Identification of a non-conformal chiral transition in various 2D classical models with CTMRG

EPFL

SN, J. Colbois, F. Mila, Nuclear Phys. B 2021

SN, F. Mila, PRR 2022

Outline

1. Introduction
2. Three-state chiral Potts model
3. Chiral Ashkin-Teller model
4. Hard-square model
5. Conclusion

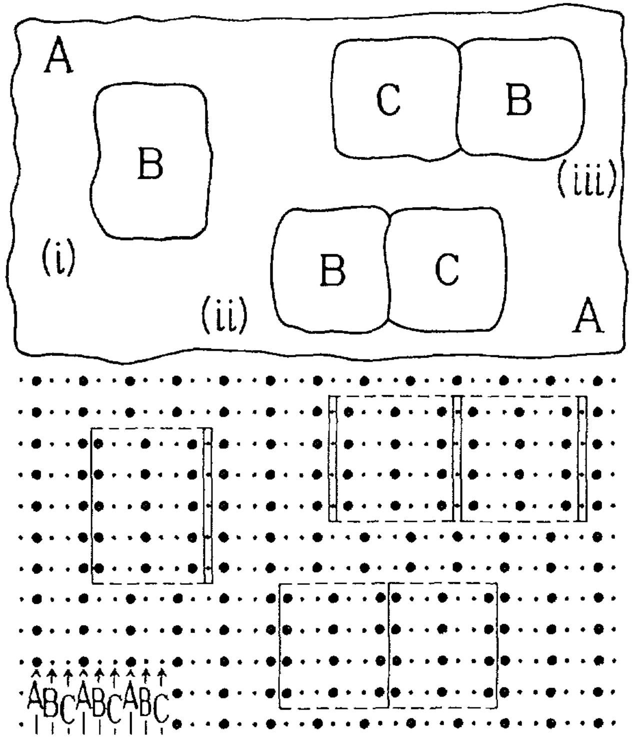
1. C-IC transition problem in 2D classical systems

- Originally introduced independentely by Osltund, and Huse and Fisher.

Question: How does the melting of an ordered period-p phase into an incommensurate one occurs and what is its nature ?

S. Ostlund, PRB 1981

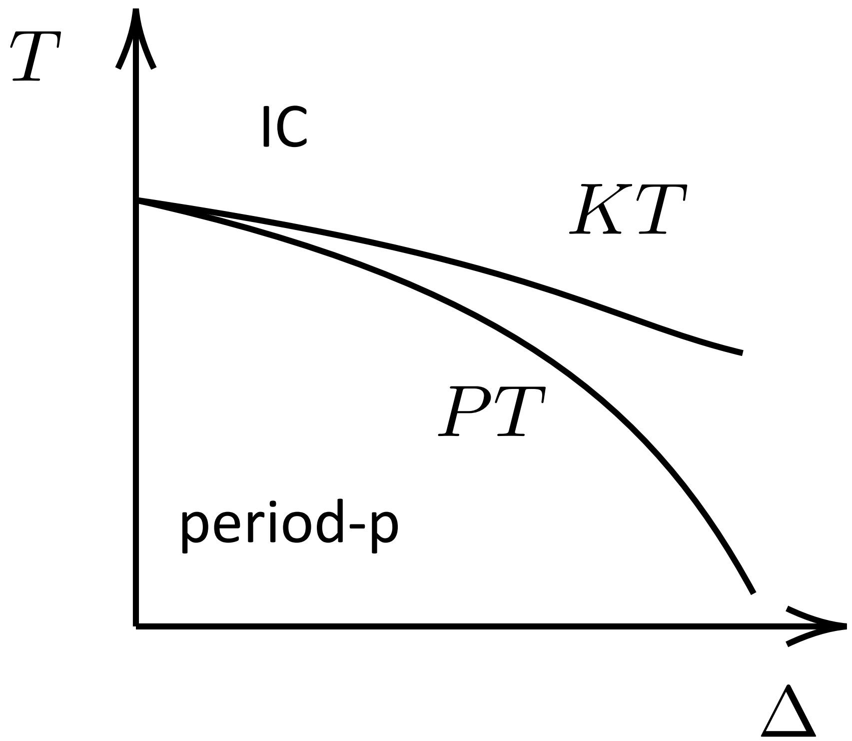
- Adsorbed atoms on substrate lattices

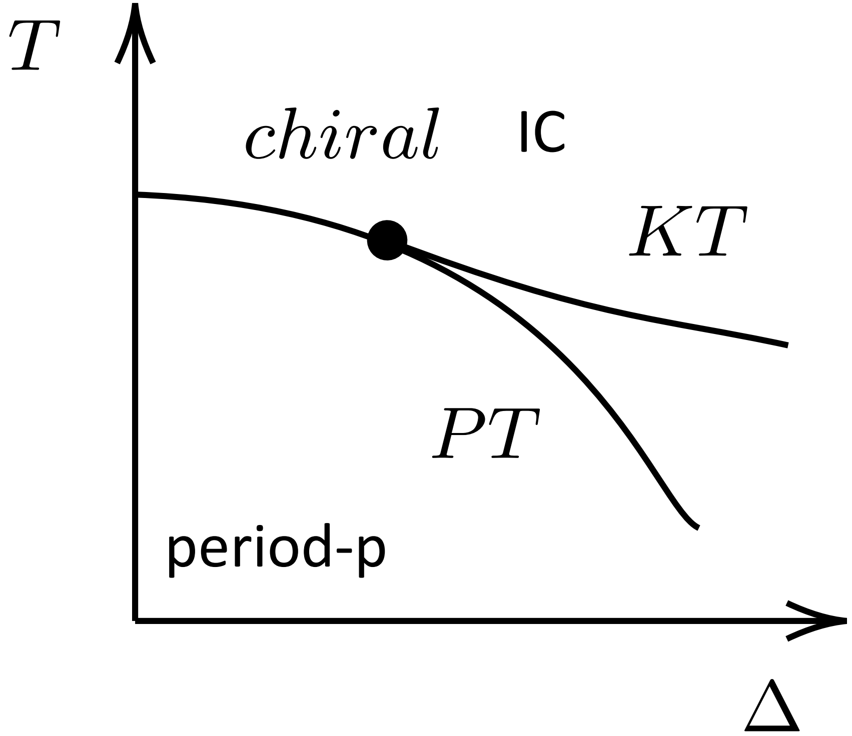


David A. Huse and Michael E. Fisher, PRL 1982

Commensurate-Incommensurate transitions

- p > 4 : Either first order or two-step transition.
- p = 3,4 ?





$$q - q_0 \propto |t|^{\overline{\beta}}$$

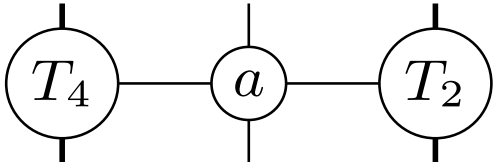
- p = 2: no floating phase.

Pokrovsky-Talapov transition : $\nu_x = 1/2 \quad \nu_y = 1 \quad \bar{\beta} = 1/2$

Chiral transition : $\nu_x = \bar{\beta}$ Huse and Fisher PRL 1982

Method : extrapolation

Transfer matrix in one direction



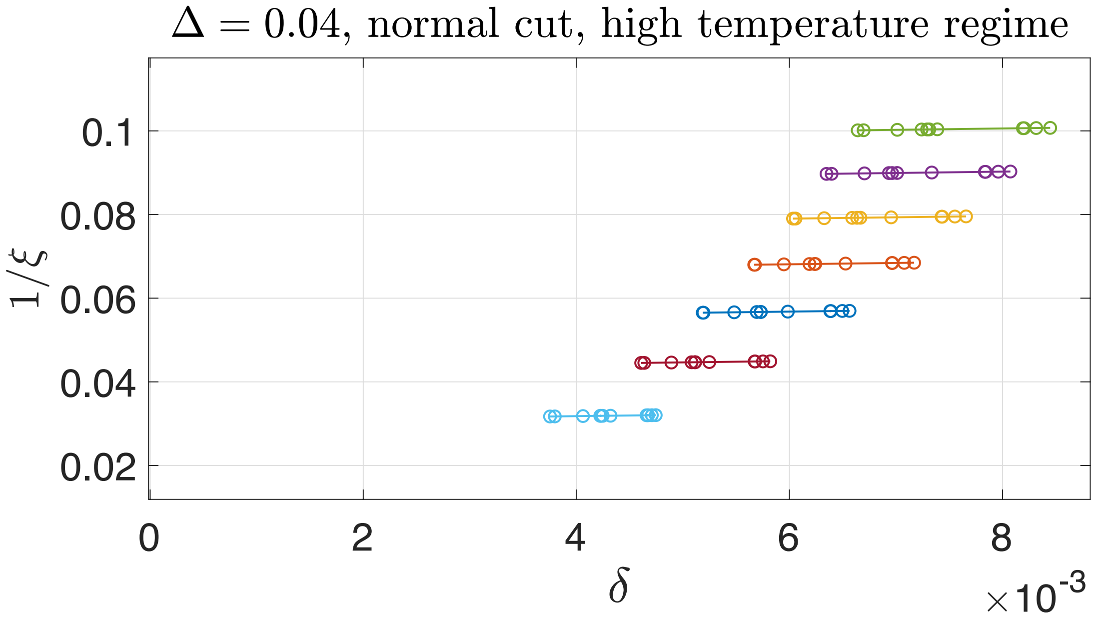
$$\lambda_j = e^{-\epsilon_j - i\phi_j}, \qquad j \in \mathbb{N}^*$$

$$\xi = \frac{1}{\epsilon_2}, \qquad q = \phi_2$$

Linear extrapolation:

$$\frac{1}{\xi(\chi)} = \frac{1}{\xi_{\text{exact}}} + b\delta(\chi) \qquad \delta = \epsilon_4 - \epsilon_2$$

$$q(\chi) = q_{\text{exact}} + b'\delta'(\chi) \qquad \delta' = \phi_4 - \phi_2$$



