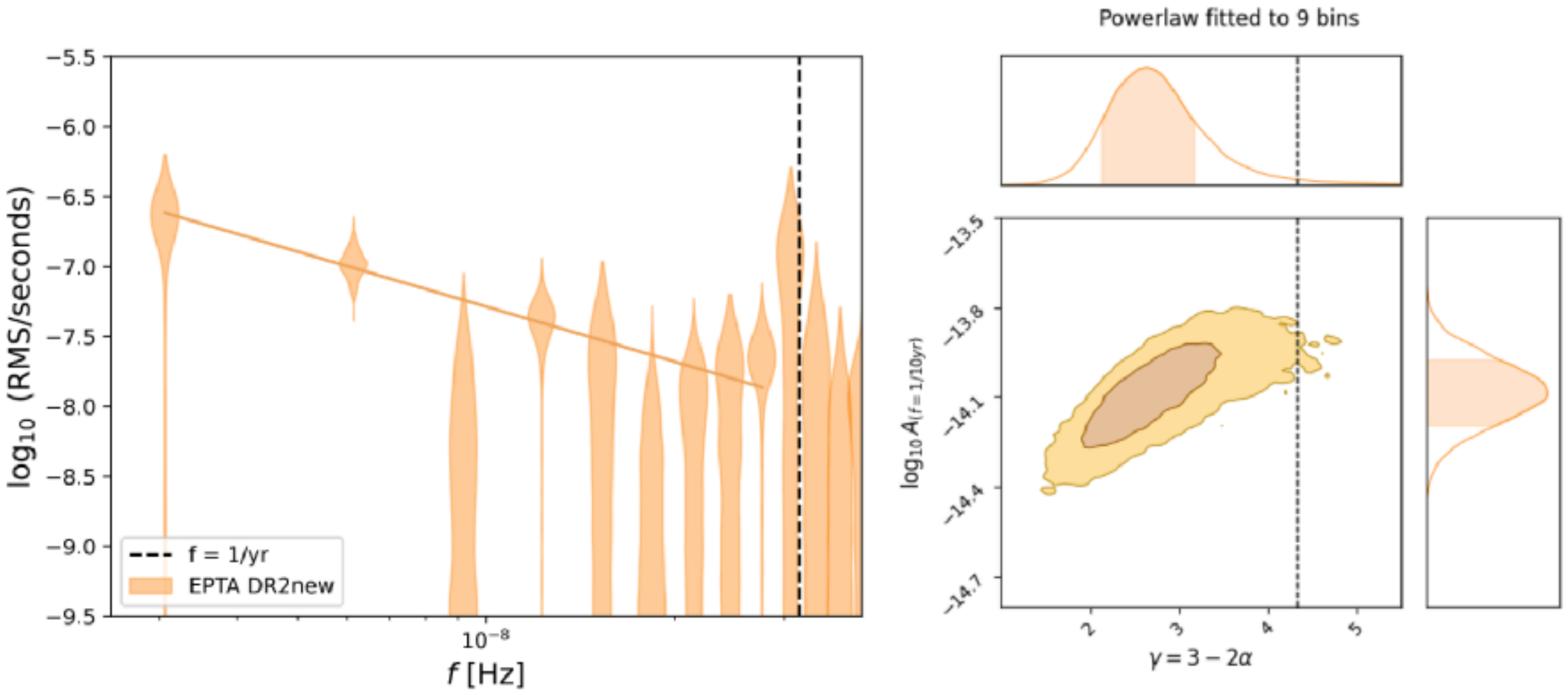
NANOGrav: Agazie+2023

EPTA: Antoniadis+2023

PPTA: Reardon+2023

Pulsar Timing Arrays: was the background detected?

[Antoniadis+2023]



NANOGrav: Agazie+2023

EPTA: Antoniadis+2023

PPTA: Reardon+2023

Massive black hole binaries

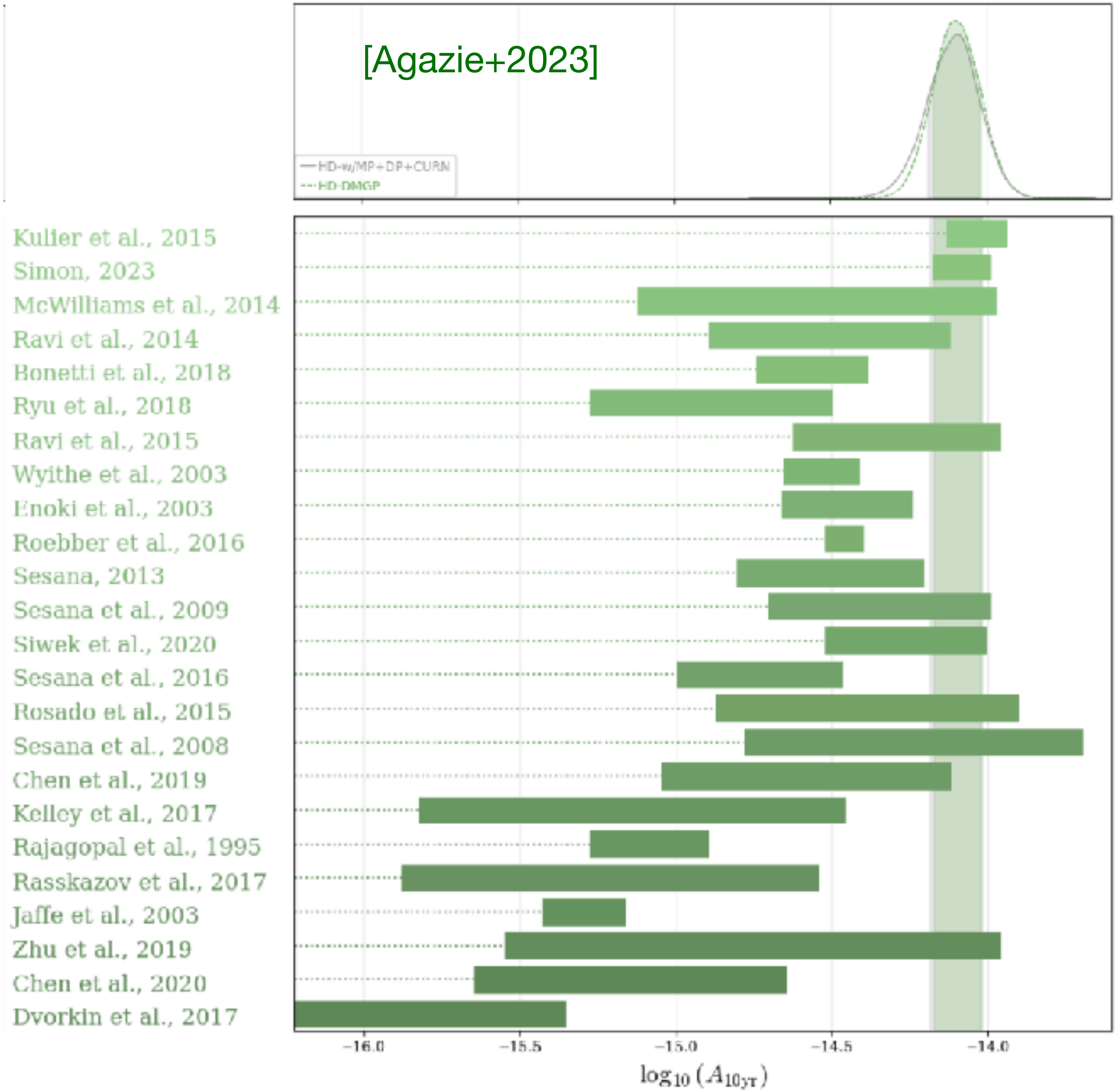
Evolution of massive BH binaries:

$M_{BH} \sim 10^5 - 10^9 M_\odot$

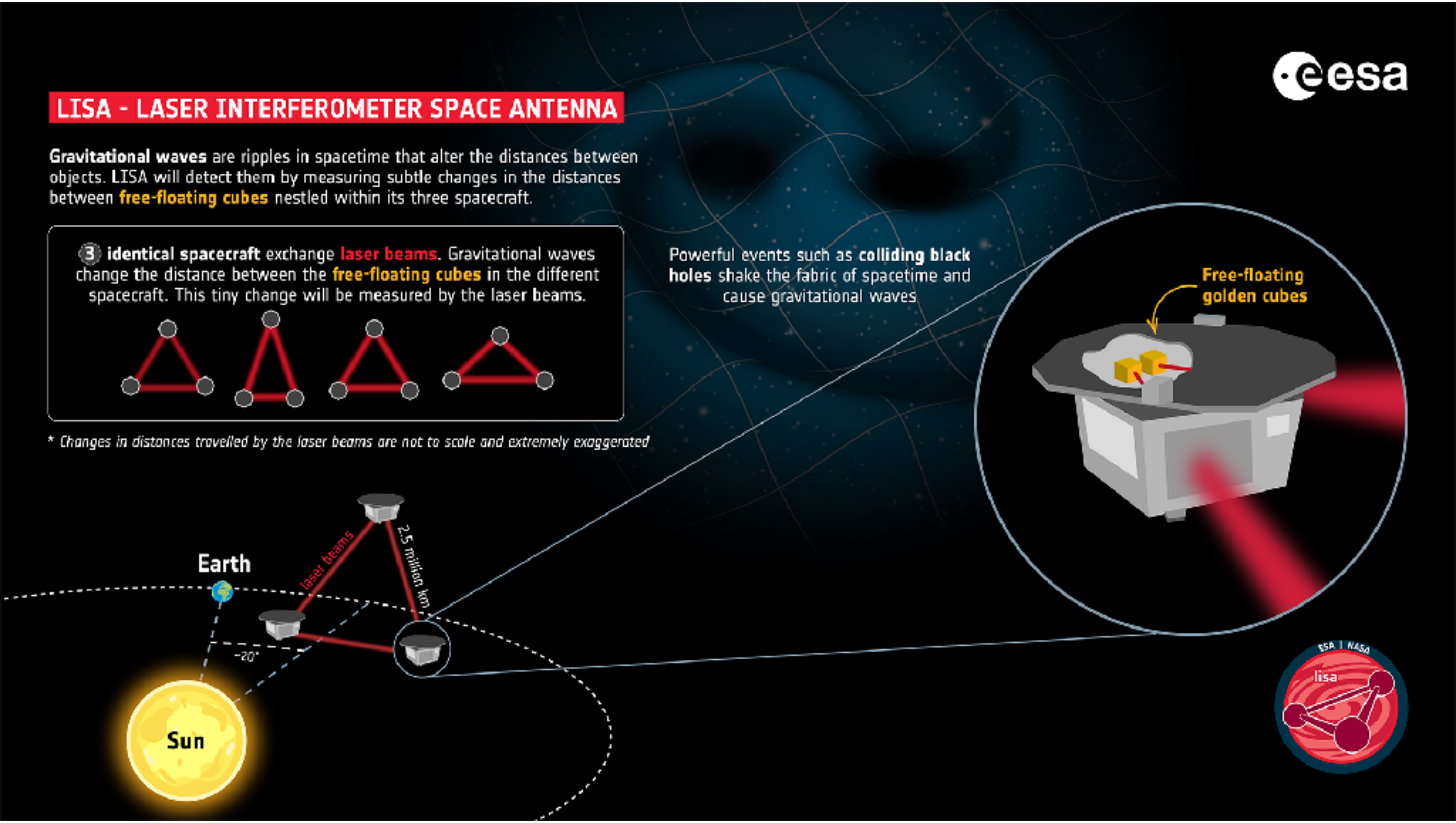
- Seed BHs grow through accretion in galactic centers
- Two galaxies that host BHs merge ***(10-100 kpc)***
- Dynamical friction of BHs with surrounding gas —> bound BH binary ***(kpc)***
- Orbit decay through interactions with surrounding gas and stars ***(pc)***
- Emission of GW —> merger ***(milli-pc)***

- **Key unknown ingredients:**
  - Seeds of massive black holes
  - Co-evolution with host galaxies
  - Interactions with surrounding gas and stars

Pulsar Timing Arrays: was the background detected?



LISA: Laser Interferometry Space Antenna



LISA - LASER INTERFEROMETER SPACE ANTENNA

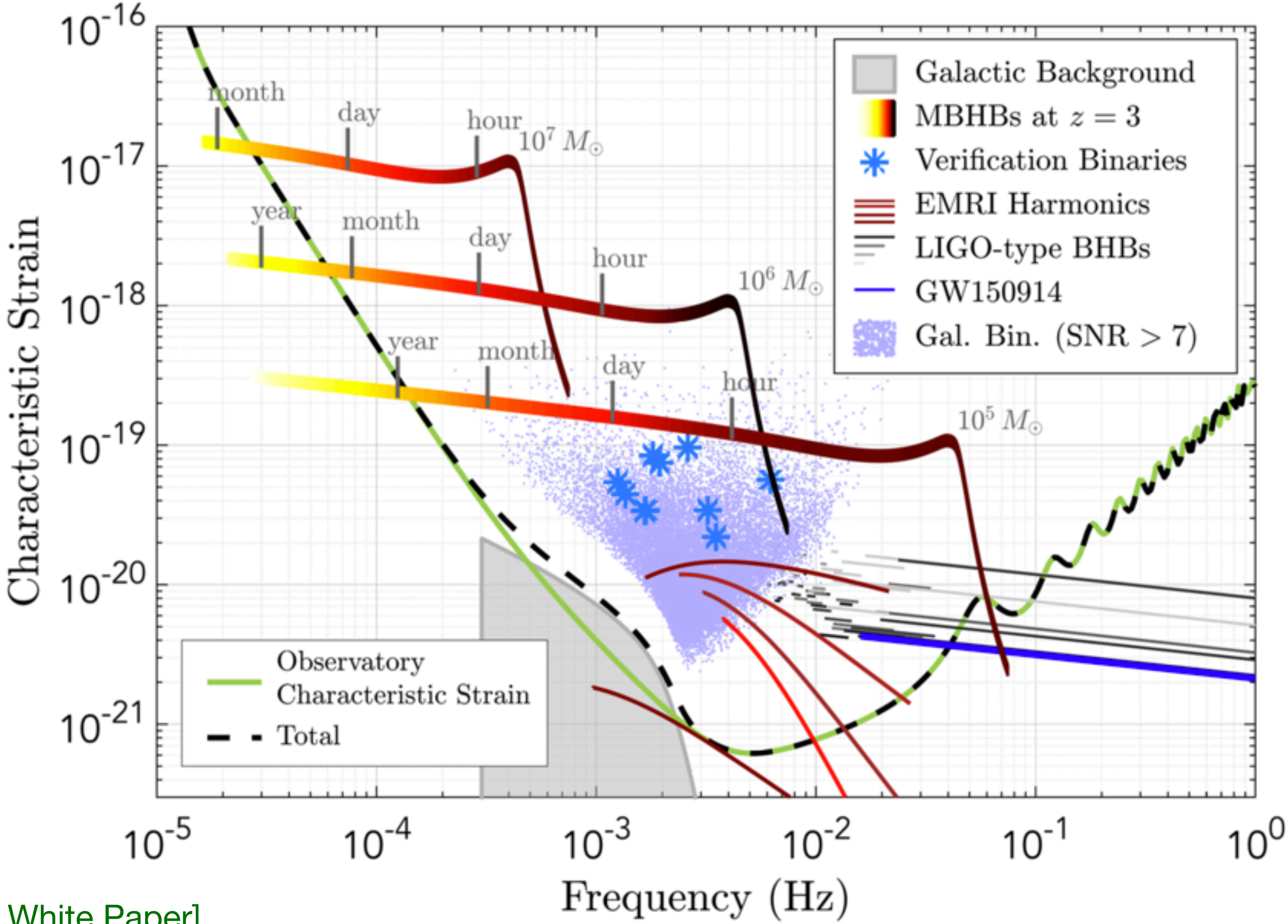
Gravitational waves are ripples in spacetime that alter the distances between objects. LISA will detect them by measuring subtle changes in the distances between **free-floating cubes** nestled within its three spacecraft.

③ **identical spacecraft** exchange **laser beams**. Gravitational waves change the distance between the **free-floating cubes** in the different spacecraft. This tiny change will be measured by the laser beams.

** Changes in distances travelled by the laser beams are not to scale and extremely exaggerated*

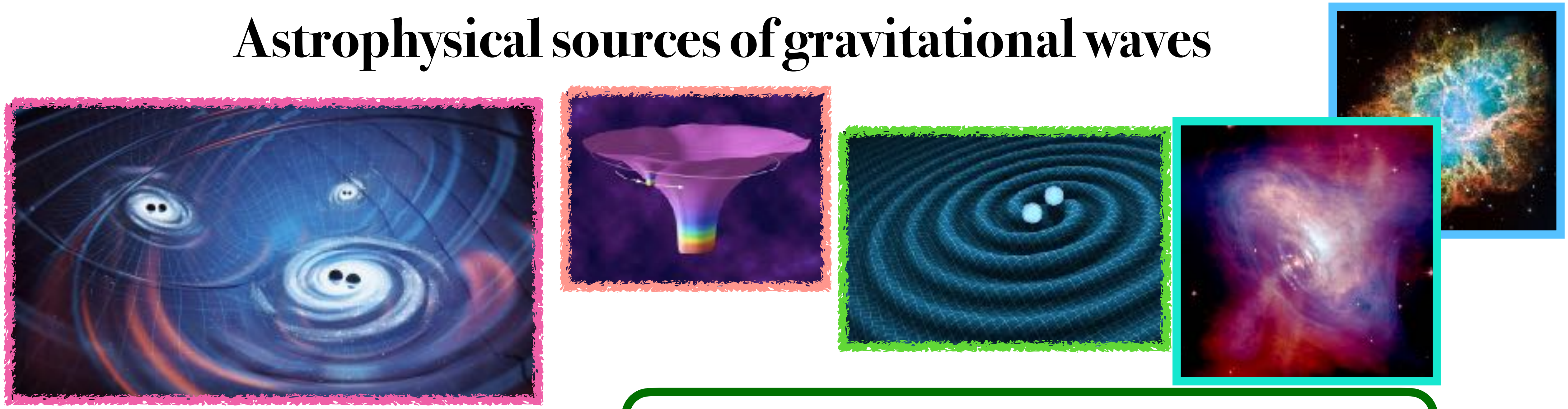
Powerful events such as **colliding black holes** shake the fabric of spacetime and cause gravitational waves

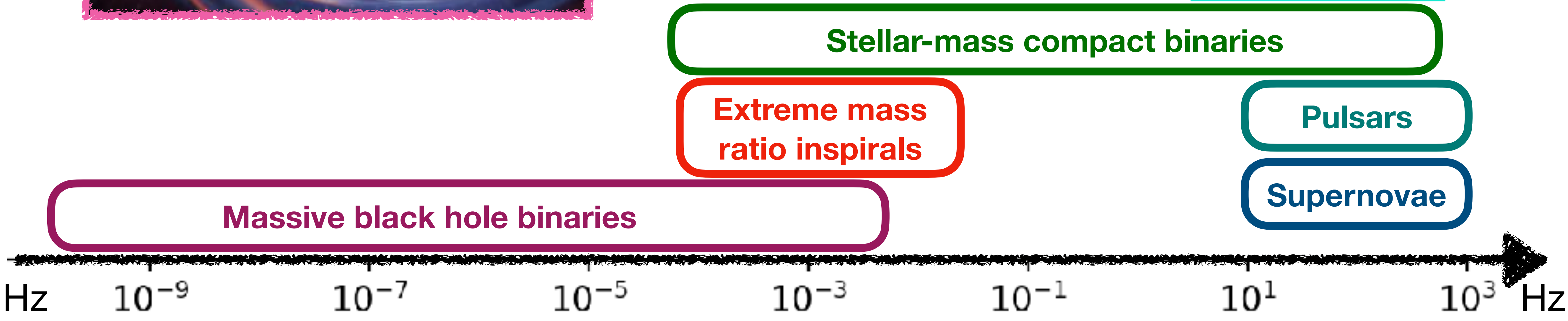
LISA: Laser Interferometry Space Antenna



[LISA White Paper]

Astrophysical sources of gravitational waves





Stellar-mass compact binaries

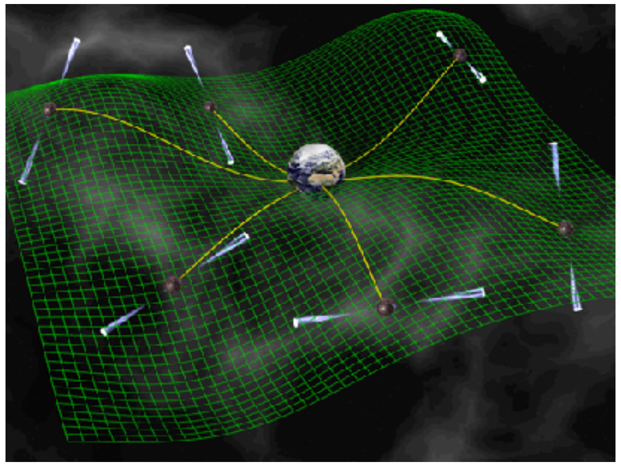
Extreme mass ratio inspirals

Pulsars

Supernovae

Massive black hole binaries

Pulsar Timing Arrays



Space-based (LISA)

