Neutrino-Pair Exchange Long-Range Force Between Aggregate Matter

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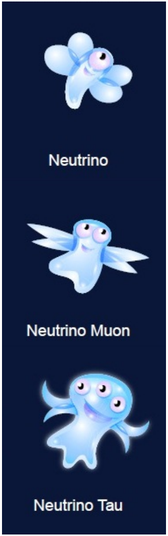
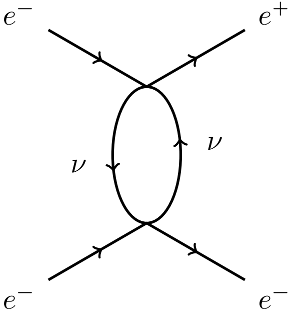
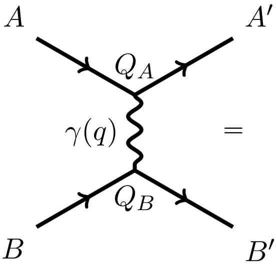
Motivation





Motivation

$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, e^{i\vec{q}\,\vec{r}}\, \frac{1}{q^2} = \frac{-1}{4\pi r}$$



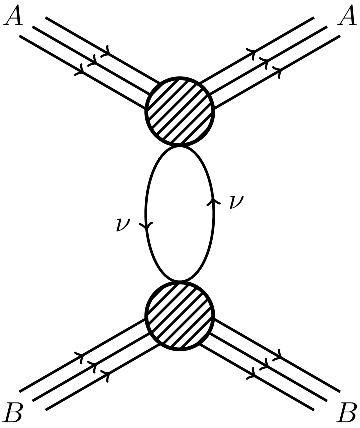
$$= \quad ie^2 Q_A Q_B \left[\bar{u}'_A \gamma^\mu u_A\right] \tfrac{1}{q^2} \left[\bar{u}'_B \gamma_\mu u_B\right]$$

$$\Longrightarrow \quad M(q^2) = e^2 Q_A Q_B \tfrac{1}{q^2}$$

$$V(r) = \frac{e^2}{4\pi} \frac{Q_A Q_B}{r}$$

$$V(r) = -\mathcal{F}\left\{M(q^2)\right\}$$

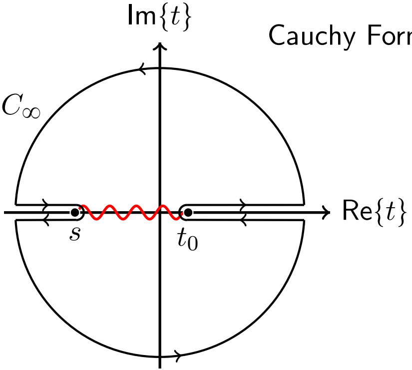
Motivation



$$V(r) = -\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, e^{i\vec{q}\,\vec{r}}\, M(q^2)$$


Let's start!

Low-Energy Dispersion Relation



Cauchy Formula: $f(z) = \frac{1}{2\pi i} \int_C \mathrm{d}z' \, \frac{f(z')}{z' - z}$

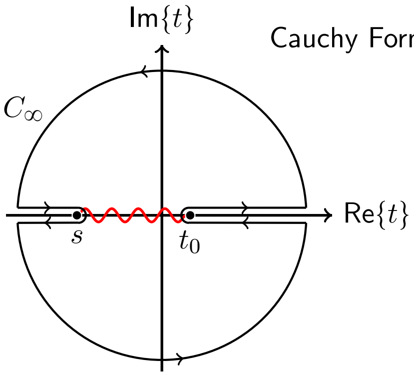
Physical Region:

$-s \leqslant t \leqslant t_0$

$-M_A^2 \lesssim t \leqslant 4m_\nu^2$

$$M(t;s) = \frac{1}{\pi} \int_{t_0}^{\infty} \mathrm{d}t' \, \frac{\mathrm{Im}\{M(t')\}}{t' - t} + \frac{1}{\pi} \int_{-\infty}^{-s} \mathrm{d}t' \, \frac{\mathrm{Im}\{M(t')\}}{t' - t} + \int_{C_\infty}$$

