Simulating chiral spin liquids with projected entangled-pair states

(JH, M. Van Damme, D. Poilblanc, L. Vanderstraeten, arXiv:2201.07758)

Outline

1) Introduction & Motivation

- Fractional quantum Hall states in lattice models – chiral spin liquids (CSL)
- Lattice hamiltonians hosting CSL phase

2) iPEPS perspective

- CSL via symmetric iPEPS construction
- CSL via unrestricted iPEPS

3) Conclusions & Future directions

Fractional quantum Hall effect

Quantum matter beyond Landau-Ginzburg paradigm

Discovery: D. C. Tsui, H. L. Stormer, & A. C. Gossard, PRL (1982)

First theory: R. B. Laughlin PRL (1983)

Topological order

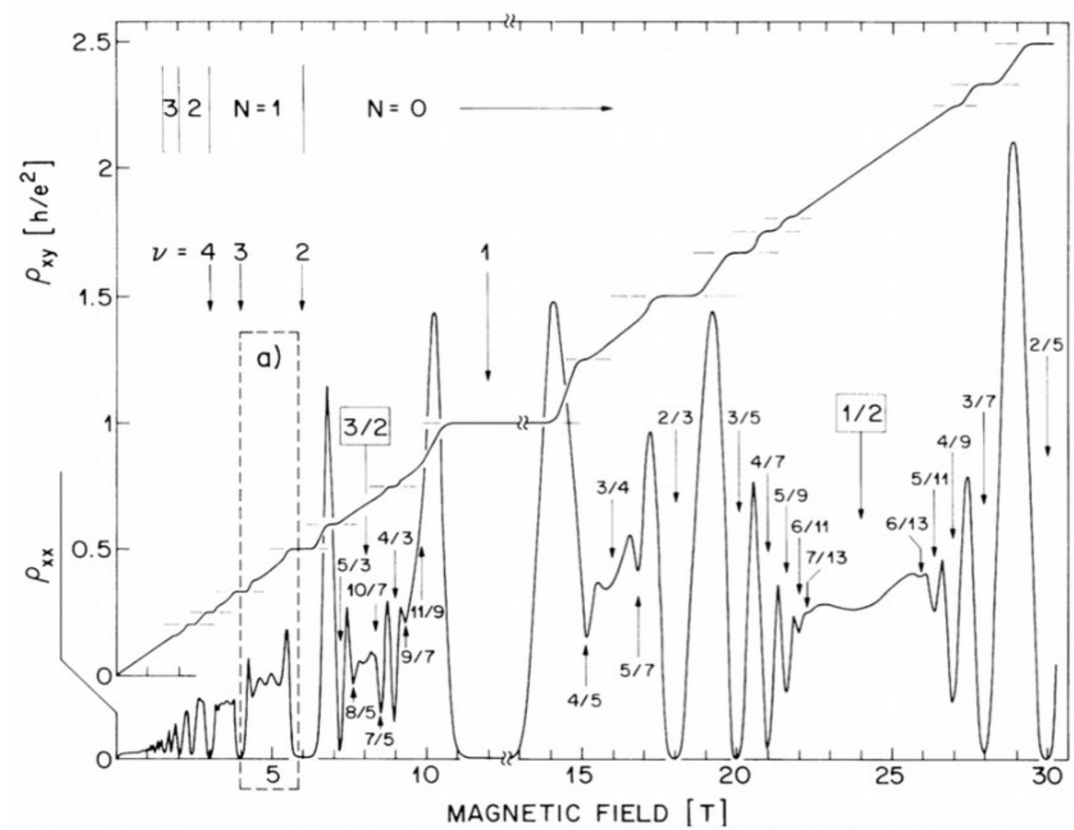
- Fractionally charged quasiparticles

Goldman, Su, Science (1995)
Saminadayar, Glattli, Jin, and Etienne, PRL (1997)
de-Picciotto et al, Nature (1997)
Martin et al, Science (2004)

- Anyonic exchange statistics

Nobel Prize 1998

Laughlin, Störmer, Tsui



R. Willett, J. P. Eisenstein, H. L. Stormer, D. C. Tsui, A. C. Gossard and H. English, PRL (1987)

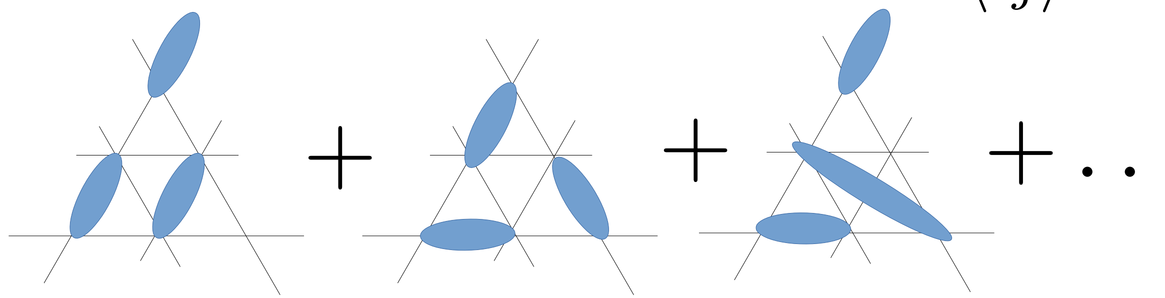
FQHE on a lattice

- **Magnetic field not essential ? Occurence in Nature ?**

Equivalence of Anderson's RVB state to FQH state for triangular lattice spin-1/2 Heisenberg antiferromagnet

$$H = J \sum_{\langle ij \rangle} \mathbf{S_i} \cdot \mathbf{S_j}$$

$$|\mathrm{RVB}\rangle = \sum_c \phi_c |c\rangle =$$



 $= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

$$\Updownarrow$$

$$\psi_{KL}(z_1, ..., z_N) = \prod_{j<k} (z_j - z_k)^2 exp(-\frac{1}{4l_0^2} \sum_i^N |z_i|)$$

Extension to square lattice soon after

Kalmeyer, Laughlin, PRL (1987)
Zou, Doucot, Shastry, PRL (1989)

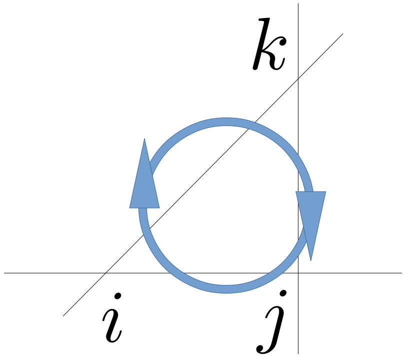
FQHE on a lattice

Fractional statistics ⇔ violation of **P** and **T** symmetry

Chiral spin states: (spontaneously) violate **P** and **T** but preserve **PT**

$$\langle \mathbf{S_i} \cdot (\mathbf{S_j} \times \mathbf{S_k}) \rangle \neq 0$$

$$\Leftrightarrow P_{ijk} - P_{ijk}^{-1} \neq 0$$



$$P|ijk\rangle = |jki\rangle$$

Chiral spin liquids (CSL) – lattice analogues of FQH states, where **large loops** γ are ordered

$$\mathrm{Im}\ ln\langle P_{n1} \dots P_{23} P_{12} \rangle_\gamma \propto bA(\gamma)$$

Wen, Wilczek, Zee, PRL (1989)

Parent Hamiltonians for CSL

Physical Hamiltonians i.e. local and without explicit breaking of **P** or **T** symmetry with CSL phase

$$H = -t \sum_{\langle ij \rangle \sigma} c^\dagger_{i\sigma} c_{j\sigma} + h.c. + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Build **parent Hamiltonians**

