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COSMOLOGY: THEORY

Vacuum Energy

and the

Accelerated Universe (II)

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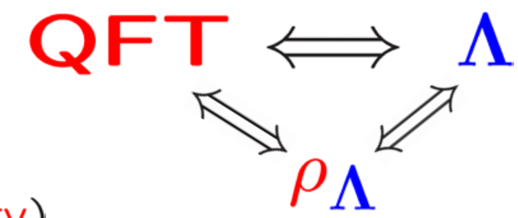
Vacuum energy: zero-point energy and some cosmic numerology

- Zeldovich (1967) first raised the connection



$$\rho_\Lambda = \frac{\Lambda}{8\,\pi\,G_N} \quad \text{(vacuum energy density)}$$

First thought: $\rho_\Lambda \propto m_p^4 \sim 1\mathrm{GeV}^4$

Impossible since $\rho_\Lambda \simeq \rho_c^0$ and $\rho_c^0 = \frac{3\,H_0^2}{8\pi\,G} \sim 10^{-47}\,\mathrm{GeV}^4$

Second thought:

$$\rho_\Lambda \simeq G\,m_p^6 = \frac{m_p^6}{M_P^2} \sim 10^{-38}\,\mathrm{GeV}^4$$

much better, but still unacceptable...

- Weinberg in 1972 "cosmic prediction" of the pion mass

$$m_\pi^3 \sim \frac{H_0}{G} = H_0\, M_P^2 \sim 10^{-4}\, \mathrm{GeV}^3 \quad \Rightarrow \quad m_\pi = \mathcal{O}(0.1)\ \mathrm{GeV}$$

Using now Zeldovich's second thought with $m_p \to m_\pi$:

$$\rho_\Lambda \sim G\, m_\pi^6 = \frac{m_\pi^6}{M_P^2} \sim H_0^2\, M_P^2 = \frac{H_0^2}{G} \sim \rho_c^0$$

correct order of magnitude **!!**

From the previous formulae, one findsthe disguised form:

$$\rho_\Lambda \sim m_\pi^3\, H_0 \sim H_0\, \Lambda_{QCD}^3$$

The last form became popular more recently by several authors

But it is **unacceptable** $\Rightarrow$ ~~covariance~~ of the effective action

Also inconsistent with observations $\rightarrow$ A. Gómez-Valent talk

- Of all SM* particles there is one that may realize the "CC dream":

$$\text{neutrino of a few meV} = 10^{-3}\ \text{eV}$$

$$\rho_\Lambda \sim m_\nu^4 \sim 10^{-11}\,\text{meV}^4 \sim 10^{-47}\,\text{GeV}^4 \quad \text{!!}$$

Fine, but why only this d.o.f.?

- Perhaps the most "cabalistic" attempt at finding the CC "number" is the following one using G, m_e and α:

$$\Lambda = \frac{G^2}{\hbar^4}\left(\frac{m_e}{\alpha}\right)^6$$

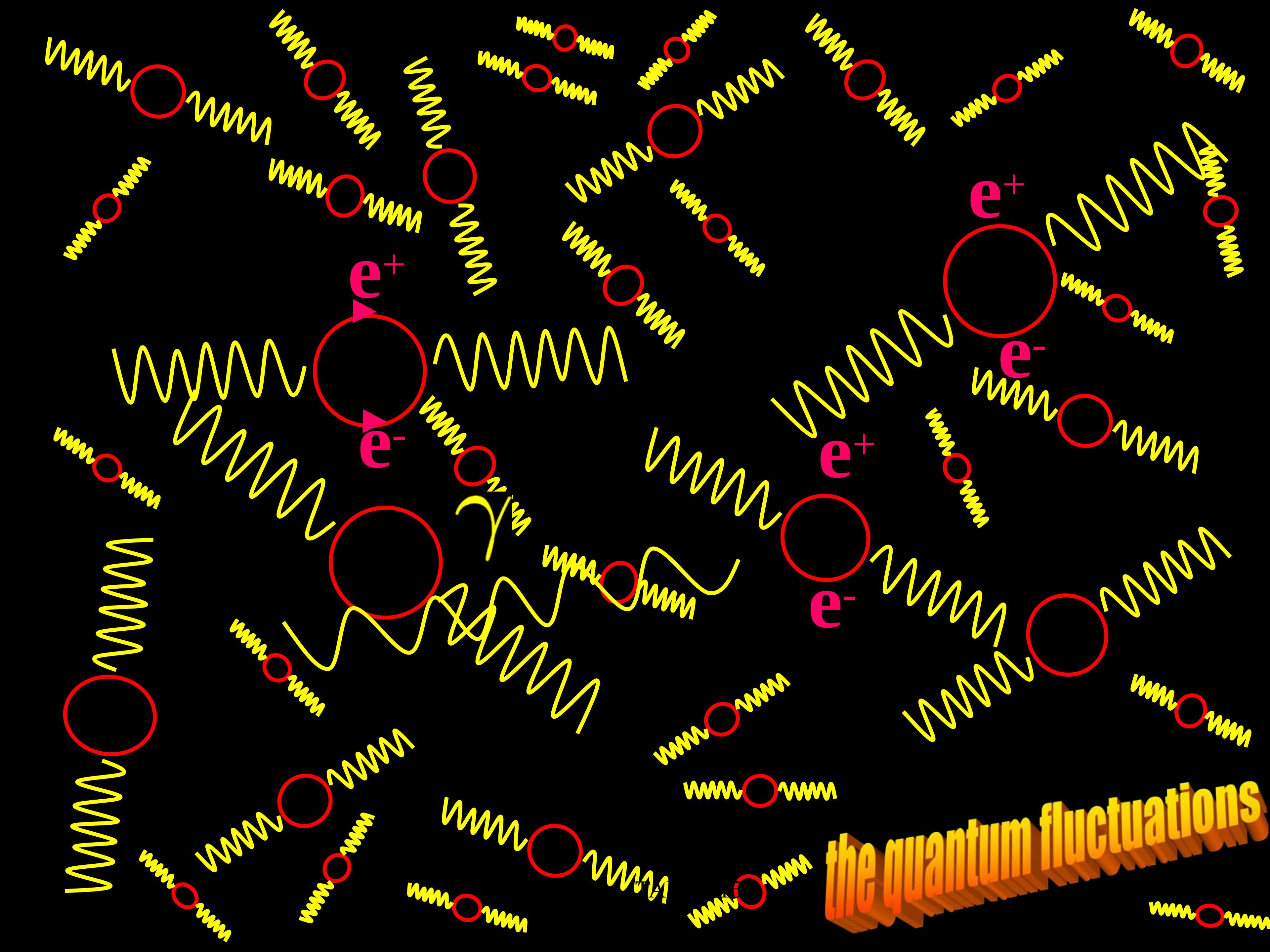
Or in nat. unit.

$$\rho_\Lambda = \frac{G}{8\pi}\left(\frac{m_e}{\alpha}\right)^6 \simeq 3\times 10^{-47}\ \text{GeV}^4 \ \text{!!}$$

Kind of "improved" form of Zeldovich's second thought

the classical vacuum

The absolute nothingness...


e+
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γ
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e-
the quantum fluctuations


...of quantum "bubles" !!
e+
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e-
$\sum_k \frac{1}{2}\hbar\omega_k$
the quantum vacuum
is full!

