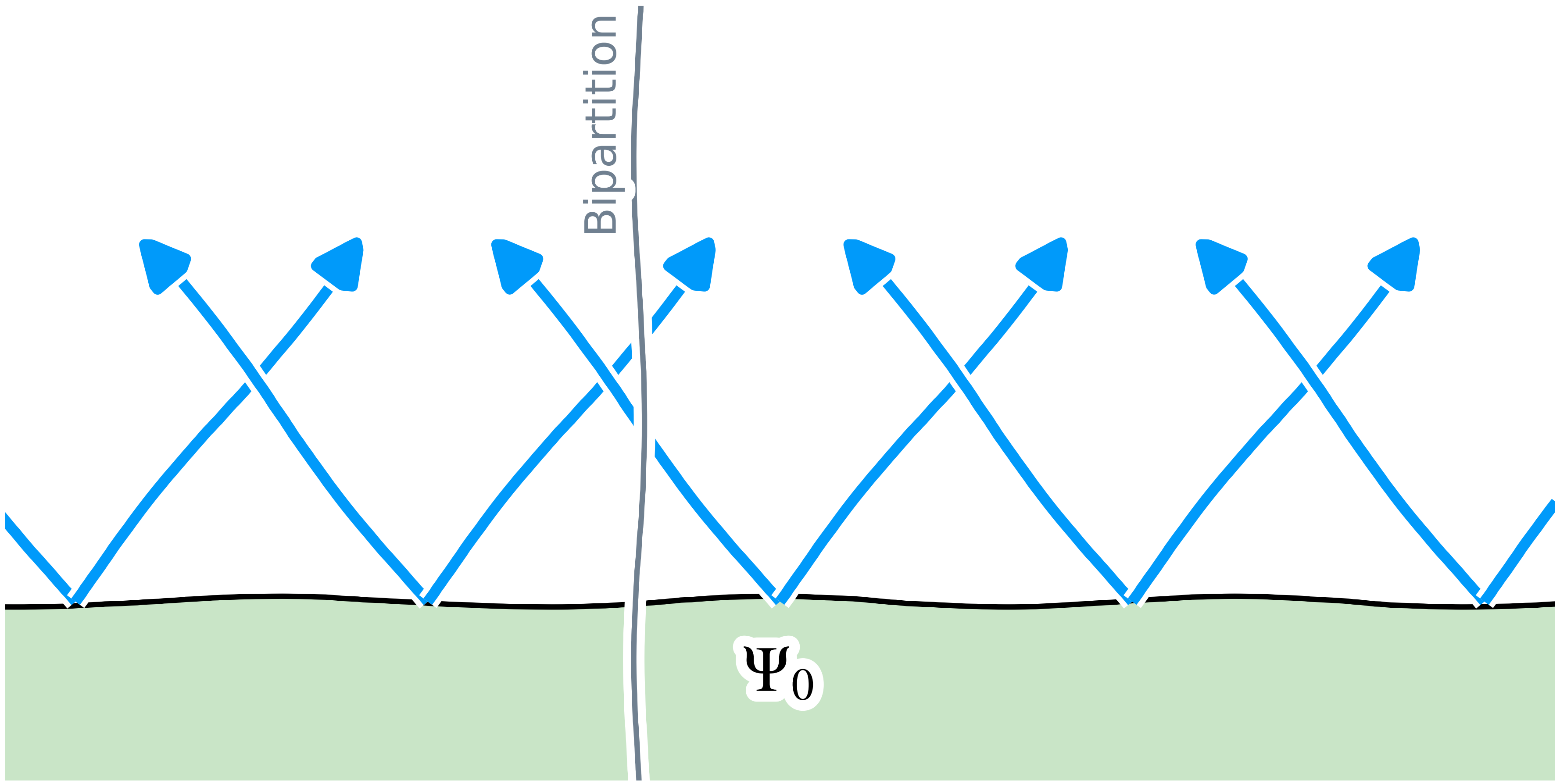
Many-body quantum dynamics & temporal entanglement