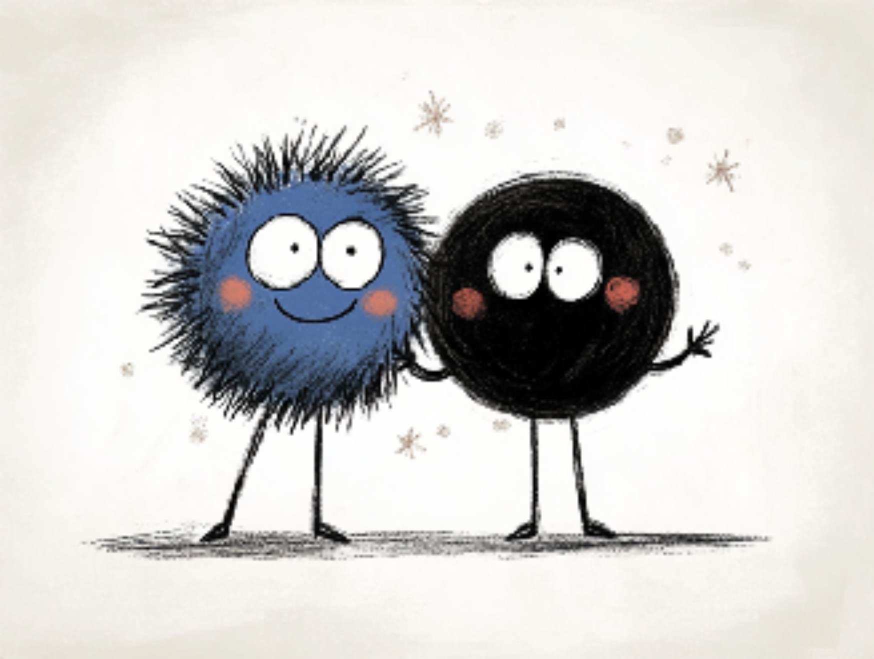
Ground-based (LIGO/Virgo/Kagra)

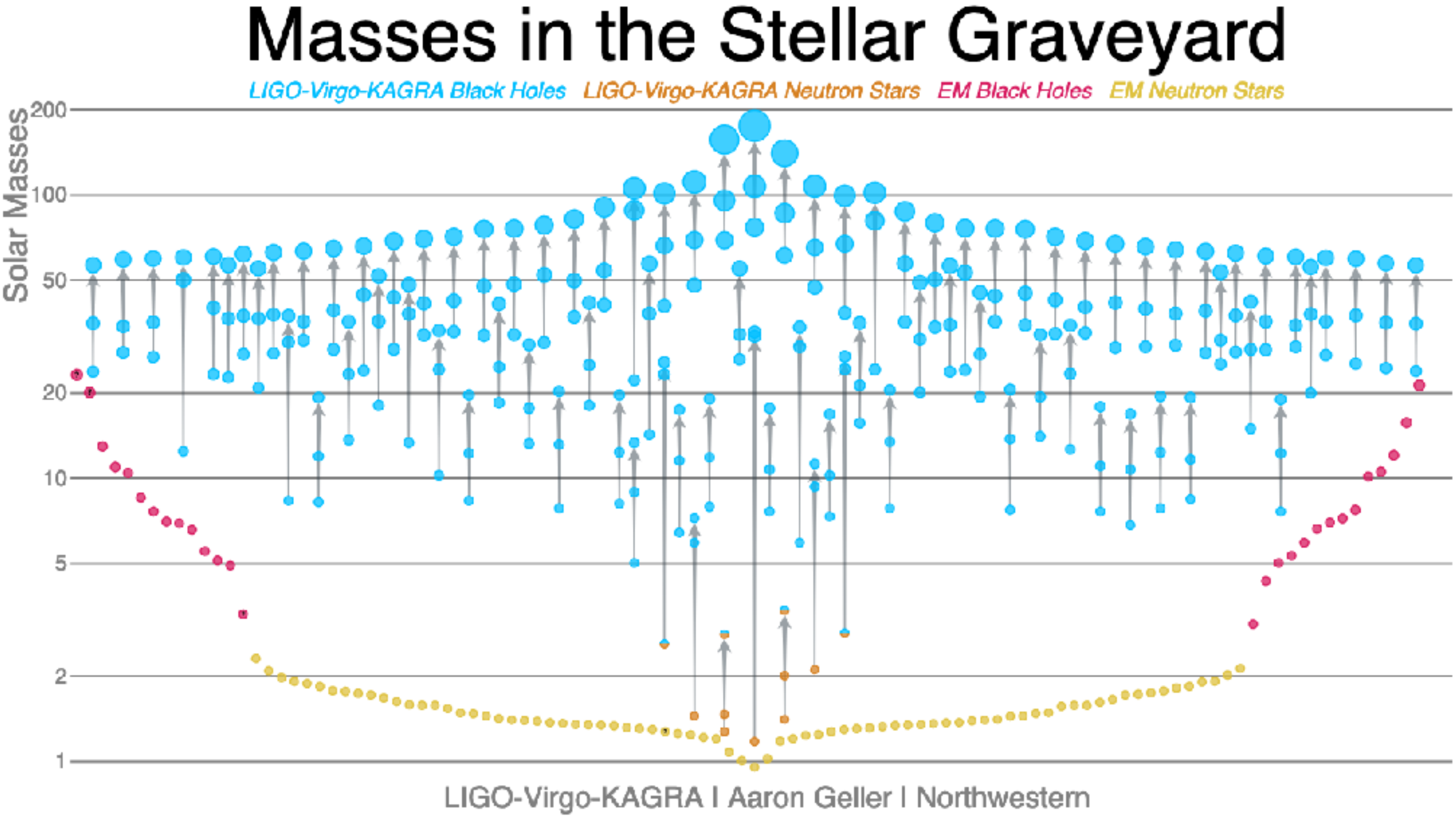
Lecture plan

- Evolution of massive stars and formation of compact objects
- Evolution of binary massive stars and formation of binary compact objects
- Gravitational-wave astronomy
- Gravitational-wave observations of binary compact objects

Lecture plan

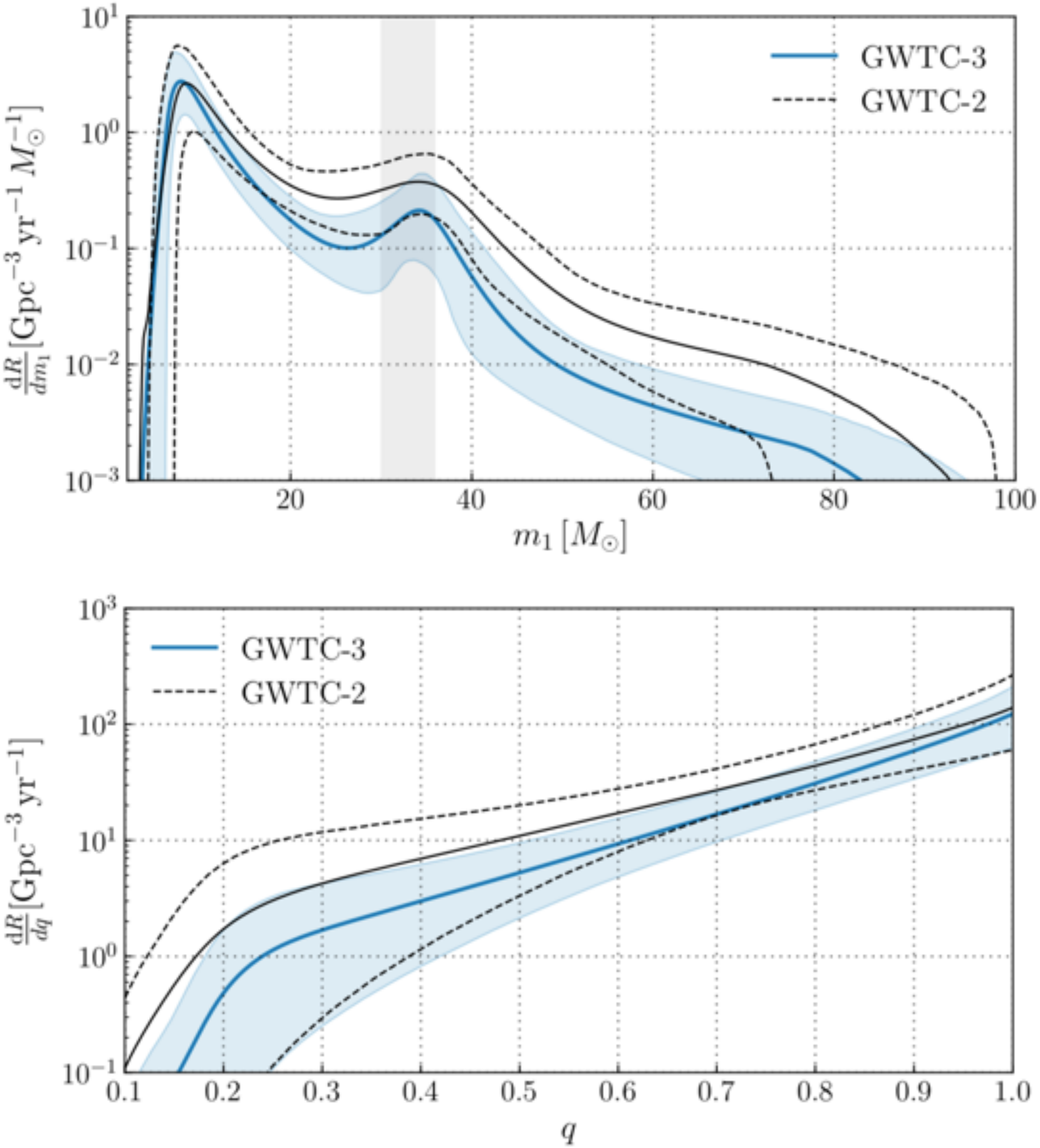
- Gravitational-wave observations of binary compact objects
    - Binary black hole mergers
    - Binary neutron star mergers: GW and EM observations
    - Stochastic GW backgrounds





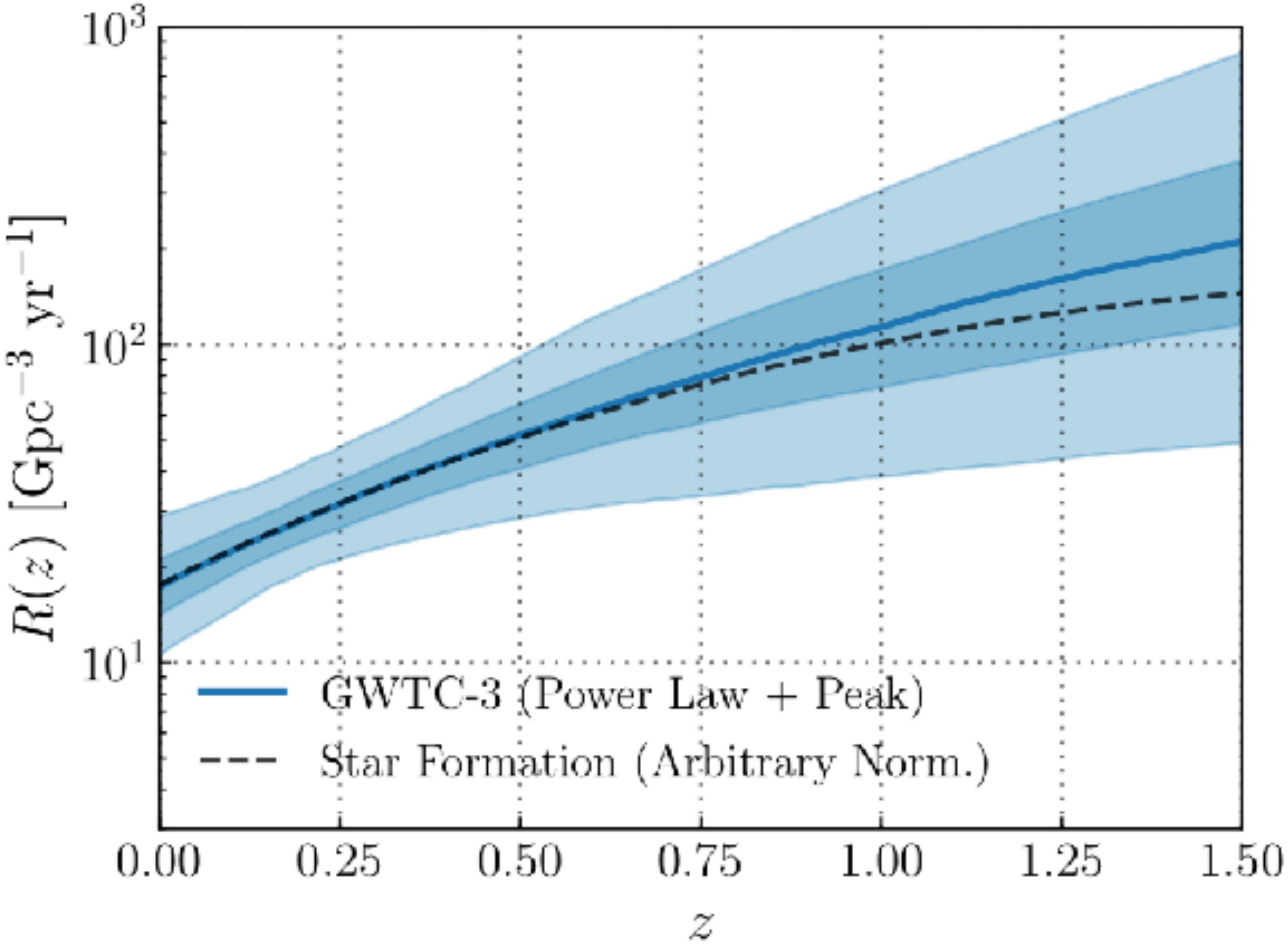
Abbott et al. 2019, PRX, 9, 031040; Abbott et al. 2021, PRX, 11, 021053;
Abbott et al. 2021, arXiv:2111.03606; Abbott et al. 2021, arXiv:2108.01045

Black hole populations: mass distribution



[Abbott et al. 2023, PRX, 13, 011048]

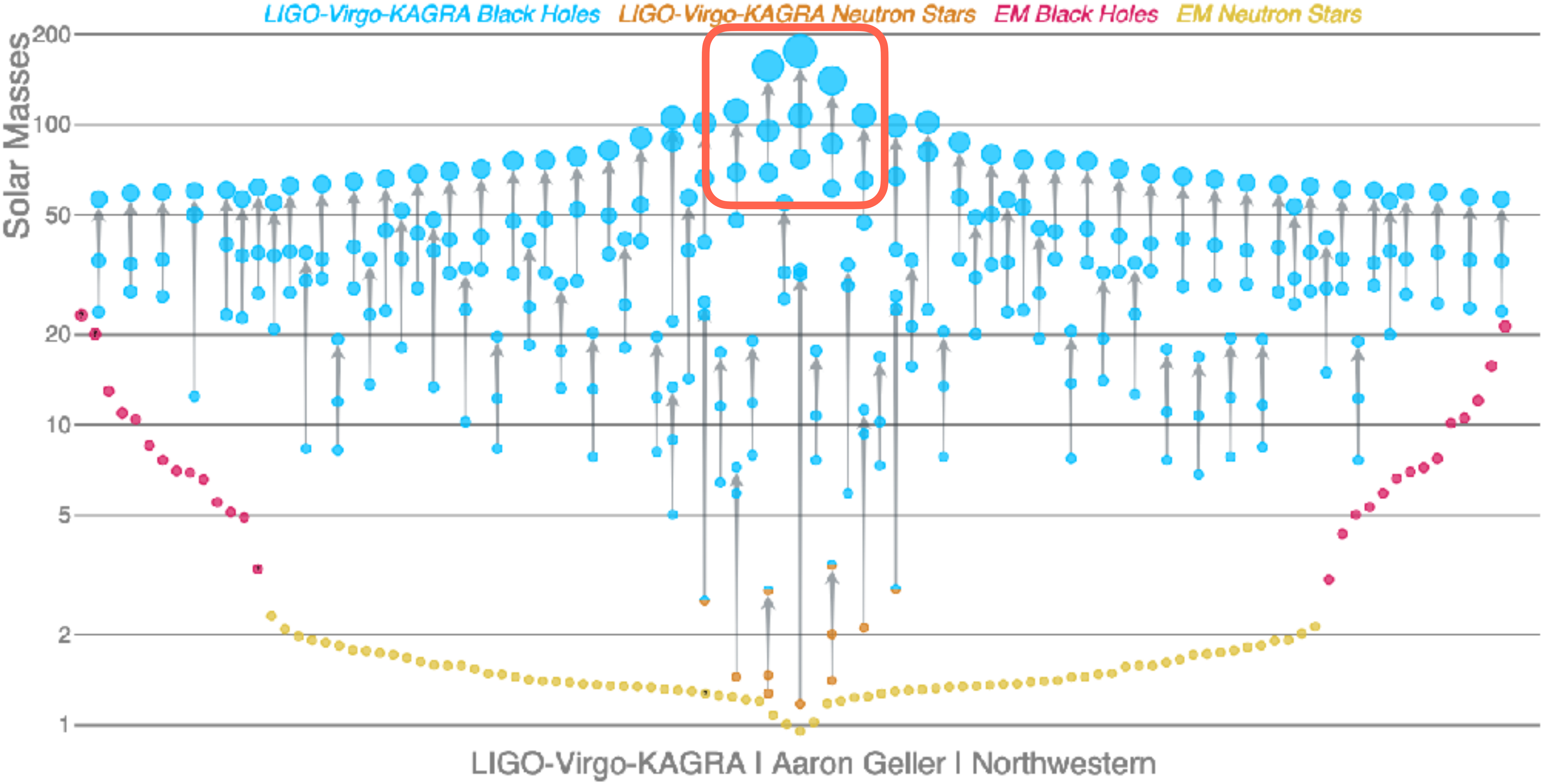
Black hole populations: merger rate evolution



$$R_{BBH}(z = 0.2) = 17.3 - 45 \quad Gpc^{-3}yr^{-1}$$

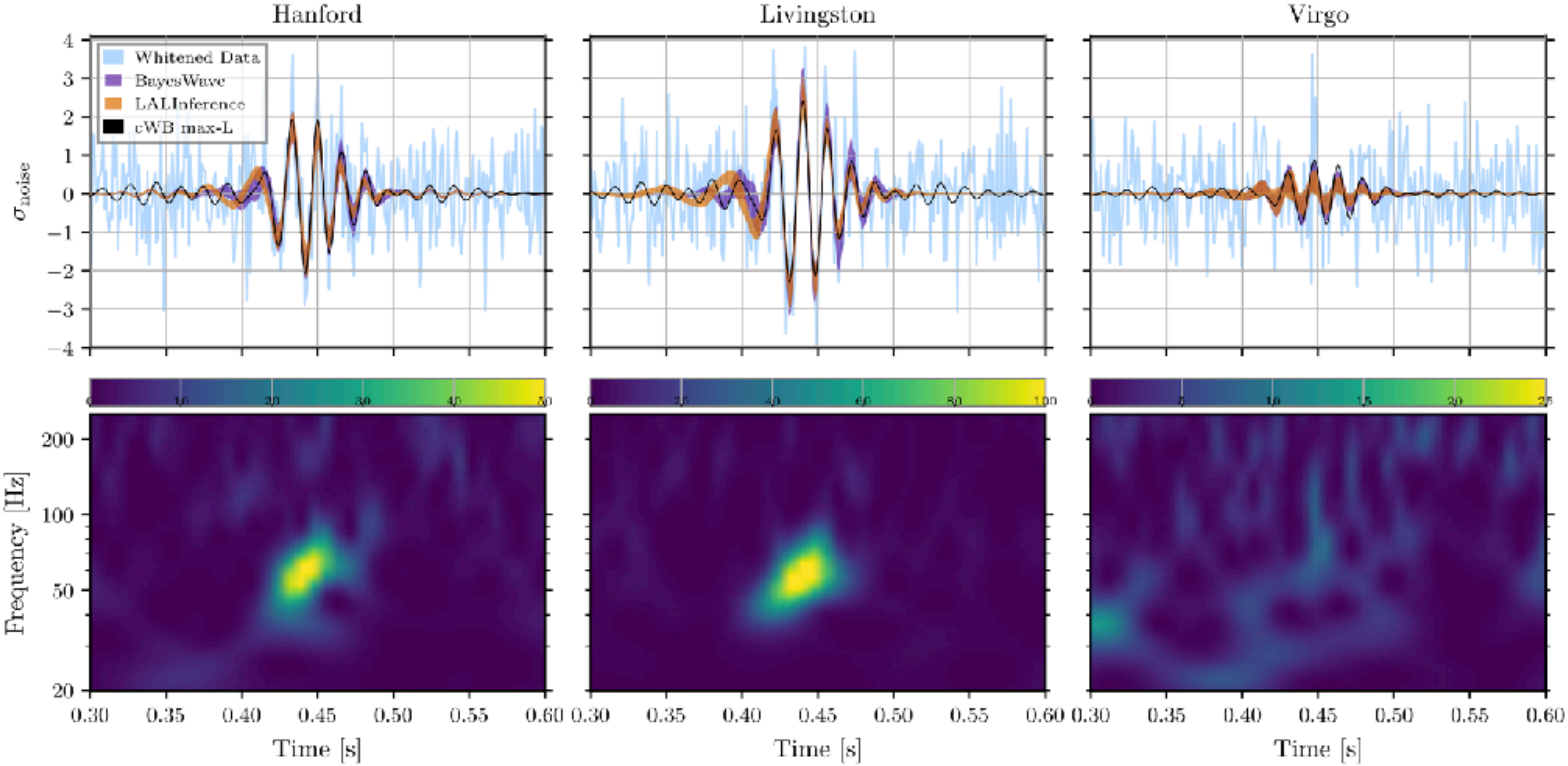
[Abbott et al. 2023, PRX, 13, 011048]