- 6-spin terms

D. F. Schroeter, E. Kapit, R. Thomale, and M. Greiter, PRL (2007)
R. Thomale, E. Kapit, D. F. Schroeter, and M. Greiter, PRB (2009)

- construction from conformal field theory (CFT) + truncation

A. E. B. Nielsen, G. Sierra, and J. I. Cirac, Nat. comm. (2013)
I. Glasser, J. I. Cirac, G. Sierra, and A. E. B. Nielsen, NJP (2015)
B. Jaworowski, A. E. B. Nielsen, arXiv:2202.09193

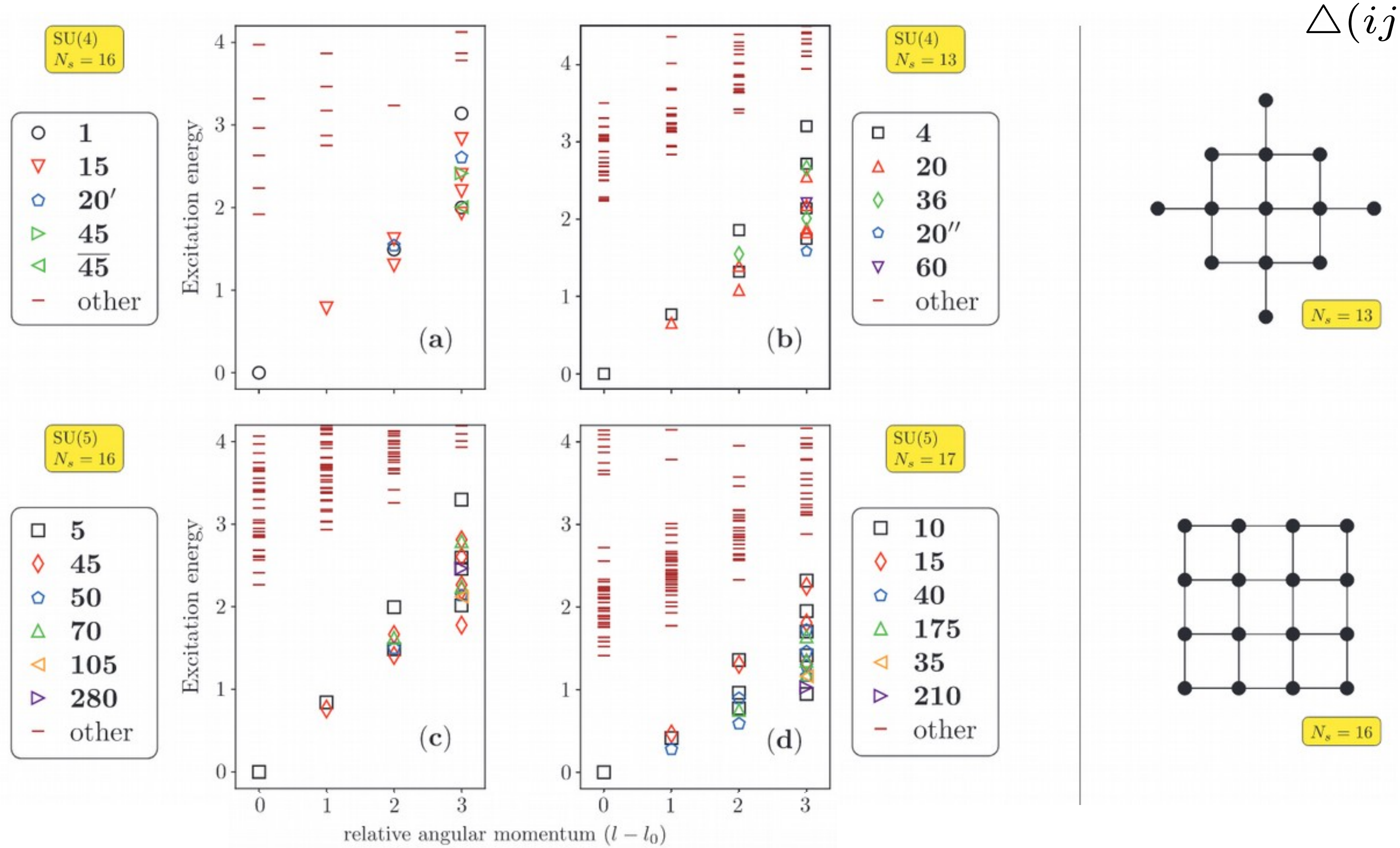
Models with scalar chirality or extended interactions

Bauer et al, Nat. comm. (2014)
S.-S. Gong, W. Zhu, and D. Sheng, Sci. rep. (2014)
A. Wietek and A. M. Läuchli, PRB (2017)

Parent Hamiltonians for CSL

SU(N) CSL phases commonly appear in the case of explicit T-symmetry breaking

$$H = H_{2-site} + K \sum_{\triangle(ijk)} (P_{ijk} - P_{ijk}^{-1})$$

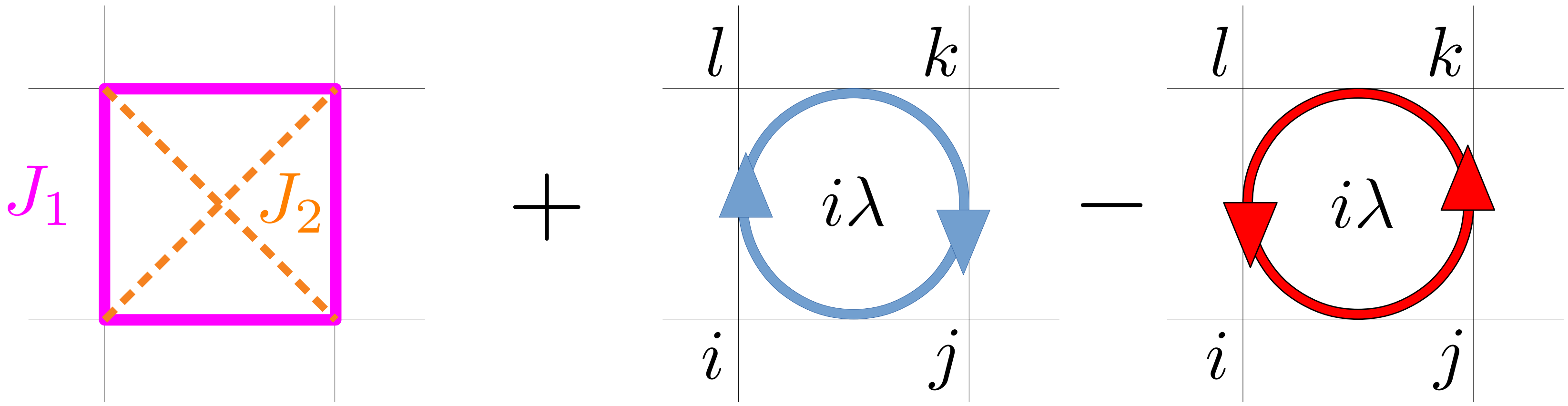


Nataf et al, PRL (2016); Chen et al PRB (2021) [ED,DMRG.PEPS]

Chiral antiferromagnet on a square lattice

Extension of the paradigmatic example of a frustrated magnet (J_1, J_2>0)

$$H = J_1 \sum_{\langle i,j \rangle} S_i . S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i . S_j + i\lambda \sum_{\square(ijkl)} (P_{ijkl} - P^{-1}_{ijkl})$$



