Entanglement- and Operator Spreading in a Thermalizing Spin Chain with Long-Range Interactions

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Overview

1. Quantum Information Scrambling
2. Mixed Field Ising Model
3. Entanglement-, and Operator Spreading
4. Conclusion and Outlook

Quantum Information Scrambling

- Mechanism behind thermalization is *quantum information scrambling*

→ Quantum information spreads under time-evolution such that **local measurements** are not sufficient to reconstruct it at later times

→ Information is *scrambled*, stored in global d.o.f., and seems lost for a local observer

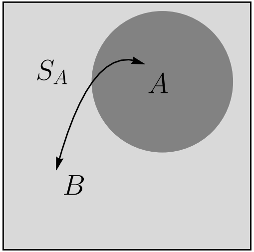
- Linked to a variety of fields, e.g., thermalization, MBL, AdS/CFT, QI-processing,...
- Here: Interactions $\sim 1/r^{\alpha}$; Interplay between entanglement-, and operator spreading

How to Diagnose Scrambling?

- **Entanglement**: Quantified via entropy in pure states
- Von Neumann entropy $S_A = -\operatorname{Tr}\left[\rho_A \log\left(\rho_A\right)\right]$

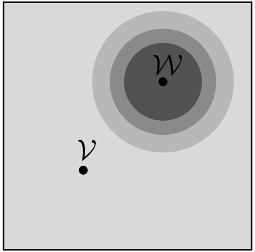
→ Probes how information 'leaks out' of region A

- Monotonic growth until saturation
- **Local**: Linear growth; *Entanglement velocity* v_{E}
- **LR**: Slowdown of growth[1,2]; no constant velocity



[1] Schachenmayer, Lanyon, Roos, Daley, **PRX** 3 031015 (2013)
[2] Lerose, Pappalardi, **PRR** 2 012041 (2020)

How to Diagnose Scrambling?



- **Operator spreading**: $\mathcal{W}(t) = \sum_{k=0}^{\infty} \frac{(it)^k}{k!} [\mathcal{H}, \dots [\mathcal{H}, \mathcal{W}] \dots]$
- Take another (local) operator $\mathcal{V}$, where $[\mathcal{V}, \mathcal{W}] = 0$
- Squared commutator: $C(t) = \left\langle [\mathcal{V}, \mathcal{W}(t)]^\dagger [\mathcal{V}, \mathcal{W}(t)] \right\rangle$

$\rightarrow$ Probes how support of operator $\mathcal{W}$ grows with time

- **Local**: Linear lightcone; *Butterfly velocity* v_B
- **LR**: Nonlinear lightcone[3,4]; fast operator spreading

[3] Colmenarez, Luitz, **PRR** 2 043047 (2020)
[4] Zhou, Xu, Chen, Guo, Swingle, **PRL** 124 180601 (2020)

Hamiltonian and Initial State

Long-Range MFI Model

$$\mathcal{H} = -\sum_{m<n} J_{mn}\mathcal{Z}_m\mathcal{Z}_n - h_x\sum_m \mathcal{X}_m - h_z\sum_m \mathcal{Z}_m$$

- Interaction strength follows powerlaw: $J_{mn} = \frac{J}{\mathcal{N}(\alpha)|m-n|^\alpha}$
- System initially in a fully polarized state $|\Psi_0\rangle = |Y+\rangle$, $h_x = -1.05$, and $h_z = 0.5$
- Model thermalizes; **infinite temperature ensemble** ($\rho_{\text{th}} \sim \mathbf{1}$), for $\alpha \to \infty$[5]

[5] Bañuls, Cirac, Hastings, **PRL** 106 050405 (2011)

Thermalization Behavior

- Thermal ensemble affected by LR interactions?
- Exact simulations up to $N = 24$ indicate that the effective thermal ensemble is not altered for $\alpha \gtrsim 1$
- For $\alpha \lesssim 1$, not clear $\rightarrow$ finite size effect?

