

# RVB STATES IN RYDBERG ATOM ARRAYS

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28/02/2022



# ACKNOWLEDGEMENTS

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*Hannes  
(IQOQI)*



*Misha  
(Harvard)*



*Giacomo  
(MPQ)*



*Federica  
(Caltech)*

# OUTLINE

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- ▶ Rydberg atom arrays as quantum simulators
- ▶ Resonating valence bond states (RVB) of dimers in Rydberg atom arrays

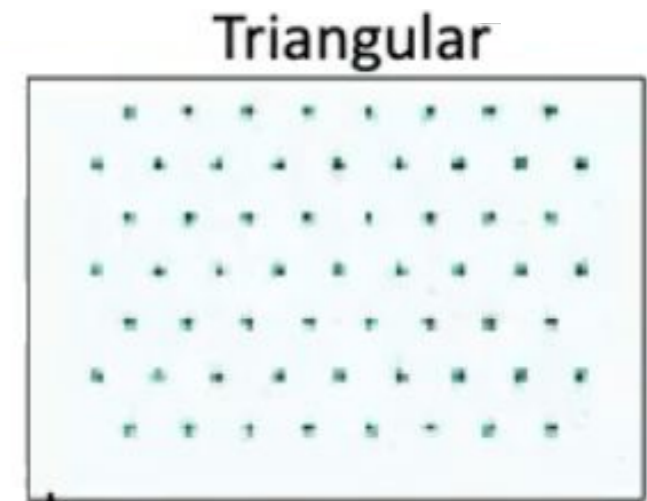
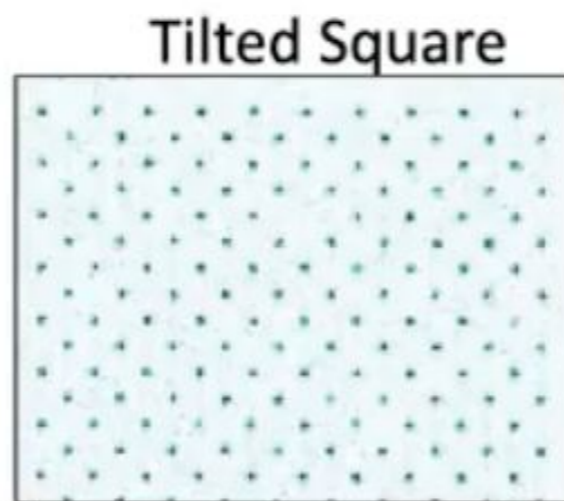
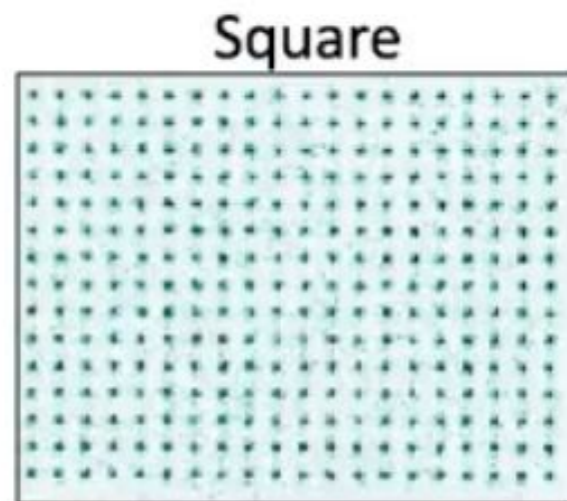
[arXiv:2201.04034](https://arxiv.org/abs/2201.04034)

- ▶ Generalization: resonating valence bond states of trimers (tRVB)
- ▶ tRVB states with Rydberg atom arrays

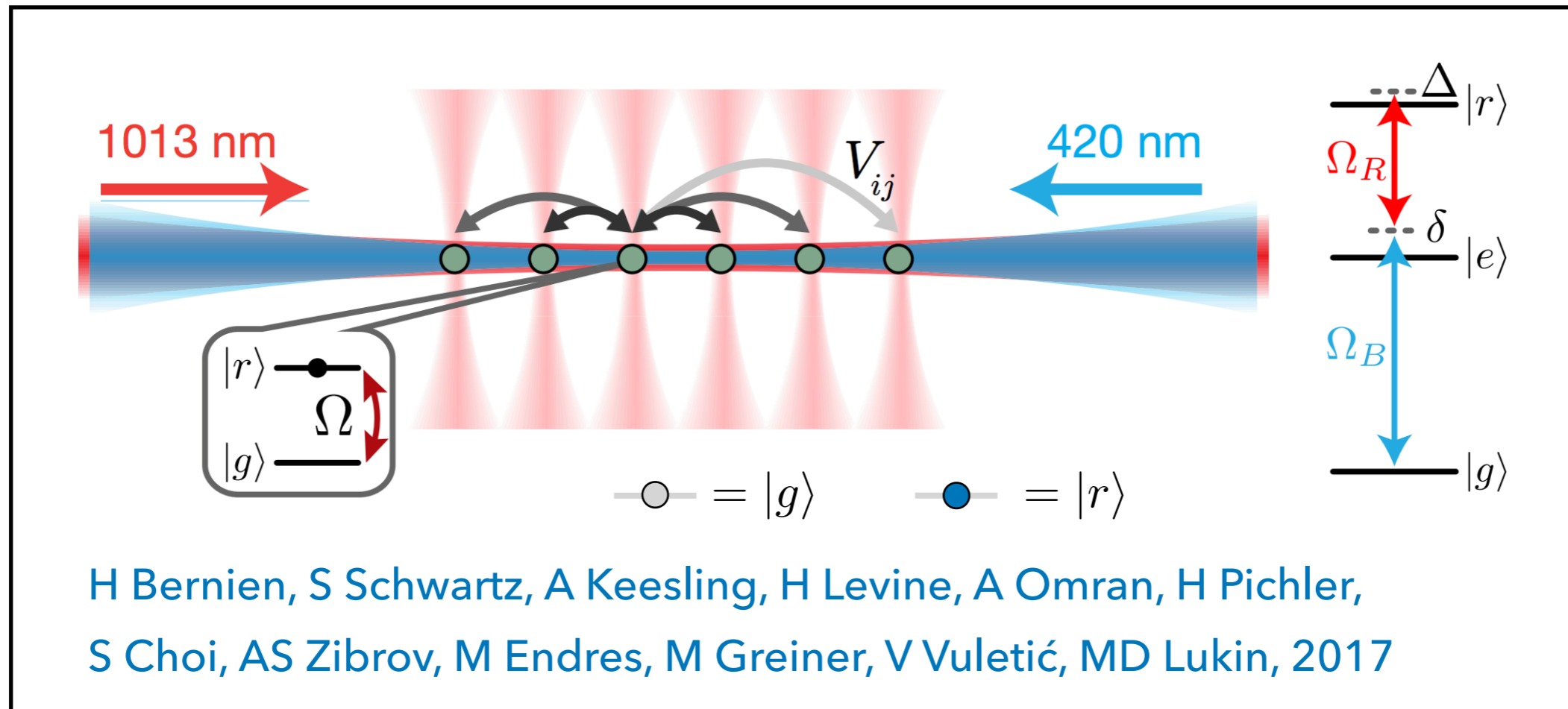
# WHAT ARE RYDBERG ATOM ARRAYS?

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- ▶ Neutral atoms
- ▶ Optically trapped into predefined lattice geometries
- ▶ Encode a qubit into two electronic states of each atom



# RYDBERG ATOMS HAMILTONIAN



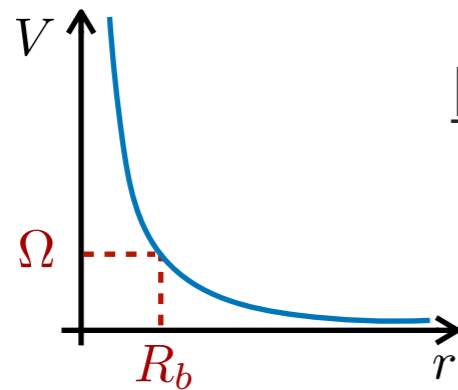
$$H = \frac{\Omega}{2} \sum_{\mathbf{i}} \sigma_{\mathbf{i}}^x - \Delta \sum_{\mathbf{i}} n_{\mathbf{i}} + \sum_{\mathbf{i}, \mathbf{j}} V(|\mathbf{i} - \mathbf{j}|) n_{\mathbf{i}} n_{\mathbf{j}}$$

$$\sim \frac{1}{|\mathbf{i} - \mathbf{j}|^6}$$

# RYDBERG ATOMS HAMILTONIAN

$$H = \frac{\Omega}{2} \sum_{\mathbf{i}} \sigma_{\mathbf{i}}^x - \Delta \sum_{\mathbf{i}} n_{\mathbf{i}} + \sum_{\substack{|\mathbf{i}-\mathbf{j}|=r \\ r \leq R_b}} V(r) n_{\mathbf{i}} n_{\mathbf{j}} + \sum_{\substack{|\mathbf{i}-\mathbf{j}|=r \\ r > R_b}} V(r) n_{\mathbf{i}} n_{\mathbf{j}}$$

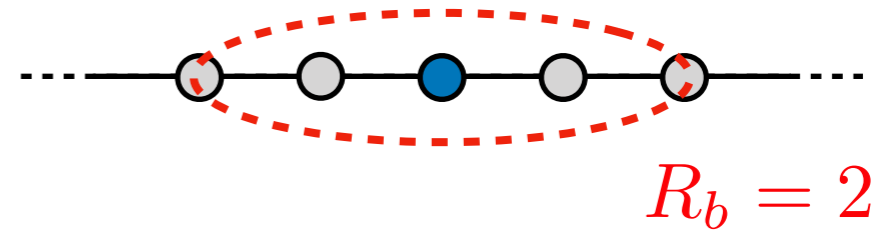
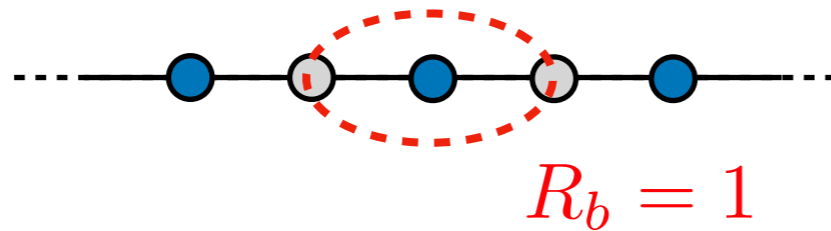
$\simeq \infty$



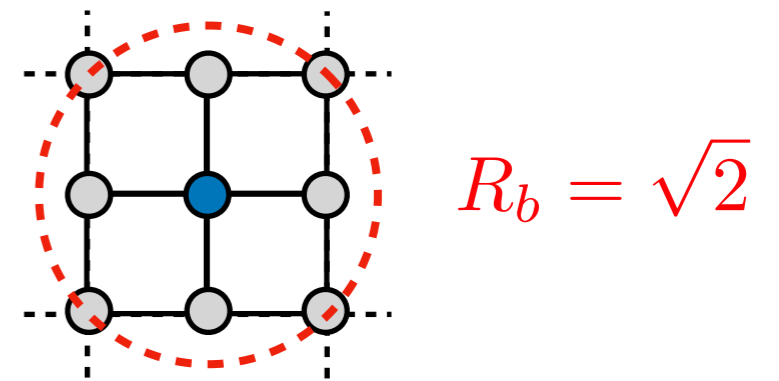
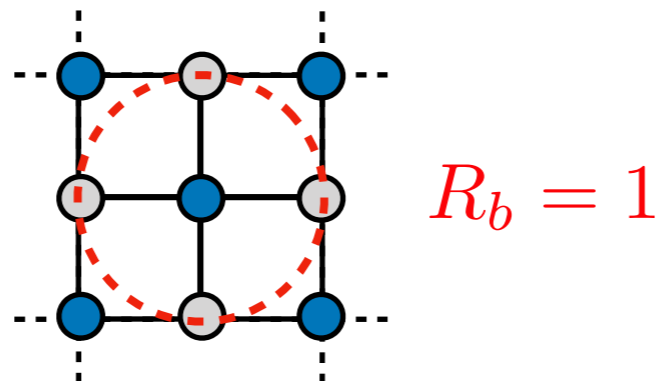
Blockade radius :  $V(R_b) := \Omega$

*tunable by varying lattice spacing!*

▶ 1D :

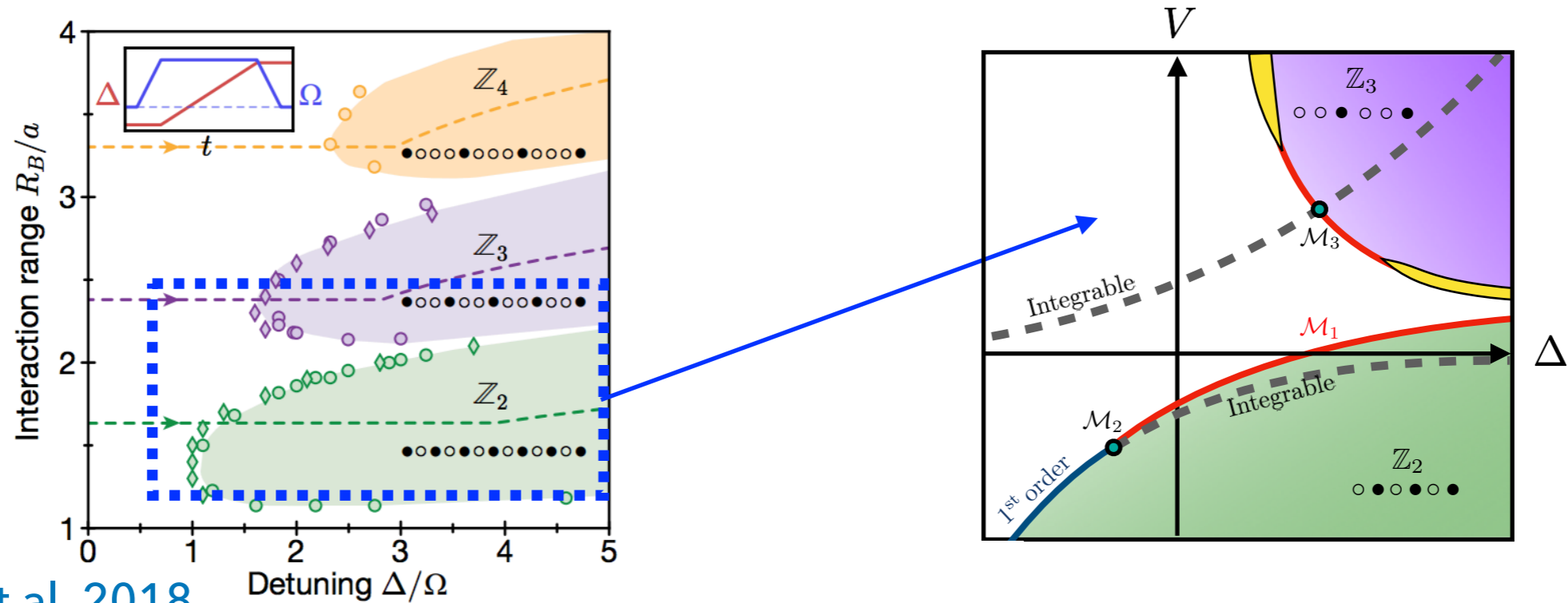


▶ 2D :



