

A classification of 1D mixed states

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David Pérez-García

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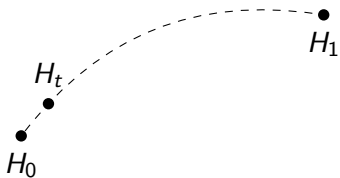
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This work was funded by the ERC (grant agreement No. 648913).

- ▶ Phases in open systems
- ▶ Matrix product density operators (MPDO)
- ▶ Renormalization fixed point MPDO
- ▶ Results:
 - ▶ Examples for RFP MPDO
 - ▶ Some of the examples are in the trivial phase

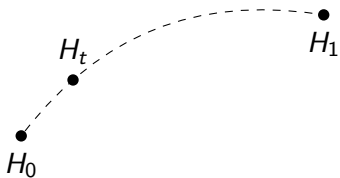
Phases in closed systems

Local, gapped Hamiltonians



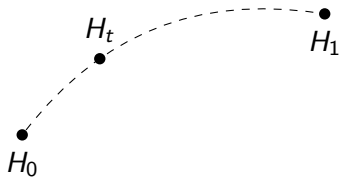
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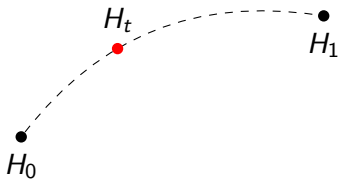
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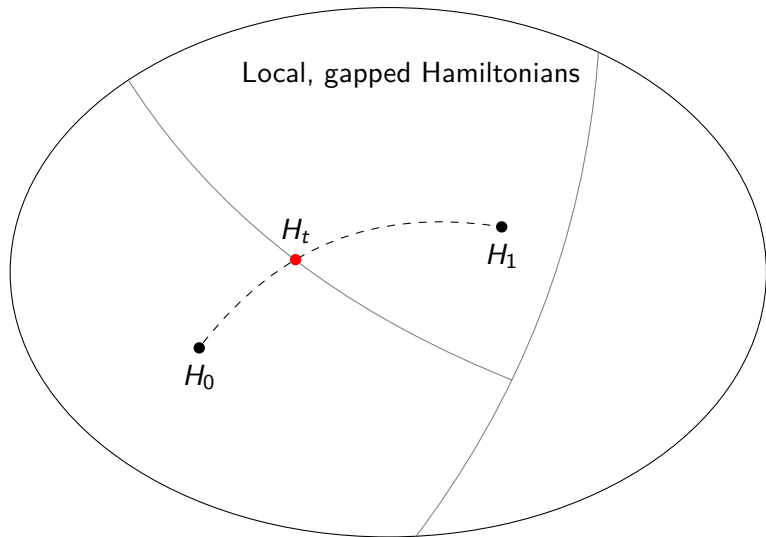


Phases in closed systems

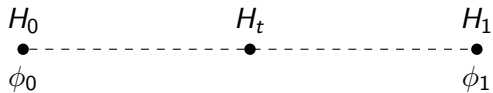
Local, gapped Hamiltonians



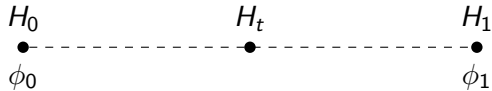
Phases in closed systems



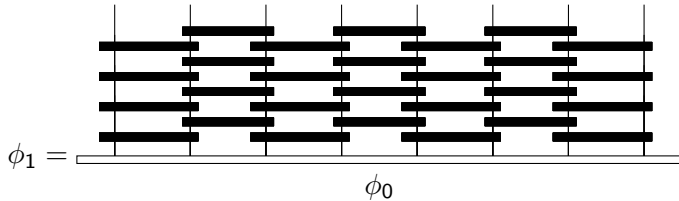
Phases: operational definition



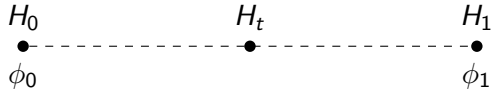
Phases: operational definition



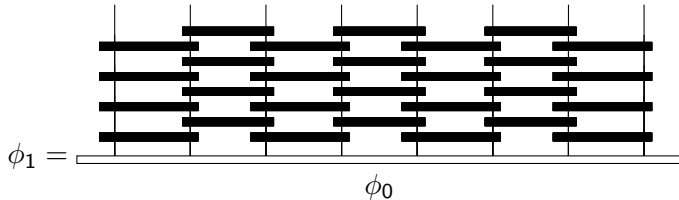
Shallow circuit of local unitaries:



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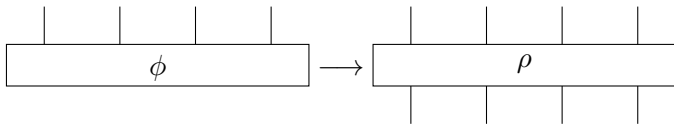


Shallow circuit of local unitaries:

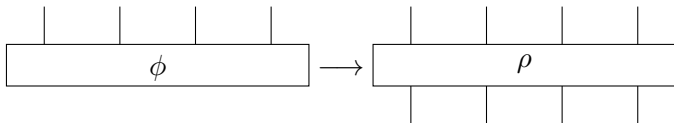


- ▶ States, not Hamiltonians
- ▶ Same phase = equally hard to prepare

Phases in open systems

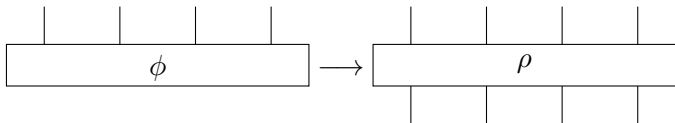


Phases in open systems



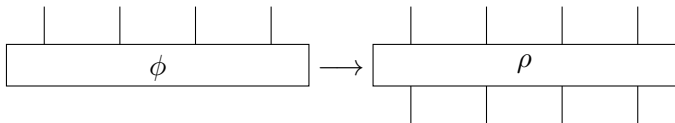
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Coser, Perez-Garcia, Quantum. 3, 174 (2019)

Phases in open systems



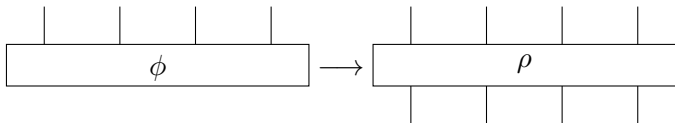
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- ▶ Operations: local Lindbladian for short time

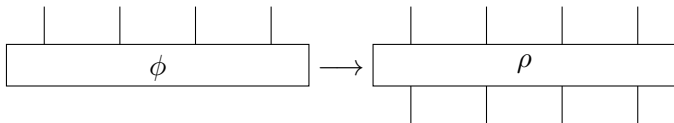
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$$\rho_1 \rightarrow \rho_2 \quad \text{if} \quad \rho_2 \approx e^{t \cdot \mathcal{L}_1}(\rho_1)$$

Phases in open systems

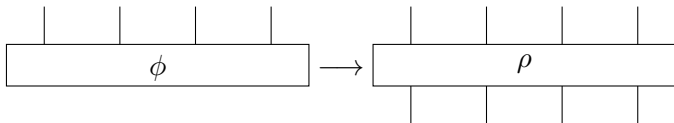


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Phases in open systems



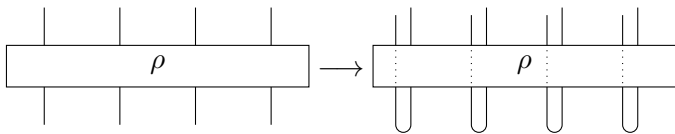
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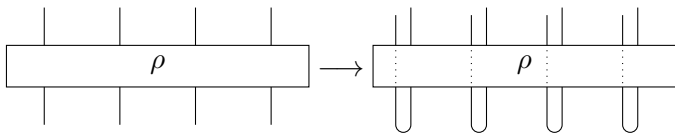
$$\rho_1 \leftarrow \rho_2 \quad \text{if} \quad \rho_1 \approx e^{t \cdot \mathcal{L}_2}(\rho_2)$$

- ▶ Same phase: $\rho_1 \leftrightarrow \rho_2$

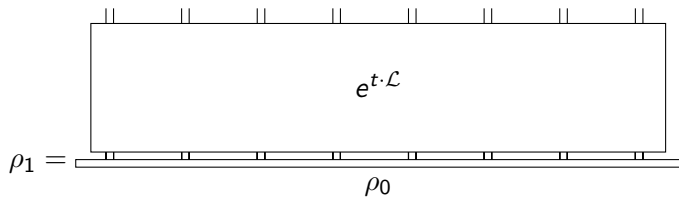
Phases in open systems: circuit of channels



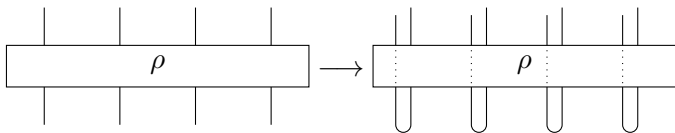
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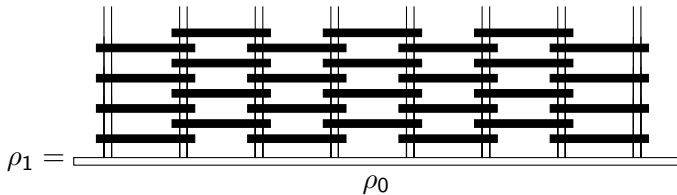
$\rho_0 \rightarrow \rho_1$ if with a local Lindbladian for short time



