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The Standard Model

Dark Matter (and Dark Energy)

Plan for today

1) Motivation for Dark Matter

2) Brief reminder of Cosmology

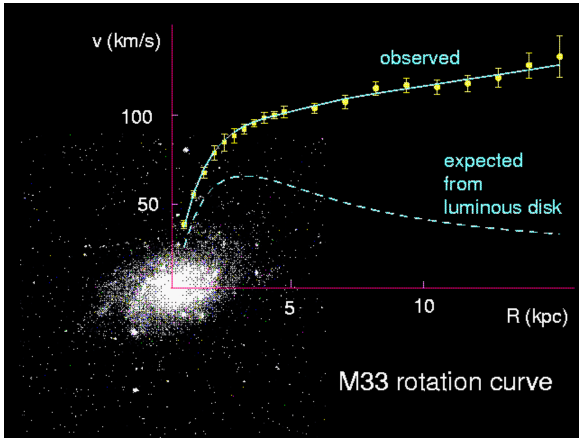
3) Decoupling of particles in the Early Universe (WIMPs)

4) 

Dark Matter is a necessary (and abundant) ingredient in the Universe

Galaxies

- Rotation curves of spiral galaxies
- Gas temperature in elliptical galaxies



Clusters of galaxies

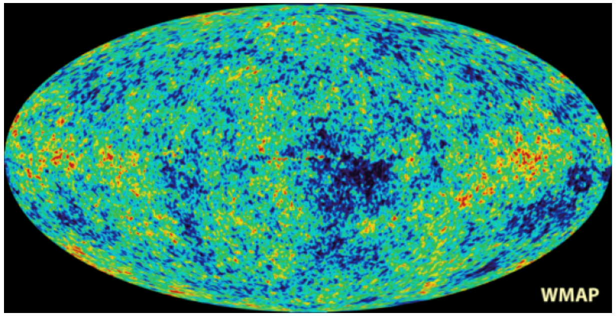
- Peculiar velocities and gas temperature
- Weak lensing
- Dynamics of cluster collision

Cosmological scales

Through the study of the anisotropies in the Cosmic Microwave Background the fundamental components of the Universe can be determined

$$\Omega_{CDM}\, h^2 = 0.1196 \pm 0.003$$

Planck 2013



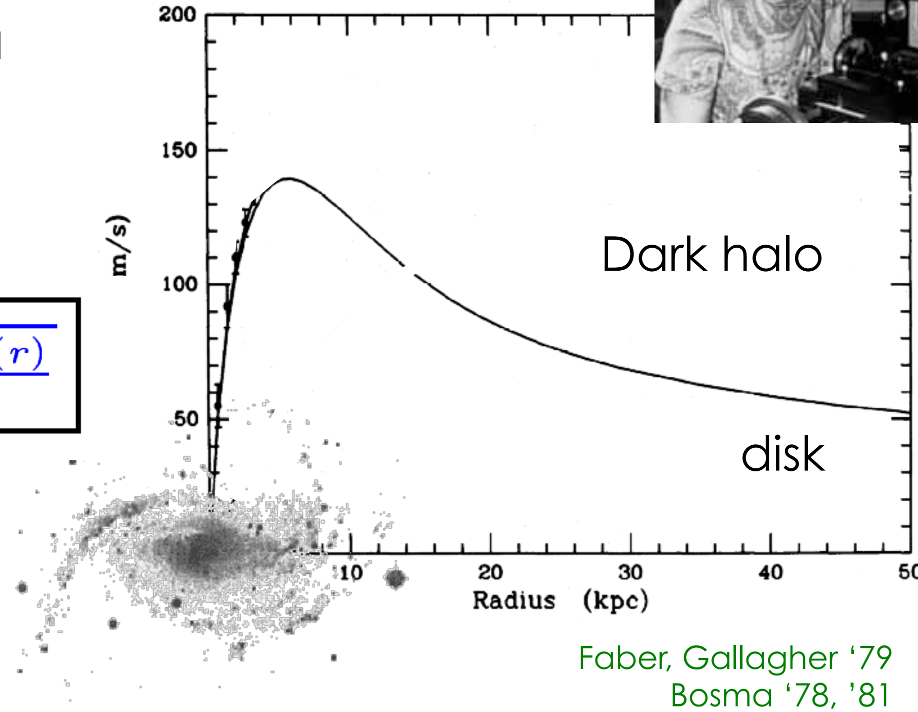
Rotation curves of spiral galaxies become flat for large distances

From the luminous matter of the disc one would expect a decrease in the velocity that is not observed

Rubin '75

$$\boxed{\frac{v_{\text{rot}}^2}{r} = \frac{G\ M(r)}{r^2}} \rightarrow \boxed{v_{\text{rot}} = \sqrt{\frac{G\ M(r)}{r}}}$$

$$M(r) = cte \rightarrow v_{\text{rot}} \propto \frac{1}{\sqrt{r}}$$



Faber, Gallagher '79
Bosma '78, '81
van Albada, Bahcall, Begeman, Sancisi '84

Galaxies contain vast amounts of non-luminous matter $M \gg M_*$

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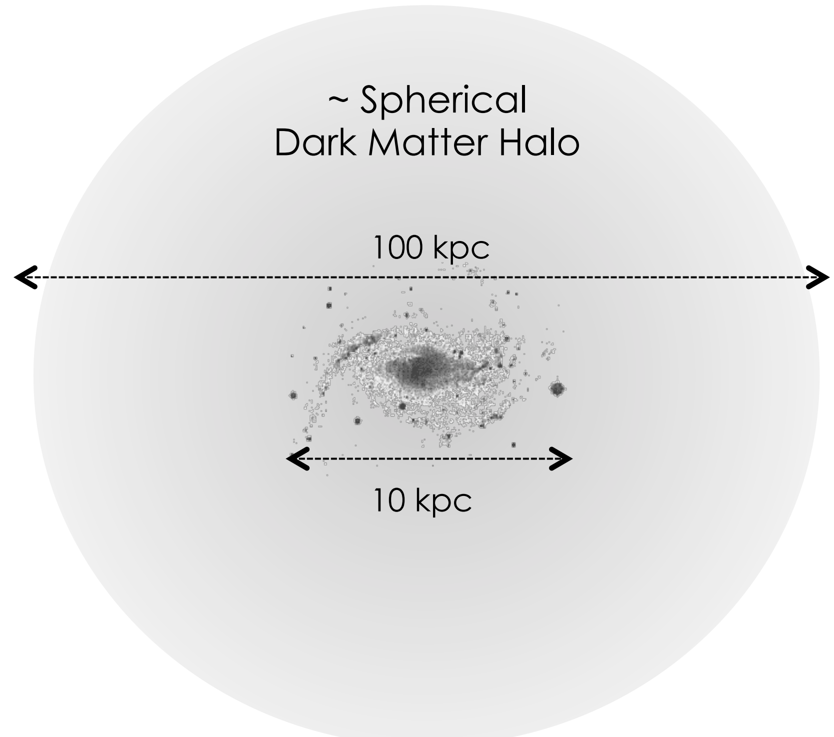