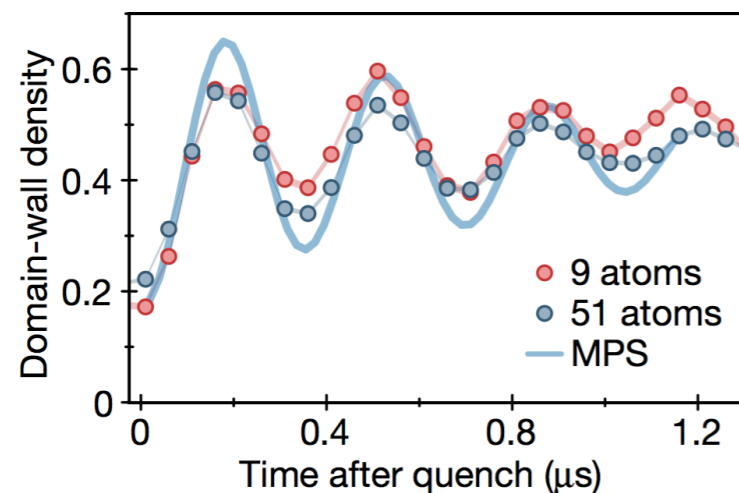
# 1D : IN & OUT OF EQUILIBRIUM

- ▶ 1D criticality: CFT universality and beyond

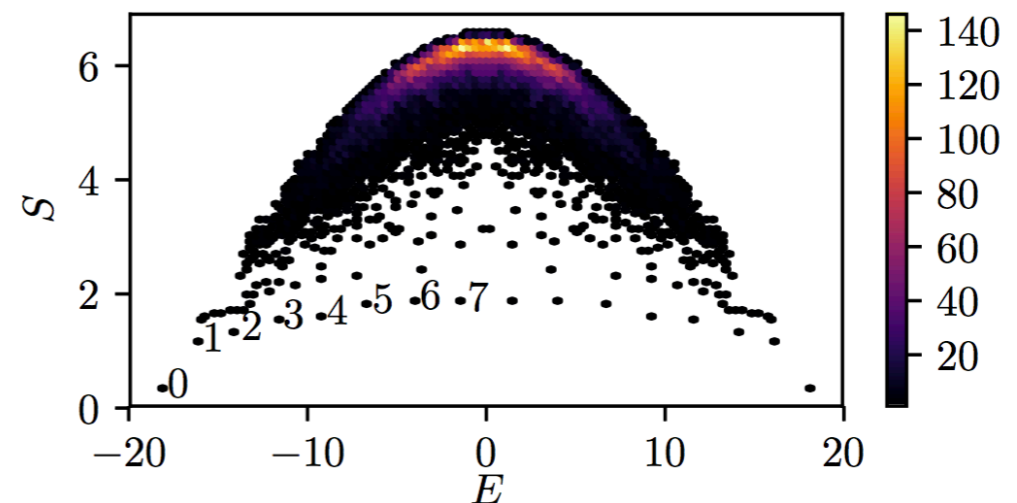


A Keesling et al, 2018

- ▶ Slow dynamics: many-body quantum scars



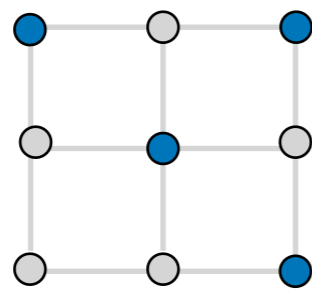
H Bernien et al, 2017



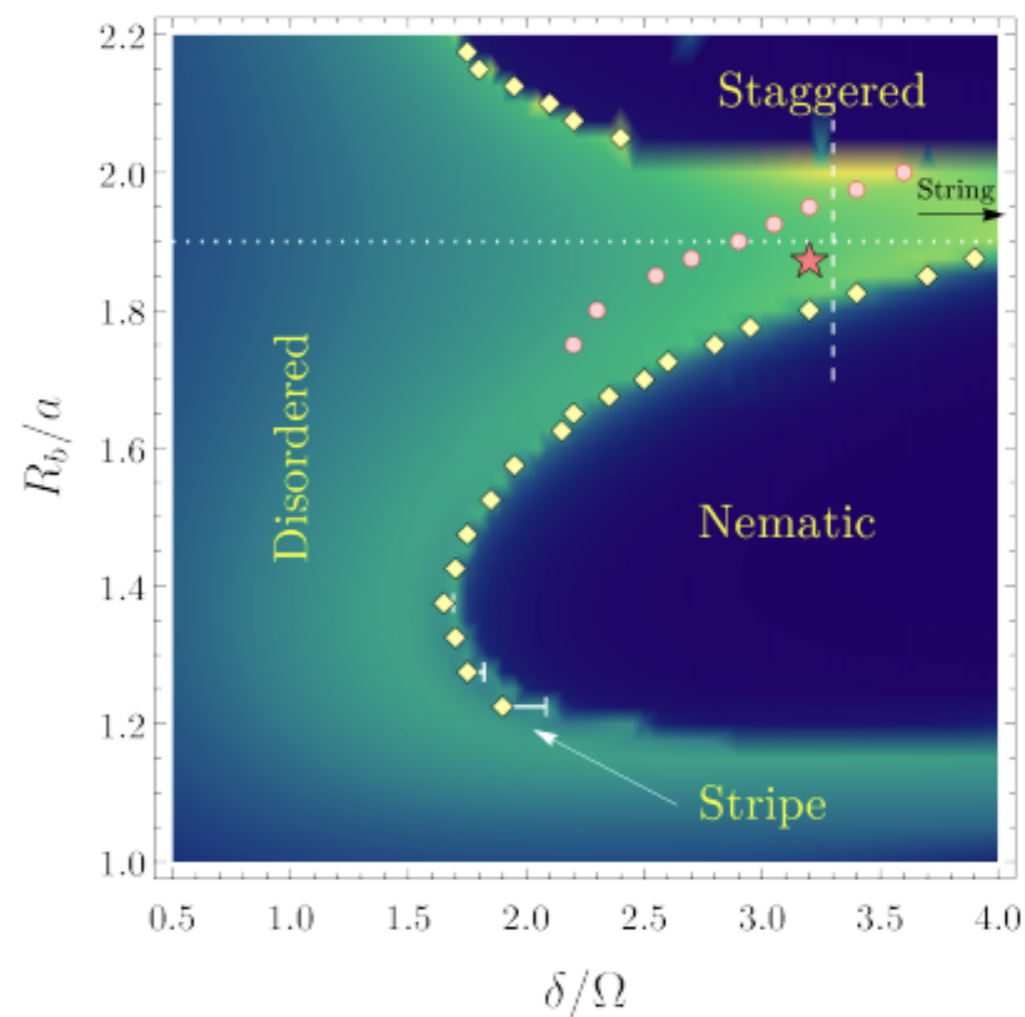
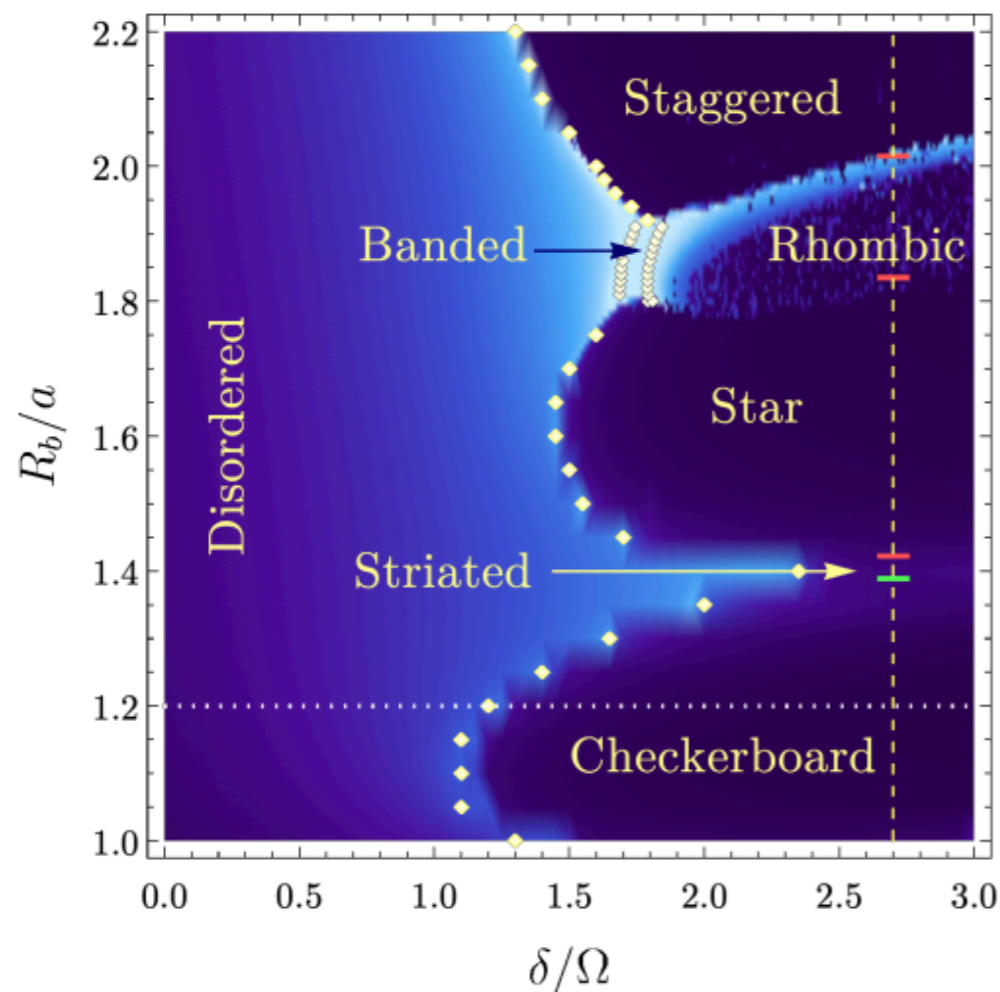
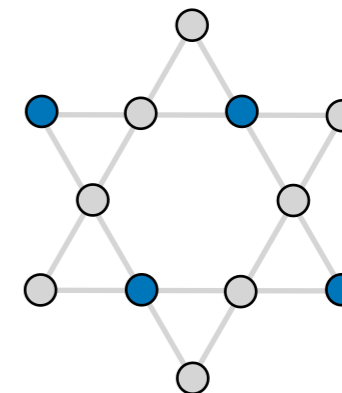
CJ Turner et al, 2017

# 2D : IN EQUILIBRIUM

Square



Kagome



*All **classically** ordered phases.. anything **quantum** ordered?*

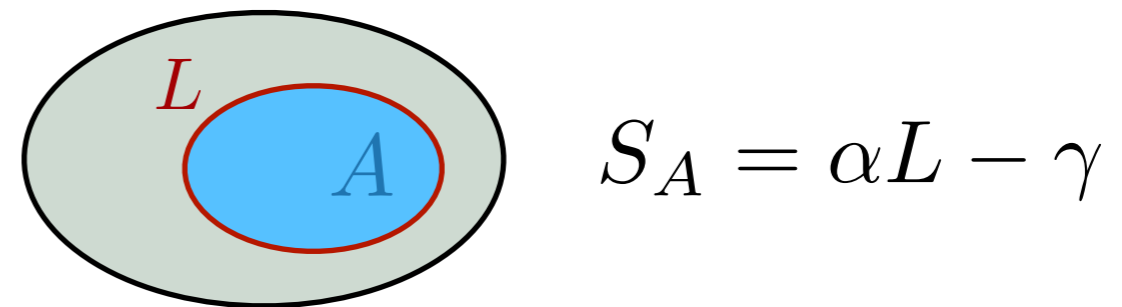


# QUANTUM SPIN LIQUIDS

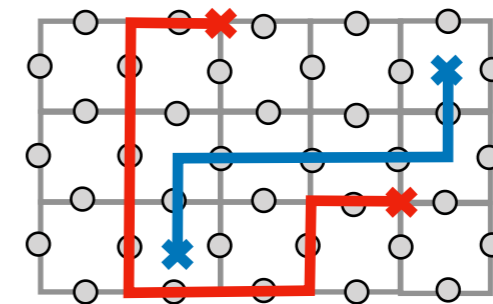
- ▶ No symmetry breaking



- ▶ Long range entanglement



- ▶ Non-local excitations



- ▶ Emergent deconfined gauge fields

- ▶ Non-local order parameters

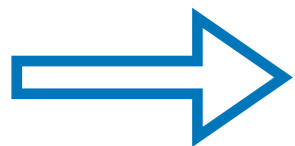
$$O = \frac{\langle \text{red path with } \times \text{ ends} \rangle}{\sqrt{\langle \text{blue loop} \rangle}}$$