Based on arXiv:2112.14264

ENTANGLEMENT GROWTH IN GLOBAL QUENCHES

$$|\Psi_t\rangle = e^{-iHt}|\Psi_0\rangle$$

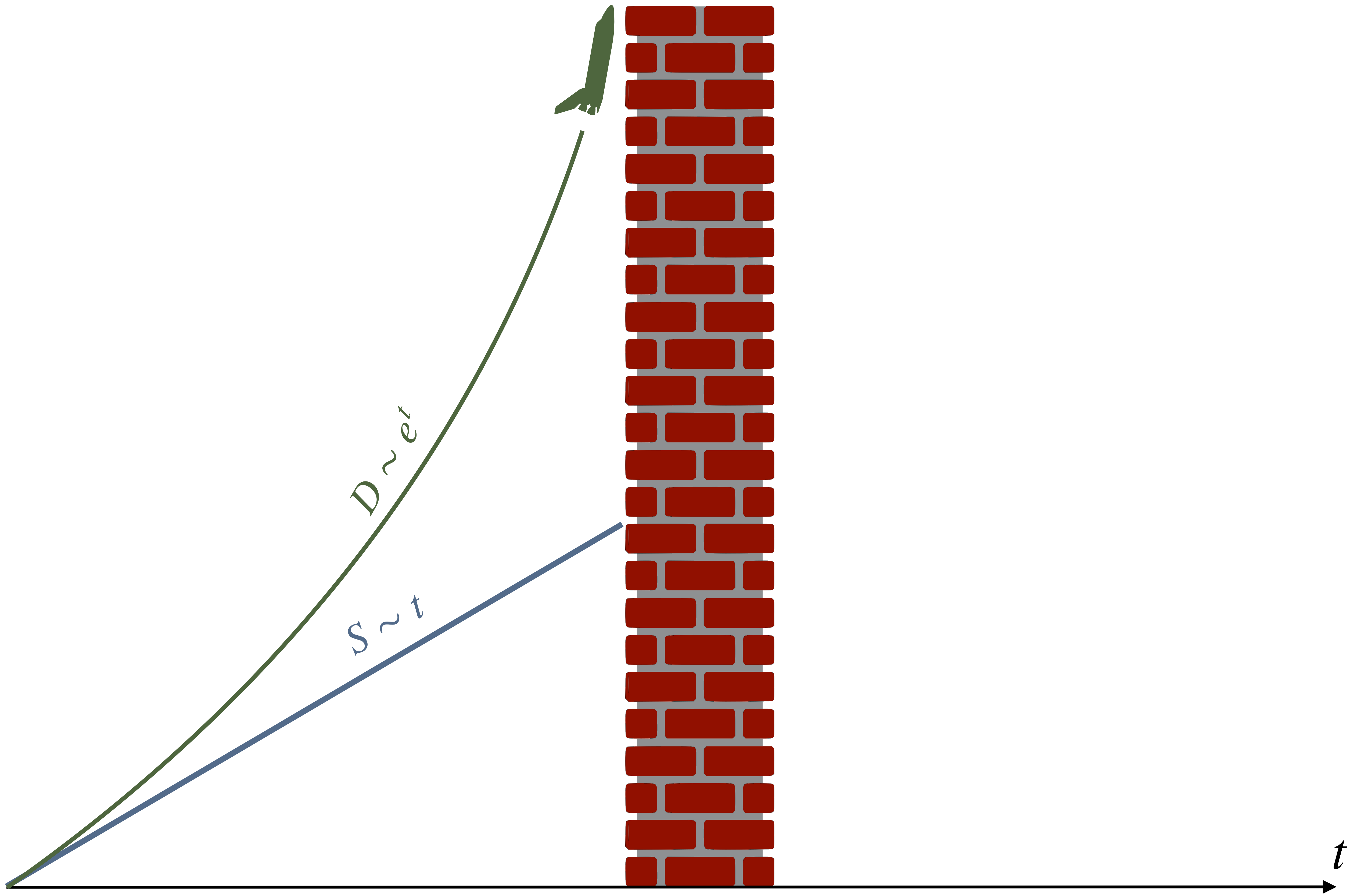
$$H = \sum_n h_{n,n+1}$$



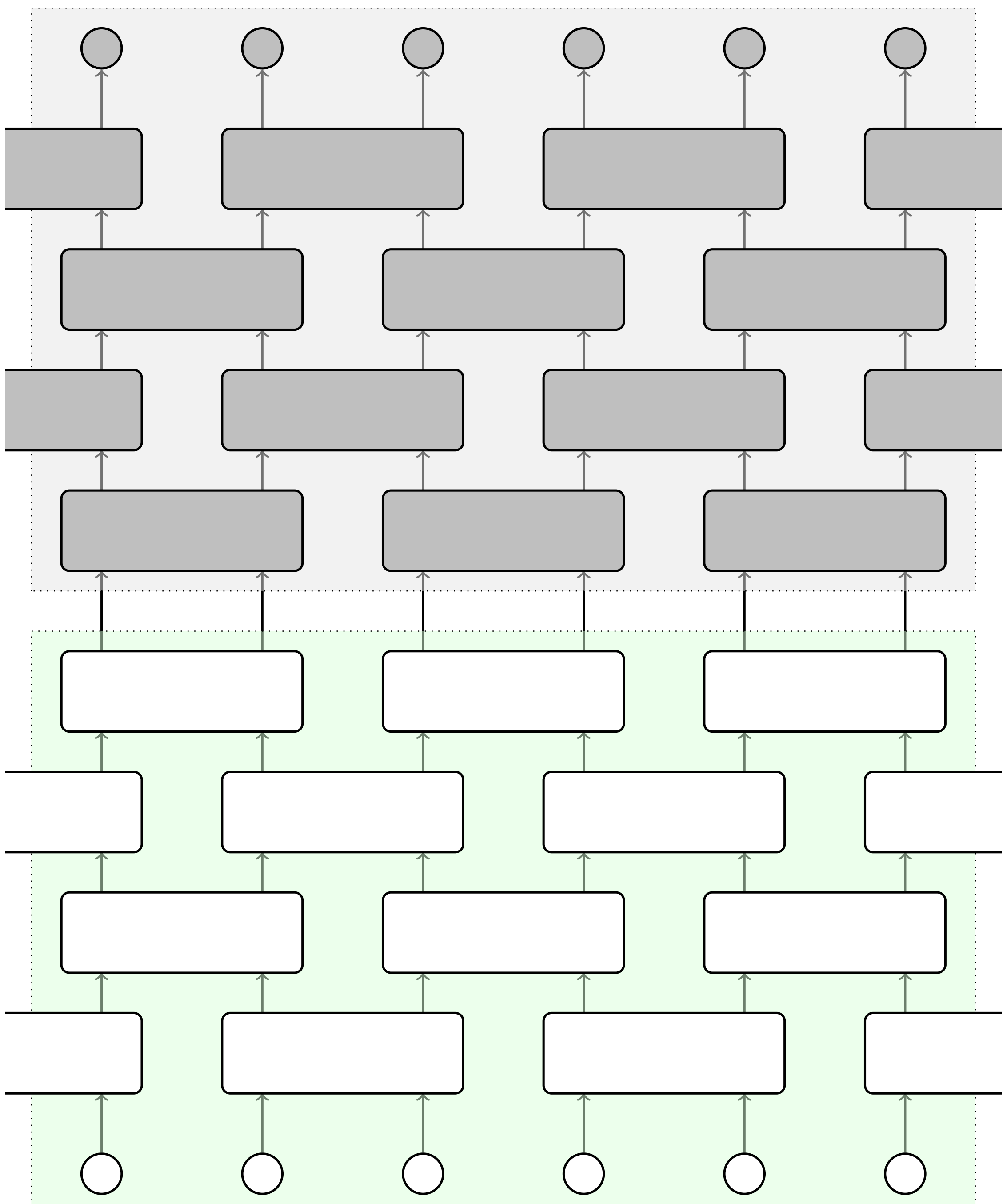
$$S(t) \sim t$$

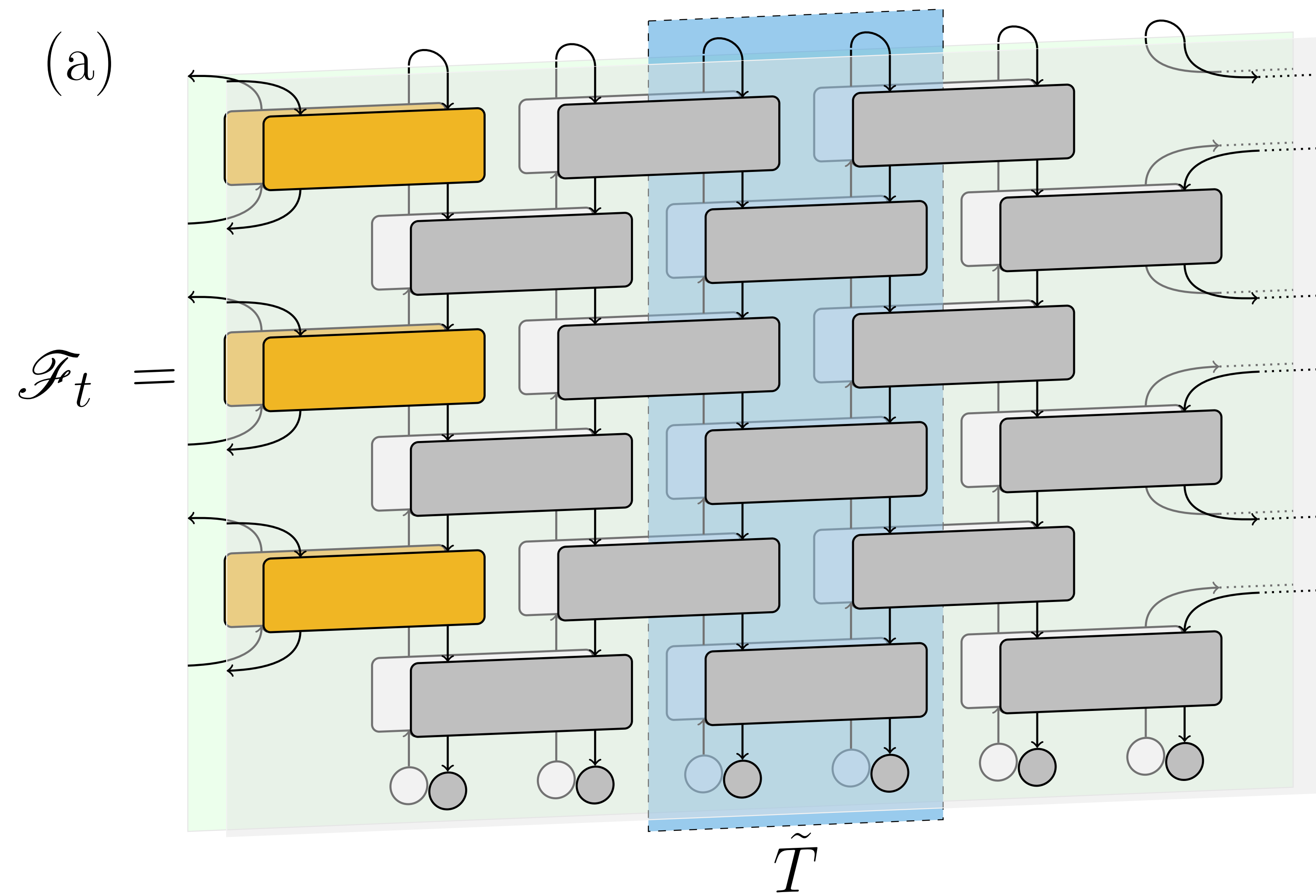
V. Alba and P. Calabrese, PNAS 114, 7947 (2017)
V. Alba and P. Calabrese, SciPost Phys. 4, 17 (2018)

But for MPS $S \leq \log D$ → Entanglement barrier!