Low-Energy Dispersion Relation



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Physical Region:

$-s \leqslant t \leqslant t_0$

$-M_A^2 \lesssim t \leqslant 4m_\nu^2$

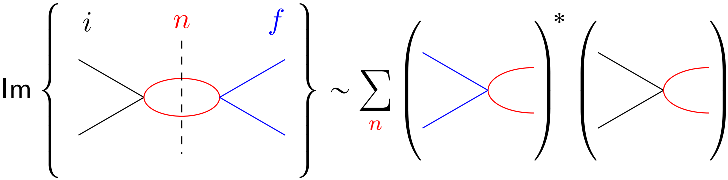
$$M(t; s) = \frac{1}{\pi} \int_{t_0}^{\infty} \mathrm{d}t' \, \frac{\mathrm{Im}\{M(t')\}}{t' - t} + \text{Short Range}$$

S Matrix Unitarity

$$S^\dagger S = 1 \quad \xrightarrow{S \equiv 1 + iT} \quad -i(T - T^\dagger) = T^\dagger T$$

$$\mathcal{M}(i \to f) \sim \langle f | \, T \, | i \rangle$$

$$\begin{aligned} 2 \,\text{Im}\left\{ \langle f | \, T \, | i \rangle \right\} &= \langle f | \, T^\dagger \, \mathbb{1} \, T \, | i \rangle \\ &= \sum_n \langle f | \, T^\dagger | n \rangle \langle n | T \, | i \rangle \end{aligned}$$



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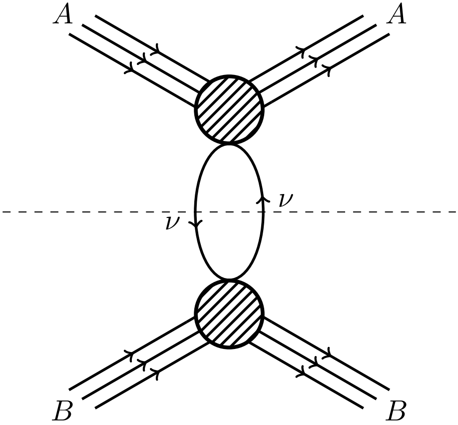

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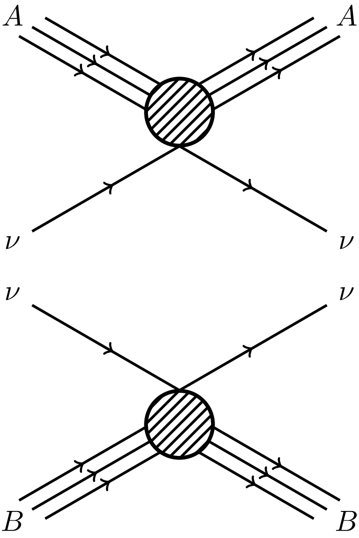
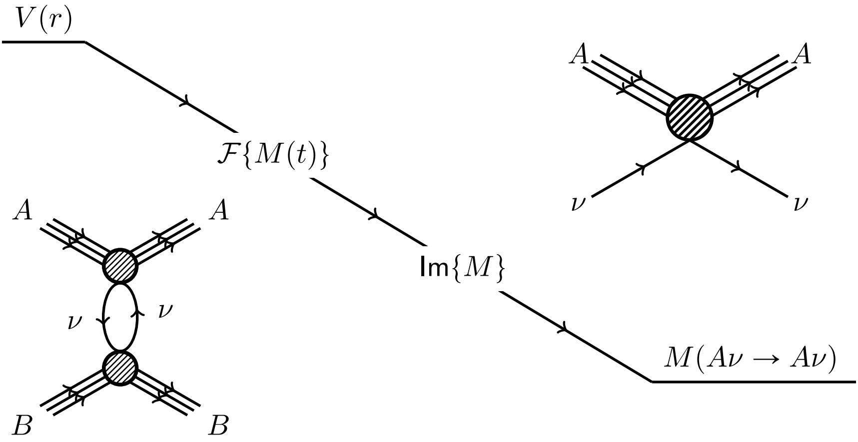
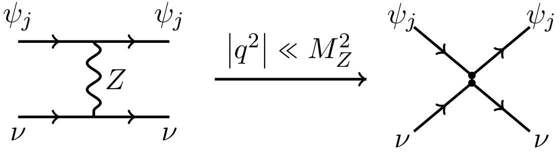