Pauli (1933)

$$\mathrm{ZPE}=\sum_k \frac{1}{2}\hbar\,\omega_k$$

He thought on the cosmological consequences of the ZPE:

$$\rho_m = \frac{1}{4\pi G\, a^2} = 2\,\rho_\Lambda \qquad (1)$$

$$\langle\rho\rangle = \frac{1}{4\pi^2}\int_0^M \mathrm{d}k k^2\sqrt{k^2+m^2} = \frac{M^4}{16\pi^2}\left(1+\frac{m^2}{M^2}+\cdots\right)$$

Taking the cutoff at the inverse of the classical electron radius,

$$M \to \omega_{\mathrm{max}} = 2\pi\nu_{\mathrm{max}} = 2\pi/\lambda_e = 2\pi m_e/\alpha$$

For photons $m=0$ and include a factor of 2:

$$\langle\rho\rangle = \frac{1}{8\pi^2}\,\omega_{\mathrm{max}}^4 \to \rho_\Lambda \text{ in } (1)$$

$$a = \frac{1}{(2\pi)^{3/2}}\,\frac{\alpha^2 M_P}{m_e^2} \simeq 26\ \mathrm{Km}$$

!!

➢Gauge Principle and the SM of Particle Physics

principle of local gauge invariance

Standard Model

symmetry group $SU(2) \times U(1) \times SU(3)_C$

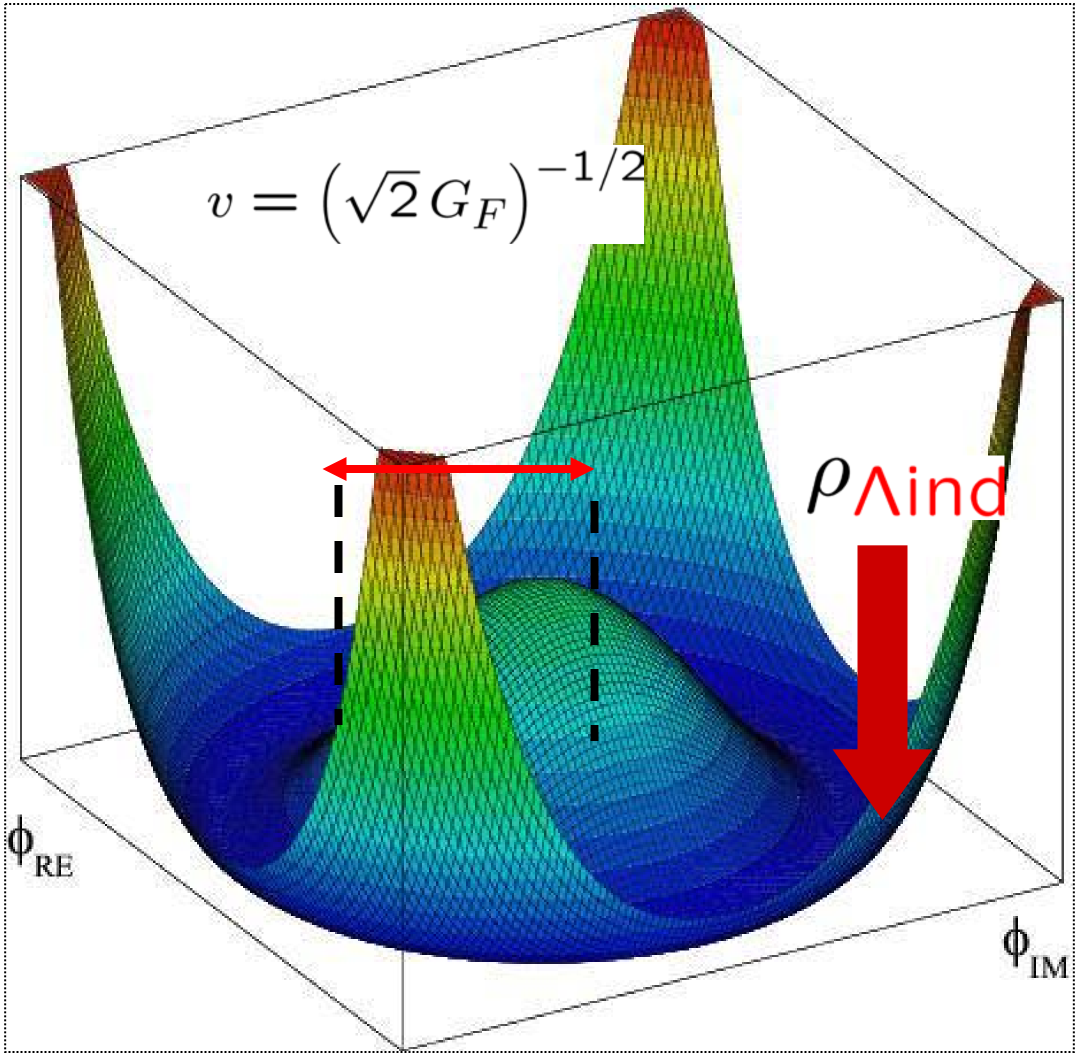
Higgs mechanism and Yukawa interactions

$\longrightarrow$ masses $M_W, \ M_Z, \ m_{\text{fermion}}$

SM
- renormalizable quantum field theory
- accurate theoretical predictions

detect deviations $\rightarrow$ "new physics" ?

Higgs Potential ⇒ Vacuum Energy



$$V(\varphi) = \frac{1}{2}\, m^2\, \varphi^2 + \frac{1}{4!}\, \lambda\, \varphi^4$$

$$m^2 < 0 \quad \Rightarrow$$

$$v \equiv \langle \varphi \rangle = \sqrt{\frac{-6\, m^2}{\lambda}}$$

$$\langle V(\varphi) \rangle = -\frac{1}{8}\, M_{\mathcal{H}}^2\, v^2$$
$$\sim -10^8\ GeV^4$$

$$M_W = \frac{1}{2}\, g\, v$$
$$M_Z = M_W / \cos\theta_w$$
$$m_f = \lambda_f\, v$$