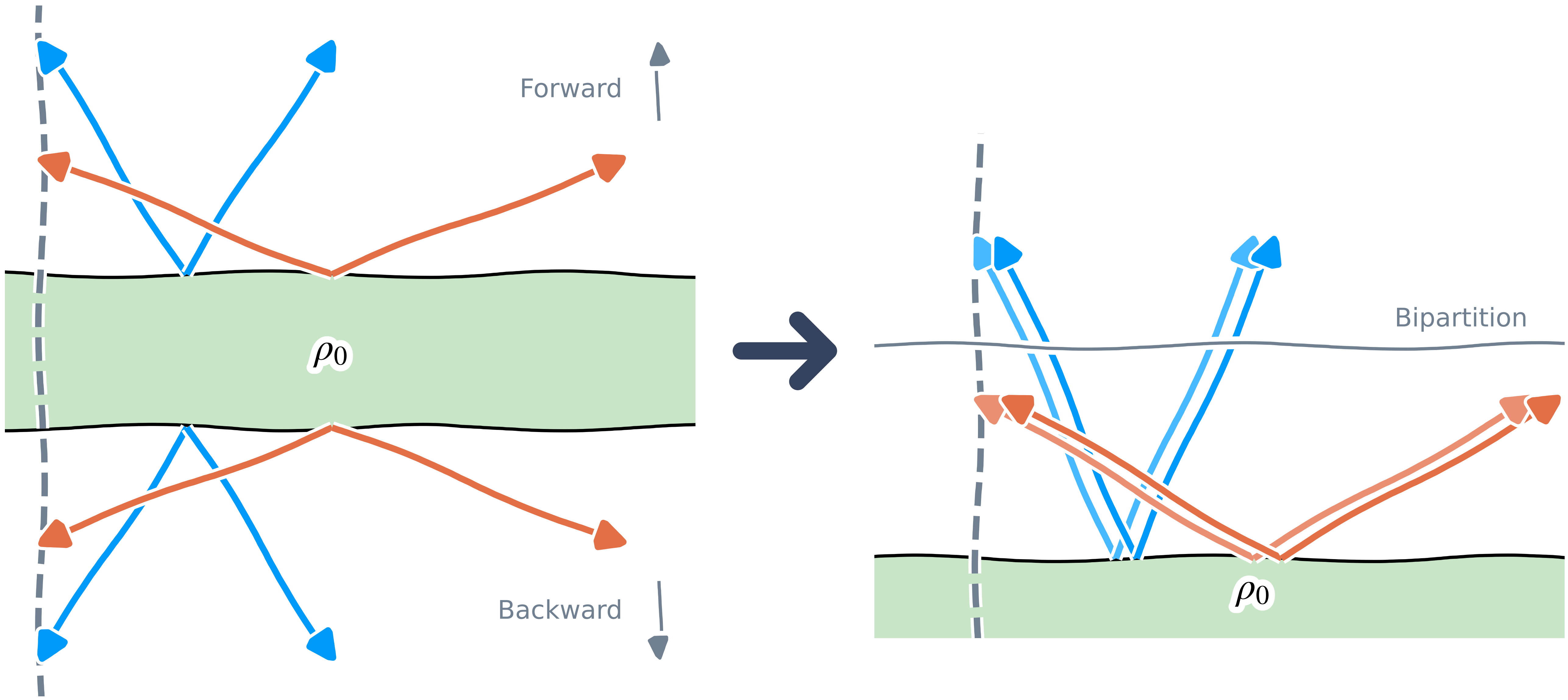
$$U_{n,n+1} = \exp\left(-i\delta t\, h_{n,n+1}\right)$$