$P|ijkl\rangle = |jkli\rangle$ $\quad$ $P^{-1}|ijkl\rangle = |lijk\rangle$

A. E. B. Nielsen, G. Sierra, J. I. Cirac, Nat. comm. (2013)

Chiral antiferromagnet on a square lattice

Parametrization on a sphere

$$J_1 = 2cos(\phi_1)cos(\phi_2)$$
$$J_2 = 2sin(\phi_1)cos(\phi_2)$$
$$\lambda = 2sin(\phi_2)$$

Point of **maximal overlap**

$$\phi_1 = 0.14\pi, \phi_2 = 0.06\pi$$

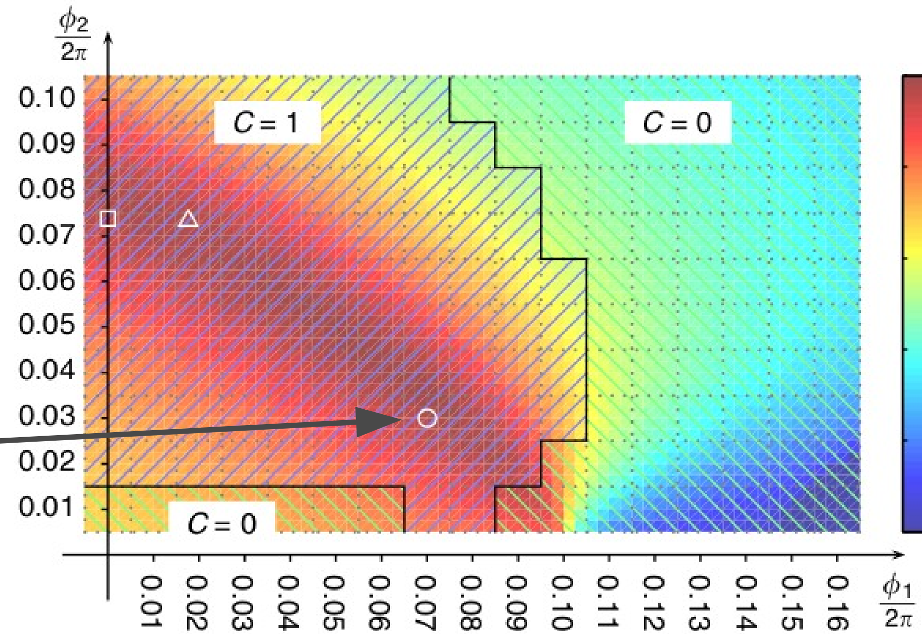


Figure 2 | Phase diagram. Phase diagram of the Hamiltonian in equation (3). The background colour gives the overlap between the CFT state in equation (2) and the ground state of the Hamiltonian in equation (3) for a 4 × 5 lattice with open boundary conditions. C is the total Chern number of the states ψ'_{T0} and ψ'_{T1} on a 4 × 5 lattice with periodic boundary conditions, where ψ'_{T0} (ψ'_{T1}) is the lowest energy state in the subspace spanned by all states with the same eigenvalues of S^z_{tot} and the translation operators in the x- and y directions as ψ^{CFT}_{T0} (ψ^{CFT}_{T1}). Within the topological phase ($C=1$), the two states are well separated from higher energy states in the same subspaces and flow into each other under flux insertion (like in Fig. 3b). The white square, triangle and circle mark possible parameter choices considered in the text. We omit $\phi_2 = 0$ because the additional symmetries present for this case may cause the lowest energy states in the considered subspaces to be degenerate.

A. E. B. Nielsen, G. Sierra, J. I. Cirac, Nat. comm. (2013)

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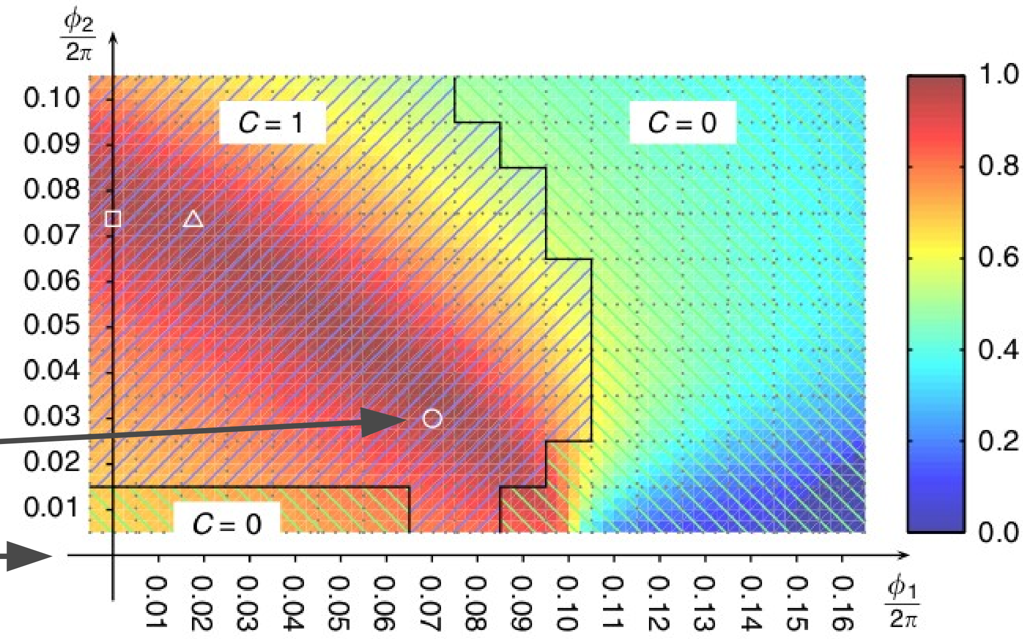


J1-J2 physics

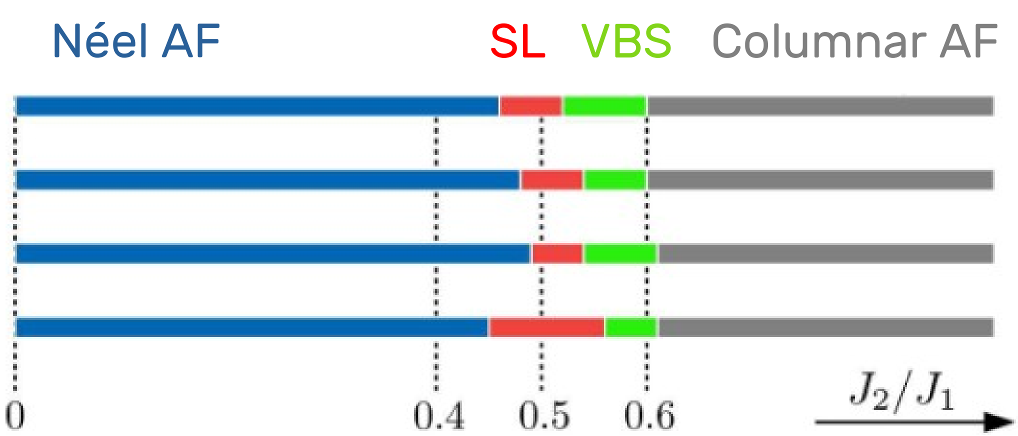
- Néel to gapless SL; DQCP ?; SL to VBS ...