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Isothermal Spherical ~~Cow~~ Halo (a.k.a. Standard Halo Model)

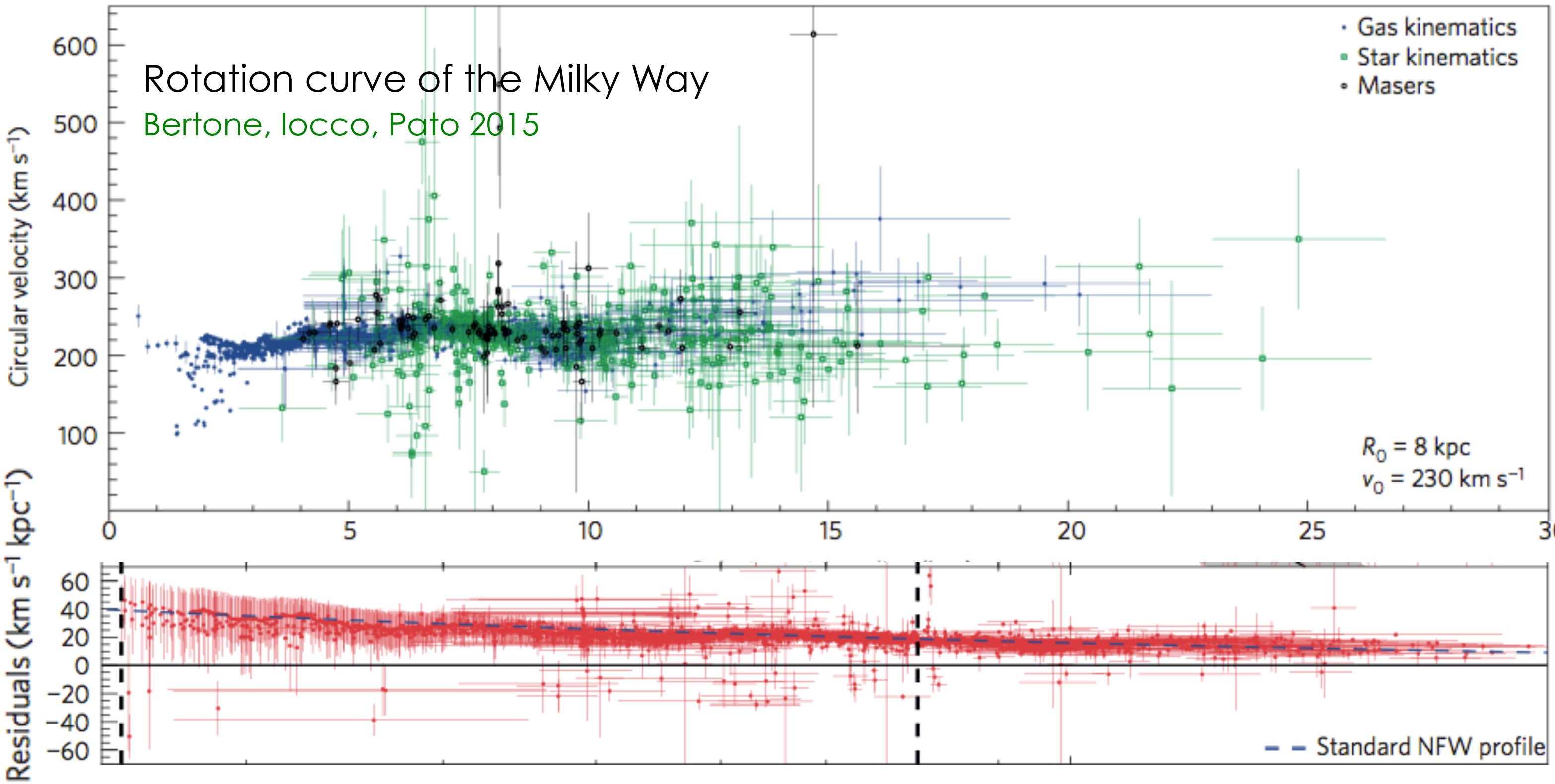
Isotropic

density distribution $\rho(r) \propto r^{-2}$

it has reached a steady state (Maxwell-Bolzmann distribution of velocities)

The effect of DM has also been observed in the Milky Way...