# RVB STATES IN DIMER MODELS

▶ Dimer model:  $\mathcal{H} = \left\{ \left| \begin{array}{cc} \vdots & \vdots \\ \vdots & \vdots \end{array} \right\rangle, \left| \begin{array}{cc} \vdots & \vdots \\ \vdots & \vdots \end{array} \right\rangle, \left| \begin{array}{cc} \vdots & \vdots \\ \vdots & \vdots \end{array} \right\rangle, \left| \begin{array}{cc} \vdots & \vdots \\ \vdots & \vdots \end{array} \right\rangle \dots \right\}$

▶ Resonating valence bond (RVB) state:  
equal weight superposition of all dimer coverings

$$|\text{RVB}\rangle = \left| \begin{array}{cc} \vdots & \vdots \\ \vdots & \vdots \end{array} \right\rangle + \left| \begin{array}{cc} \vdots & \vdots \\ \vdots & \vdots \end{array} \right\rangle + \left| \begin{array}{cc} \vdots & \vdots \\ \vdots & \vdots \end{array} \right\rangle + \left| \begin{array}{cc} \vdots & \vdots \\ \vdots & \vdots \end{array} \right\rangle + \dots$$



***Simplest representative of quantum spin liquid state***

Non-bipartite lattice

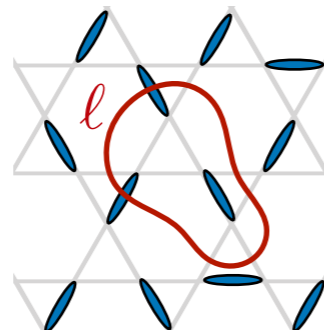
Bipartite lattice

▶ Finite correlation length

▶ Infinite correlation length

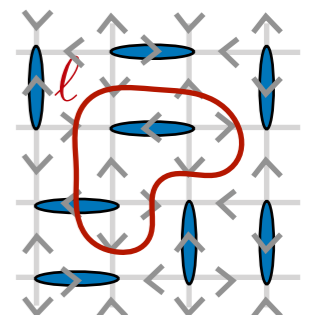
▶  $\mathbb{Z}_2$  Gauss' law

$$\prod_{i \in \ell} Z_i = (-1)^{N_v}$$



▶  $U(1)$  Gauss' law

$$\sum_{i \in \ell_{\text{in}}} Z_i - \sum_{i \in \ell_{\text{out}}} Z_i = N_v^A - N_v^B$$

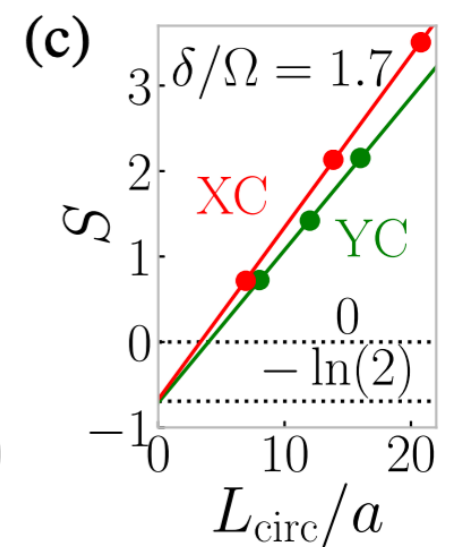
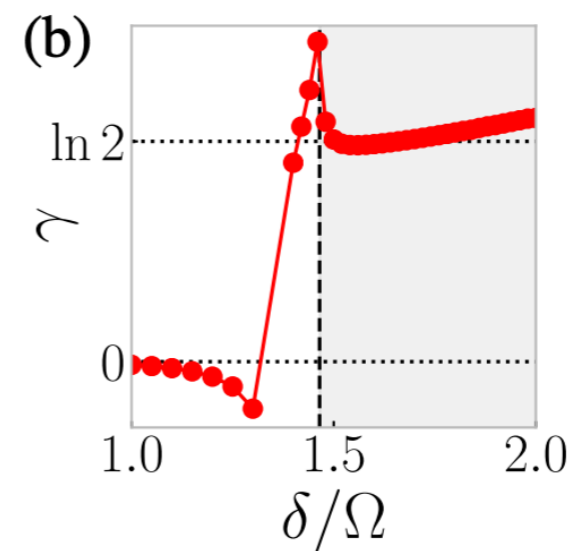
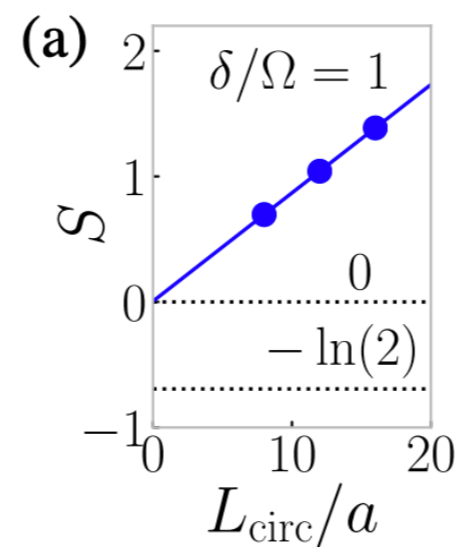
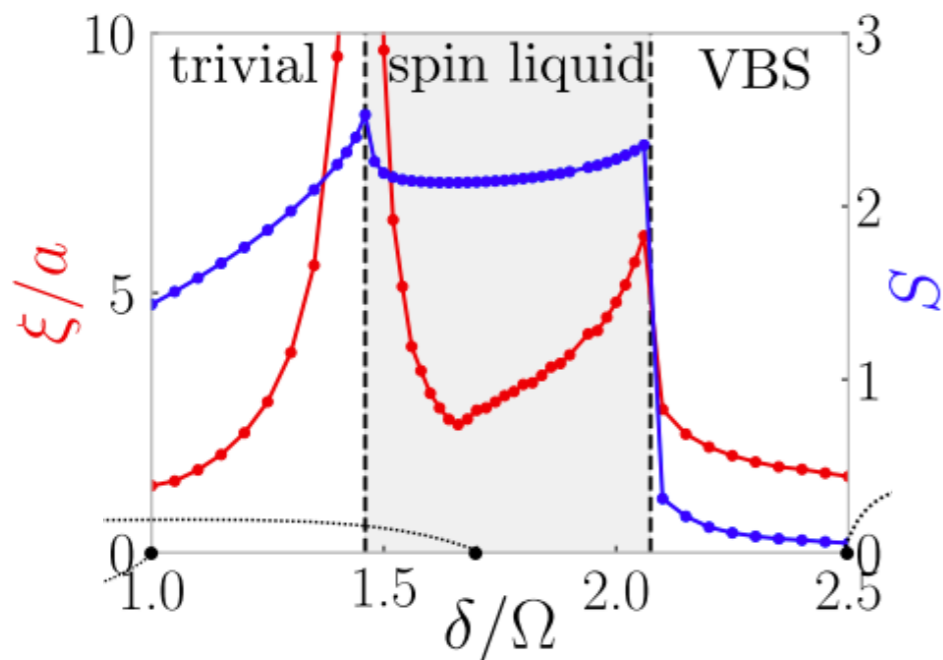
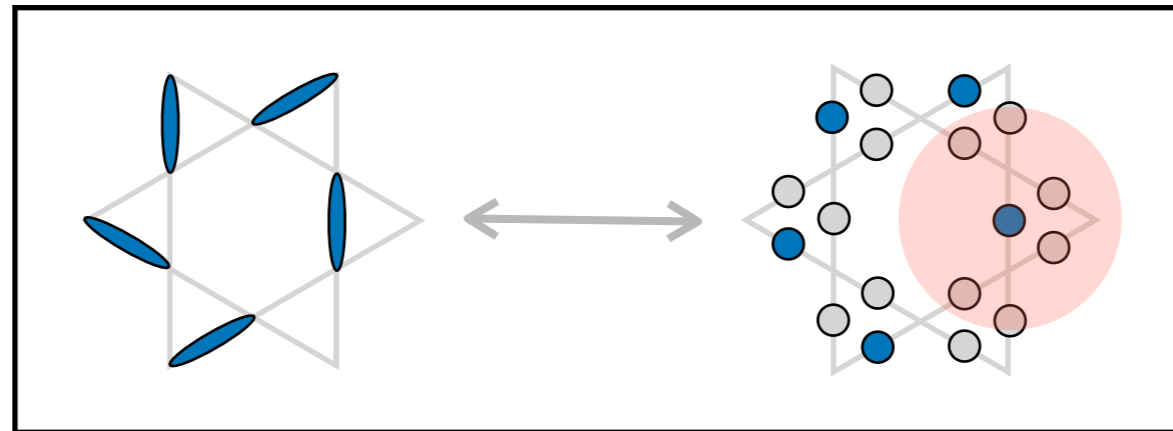


# DIMER MODELS & RYDBERG ATOMS

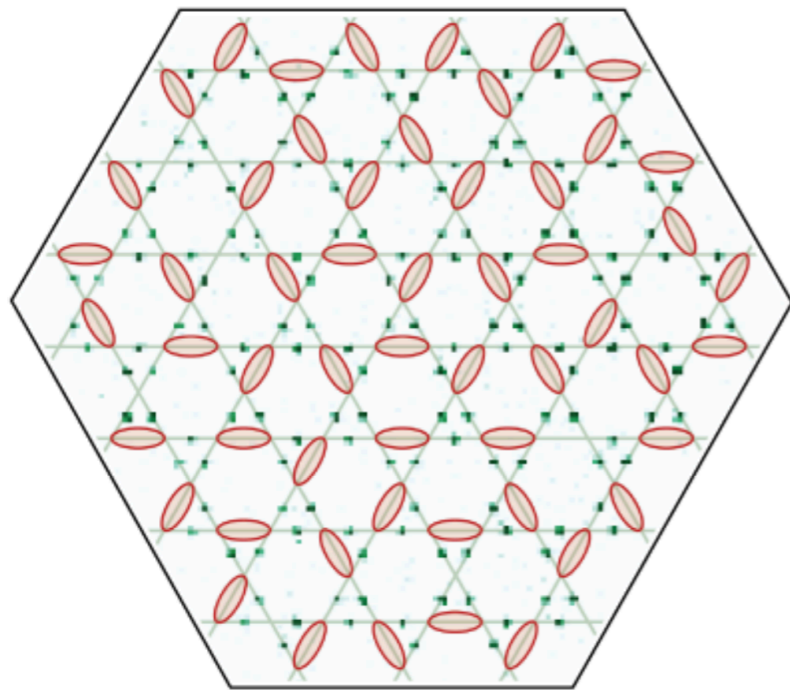
- Rydberg atoms on the links



- Rydberg blockade as dimer constraint

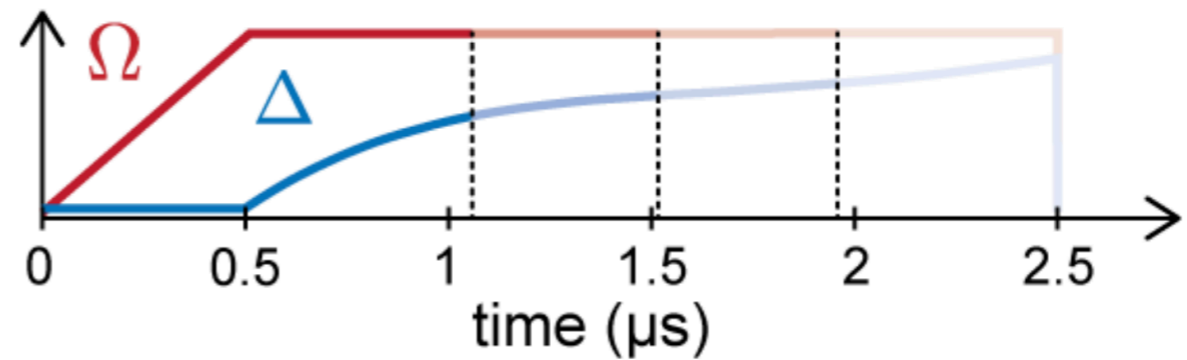


# DIMER MODELS & RYDBERG ATOMS



State preparation :  $|\psi_0\rangle = |0\rangle$

$$H(t) = \frac{\Omega(t)}{2} \sum_i \sigma_i^x - \Delta(t) \sum_i n_i + \sum_{r=|i-j|} V(r) n_i n_j$$



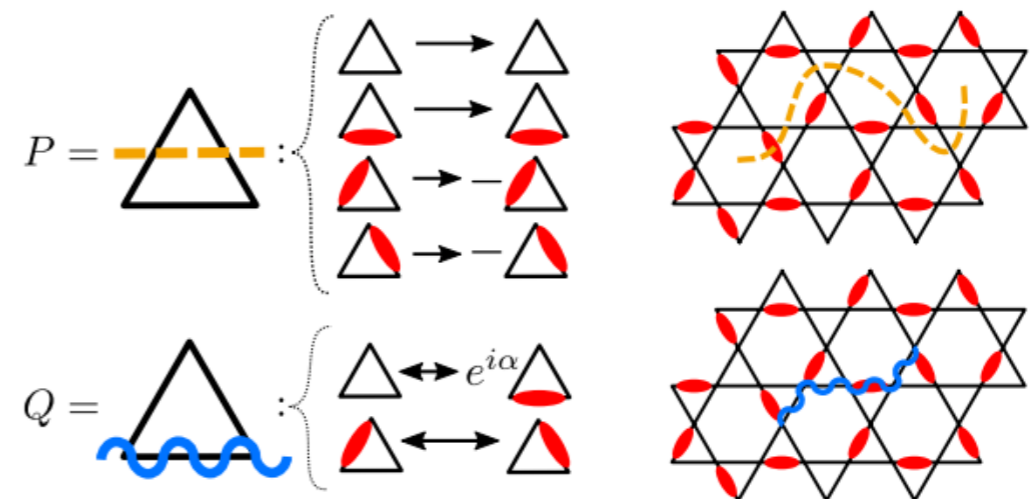
Experimental detection : non-local order parameters