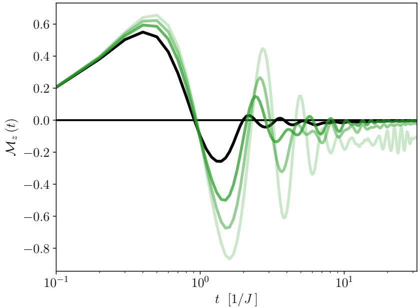
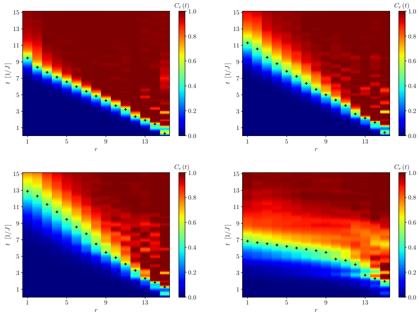


Figure: Magnetization $\mathcal{M}_z$ for $\alpha = \{\infty, 2.3, 1.5, 0.5\}$, darker colors indicate larger values

Operator Spreading: Lightcones



- $\alpha \gtrsim 2$: System **effectively local**[6]; Linear lightcone

$\rightarrow$ Broadening of the wavefront with decreasing α

$\rightarrow$ LR-interactions slow down entanglment-, and operator spreading; **Similar renormalization**

- $\alpha \lesssim 2$: **Nonlinear regime**

Figure: Squared commutator for various values of α (upper left: $\alpha = \infty$, upper right: $\alpha = 3.0$, lower left: $\alpha = 2.3$, lower right: $\alpha = 1.2$).

[6] Kuwahara, Saito, **PRX** 10 031010 (2020)

Linear Regime: Entanglement-, and Operator Spreading

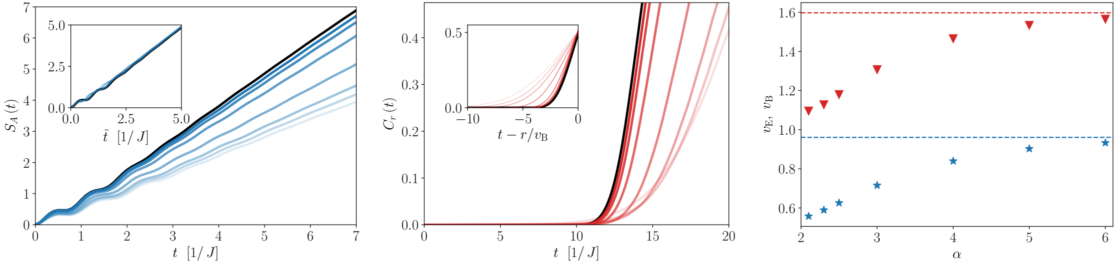


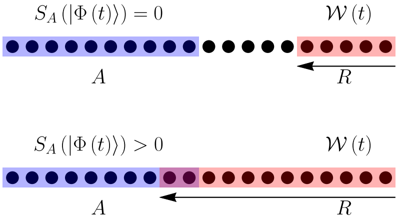
Figure: Left/Center: Entropy, and squared commutator for $\alpha = \{\infty, 6.0, 5.0, 4.0, 3.0, 2.5, 2.3, 2.1\}$. Darker colors indicate larger values. Right: Entanglement velocity v_E, and butterfly velocity v_B. Dashed lines indicate asymptotic values in the local limit, $\alpha \to \infty$.

Connecting Entanglement-, and Operator Spreading

- It is useful to define the state $|\Phi(t)\rangle := \mathcal{W}(t)|\Psi_0\rangle$
- At a given t^*, approximate $\mathcal{W}(t^*) \approx \mathbf{1}_{\Omega/R} \otimes \mathcal{W}_R$

→ $S_A(|\Phi(t)\rangle) \approx 0$ as long as $A \cap R = \emptyset$

→ Entanglement in $|\Phi(t)\rangle$ is a result of operator spreading!