Phases in open systems: circuit of channels



$\rho_0 \rightarrow \rho_1$ if with a shallow circuit of local channels



Circuit of channels: consequences

► $\rho \rightarrow (\text{Id}/d)^{\otimes N}$ for all ρ :

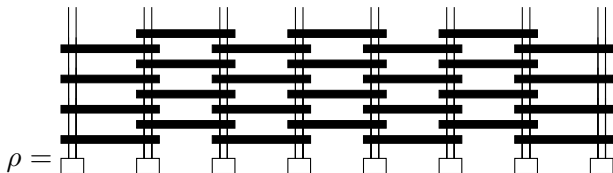
$$\text{Id}^{\otimes n}/d^n = T^{\otimes n}(\rho) \quad \text{with} \quad T(X) = \text{Tr}\{X\} \cdot \text{Id}/d.$$

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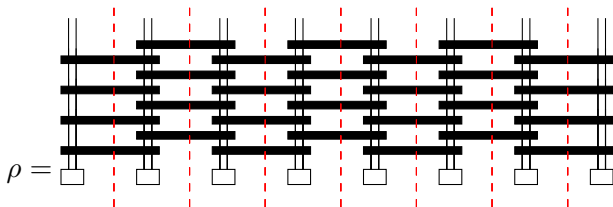


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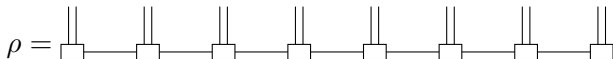


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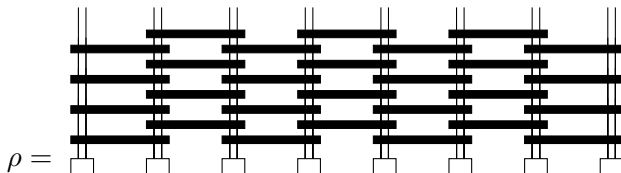
- ▶ If $(\text{Id}/d)^{\otimes N} \rightarrow \rho$, then ρ is MPDO:

$$\rho = \begin{array}{cccccccc} & \parallel & & \parallel & & \parallel & & \parallel & & \parallel & & \parallel & & \parallel & & \parallel & & \parallel \\ & | & & | & & | & & | & & | & & | & & | & & | & & | \\ \square & \text{---} & \square & \text{---} & \square & \text{---} & \square & \text{---} & \square & \text{---} & \square & \text{---} & \square & \text{---} & \square & \text{---} & \square & \text{---} & \square \end{array}$$

- ▶ Trivial phase: MPDO

So far...

- ▶ Phase: shallow circuit of local channels



- ▶ What are the MPDOs in the trivial phase?

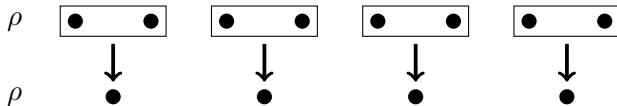
Renormalization fixed points

Pure systems: RFP captures relevant physics.

ρ ● ● ● ● ● ● ● ●

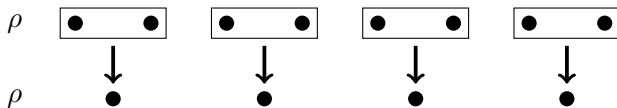
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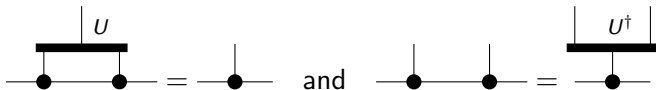


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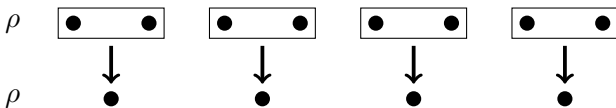


An MPS is a RFP if \exists isometry U :

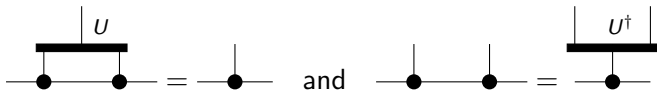


Renormalization fixed points

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An MPS is a RFP if \exists isometry U :



Cirac, Perez-Garcia, Schuch, Verstraete, *Ann. Phys.* **378**:

RFP = ZCL = GS of commuting PH

Renormalization fixed point MPDO

Cirac, Perez-Garcia, Schuch, Verstraete, *Ann. Phys.* **378**:

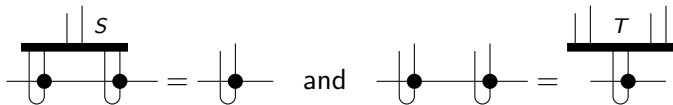
ZCL \neq No length scale \approx RFP

Renormalization fixed point MPDO

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ZCL \neq No length scale \approx RFP

Def.: an MPDO is a RFP if \exists CPTP maps T, S :

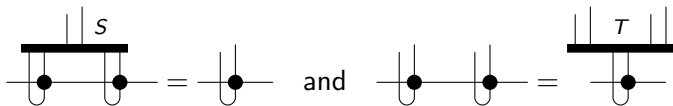


Renormalization fixed point MPDO

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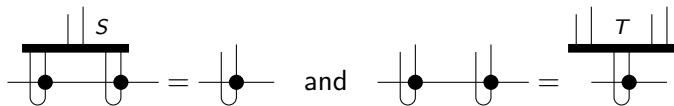
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Renormalization fixed point MPDO

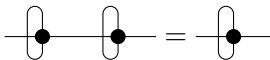
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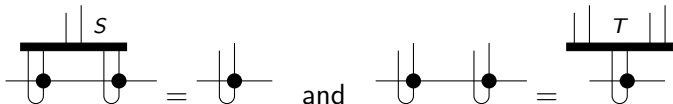


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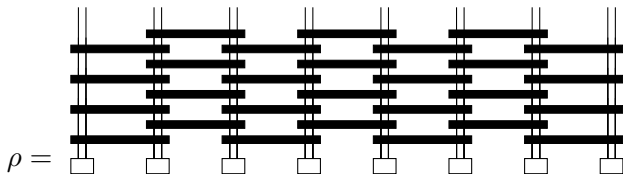


So far...

- ▶ RFP MPDO: \exists CPTP maps T, S :



- ▶ What are the RFP MPDOs in the trivial phase?
- ▶ Prepare w/ shallow circuit of local channels



Renormalization fixed point MPDO: Examples

- ▶ Boundaries of topological order

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Renormalization fixed point MPDO: Examples

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- ▶ MPO tensors: Sahinoglu, et. al., **1409.2150**

$$\begin{array}{c}
 \begin{array}{c}
 f' \quad a' \quad b \\
 \begin{array}{c}
 \text{---} f \text{---} \\
 \text{---} l \text{---} \\
 \text{---} e' \text{---} \\
 e \quad a \quad k' \\
 \text{---} e' \text{---} \\
 \text{---} l \text{---} \\
 \text{---} f \text{---} \\
 b' \\
 \text{---} k \text{---}
 \end{array}
 \end{array}
 \end{array}
 = [F_{abl}^e]_k^f \cdot \delta_{aa'} \delta_{bb'} \delta_{ll'} \delta_{ee'} \delta_{ff'} \delta_{kk'}
 \end{array}$$

$$\begin{array}{c}
 \begin{array}{c}
 e' \quad f \quad l' \\
 \text{---} e' \text{---} \\
 \text{---} e \text{---} \\
 \text{---} a \text{---} \\
 \text{---} k \text{---} \\
 \text{---} k \text{---} \\
 \text{---} b \text{---} \\
 \text{---} l \text{---} \\
 \text{---} l' \text{---}
 \end{array}
 \end{array}
 = [F_{abl}^e]_k^f \cdot \delta_{ff'} \delta_{gg'} \delta_{ee'}$$

$$\begin{array}{c}
 \begin{array}{c}
 g \quad a \quad h \\
 \text{---} g \text{---} \\
 \text{---} g' \text{---} \\
 \text{---} t \text{---} \\
 \text{---} c' \text{---} \\
 \text{---} h \text{---} \\
 \text{---} h \text{---} \\
 \text{---} b \text{---} \\
 \text{---} c \text{---}
 \end{array}
 \end{array}
 = [(F_{abc}^g)^{-1}]_f^h \cdot \delta_{ff'} \delta_{gg'} \delta_{ee'}$$