Masses in the Stellar Graveyard



Abbott et al. 2019, PRX, 9, 031040; Abbott et al. 2021, PRX, 11, 021053;
Abbott et al. 2021, arXiv:2111.03606; Abbott et al. 2021, arXiv:2108.01045

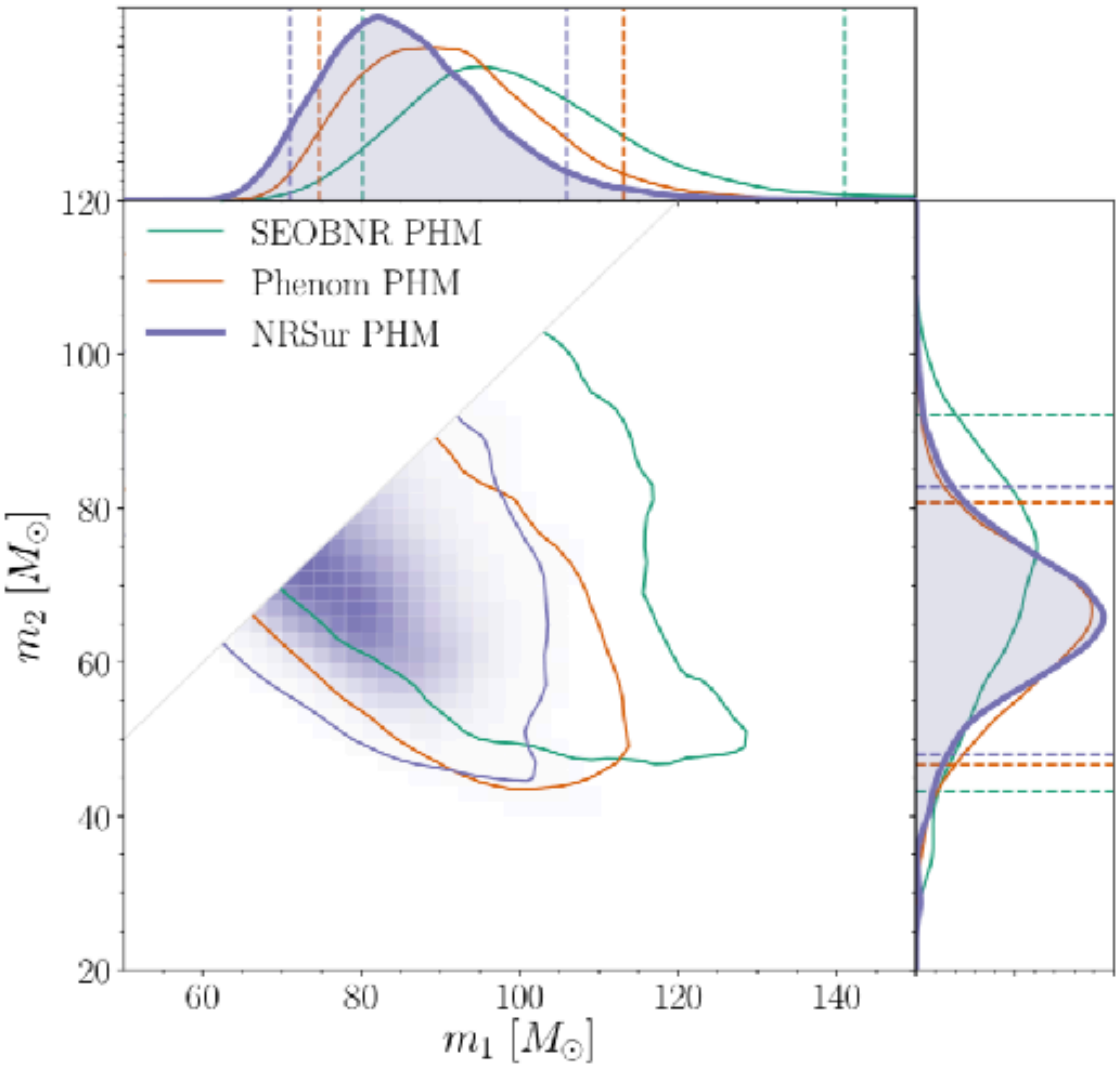
GW190521



[Abbott et al. 2020, PRL **125**, 101102]

[Abbott et al. 2020, ApJL, 900, 13]

Black holes in the upper mass gap



[Abbott et al. 2020, PRL **125**, 101102]

[Abbott et al. 2020, ApJL, 900, 13]

GW190521

$$m_1 = 85^{+21}_{-14}\ M_\odot$$

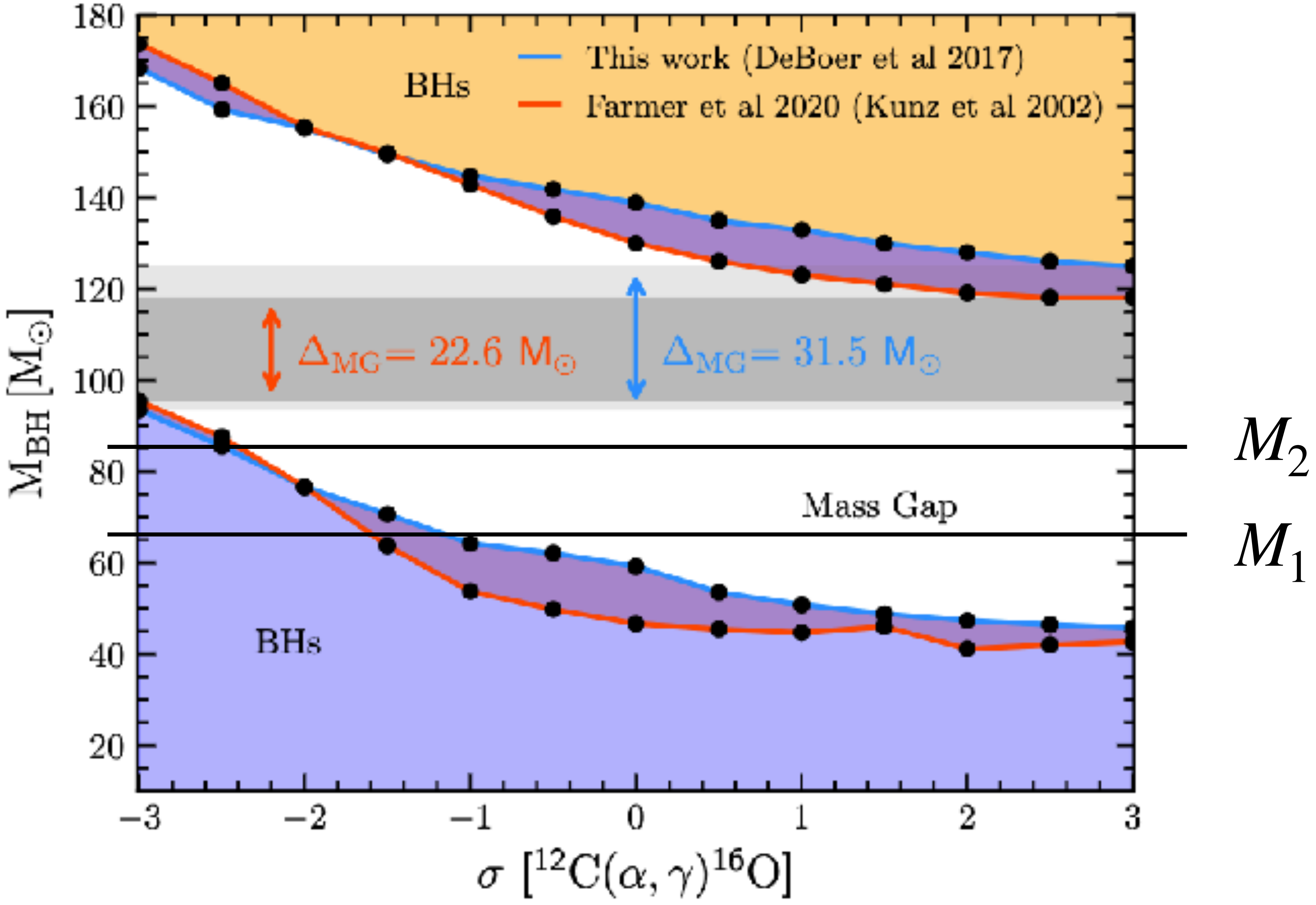
$$m_2 = 66^{+17}_{-18}\ M_\odot$$

Hierarchical merger?

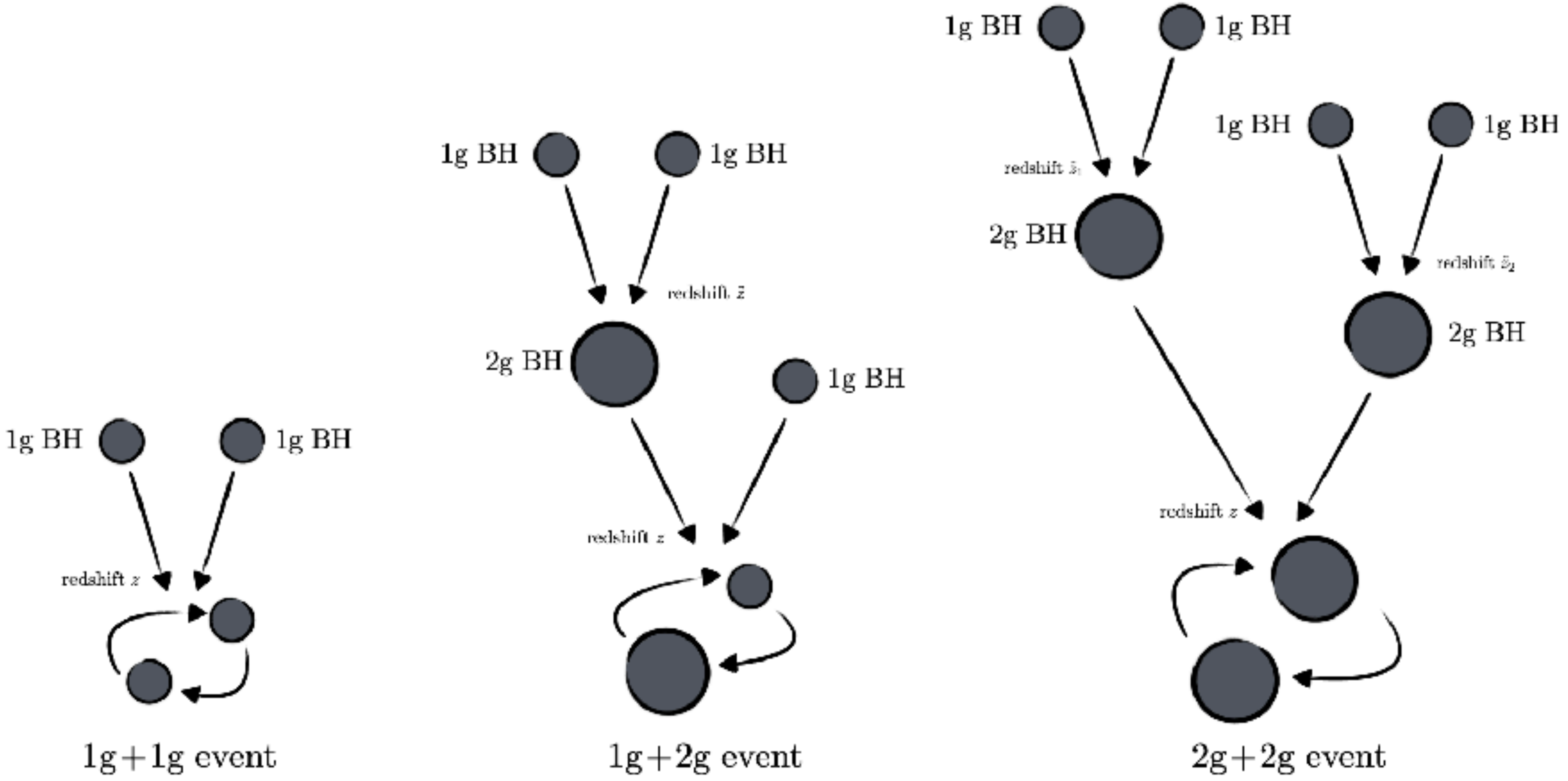
Black hole formed in the mass gap?

PISN: Uncertainty in nuclear reaction rates

$$^{12}\mathrm{C} + ^{4}\mathrm{He} \rightarrow ^{16}\mathrm{O}$$



Dynamical formation

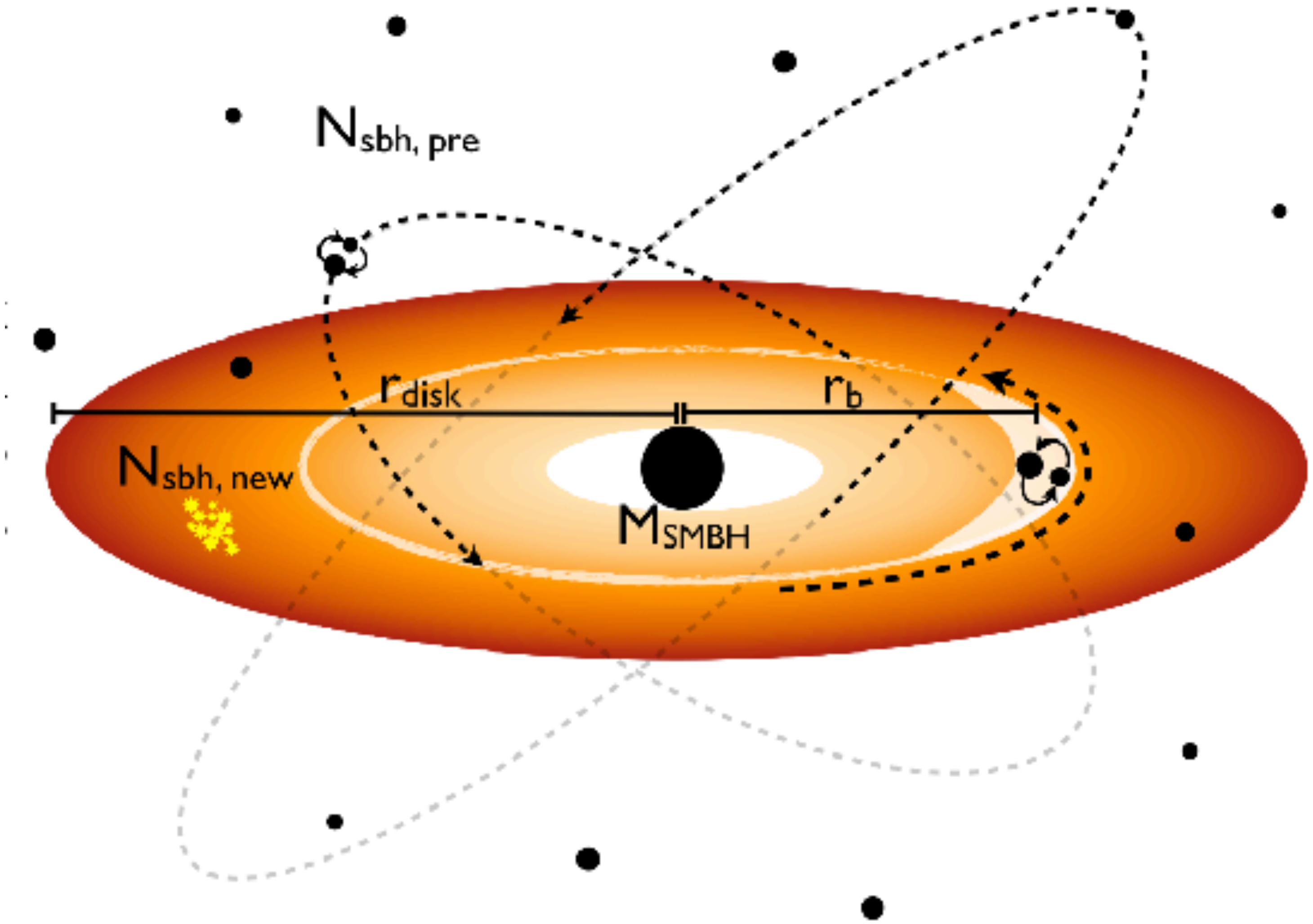


[figure: Gerosa&Berti 2017]

BBH mergers in AGN disks?

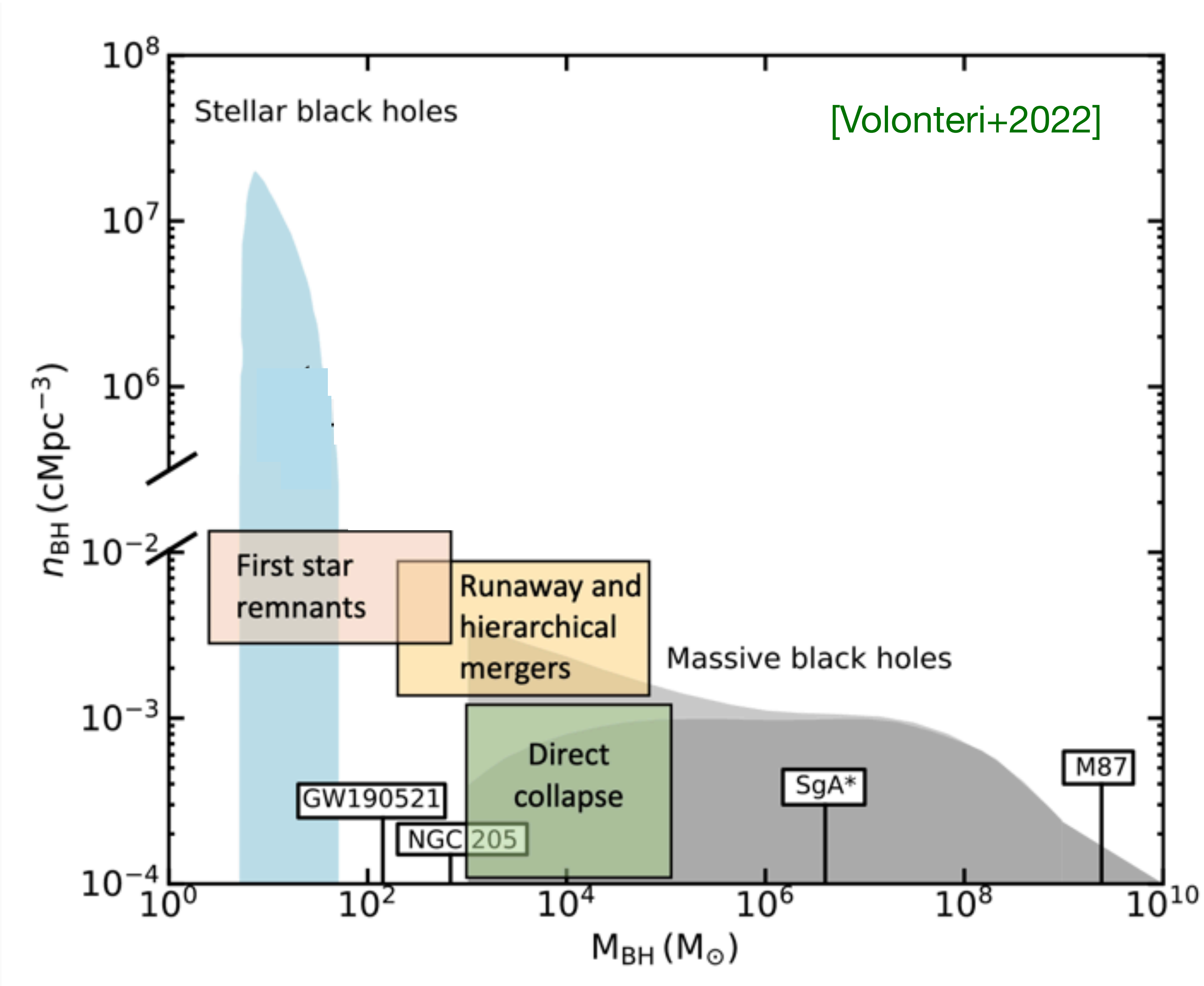
- AGN + gaseous disk + distribution of BHs
- Some BHs get trapped in the disk
- Torques from gas: BHs migrate within the disk and merge
- BH can grow by gas accretion —> IMBH

Optical counterpart to GW190521: J124942.3+344929 ? [Graham+2020]