- There is DM in the central region of our Galaxy



- Observations also show that there is need for DM in the solar neighbourhood

Bovy, Tremaine 2012

There are substantial uncertainties in the description of our DM halo

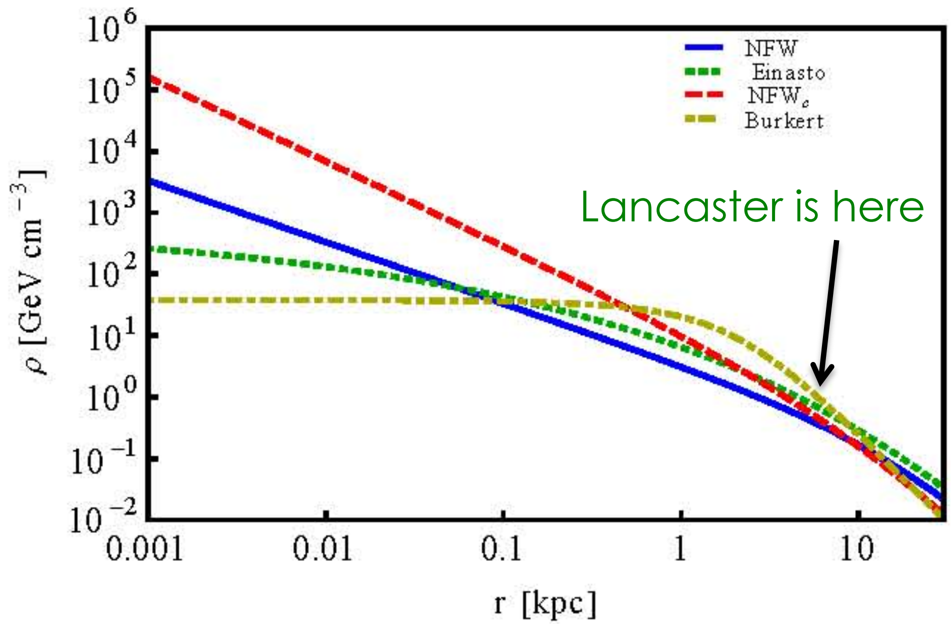
- local DM density

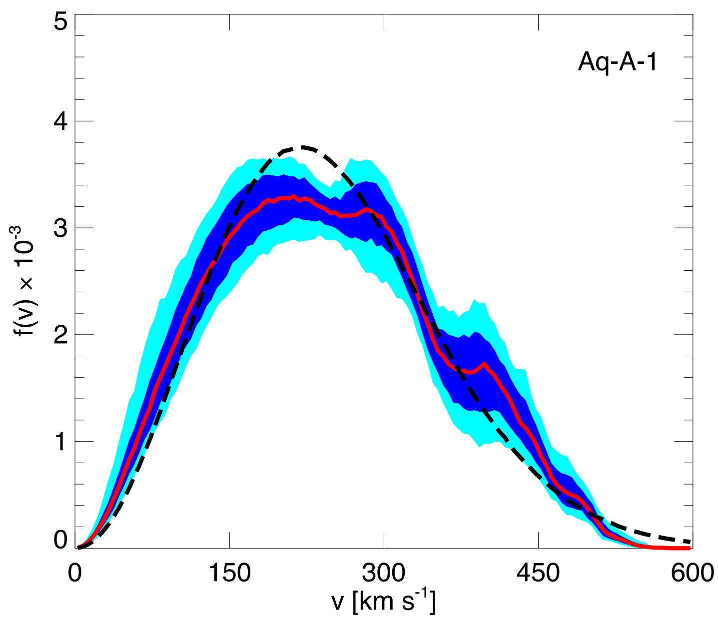
$\rho_{DM}(R_0) = 0.43(0.11)(0.10)\,\mathrm{GeV/cm^3}$ Nesti, Salucci 2012

$\rho_{DM}(R_0) = 0.32 \pm 0.07\ \mathrm{GeV/cm^3}$ Strigari, Trotta 2009

$\rho_{DM}(R_0) = 1.3 \pm 0.3\ \mathrm{GeV/cm^3}$ De Boer, Webber 2011

- DM density profile
  (DM density at the galactic centre)





- Velocity distribution of DM particles
  Central and escape velocities
  Deviations from Maxwellian distribution

The main questions concerning dark matter are whether it is really present in the first place and, if so, how much is there, where is it and what does it consist of.

How much. In general one wants to know the amount of dark matter relative to luminous matter. For cosmology the main issue is whether there is enough dark matter to close the universe. Is the density parameter Ω equal to 1?

Where. The problem of the distribution of dark matter with respect to luminous matter is fundamental for understanding its origin and composition. Is it associated with individual galaxies or is it spread out in intergalactic and intracluster space? If associated with galaxies how is it distributed with respect to the stars?

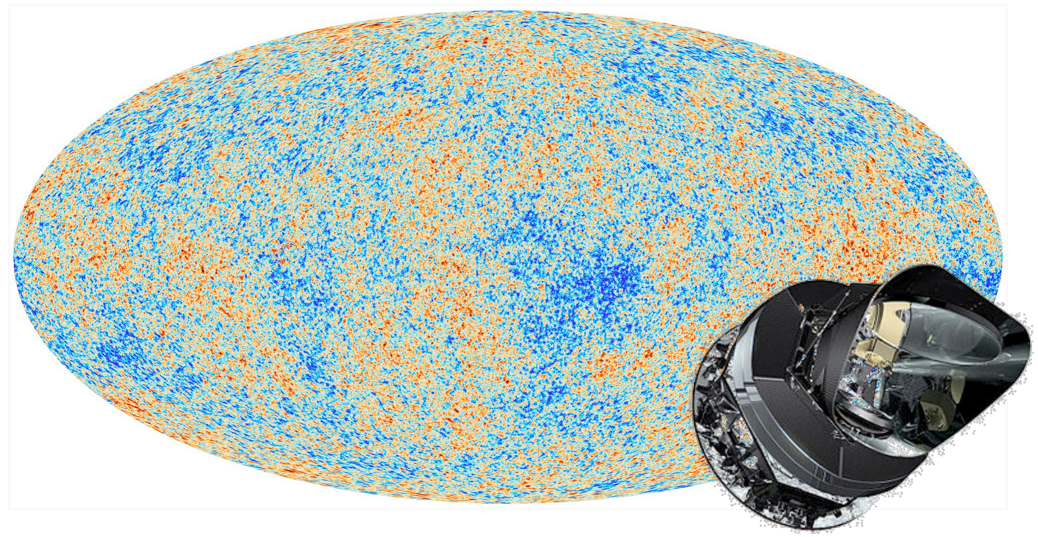
What. What is the nature of dark matter? Is it baryonic or non-baryonic or is it both?

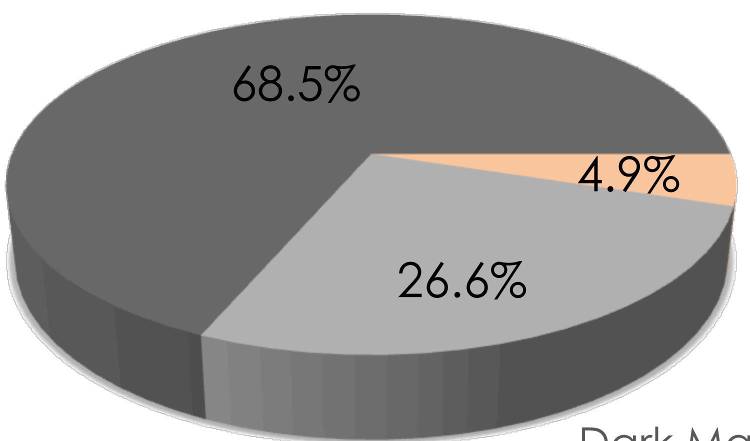
van Albada, Sancisi '87

Observations of the Cosmic microwave Background can be used to determine the components of our Universe

WMAP and Planck precision data of the CMB anisotropies allow the determination of cosmological parameters

COBE, WMAP, Planck





The dark matter abundance is measured accurately

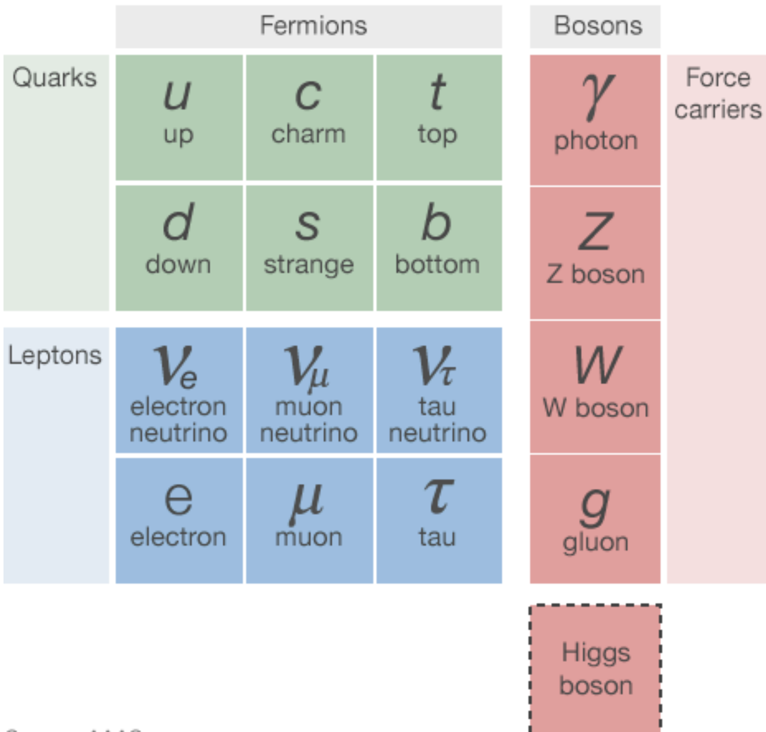
$$\Omega_\Lambda h^2 = 0.3116 \pm 0.009$$

$$\Omega_c h^2 = 0.1196 \pm 0.003$$

$$\Omega_b h^2 = 0.02207 \pm 0.00033$$

Planck 2013

The Standard Model does not contain any viable candidate for DM



Source: AAAS

Neutrinos constitute a tiny part of (Hot) dark matter

$$\Omega_\nu h^2 = \frac{\sum_i m_{\nu_i}}{91.5\mathrm{eV}} \lesssim 0.003$$

Hot dark matter not consistent with observations on structure formation.