Marek M. Rams, Piotr Czarnik, and Lukasz Cincio Phys. Rev. X 8, 041033

2. Three-state chiral Potts model : $p = 3$

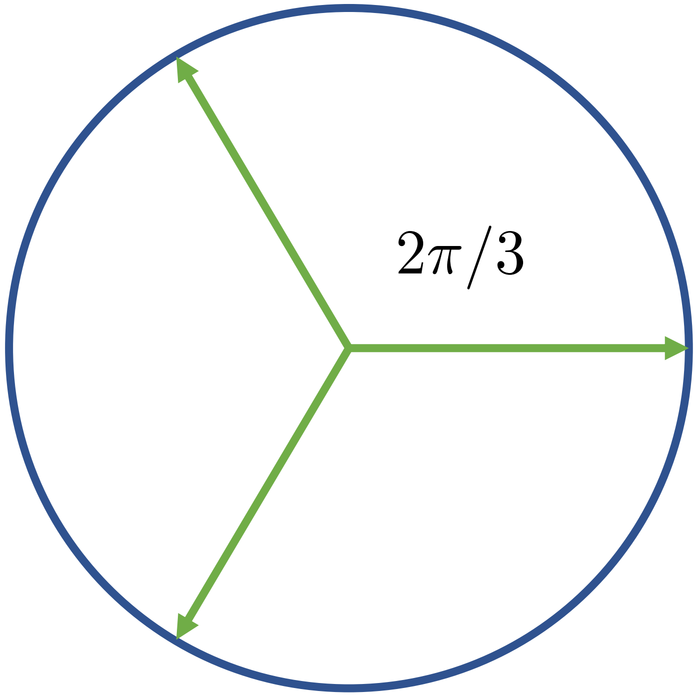
$$E = \sum_{\vec{r}} \cos(\theta_{\vec{r}+\vec{x}} - \theta_{\vec{r}} + \Delta_\theta) + \cos(\theta_{\vec{r}+\vec{y}} - \theta_{\vec{r}})$$

$$\theta \in \{0, 2\pi/3, 4\pi/3\}$$

$$\Delta_\theta = 2\pi/3\Delta$$

- Three-state Potts point : $\Delta = 0$

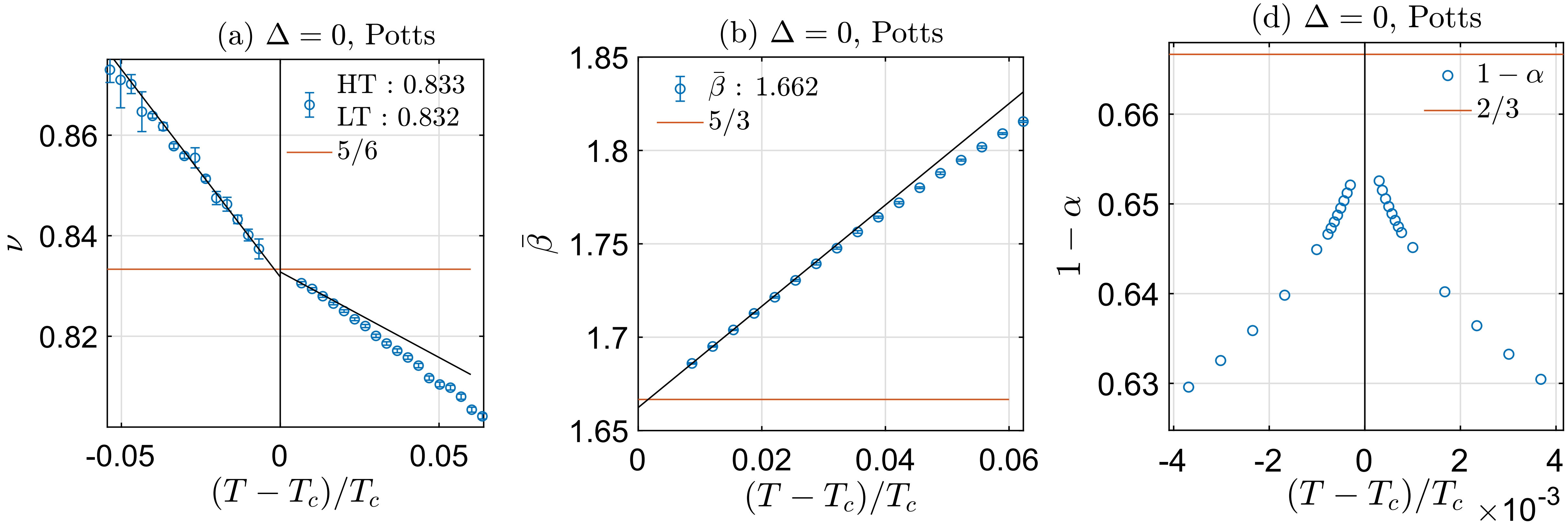
$$T_c = 2/[2\log(\sqrt{3}+1)]$$
$$\nu = 5/6$$
$$\bar{\beta} = 5/3$$
$$\alpha = 1/3$$



effective exponent

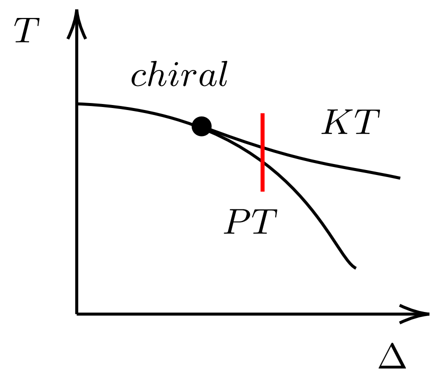
$$A \propto |t|^{-\theta} \qquad \theta(|t|) = -\frac{d\ln A}{d\ln|t|}. \qquad \theta = \lim_{|t|\to 0} \theta(|t|)$$

- If the transition is unique, can determine the critical temperature.

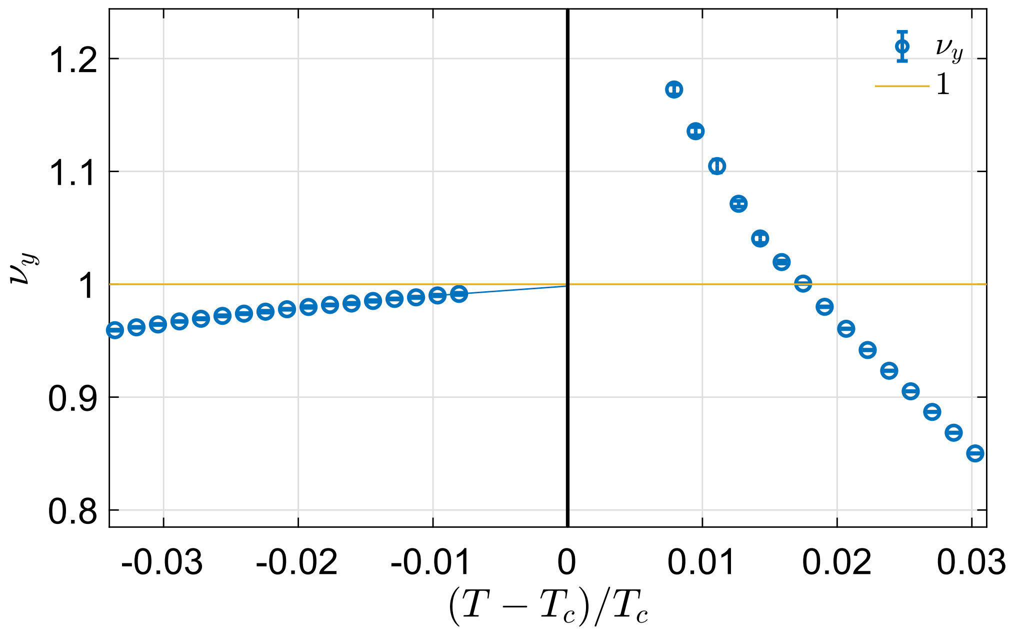


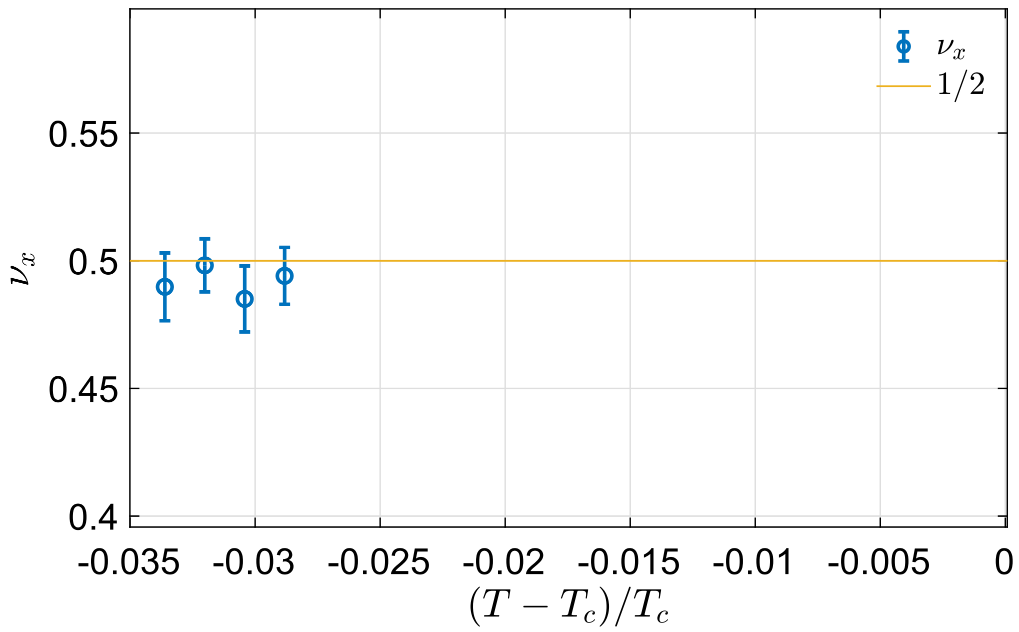
KT and PT transitions at large chirality