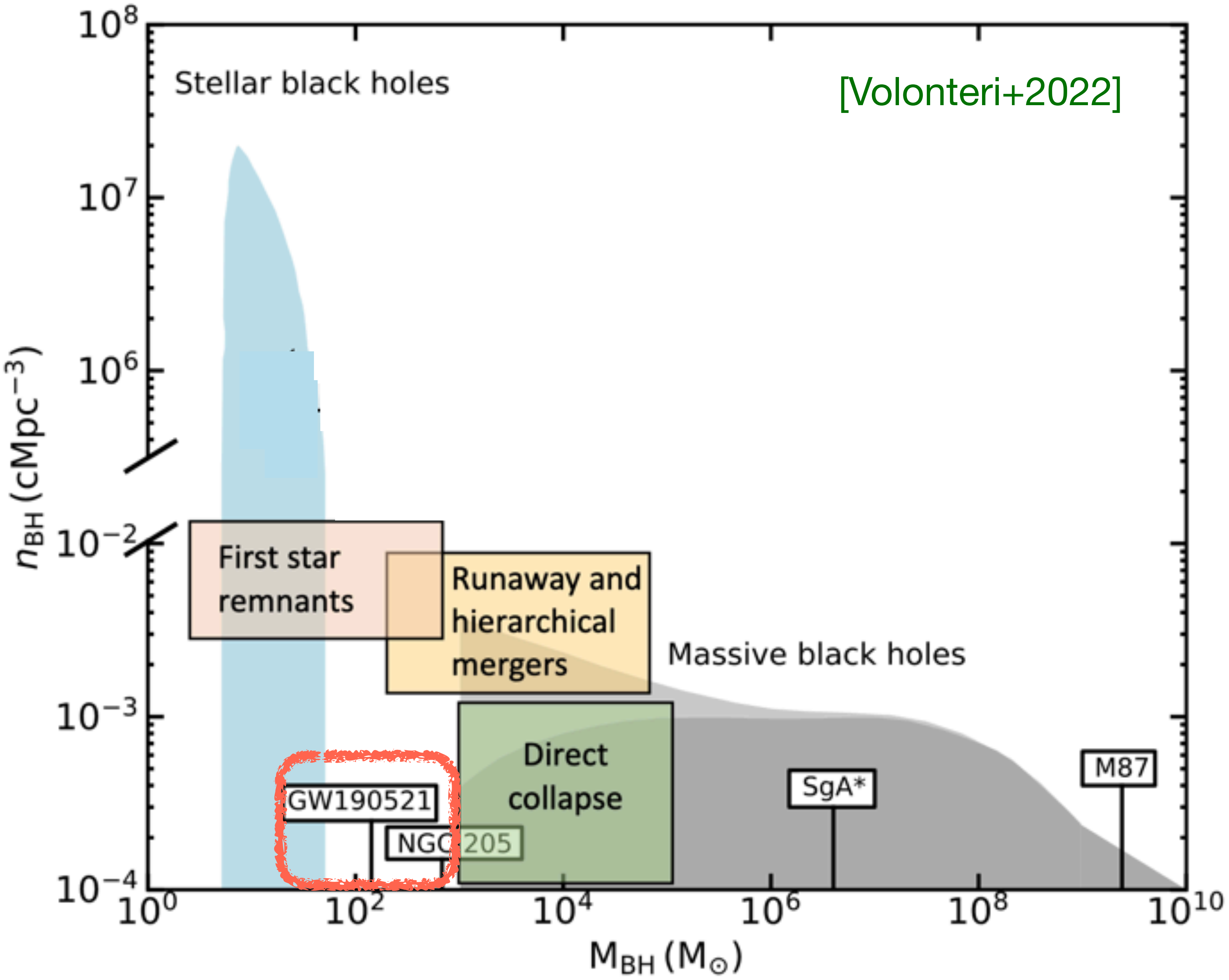


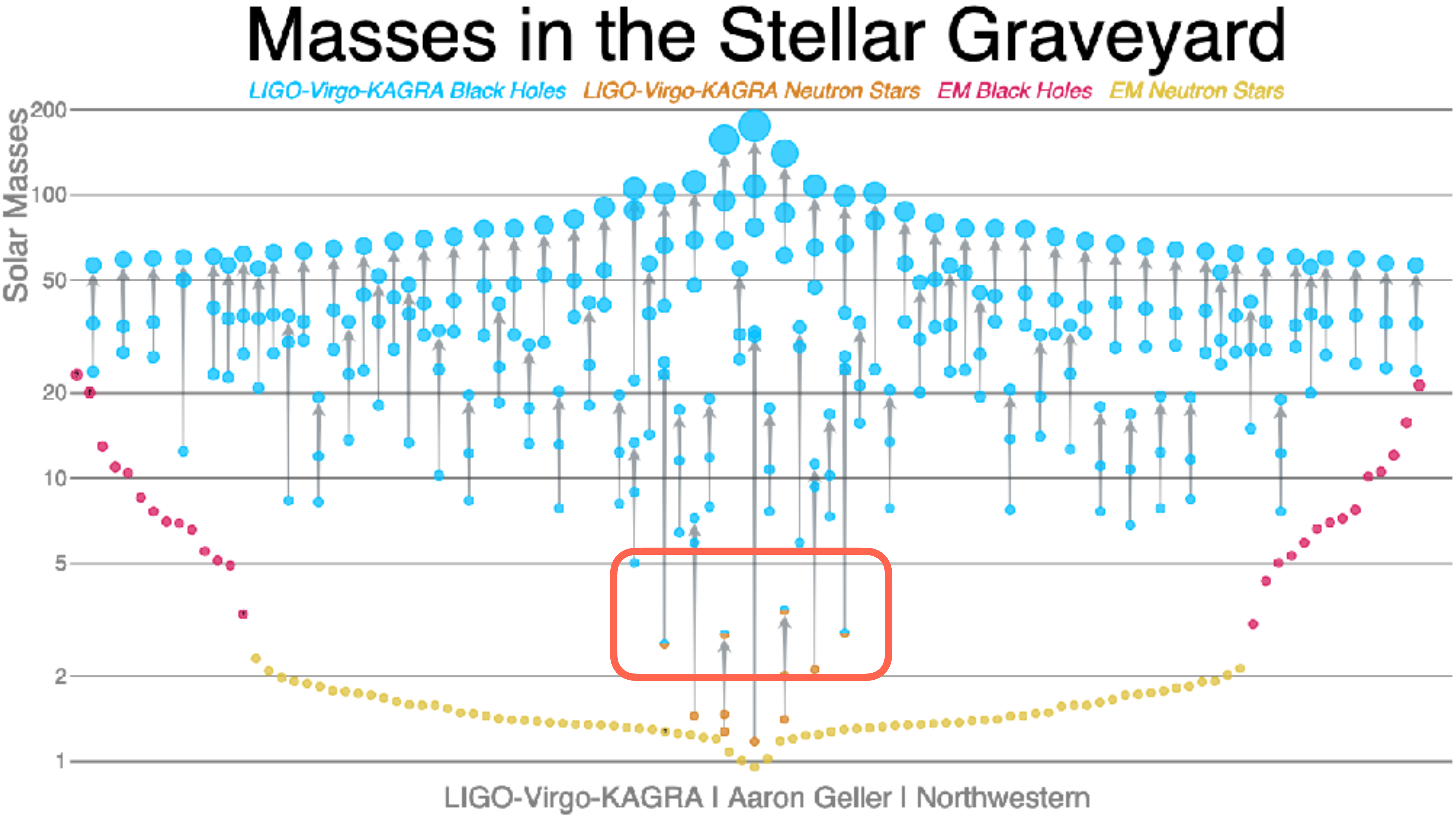
[Saavik Ford+2019: Astro2020 White Paper]

The link between stellar-mass and massive black holes?



The link between stellar-mass and massive black holes?





Abbott et al. 2019, PRX, 9, 031040; Abbott et al. 2021, PRX, 11, 021053;
Abbott et al. 2021, arXiv:2111.03606; Abbott et al. 2021, arXiv:2108.01045

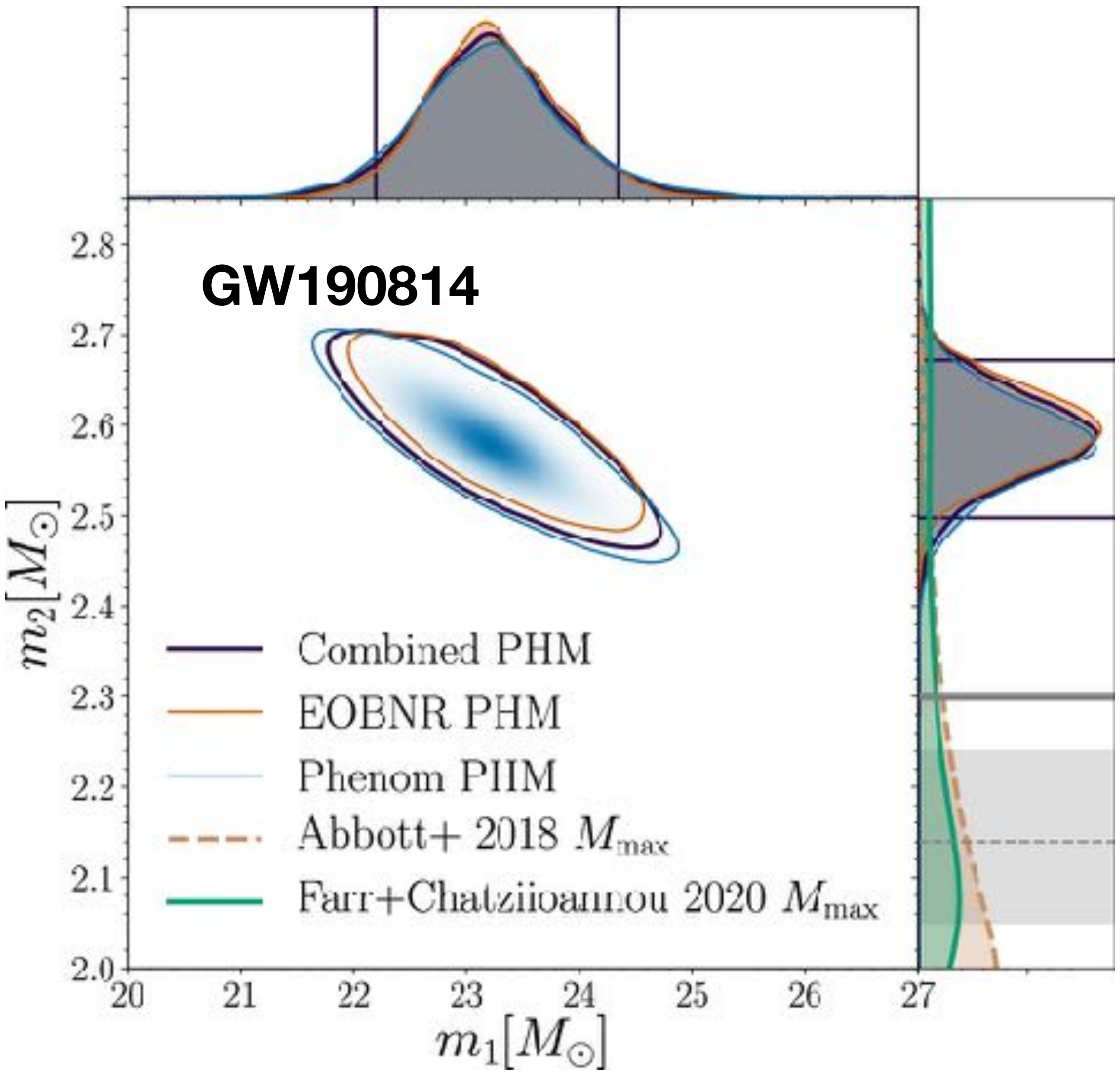
Black holes in the lower mass gap

$$m_1 = 23.3^{+1.1}_{-1.0}\ M_\odot$$

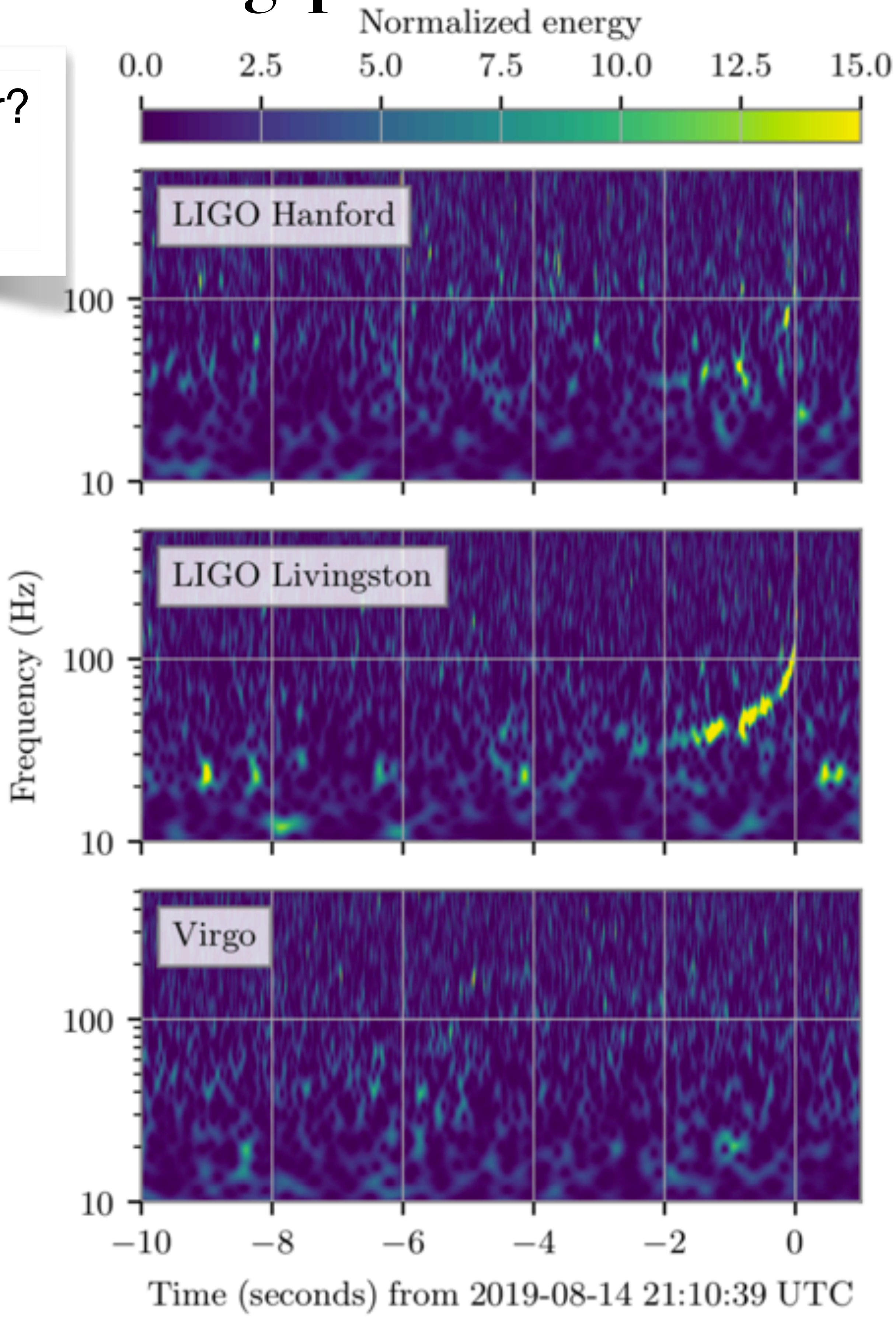
$$m_2 = 2.59^{+0.08}_{-0.09}\ M_\odot$$

Heaviest neutron star?

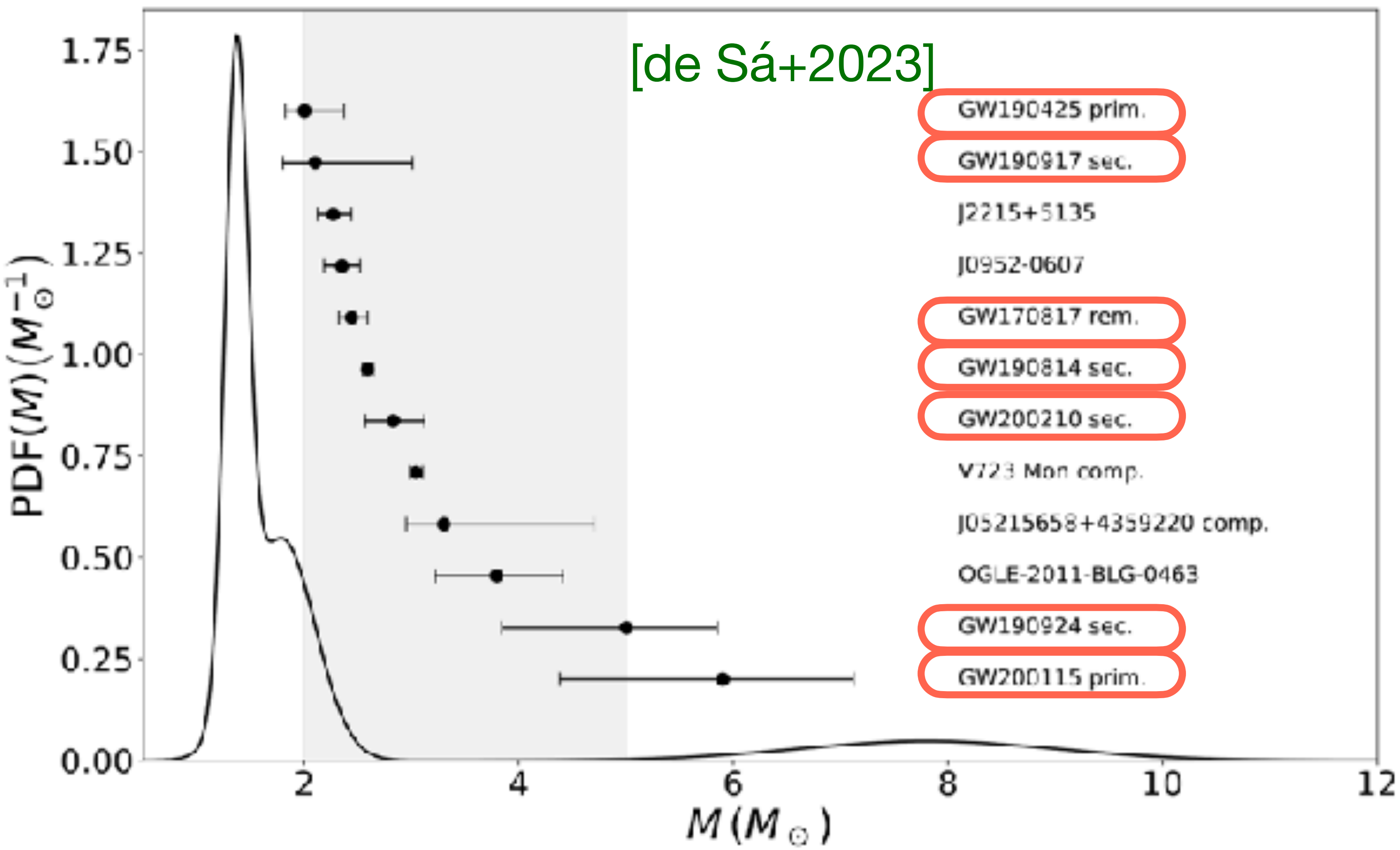
Lightest black hole?



