A classification of 1D mixed states

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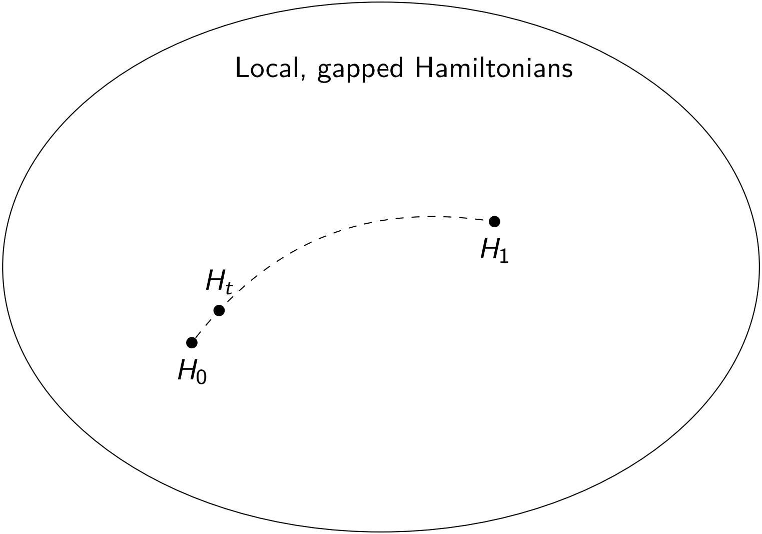
David Pérez-García

March 3, 2022

This work was funded by the ERC (grant agreement No. 648913).

- Phases in open systems
- Matrix product density operators (MPDO)
- Renormalization fixed point MPDO
- Results:
  - Examples for RFP MPDO
  - Some of the examples are in the trivial phase

Phases in closed systems



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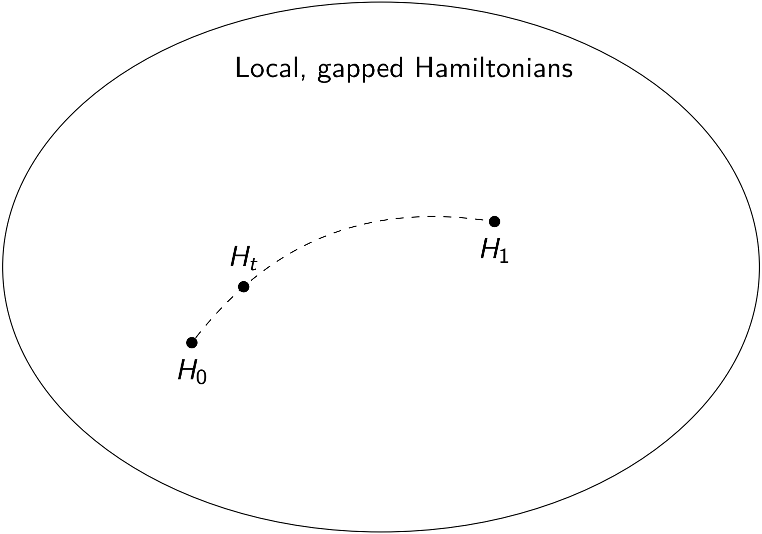

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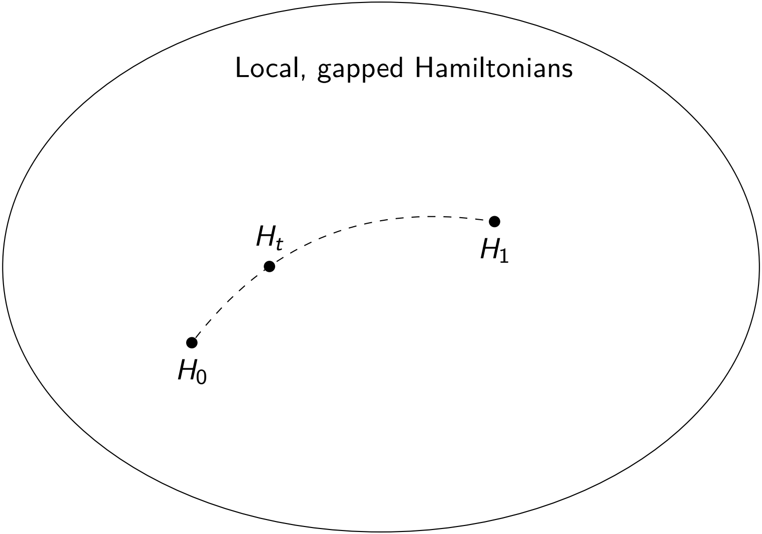