- ▶ Closed loops : perimeter law

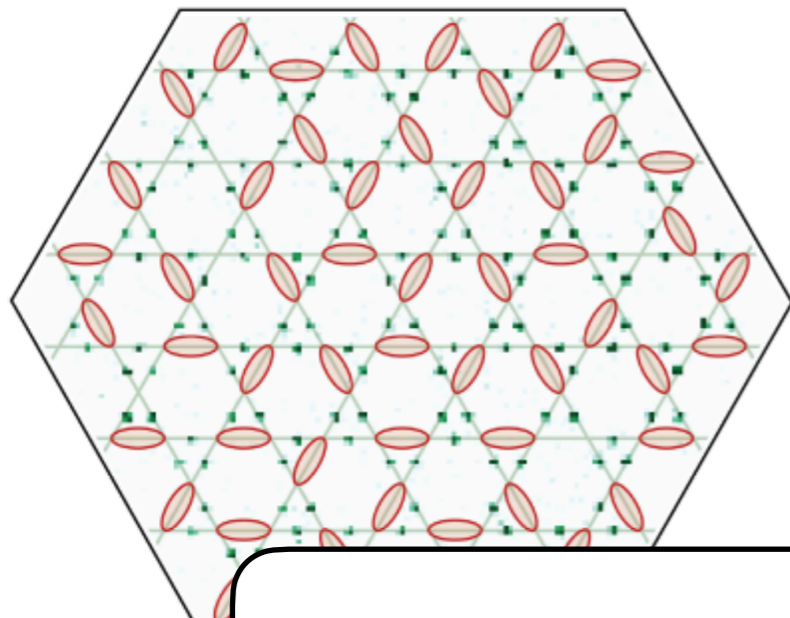
$$\langle \text{loop} \rangle \sim e^{-\alpha l}$$

- ▶ BFFM order parameters :

$$\frac{\langle \text{loop with } * \rangle}{\sqrt{\langle \text{loop} \rangle}} \xrightarrow{l \rightarrow \infty} 0$$

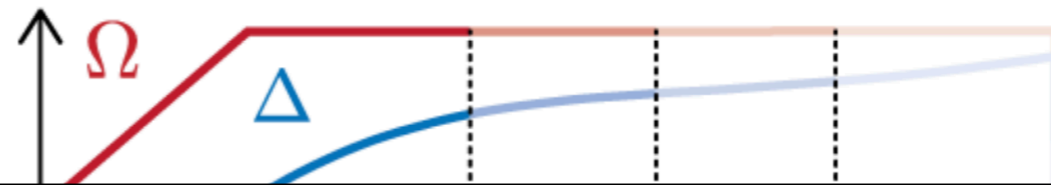


# DIMER MODELS & RYDBERG ATOMS

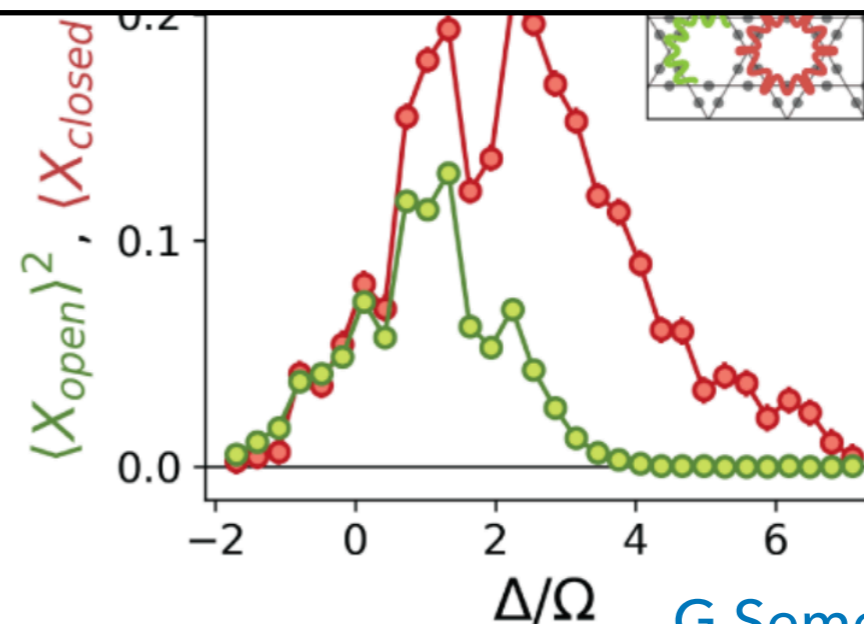
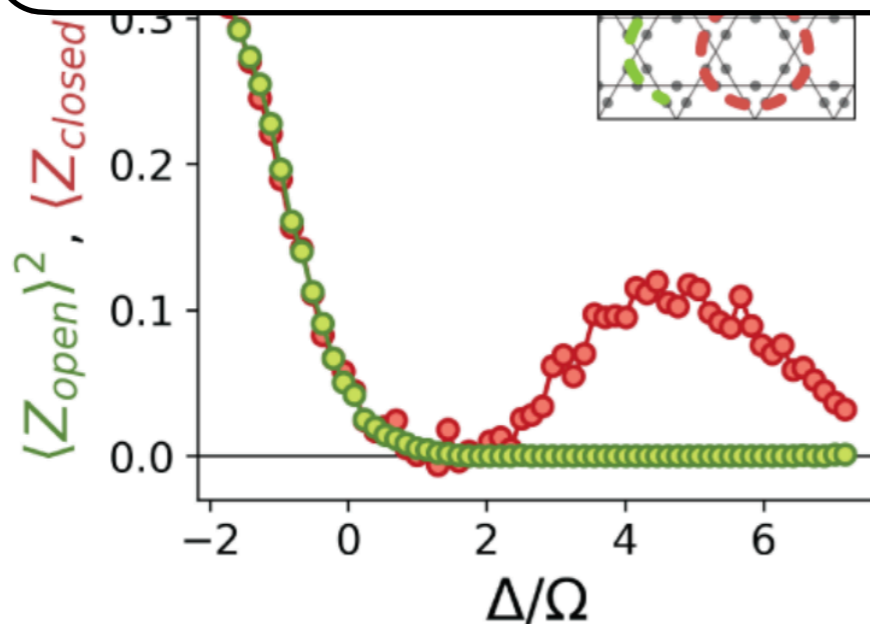


State preparation :  $|\psi_0\rangle = |0\rangle$

$$H(t) = \frac{\Omega(t)}{2} \sum_i \sigma_i^x - \Delta(t) \sum_i n_i + \sum_{r=|i-j|} V(r) n_i n_j$$

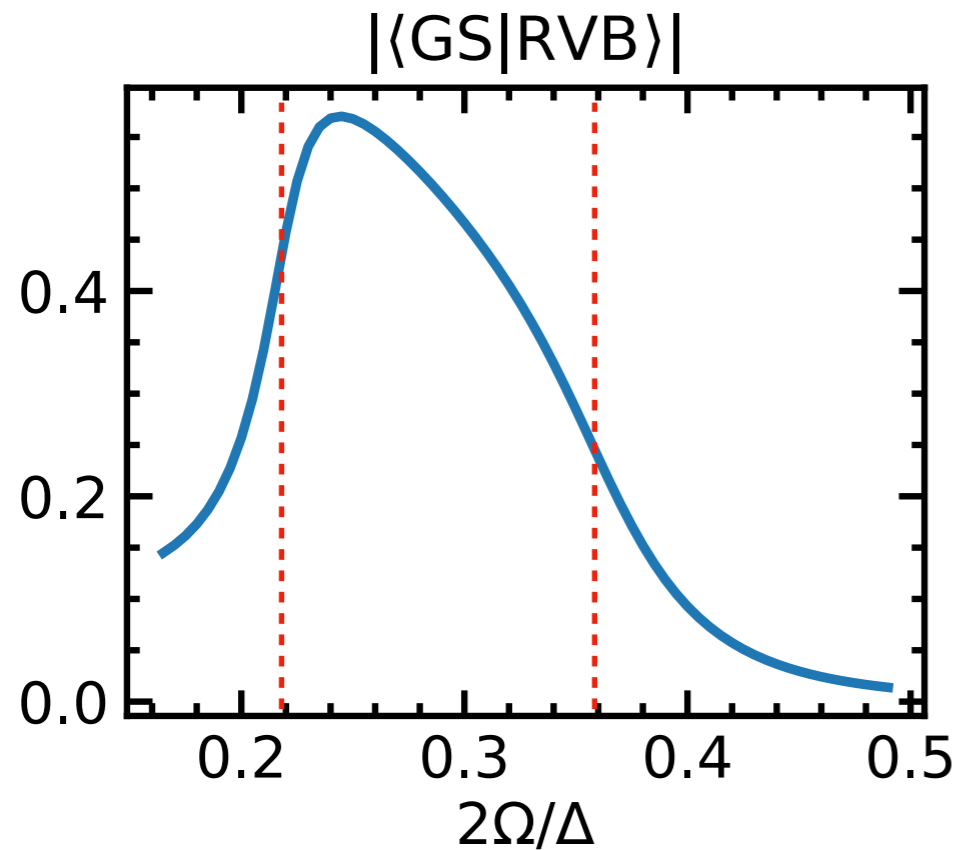
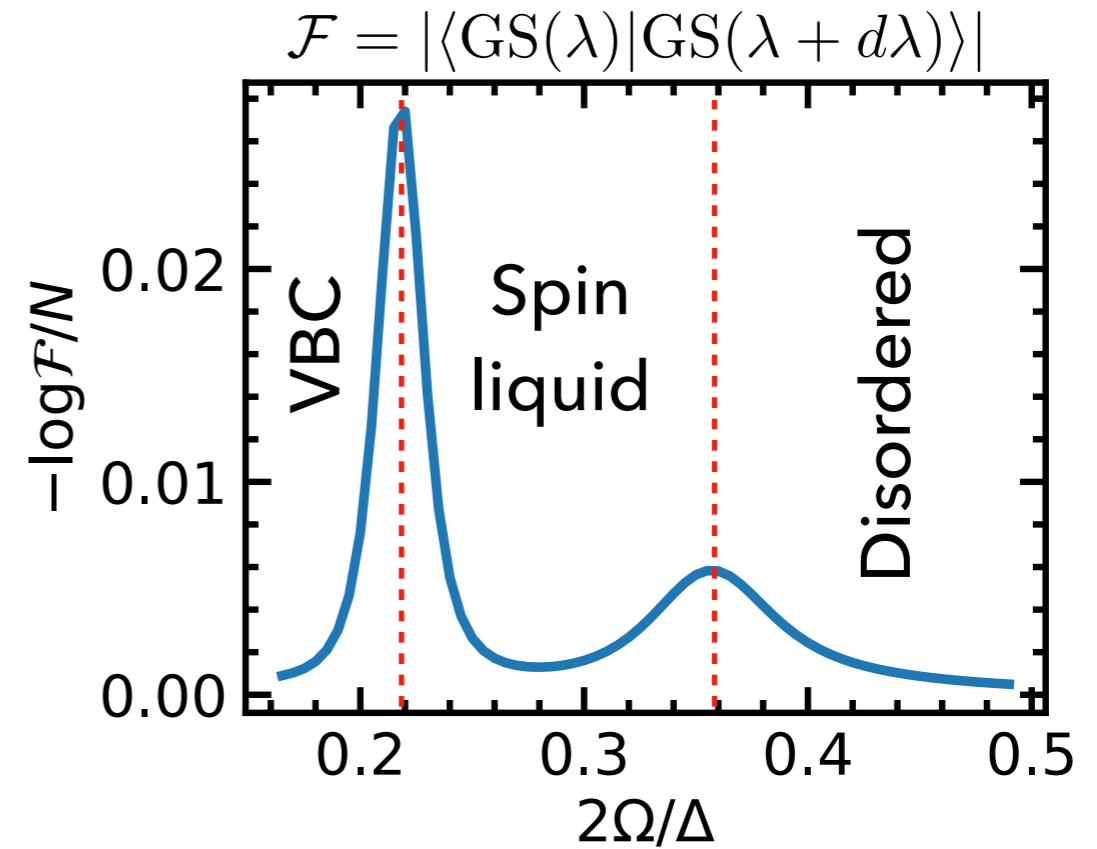
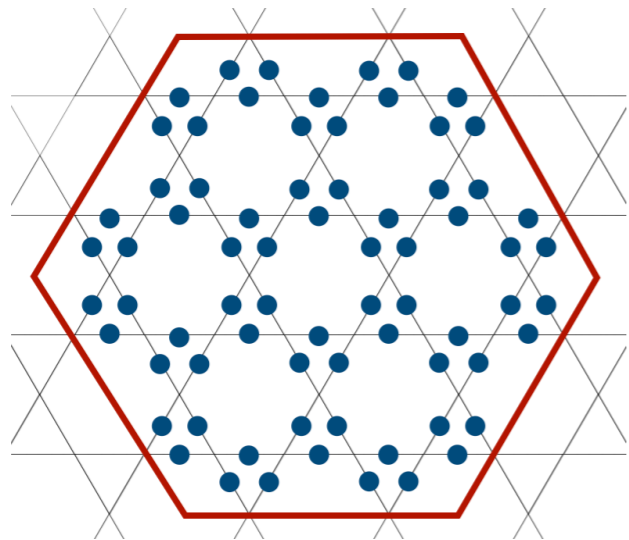


- ➔ *Relation between this many-body state and RVB?*
- ➔ *Simple representation for large-scale calculations?*

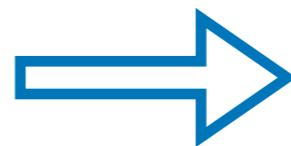


# GROUND STATE

ED on periodic clusters ( $N = 72$ )