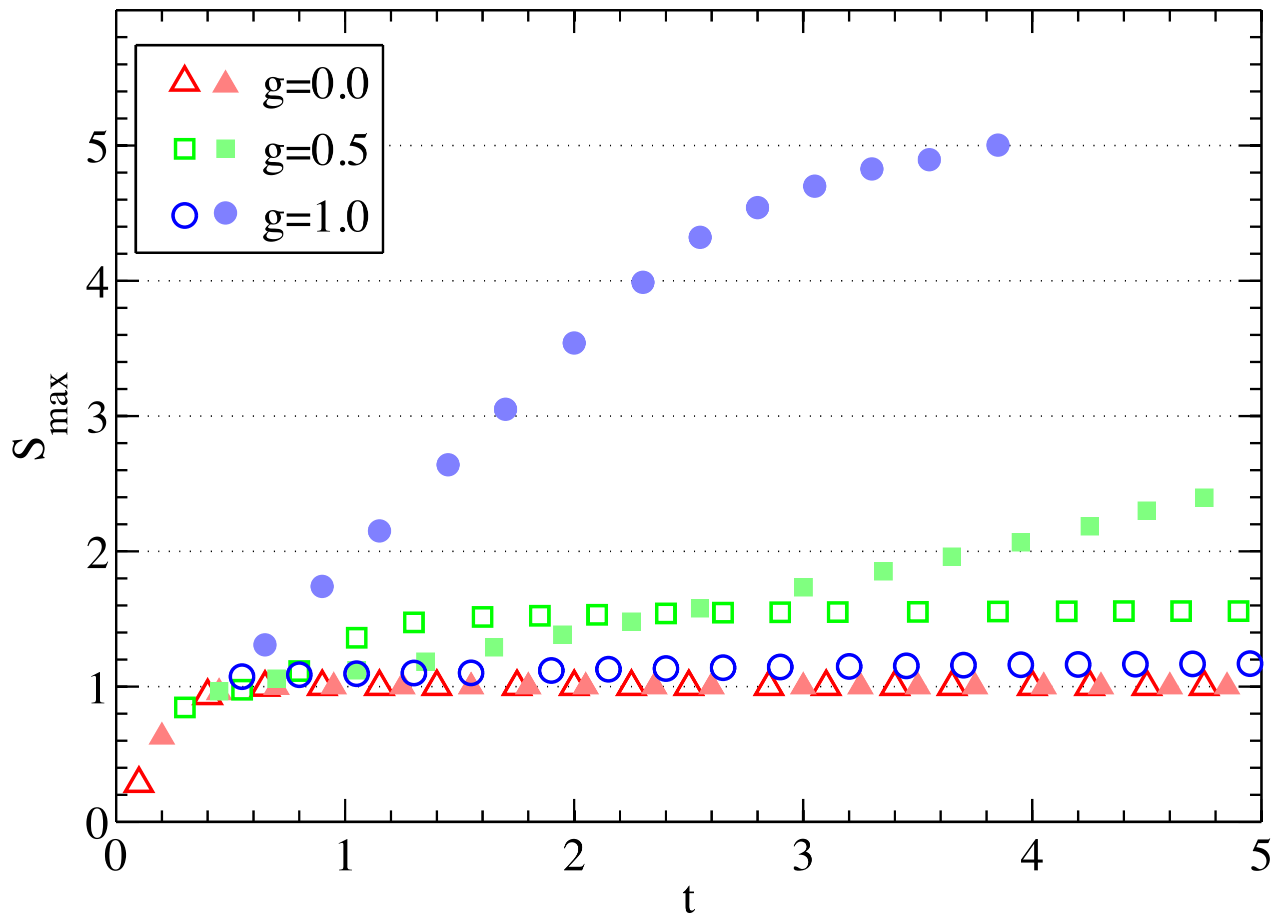


M. C. Bañuls, M. B. Hastings, F. Verstraete, and J. I. Cirac, Phys. Rev. Lett. 102, 240603 (2009)

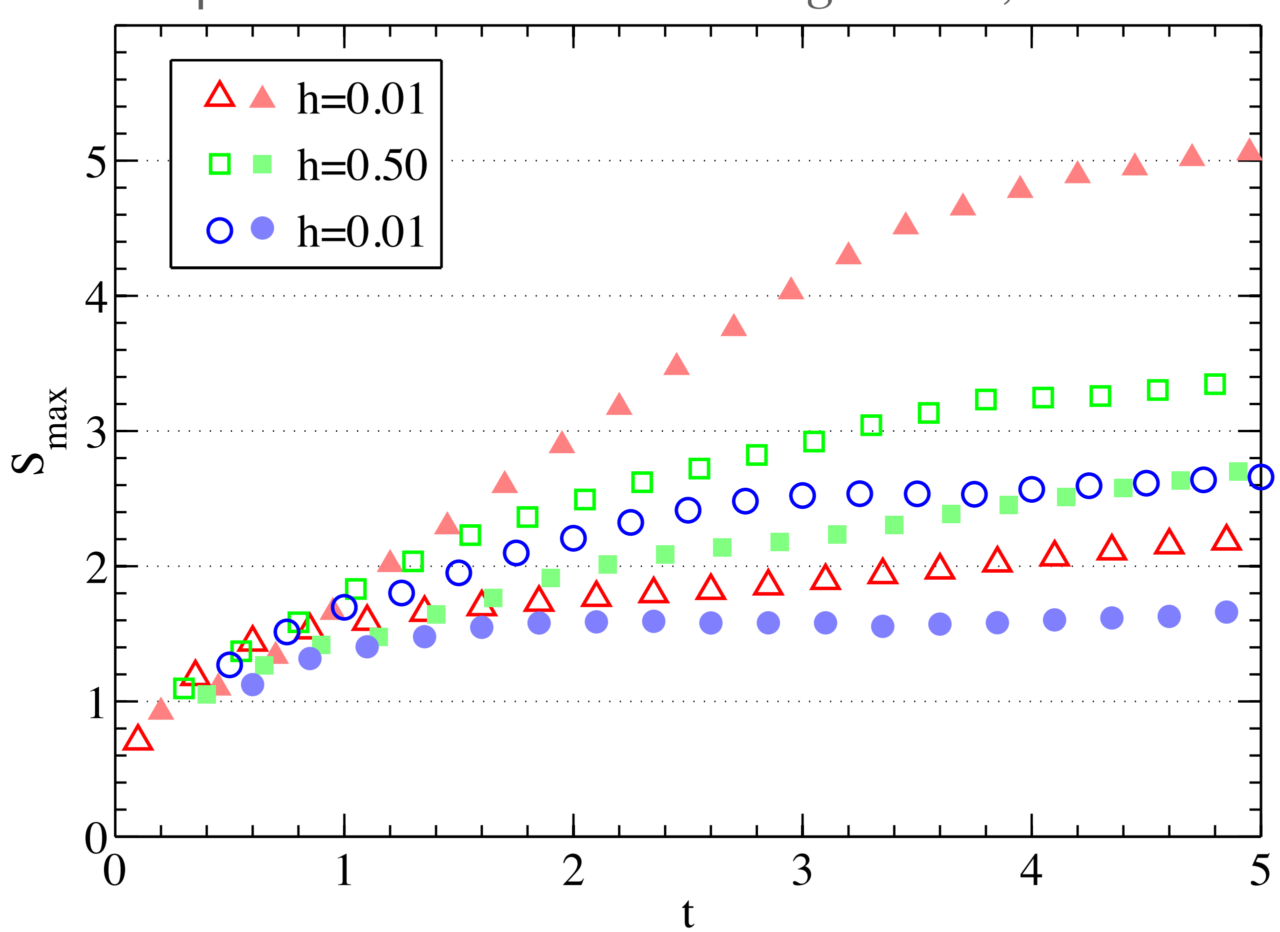
Cartoon picture



$|\Psi_0\rangle = |+\rangle^{\otimes N}$



quench from GS with $g = 1.1$, $h = 0$



Solid → unfolded
Empty → folded

A. Müller-Hermes, J. I. Cirac, and M. C. Bañuls, New J. Phys. 14, 075003 (2012)

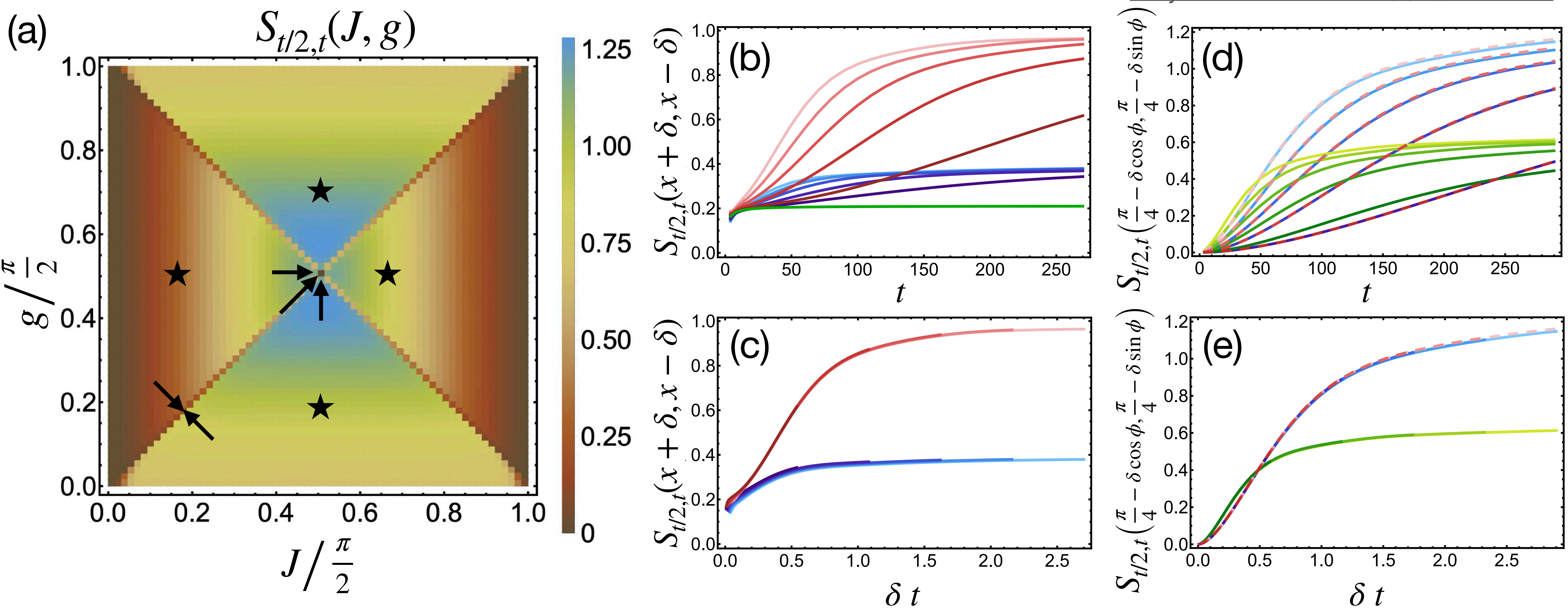
Continuum is tricky

↓

What about discrete dynamics?