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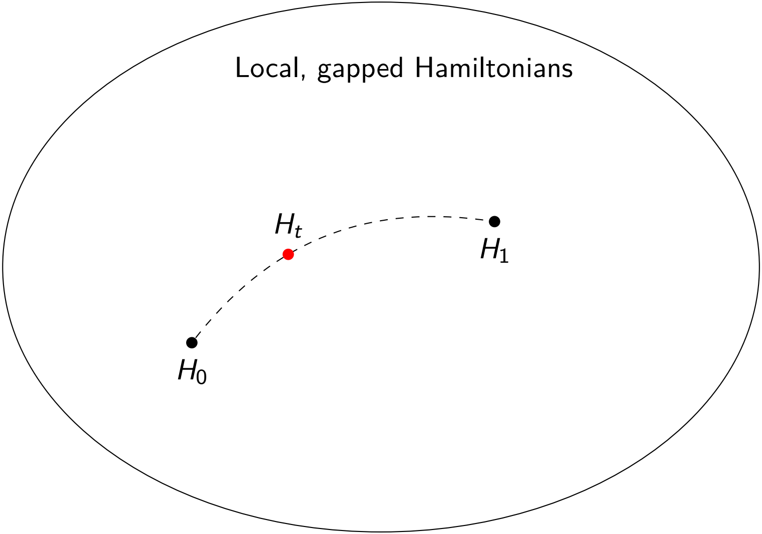

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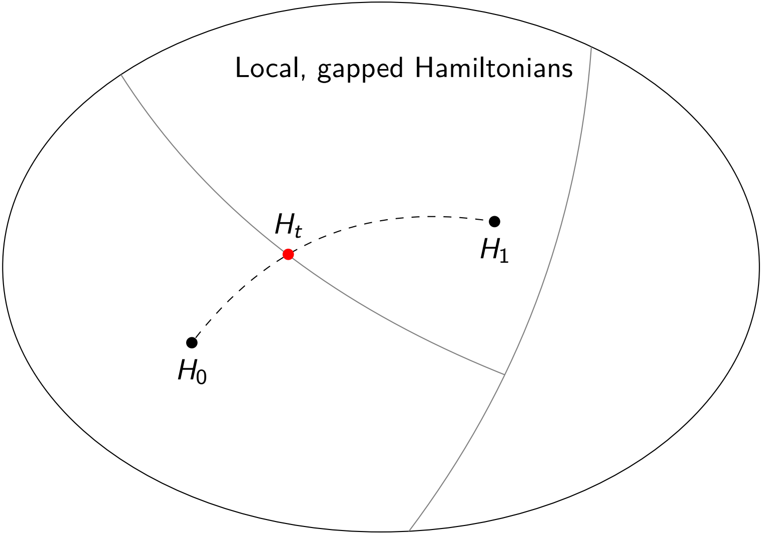
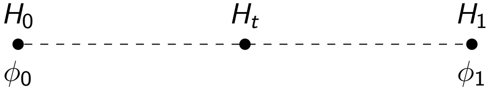


Phases: operational definition



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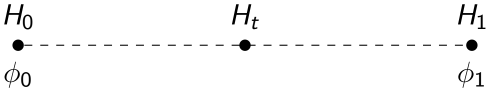
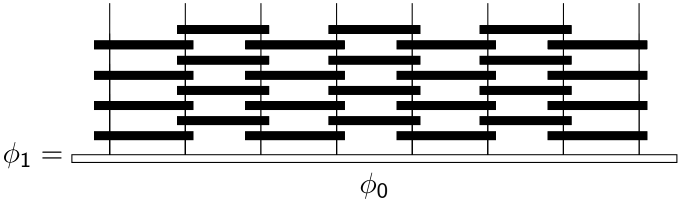