Dark Matter is one of the clearest hints of Physics Beyond the SM

Cosmology 101

Friedmann-Lemaître-Robertson-Walker (FLRW) metric for a homogeneous and isotropic universe that is expanding (or contracting)

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) = g_{\mu\nu}dx^\mu dx^\nu$$

k = curvature

Components of the metric

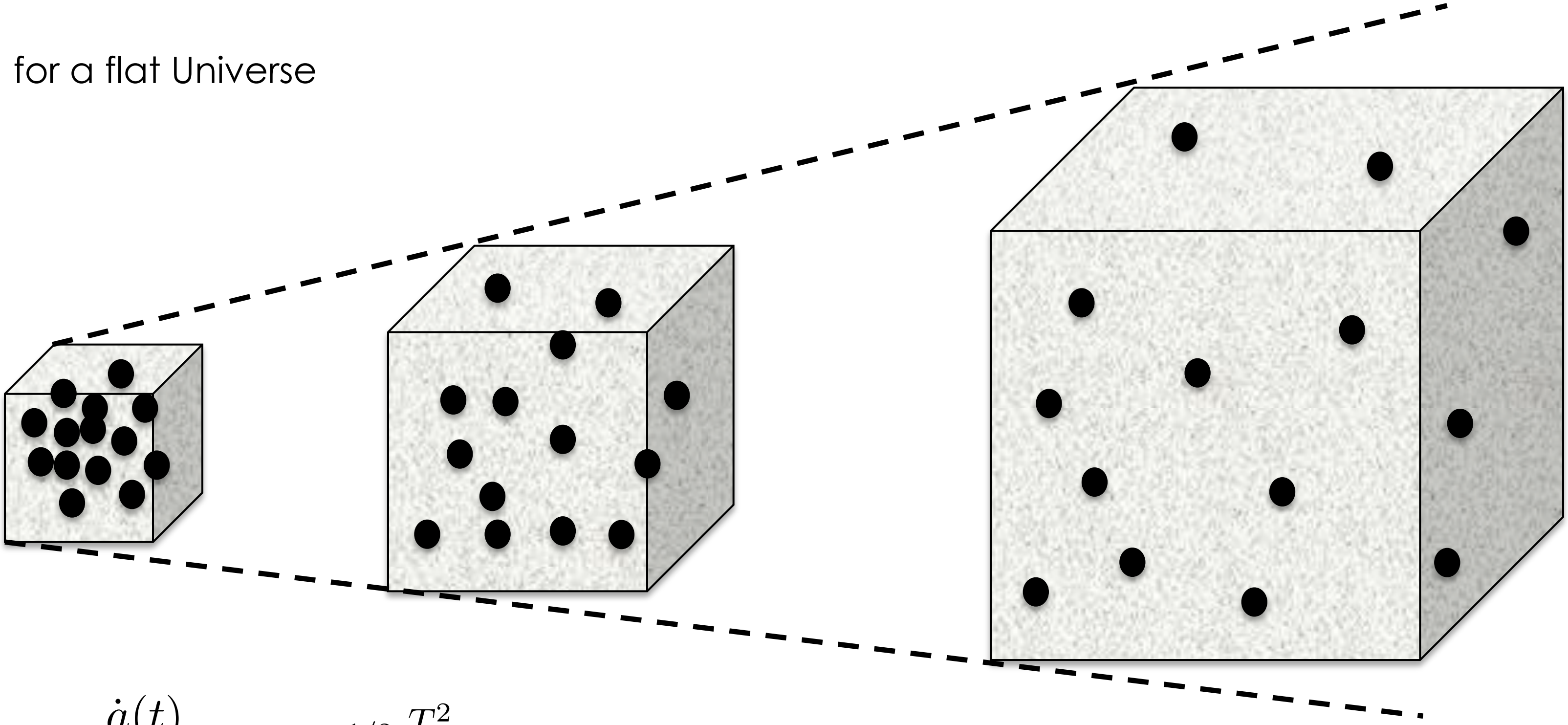
$$\begin{aligned} g_{00} &= 1 \\ g_{11} &= \frac{-a(t)^2}{1-kr^2} \\ g_{22} &= -r^2 a(t)^2 \\ g_{33} &= -r^2 \sin^2\theta a(t)^2 \end{aligned}$$

a(t) is the scale parameter

WIMP dilution

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

k=0 for a flat Universe



$$H = \frac{\dot{a}(t)}{a(t)} = 1.66\, g_*^{1/2}\frac{T^2}{M_P}$$

A system of particles in kinetic equilibrium has a phase space occupancy f given by the Bose-Einstein or Fermi-Dirac distributions at temperature T:

$$f(\mathbf{p}) = \frac{1}{e^{\frac{E-\mu}{T}} \pm 1}$$

The phase space distribution allows one to compute the associated number density n, energy density ρ and pressure p for a dilute and weakly-interacting gas of particles with g internal degrees of freedom:

$$\begin{aligned} n &= g\int \frac{d^3p}{(2\pi)^3} f(\mathbf{p}) \ , \\ \rho &= g\int \frac{d^3p}{(2\pi)^3} E(\mathbf{p}) f(\mathbf{p}) \ , \\ p &= g\int \frac{d^3p}{(2\pi)^3} \frac{|\mathbf{p}|^2}{3E(\mathbf{p})} f(\mathbf{p}) \ . \end{aligned}$$

Relativistic particles

$$T \gg m \qquad E \sim |\mathbf{p}|$$

$$n_b = \frac{g}{\pi^2}\zeta(3)T^3 \qquad \rho_b = \frac{\pi^2}{30} g T^4$$

$$n_f = \frac{3}{4}\frac{g}{\pi^2}\zeta(3)T^3 \qquad \rho_f = \frac{7}{8}\frac{\pi^2}{30} g T^4$$