*Defects on full dimer coverings  
are allowed (**diluted RVB**)*

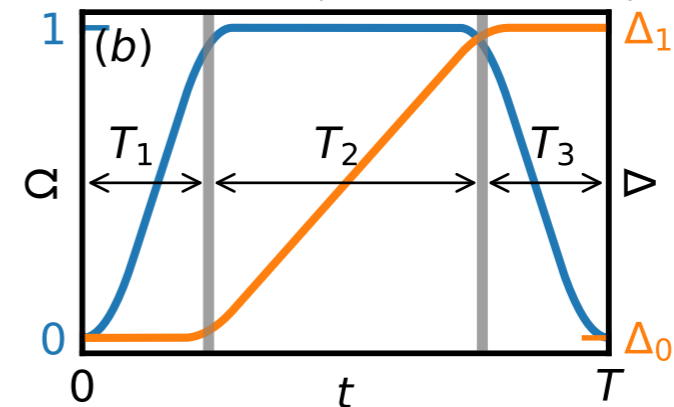


$$|\text{GS}\rangle = |\text{RVB}\rangle + \dots$$

# DYNAMICAL PREPARATION

- Optimized dynamical preparation protocol for **pure** RVB state

$$H(t) = \frac{\Omega(t)}{2} \sum_i \sigma_i^x - \Delta(t) \sum_i n_i$$



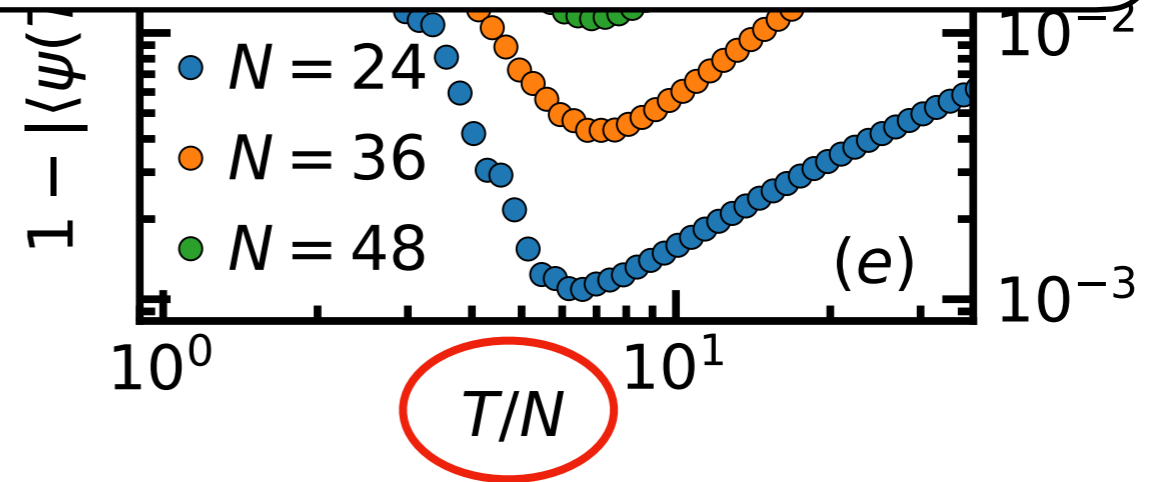
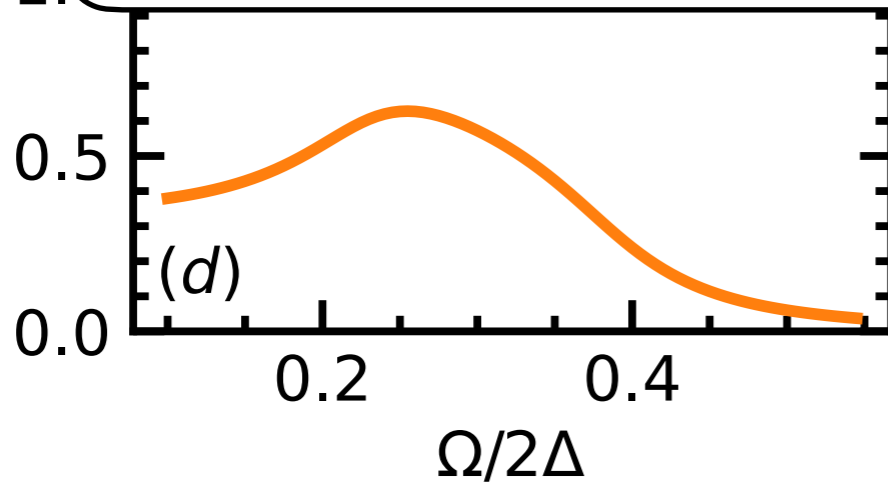
$$\mathcal{F} = 1 - |\langle \text{GS}(\lambda) | \text{GS}(\lambda + d\lambda) \rangle|$$



How to describe the experimentally prepared state?

$\mathcal{F}$

$|\langle \text{GS} | \text{RVB} \rangle|$



- Optimal T scales linearly with the number of atoms

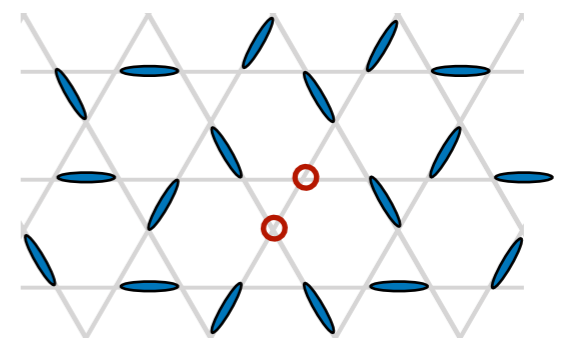
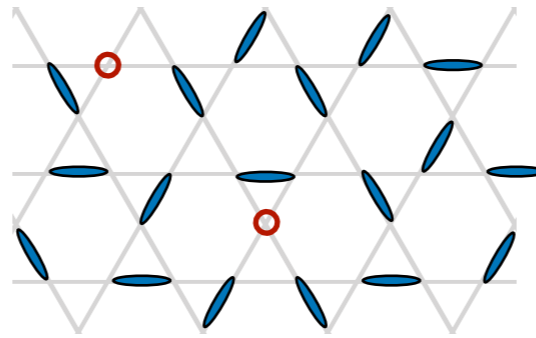
# DILUTED RVB ANSATZ

$$|\phi(z_1, z_2)\rangle = \mathcal{P} \left[ \bigotimes_{i=1}^N (1 + z_2 \sigma_i^+) (1 + z_1 \sigma_i^-) \right] |\text{RVB}\rangle$$

Project onto constrained  
Hilbert space

Allow monomer pairs to  
separate by creating dimers

Create monomer pairs  
by destroying dimers



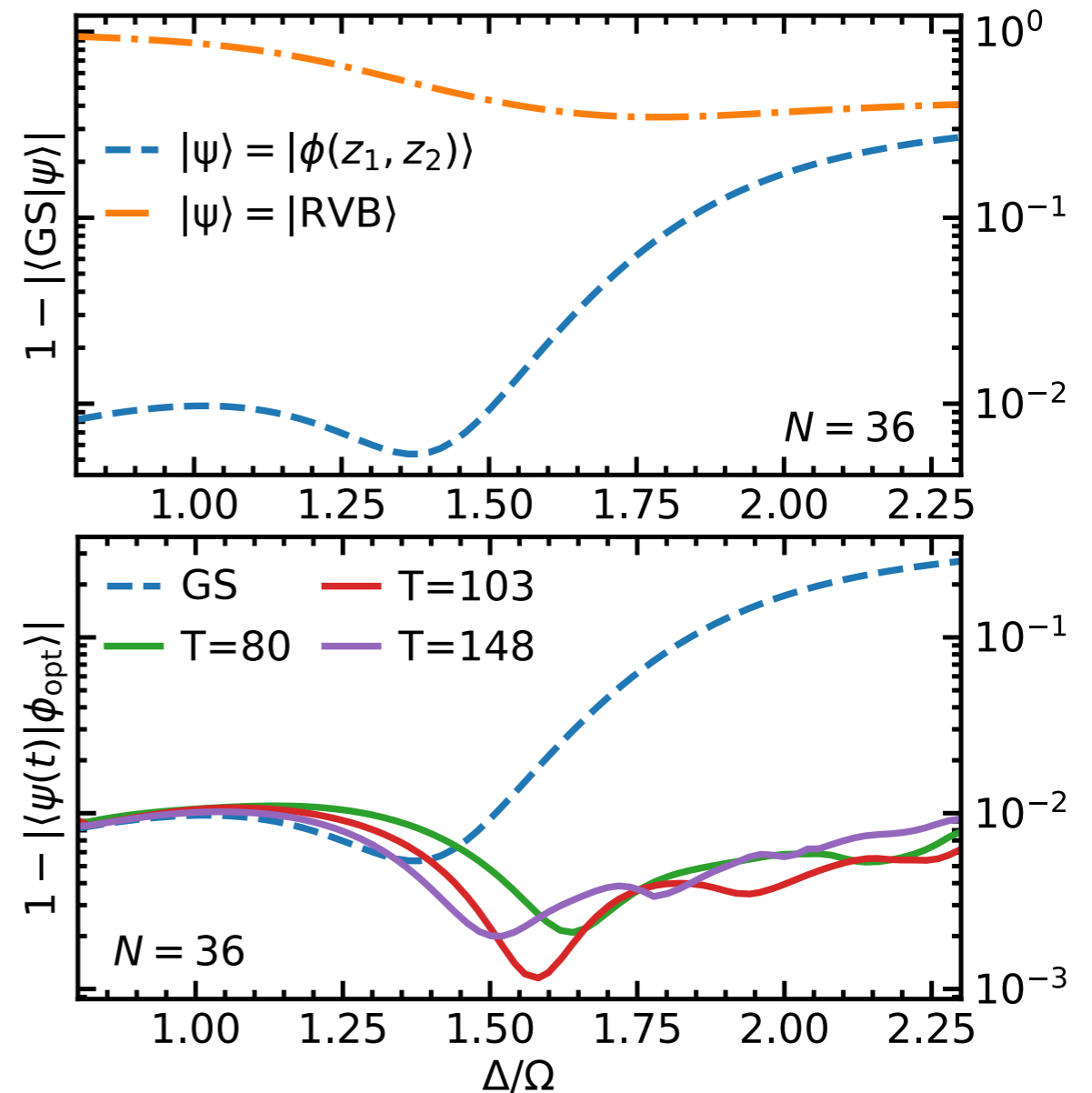
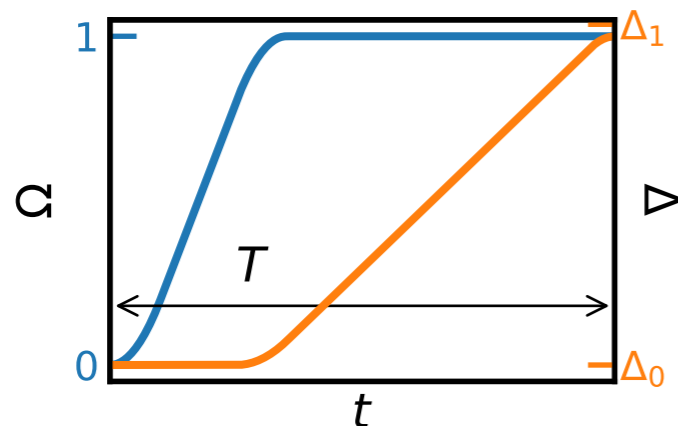


# DILUTED RVB ANSATZ

$$|\phi(z_1, z_2)\rangle = \mathcal{P} \left[ \bigotimes_{i=1}^N (1 + z_2 \sigma_i^+) (1 + z_1 \sigma_i^-) \right] |\text{RVB}\rangle$$

- ▶ Good variational ansatz for the **ground state**
- ▶ Even better ansatz for the **preparation dynamics**

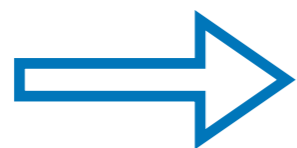
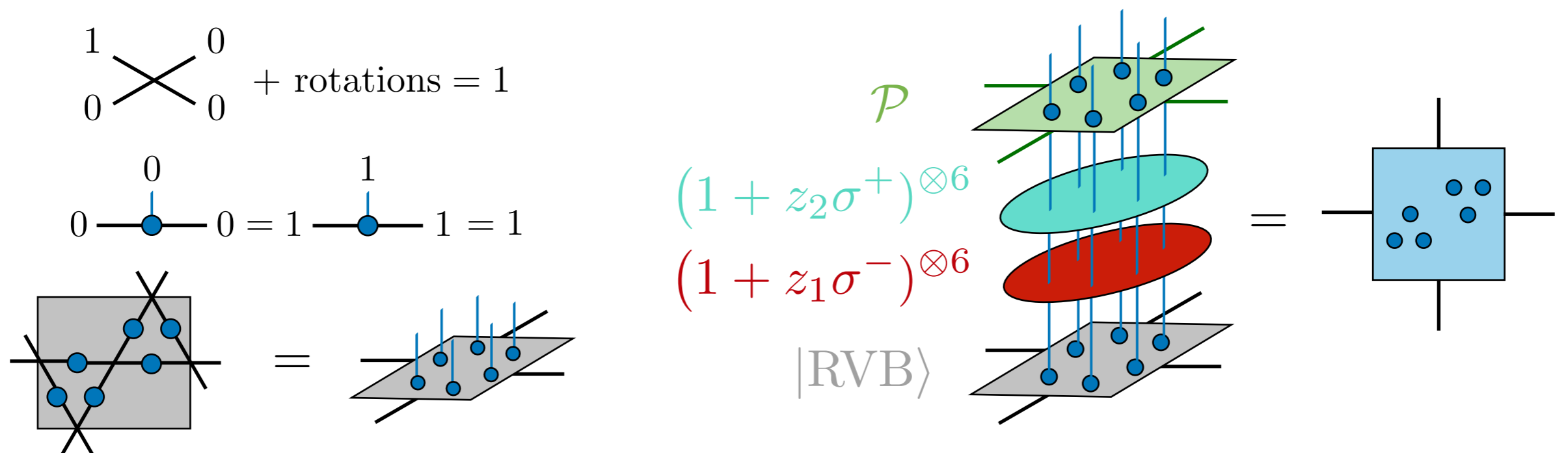
$$H(t) = \frac{\Omega(t)}{2} \sum_i \sigma_i^x - \Delta(t) \sum_i n_i$$