Shallow circuit of local unitaries:



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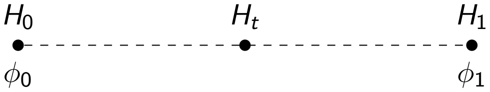

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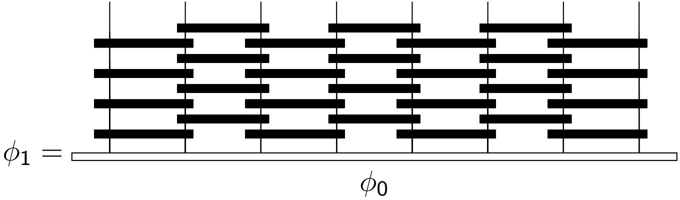
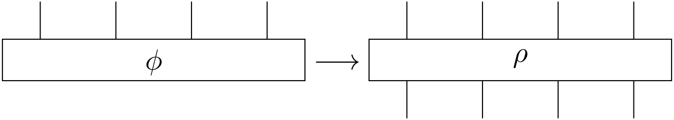


- States, not Hamiltonians
- Same phase = equally hard to prepare

Phases in open systems



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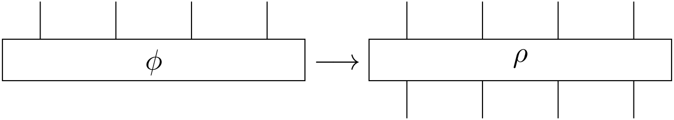

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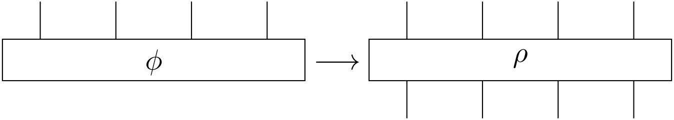

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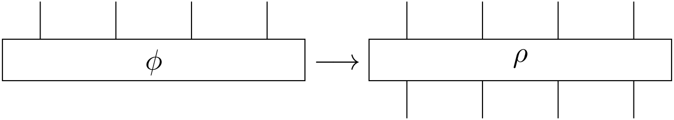

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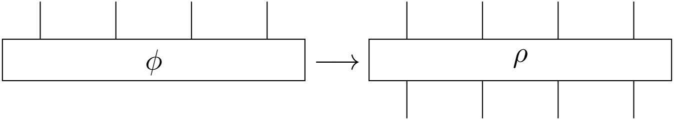

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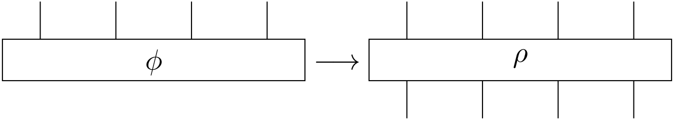

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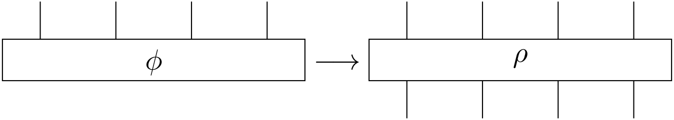

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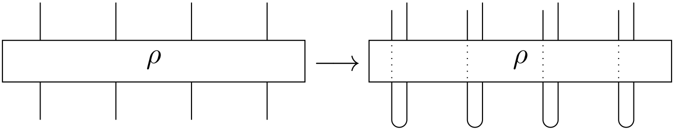
- Same phase: $\rho_1 \leftrightarrow \rho_2$

Phases in open systems: circuit of channels



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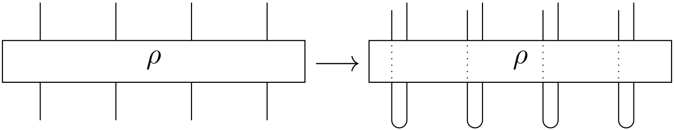

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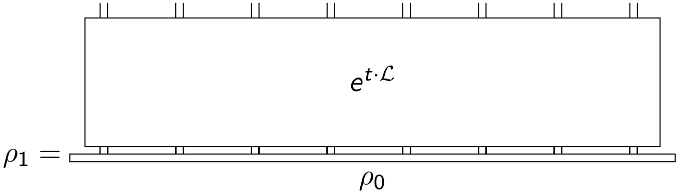