L. Wang and A. W. Sandvik, PRL (2018) [iDMRG]

Ferrari and F. Becca, PRB (2020) [VMC]

Y. Nomura and M. Imada, PRX (2021) [VMC]

W.-Y. Liu, S.-S. Gong, Y.-B. Li, D. Poilblanc, W.-Q. Chen, and Z.-C. Gu, arXiv:2009.01821 [PEPS]



H. Shackleton, A. Thomson, and S. Sachdev, PRB (2021)

iPEPS

Variational states targeting GS of lattice models

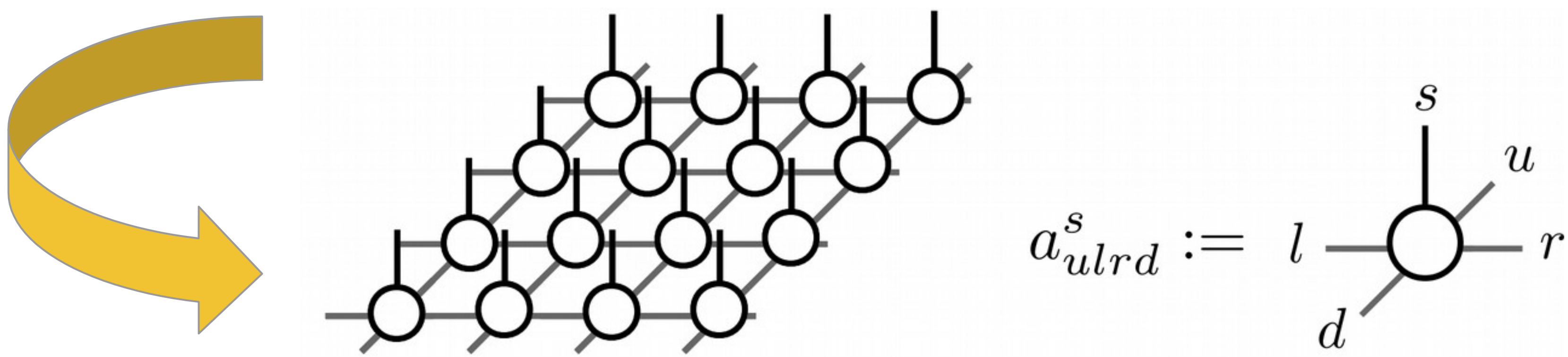
F. Verstraete and J. I. Cirac, arXiv:cond-mat/0407066, (2004)

- **area law** by construction BUT **can support algebraic corr.**
- **no FS effects (iPEPS)**
- impose **internal** symmetry [**i.e.** SU(N)] and **spatial** sym.

$$|\psi\rangle = \sum_{s_1 s_2 \dots} c_{s_1 s_2 \dots} |s_1 s_2 \dots\rangle$$

#parameters: 2^#spins

iPEPS



$\mathrm{Tr}_{aux}(a^{s_1} a^{s_2} \dots)$

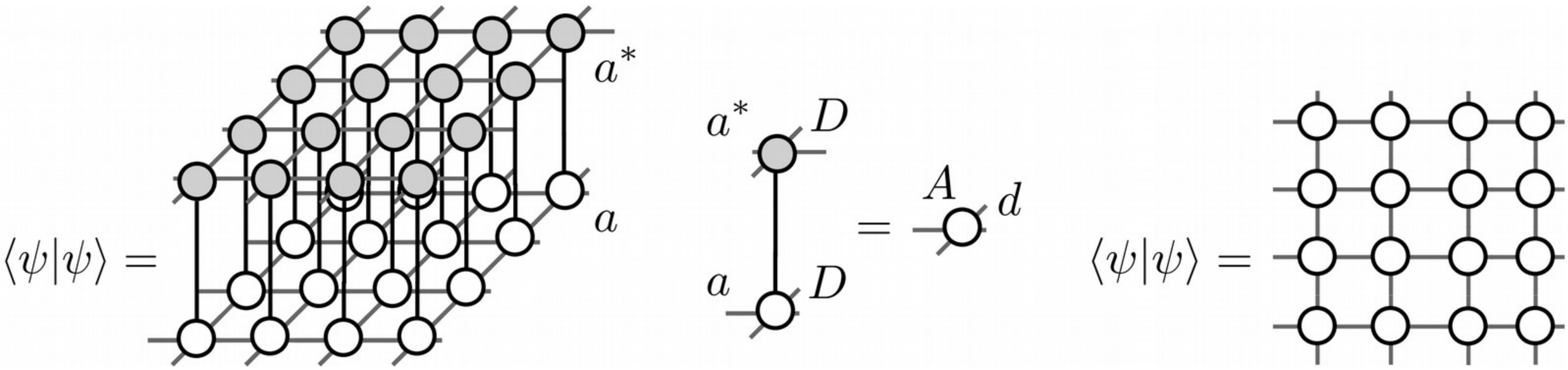
dimension(*u,l,d,r*) = D

$$|\psi\rangle = \sum_{s_1 s_2 \dots} Tr_{aux}(a^{s_1} a^{s_2} \dots) |s_1 s_2 \dots\rangle$$

#parameters: **2D^4 per tensor**

Observables of iPEPS

Expectation values must be **approximated**



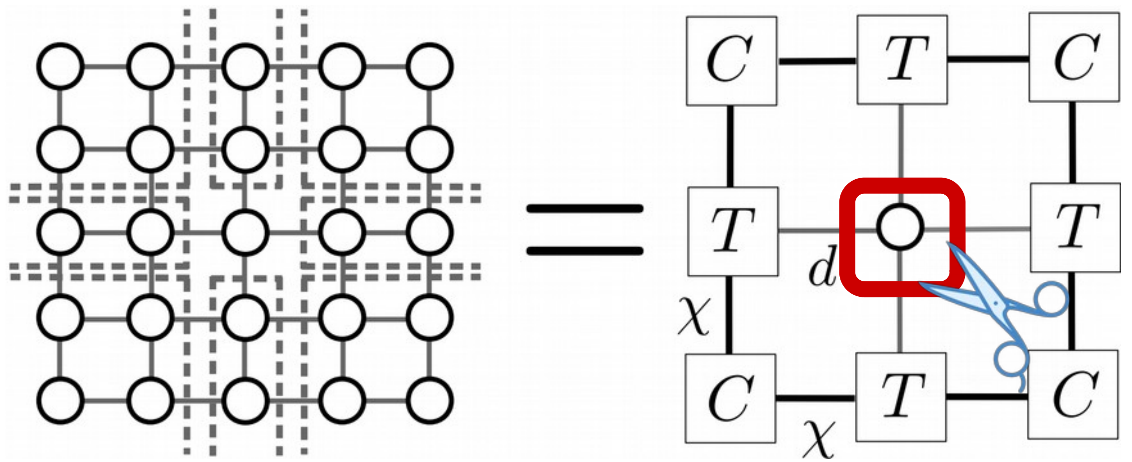
Construct **environment**

- corners **C**
- half-row/-columns **T**

Baxter, J. Stat. Phys. 17, 1 (1977)

New control parameter:

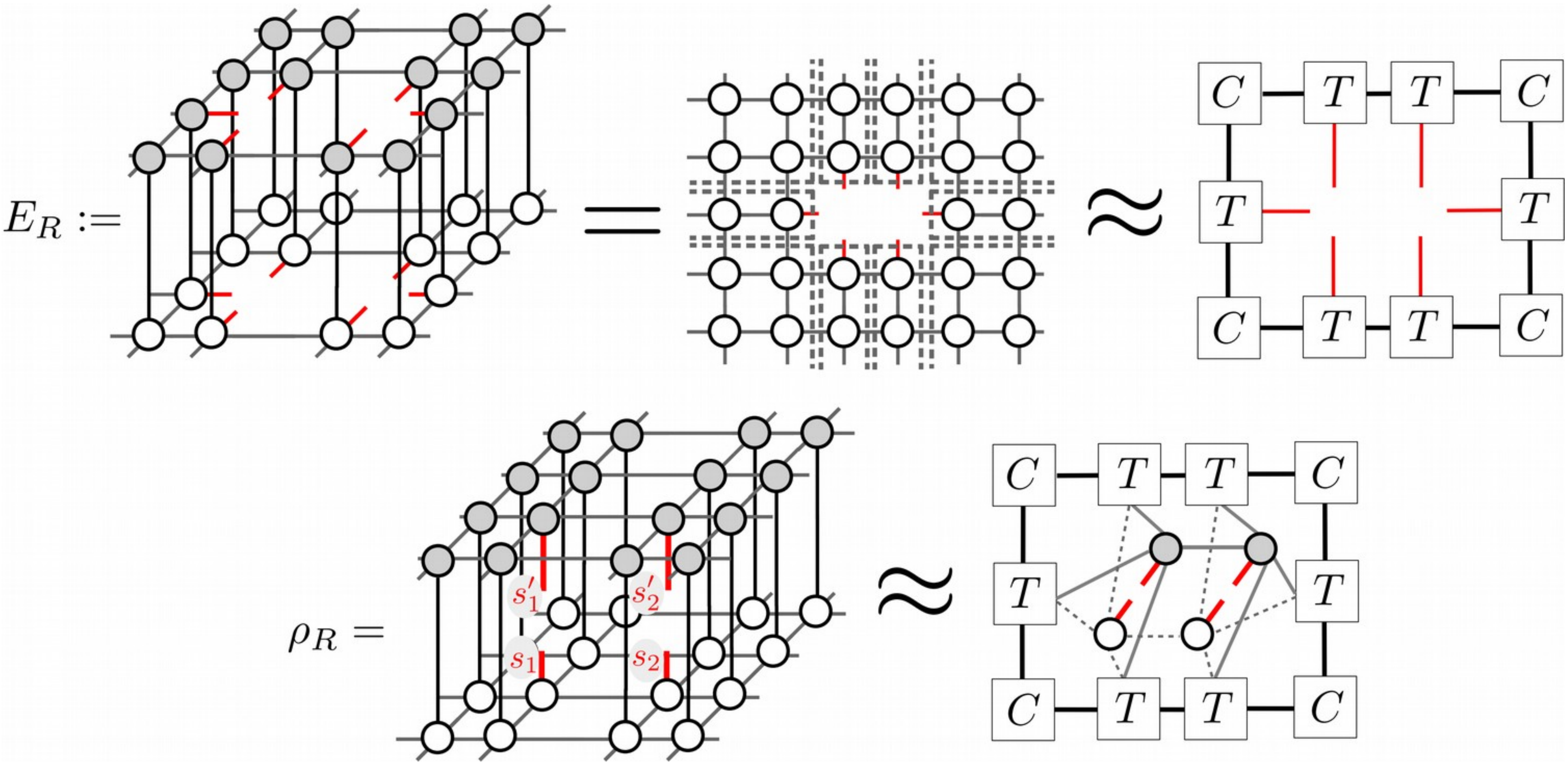
env. dimension χ



Alternative: Channels Vanderstraeten et al. (2015, 2016)

Observables of iPEPS

From **reduced environments** (E_R) of region R build **reduced density matrices** (ρ_R) of region R



Any observable inside the region R is: $\langle \mathcal{O} \rangle_\chi \approx \mathrm{Tr}(\rho_R(\chi)\mathcal{O})$

Constructing CSL via iPEPS

"no-go theorem": Any short-range quadratic parent Hamiltonian for chiral non-interacting PEPS is gapless.

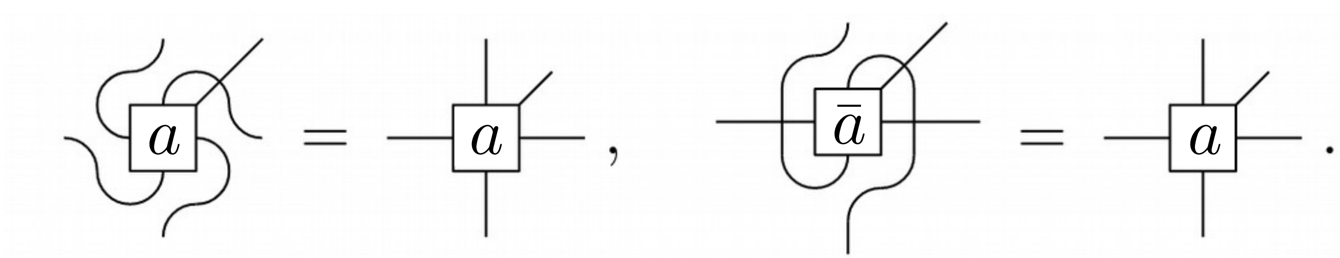
J. Dubail and N. Read, PRB (2015)

Free-fermion (Gaussian) PEPS

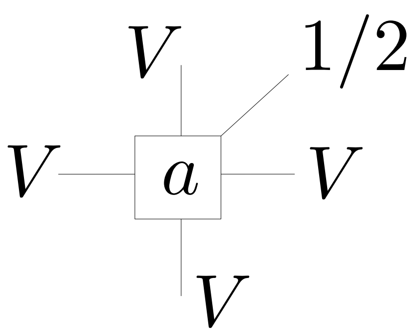
T. B. Wahl, H.-H. Tu, N. Schuch, and J. I. Cirac, PRL (2013)

Symmetric construction

- Point-group symmetry: break **P** and **T** but preserve **PT**



- Internal symmetry: *a* transforms **covariantly** under G



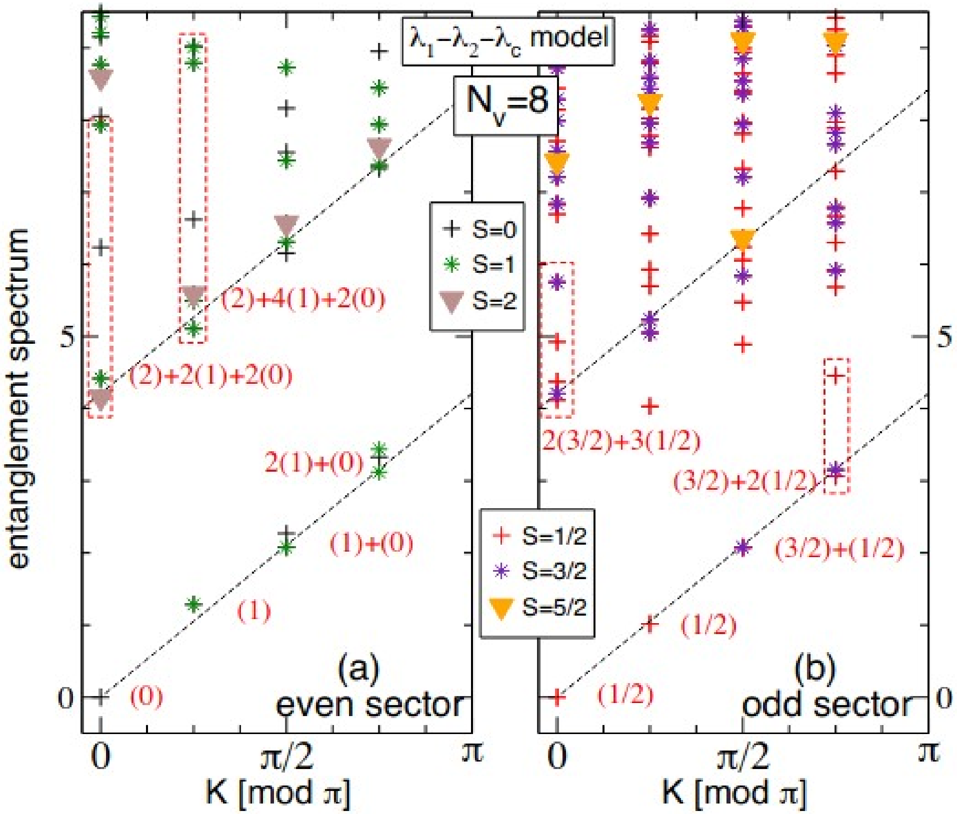
$$a(\vec{\lambda}) = \sum_i \lambda_i a_R^i + i \sum_j \lambda_j a_I^j$$