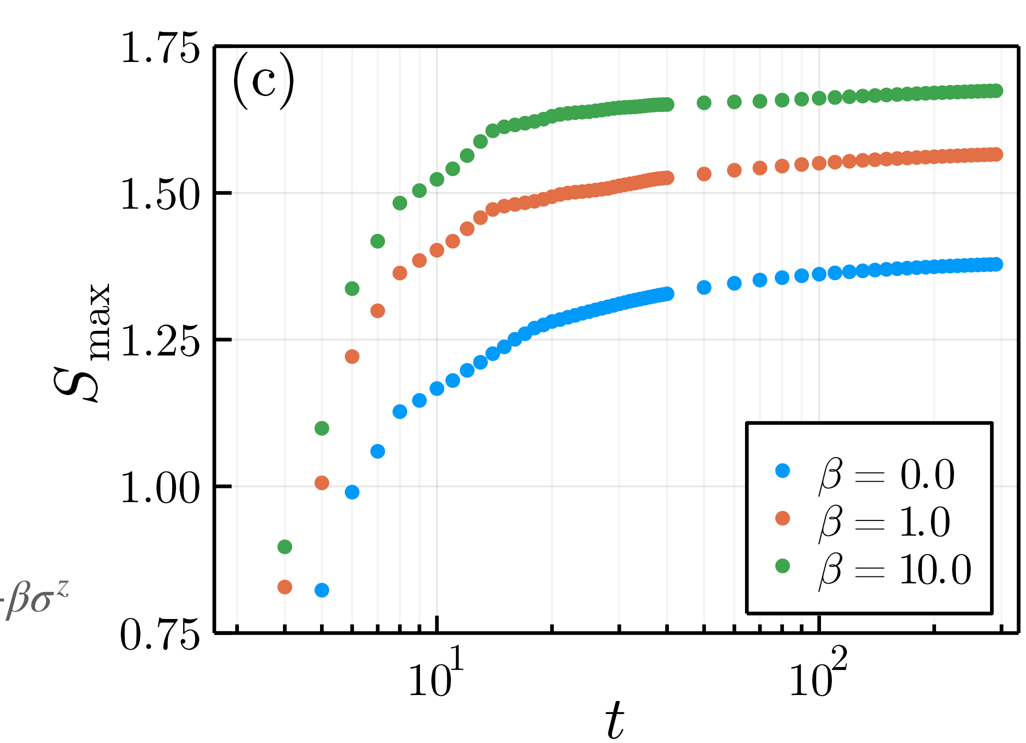
Non-interacting systems

A. Lerose, M. Sonner, and D. A. Abanin, Phys. Rev. B 104, 035137 (2021)



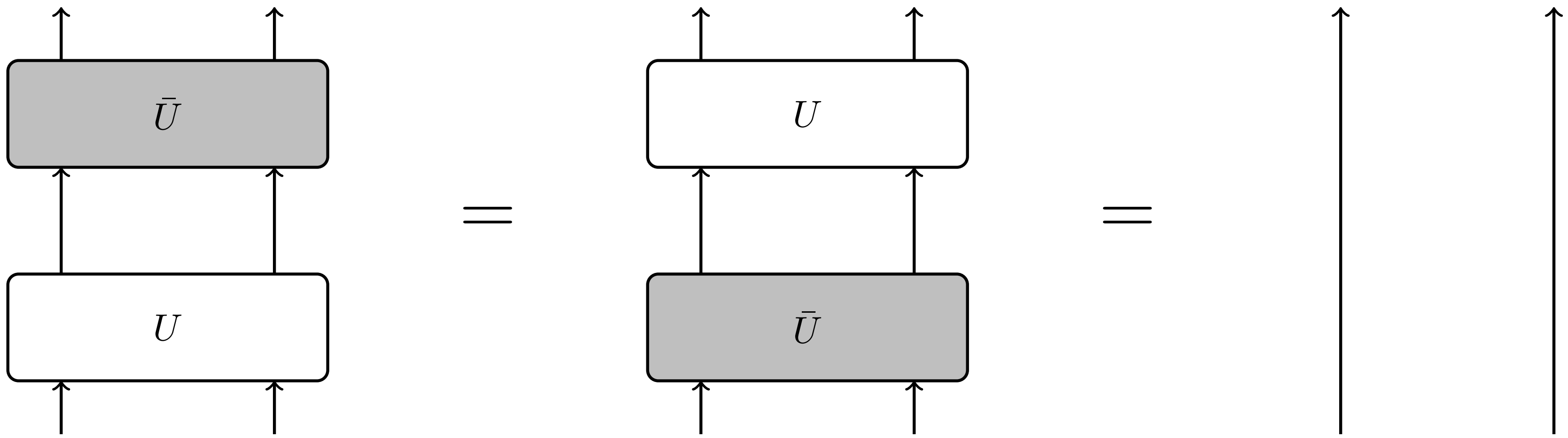
Generalizable to Gaussian initial states

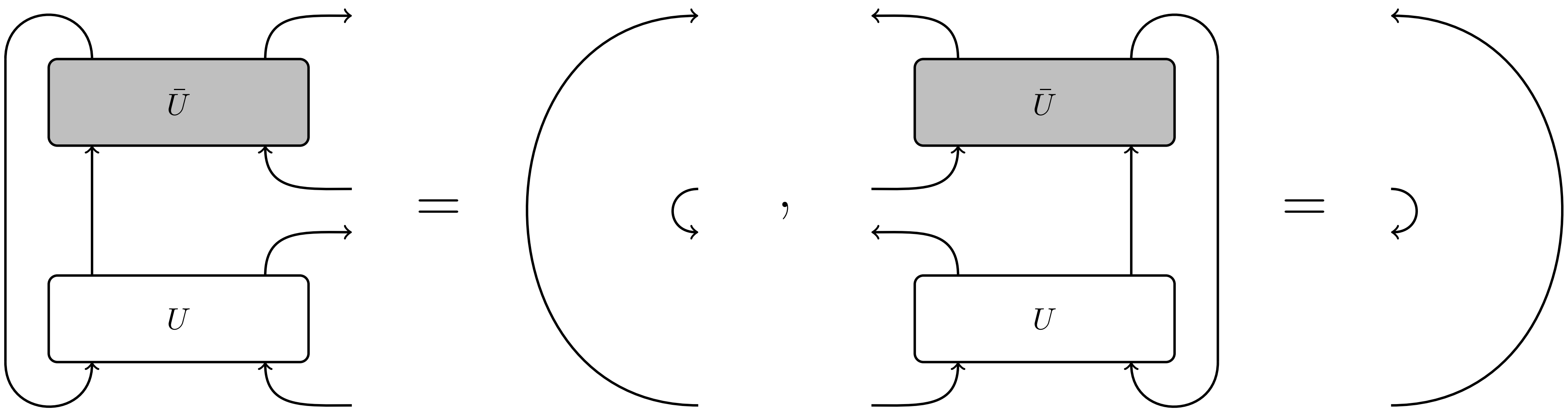
$\rho_0 \propto \bigotimes e^{-\beta\sigma^z}$



Calculations by J. Thönniß

Dual-unitary gates





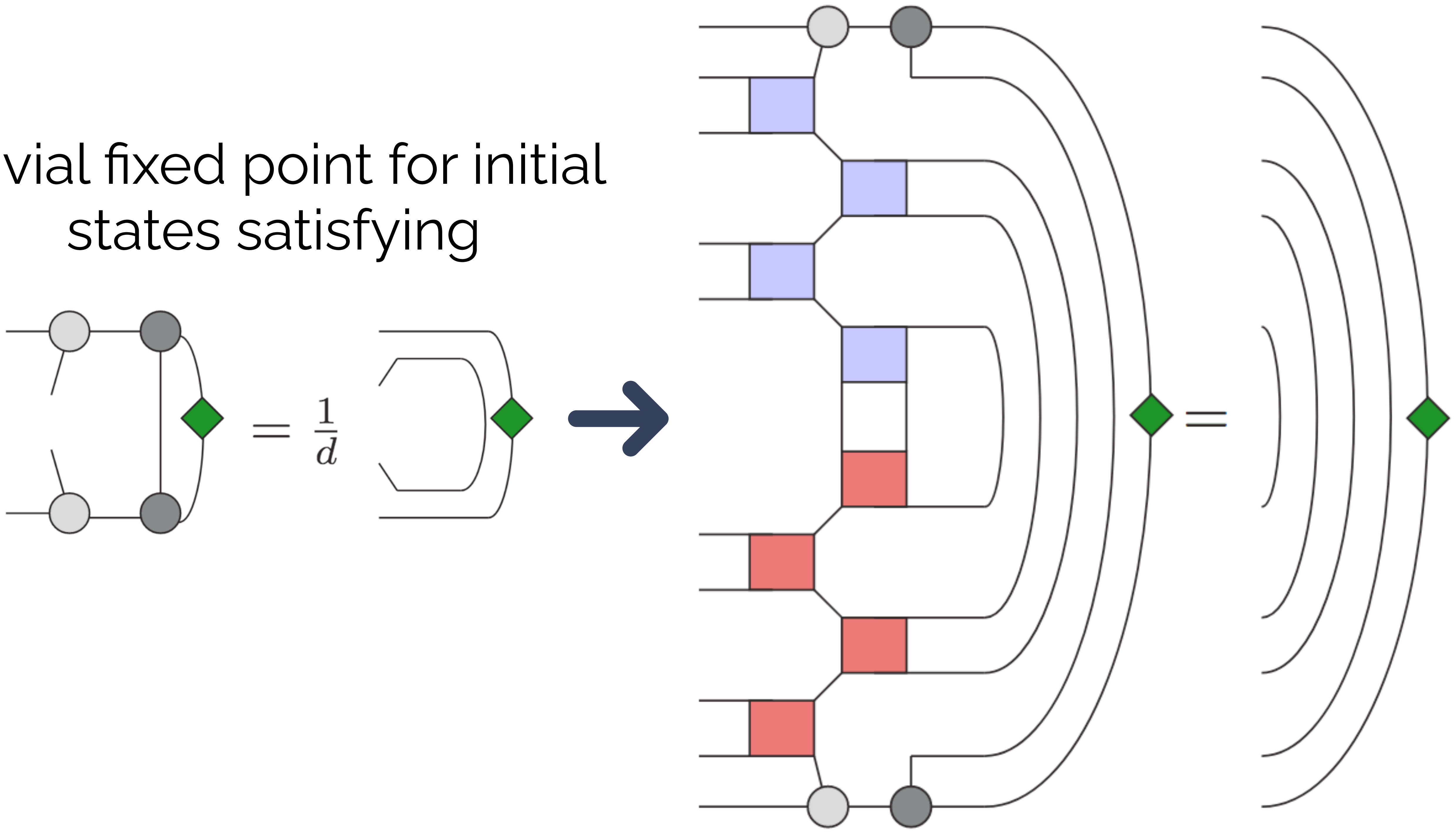
Parametrization for spin-½

$$U = e^{i\phi}\left(u_+^{(l)} \otimes u_+^{(r)}\right) V_J \left(u_-^{(l)} \otimes u_-^{(r)}\right), \quad V_J = \exp\left(\frac{\pi}{4}\sigma^x \otimes \sigma^x + \frac{\pi}{4}\sigma^y \otimes \sigma^y + J\sigma^z \otimes \sigma^z\right), \quad u_\pm^{(l,r)} \in SU(2)$$