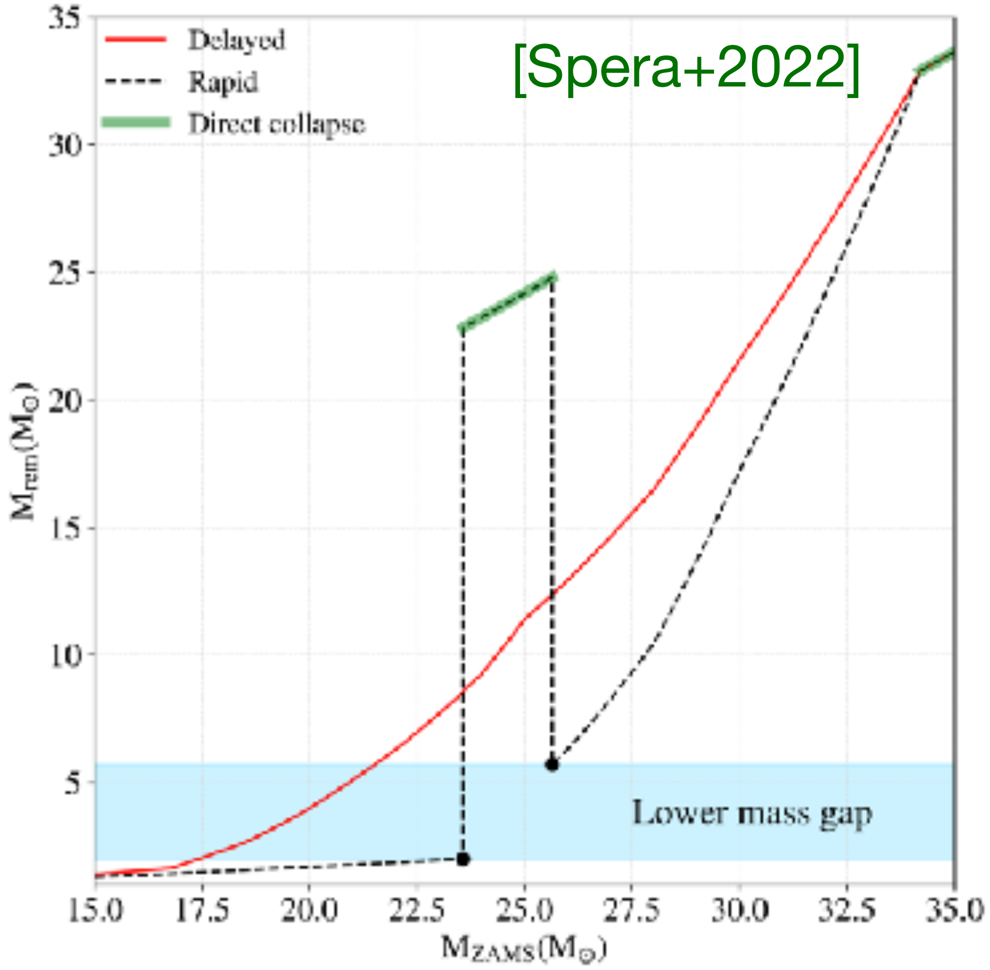
[Abbott et al. 2020, ApJL, 896, 44]



Black holes in the lower mass gap



[de Sá+2023]

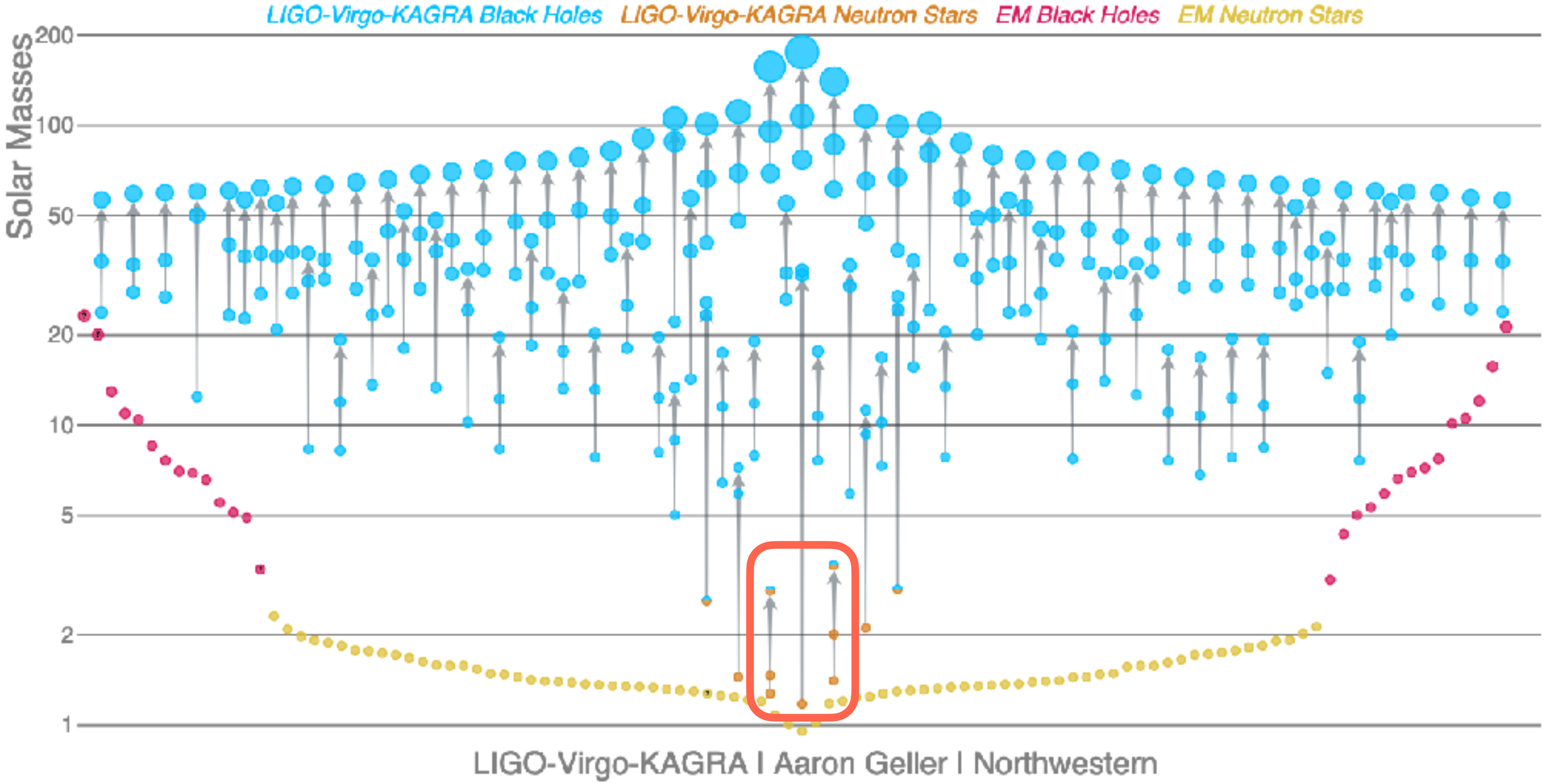


[Spera+2022]

Is the mass gap real?
Is it an observational effect?

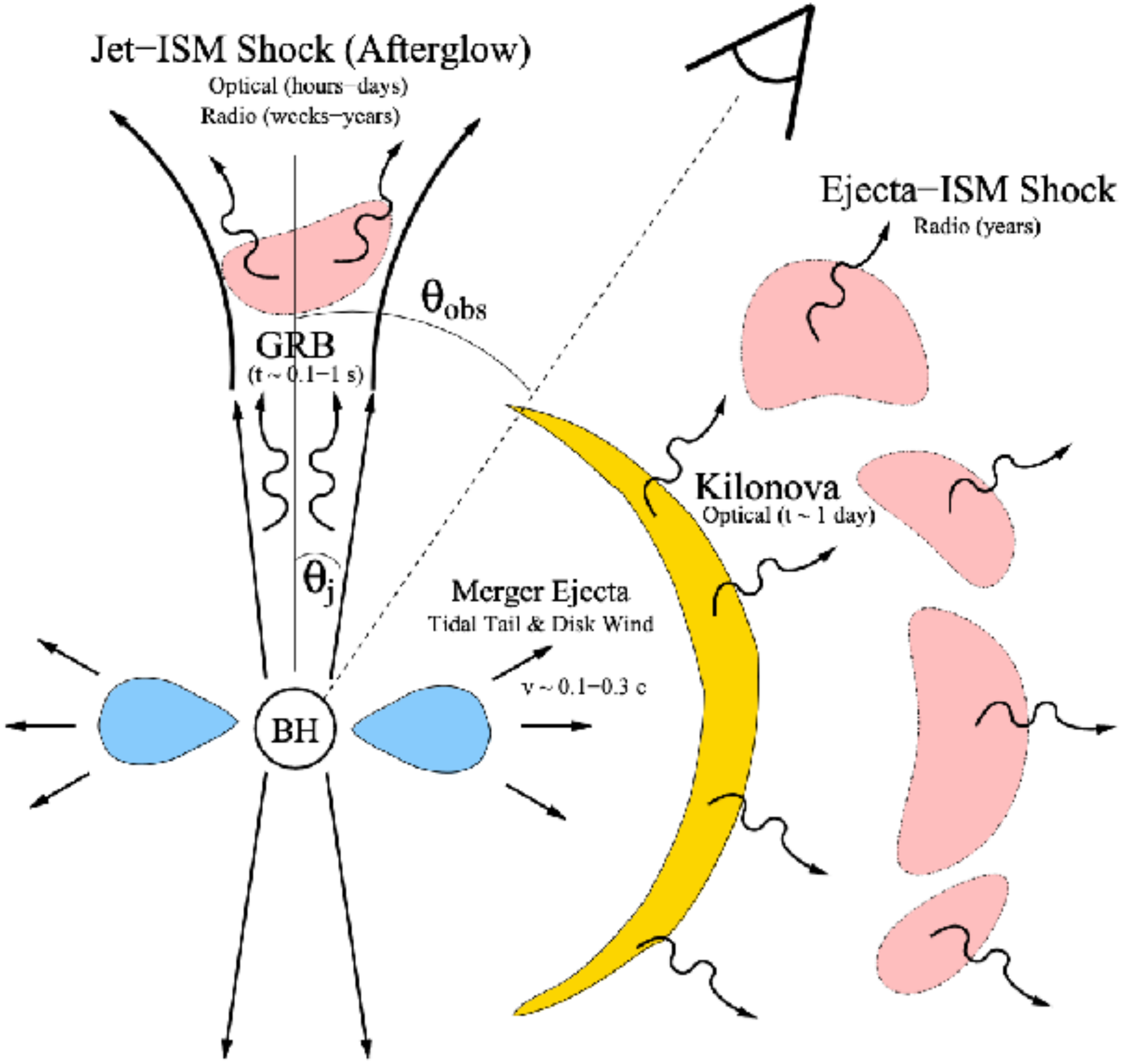
Implications for supernova explosion mechanism?

Masses in the Stellar Graveyard



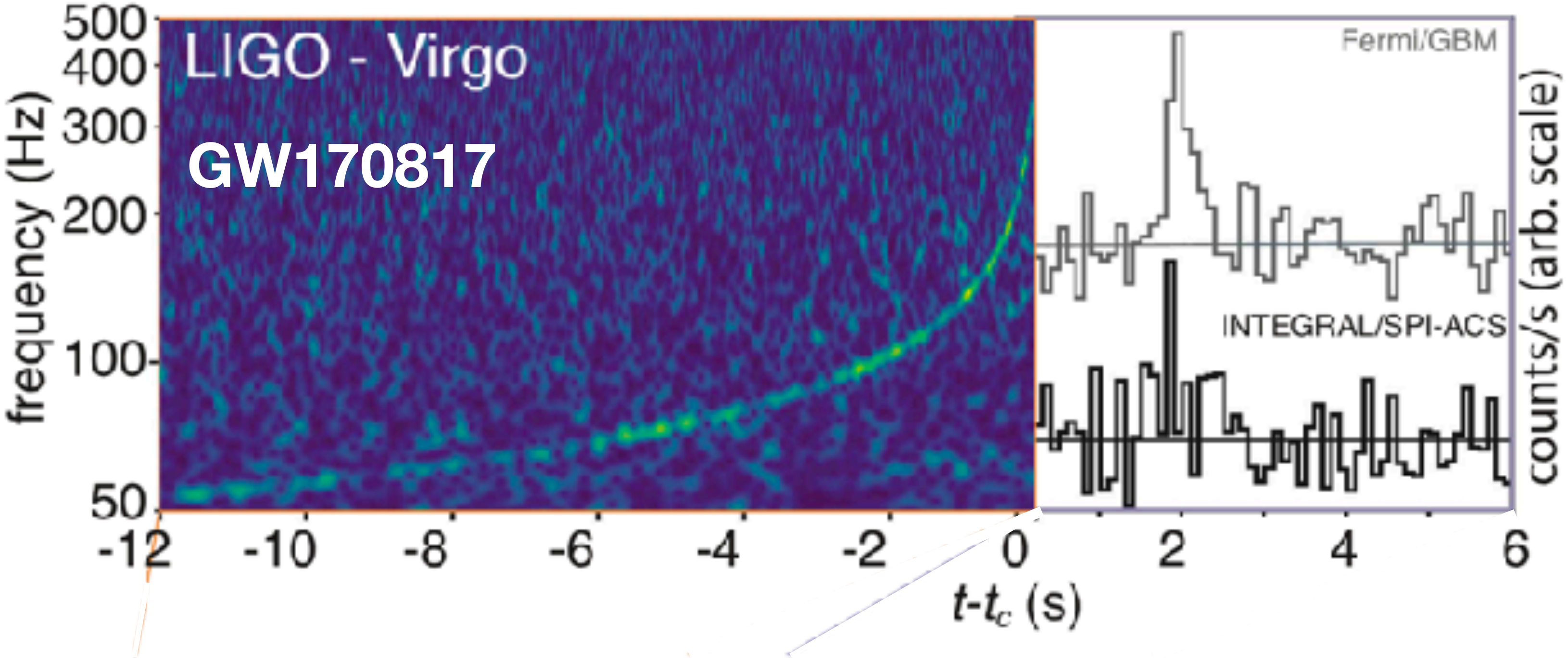
Abbott et al. 2019, PRX, 9, 031040; Abbott et al. 2021, PRX, 11, 021053;
Abbott et al. 2021, arXiv:2111.03606; Abbott et al. 2021, arXiv:2108.01045

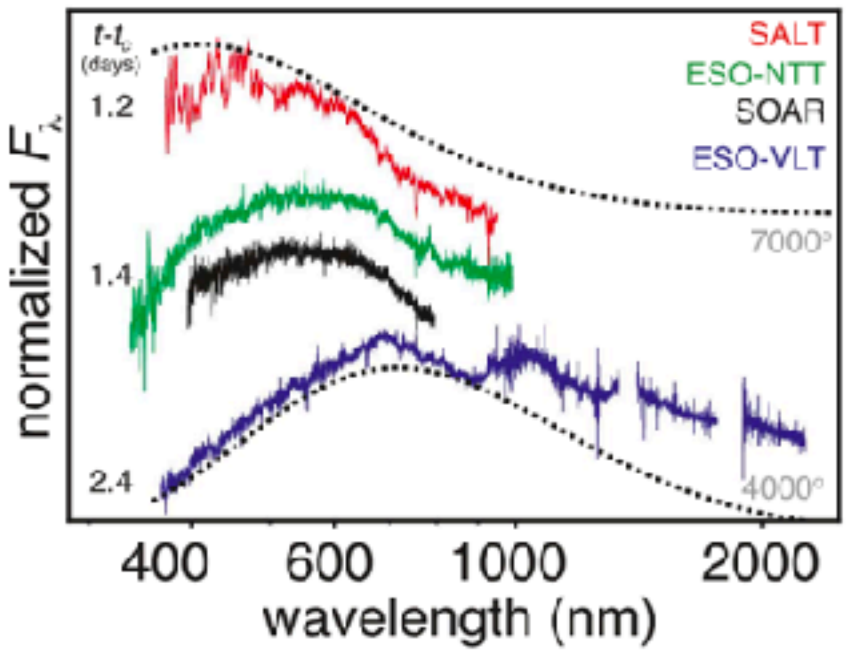
Merger of binary neutron stars

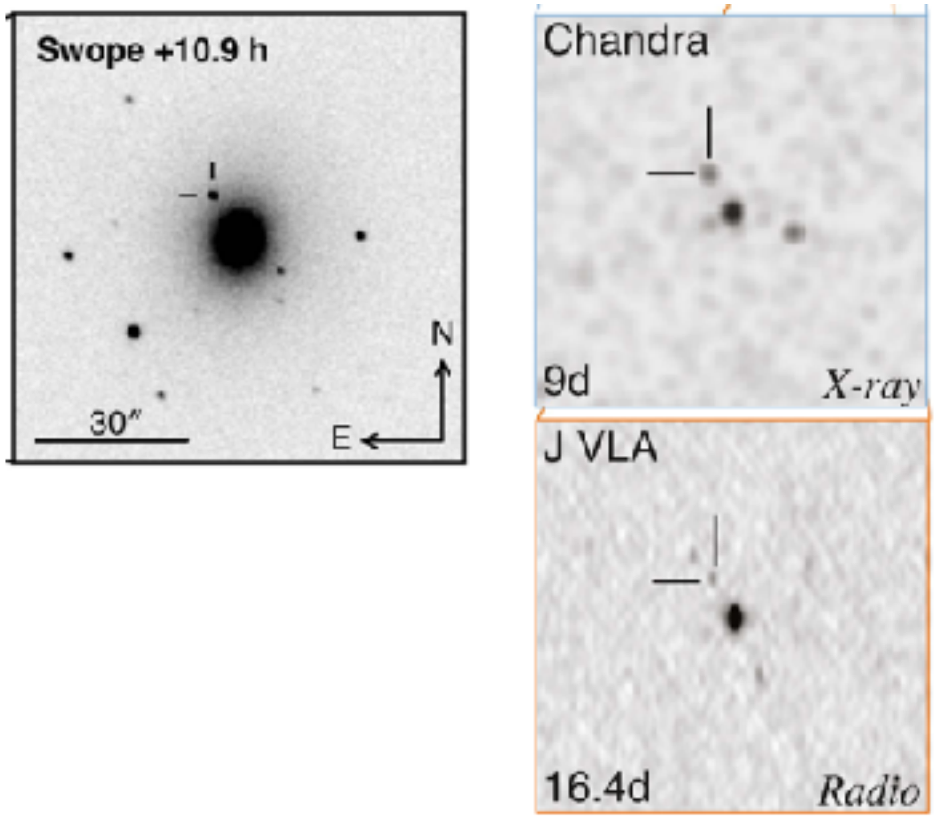


[Metzger & Berger. 2012, ApJ, 746, 48]

Binary neutron stars: multi-messenger observations!







- Gravitational Waves + Electromagnetic
- Connection with short gamma ray bursts (GRB)
- Kilonova: synthesis of heavy elements
- Identification of host galaxy: 40 Mpc away

[Abbott et al. 2017, ApJ Letters, 848, 2]

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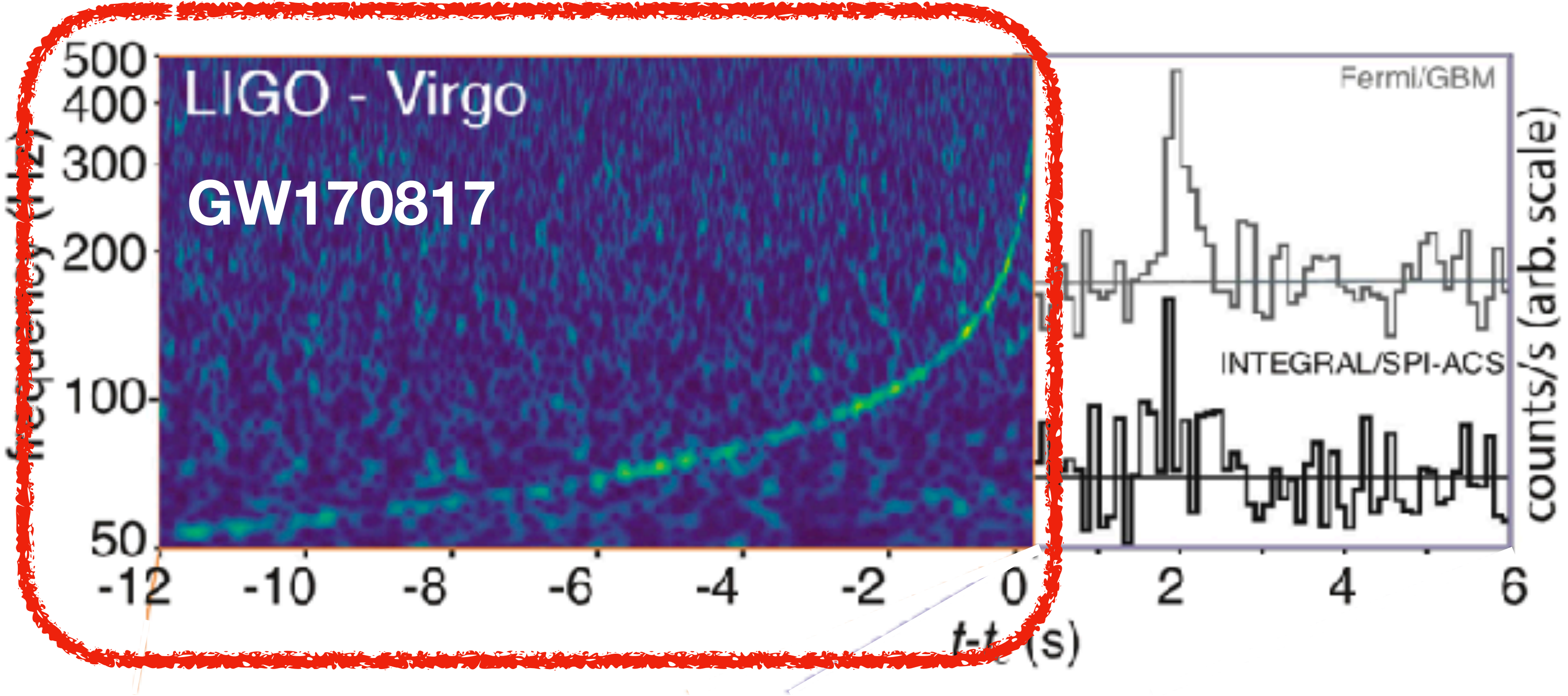

