Entanglement scaling for $\lambda\phi_2^4$ @ Benasque 2022

Bram Vanhecke

University of Vienna

February 28, 2022

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[1]based on 'A scaling hypothesis for matrix product states' by BV, Jutho Haegeman, Karel Van Acoleyen, Laurens Vanderstraeten, Frank Verstraete Phys.Rev.Lett. 123 (2019) no.25, 250604
and 'Entanglement scaling for $\lambda\phi_2^4$'. by BV, Frank Verstraete, Karel Van Acoleyen arXiv: 2104.10564 [hep-lat]


Gear VR

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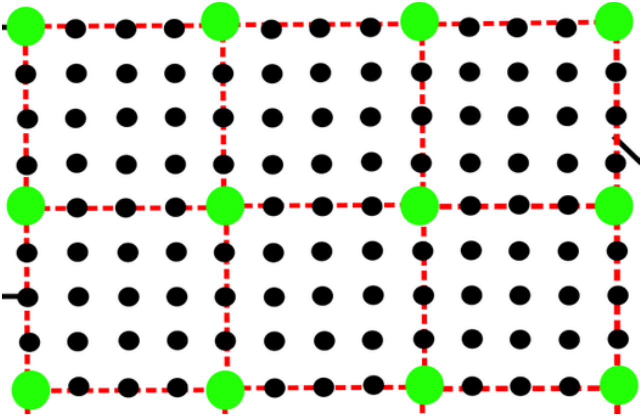

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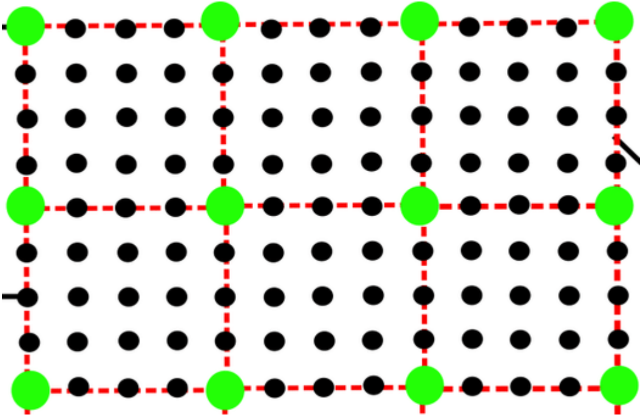

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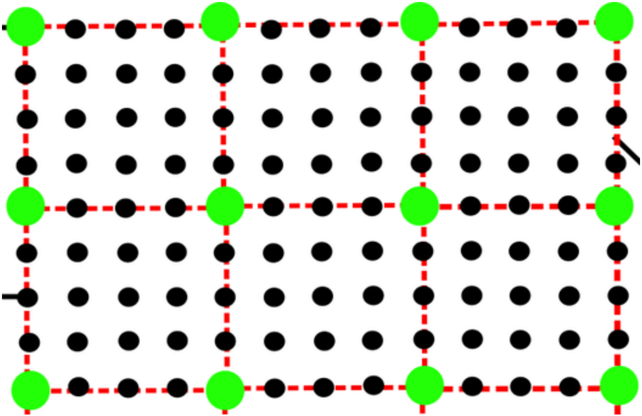

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- Lattice spacing a is a UV-regulator

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  - can be calculated with perturbation theory for asymptotically free theories

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$$\mu^2 = a^2\alpha - 3a^2 A(a^2\alpha) \qquad (2)$$

$$\phi(\alpha) = \phi(\mu^2,\lambda) \;\propto\; \phi^3(\mu^2,\lambda) - \frac{3}{4\pi}\log(\lambda)\phi(\mu^2,\lambda) \qquad (3)$$

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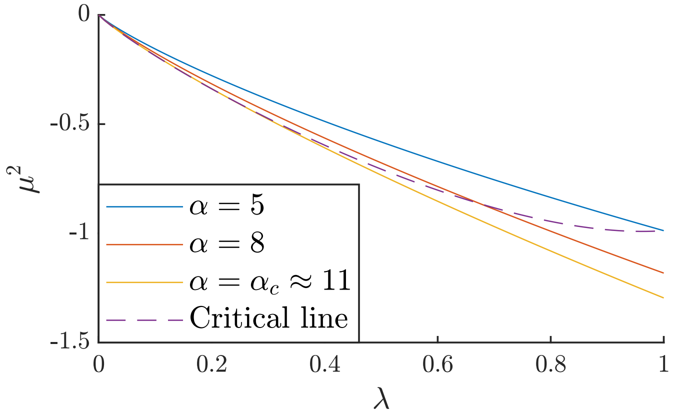

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[2] Daisuke Kadoh, Yoshinobu Kuramashi, Yoshifumi Nakamura, Ryo Sakai, Shinji Takeda, and Yusuke Yoshimura, "Tensor network analysis of critical coupling in two dimensional 4 theory," JHEP 2019, 184 (2019).