Pokrovsky-Talapov transition : $\nu_x = 1/2 \quad \nu_y = 1$

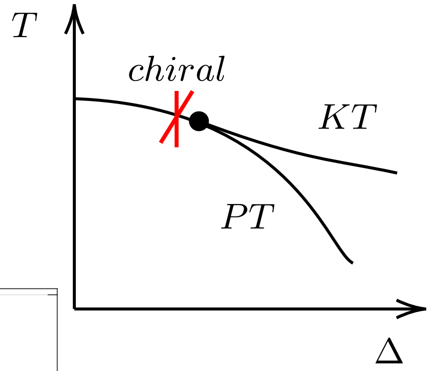


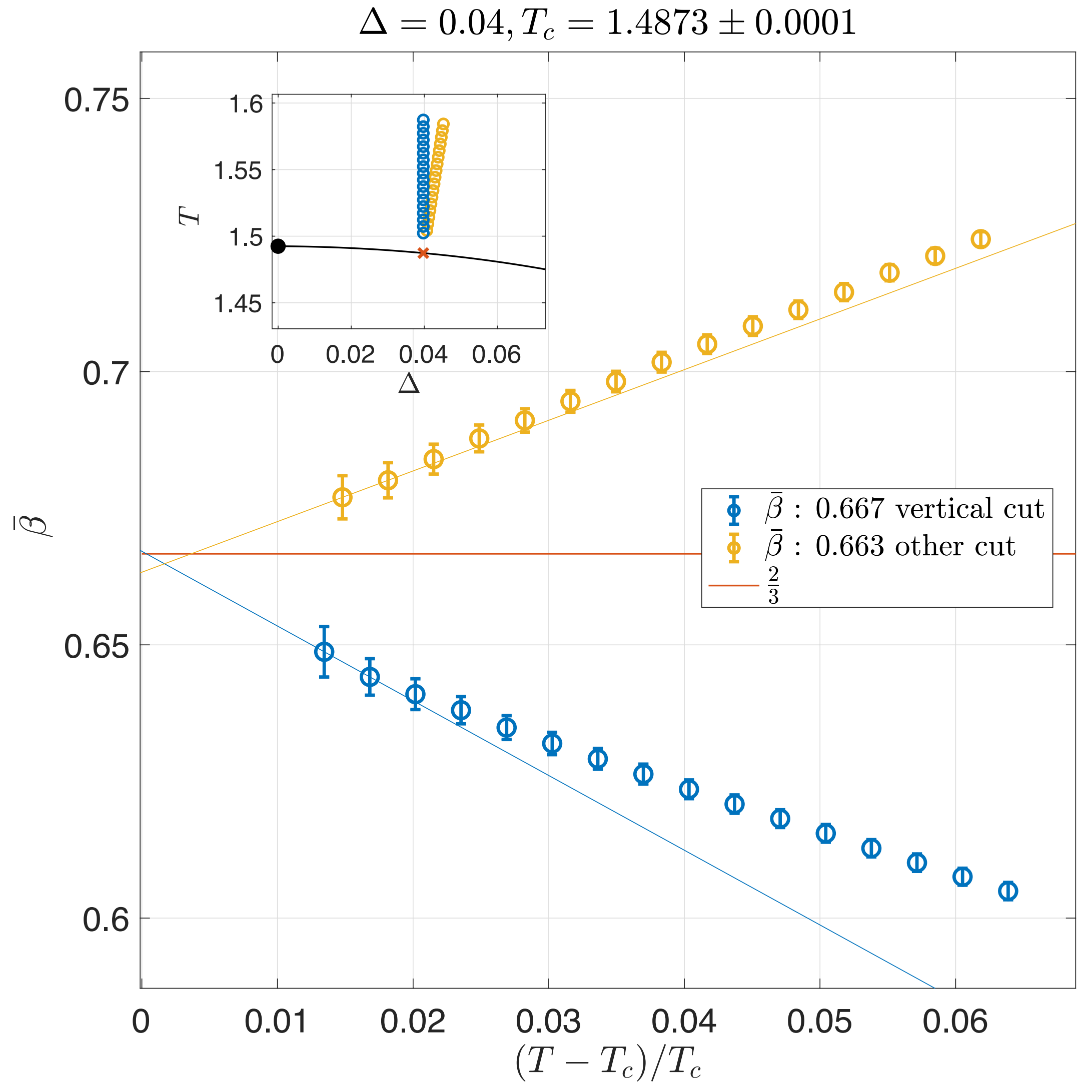
$\Delta = 0.28, T_c = 1.2266 \pm 0.0001$ normal cut