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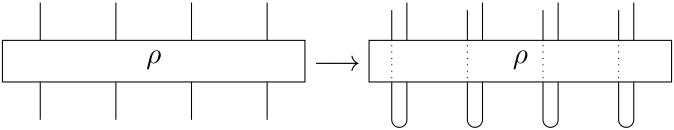
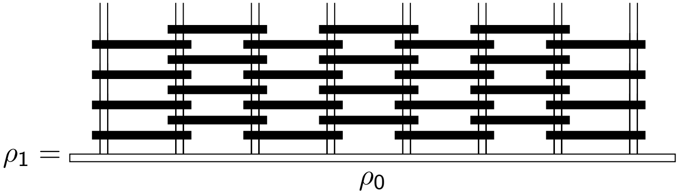


$\rho_0 \to \rho_1$ if with a shallow circuit of local channels



Circuit of channels: consequences

- $\rho \to (\mathrm{Id}/d)^{\otimes N}$ for all ρ:

$$\mathrm{Id}^{\otimes n}/d^n = T^{\otimes n}(\rho) \quad \text{with} \quad T(X) = \mathrm{Tr}\{X\} \cdot \mathrm{Id}/d.$$

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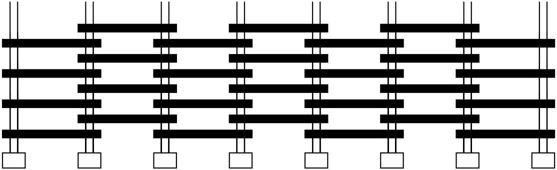
$\rho =$ 

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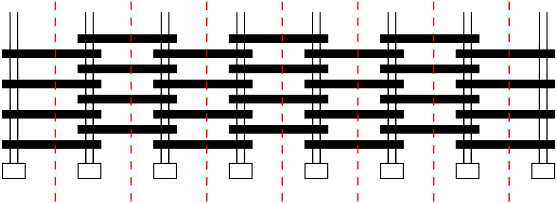
$\rho =$ 

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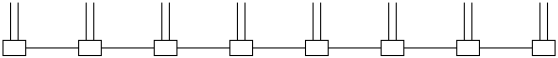
$\rho =$ 

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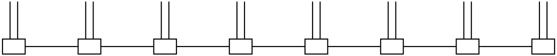
$\rho =$ 

- Trivial phase: MPDO

So far...

- Phase: shallow circuit of local channels

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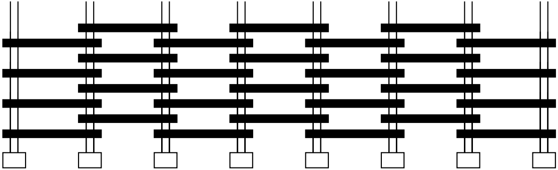

- What are the MPDOs in the trivial phase?

Renormalization fixed points

Pure systems: RFP captures relevant physics.

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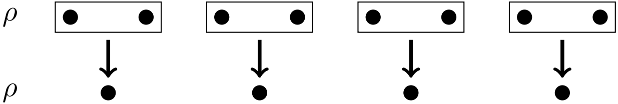

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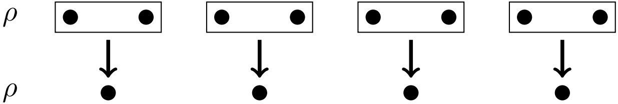
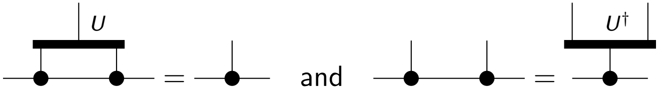


An MPS is a RFP if $\exists$ isometry U:



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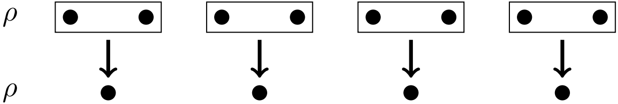