# DILUTED RVB ANSATZ

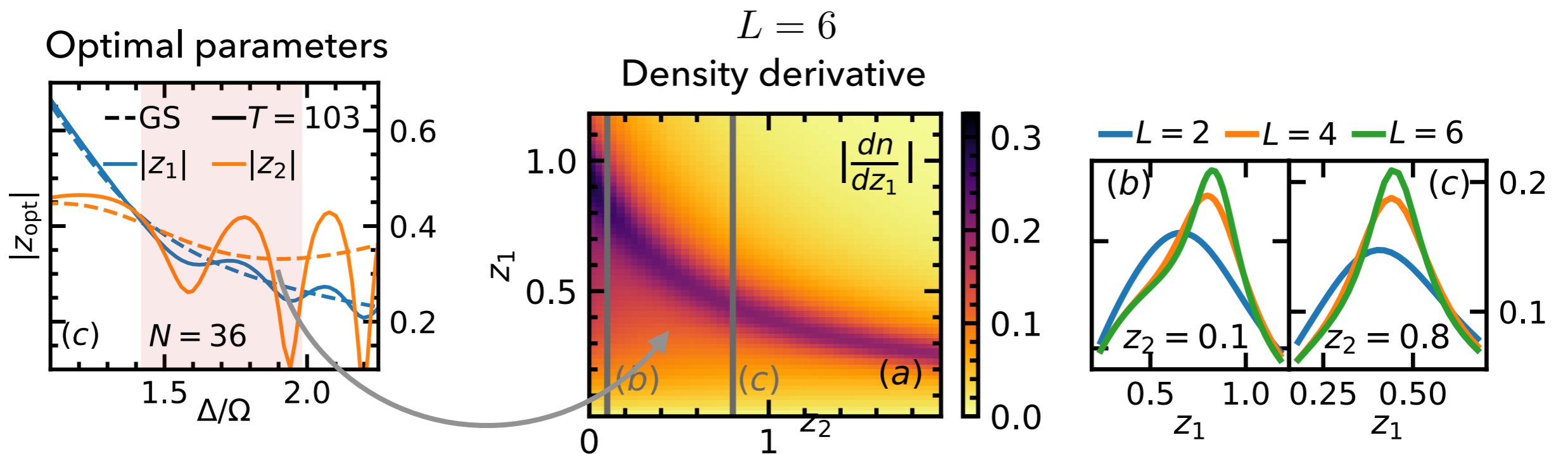
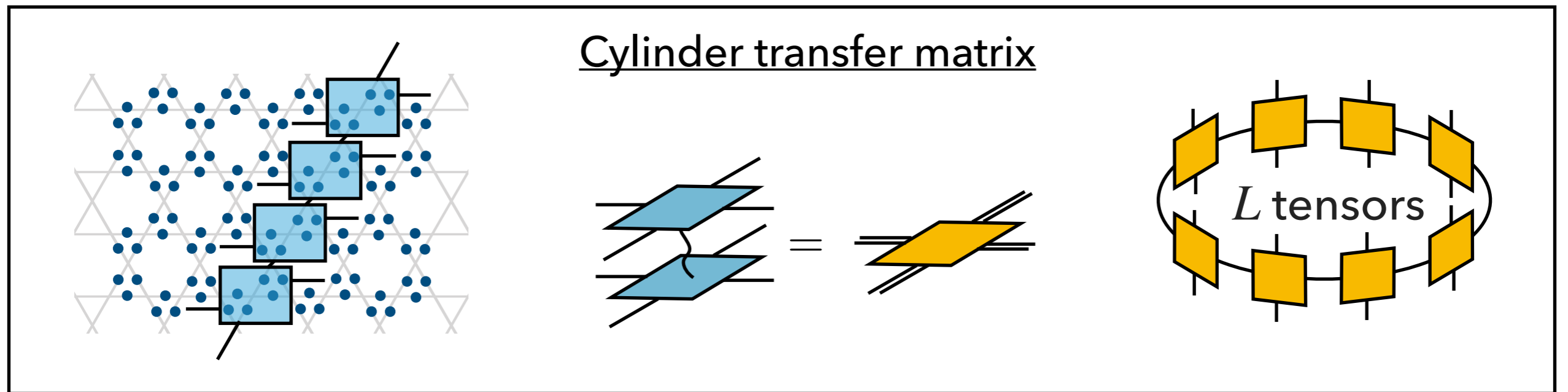
$$|\phi(z_1, z_2)\rangle = \mathcal{P} \left[ \bigotimes_{i=1}^N (1 + z_2 \sigma_i^+) (1 + z_1 \sigma_i^-) \right] |\text{RVB}\rangle$$

- ▶ It is a **tensor network state**



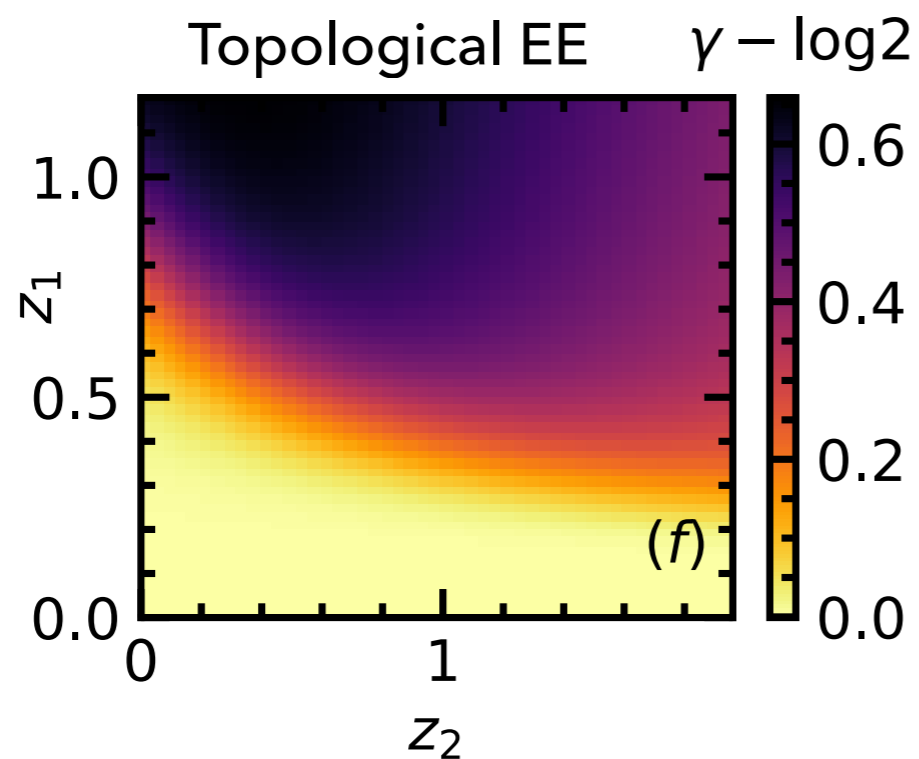
Suitable for large scale calculations!

# STATE PHASE DIAGRAM

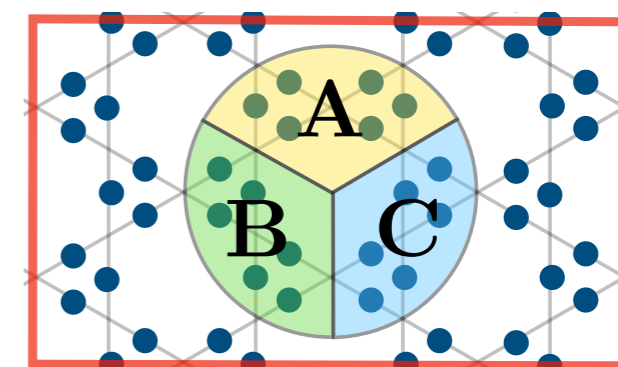


- ▶ Topologically ordered phase in the state phase diagram

# STATE PHASE DIAGRAM

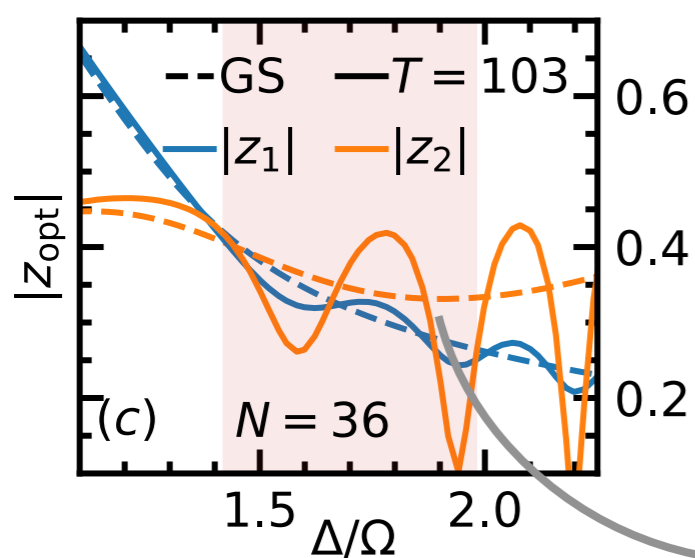


$$\gamma = S_{AB} + S_{BC} + S_{AC} - S_A - S_B - S_C - S_{ABC}$$

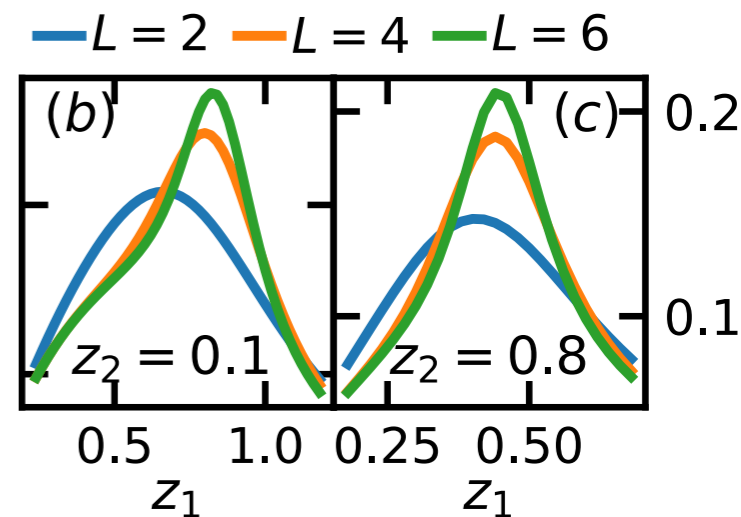
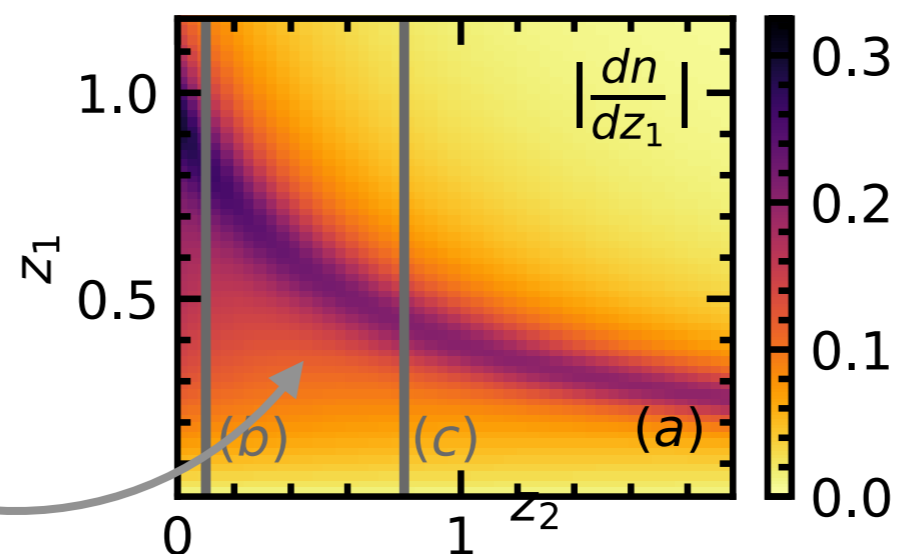


$L = 6$

Optimal parameters



Density derivative



- ▶ Topologically ordered phase in the state phase diagram

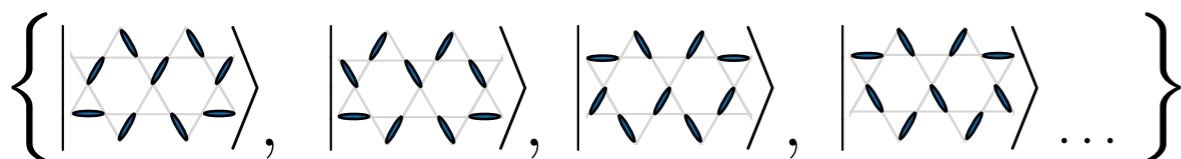
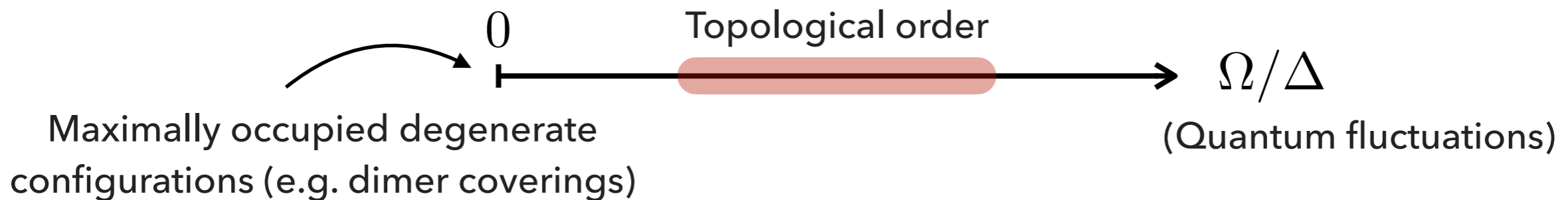
# CONCLUSIONS (1)