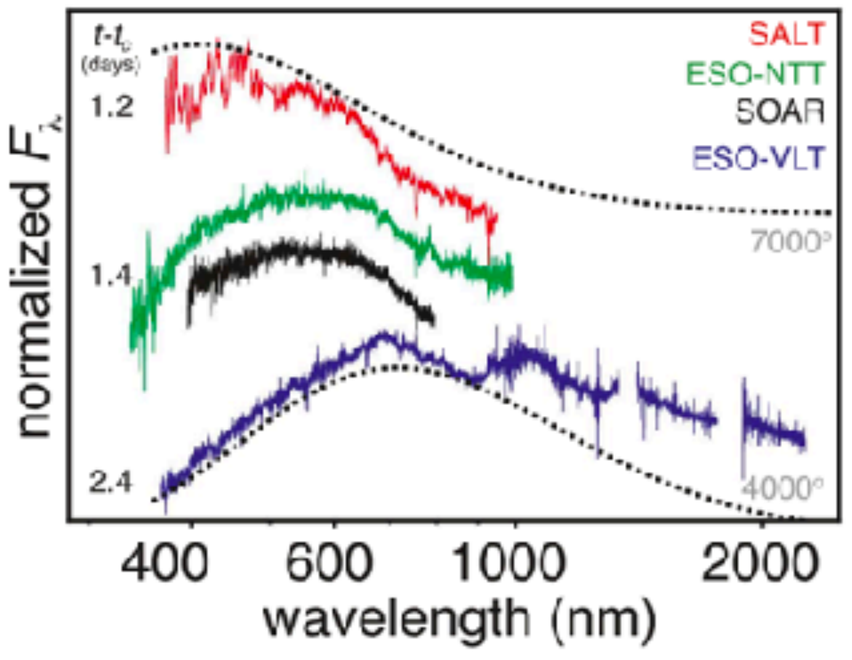

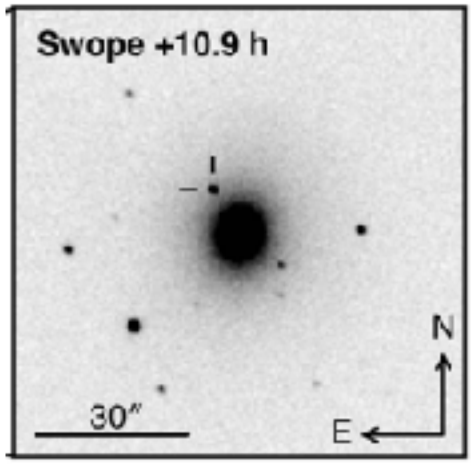

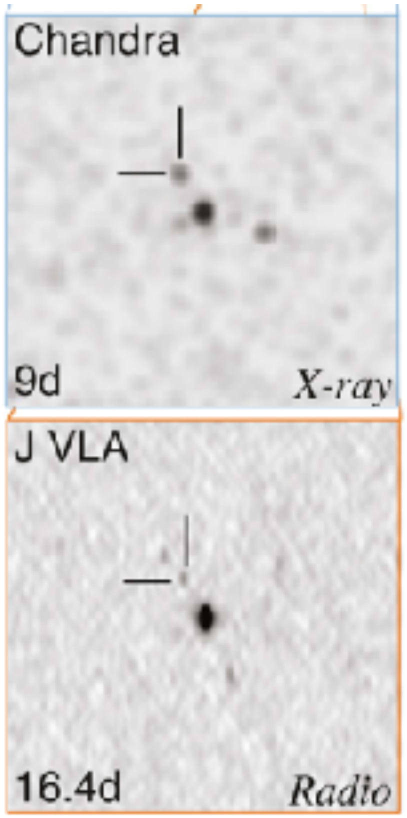

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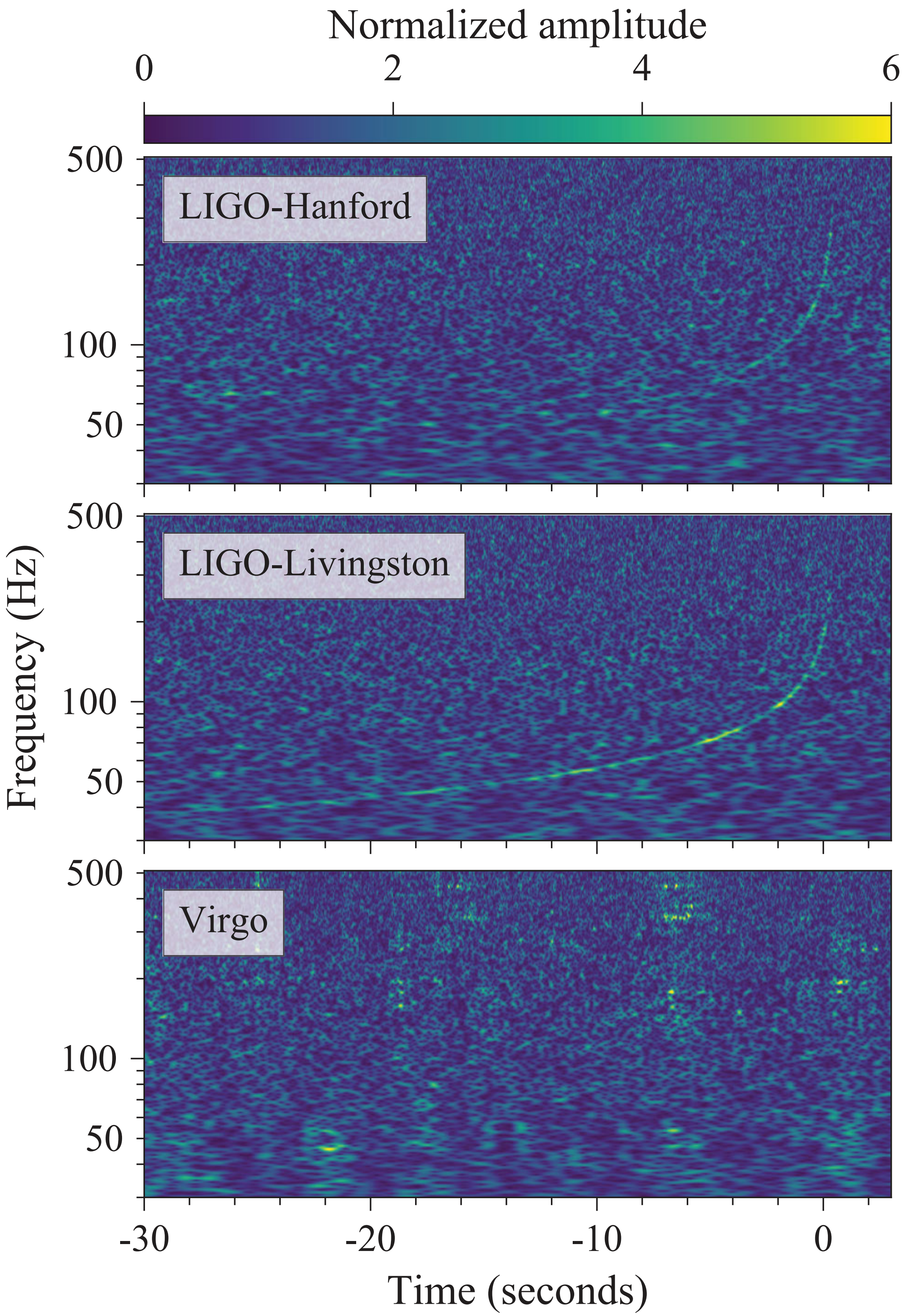
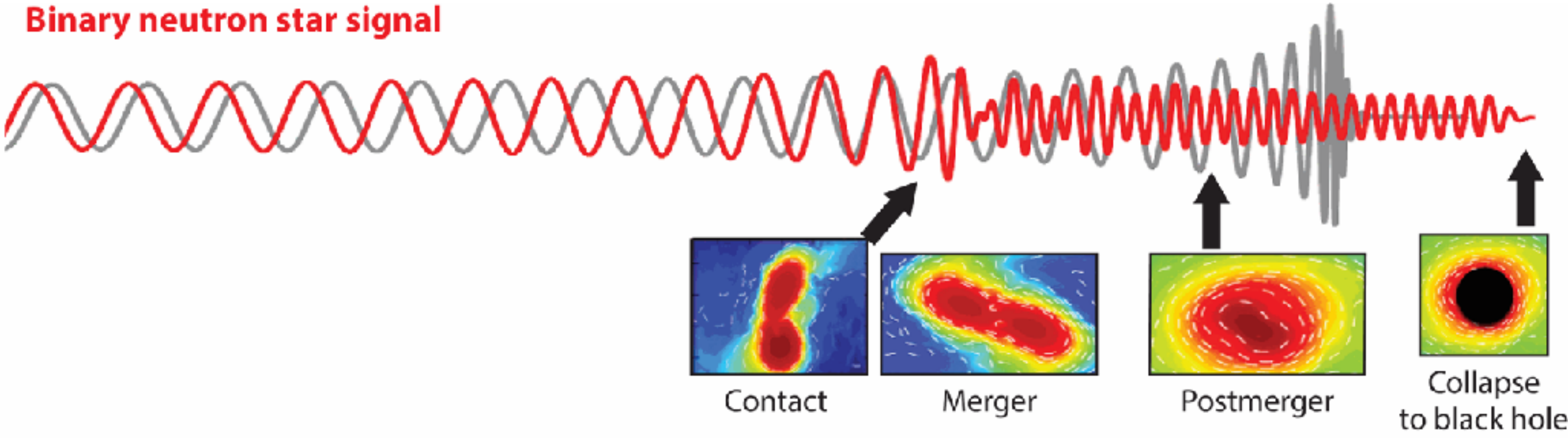
GW170817: gravitational-wave detection



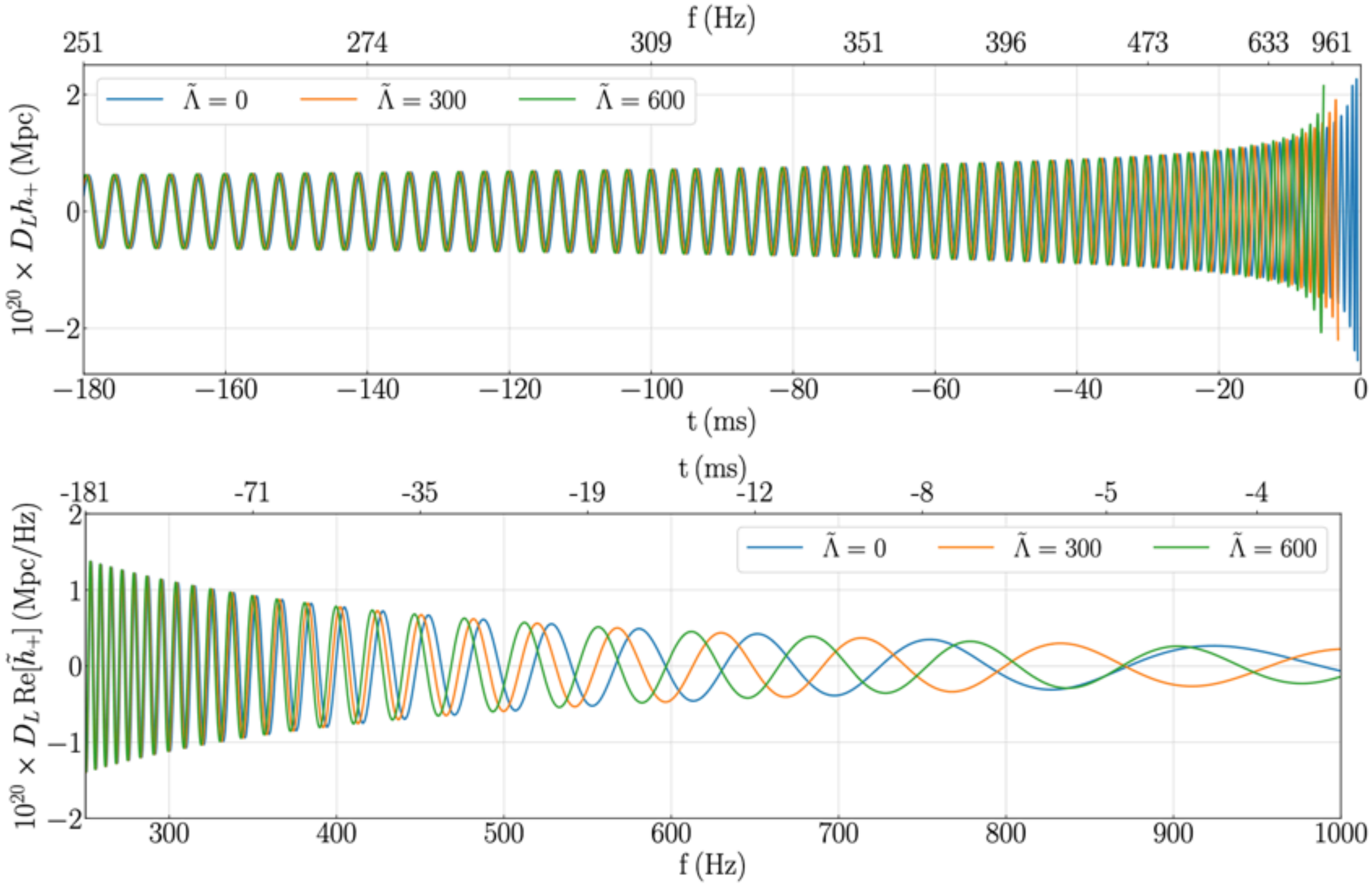
Binary neutron stars: waveform



Lattimer JM. 2021
Annu. Rev. Nucl. Part. Sci. 71:433–64

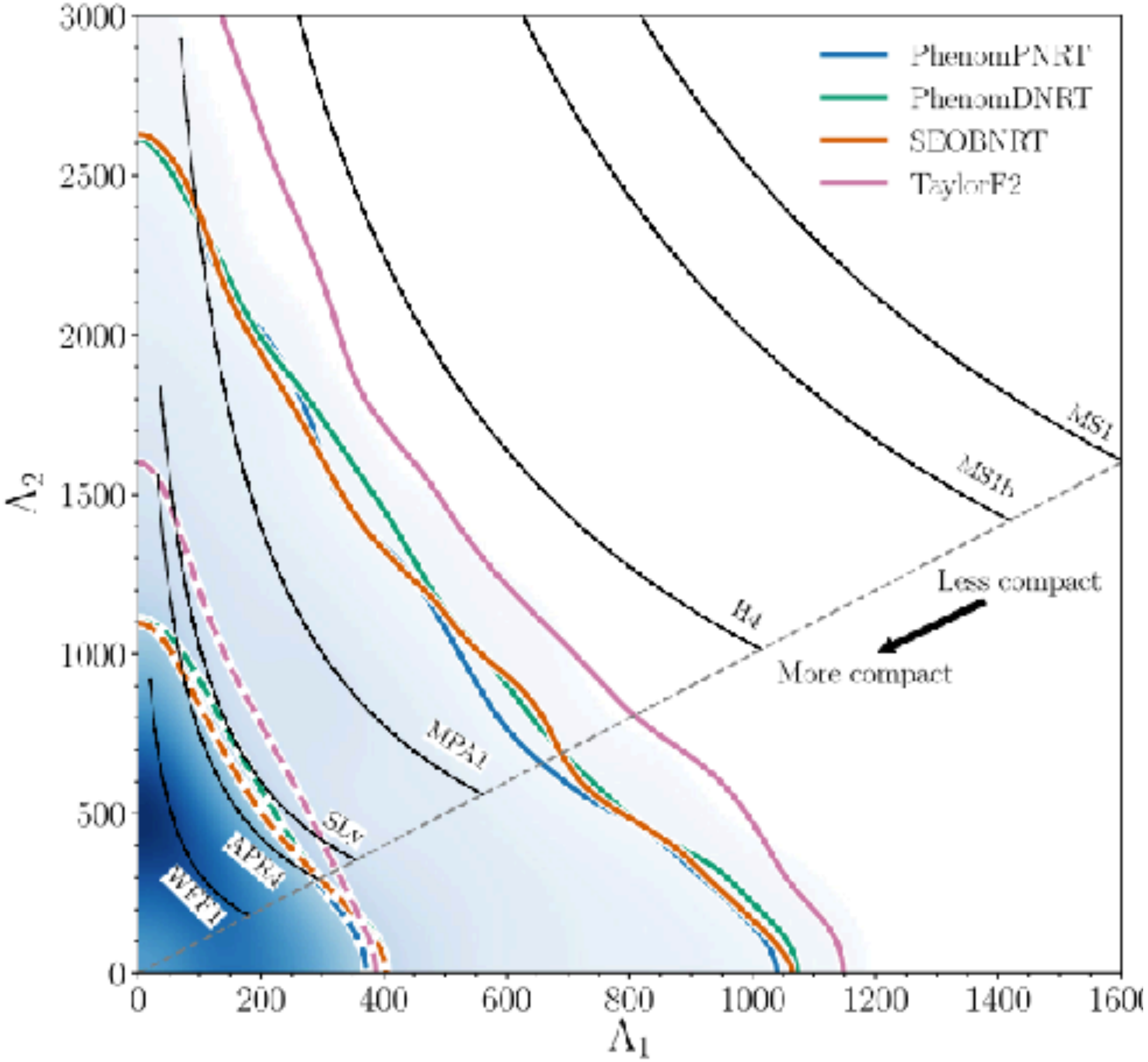
- Some energy goes into deforming the stars
- GW emission from time-varying quadrupole sources

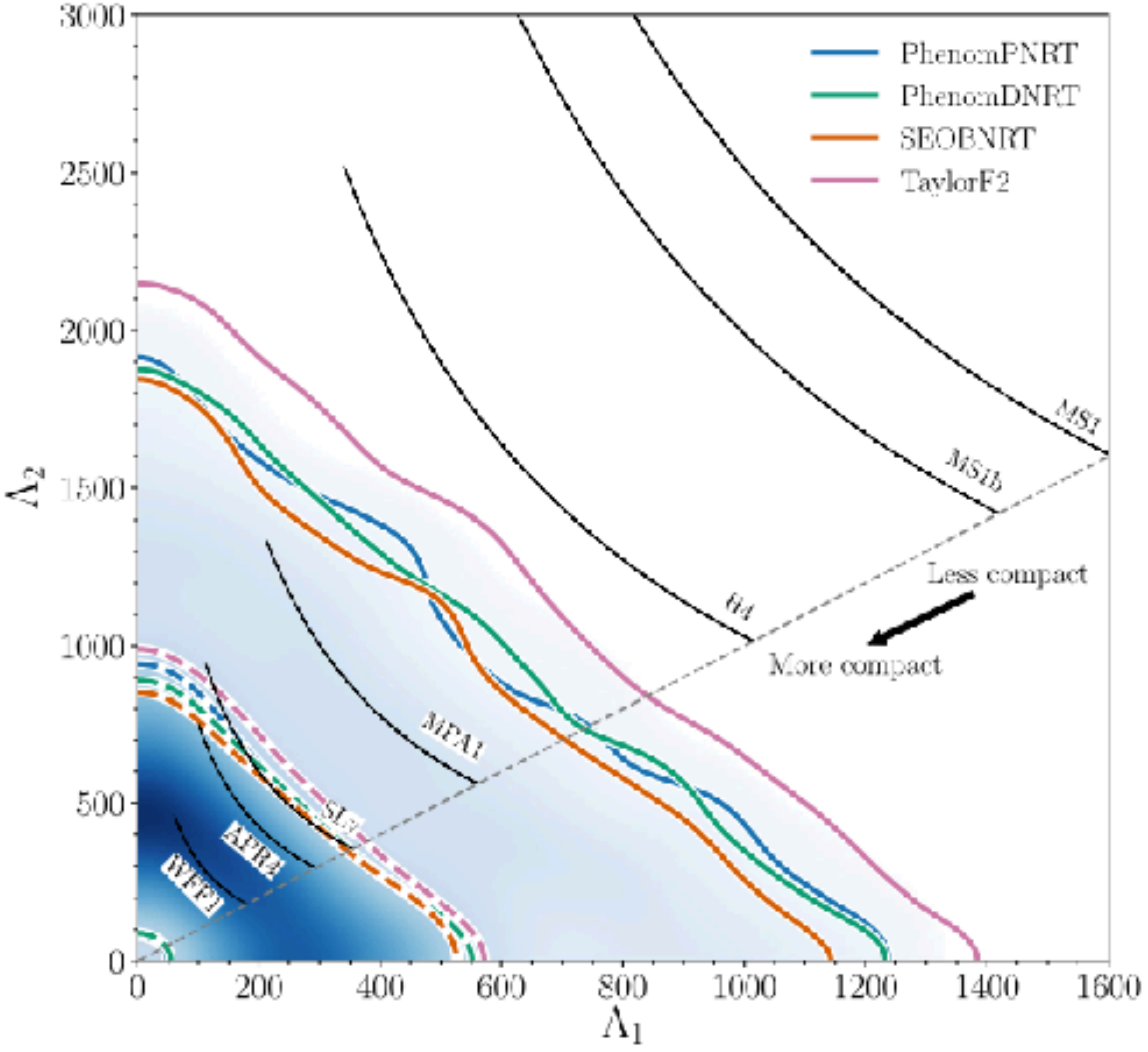
Binary neutron stars: waveform



[Chatziioannou et al., arXiv:2407.11153]