Summing up...



Low-Energy Effective Neutral Currents



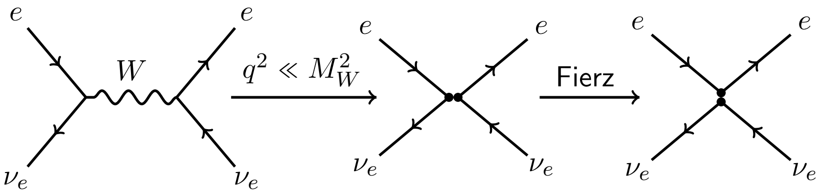
$$g_V = 2T_3 - 4Q\sin^2\theta_W$$
$$g_A = 2T_3$$

$$\mathscr{L}_{\mathsf{NC}} = \frac{1}{2}M_Z^2 Z_\mu Z^\mu - \frac{e}{4\sin\theta_W\cos\theta_W} Z_\mu\,\bar\psi_j\,\gamma^\mu\,\left(g_{V_j} - g_{A_j}\gamma_5\right)\psi_j$$

$$0 = \frac{\partial\mathscr{L}_{\mathsf{NC}}}{\partial Z_\mu} = M_Z^2 Z^\mu - \frac{e}{4\sin\theta_W\cos\theta_W}\bar\psi_j\,\gamma^\mu\,\left(g_{V_j} - g_{A_j}\gamma_5\right)\psi_j$$

$$\mathscr{L}_{\mathsf{NC}}^{\mathsf{eff}} = \frac{G_F}{2\sqrt{2}}\left[\bar\nu\,\gamma^\mu\,(1-\gamma_5)\,\nu\right]\left[\bar\psi_j\,\gamma_\mu\,\left(g_{V_j} - g_{A_j}\gamma_5\right)\psi_j\right]$$

Low-Energy Effective Charged Currents

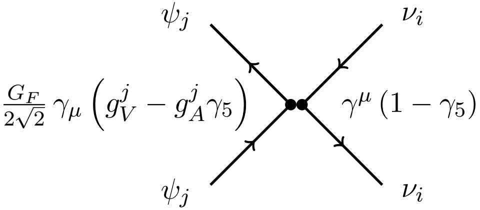


$$\mathscr{L}_{\mathsf{CC}} = M_W^2 W_\mu^\dagger W^\mu - \frac{e}{2\sqrt{2}\sin\theta_W} W_\mu \, \bar{e}\, \gamma^\mu (1-\gamma_5)\, \nu_e + h.c.$$

$$\mathscr{L}_{\mathsf{CC}}^{\mathsf{eff}} = -\frac{G_F}{\sqrt{2}} \left[\bar{\nu}_e \, \gamma^\mu \, (1-\gamma_5)\, e\right] \left[\bar{e}\, \gamma_\mu \, (1-\gamma_5)\, \nu_e\right]$$

$$\mathscr{L}_{\mathsf{CC}}^{\mathsf{eff}} = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_e \, \gamma^\mu \, (1-\gamma_5)\, \nu_e\right] \left[\bar{e}\, \gamma_\mu \, (1-\gamma_5)\, e\right]$$