Non-Relativistic particles

$$T \ll m, \qquad E = (|\mathbf{p}|^2 + m^2)^{1/2} = m\left(1 + \frac{|\mathbf{p}|^2}{m^2}\right)^{1/2} \simeq m + \frac{|\mathbf{p}|^2}{2m}$$

$$n \simeq g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$$

The energy total density and entropy density can be easily computed

$$s = \frac{\rho + p}{T}$$

$$\begin{aligned} \rho(T) &= \frac{\pi^2}{30} g_*(T) T^4 \\ s(T) &= \frac{2\pi^2}{45} g_{*s}(T) T^3 \end{aligned}$$

Thermal bath: relativistic species in thermal equilibrium with the photons

Decoupled species: relativistic species not in thermal equilibrium (Ti)

$$\begin{aligned} g_*(T) &= \sum_{\text{bos}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\text{fer}} g_i \left(\frac{T_i}{T}\right)^4 \\ g_{*s}(T) &= \sum_{\text{bos}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fer}} g_i \left(\frac{T_i}{T}\right)^3 \end{aligned}$$

It is customary to define the Yield (equivalent to the number density but in a comoving volume) in terms of the entropy density (which scales as $a^3(t)$)

$$Y = \frac{n}{s} \qquad s = \frac{2\pi^2}{45} g_{*s} T^3$$

For relativistic particles, we have

$$n = \frac{g_{eff}}{\pi^2}\zeta(3)T^3 \longrightarrow Y_{eq} = \frac{45}{2\pi^4}\zeta(3)\frac{g_{eff}}{g_{*s}}$$

For non-relativistic particles, we have

$$n = g_{eff}\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \longrightarrow Y_{eq} = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{1/2}\frac{g_{eff}}{g_{*s}}\left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$

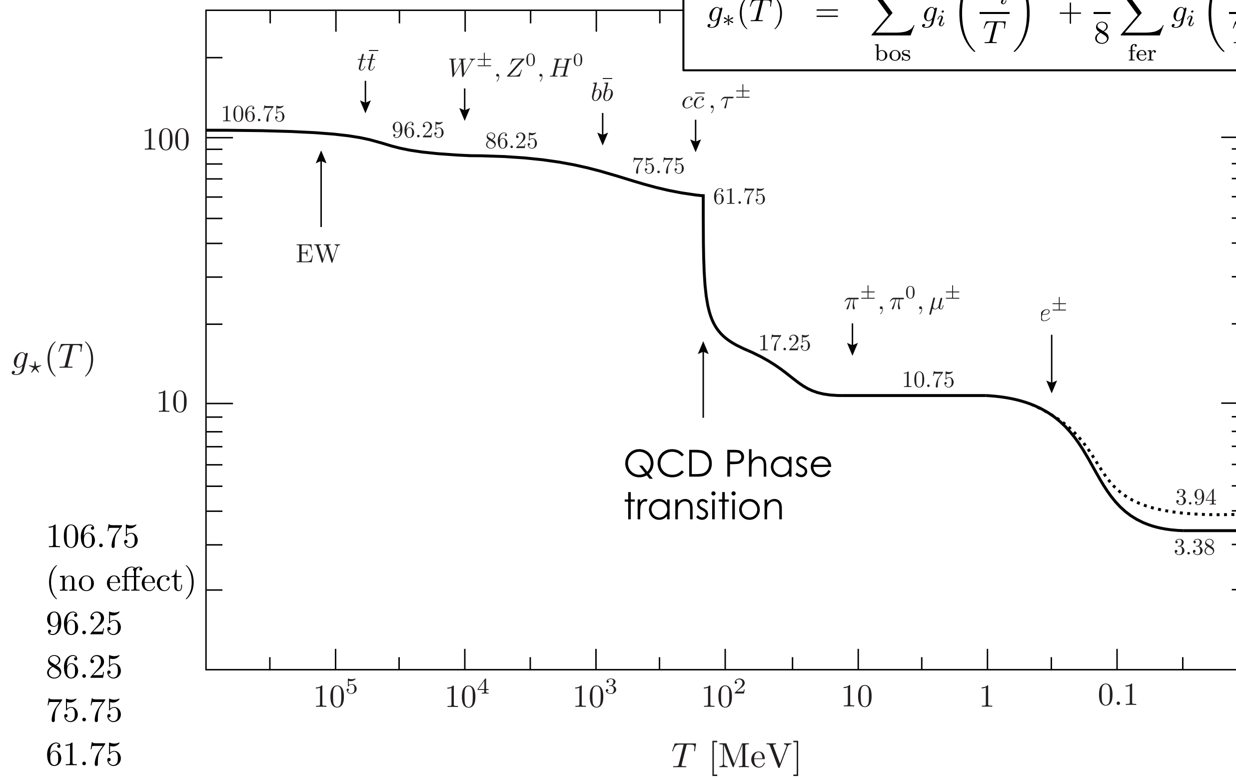
Number of relativistic degrees of freedom in the Standard Model

Quarks	t	$174.2 \pm 3.3\text{GeV}$	$\bar{t}$	spin=$\frac{1}{2}$	$g = 2 \cdot 2 \cdot 3 = 12$	
	b	$4.20 \pm 0.07\text{GeV}$	$\bar{b}$	3 colors		
	c	$1.25 \pm 0.09\text{GeV}$	$\bar{c}$			
	s	$95 \pm 25\text{MeV}$	$\bar{s}$			
	d	3–7MeV	$\bar{d}$			
	u	1.5–3.0MeV	$\bar{u}$			
						72
Gluons	8 massless bosons			spin=1	$g = 2$	16
Leptons	τ^-	$1777.0 \pm 0.3\text{MeV}$	τ^+	spin=$\frac{1}{2}$	$g = 2 \cdot 2 = 4$	
	μ^-	105.658MeV	μ^+			
	e^-	510.999keV	e^+			
						12
	ν_τ	$< 18.2\text{MeV}$	$\bar{\nu}_\tau$	spin=$\frac{1}{2}$	$g = 2$	
	ν_μ	$< 190\text{keV}$	$\bar{\nu}_\mu$			
	ν_e	< 2 eV	$\bar{\nu}_e$			
						6
Electroweak gauge bosons	W^+	$80.403 \pm 0.029\text{GeV}$		spin=1	$g = 3$	
	W^-	$80.403 \pm 0.029\text{GeV}$				
	Z^0	$91.1876 \pm 0.0021\text{GeV}$				
	γ	$0 \quad (< 6 \times 10^{-17}\text{eV})$			$g = 2$	
						11
Higgs boson (SM)	H^0	*125.5* GeV		spin=0	$g = 1$	1

$$g_f = 72 + 12 + 6 = 90$$
$$g_b = 16 + 11 + 1 = 28$$

Number of relativistic degrees of freedom in the Standard Model

$$g_*(T) = \sum_{\text{bos}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\text{fer}} g_i \left(\frac{T_i}{T}\right)^4$$