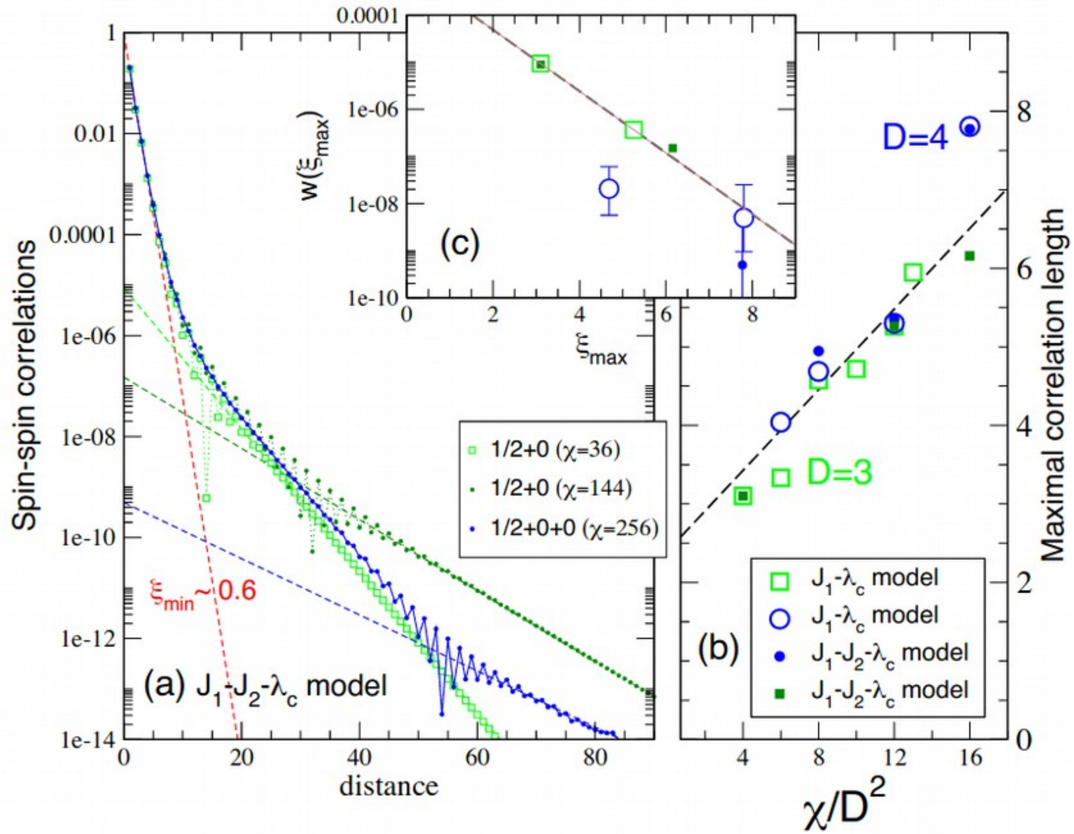
D. Poilblanc, J. I. Cirac, and N. Schuch, PRB (2015)
M. Mambrini, R. Orús, and D. Poilblanc, PRB (2016)

Constructing CSL via iPEPS

Results from **symmetric construction**

$$H = J_1 \sum_{\langle i,j \rangle} S_i . S_j + J_2 \sum_{\langle\langle i,j \rangle\rangle} S_i . S_j + i\lambda \sum_{\square(ijkl)} (P_{ijkl} - P^{-1}_{ijkl})$$





SU(2)$_1$ edge spectra; **long-range tails** in bulk correlations

Poilblanc, PRB (2017); Chen, Vanderstraeten, Capponi, and Poilblanc PRB (2018)

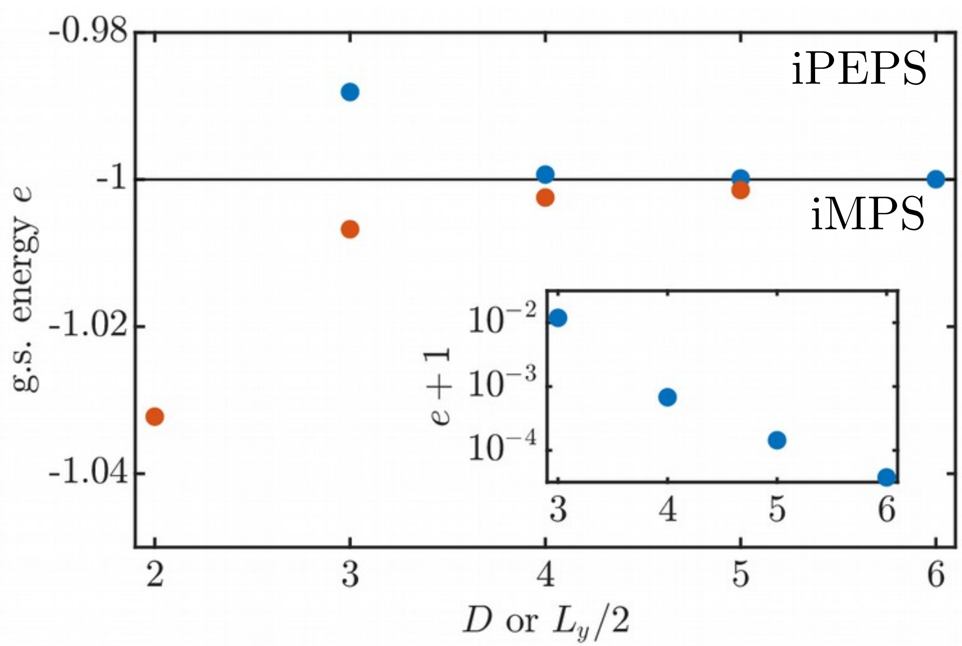
Simulating CSL via iPEPS

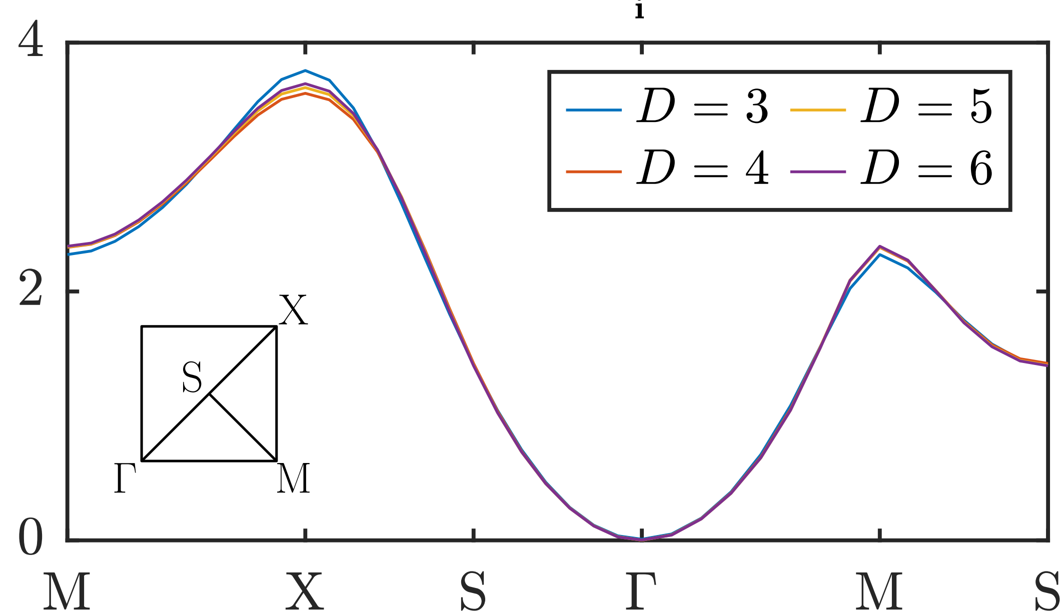
Abandon internal symmetry

- Gradient-based optimization (AD/autodiff/autograd)
- iPEPS for J1-J2 develop finite magnetization