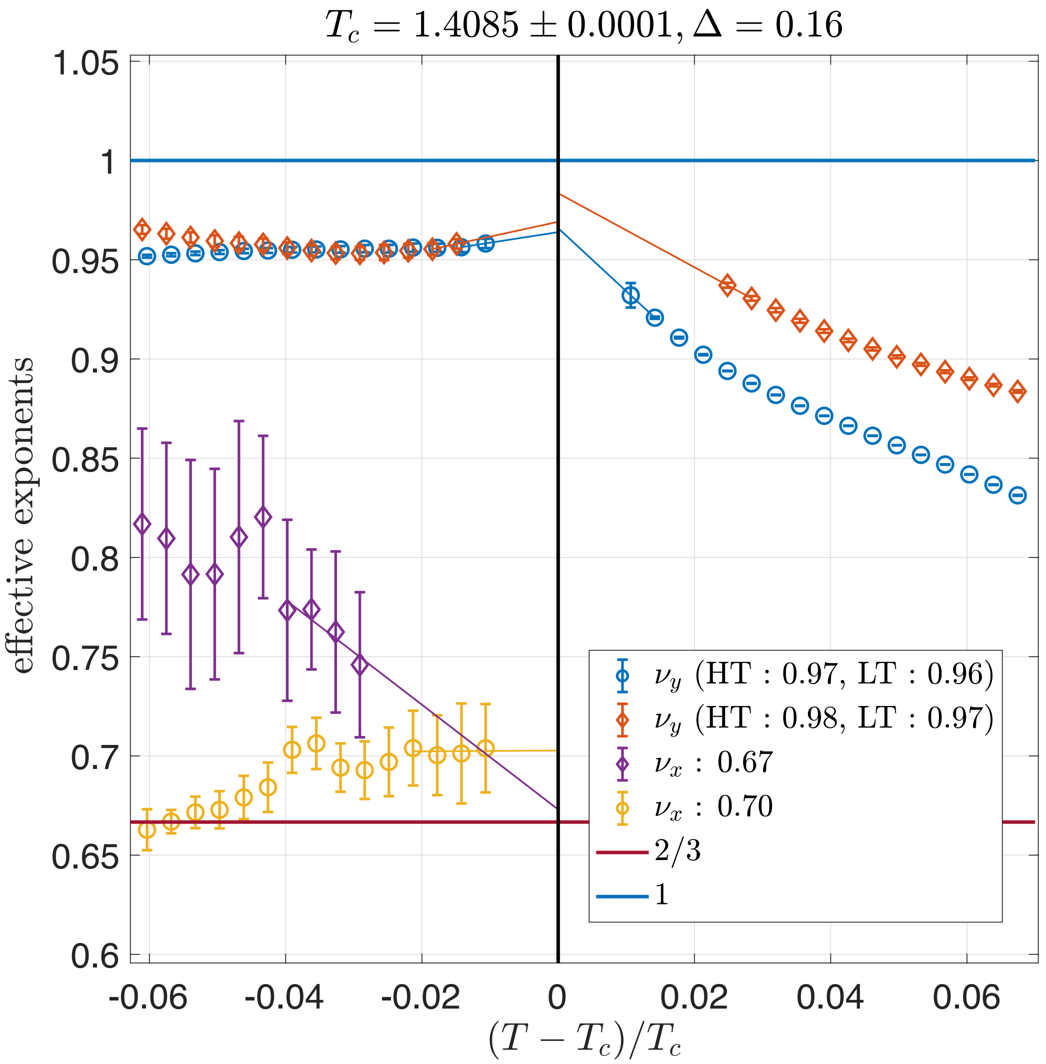




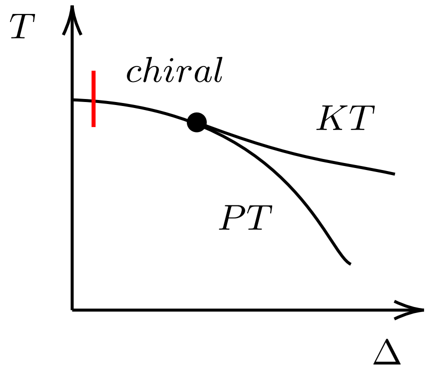
Close to the Potts point

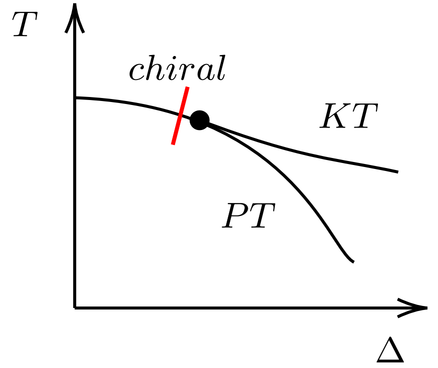


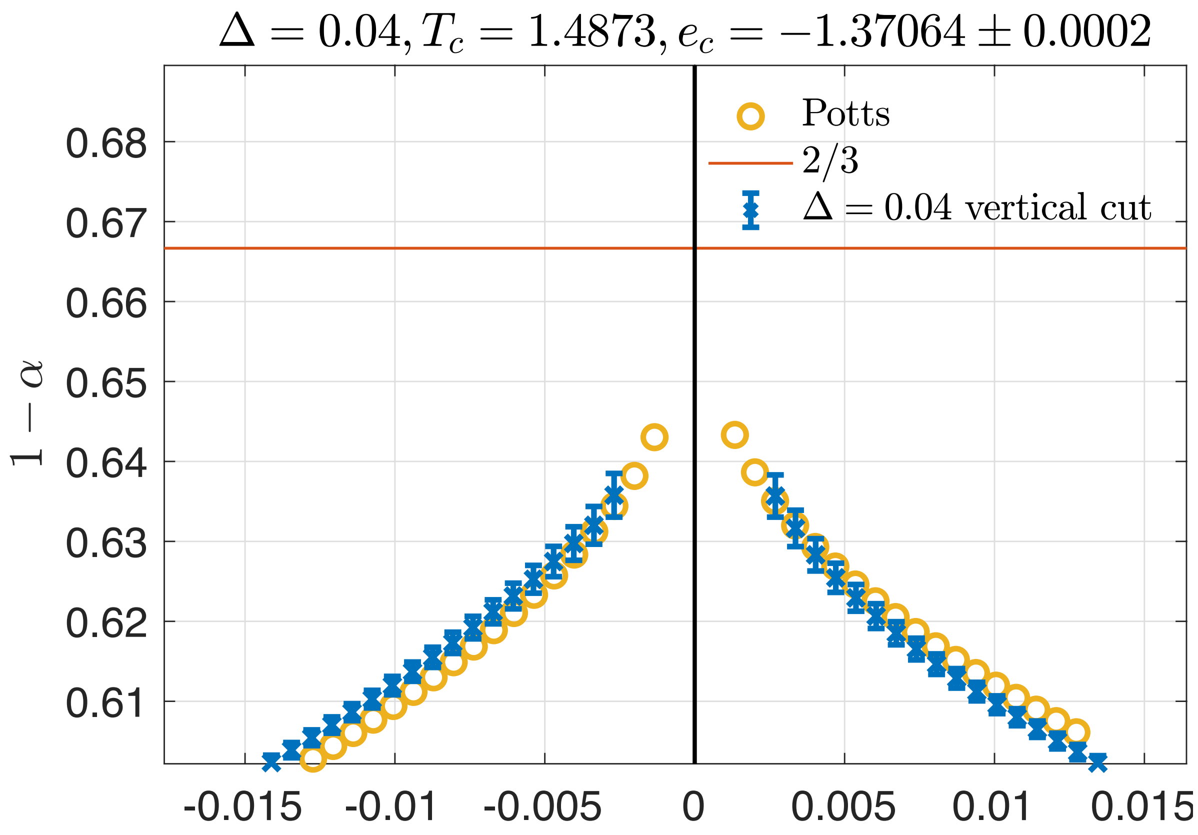


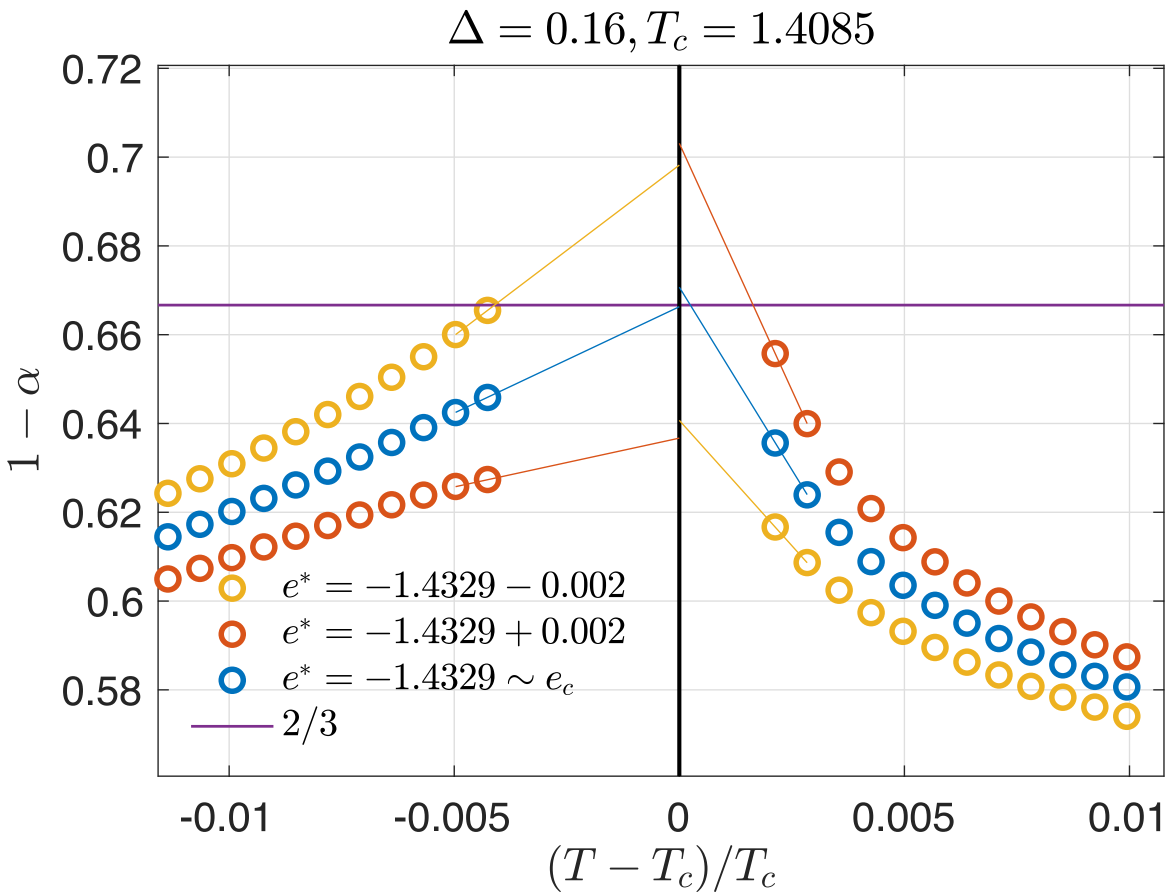


Exponent α









Respect the hyper-scaling relation : $\nu_x + \nu_y = 2 - \alpha$

Phase Diagram

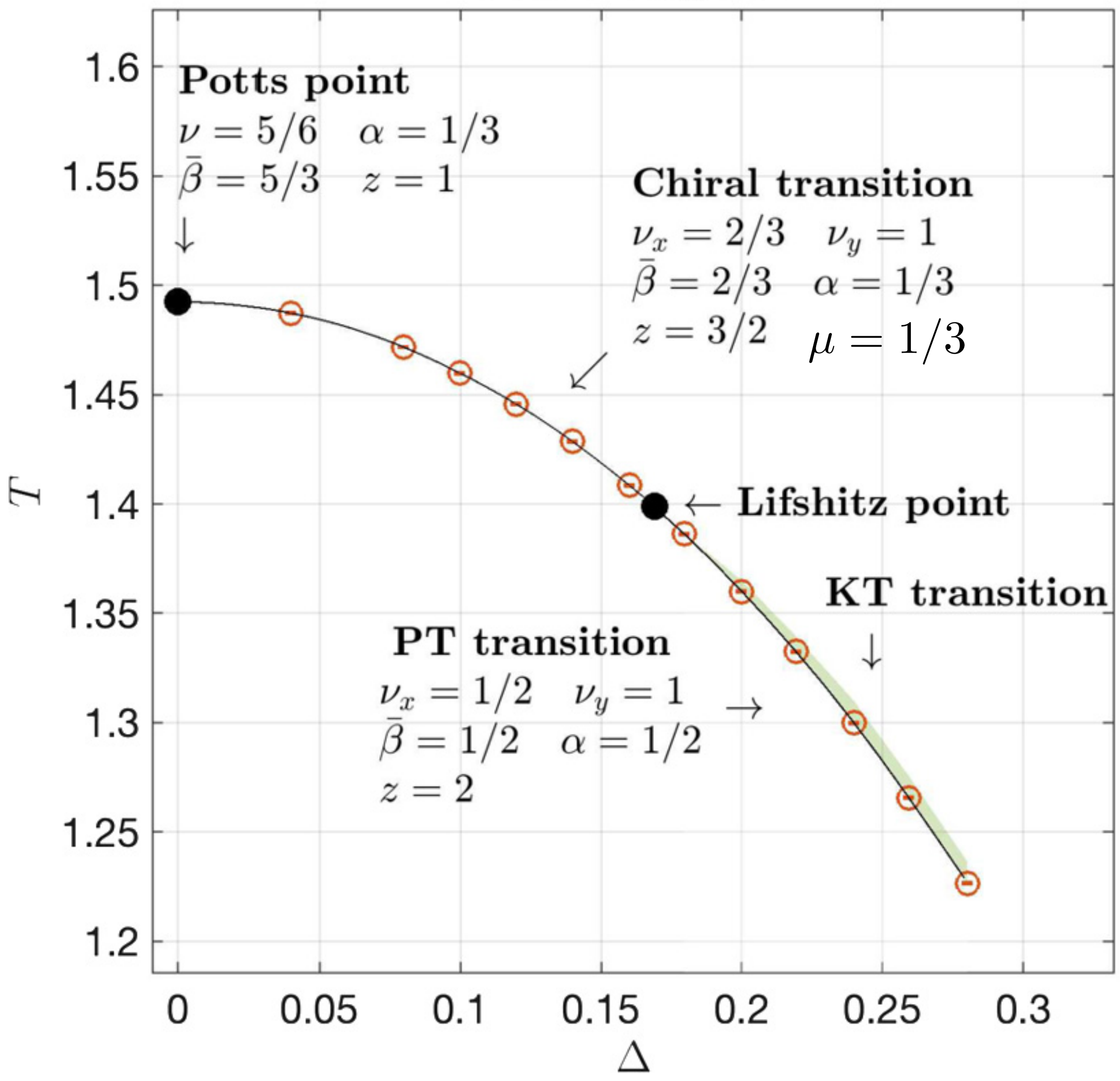
- Agreement with x-ray experiment

(Abernathy et al. PRB 1994)

$$\bar{\beta} = 0.66 \pm 0.05$$
$$\nu_x = 0.65 \pm 0.07$$
$$\nu_y = 1.06 \pm 0.07$$

- Agreement with experimental realisation on Rydberg atoms $\mu \simeq 0.38$

(Lukin et al. Nature 2019)