$T \sim 200$ GeV	all present	106.75	
$T \sim 100$ GeV	EW transition	(no effect)	
$T < 170$ GeV	top annihilation	96.25	
$T < 80$ GeV	$W^\pm$, Z^0, H^0	86.25	
$T < 4$ GeV	bottom	75.75	
$T < 1$ GeV	charm, τ^-	61.75	
$T \sim 150$ MeV	QCD transition	17.25	(u,d,g$\to \pi^{\pm,0}$, $37 \to 3$)
$T < 100$ MeV	$\pi^\pm$, π^0, μ^-	10.75	$e^\pm$, ν, $\bar{\nu}$, γ left
$T < 500$ keV	e^- annihilation	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$

QCD Phase transition $T \sim 150\ \mathrm{MeV}, t \sim 20\ \mu\mathrm{s}$

The temperature and thus the quark energies have fallen so that the quarks lose their asymptotic freedom

There are no more free quarks and gluons; the quark-gluon plasma has become a hadron gas

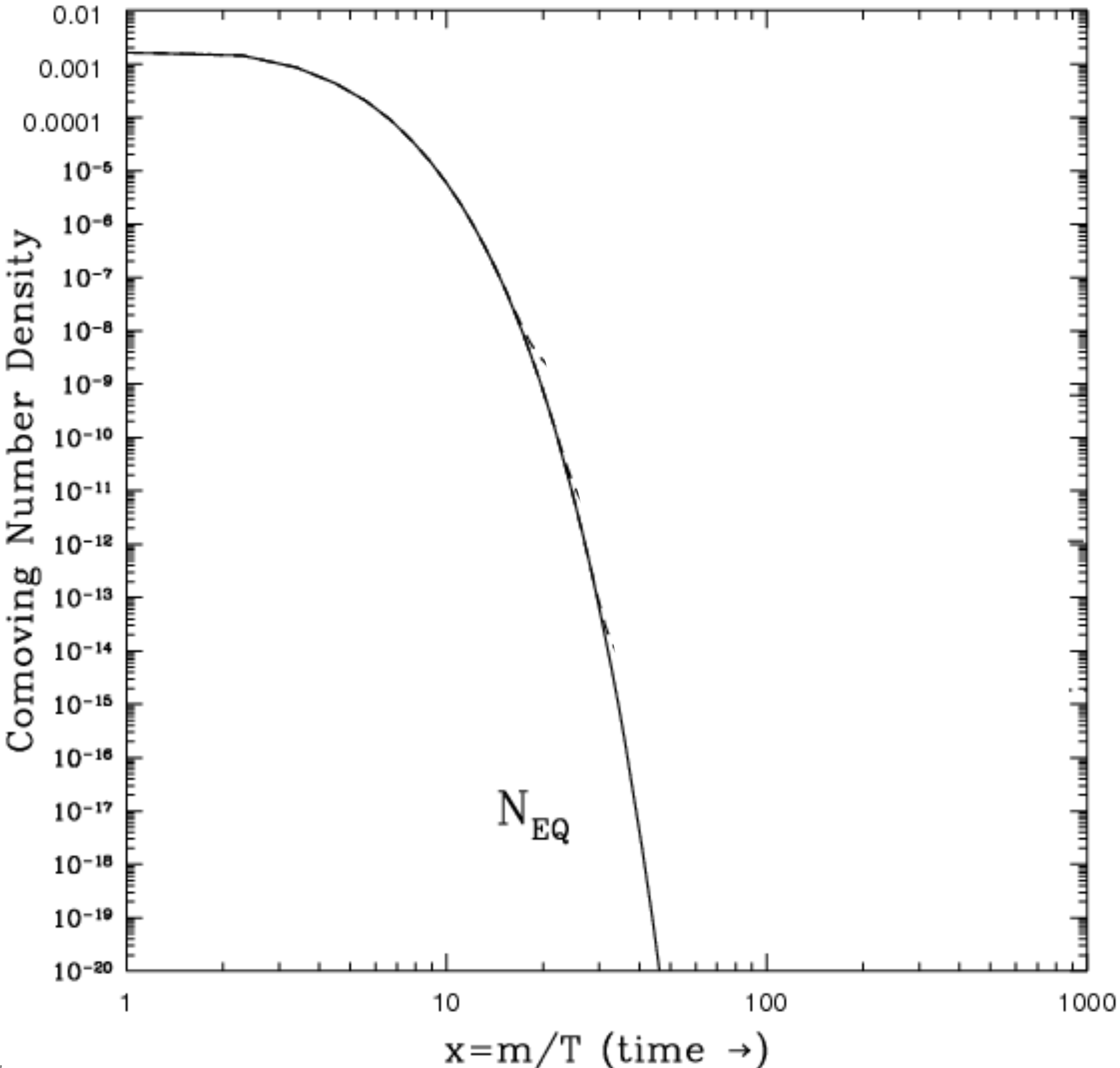
The lightest baryons are the nucleons: the proton and the neutron. The lightest mesons are the pions

all except pions are nonrelativistic below the QCD phase transition temperature.

Thus the only particle species left in large numbers are the pions (g=3), muons (4), electrons (4), neutrinos (2x3), and the photons (2).

$g_*=17.25$

$$Y_{eq} = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{1/2}\frac{g_{eff}}{g_{*s}}\left(\frac{m}{T}\right)^{3/2}e^{-m/T}$$



EXAMPLE 1.1

It is easy to estimate the value of the Yield that we need in order to reproduce the correct DM relic abundance, $\Omega h^2 \approx 0.1$, since

$$\Omega h^2 = \frac{\rho_\chi}{\rho_c} h^2 = \frac{m_\chi n_\chi h^2}{\rho_c} = \frac{m_\chi Y_0 s_0 h^2}{\rho_c} , \tag{1.9}$$