B. Bertini, P. Kos, T. Prosen, Phys. Rev. Lett. 123, 210601 (2019)
L. Piroli, B, Bertini, J. I. Cirac, T. Prosen, Phys. Rev. B 101, 094304 (2020)
P. W. Claeys, A, Lamarcraft, Phys. Rev. Lett. 126, 100603 (2021)

EXACT SOLUTION

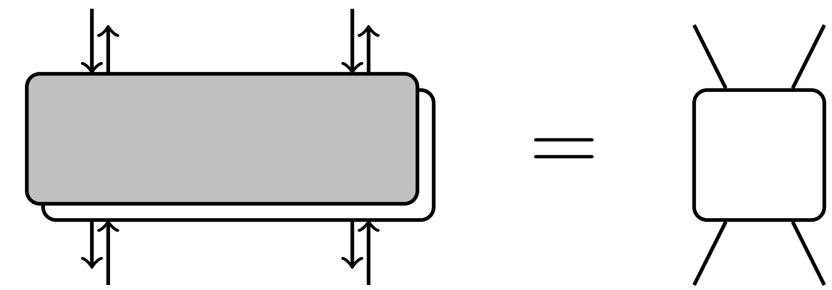
Trivial fixed point for initial states satisfying

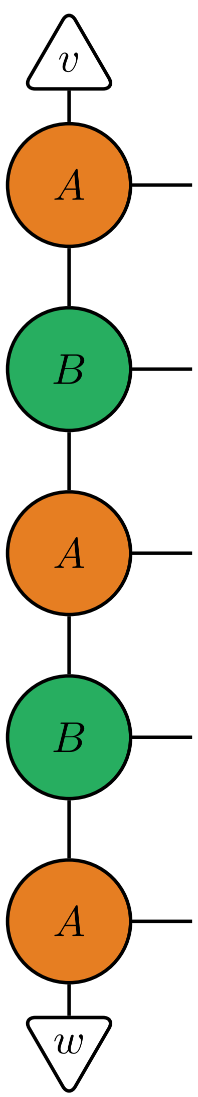


L. Piroli, B. Bertini, J. I. Cirac, and T. Prosen, Phys. Rev. B 101, 094304 (2020)

Zipper condition

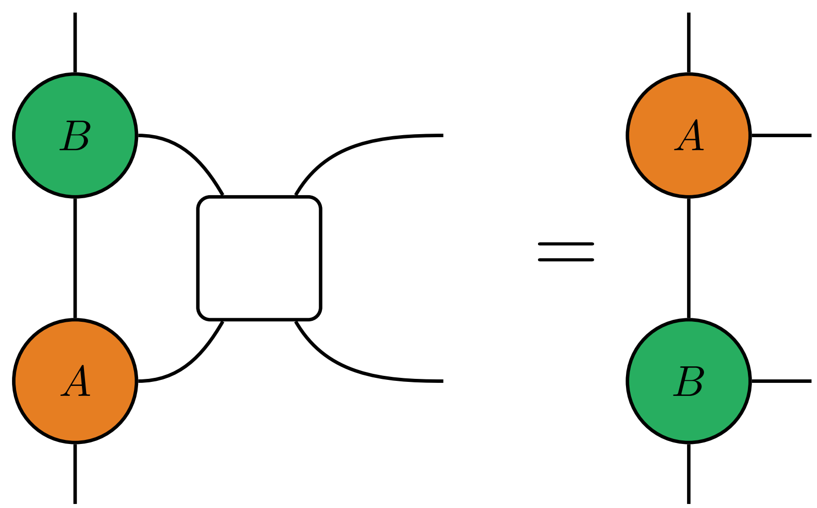
$$U_{n,n+1} = \exp\left[-iJ\left(\sigma_n^x \otimes \sigma_{n+1}^x + \sigma_n^y \otimes \sigma_{n+1}^y\right) - iJ'\sigma_n^z \otimes \sigma_{n+1}^z\right]$$



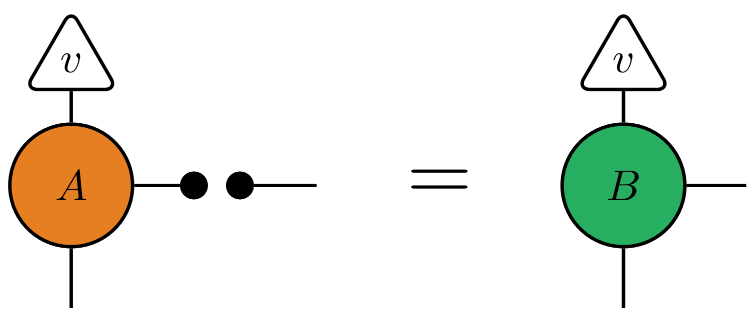


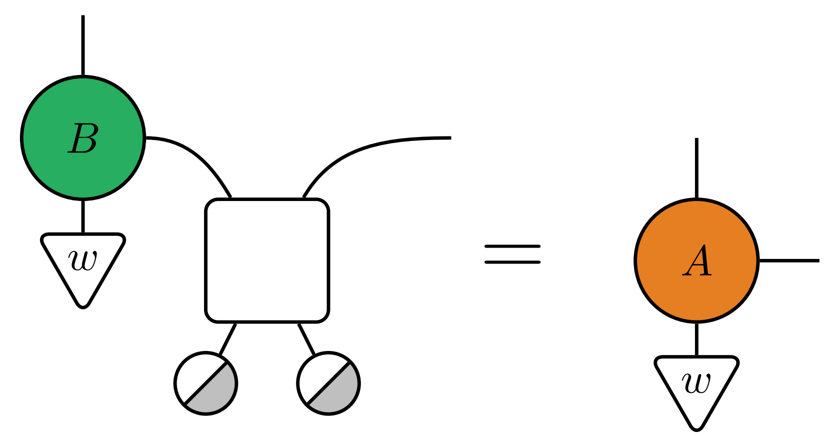
for $J = \frac{\pi}{4}$, $J' = \frac{\pi}{4} + K$

s.t.



$$B_\ell A_k e^{2iKf(k,\ell)} = A_k B_\ell$$





Similar to ASEP equations in: J. Haegeman and F. Verstraete, Ann. Rev. Cond. Matt. Phys. 8, 355 (2017)

EXACT SOLUTION

$$[A_{00}]_{\alpha,\beta} = \delta_{\alpha,\beta} \cos[2K\alpha]\,,$$
$$[A_{01}]_{\alpha,\beta} = \delta_{1,\alpha-\beta} \cos[2K(\alpha-1)]\,,$$
$$[A_{10}]_{\alpha,\beta} = \delta_{1,\beta-\alpha} \cos[2K\beta]\,,$$
$$[A_{11}]_{\alpha,\beta} = [A_{00}]_{\alpha,\beta}\,,$$

$$[B_{00}]_{\alpha,\beta} = \delta_{\alpha,\beta} \exp[2Ki\alpha]\,,$$
$$[B_{11}]_{\alpha,\beta} = \delta_{\alpha,\beta} \exp[-2Ki\alpha]\,,$$
$$[B_{01}]_{\alpha,\beta} = [B_{10}]_{\alpha,\beta} = 0\,,$$