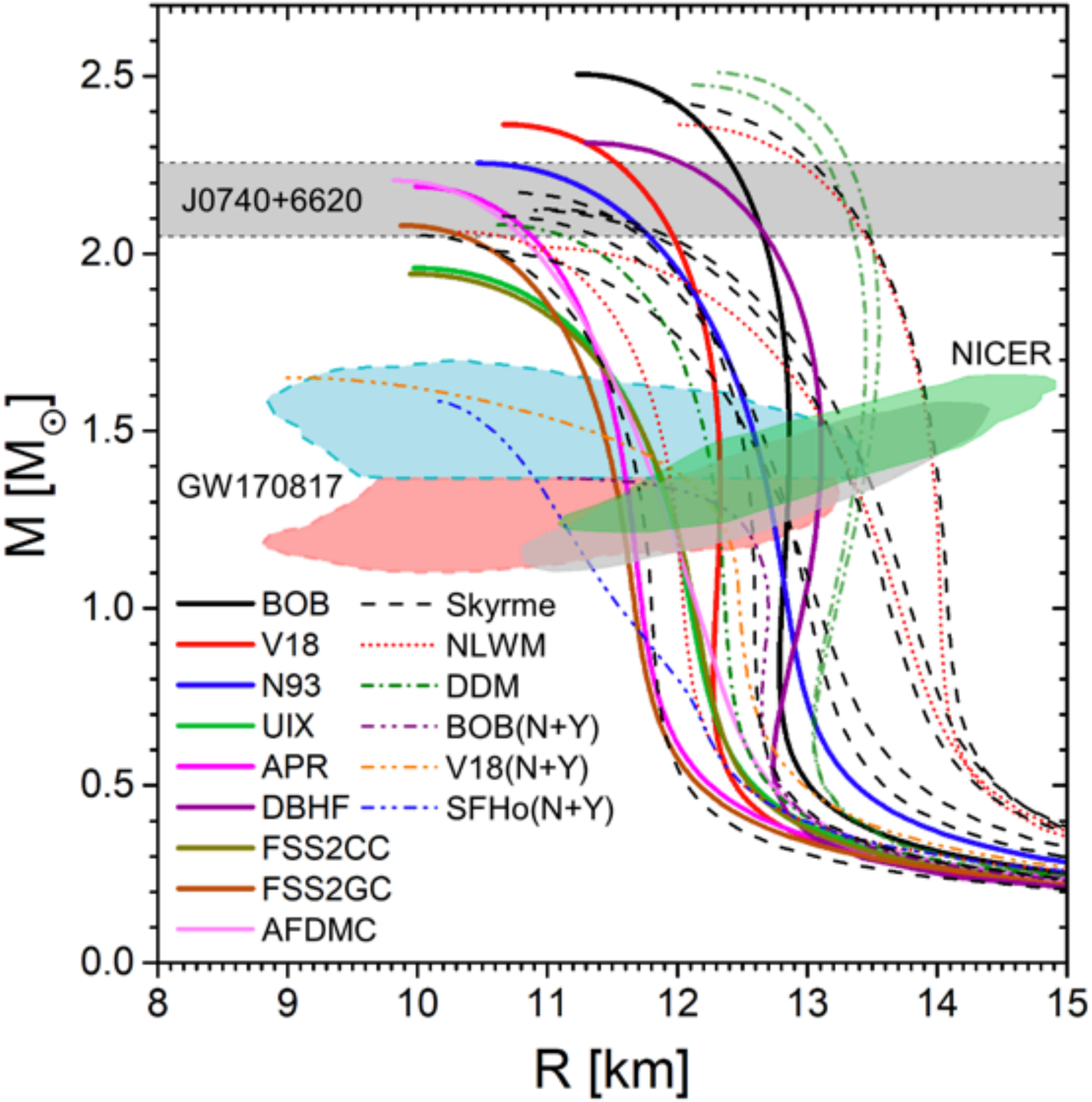
Binary neutron stars: constraining the EOS





Neutron star mass-radius relation

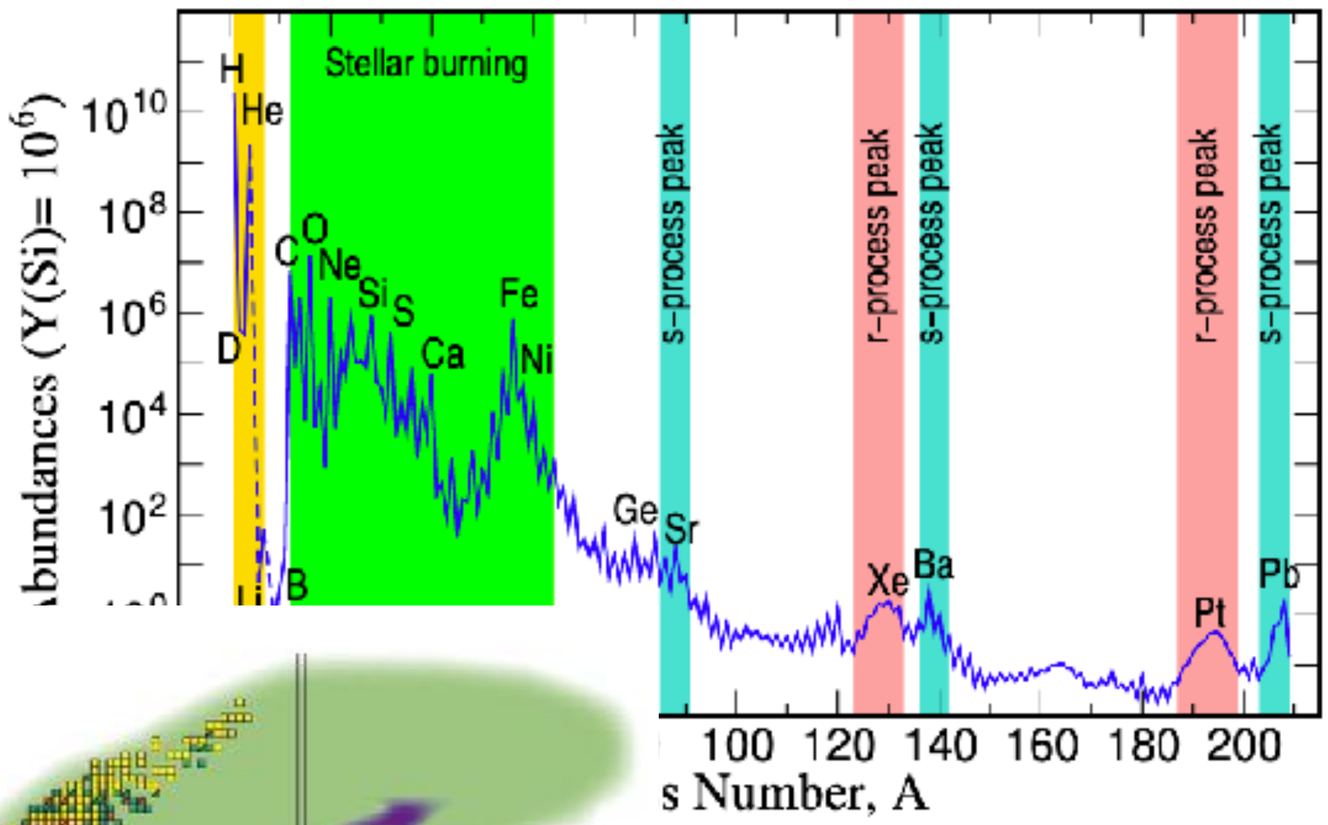
- Stiff EOS: high maximum mass, large radii
- Soft EOS: low maximum mass, small radii



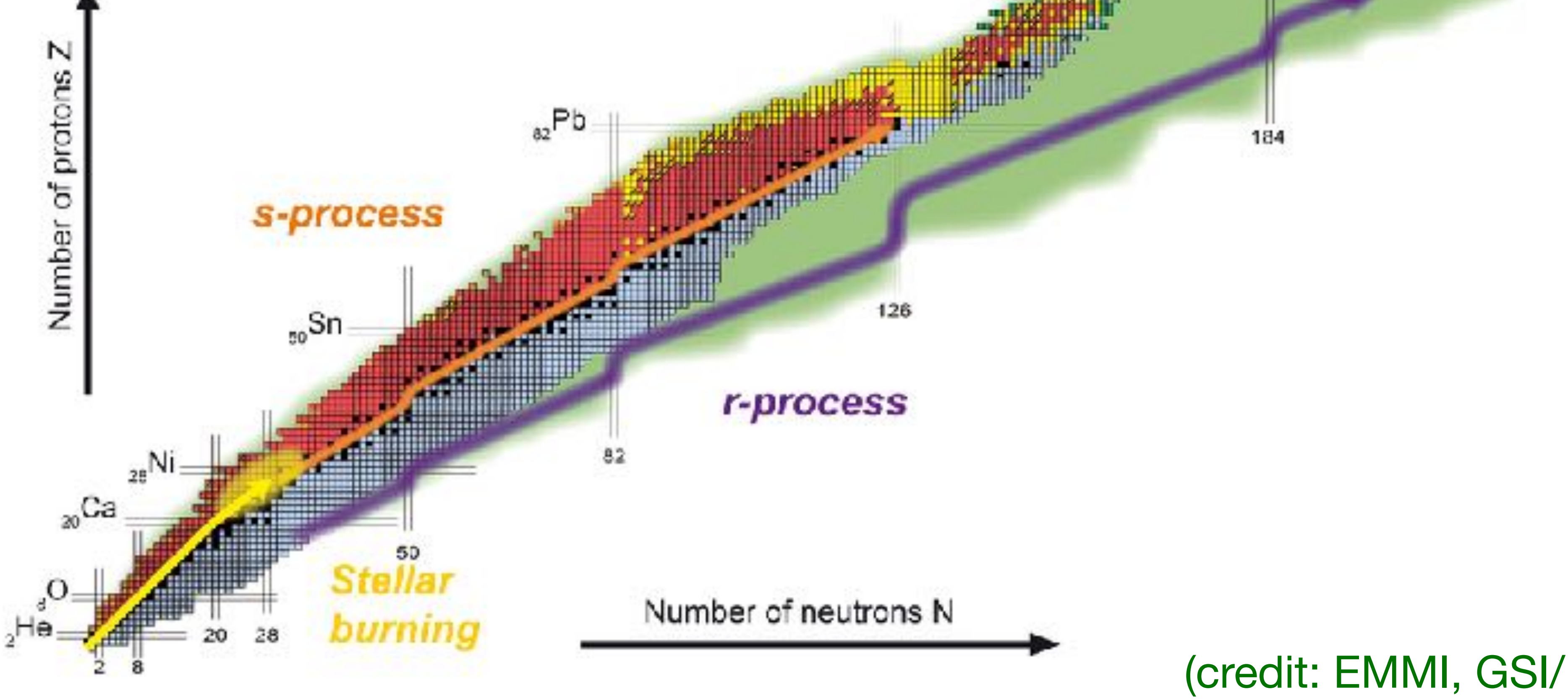
Burgio et al., arXiv:2105.03747

r-process elements

[Cowan+2020]



- Rapid process: neutron capture on a timescale of seconds, requires high neutron densities
- Some elements form (almost) exclusively via r-process (Eu, Pt, U, …)

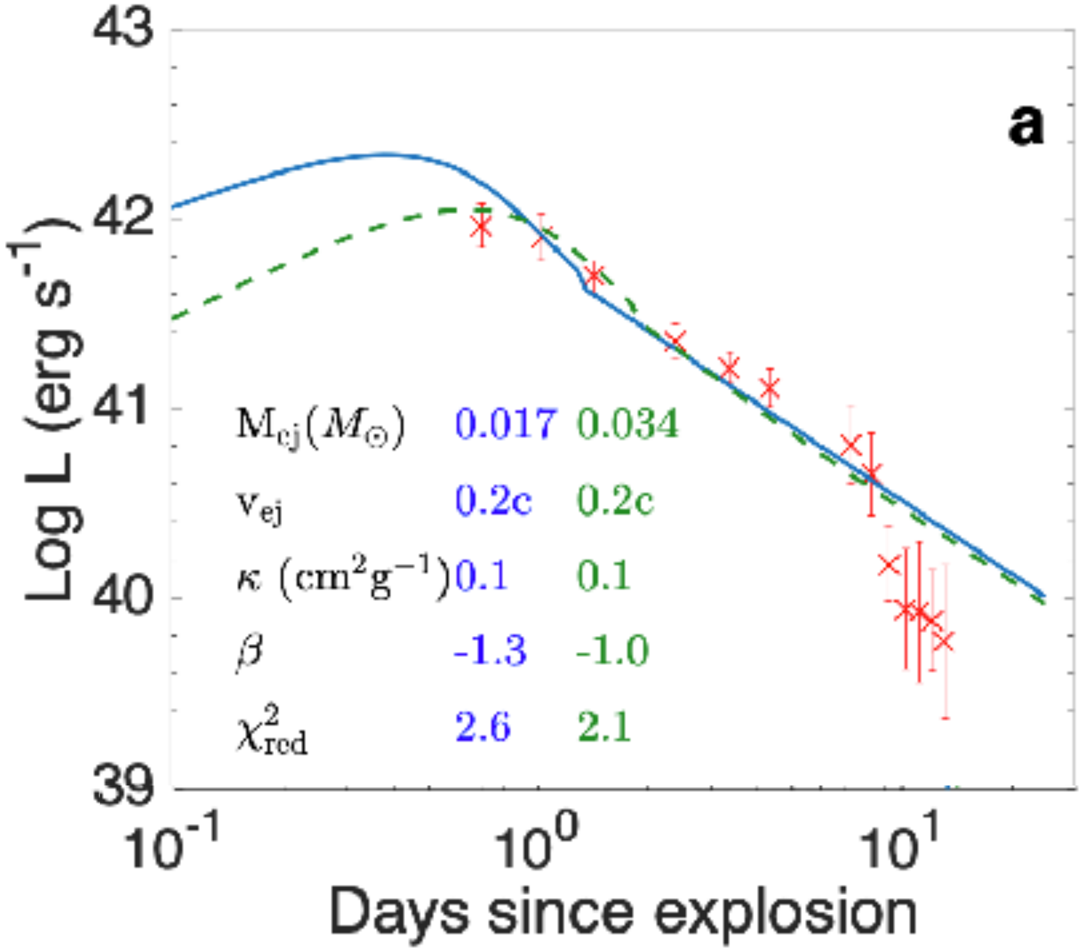


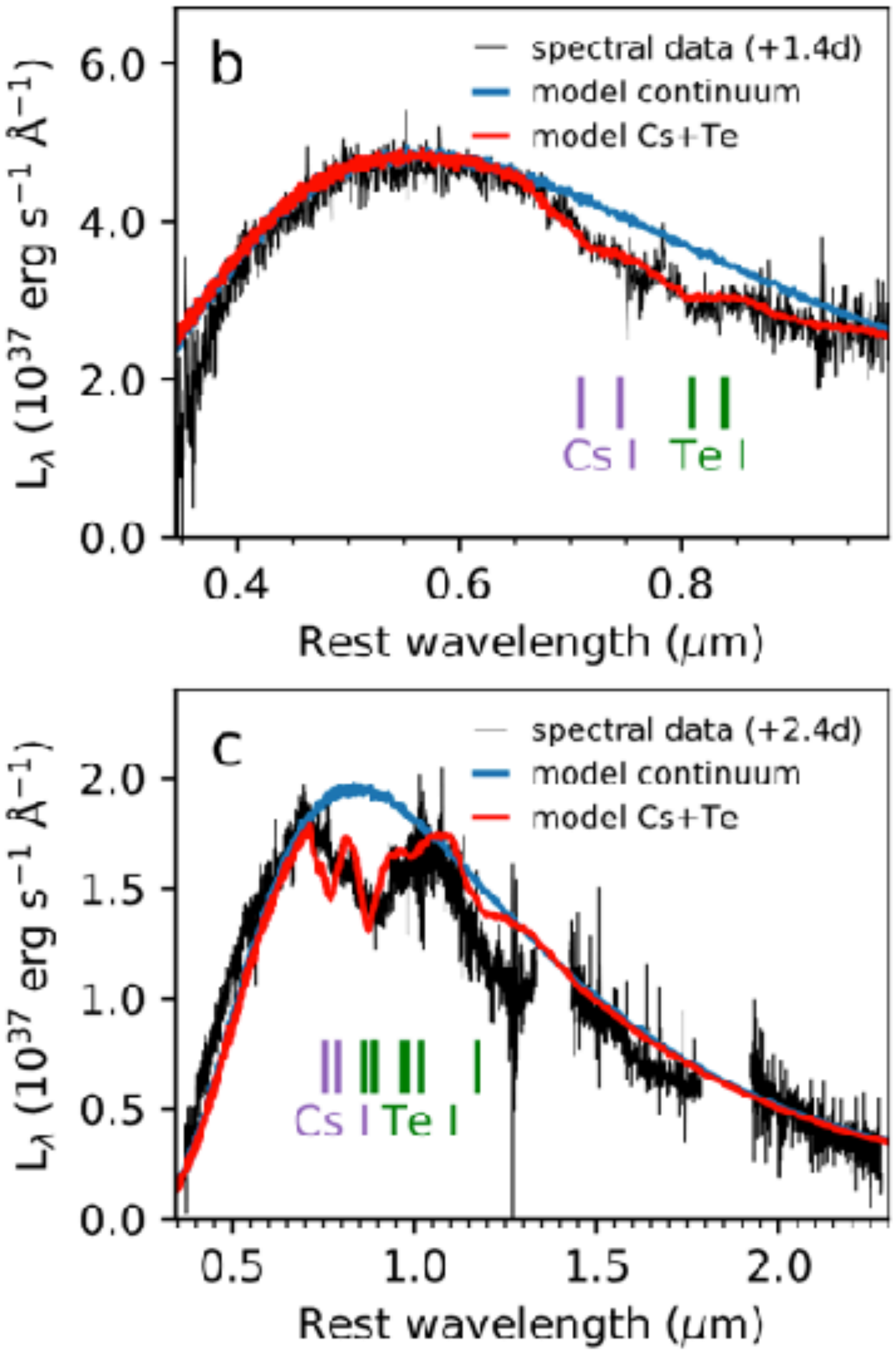
(credit: EMMI, GSI/Different Arts)

AT 2017gfo kilonova

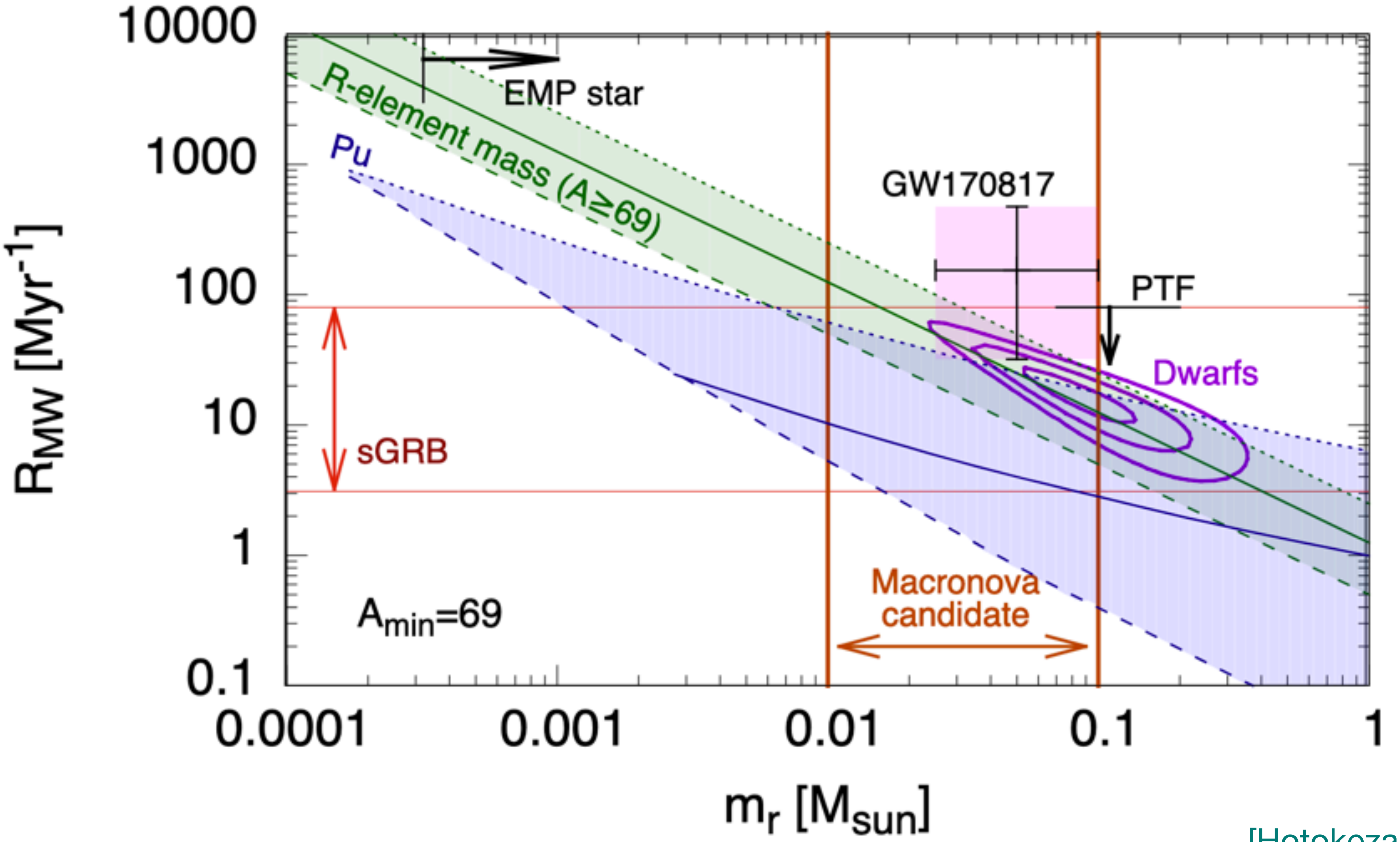
[Smartt+2017]

- Kilonova: UV-optical-IR transient powered by the radioactive decay of r-process elements synthesized in the merger ejecta
- Opacity depends on the chemical composition of the ejecta
- Ejected mass: $0.04\,M_\odot$





Did kilonovae produce all the r-process elements?