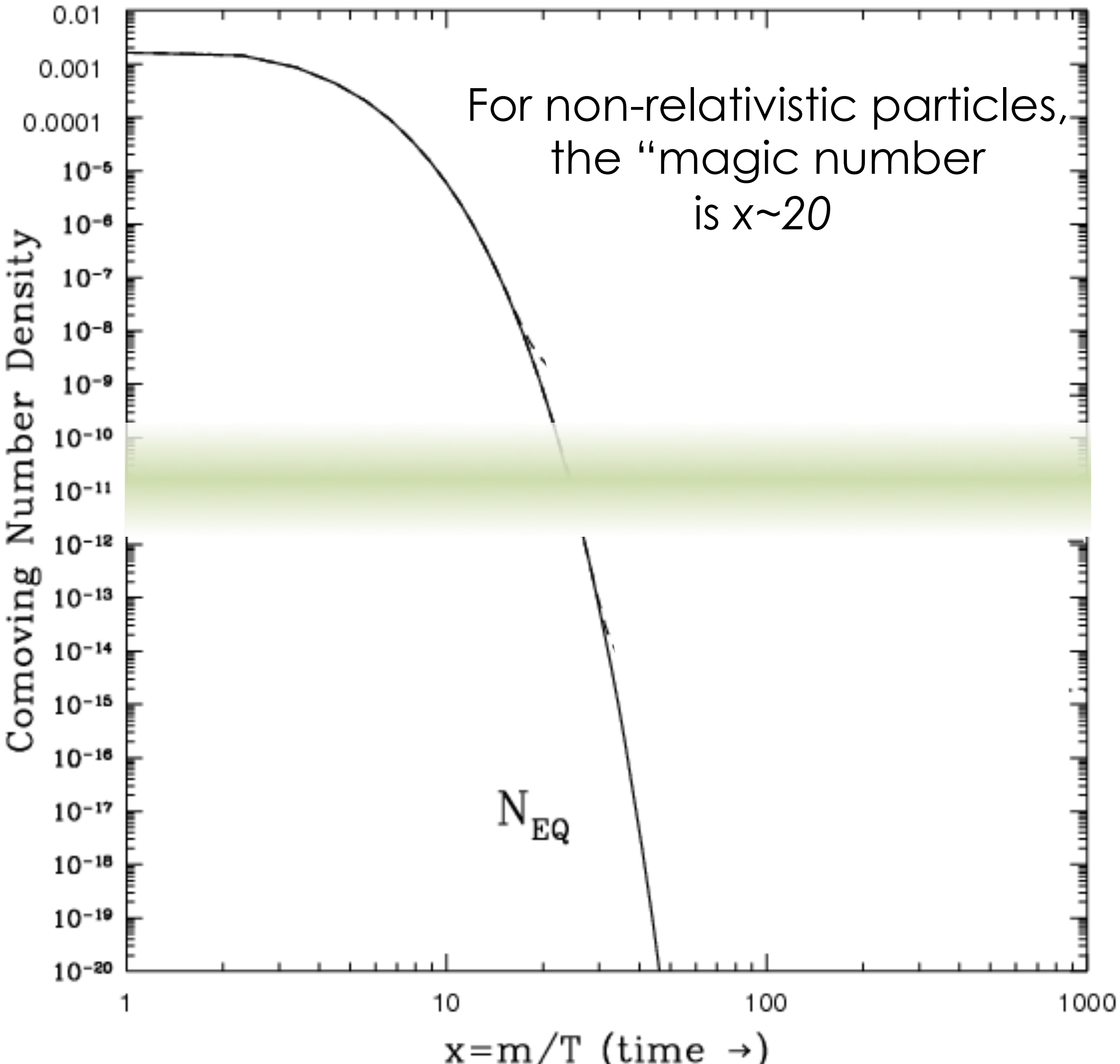
where Y_0 corresponds to the DM Yield today and s_0 is today's entropy density. We can assume that the Yield did not change since DM freeze-out and therefore

$$\Omega h^2 = \frac{m_\chi Y_f s_0 h^2}{\rho_c} . \tag{1.10}$$

Using the measured value $s_0 = 2970\ \text{cm}^{-3}$ and the value of the critical density $\rho_c = 1.054 \times 10^{-5} h^2\ \text{GeV}\,\text{cm}^{-3}$, as well as Planck's result on the DM relic abundance we arrive at

$$Y_f \approx 3.55 \times 10^{-10} \left(\frac{1\ \text{GeV}}{m_\chi} \right) . \tag{1.11}$$

$$Y_{eq} = \frac{45}{2\pi^4} \left(\frac{\pi}{8}\right)^{1/2} \frac{g_{eff}}{g_{*s}} \left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$



For DM masses in the range 1 GeV – 1 TeV

The time evolution of the phase space distribution function is dictated by Liouville's operator (which ensures conservation of density in the phase space) and the Collisional operator, which encodes number changing processes

$$\hat{L}[f] = C[f]$$

The Liouville operator can be written in a covariant way

$$\hat{L} = \frac{d}{d\tau} = p^{\mu}\frac{\partial}{\partial x^{\mu}} - \Gamma^{\mu}_{\sigma\rho}p^{\sigma}p^{\rho}\frac{\partial}{\partial p^{\mu}}$$

Where the affine connection is related to derivatives of the metric as follows

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2}g^{\mu\sigma}(g_{\sigma\nu,\lambda} + g_{\sigma\lambda,\nu} - g_{\nu\lambda,\sigma})$$

Notice that this terms incorporates gravity and the actual geometry of space-time.

If we apply this to the FRW metric, which only depends on *t* and *E*

$$f(x^\mu, p^\mu) = f(t, E)$$

We find that Liouville operator can be greatly simplified

Exercise 1

$$\begin{aligned}\hat{L} &= E\frac{\partial}{\partial t} - \Gamma^0_{\sigma\rho} p^\sigma p^\rho \frac{\partial}{\partial E} \\ &= E\frac{\partial}{\partial t} - H|\mathbf{p}|^2 \frac{\partial}{\partial E}\end{aligned}$$

Ultimately, we are interested in the time evolution of the number density

$$n = \frac{g}{2\pi^3}\int f(\mathbf{p}) d^3p$$

Thus, we integrate Liouville's operator in the momentum space

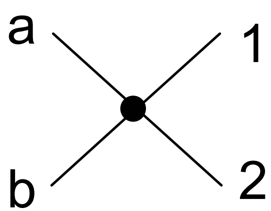
$$\frac{g}{2\pi^3}\int \hat{L}[f]\, d^3\mathbf{p} = \frac{g}{2\pi^3}\int C[f]\, d^3\mathbf{p}$$

Exercise 2

Prove the following relation

$$\frac{g}{(2\pi)^3}\int \frac{d^3\vec{p}}{E}\left[E\frac{\partial f}{\partial t} - H|\vec{p}|^2\frac{\partial f}{\partial E}\right] = \frac{dn}{dt} + 3Hn$$

Where we have divided by E for convenience



a,b=WIMP
1,2=SM (light) particles

$$d\Pi_i = \frac{g_i}{2\Pi^3}\frac{d^3\mathbf{p_i}}{2E_i}$$

No CP violation in DM sector

$$|\mathcal{M}_{12\to AB}|^2 = |\mathcal{M}_{AB\to 12}|^2$$

Energy Conservation

$$f_A f_B = f_A^{eq} f_B^{eq} = e^{-\frac{E_A+E_B}{T}} = e^{-\frac{E_1+E_2}{T}} = f_1^{eq} f_2^{eq}$$

$$\frac{g}{2\pi^3}\int \frac{C[f]}{E}\, d^3\mathbf{p} \;=\; -\int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta\left(p_A + p_B - p_1 - p_2\right) \left[|\mathcal{M}_{12\to AB}|^2 f_1 f_2 - |\mathcal{M}_{AB\to 12}|^2 f_A f_B\right]$$

$$= \; -\langle\sigma v\rangle \left(n^2 - n_{eq}^2\right)$$

We have defined the thermally averaged annihilation cross section

$$\langle\sigma v\rangle \equiv \frac{1}{n_{eq}^2}\int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta\left(p_A + p_B - p_1 - p_2\right) |\mathcal{M}|^2 f_1^{eq} f_2^{eq}$$