Connecting Entanglement-, and Operator Spreading

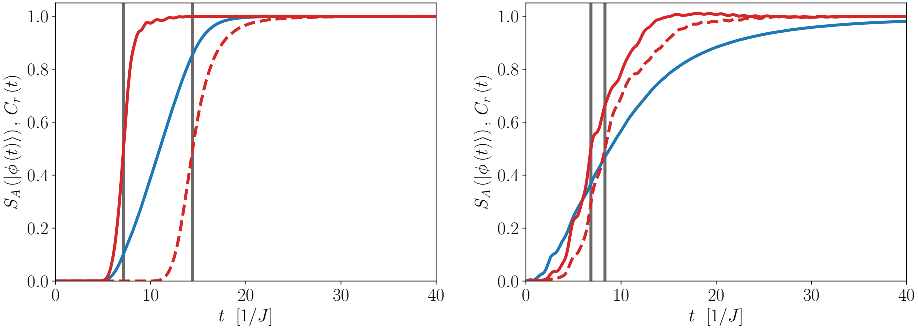


Figure: Blue: Entanglement entropy of the state $|\Phi(t)\rangle$. Red: Squared commutator $C_r(t)$, where the operator $\mathcal{W}$ is located at the right boundary of the system, and $\mathcal{V}_r$ at the right (solid) or left (dashed) boundary of A. Left plot is data for $\alpha = \infty$, and right for $\alpha = 1.1$.

Connecting Entanglement-, and Operator Spreading

- In the linear regime, most of the entanglement growth happens **while** the wavefront propagates through the region A
- In the nonlinear regime, however, entanglement continues to grow **after** the operator has spread over the entire system (as measured by the squared commutator)
- **Idea**: Expand operator in Pauli strings $\mathcal{W}(t) = \sum_{\nu} c_{\nu}(t)\, \mathcal{S}_{\nu}$

→ *Operator density*[7][8] : $\rho_l(t) = \sum_{\nu, L(\nu)=l} |c_{\nu}(t)|^2$, where $\sum_l \rho_l(t) = 1,\ \forall\, t$

[7] Roberts, Stanford, Susskind, **JHEP** 51 (2015)

[8] Keyserlingk, Rakovszky, Pollmann, Sondhi, **PRX** 8 021013 (2018)

Connecting Entanglement-, and Operator Spreading

→ First results: Small operators have high weight for a longer time in LR systems!

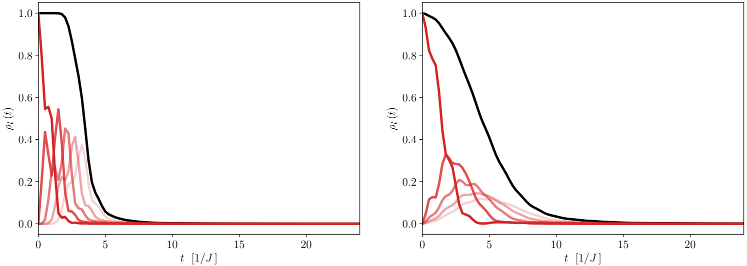


Figure: Operator density: Black curves correspond to the total weight of all operators up to $l = 5$. Red curves correspond to total weight of all operators with particular length l. Darker colors indicate smaller values. Left plot is data for $\alpha = \infty$, and right for $\alpha = 1.1$.

Conclusion and Outlook

- System **effectively local** for $\alpha \gtrsim 2$; Velocities v_{E}, and v_{B} are similary renormalized

→ Local dynamics accessible in systems of trapped ions for a wider range of α

→ TDVP seems to work well for intermediate timescales

- For $\alpha \lesssim 2$, operator structure changes

→ **Connection between slow entanglement growth and operator structure!**

- More quantitative analysis needed...; interplay is important!

→ Might give insights into different models, e.g., MBL, or fast scrambling