[Hotokezaka+2018]

GW190425: BNS without EM counterparts

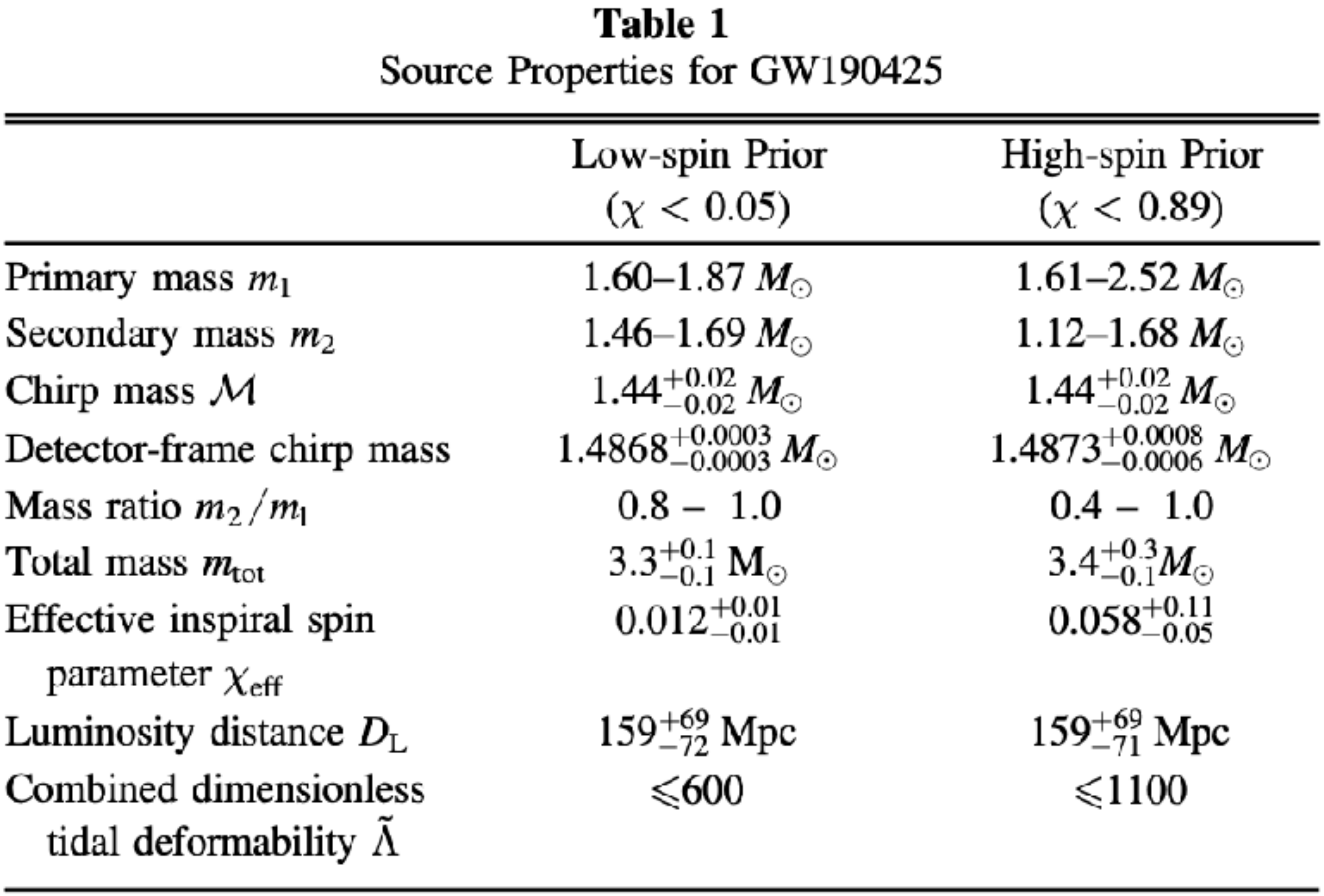
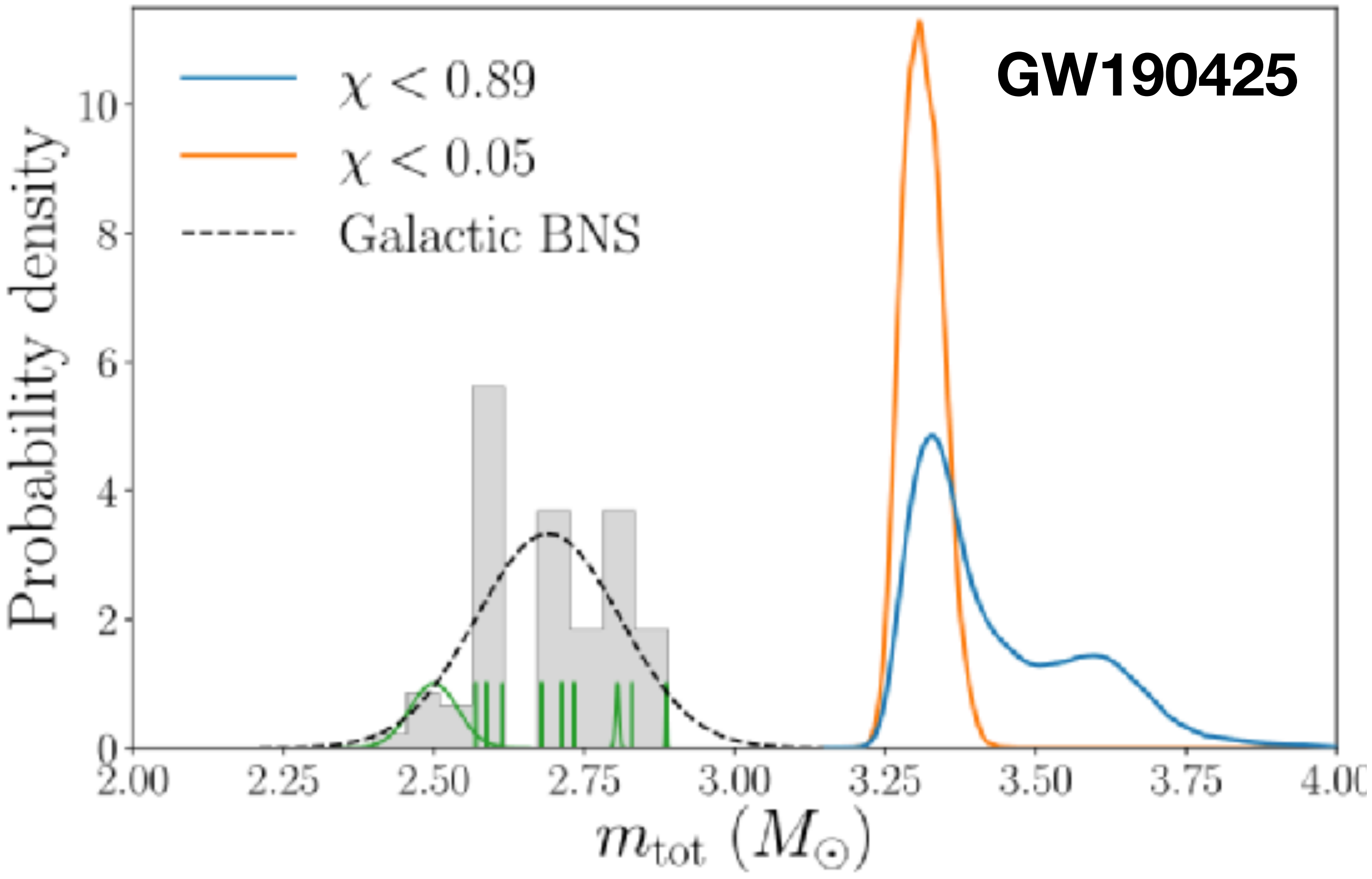


Table 1
Source Properties for GW190425

	Low-spin Prior ($\chi < 0.05$)	High-spin Prior ($\chi < 0.89$)
Primary mass m_1	1.60–1.87 $M_\odot$	1.61–2.52 $M_\odot$
Secondary mass m_2	1.46–1.69 $M_\odot$	1.12–1.68 $M_\odot$
Chirp mass $\mathcal{M}$	$1.44^{+0.02}_{-0.02}$ $M_\odot$	$1.44^{+0.02}_{-0.02}$ $M_\odot$
Detector-frame chirp mass	$1.4868^{+0.0003}_{-0.0003}$ $M_\odot$	$1.4873^{+0.0008}_{-0.0006}$ $M_\odot$
Mass ratio m_2/m_1	0.8 – 1.0	0.4 – 1.0
Total mass $m_{\rm tot}$	$3.3^{+0.1}_{-0.1}$ $\mathrm{M}_\odot$	$3.4^{+0.3}_{-0.1}M_\odot$
Effective inspiral spin parameter $\chi_{\rm eff}$	$0.012^{+0.01}_{-0.01}$	$0.058^{+0.11}_{-0.05}$
Luminosity distance $D_{\rm L}$	159^{+69}_{-72} Mpc	159^{+69}_{-71} Mpc
Combined dimensionless tidal deformability $\tilde{\Lambda}$	$\leqslant 600$	$\leqslant 1100$

[Abbott et al. 2020, Astrophys. J. Lett. 892, L3]

Binary neutron stars: masses



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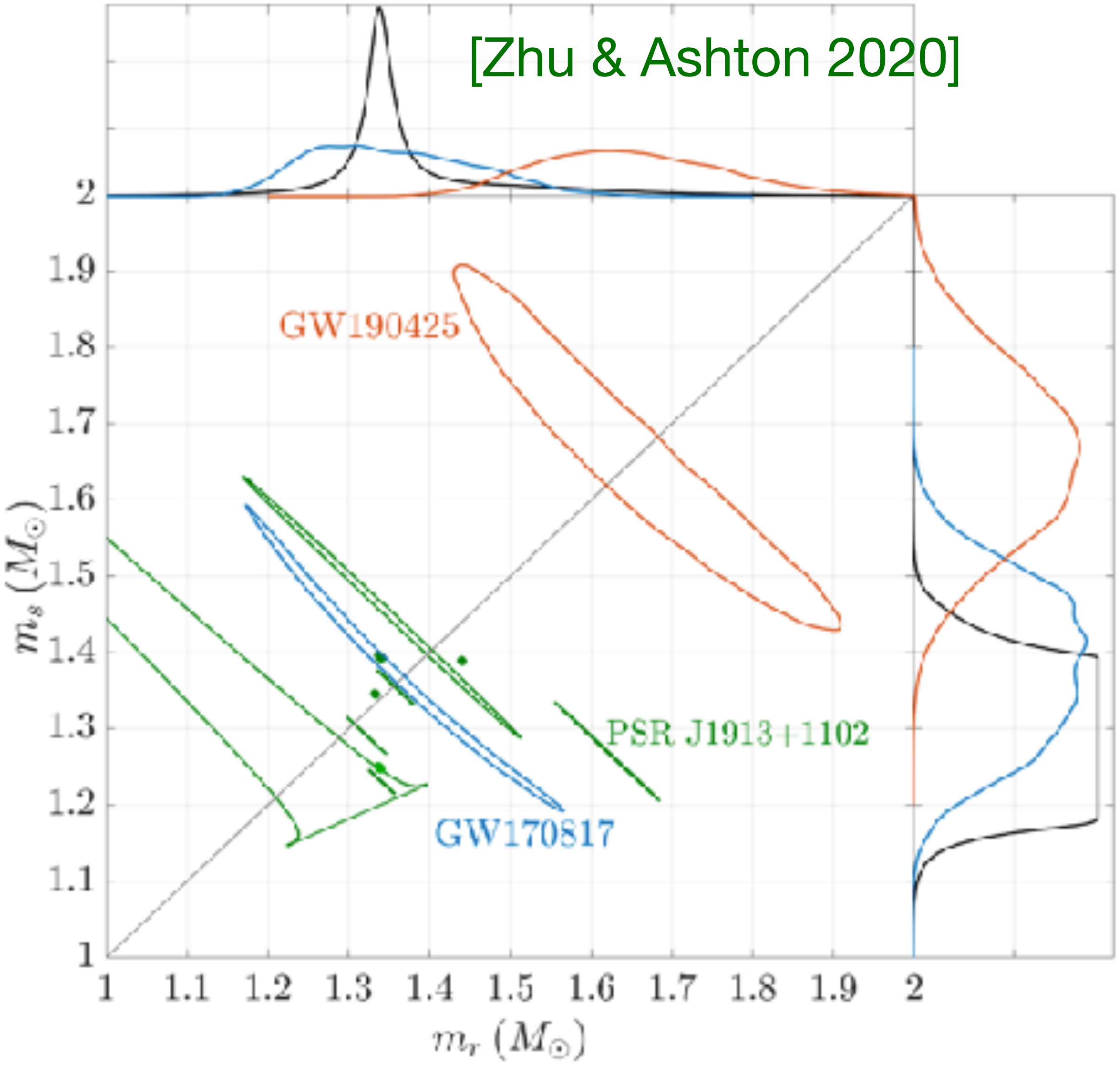
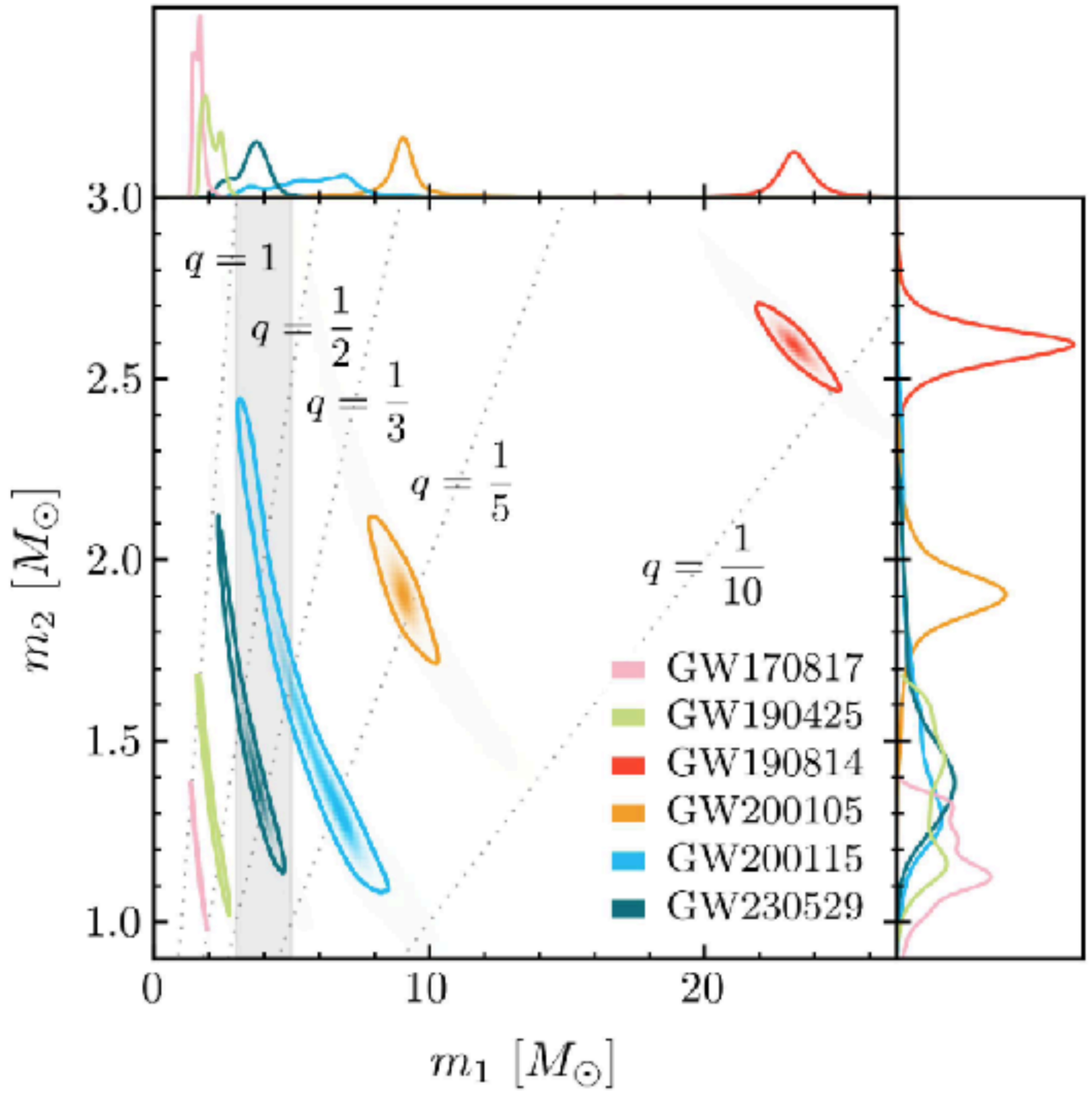


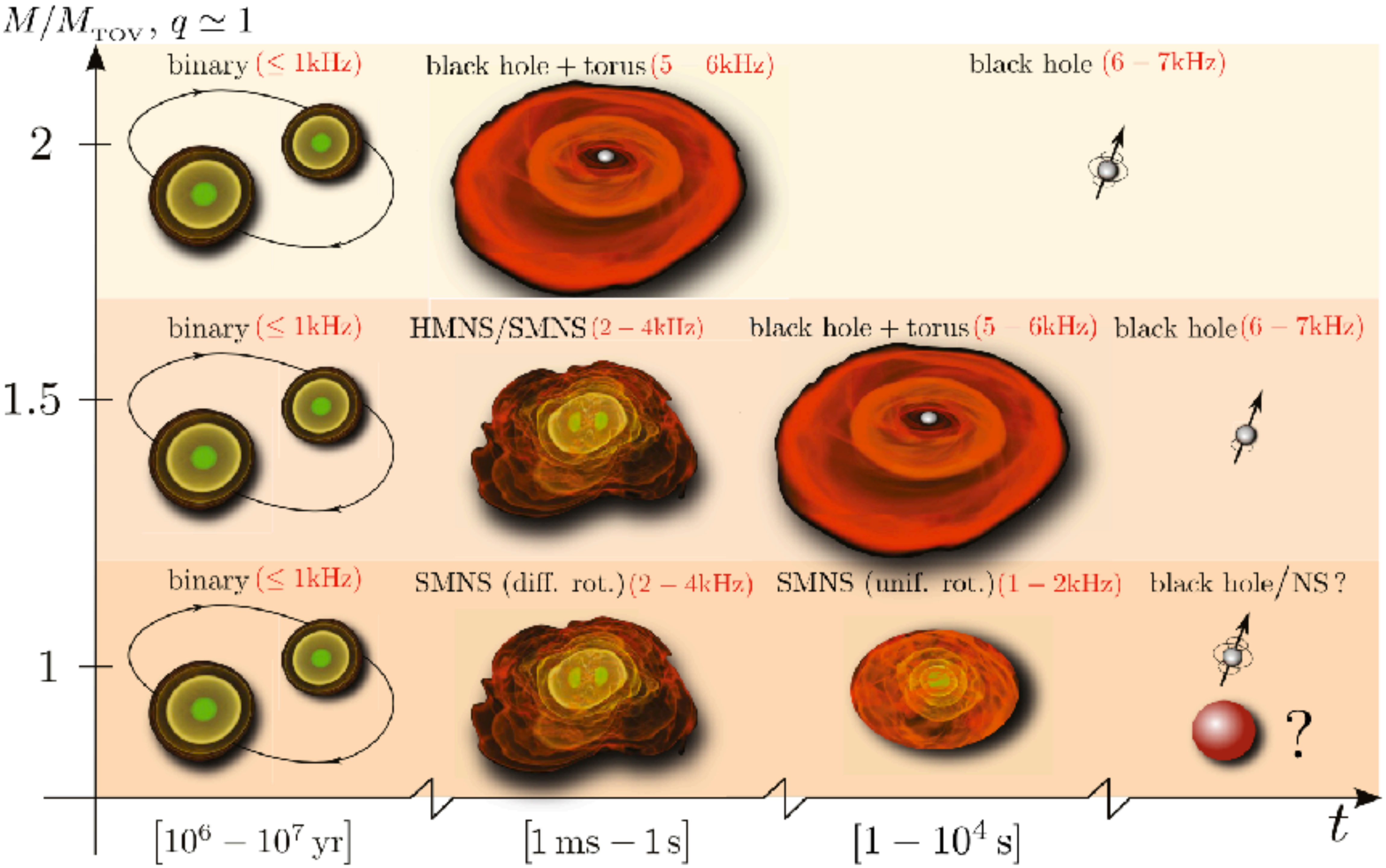
[Zhu & Ashton 2020]

Binaries with neutron stars: masses



[Chatziioannou et al., arXiv:2407.11153]

Binary neutron stars: after the merger



Baiotti and Rezzolla, arXiv:1607.03540


Merci !