$$G_F/\sqrt{2} = g^2/8 M_W^2$$

Vacuum energy = bubbles+hot chocolade

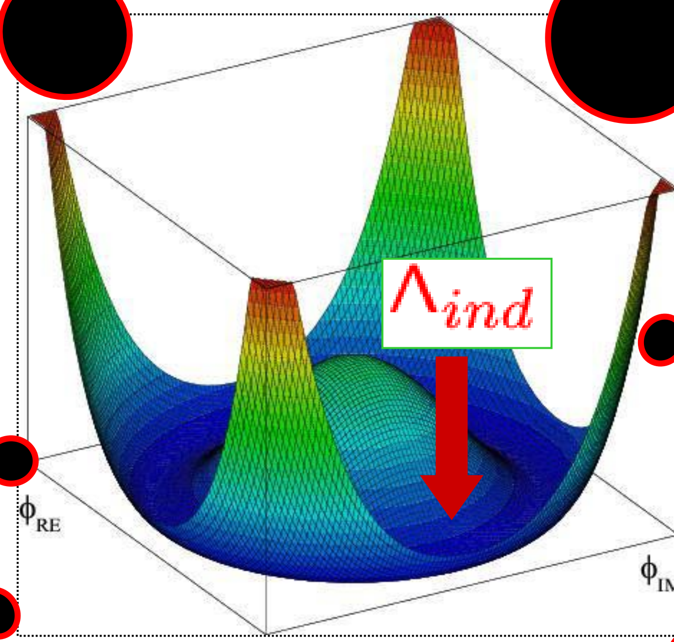
e+ e-

e+ e-

e+ e-

e+ e-

$$\frac{1}{2}\hbar\omega_k$$



The CC problem (s)

Problem 1 : the "old" CC problem

Why all the big contributions to the DE add up to such a small value in Particle Physics units?

In the SM, $\Lambda_{\rm ph} = \Lambda_v + \Lambda_{SM}$ $\quad (\frac{\Lambda_{SM}}{\Lambda_{\rm ph}} \simeq \frac{10^8}{10^{-47}} \simeq 10^{55})$

Problem 2: the cosmic "coincidence" problem

Why the currently observed DE density is so close to the matter densisty?

coincidence ratio now:

$$r \equiv \frac{\rho_\Lambda^0}{\rho_M^0} = \frac{\Omega_\Lambda^0}{\Omega_M^0} \simeq \frac{7}{3} = \mathcal{O}(1)$$

Λ in the SM and beyond

Source	**Effect** (GeV^4)	Λ/Λ_{exp}
electron 0-point	10^{-16}	10^{31}
QCD chiral	10^{-4}	10^{43}
QCD gluon	10^{-2}	10^{45}
Electroweak SM	10^{+9}	10^{56}
typical GUT	10^{+64}	10^{111}
Quantum Gravity	10^{+76}	10^{123} !!

!!

$$\rho_\Lambda^0 = \Omega_\Lambda^0\, \rho_c^0 \simeq 6\, h^2 \times 10^{-47}\, \mathrm{GeV}^4 \simeq 3 \times 10^{-47}\, \mathrm{GeV}^4$$

$$m_\Lambda \equiv \sqrt[4]{\rho_\Lambda^0} \simeq 2 \text{ meV}$$

◇ The contribution from QCD comes from the VEV of the anomalous trace

$$< T^{\mu}_{\mu} >=< \frac{\beta_3(g_s)}{2\,g_s} F^{\mu\nu}_{a}\, F^{a}_{\mu\nu} > + (1+\gamma_q) < m_q\,\bar{q}\,q >$$

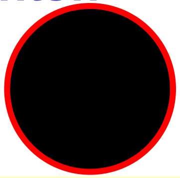
with

$$< m_q\,\bar{q}\,q >= -f_{\pi}^2\, m_{\pi}^2 \sim 10^{-4}\, GeV^4$$

$$< \frac{\beta_3(g_s)}{2\,g_s} F^{\mu\nu}_{a}\, F^{a}_{\mu\nu} >\sim 10^{-2}\, GeV^4$$

$$\frac{<\text{QCD chiral}>}{\rho^0_{\Lambda}} \simeq 10^{43} \qquad \frac{<\text{QCD gluon}>}{\rho^0_{\Lambda}} \simeq 10^{45}$$

Aside the hot chocolade…the donoughts!!

Bubbles at the Planck scale !!:  M_P^4

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$\Downarrow$

10^{123}

Opening Pandora's box of Cosmology (DM+DE)!!...

SM (et al) becomes a "bit" shaken
and even badly beaten by cosmology issues ...
despite its present "glory" ...

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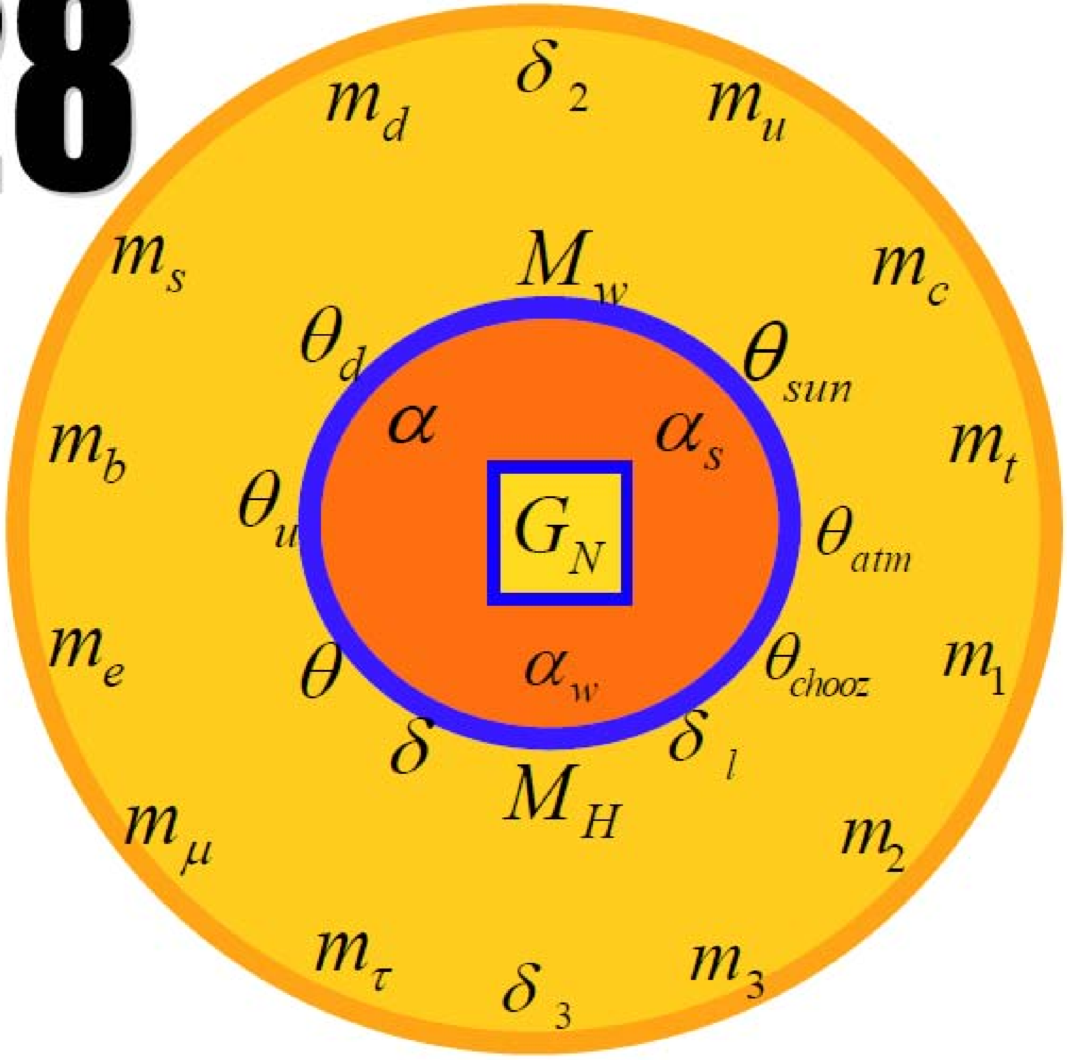