Liao et al., PRX (2019); JH, D. Poilblanc, and F. Becca, SciPost Phys. (2021)

$$s(\mathbf{k}) = \sum_{\mathbf{i}} \mathbf{e}^{\mathbf{ik.r_i}} \langle \mathbf{S}(\mathbf{r_i}) \cdot \mathbf{S}(\mathbf{0}) \rangle$$

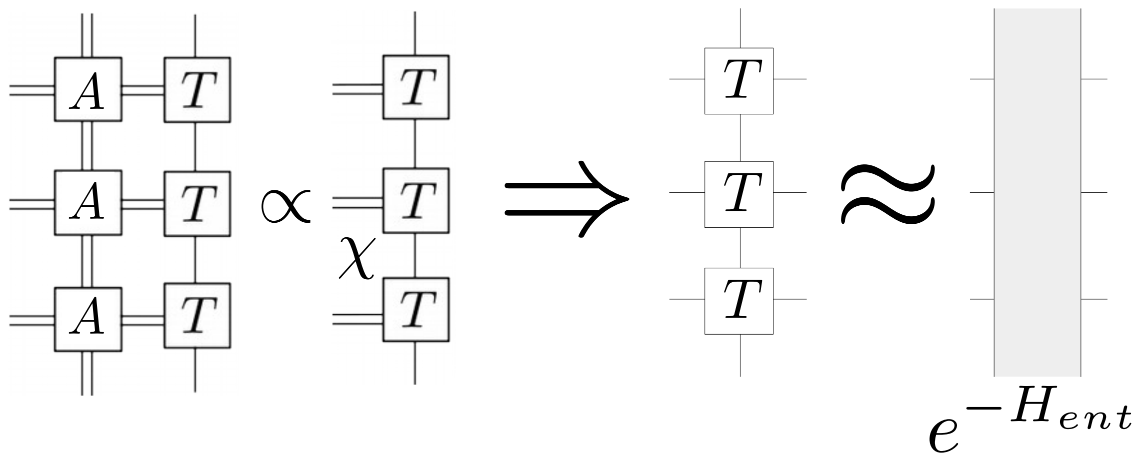




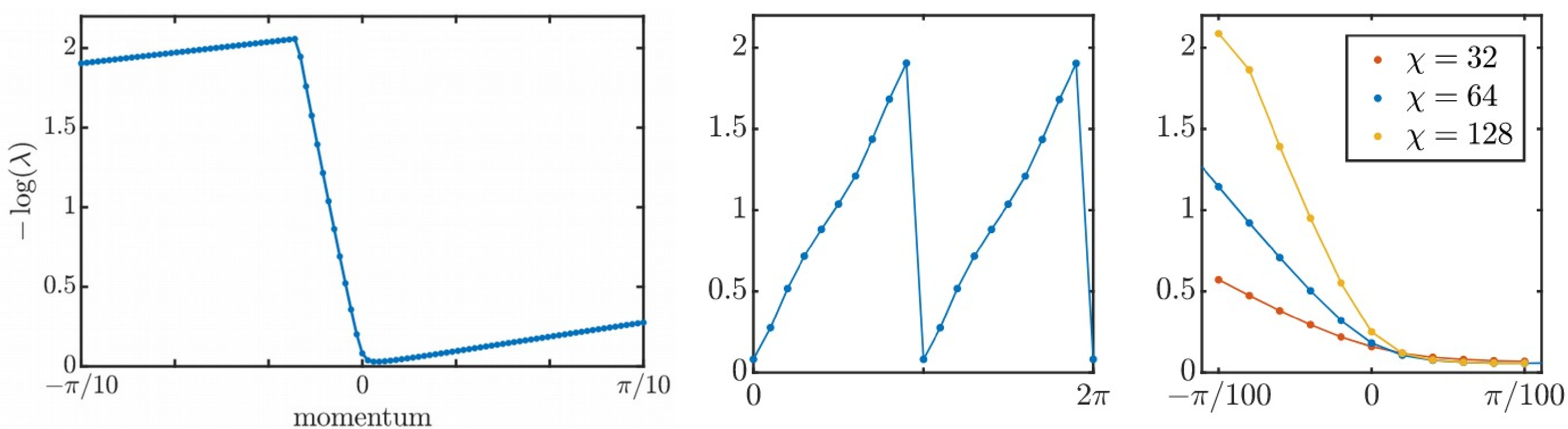
- SU(2) breaking: from m ≈ 10^{-3} at D=3 down to **m ≈ 10^{-5}** at D=6
- Energetics: iPEPS O(10^3) vs iMPS O(10^7) variational parameters

Simulating CSL via iPEPS

Edge spectra – smoking gun for diagnosing CSL



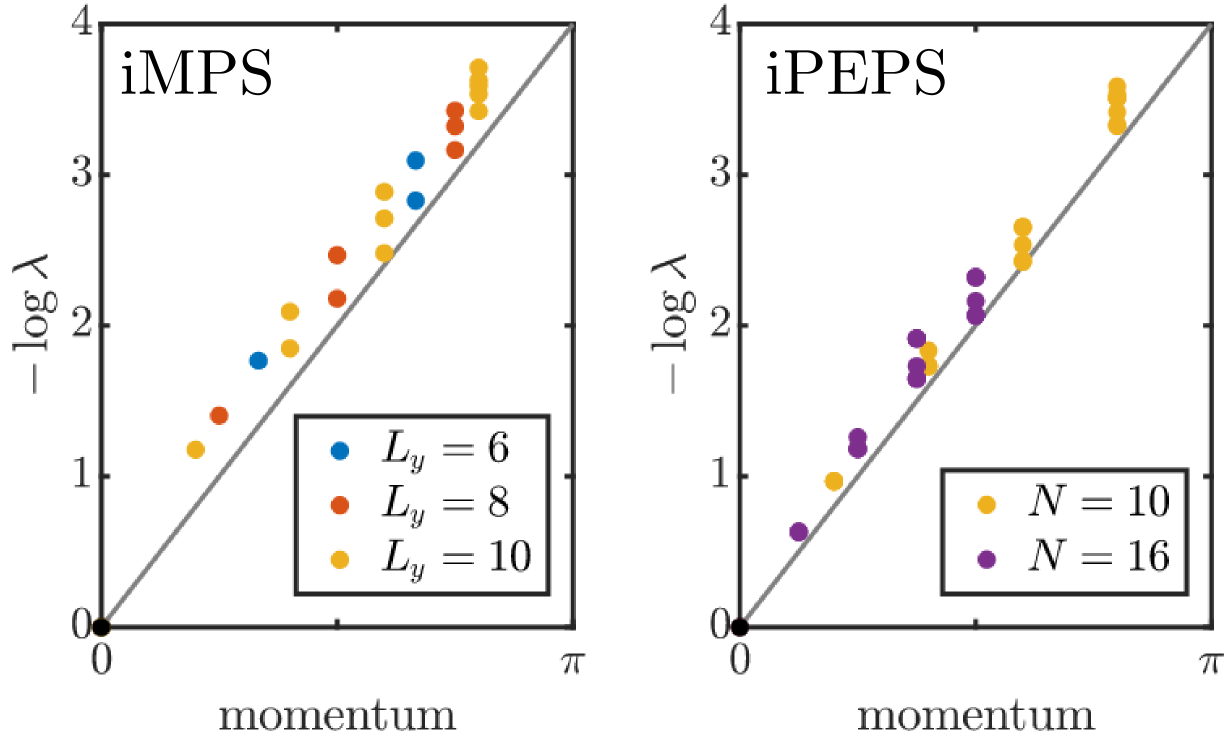
J. I. Cirac, D. Poilblanc, N. Schuch, and F. Verstraete, PRB (2011)
L. Vanderstraeten, J. Haegeman, and F. Verstraete, SciPost Phys. Lect. Notes (2019)