Low-Energy Effective Weak Theory

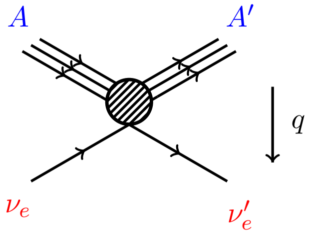


$$\psi_j = e, p, n$$

$$\nu_i = \nu_e, \nu_\mu, \nu_\tau$$

NC:	$p\,\nu_{e,\mu,\tau}$	$\longrightarrow$	$g_V^p = 1 - 4\sin^2\theta_W$
NC:	$n\,\nu_{e,\mu,\tau}$	$\longrightarrow$	$g_V^n = -1$
NC:	$e\,\nu_{\mu,\tau}$	$\longrightarrow$	$g_V^e = -1 + 4\sin^2\theta_W = -g_V^p$
NC+ CC:	$e\,\nu_e$	$\longrightarrow$	$\tilde{g}_V^e = 2 + g_V^e = 2 - g_V^p$

Scattering Amplitude $A\nu_e \to A\nu_e$



$$A = \begin{cases} Z \text{ electrons} \\ Z \text{ protons} \\ N \text{ neutrons} \end{cases}$$

$$T_{\nu_e} = \int \mathrm{d}^4x \, \langle A' \, \nu_e' | \, \mathscr{L}(x) \, | A \, \nu_e \rangle = \frac{G_F}{2\sqrt{2}} \int \mathrm{d}^4x \, j^\mu(x) \, J_\mu(x)$$

$$j^\mu(x) = \left[\bar{u}' \, \gamma^\mu \, (1 - \gamma_5) \, u \right] \, e^{-i(k-k')x}$$

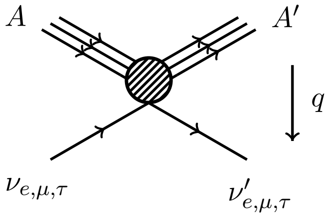
Molecular matrix element

$$J_\mu(x) = \tilde{g}_V^e \langle A'|\,\bar{e}(x)\gamma_\mu e(x)\,|A\rangle - g_A^e \langle A'|\,\bar{e}(x)\gamma_\mu\gamma_5 e(x)\,|A\rangle$$

- Scalar, γ^0: number operator, $e^\dagger e$.
  COHERENT
- Pseudo-scalar, $\gamma^0\gamma_5$: matrix element $\sim \vec{\sigma}\vec{q}/M$.
  INCOHERENT, RELATIVISTIC
- Polar vector, $\vec{\gamma}$: matrix element $\sim \vec{q}/M$.
  RELATIVISTIC
- Axial vector, $\vec{\gamma}\gamma_5$: matrix element $\sim \vec{\sigma}$.
  INCOHERENT

$$J_0(x) = Z\,\tilde{g}_V^e\,e^{iqx}$$

Scattering Amplitude $A\nu \to A\nu$



$$A = \begin{cases} Z \text{ electrons} \\ Z \text{ protons} \\ N \text{ neutrons} \end{cases}$$

$$\mathcal{M}(A\nu_e \to A\nu_e) = \frac{G_F}{2\sqrt{2}} J_\mu \left[\bar{u}' \gamma^\mu (1 - \gamma_5) u\right]$$