- ▶ **Pure** RVB states can be prepared dynamically (quasi-adiabatically) in Rydberg atom Hamiltonians with high fidelity
- ▶ **Diluted** RVB states arising in Rydberg atom arrays can be accurately described with local perturbations of tensor network states that host topological order

General recipe for topological order :

$$H = \frac{\Omega}{2} \sum_i \sigma_i^x - \Delta \sum_i n_i$$

Effective "PXP"  
Rydberg Hamiltonian  
with exact constraint



*Can this be generalized?*

# TRIMERS & RYDBERG ATOMS

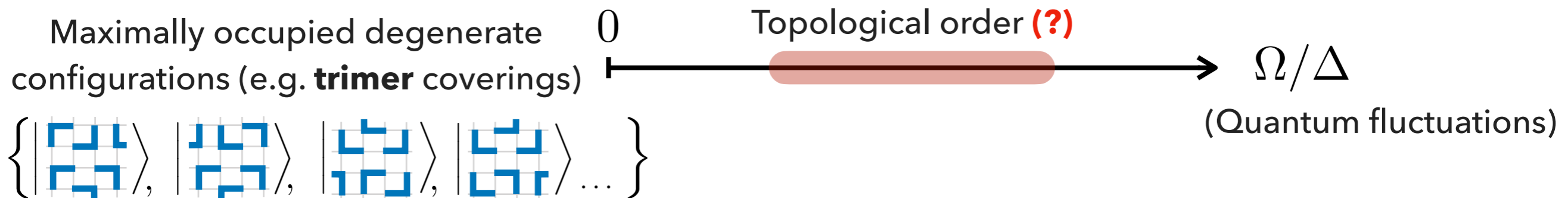
- ▶ **Trimer** : set of 3 nearest neighbour vertices
- ▶ **Constraint** : one trimer on each vertex

Implementation with Rydberg atoms

$\text{---}\circ\text{---} = |g\rangle$   
 $\text{---}\bullet\text{---} = |r\rangle$

$$H = \frac{\Omega}{2} \sum_{\square} |\square \times \square| + R_{\frac{\pi}{2}} + \text{h.c.} - \Delta \sum_{\square} |\square \times \square| + R_{\frac{\pi}{2}}$$

*Check the arXiv in a couple of weeks*



# TRVB STATES IN TRIMERS MODELS

▶ Trimer model :  $\mathcal{H} = \left\{ \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle, \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle, \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle, \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle \dots \right\}$

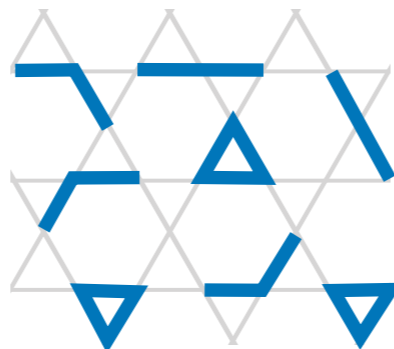
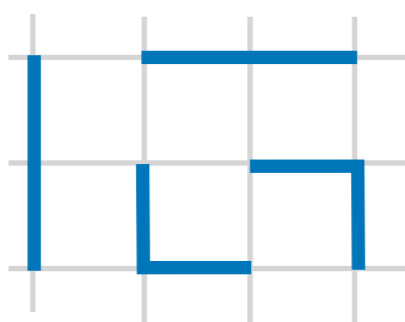
▶ (Trimer) resonating valence bond (tRVB) state:

equal weight superposition of all trimer coverings

$$|tRVB\rangle = \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle + \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle + \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle + \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle + \dots$$

**Do they provide topological order?**

Much more freedom than with dimers ...



...

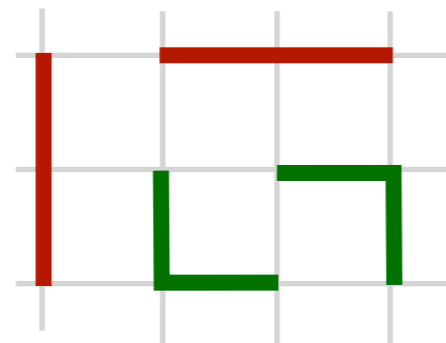
# TRVB: WHAT DO WE KNOW?

- ▶ Admit simple tensor network representations for large system calculations

## ▶ Square lattice

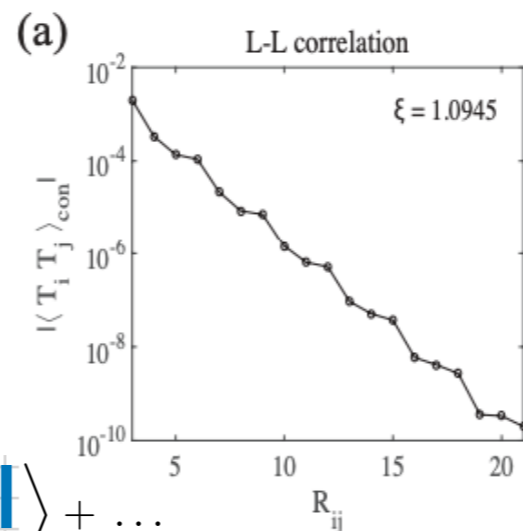
type 1 trimers

type 2 trimers

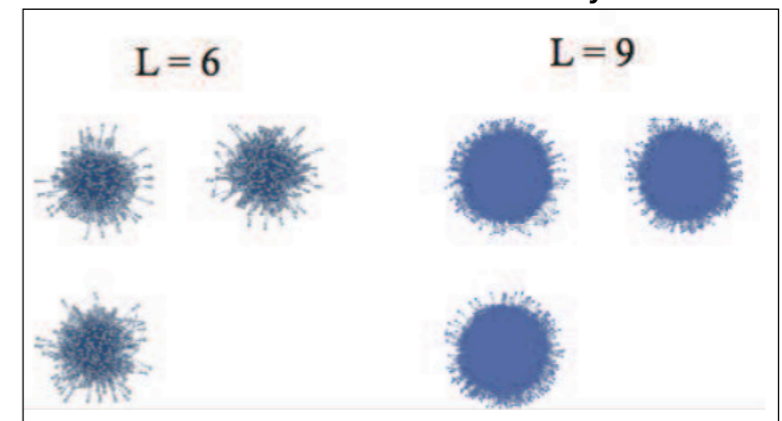


$$|\text{tRVB}\rangle = \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\rangle + \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\rangle + \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\rangle + \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\rangle + \dots$$

**$|\text{tRVB}\rangle = \text{type 1} + \text{type 2}$**



Transfer matrix connectivity



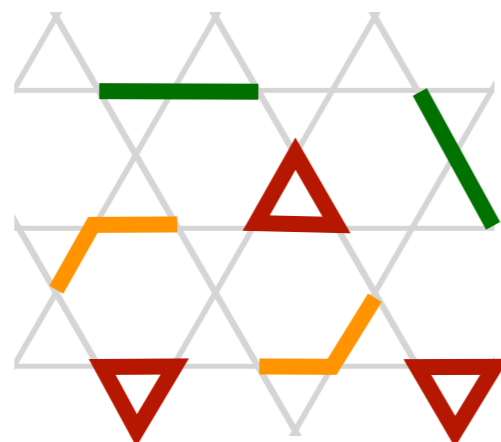
H Lee, Y Oh, JH Han, H Katsura, 2017

## ▶ Kagome lattice

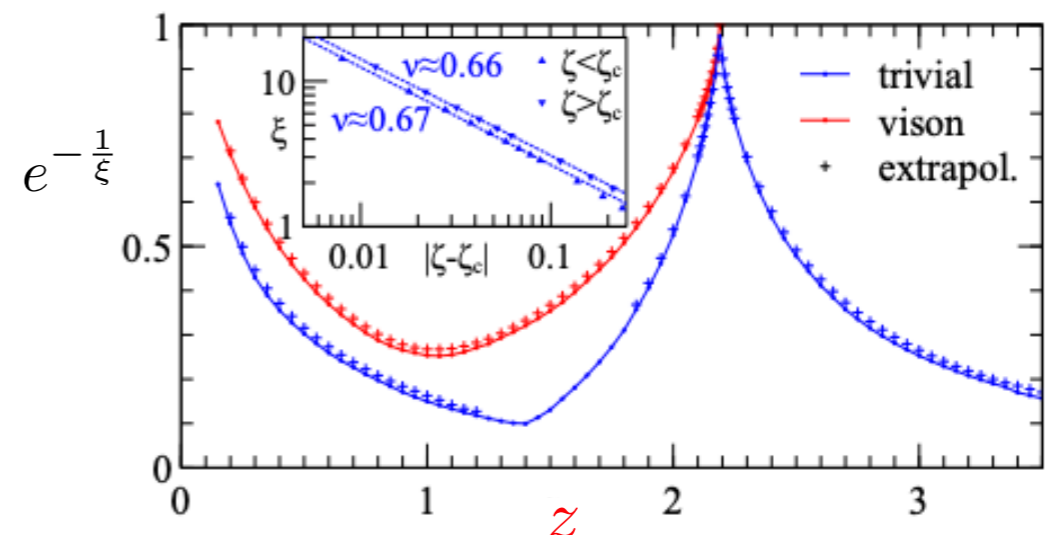
type 1 trimers

type 2 trimers

type 3 trimers



**$|\text{tRVB}\rangle = \text{type 1} + \text{type 2} + z^* \text{type 3}$**



S Jandura, M Iqbal, N Schuch, 2020

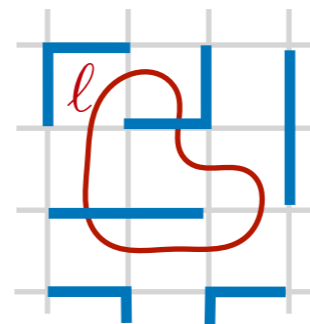
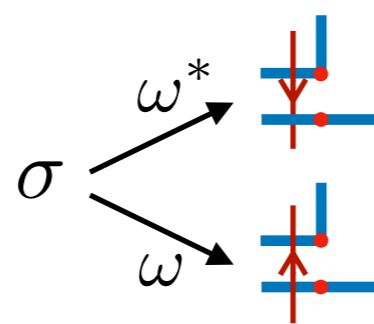


# TRVB: SOME NEW RESULTS

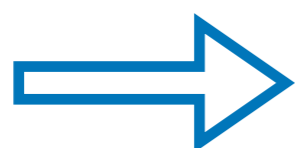
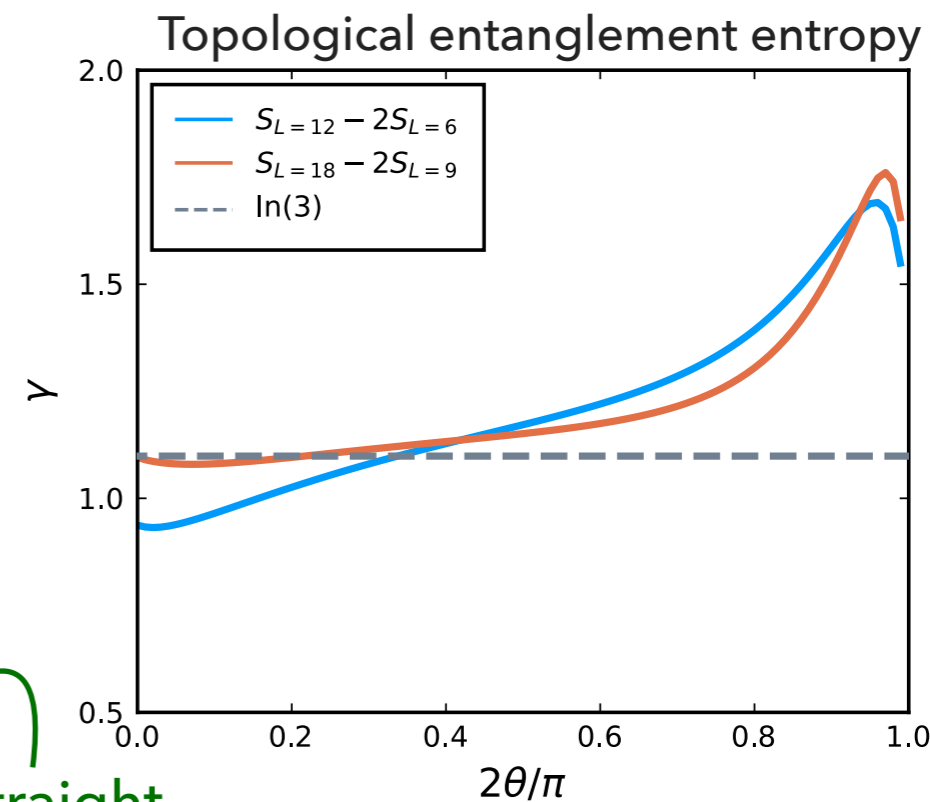
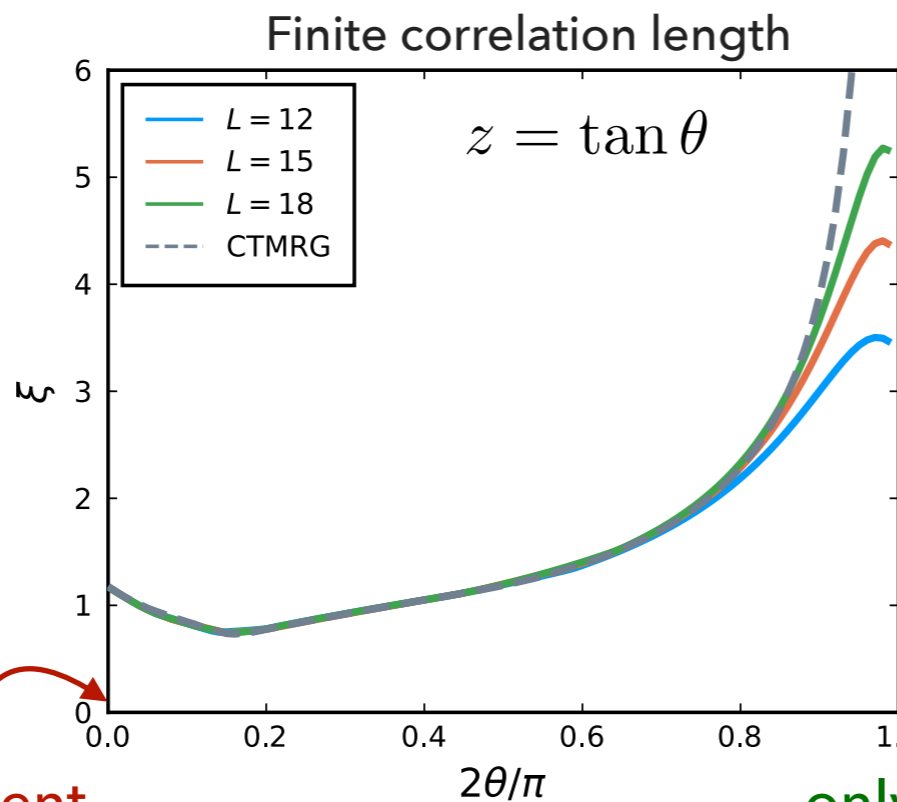
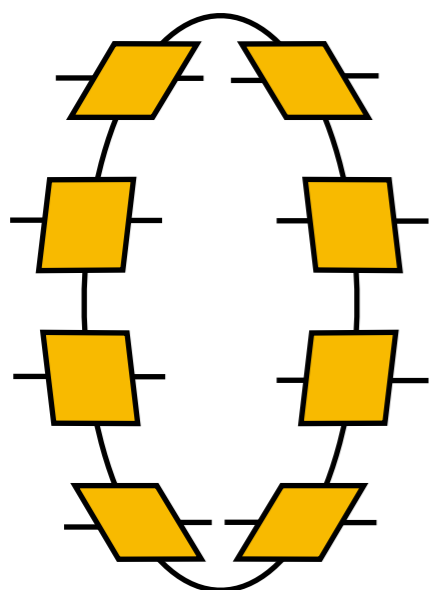
$$|\text{tRVB}\rangle = \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle + z \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle + z^2 \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle + z^3 \left| \begin{array}{ccc} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{array} \right\rangle + \dots$$