3. Chiral Ashkin-Teller model : $p = 4$

- Ashkin-Teller model $\tau, \sigma \in \{\pm 1\}$

$$H_0 = -\sum_{\langle i,j \rangle} \sigma_i \sigma_j + \tau_i \tau_j + \lambda \sigma_i \sigma_j \tau_i \tau_j$$

$$\nu = \frac{1}{2 - \frac{\pi}{2} \arccos(-\lambda)^{-1}}$$

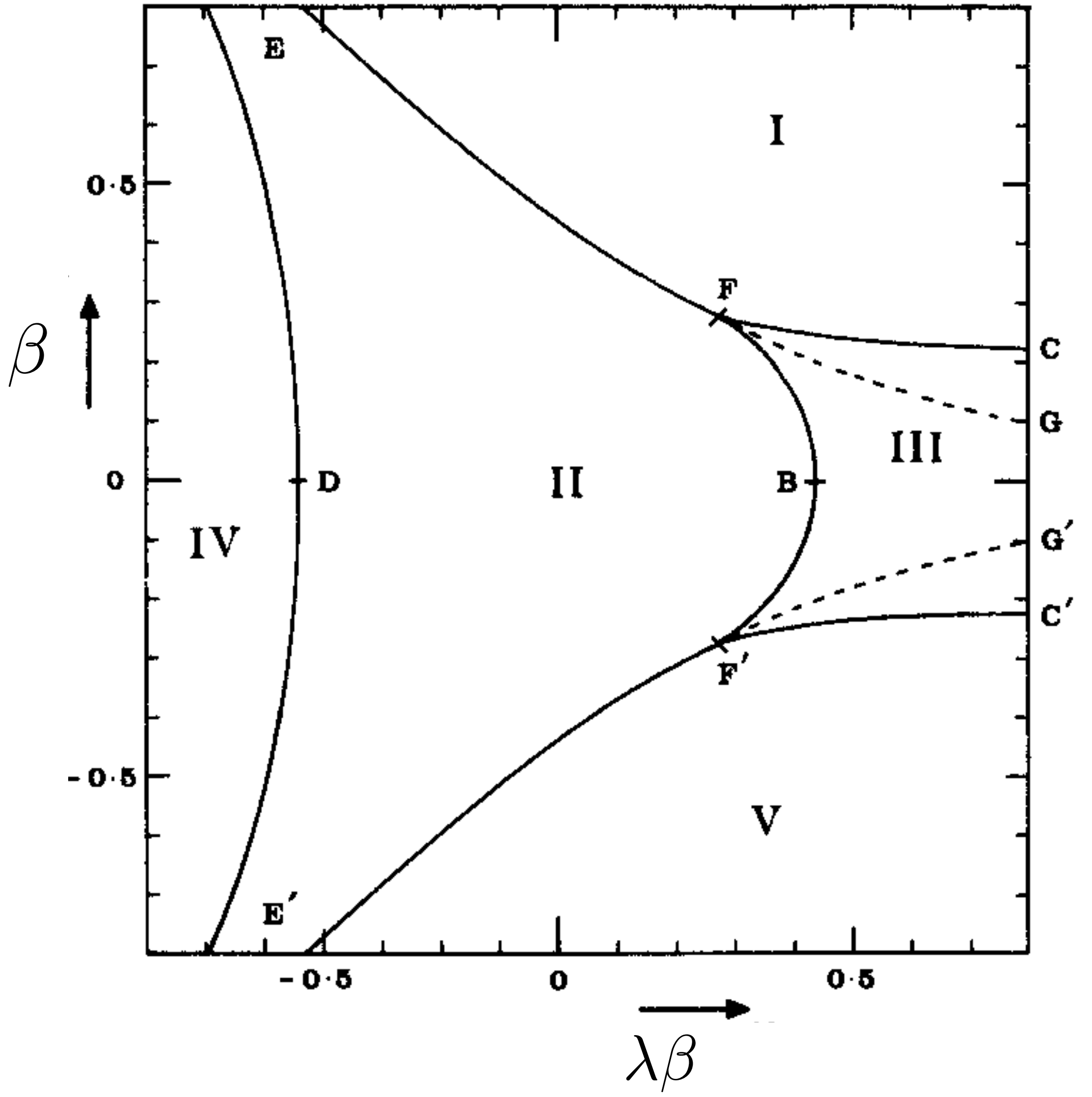
$$\lambda = 0 \quad \nu = 1 \quad \xrightarrow{\bar{\beta} > \nu} \quad \lambda = 1 \quad \nu = 2/3$$

Ising universality class

Four-state Potts universality class

- Chiral perturbation

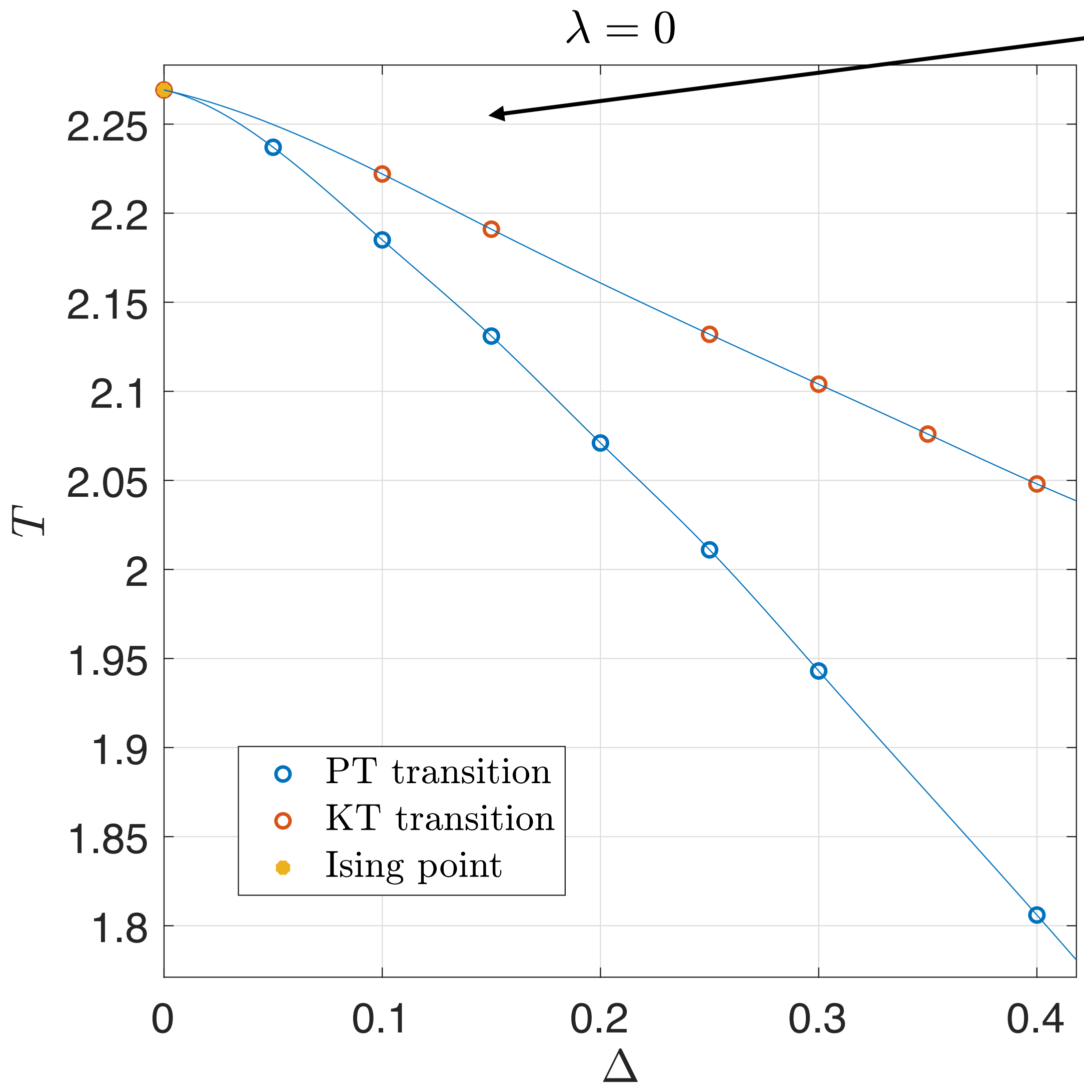
$$H = H_0 + \Delta \sum_{x,y} (\tau_{x+1,y} \sigma_{x,y} - \sigma_{x+1,y} \tau_{x,y})$$