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Cirac, Perez-Garcia, Schuch, Verstraete, *Ann. Phys.* **378**:

$$\text{RFP} = \text{ZCL} = \text{GS of commuting PH}$$

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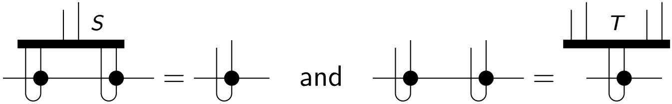

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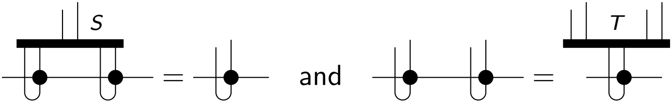

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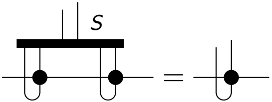

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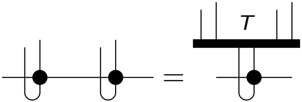

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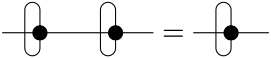

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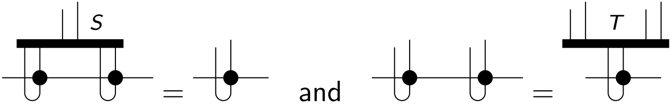
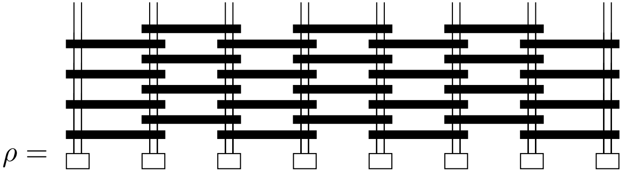
- RFP MPDO: ∃ CPTP maps T, S:



- What are the RFP MPDOs in the trivial phase?
- Prepare w/ shallow circuit of local channels



Renormalization fixed point MPDO: Examples

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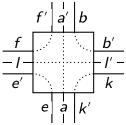
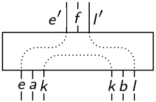


$$= [F^{e}_{abl}]^{f}_{k} \cdot \delta_{aa'}\delta_{bb'}\delta_{ll'}\delta_{ee'}\delta_{ff'}\delta_{kk'}$$



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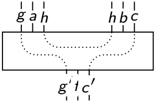

$$= \left[\left(F^{g}_{abc}\right)^{-1}\right]^{h}_{f} \cdot \delta_{ff'}\delta_{gg'}\delta_{ee'}$$

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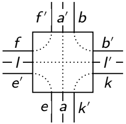

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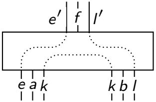

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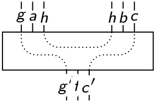

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- Bimodule categories: Lootens et al., *Scipost Physics*, **10**, 3

Renormalization fixed point MPDO

Pentagon equation:

$$[F^{e}_{ahd}]^{g}_{k}\left[F^{k}_{bcd}\right]^{h}_{l}=\sum_{f}[F^{e}_{fcd}]^{g}_{l}\,[F^{e}_{abl}]^{f}_{k}\left[\left(F^{g}_{abc}\right)^{-1}\right]^{h}_{f}.$$

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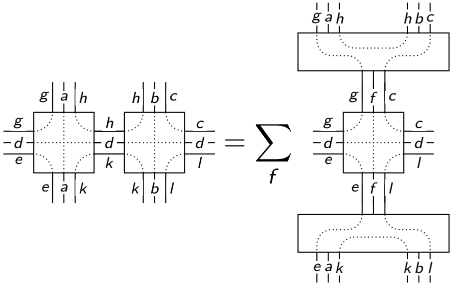
With the MPO tensors:



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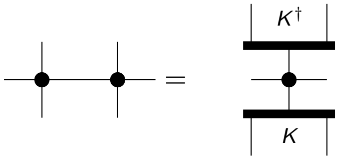

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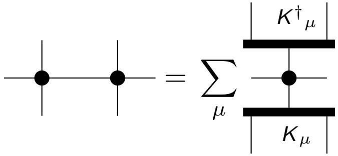