▶  $\mathbb{Z}_3$  Gauss' law

$$\omega = e^{i2\pi/3}$$



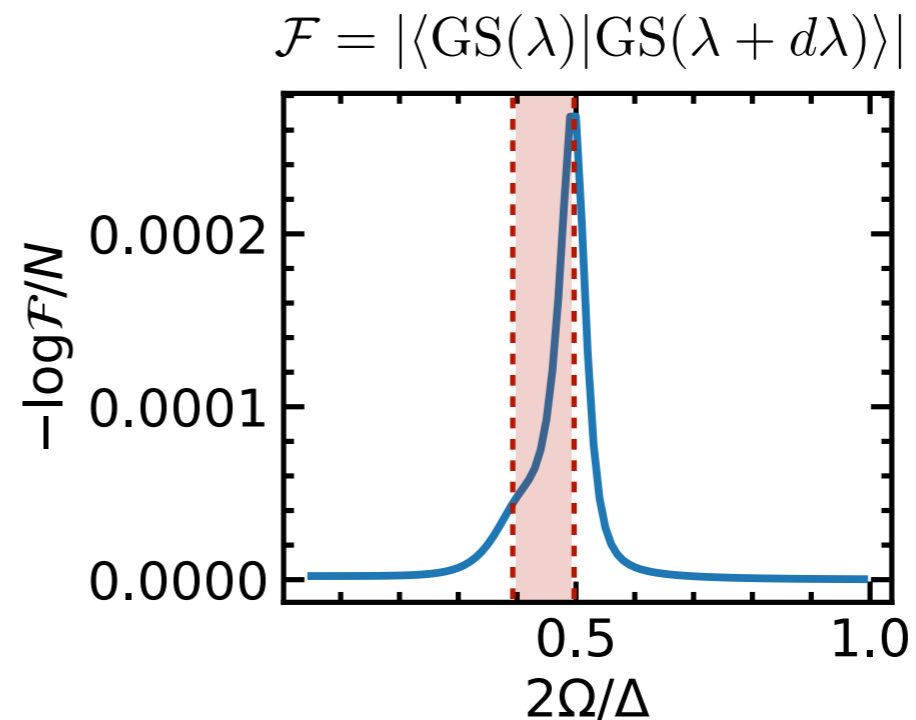
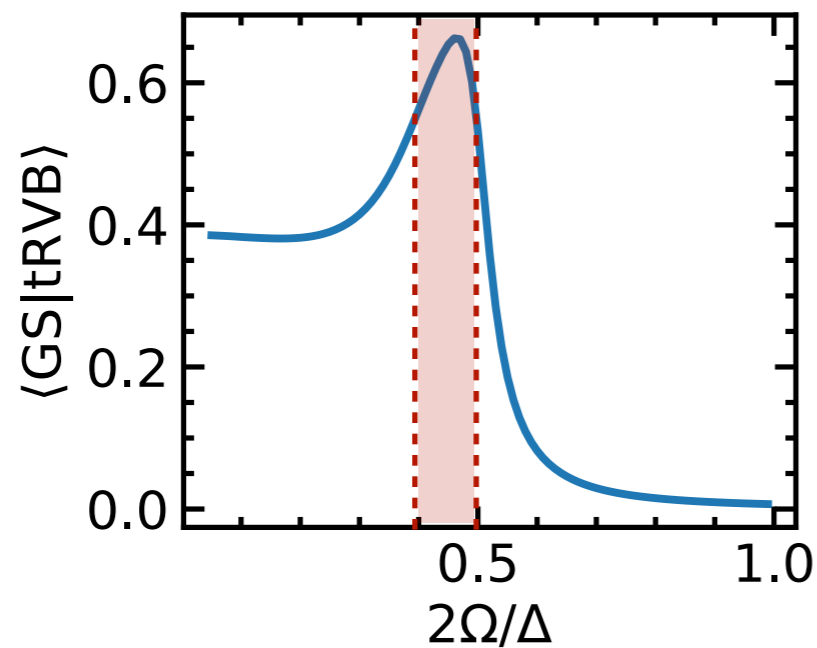
$$\prod_{i \in l} \sigma_i = \omega^{N_v}$$



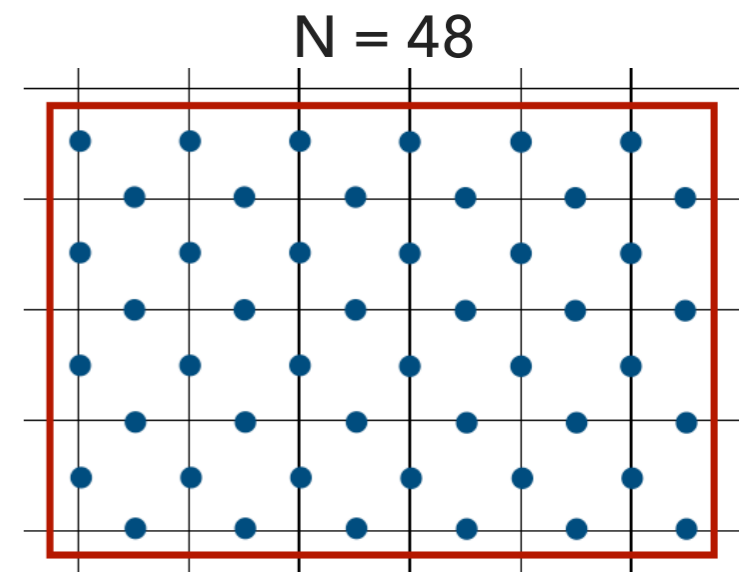
$\mathbb{Z}_3$  topologically ordered for all  $\theta \neq \frac{\pi}{2}$

# BENT TRIMERS ON THE SQUARE LATTICE

$$H = \frac{\Omega}{2} \sum_{\square} | \begin{array}{|c|} \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \end{array} | + R_{\frac{\pi}{2}} + \text{h.c.} - \Delta \sum_{\square} | \begin{array}{|c|} \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \end{array} | + R_{\frac{\pi}{2}}$$

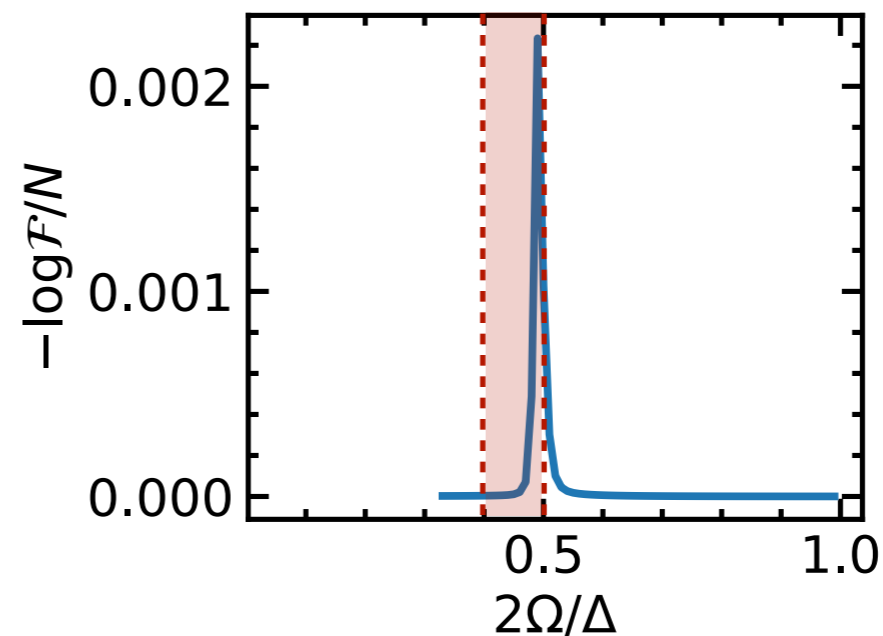
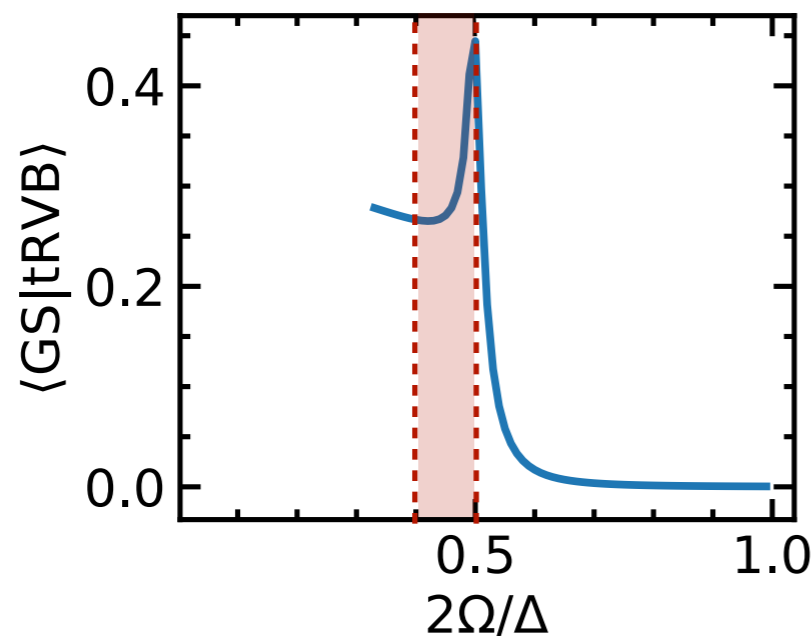


- ▶ Intermediate phase with  $\mathbb{Z}_3$  topological order?



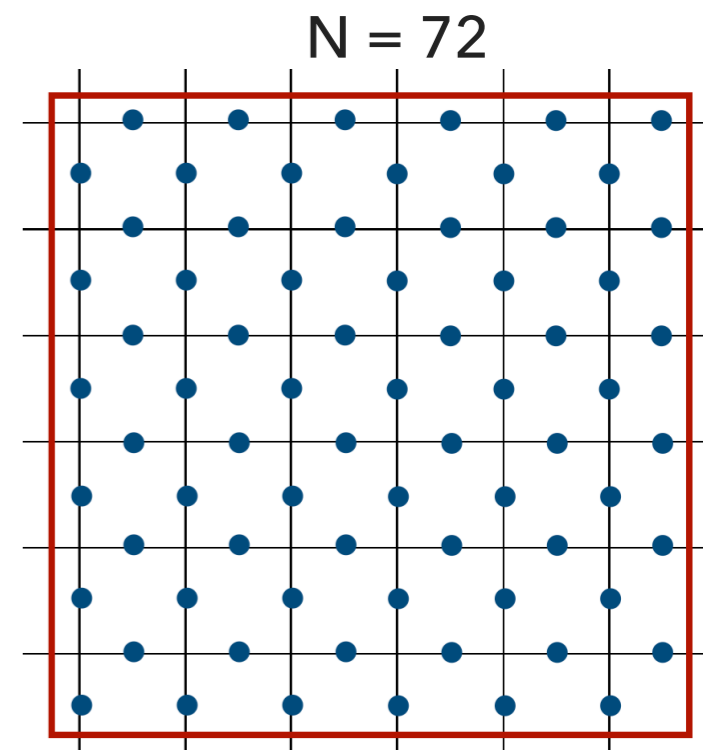
# BENT TRIMERS ON THE SQUARE LATTICE

$$H = \frac{\Omega}{2} \sum_{\square} | \begin{array}{|c|} \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \end{array} | + R_{\frac{\pi}{2}} + \text{h.c.} - \Delta \sum_{\square} | \begin{array}{|c|} \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \end{array} | + R_{\frac{\pi}{2}}$$



- ▶ Intermediate phase with  $\mathbb{Z}_3$  topological order?
- ▶ Unstable with the system size?
- ▶ Can we prepare this tRVB states quasi-adiabatically?

*... check the arXiv in a couple of weeks*



## CONCLUSIONS (2)

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- ▶ tRVB states provide quantum spin liquids with  $\mathbb{Z}_3$  topological order
- ▶ tRVB-like phases might arise from Rydberg atom Hamiltonians
- ▶ Are there other topological states that can be engineered?

**Thank you!**