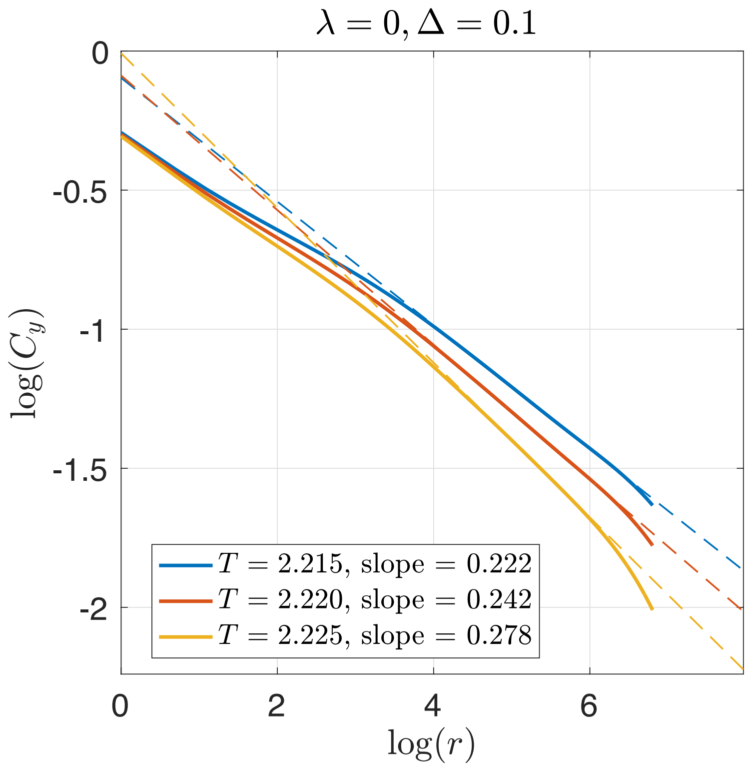


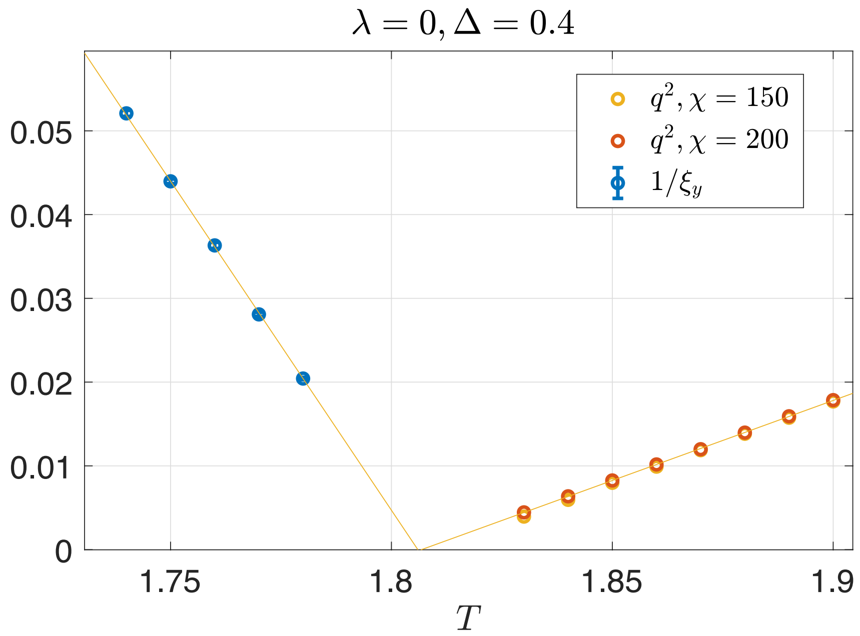
Baxter, Exactely solved models in Stat. Phys. 1992

$\lambda = 0$ cut

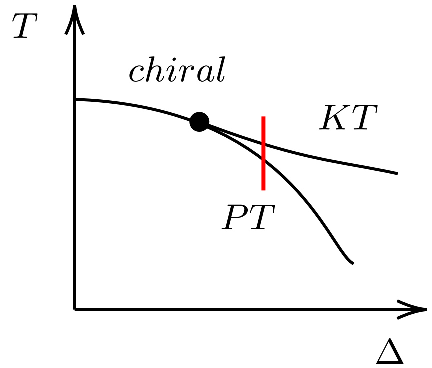
No evidence of a unique transition!