δ_2
m_d
m_u
m_s
M_W
m_c
θ_d
θ_{sun}
α
α_s
m_b
m_t
θ_u
G_N
θ_{atm}
m_e
θ
α_w
θ_{chooz}
m_1
δ
δ_l
M_H
m_μ
m_2
m_τ
δ_3
m_3

$+ \Lambda$ **?**

Λ in QFT: the Vacuum Energy

◇ Action integral for a scalar QFT:

$$S[\phi] = \int d^4x \sqrt{-g}\,\mathcal{L}(\phi, \partial_\mu \phi)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi - V_{eff}(\phi)$$

◇ Matter field energy-momentum tensor:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = 2 \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} - g_{\mu\nu}\, \mathcal{L}$$

$$= \left[\partial_\mu \phi\, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu}\ \partial_\alpha \phi\, \partial^\alpha \phi \right] + g_{\mu\nu}\, V_{eff}$$

◇ For static equilibrium configurations $\Rightarrow$

$$< T_{\mu\nu} >= g_{\mu\nu}\ < V_{eff} >$$

generic **QFT** **problem** **!!**

➢ ''Canonical'' definition of Dynamical Dark Energy

One popular possibility is the idea of quintessence, where there is no "true" Λ

The total energy-momentum tensor on the *r.h.s.* of Einstein eqs. is the sum

$$\tilde{T}_{\mu\nu} \equiv T_{\mu\nu}^{M} + T_{\mu\nu}^{D} \,.$$

One assumes that both tensors are separately conserved, and so $\bigtriangledown^{\mu}\,\tilde{T}_{\mu\nu} = 0$ is equivalent to

$$\bigtriangledown^{\mu}\,T_{\mu\nu}^{M} = 0 \Longleftrightarrow \frac{d\rho}{dt} + 3\,H\,(\rho + p) = 0,$$

and

(unmixed conservation laws)

$$\bigtriangledown^{\mu}\,T_{\mu\nu}^{D} = 0 \Longleftrightarrow \frac{d\rho_D}{dt} + 3\,H\,(\rho_D + p_D) = 0$$

• Subsequently one introduces an "effective" equation of state

$$p_D = \omega_D\, \rho_D$$

to describe a phenomenological relationship between p_D and ρ_D.

• Finally one also assumes that at present:

$$\omega_D = \frac{p_D}{\rho_D} = \frac{\frac{1}{2}\xi\dot{\chi}^2 - V(\chi)}{\frac{1}{2}\xi\dot{\chi}^2 + V(\chi)} \begin{cases} \gtrsim -1 & \xi > 0 \text{ (quintessence)} \\ \lesssim -1 & \xi < 0 \text{ (phantom DE)} \end{cases}$$

• For Λ the only possible equation of state is

$$p_\Lambda = -\rho_\Lambda\,.$$

Nice feature of quintessence field:

$$\omega_D = \frac{p_D}{\rho_D} = \frac{\frac{1}{2}\xi\dot{\chi}^2 - V(\chi)}{\frac{1}{2}\xi\dot{\chi}^2 + V(\chi)} \simeq -1 + \xi\,\dot{\chi}^2/V(\chi)$$

Problems with quintessence field: forgets SM vacuum!!

Even taking the simplest form $V(\chi) = (1/2)\, m_\chi^2\, \chi^2$

$$\rho_\Lambda^0 = V(\chi)$$

$$\chi \simeq M_X \simeq 10^{16-19}\, GeV \quad \Rightarrow \quad m_\chi \simeq H_0 \simeq 10^{-33}\, eV$$

$$\chi \simeq M_F = G_F^{-1/2} \quad \Rightarrow \quad m_\chi \simeq 10^{-12}\, eV$$

(Recall that $m_\Lambda \sim$ meV $\Rightarrow$ billion times) **!**

With phantoms the situation is worst:

$$\rho_X = \rho_X^0 \,(1+z)^{3\,(1+\omega_X)}\,, \qquad p = \omega_X \rho$$

$$-(\rho + 3p) \to -(1 + 3\,\omega_X)\rho > 0 \qquad \text{for } \omega_X < -1$$

increases with time. A planet in orbit R around a star of mass M will become unbound when