$$\alpha,\beta = -t, -(t-1), \ldots, t-1, t$$

Generalizable to all initial states between |++⟩ and Bell pairs

- Upper bound $$\max_\tau[S_\tau(t)] \leq \log(2t+1) \sim \log(t)$$

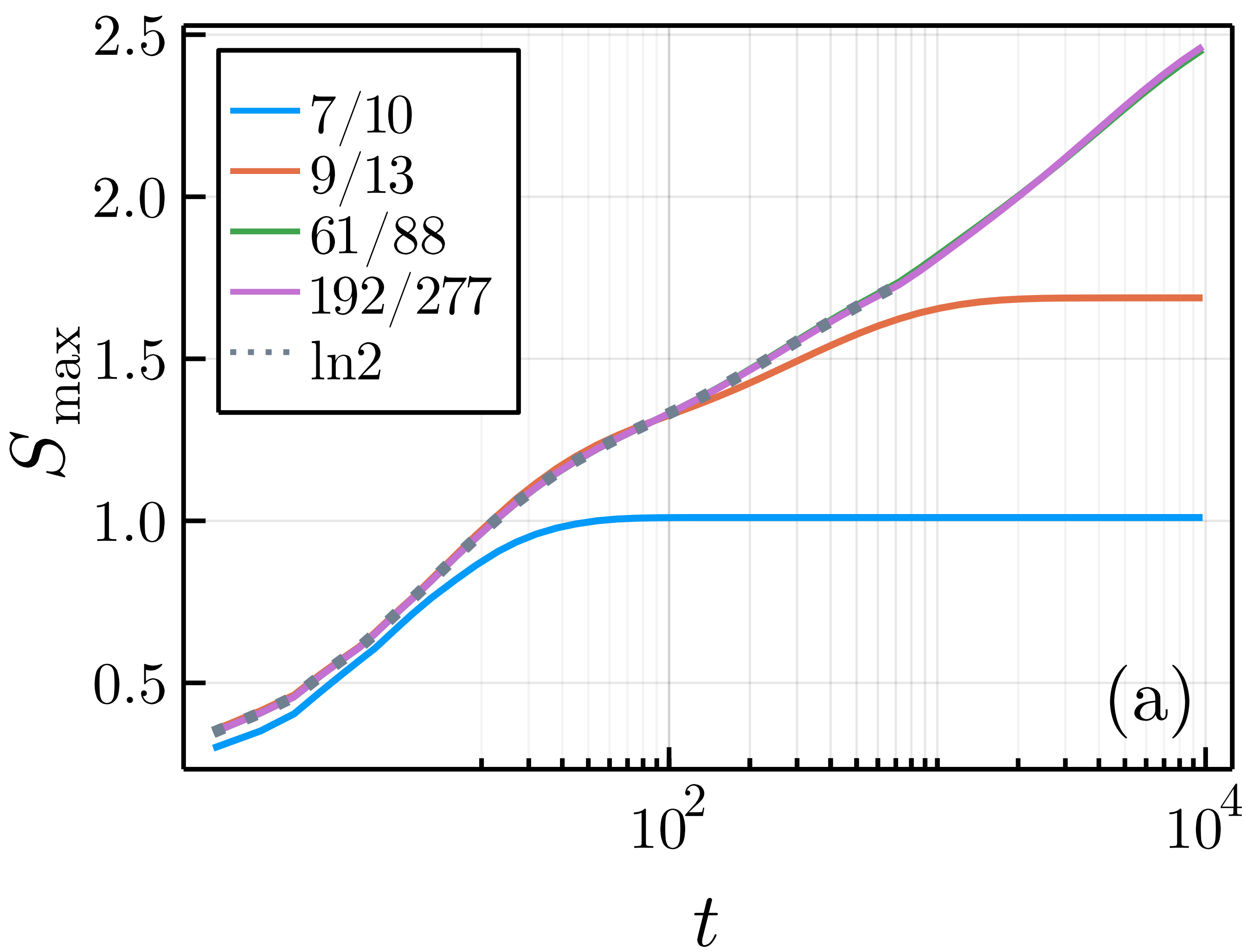
- The MPS can be *compressed* to bond dimension m if

$$K = \frac{n}{m}\pi, \quad n, m \in \mathbb{Z}$$

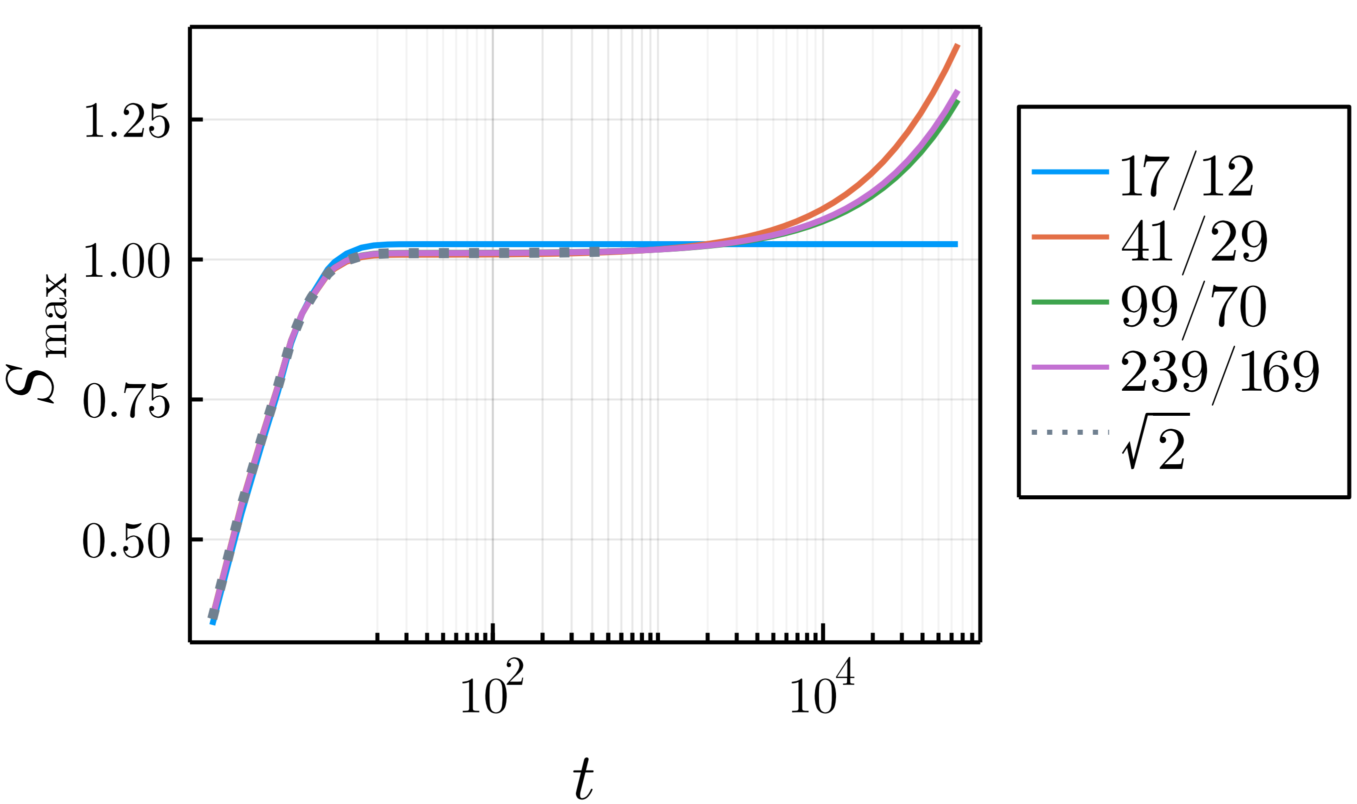
→ Bounded entanglement entropy

TE ENTROPY OF THE EXACT SOLUTION

- Finite TE entropy for rational K/π
- A priori infinite TE entropy for irrational K/π