$\tilde{g}_V^e Z + g_V^p Z + g_V^n N$

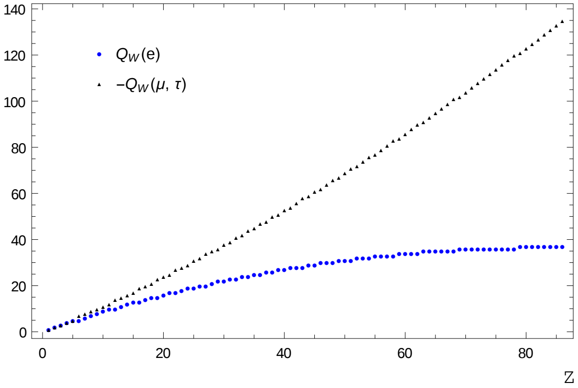
$$\boxed{J_0 = 2Z - N} \equiv Q_W^e$$

$$\mathcal{M}(A\nu_{\mu,\tau} \to A\nu_{\mu,\tau}) = \frac{G_F}{2\sqrt{2}} J_\mu \left[\bar{u}' \gamma^\mu (1 - \gamma_5) u\right]$$

$$\boxed{J_0 = -N} \equiv Q_W^{\mu,\tau}$$

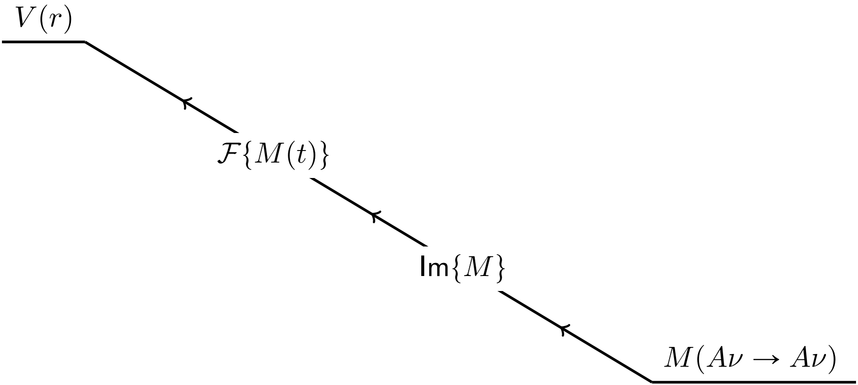
$g_V^e Z + g_V^p Z + g_V^n N$

Weak Flavor Charges of Stable Atoms



$$Q_W^e = 2Z - N \qquad Q_W^\mu = Q_W^\tau = -N$$

Finally...



Conclusion: Long-Range Weak Interaction

Interaction Potential:

$$V(r) = \frac{G_F^2}{8\pi^3} \left(\sum_f Q_{W,A}^f \, Q_{W,B}^f \right) \frac{1}{r^5}$$

Force:

$$\mathbf{F} = \frac{5G_F^2}{8\pi^3} \left(\sum_f Q_{W,A}^f Q_{W,B}^f \right) \frac{\hat{\mathbf{r}}}{r^6}$$

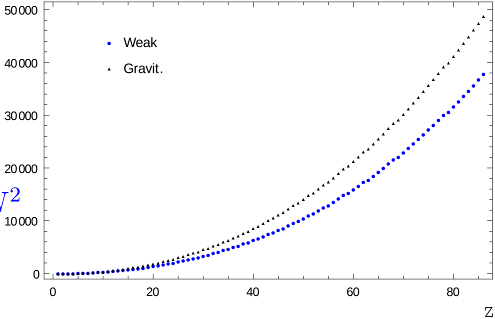
Conclusion: Long-Range Weak Interaction

Interaction Potential:

$$V(r) = \frac{G_F^2}{8\pi^3} \left(\sum_f Q_{W,A}^f \, Q_{W,B}^f \right) \frac{1}{r^5}$$

Gravit $\approx (Z+N)^2$

Weak $= (2Z-N)^2 + 2N^2$



Perspectives

- Interesting ranges:

$$r_{\text{min}} \sim 1 \text{ nm}.$$
$$r_{\text{max}} \sim m_\nu^{-1} \sim (0.1 \text{ eV})^{-1} \sim 1 \ \mu\text{m}.$$

a_0

e^{-2mr}

- Competitors:

  Electromagnetic residual interactions

  Gravitation

- Measurement of the **absolute** mass of the neutrino!

Dirac or Majorana?