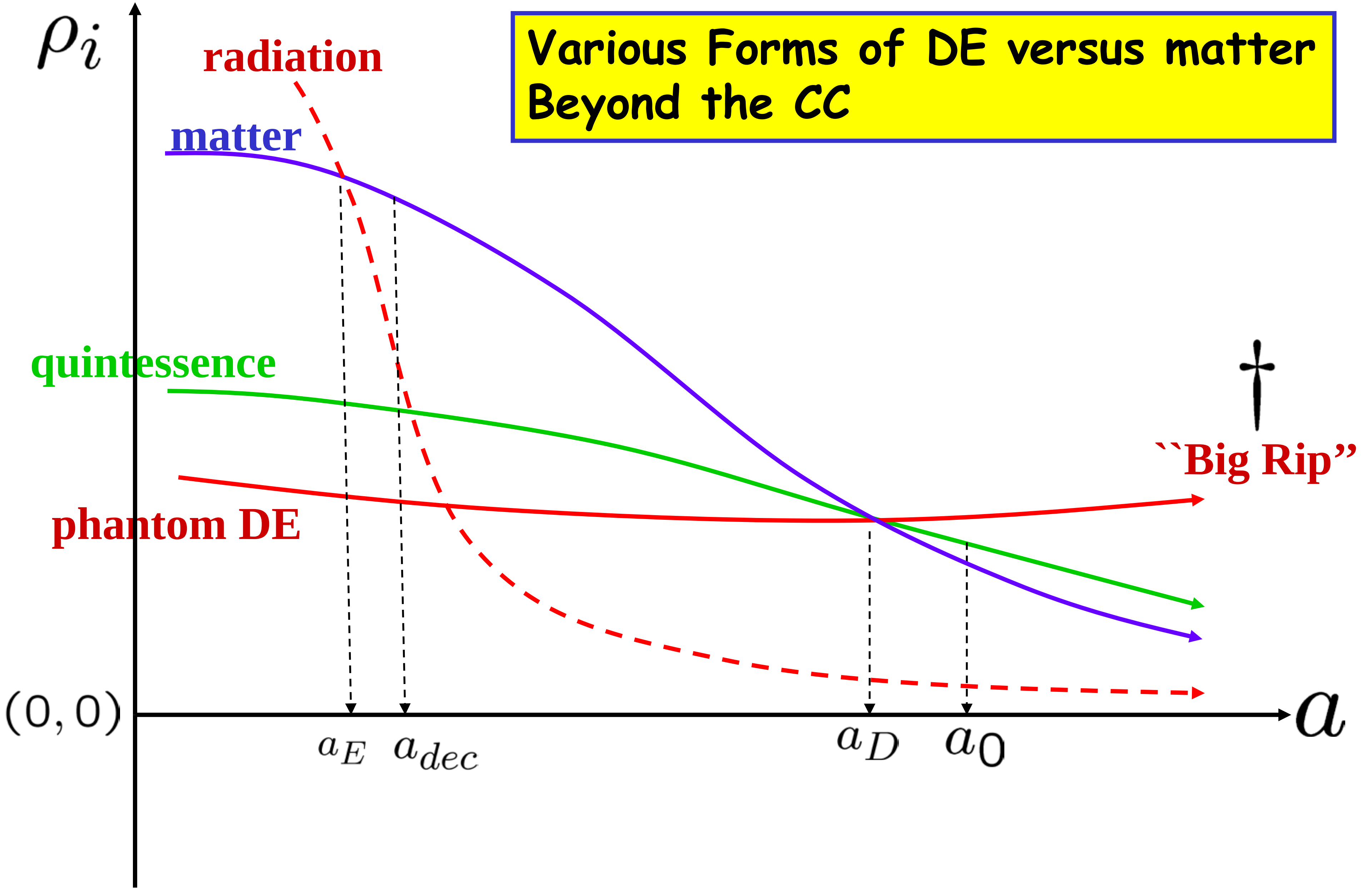
$$-\frac{4\pi}{3}\,(\rho + 3p)\,R^3 \simeq M$$

For $\omega_X > -1$ the LHS of the previous equation decreases with time, so if it was less than M it will remain always so. But if $\omega_X < -1$ it will surely overshoot the limit, and then

↓

``Big Rip"

Various Forms of DE versus matter Beyond the CC



Vacuum Energy and Quantum fluctuations

Heisenberg's Uncertainty Principle

$$\begin{cases} \Delta p \, \Delta x \sim \hbar \\ \\ \Delta E \, \Delta t \sim \hbar \end{cases}$$

Planck const. h $\boxed{\hbar = \frac{h}{2\pi}}$ (**quantum of action**)

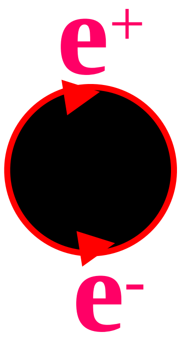
Zero point energy

Planck+Einstein:

$$E = \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} + \frac{1}{2}\hbar\,\omega$$

$$T = 0 \quad ?$$

$$E_0 = \frac{1}{2}\hbar\,\omega$$



Nullpunktsenergie

- For the average energy of an oscillator of frequency ω in equilibrium with radiation at temperature T, Planck obtained

$$E_\omega = \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} + \frac{1}{2}\,\hbar\omega$$

$$E_\omega \to (1/2)\ \hbar\omega \text{ for } T \to 0 \quad \textit{Nullpunktsenergie}$$ **!!**

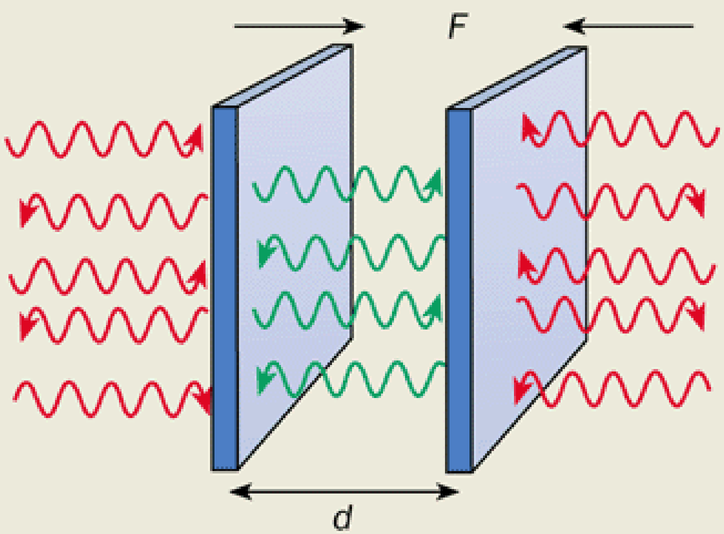
Then Einstein and Stern also noted:

$$kT \gg \hbar\omega \implies E_\omega \simeq kT - (1/2)\ \hbar\omega + (1/2)\ \hbar\omega = kT$$

classical limit **!!**

- First quantum implications of the **ZPE** date back to W. Nernst in 1916, and later by Pauli in the 1920s and then in 1933

Hendrik Casimir (1909-2000)



Casimir effect

Proposed: 1948

Electrically neutral conducting plates attract each other in the vacuum!

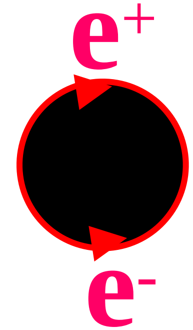
Measuring the energy of the quantum vacuum !!

$$E_0(k) = \frac{1}{2}\hbar\,\omega_k$$

$$\omega_n = c\sqrt{k_x^2 + k_y^2 + \frac{n^2\pi^2}{d^2}}$$

$$E_0 = \frac{1}{2}\hbar\sum_k \omega_k \rightarrow \frac{1}{2}\hbar\, A\int \frac{dk_x dk_y}{(2\pi)^2}\;\omega_n$$

The Casimir effect



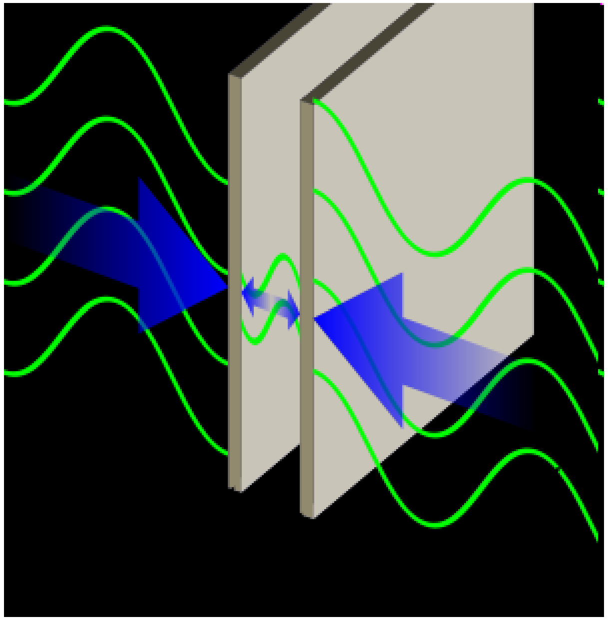
- has been claimed since long as an spectacular evidence of the quantum vacuum. S. Weinberg, in his famous cosmological constant review *

"Perhaps surprisingly, it was a long time before particle physicists began seriously to worry about this problem, despite the demonstration in the Casimir effect of the reality of zero-point energies."