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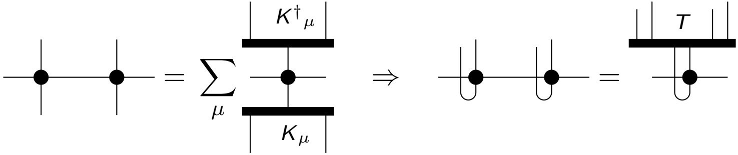

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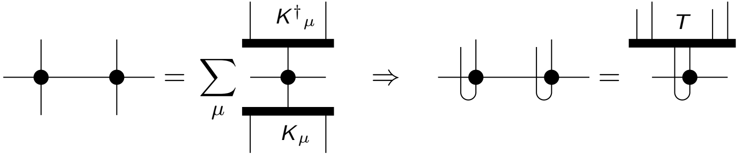

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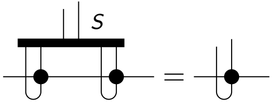

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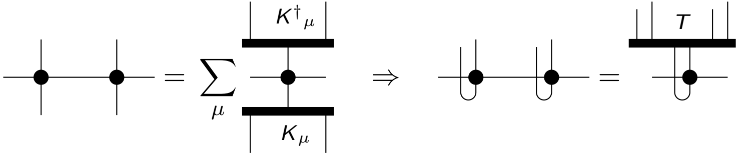

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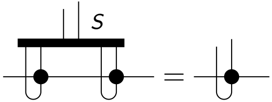
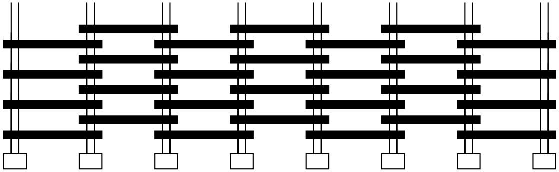


- S and T are CP, but not TP
- The MPO is not ZCL
- After normalization, MPO is ZCL, and S,T are CPTP

So far...

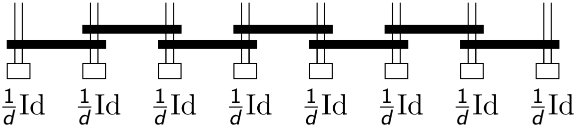
- RFP MPDO from string-nets
- Which ones are in the trivial phase?
- Which ones can be prepared w/ shallow circuit of local channels?

$\rho =$



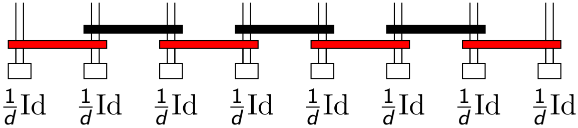
Creating RFP MPDOs with local channels

- Depth-2 channel



Creating RFP MPDOs with local channels

- Depth-2 channel
  - Throw away and prepare: $X \mapsto \mathrm{Tr} X \cdot \rho_2$



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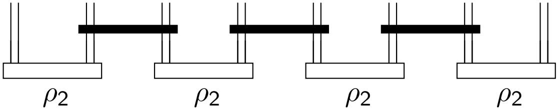
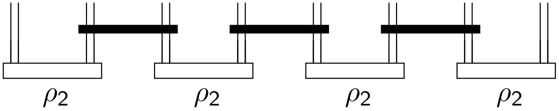


Creating RFP MPDOs with local channels

- Depth-2 channel
  - Throw away and prepare: $X \mapsto \mathrm{Tr} X \cdot \rho_2$
  - Glue



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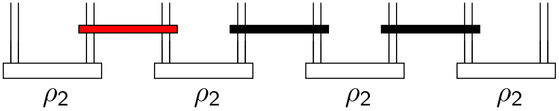

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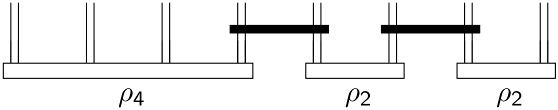

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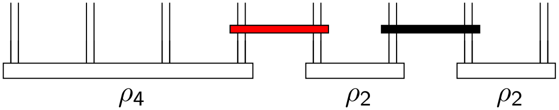

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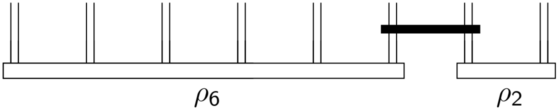