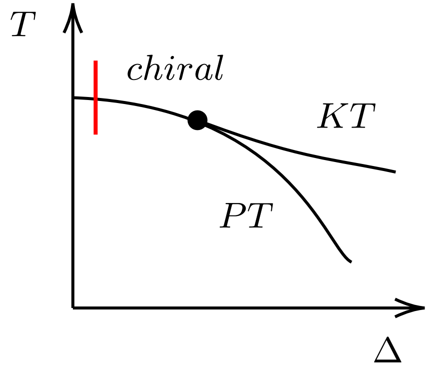


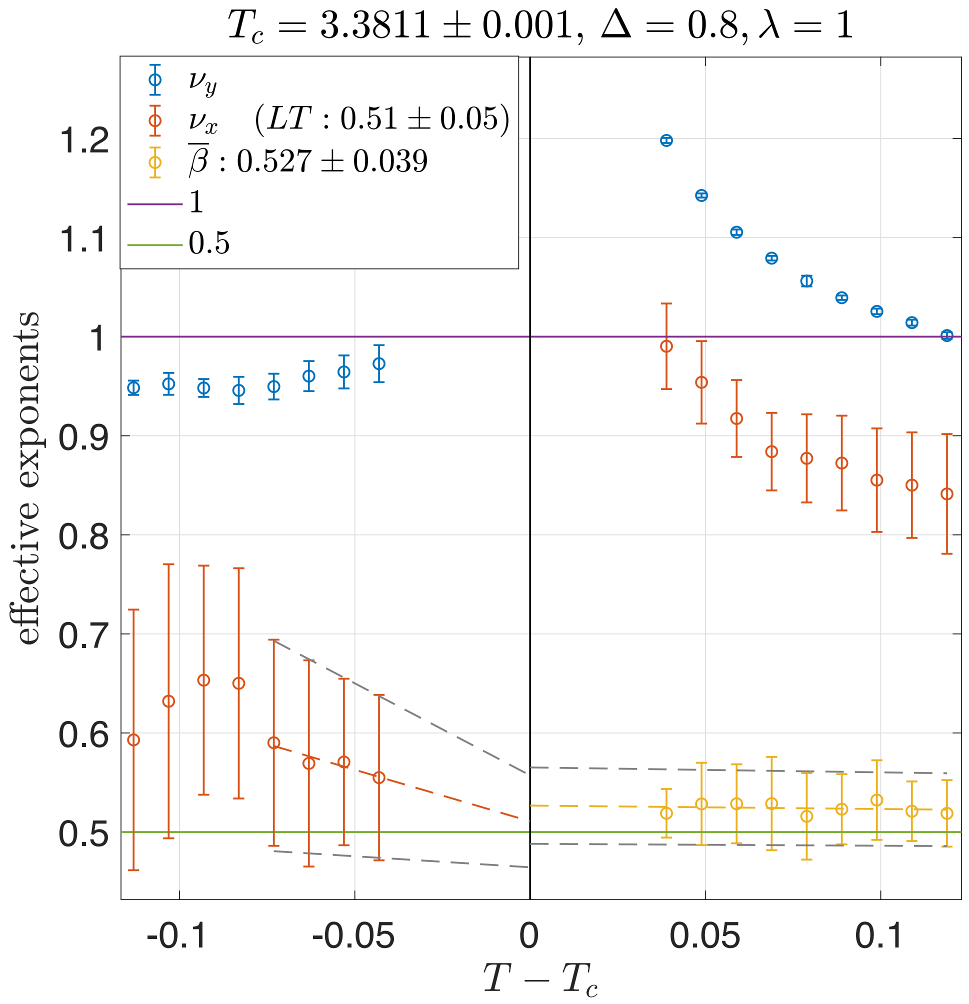


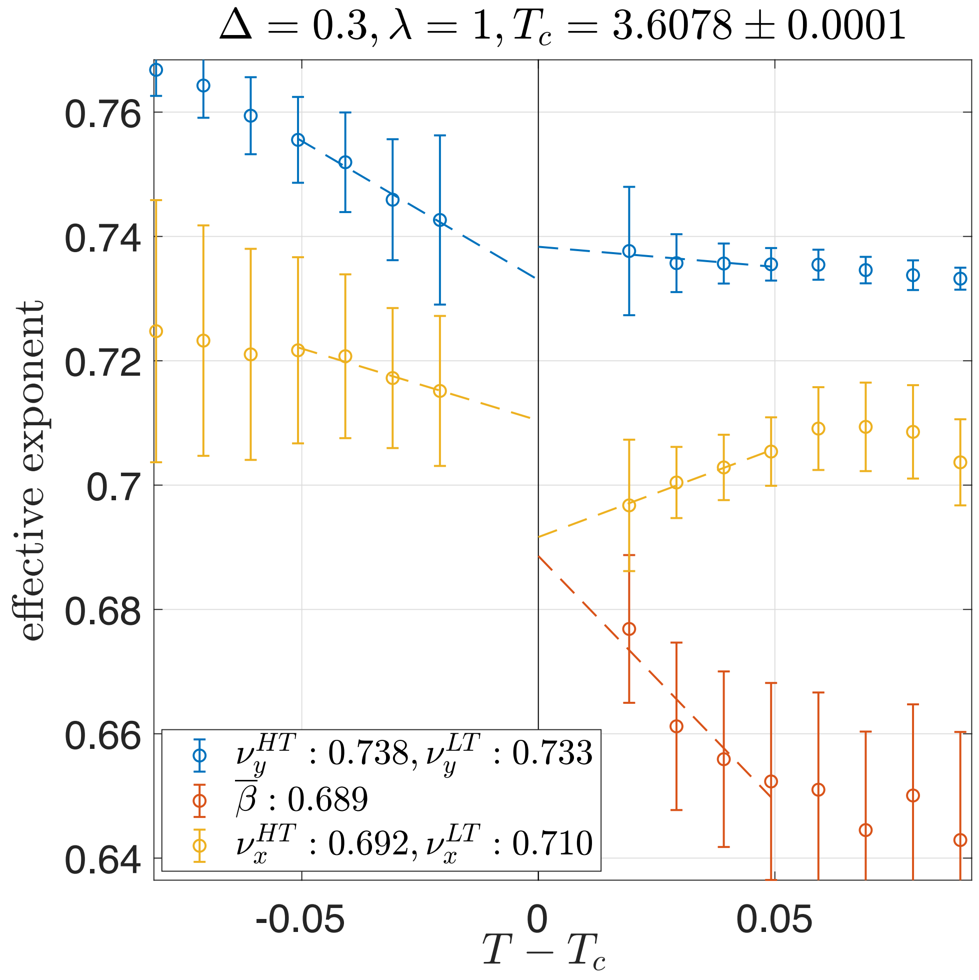


$\lambda = 1$ cut

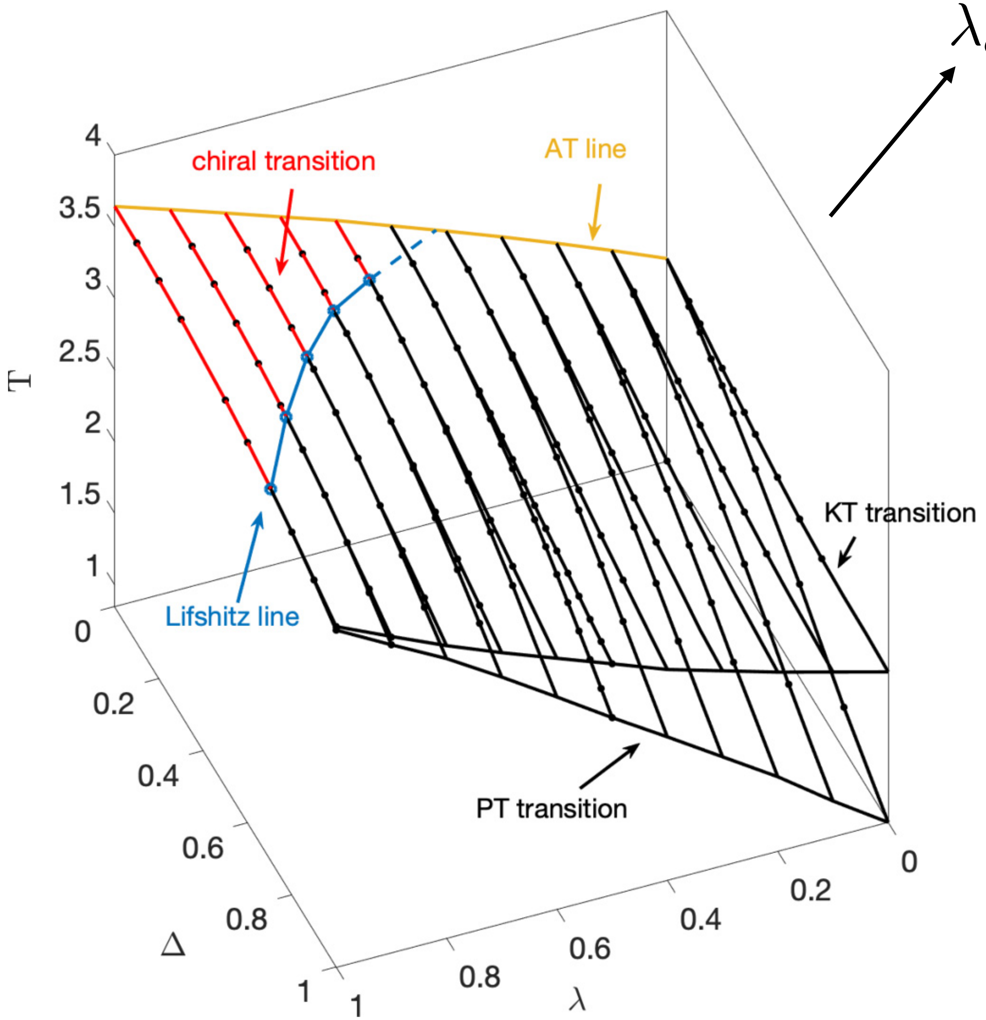

T
chiral
KT
PT
Δ


T
chiral
KT
PT
Δ

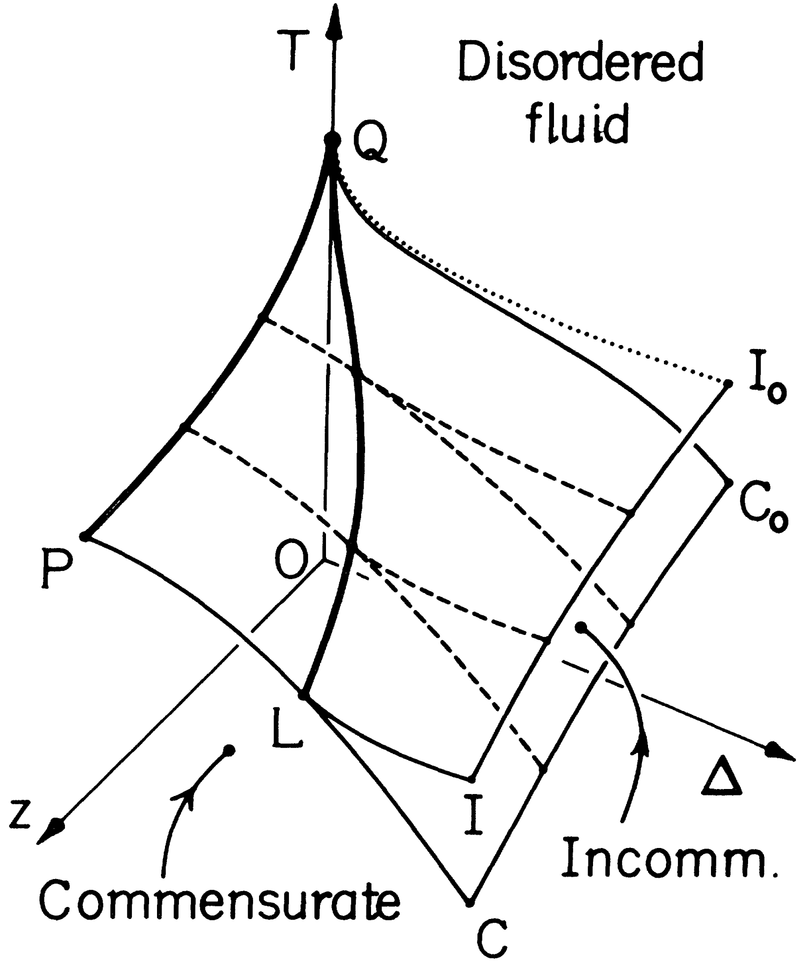

$T_c = 3.3811 \pm 0.001,\, \Delta = 0.8, \lambda = 1$
ν_y
ν_x $(LT : 0.51 \pm 0.05)$
$\overline{\beta} : 0.527 \pm 0.039$
1
0.5
effective exponents
$T - T_c$
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
-0.1
-0.05
0
0.05
0.1


$\Delta = 0.3, \lambda = 1, T_c = 3.6078 \pm 0.0001$
$\nu_y^{HT} : 0.738, \nu_y^{LT} : 0.733$
$\overline{\beta} : 0.689$
$\nu_x^{HT} : 0.692, \nu_x^{LT} : 0.710$
effective exponent
$T - T_c$
0.76
0.74
0.72
0.7
0.68
0.66
0.64
-0.05
0
0.05

Phase diagram



$\lambda_c \sim 0.42$



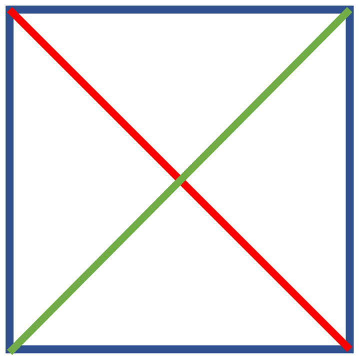
David A. Huse and Michael E. Fisher, PRB 1984