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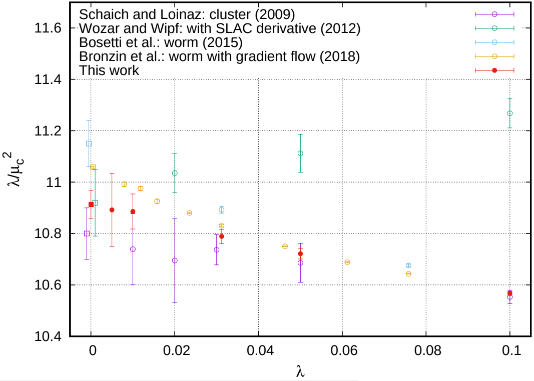

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$\alpha - \alpha_c$	0	$1/\nu = 1$
L^{-1} , δ	1	1
$l^{-1} = L^{-1}/\sqrt{\lambda}$, $\Delta = \delta/\sqrt{\lambda}$	0	1
ξ	-1	-1
ϕ	0	$\beta = 1/8$
$\phi^3 - \frac{3}{4\pi}\log(\lambda)\phi$	0	$\beta = 1/8$
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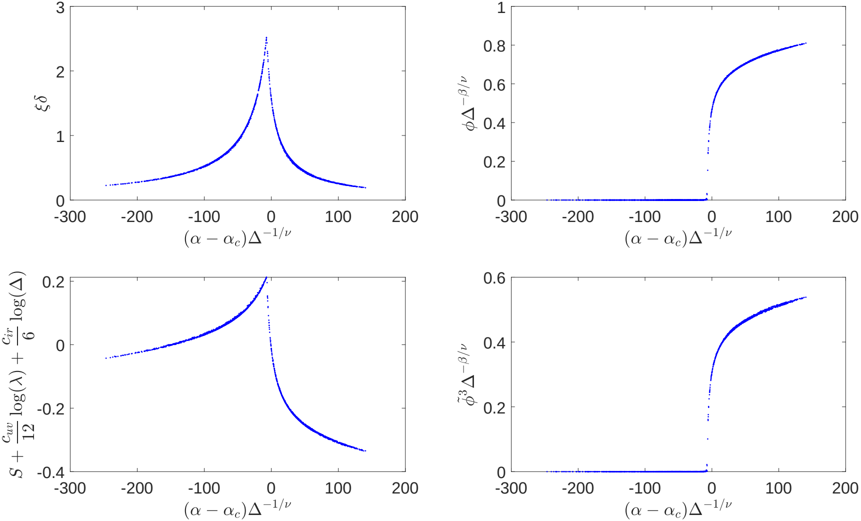
- Corrections to these properties must be taken into account

Double collapse plots



Critical point

Method	$f_c^{\text{cont.}}$	Year	Ref.
Tensor network coarse-graining	10.913(56)	2019	[9]
Borel resummation	11.23(14)	2018	[6]
Renormalized Hamil. Trunc.	11.04(12)	2017	[5]
Matrix Product States	11.064(20)	2013	[7]
Monte Carlo	11.055(20)	2019	[15]
This work	11.0861(90)	2020	[3]

$$\alpha_c = 11,09698(31)$$

[3] Clement Delcamp and Antoine Tilloy, "Computing the renormalization group flow of two-dimensional 4 theory with tensor networks," Phys. Rev. Research 2, 033278 (2020)

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- and matrices with the Boltzmann weights for all the interactions:

$$t(i,j) = e^{-\beta H(i,j)}\,.$$

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- An efficient, arbitrarily precise MPO is thus:

$$MPO(i,j,k,p) = \int d\phi \quad \langle v_i|\phi\rangle\,\langle v_j|\phi\rangle\,\langle\phi|v_k\rangle\,\langle\phi|v_p\rangle\,\sqrt{\Lambda_i\Lambda_j\Lambda_k\Lambda_p}\,.$$

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- Construction can be readily generalized
  $\rightarrow$ fermions, weird constraints on fields, topological terms,...

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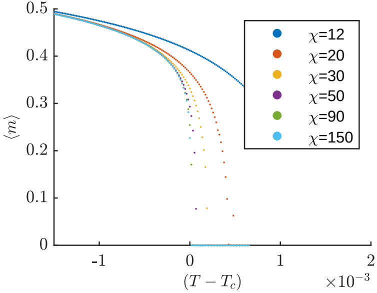
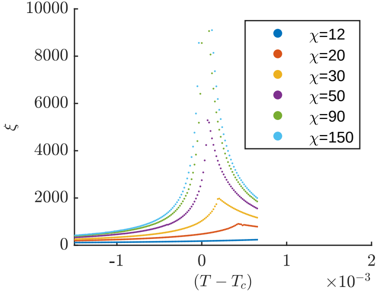
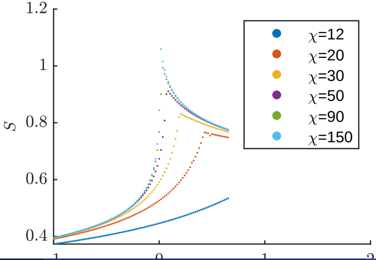
Construct RG-invariant quantities to make a collapse

Ising model example







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