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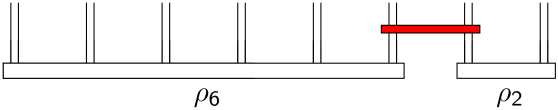

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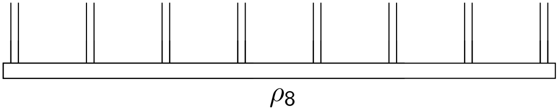

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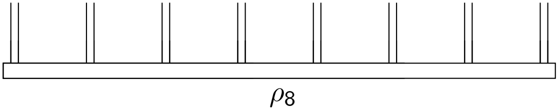
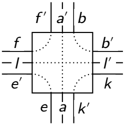


- Gluing only works in special cases:
  - 1D vacuum sector
  - Only vacuum sector

On the structure of the MPO tensor



$$= [F^{e}_{abl}]^{f}_{k} \cdot \delta_{aa'}\delta_{bb'}\delta_{ll'}\delta_{ee'}\delta_{ff'}\delta_{kk'}$$

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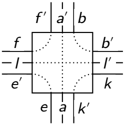

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$$a \times b = \sum_{c} N^{c}_{ab} c$$

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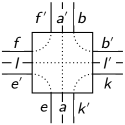

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$$e \times a = a \times e = a$$

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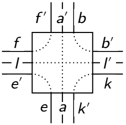

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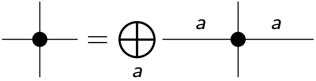

$$\Rightarrow \quad O_a$$

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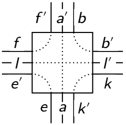

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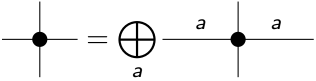


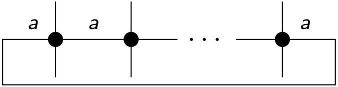
$$\Rightarrow \quad O_a$$

- 1D Vacuum: $O_e = \mathrm{Id}^{\otimes n}$

The gluing map (1D vacuum)

- Special MPO:

$$P = \sum_a \frac{d_a}{d^2} \cdot$$



The gluing map (1D vacuum)

- Special self-adjoint projector MPO:

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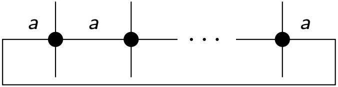

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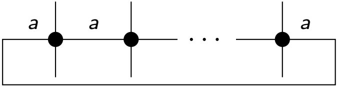
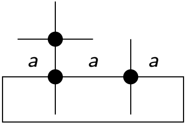
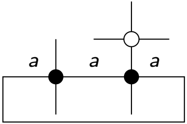
$$P = \sum_a \frac{d_a}{d^2} \cdot$$ 

- Pulling-through equation:

$$P = \sum_a \frac{d_a}{d^2} \cdot$$ 

$$= \sum_a \frac{d_a}{d^2} \cdot$$ 

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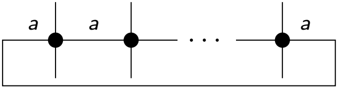

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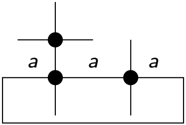

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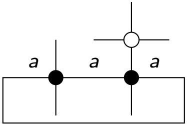
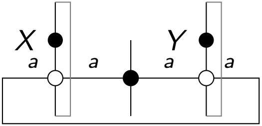


Bultinck et al., *Ann. Phys.* **378** 183-233

The gluing map (1D vacuum)

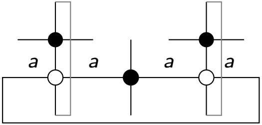
The gluing map:

$$T(X \otimes Y) \approx \sum_a \frac{d_a}{d^2} \cdot$$

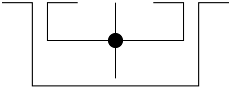


The way it works:

$$\sum_a \frac{d_a}{d^2} \cdot$$



$$=$$



Conclusion

- Phase classification for mixed state
- Renormalization fixed point MPDO
- Examples from string-net
- A subset of them is in the trivial phase
- Conjecture: MPDO from Fibonacci is not in the trivial phase