4. Hard-square model

$n \in \{1, 0\}$

$$\beta H = -M \sum_{x,y} n_{x,y} n_{x+1,y+1} - L \sum_{x,y} n_{x,y} n_{x+1,y-1}$$

$$Z = \sum_{\{n\}} \prod_{\langle i,j \rangle} (1 - n_i n_j) e^{-\beta H} \prod_i z^{n_i}$$

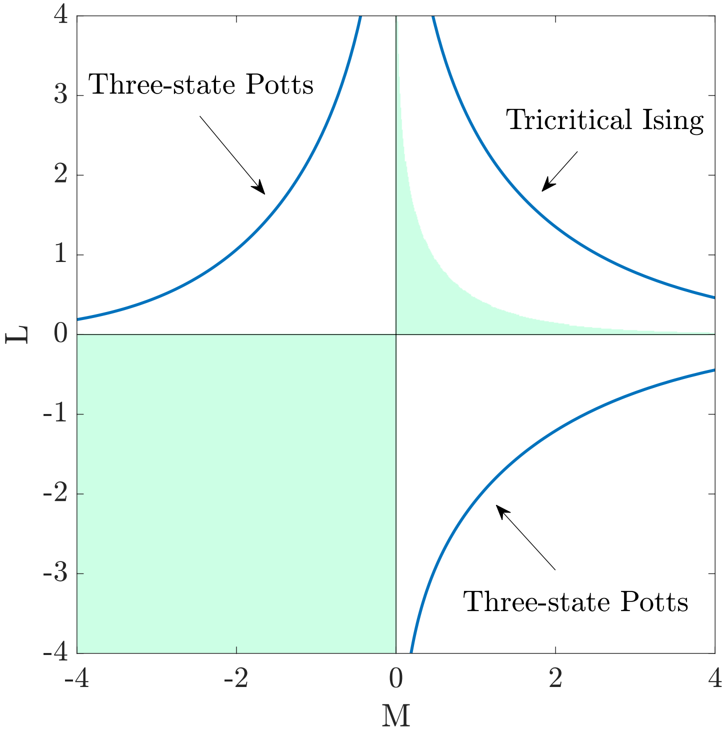


- Integrable manifold

$$z = (1 - e^{-L})(1 - e^{-M})/(e^{L+M} - e^L - e^M).$$

$$\frac{z}{(1 - z e^{L+M})^2} = \frac{1}{2}(11 + 5\sqrt{5}).$$

R. J. Baxter, Journal of Physics A, 1980



- Tricritical Ising

$M, L > 0$

D. A. Huse, Journal of Physics A, 1983

- Three-state Potts

$M < 0, L > 0$

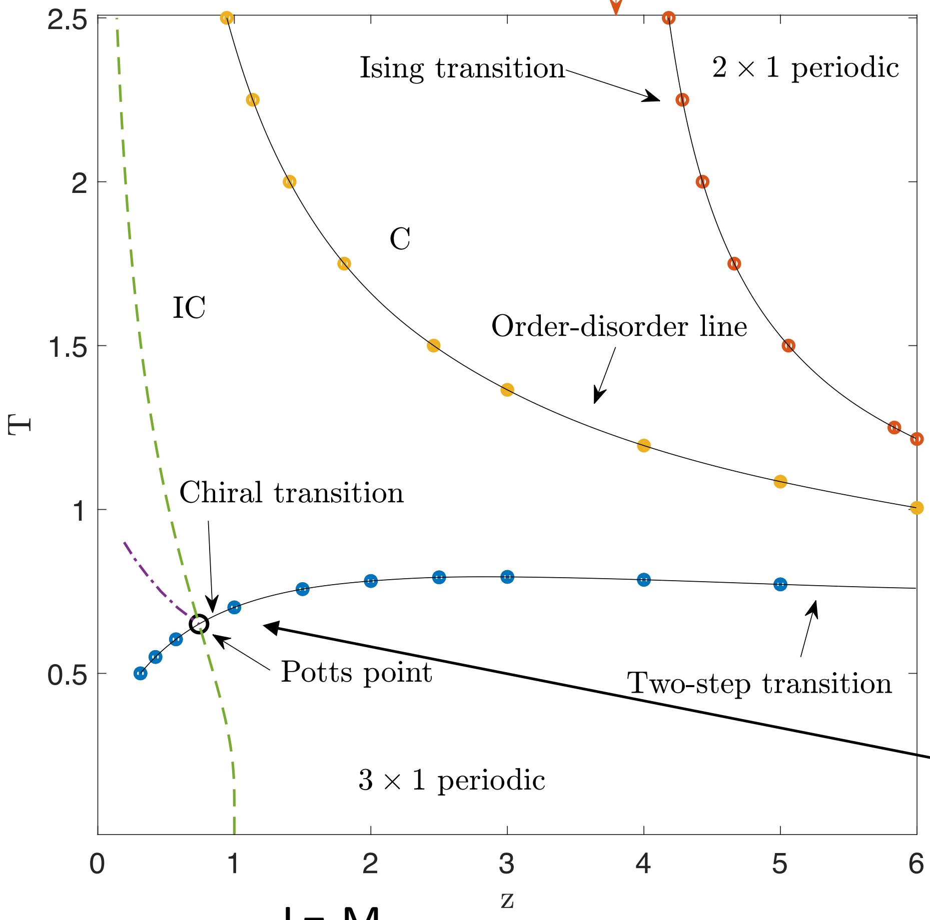
$M > 0, L < 0$

- Classical equivalent to the hard-boson model

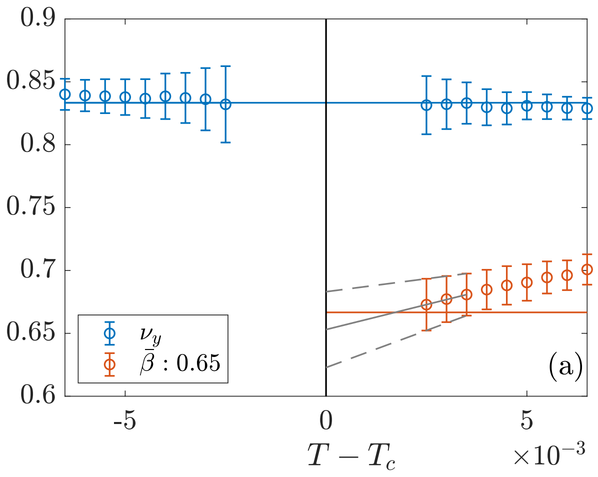
$$\mathcal{H} = \sum_j \left[-w \left(d_j + d_j^\dagger \right) + U n_j + V n_j n_{j+2} \right]$$

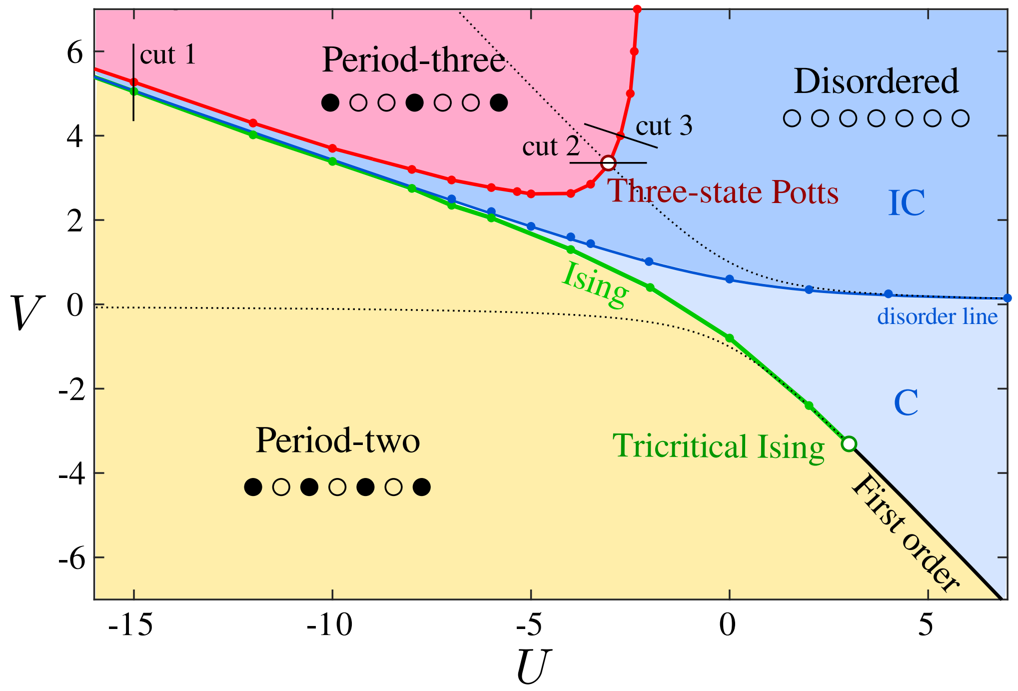
$$n_j n_{j+1} = 0. \quad n_j \equiv d_j^\dagger d_j.$$

P. Fendley, K. Sengupta, and S. Sachdev, PRB 2004



L=-M





N. Chepiga, F. Mila, PRL 2019

Conclusion

- Evidence of a chiral transition for p=3 of universality class:

$$(\alpha, \nu_x, \nu_y, \bar{\beta}) = (1/3, 2/3, 1, 2/3)$$

- Evidence of the existence of a chiral transition for p=4.

Thank you