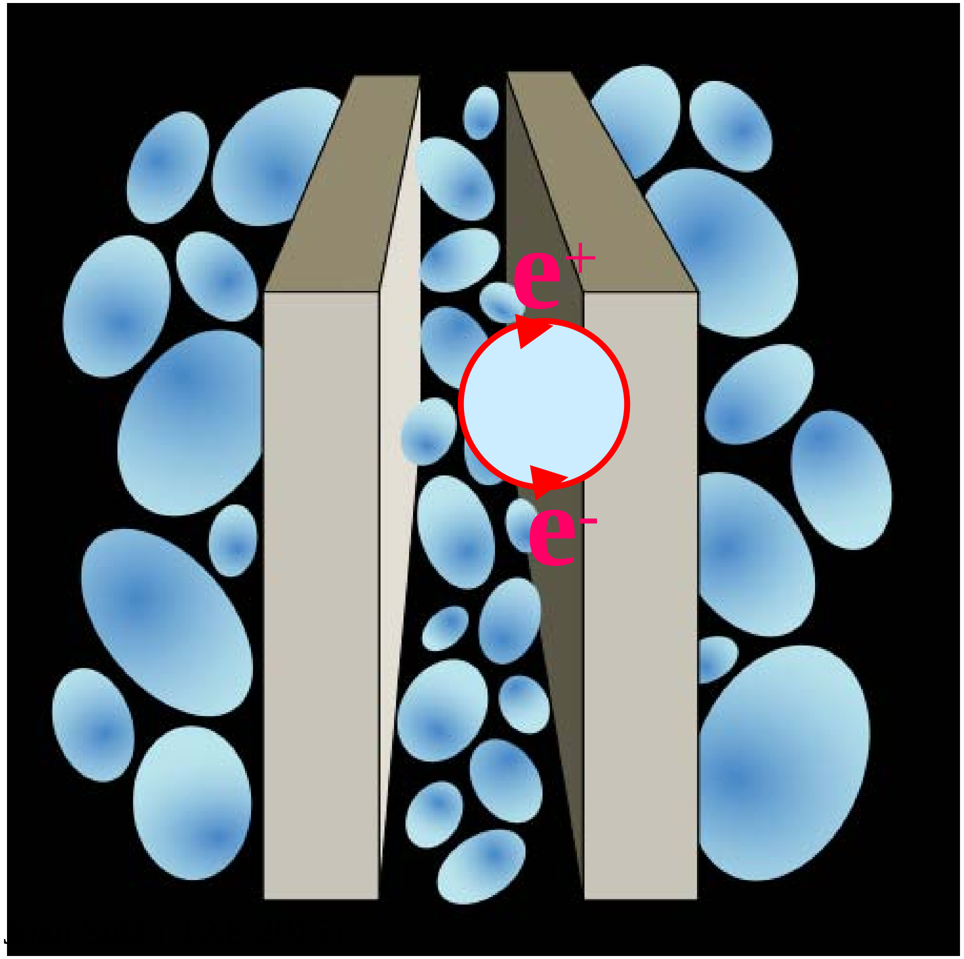
- In 1997 S. Lamoreaux was able to perform a measurement of the Casimir force between a sphere and a plate using a torsion pendulum to an unprecedented accuracy of $\sim 5\%$
- Shortly afterwards Mohideen and Roy reached 1% accuracy

*S. Weinberg, *The Cosmological Constant Problem*, Rev. Mod. Phys. **61** (1989) 1

Casimir effect



$$\frac{F}{A} = -\frac{\hbar c \pi^2}{240}\frac{1}{d^4}$$ (pressure)



$$F \sim -\frac{\hbar}{d^4}$$

Mesurements: 1958, 1997, 2001…

➢ The old CC problem as a fine tuning problem

The CC problem stems from realizing that the effective or physical vacuum energy is the sum of two terms:

$$\rho_{\Lambda \mathrm{phys}} = \rho_{\Lambda \mathrm{vac}} + \rho_{\Lambda \mathrm{ind}}$$

$$S_{EH} = \frac{1}{16\pi G_N}\int d^4x\sqrt{|g|}\,(R - 2\Lambda_{\mathrm{vac}}) = \int d^4x\sqrt{|g|}\left(\frac{1}{16\pi G_N}R - \rho_{\Lambda \mathrm{vac}}\right)$$

$$\rho_{\Lambda \mathrm{vac}} = \frac{\Lambda}{8\pi G_N}$$

Vacuum bare term in Einstein eqs.

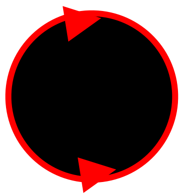
$$R_{ab} - \frac{1}{2}g_{ab}R = -8\pi G_N\left(\langle \tilde{T}_{ab}^{\varphi}\rangle + T_{ab}\right) = -8\pi G_N\, g_{ab}\,(\rho_{\Lambda \mathrm{vac}} + \rho_{\Lambda \mathrm{ind}} + T_{ab})$$

Quantum effects $\Rightarrow$ $\rho_{\Lambda \mathrm{ind}} = \langle V(\varphi)\rangle + \mathrm{ZPE}$

Zero-point energy in quantum field theory in flat spacetime

JS, arXiv:1306.1527

$$V_{\mathrm{ZPE}}(P) = \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} + \ldots.$$



Real scalar field, one loop:

$$V_P^{(1)} = (1/2)\sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} = (1/2)\sum_{\mathbf{k}} \hbar\sqrt{\mathbf{k}^2+m^2} \;\rightarrow$$

$$= \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2+m^2} = \frac{1}{4\pi^2}\int_0^{\Lambda_{\mathrm{UV}}} dk\, k^2\sqrt{k^2+m^2}$$

$$= \frac{\Lambda_{\mathrm{UV}}^4}{16\pi^2}\left(1+\frac{m^2}{\Lambda_{\mathrm{UV}}^2}-\frac{1}{4}\frac{m^4}{\Lambda_{\mathrm{UV}}^4}\ln\frac{\Lambda_{\mathrm{UV}}^2}{m^2}+\cdots\right),$$

renormalization

$$V_P^{(1)\mathrm{renorm}} = -\frac{m^4}{64\pi^2}\ln\frac{\mu^2}{m^2}+\cdots$$

(Pauli's result was unrenormalized)