Renormalization fixed point MPDO: Examples

- ▶ Boundaries of topological order
- ▶ String-net: Unitary fusion category
- ▶ MPO tensors: Sahinoglu, et. al., **1409.2150**

$$\begin{array}{c}
 \begin{array}{c}
 f' \quad | \quad a' \quad | \quad b \\
 \hline
 f \quad | \quad \quad | \quad b' \\
 \hline
 e' \quad | \quad \quad | \quad k \\
 \hline
 e \quad | \quad a \quad | \quad k'
 \end{array}
 \end{array}
 = [F_{abl}^e]_k^f \cdot \delta_{aa'} \delta_{bb'} \delta_{ll'} \delta_{ee'} \delta_{ff'} \delta_{kk'}$$

$$\begin{array}{c}
 e' \quad | \quad f \quad | \quad l' \\
 \hline
 \text{---} \quad \text{---} \quad \text{---} \\
 \hline
 e \quad a \quad k \quad \quad \quad k \quad b \quad l
 \end{array}
 = [F_{abl}^e]_k^f \cdot \delta_{ff'} \delta_{gg'} \delta_{ee'}$$

$$\begin{array}{c}
 g \quad a \quad h \quad \quad \quad h \quad b \quad c \\
 \hline
 \text{---} \quad \text{---} \quad \text{---} \\
 \hline
 g' \quad t \quad c'
 \end{array}
 = [(F_{abc}^g)^{-1}]_f^h \cdot \delta_{ff'} \delta_{gg'} \delta_{ee'}$$

- ▶ Bimodule categories: Lootens et al., *SciPost Physics*, **10**, 3

Renormalization fixed point MPDO

Pentagon equation:

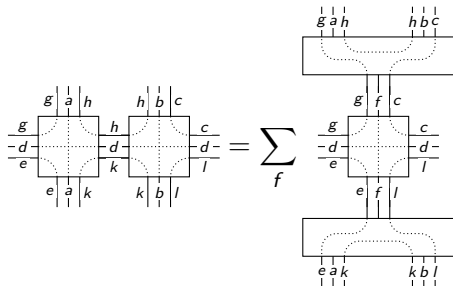
$$[F_{ahd}^e]_k^g [F_{bcd}^k]_l^h = \sum_f [F_{fcd}^e]_l^g [F_{abl}^e]_k^f [(F_{abc}^g)^{-1}]_f^h.$$

Renormalization fixed point MPDO

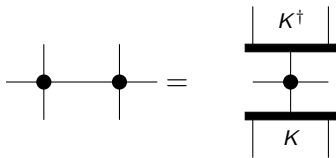
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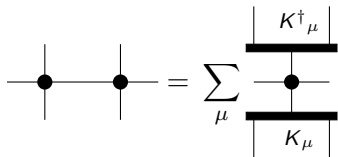
With the MPO tensors:



Renormalization fixed point MPDO



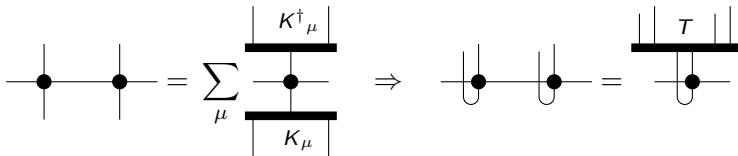
Renormalization fixed point MPDO



The diagram shows an equality between two tensor network expressions. On the left, two black dots are connected by a horizontal line, with a vertical line extending from each dot. On the right, a summation over μ is shown. The summand consists of a central black dot with a horizontal line passing through it. This dot is connected to two thick horizontal bars, one above and one below. The top bar is labeled K^\dagger_μ and the bottom bar is labeled K_μ . Vertical lines extend from the top and bottom bars.

$$\text{---} \bullet \text{---} \text{---} \bullet \text{---} = \sum_{\mu} \begin{array}{c} | \\ \text{---} K^\dagger_\mu \text{---} \\ | \\ \bullet \\ \text{---} \\ | \\ \text{---} K_\mu \text{---} \\ | \end{array}$$

Renormalization fixed point MPDO



The diagram illustrates a renormalization fixed point equation for Matrix Product Density Operators (MPDO). It consists of two parts connected by an implication arrow (\Rightarrow).

Left side: A diagram showing two black dots on a horizontal line, each with a vertical line passing through it. This is equal to a summation over μ of a diagram where a single black dot is on a horizontal line, with two thick horizontal bars above and below it. The top bar is labeled K^\dagger_μ and the bottom bar is labeled K_μ .

Right side: A diagram showing two black dots on a horizontal line, each with a vertical line passing through it and a U-shaped line below it. This is equal to a diagram where a single black dot is on a horizontal line, with a U-shaped line below it and a thick horizontal bar above it labeled \mathcal{T} .