Simulating CSL via iPEPS

Edge spectra – smoking gun for diagnosing CSL

- Expected SU(2)$_1$ counting
- Small multiplet breaking ~ 10^{-3} to 10^{-4}



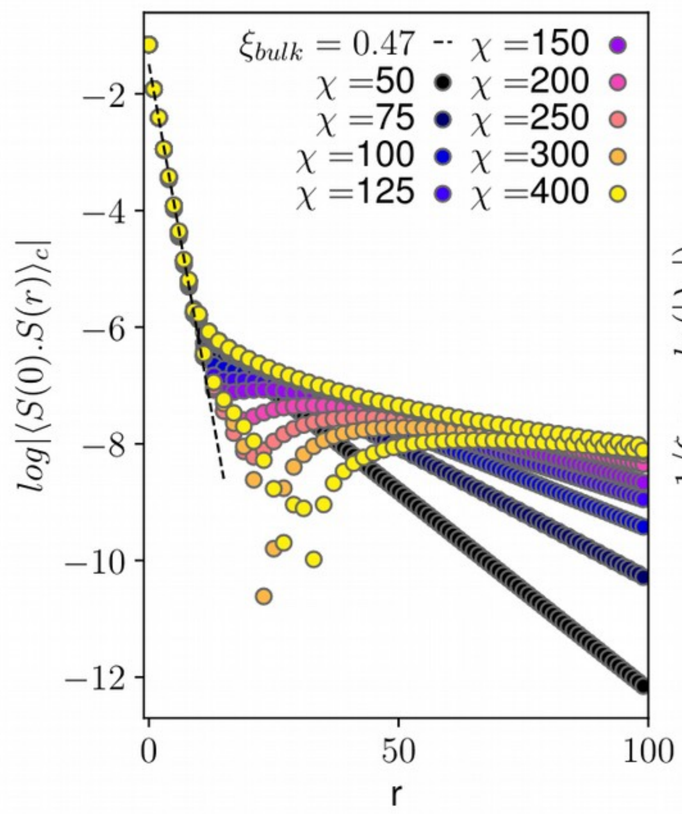
M. Van Damme, R. Vanhove, J. Haegeman, F. Verstraete, and L. Vanderstraeten PRB (2021)

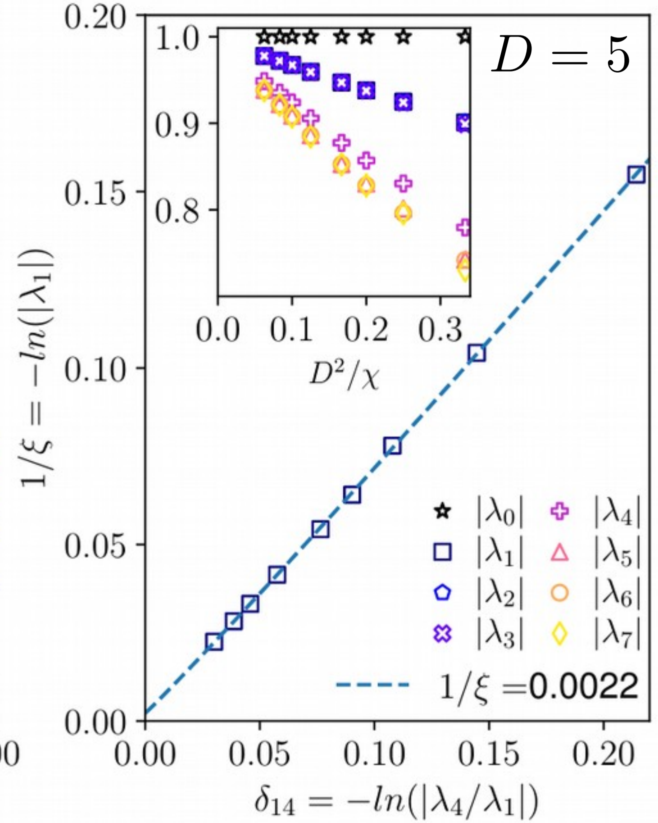
Restored SU(2) symmetry in the bulk and SU(2)$_1$ edge spectra indicate genuine CSL (KL) state obtained via generic bosonic iPEPS

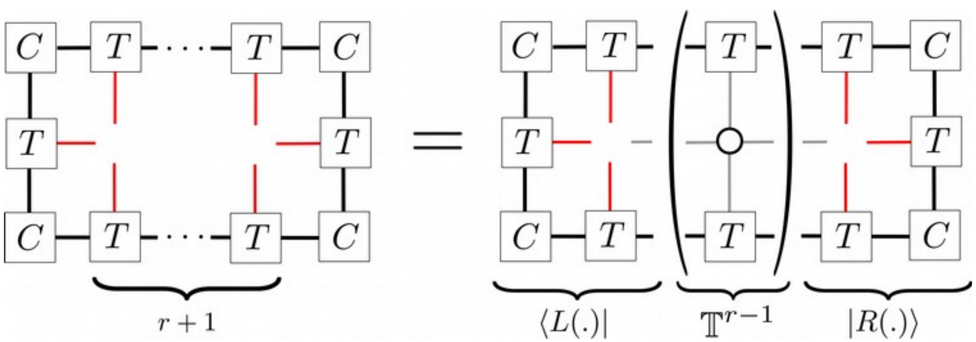
Simulating CSL via iPEPS

Bulk spin-spin correlations

- **two distinct regimes**
  I. gapped
  II. (almost) algebraic tail
- similar to symmetric iPEPS







$$\Leftrightarrow \quad f^C(r)_{O_1O_2} = \sum_{i>0} \lambda_i^{r-1} \langle L|l_i\rangle\langle r_i|R\rangle$$

Qualitatively separate form-factors ?

Concluding remarks

- Generic iPEPS can simulate (by optimization) CSL
- CSL in iPEPS ⇔ algebraic decay in transfer matrix
  - Can we eliminate algebraic tail "artefact" from physical observables ?
- More apt then iMPS ? Attack spontaneous CSL in triangular lattice Hubbard model

A. Szasz, J. Motruk, M. P. Zaletel, and J. E. Moore, PRX (2020); Chen et al, arXiv:2102.05560 (2021)
L. Tocchio, A. Montorsi, F. Becca, PRR (2021)

- Open-source implementation available

github.com/jurajHasik/peps-torch
J. Hasik and G. Mbeng

quantumghent.github.io/software/