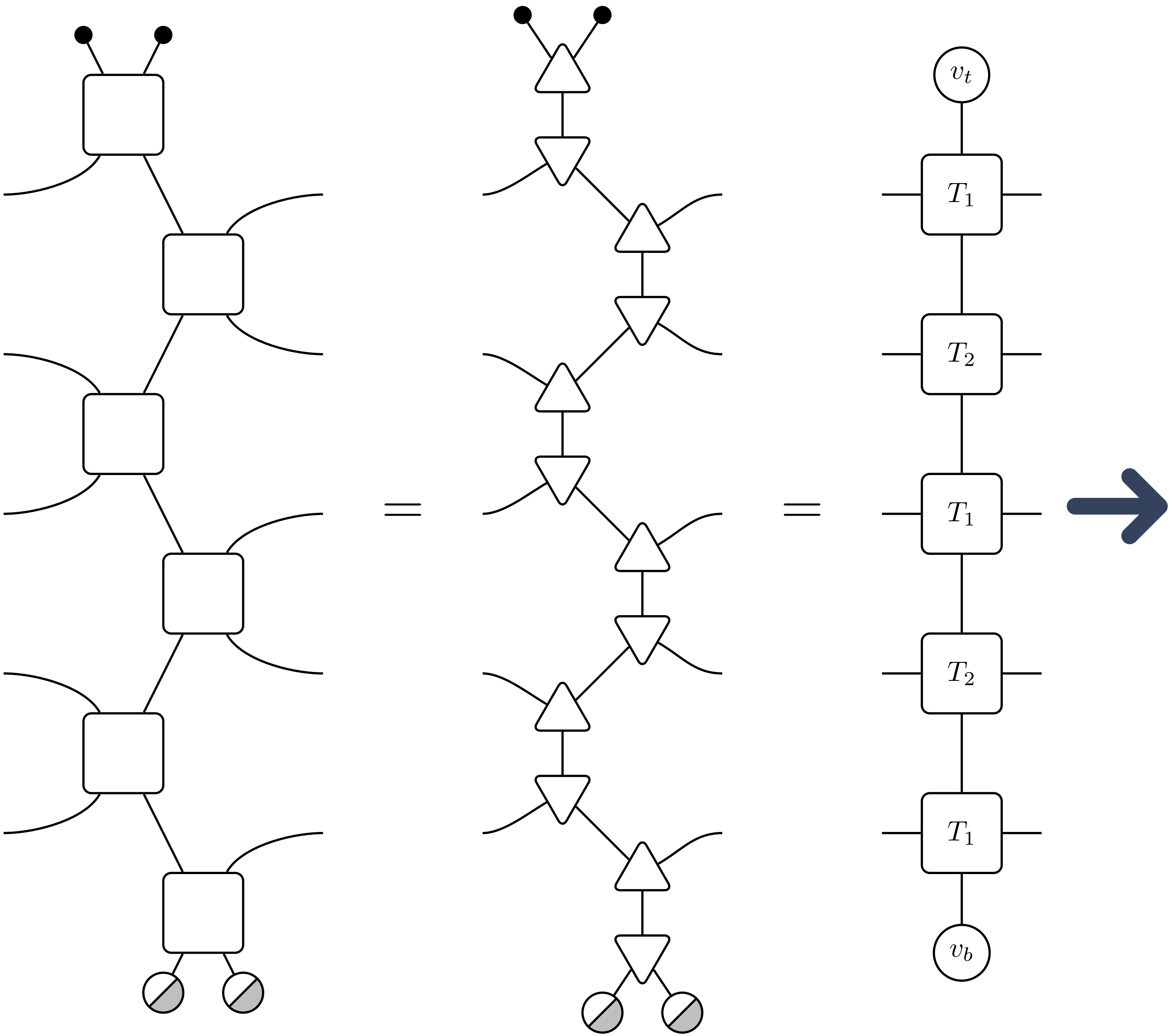


Quench from $|+\rangle^{\otimes L}$



Fractal Drude weights: M. Ljubotina, L. Zadnik, T. Prosen, Phys. Rev. Lett. 122, 150605 (2019)

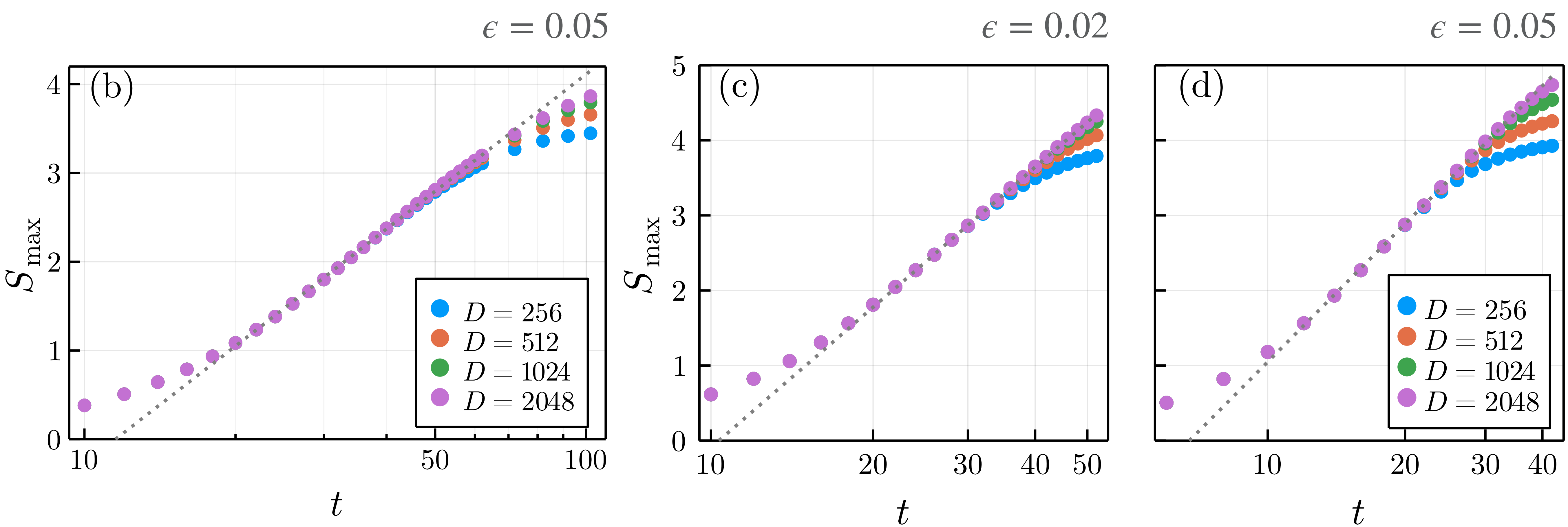
Transfer matrix as an MPO



Fixed point with DMRG

Numerical results

$J = \frac{\pi}{4} + \epsilon, \quad J' = 1$



$\rho_0 = \frac{\mathbb{I}}{2^L}$

$|\psi_0\rangle = |01\rangle^{\otimes L/2}$

Outlook

- Generality of $\log(t)$ growth?
- Understand continuous limit
- Efficient classical simulation? MERA?

Collaborators

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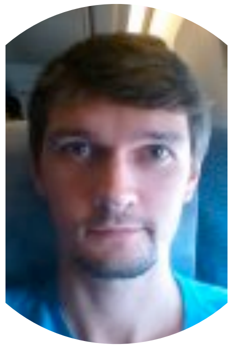
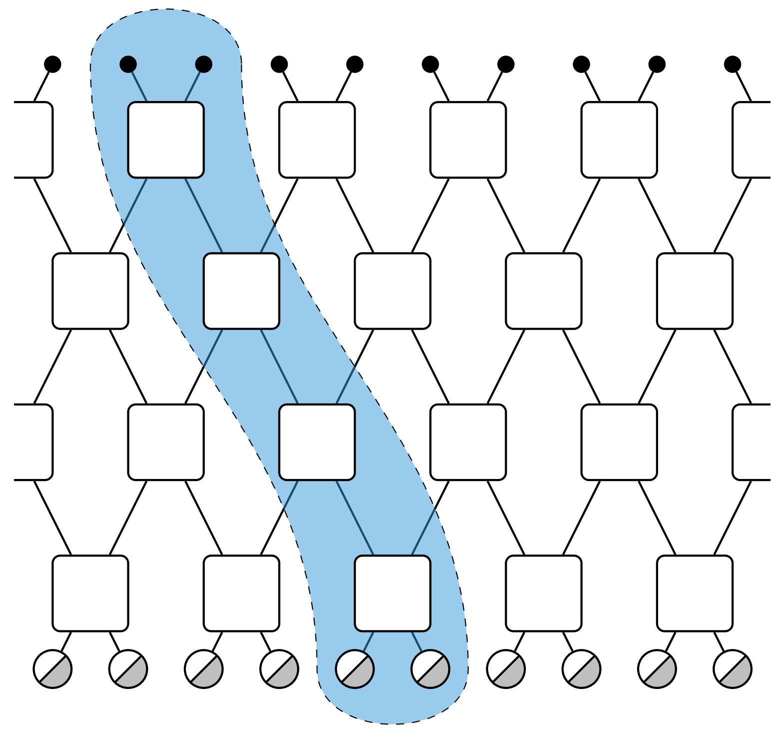
LMU Munich

Lorenzo Piroli
ENS Paris



Michael Sonner, Julian Thoenniss, Alessio Lerose, Dimitri Abanin
University of Geneva

LIGHT-CONE TRANSFER MATRIX



$$\tilde{T}_{\mathrm{LC}} =$$