Renormalization fixed point MPDO

The diagram shows an equality between two expressions. On the left, two vertical lines with dots are connected by a horizontal line. This is equal to a summation over μ of a diagram where a horizontal line with a dot is sandwiched between two thick horizontal bars labeled K_μ^\dagger (top) and K_μ (bottom). This is then shown to be equivalent to two vertical lines with dots, each having a loop below it. Finally, this is equal to a diagram with a thick horizontal bar labeled \mathcal{T} above a vertical line with a dot and a loop below it.

Similarly:

The diagram shows an equality between two expressions. On the left, a thick horizontal bar labeled S is above two vertical lines with dots, each having a loop below it. This is equal to a single vertical line with a dot and a loop below it.

Renormalization fixed point MPDO

The diagram shows an equality between two expressions. On the left, two vertical lines with black dots on a horizontal line are equal to a sum over μ of a diagram with two thick horizontal bars labeled K^\dagger_μ and K_μ above and below a central dot. An arrow points to the right, where two vertical lines with loops are equal to a diagram with a thick horizontal bar labeled T above a central dot.

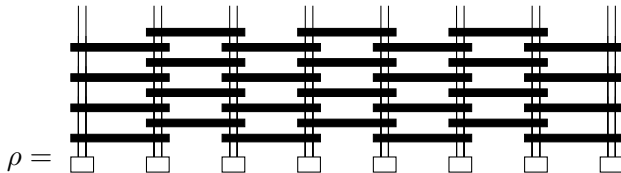
Similarly:

The diagram shows a diagram with two thick horizontal bars labeled S above two dots, with loops below each dot, equal to a single diagram with one dot and a loop below it.

- ▶ S and T are CP, but not TP
- ▶ The MPO is not ZCL
- ▶ After normalization, MPO is ZCL, and S, T are CPTP

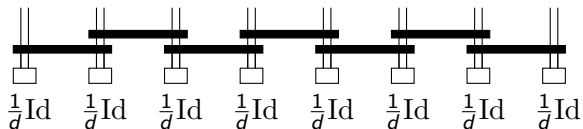
So far...

- ▶ RFP MPDO from string-nets
- ▶ Which ones are in the trivial phase?
- ▶ Which ones can be prepared w/ shallow circuit of local channels?



Creating RFP MPDOs with local channels

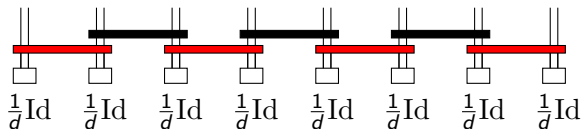
► Depth-2 channel



Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

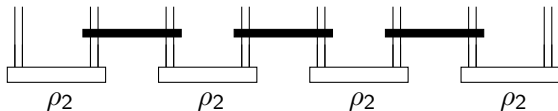
- ▶ Throw away and prepare: $X \mapsto \text{Tr}X \cdot \rho_2$



Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

- ▶ Throw away and prepare: $X \mapsto \text{Tr}X \cdot \rho_2$

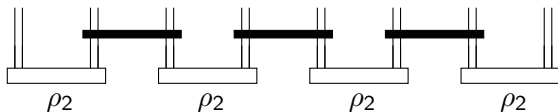


Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

- ▶ Throw away and prepare: $X \mapsto \text{Tr}X \cdot \rho_2$

- ▶ Glue

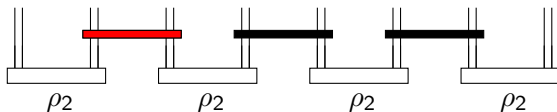


Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

- ▶ Throw away and prepare: $X \mapsto \text{Tr}X \cdot \rho_2$

- ▶ Glue

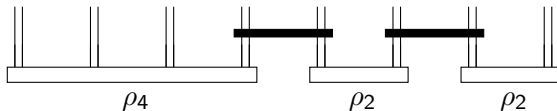


Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

- ▶ Throw away and prepare: $X \mapsto \text{Tr}X \cdot \rho_2$

- ▶ Glue

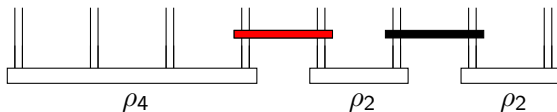


Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

- ▶ Throw away and prepare: $X \mapsto \text{Tr}X \cdot \rho_2$

- ▶ Glue

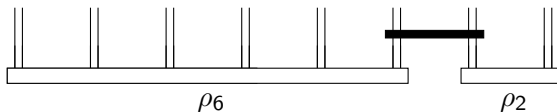


Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

- ▶ Throw away and prepare: $X \mapsto \text{Tr}X \cdot \rho_2$

- ▶ Glue

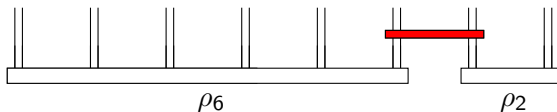


Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

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- ▶ Glue

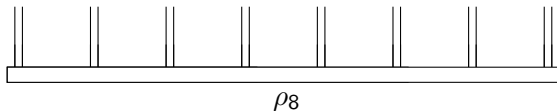


Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

- ▶ Throw away and prepare: $X \mapsto \text{Tr}X \cdot \rho_2$

- ▶ Glue

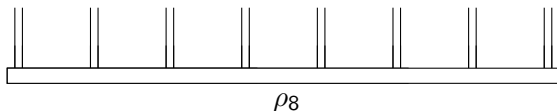


Creating RFP MPDOs with local channels

- ▶ Depth-2 channel

- ▶ Throw away and prepare: $X \mapsto \text{Tr}X \cdot \rho_2$

- ▶ Glue



- ▶ Gluing only works in special cases:

- ▶ 1D vacuum sector

- ▶ Only vacuum sector

On the structure of the MPO tensor

The diagram shows a square MPO tensor with four legs. The top leg is labeled f , the bottom leg is labeled e , the left leg is labeled f' , and the right leg is labeled b' . The tensor is divided into four quadrants by dashed lines. The top-left quadrant is labeled f' , the top-right quadrant is labeled a , the bottom-left quadrant is labeled b , and the bottom-right quadrant is labeled k' . The tensor is equated to the expression $[F_{abl}^e]^f \cdot \delta_{aa'} \delta_{bb'} \delta_{ll'} \delta_{ee'} \delta_{ff'} \delta_{kk'}$.

$$\begin{array}{c}
 f' \quad | \quad a \quad | \quad b \\
 \hline
 f \quad | \quad \square \quad | \quad b' \\
 \hline
 e' \quad | \quad \square \quad | \quad k \\
 \hline
 e \quad | \quad a \quad | \quad k'
 \end{array}
 = [F_{abl}^e]^f \cdot \delta_{aa'} \delta_{bb'} \delta_{ll'} \delta_{ee'} \delta_{ff'} \delta_{kk'}$$

On the structure of the MPO tensor

The diagram shows a square MPO tensor with four legs. The top leg has indices f and a on the left and b on the right. The bottom leg has indices e and a on the left and k' on the right. The left leg has indices f and e' on the top and l on the bottom. The right leg has indices b' and l' on the top and k on the bottom. Dashed lines represent internal connections between a and a , b and b' , l and l' , and e and e' . The tensor is equated to the expression $[F_{abl}^e]_k^f \cdot \delta_{aa'} \delta_{bb'} \delta_{ll'} \delta_{ee'} \delta_{ff'} \delta_{kk'}$.

$$= [F_{abl}^e]_k^f \cdot \delta_{aa'} \delta_{bb'} \delta_{ll'} \delta_{ee'} \delta_{ff'} \delta_{kk'}$$

- ▶ $\{a, b, c, \dots\}$: objects in unitary fusion category. Fusion rules:

$$a \times b = \sum_c N_{ab}^c c$$

On the structure of the MPO tensor

The diagram shows a square MPO tensor with four legs. The top leg is labeled f on the left and b on the right. The bottom leg is labeled e on the left and k' on the right. The left vertical leg is labeled f' at the top and e' at the bottom. The right vertical leg is labeled b' at the top and k at the bottom. A central vertical line is labeled a' at the top and a at the bottom. Dashed lines connect the corners of the square to the central vertical line. To the right of the diagram is the equation:
$$= [F_{abl}^e]^f \cdot \delta_{aa'} \delta_{bb'} \delta_{ll'} \delta_{ee'} \delta_{ff'} \delta_{kk'}$$

- ▶ $\{a, b, c, \dots\}$: objects in unitary fusion category. Fusion rules:

$$a \times b = \sum_c N_{ab}^c c$$

- ▶ Vacuum:

$$e \times a = a \times e = a$$

On the structure of the MPO tensor

$$= [F_{abl}^e]_k^f \cdot \delta_{aa'} \delta_{bb'} \delta_{ll'} \delta_{ee'} \delta_{ff'} \delta_{kk'}$$

- ▶ $\{a, b, c, \dots\}$: objects in unitary fusion category. Fusion rules:

$$a \times b = \sum_c N_{ab}^c c$$

- ▶ Vacuum:

$$e \times a = a \times e = a$$

- ▶ MPO is block-diagonal, each block is injective

$$\text{---} \bullet \text{---} = \bigoplus_a \text{---} \bullet \text{---} \Rightarrow O_a$$