Non-relativistic species

$$\frac{dn}{dt} + 3Hn - \langle \sigma v \rangle \left(n^2 - n_{eq}^2\right)$$

- $$\frac{dY}{dt} = \frac{d}{dt}\left(\frac{n}{s}\right) = \frac{d}{dt}\left(\frac{a^3 n}{a^3 s}\right) = \frac{1}{a^3 s}\left(3a^2 \dot{a} n + a^3 \frac{dn}{dt}\right) = \frac{1}{s}\left(3Hn + \frac{dn}{dt}\right)$$

- $$x = \frac{m}{T}$$

$$\frac{d}{dt}(a^3 s) = 0 \;\rightarrow\; \frac{d}{dt}(aT) = 0 \;\rightarrow\; \frac{d}{dt}\left(\frac{a}{x}\right) = 0 \qquad \longrightarrow \qquad \frac{dx}{dt} = Hx$$

$$\frac{dY}{dt} = \frac{dY}{dx}\frac{dx}{dt} = \frac{dY}{dx} Hx$$

$$\frac{dY}{dx} \;=\; -\frac{\lambda \langle \sigma v \rangle}{x^2}\left(Y^2 - Y_{eq}^2\right) \qquad\qquad \lambda \;\equiv\; \frac{2\pi^2}{45}\frac{M_P\, g_{*s}}{1.66\, g_*^{1/2}}\, m$$

Exercise 3

$$\lambda \equiv \frac{2\pi^2}{45} \frac{M_P\, g_{*s}}{1.66\, g_*^{1/2}}\, m$$

$$\frac{dY}{dx} = -\frac{\lambda \langle \sigma v \rangle}{x^2} \left(Y^2 - Y_{eq}^2\right)$$

$$\Delta_Y \equiv Y - Y_{eq}$$

$$\downarrow$$

$$\Delta_Y = -\frac{\frac{dY_{eq}}{dx}}{Y_{eq}} \frac{x^2}{2\lambda \langle \sigma v \rangle}, \qquad 1 < x \ll x_f$$

$$\Delta_{Y_\infty} = Y_\infty = \frac{x_f}{\lambda \left(a + \frac{b}{3\, x_f}\right)}, \qquad x \gg x_f$$

This leads to :

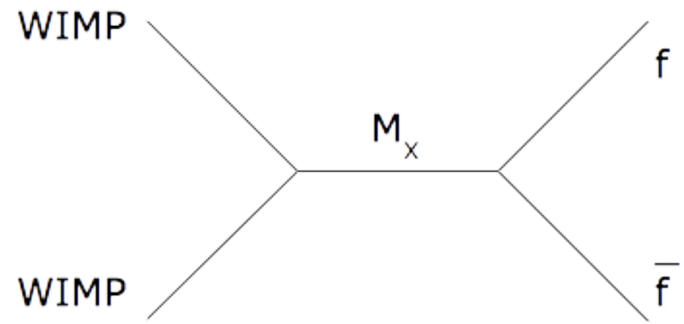
$$\Omega h^2 = \frac{m_\chi Y_\infty s_0 h^2}{\rho_c} \approx \frac{10^{-10}\ \mathrm{GeV}^{-2}}{(a + \frac{b}{60})} \approx \frac{10^{-27}\ \mathrm{cm}^3\ \mathrm{s}^{-1}}{(a + \frac{b}{60})}$$

- Very different scales conjure up to lead to the electroweak scale

A typical electroweak scale cross section for a non-relativistic particle

$$\sigma v \approx \alpha^2 \frac{m^2}{M_W} = G_F^2 m^2$$

$$G_F \approx 10^{-5}\,\mathrm{GeV}^{-2}$$



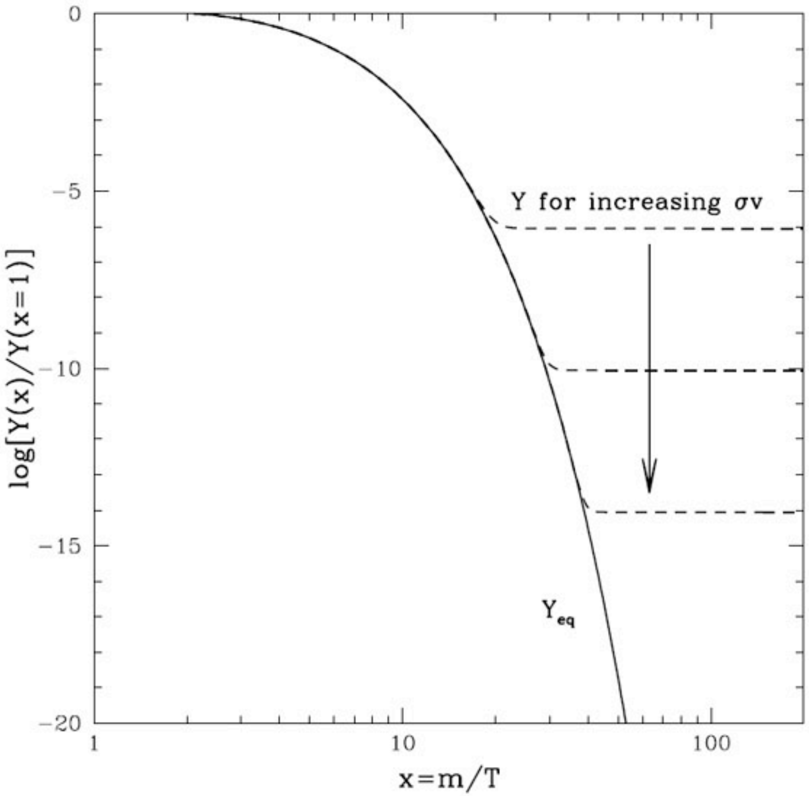
Notice that this implies $$\Omega h^2 \sim \frac{1}{\langle \sigma_A v \rangle} \sim \frac{1}{m^2}$$ (non-relativistic particle)

Imposing $$\Omega \leq 1 \quad \rightarrow \quad m \leq 340\,\mathrm{TeV}$$ (Griest, Kamionkowski '90)

WIMPs can be thermally produced in the early universe in just the right amount

The freeze-out temperature (and hence the relic abundance) depends on the DM annihilation cross-section

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\varphi}^2 \right)$$



$$\Omega_\chi h^2 \simeq const. \cdot \frac{T_0^3}{M_{\rm Pl}^3 \langle \sigma_A v \rangle} \simeq \frac{0.1\ {\rm pb} \cdot c}{\langle \sigma_A v \rangle}$$

$$T_o \approx 10^{-13}\,{\rm GeV}$$

$$H_{100} = 100\,{\rm km\,sec^{-1}\,Mpc} \approx 10^{-42}\,{\rm GeV}$$

$$M_{Planck} = 1/G_N^{1/2} = 10^{19}\,{\rm GeV}$$

A generic (electro)Weakly-Interacting Massive Particle can reproduce the observed relic density.

$$Y_{eq} = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{1/2}\frac{g_{eff}}{g_{*s}}\left(\frac{m}{T}\right)^{3/2}e^{-m/T}$$