On the structure of the MPO tensor

$$\begin{array}{c}
 \begin{array}{|c|c|c|}
 \hline
 f' & a' & b \\
 \hline
 f & \text{---} & b' \\
 \hline
 l & \text{---} & l' \\
 \hline
 e' & \text{---} & k \\
 \hline
 e & a & k' \\
 \hline
 \end{array}
 & = & [F_{abl}^e]_k^f \cdot \delta_{aa'} \delta_{bb'} \delta_{ll'} \delta_{ee'} \delta_{ff'} \delta_{kk'}
 \end{array}$$

- ▶ $\{a, b, c, \dots\}$: objects in unitary fusion category. Fusion rules:

$$a \times b = \sum_c N_{ab}^c c$$

- ▶ Vacuum:

$$e \times a = a \times e = a$$

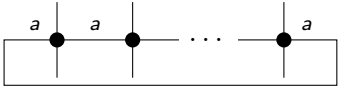
- ▶ MPO is block-diagonal, each block is injective

$$\begin{array}{c} | \\ \bullet \\ | \end{array} = \bigoplus_a \begin{array}{c} a \\ \oplus \\ a \end{array} \begin{array}{c} a \\ \text{---} \\ a \end{array} \bullet \begin{array}{c} a \\ \text{---} \\ a \end{array} \Rightarrow O_a$$

- ▶ 1D Vacuum: $O_e = \text{Id}^{\otimes n}$

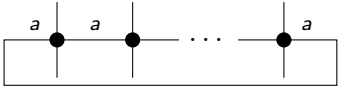
The gluing map (1D vacuum)

► Special MPO:

$$P = \sum_a \frac{d_a}{d^2} \cdot \text{[Diagram]}$$


The gluing map (1D vacuum)

- ▶ Special self-adjoint projector MPO:

$$P = \sum_a \frac{d_a}{d^2} \cdot \text{[Diagram]}$$


The gluing map (1D vacuum)

- ▶ Special self-adjoint projector MPO:

$$P = \sum_a \frac{d_a}{d^2} \cdot \boxed{\begin{array}{c} a \quad a \quad \dots \quad a \\ | \quad | \quad \dots \quad | \\ \bullet \quad \bullet \quad \dots \quad \bullet \end{array}}$$

- ▶ Pulling-through equation:

$$P = \sum_a \frac{d_a}{d^2} \cdot \boxed{\begin{array}{c} | \\ \bullet \\ a \quad a \\ | \quad | \\ \bullet \quad \bullet \end{array}} = \sum_a \frac{d_a}{d^2} \cdot \boxed{\begin{array}{c} a \quad a \\ | \quad | \\ \bullet \quad \bullet \\ | \\ \circ \end{array}}$$

The gluing map (1D vacuum)

- ▶ Special self-adjoint projector MPO:

$$P = \sum_a \frac{d_a}{d^2} \cdot \boxed{\begin{array}{c} a \quad a \quad \dots \quad a \\ | \quad | \quad \dots \quad | \\ \bullet \quad \bullet \quad \dots \quad \bullet \end{array}}$$

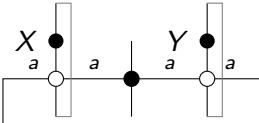
- ▶ Pulling-through equation:

$$P = \sum_a \frac{d_a}{d^2} \cdot \boxed{\begin{array}{c} | \\ \bullet \\ a \quad a \\ | \quad | \\ \bullet \quad \bullet \end{array}} = \sum_a \frac{d_a}{d^2} \cdot \boxed{\begin{array}{c} | \\ \bullet \\ a \quad a \\ | \quad | \\ \bullet \quad \bullet \end{array}}$$

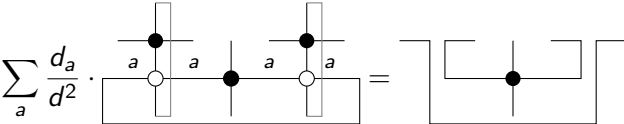
Bultinck et al., *Ann. Phys.* **378** 183-233

The gluing map (1D vacuum)

The gluing map:

$$T(X \otimes Y) \approx \sum_a \frac{d_a}{d^2} \cdot \text{Diagram}$$


The way it works:

$$\sum_a \frac{d_a}{d^2} \cdot \text{Diagram} = \text{Diagram}$$


Conclusion

- ▶ Phase classification for mixed state
- ▶ Renormalization fixed point MPDO
- ▶ Examples from string-net
- ▶ A subset of them is in the trivial phase
- ▶ Conjecture: MPDO from Fibonacci is not in the trivial phase