- In dimensional regularization:

$$V_P^{(1)} = \frac{1}{2}\,\mu^{4-n}\int\frac{d^{n-1}k}{(2\pi)^{n-1}}\,\sqrt{\mathbf{k}^2+m^2} = \frac{1}{2}\,\beta_\Lambda^{(1)}\,\left(-\frac{2}{4-n}-\ln\frac{4\pi\mu^2}{m^2}+\gamma_E-\frac{3}{2}\right)$$

with
$$\beta_\Lambda^{(1)} = \frac{m^4}{2\,(4\pi)^2}$$
(β-function coeff. for running Λ)

How to get rid of the UV-part? Recall that

$$S_{\rm EH} = \frac{-1}{16\pi\,G^{(\rm b)}}\int d^4x\sqrt{-g}\,\left(R+2\,\Lambda^{(\rm b)}\right) = -\int d^4x\sqrt{-g}\,\left(\frac{1}{16\pi\,G^{(\rm b)}}\,R+\rho_\Lambda^{(\rm b)}\right)$$

Next we split $\rho_\Lambda^{(\rm b)}$ into $\rho_\Lambda^{(\rm b)} = \rho_\Lambda(\mu)+\delta\rho_\Lambda$

$$\delta\rho_\Lambda^{\overline{\rm MS}} = \frac{m^4\,\hbar}{4\,(4\pi)^2}\,\left(\frac{2}{4-n}+\ln 4\pi-\gamma_E\right)$$

We obtain:
$$\rho_\Lambda^{(\rm b)}+V_{\rm ZPE}^{(\rm b)} = \rho_\Lambda(\mu)+V_{\rm ZPE}(\mu)$$

where

$$V_{\rm ZPE}^{(1)}(\mu) = \hbar\, V_P^{(1)} + \delta\rho_\Lambda^{\overline{\rm MS}} = \frac{m^4\,\hbar}{4\,(4\,\pi)^2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

$$\rho_{\rm vac}^{(1)} = \rho_\Lambda(\mu) + V_{\rm ZPE}^{(1)}(\mu) = \rho_\Lambda(\mu) + \frac{m^4\,\hbar}{4\,(4\,\pi)^2}\left(\ln\frac{m^2}{\mu^2} - \frac{3}{2}\right)$$

overall μ-independent (RG-invariance)

$$d\rho_{\rm vac}^{(1)}/d\ln\mu = 0 \qquad \Rightarrow \qquad \mu\frac{d\rho_\Lambda(\mu)}{d\mu} = \frac{\hbar\, m^4}{2\,(4\pi)^2} = \beta_\Lambda^{(1)}$$

We have found the expected result:

$$V_P^{(1)\rm renorm} = -\frac{m^4}{64\pi^2}\,\ln\frac{\mu^2}{m^2} + \cdots$$

- Alternative calculation that generalizes to curved space-time:

  Effective action at one'loop:

$$\begin{cases} \Gamma_{\rm eff}[\phi_c] = S[\phi_c] + \dfrac{i\,\hbar}{2}\, Tr\,\ln \mathcal{K}(x,x') \\ \mathcal{K}(x,x') = \left[\Box_x + V''(\phi_c)\right]\,\delta(x-x') \\ \displaystyle\int d^4x'\,\mathcal{K}(x,x')\,G_F(x',x'') = -\delta(x-x'') \quad \Leftrightarrow \quad \mathcal{K} = -G_F^{-1} \end{cases}$$

$$S[\phi_c] = \int d^4x\,\mathcal{L} = \int d^4x\,\left[\frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi_c\,\partial_\nu\phi_c - V_c(\phi_c)\right]$$

$$V(\phi) = \frac{1}{2}\,m^2\,\phi^2 + \frac{1}{4!}\,\lambda\,\phi^4$$

For a free field $(\lambda = 0)$ $\Rightarrow$ $\mathcal{K}(x,x') = \left[\Box_x + m^2\right]\,\delta(x-x')$

In dim. regularization: $Tr\,\ln\mathcal{K} = \Omega\displaystyle\int \frac{d^nk}{(2\pi)^n}\,\ln\left(-k^2+m^2\right)$

Setting ϕ_c =const. $S[\phi_c] = -\int d^4x V(\phi_c) = -\Omega V(\phi_c)$

$$V^{(1)} = -\frac{i}{2}\,\Omega^{-1}\,Tr\ln\mathcal{K}$$

$$= -\frac{i}{2}\int\frac{d^4k}{(2\pi)^4}\ln\left(-k^2+m^2\right) = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\oint\frac{d\tilde{k}_0}{2\pi}\ln\left(\tilde{k}_0^2+\omega_{\mathbf{k}}^2\right)$$

$$= \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\oint\frac{d\tilde{k}_0}{2\pi}\int_0^1 dz\,\frac{\omega_{\mathbf{k}}^2}{\tilde{k}_0^2+\omega_{\mathbf{k}}^2\,z}$$

$$= \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\,\omega_{\mathbf{k}} = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\sqrt{\mathbf{k}^2+m^2}$$

or using:

$$\mu^{4-n}\int\frac{d^nk}{(2\pi)^n}\ln\left(-k^2+A^2\right) \stackrel{(n\to 4)}{\longrightarrow} \left(\frac{i}{32\pi^2}\right)\left(A^2\right)^2\left(\frac{-2}{4-n}-\ln\frac{4\pi\mu^2}{A^2}+\gamma_E-\frac{3}{2}\right)$$

➢ “Fine” details of the **Fine tuning problem** !!

A little bit more of QFT stuff...

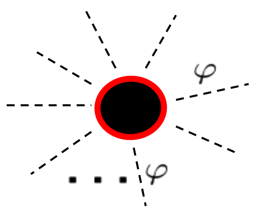
JS, arXiv:1306.1527

Take a scalar QFT with effective potential

$$V_{\rm eff} = V + \hbar\, V_1 + \hbar^2\, V_2 + \hbar^3\, V_3 + \ldots$$

where

$$V_1 = V_P^{(1)} + V_{\rm scal}^{(1)}(\varphi)\,, \qquad V_2 = V_P^{(2)} + V_{\rm scal}^{(2)}(\varphi)\,, \qquad V_3 = V_P^{(3)} + V_{\rm scal}^{(3)}(\varphi) \ldots .$$

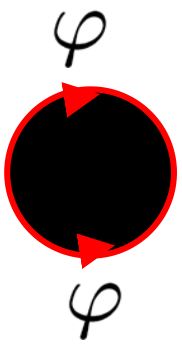


Thus,

$$\boxed{V_{\rm eff}(\varphi) = V_{\rm ZPE} + V_{\rm scal}(\varphi)}$$

with

$$V_{\rm ZPE} = \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} + \ldots .$$



The “Mother” of all the fine tuning problems

Putting everything together:

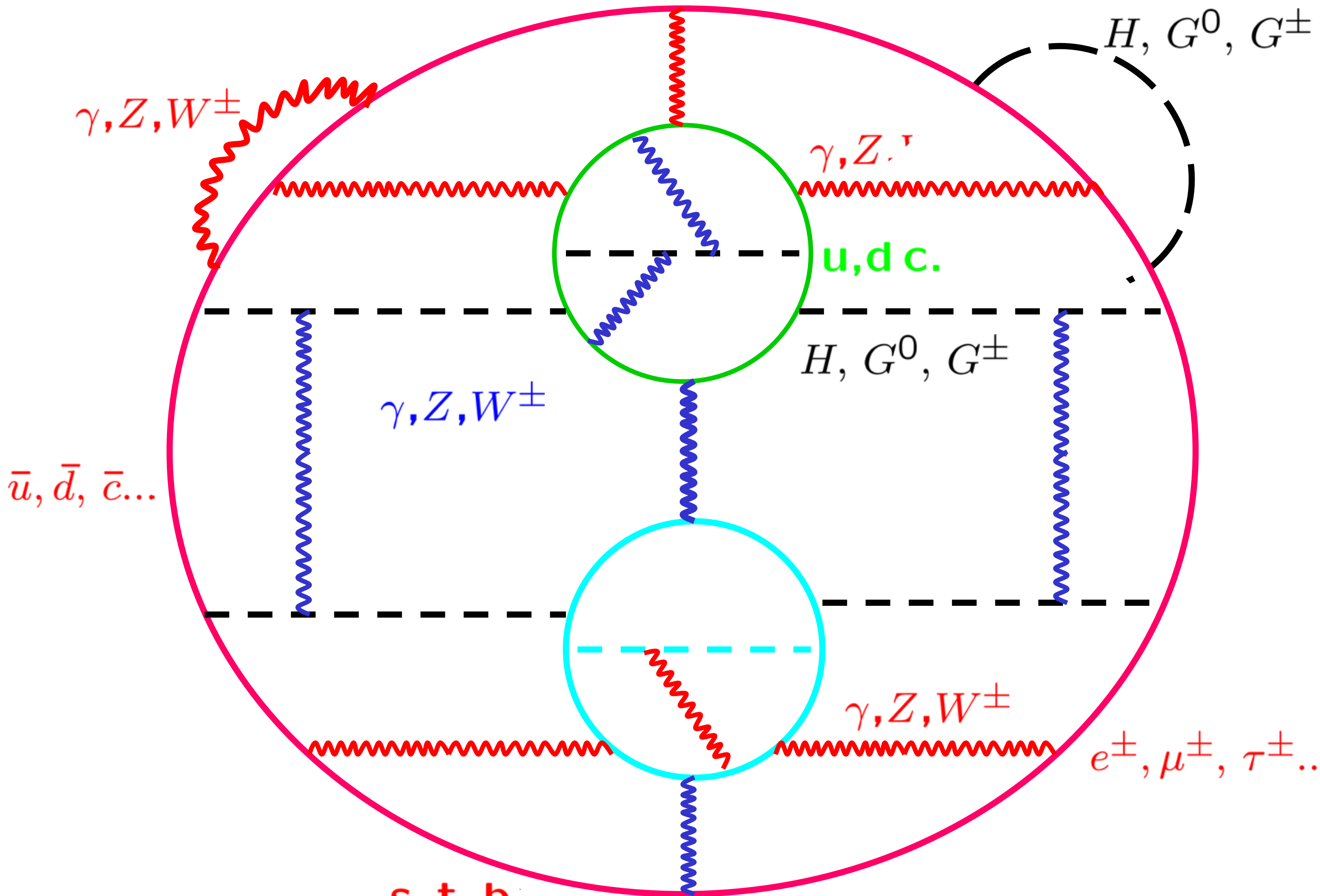
$$\rho_{\Lambda \mathrm{ph}} = \rho_{\Lambda \mathrm{vac}} + \rho_{\Lambda \mathrm{ind}} = \rho^{\mathrm{ren}}_{\Lambda \mathrm{vac}} + \langle V^{\mathrm{ren}}_{\mathrm{eff}}(\varphi)\rangle$$
$$= \rho^{\mathrm{ren}}_{\Lambda \mathrm{vac}} + V^{\mathrm{ren}}_{\mathrm{ZPE}} + \langle V^{\mathrm{ren}}_{\mathrm{scal}}(\varphi)\rangle$$

$$10^{-47}\,\mathrm{GeV}^4 = \rho_{\Lambda \mathrm{vac}} - 10^8\,\mathrm{GeV}^4 + \hbar\, V_P^{(1)} + \hbar^2\, V_P^{(2)} + \hbar^3\, V_P^{(3)} \ldots.$$
$$+ \hbar\, V^{(1)}_{\mathrm{scal}}(\varphi) + \hbar^2\, V^{(2)}_{\mathrm{scal}}(\varphi) + \hbar^3 V^{(3)}_{\mathrm{scal}}(\varphi) \ldots$$

With $v \sim 100$ GeV, which is the highest loop involved?:

$$\left(\frac{g^2}{16\,\pi^2}\right)^n v^4 = 10^{-47}\, GeV^4 \qquad \Rightarrow \boxed{n \simeq 21} \; !!$$

21th loop (one among many thousands...)



JS arXiv:1306.1527

Running Λ in **QFT**

- All the divergences in this theory can be removed by the renormalization of the bare matter fields, matter couplings, ξ, vacuum parameters $a_{1,2,3,4}$ and G_v , Λ_v.

$$S = S_{matt} + S_{nonmin} + S_v$$

where

$$S_{matt} = \int d^4x\sqrt{-g}\,\Big\{g^{\mu\nu}\,(D_\mu\Phi)^+(D_\nu\Phi) - V_0(\Phi)\Big\} + ...$$

with

$$S_{nonmin} = \int d^4x\sqrt{-g}\,\xi\;\Phi^\dagger\Phi\,R,$$

and

$$S_v = \int d^4x\sqrt{-g}\,\{\,a_1 R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} + a_2 R^{\mu\nu}R_{\mu\nu} + a_3 R^2 + a_4\Box R + \frac{1}{16\pi G_v}R - \Lambda_v\Big\}.$$

➢ Dynamical vacuum models with variable G *

JS, A. Gómez-Valent, J. de Cruz Pérez
arXiv:1506.05793, ApJ Lett. 2015

$$G1: \qquad \Lambda(H) = 3(c_0 + \nu H^2)$$

$$G2: \qquad \Lambda(H, \dot{H}) = 3(c_0 + \nu H^2 + \frac{2}{3}\alpha\, \dot{H})$$

$$\dot{\rho}_m + 3H\rho_m = 0 \text{ and } \dot{\rho}_r + 4H\rho_r = 0$$

Bianchi identity: $\nabla^\mu (G T_{\mu 0}) = 0$

$$\dot{G}(\rho_m + \rho_r + \rho_\Lambda) + G\dot{\rho}_\Lambda = 0$$

* cf. Sahni, Shafieloo & Starobinsky, arXiv:1406.2209 (ApJ Lett. 2014)

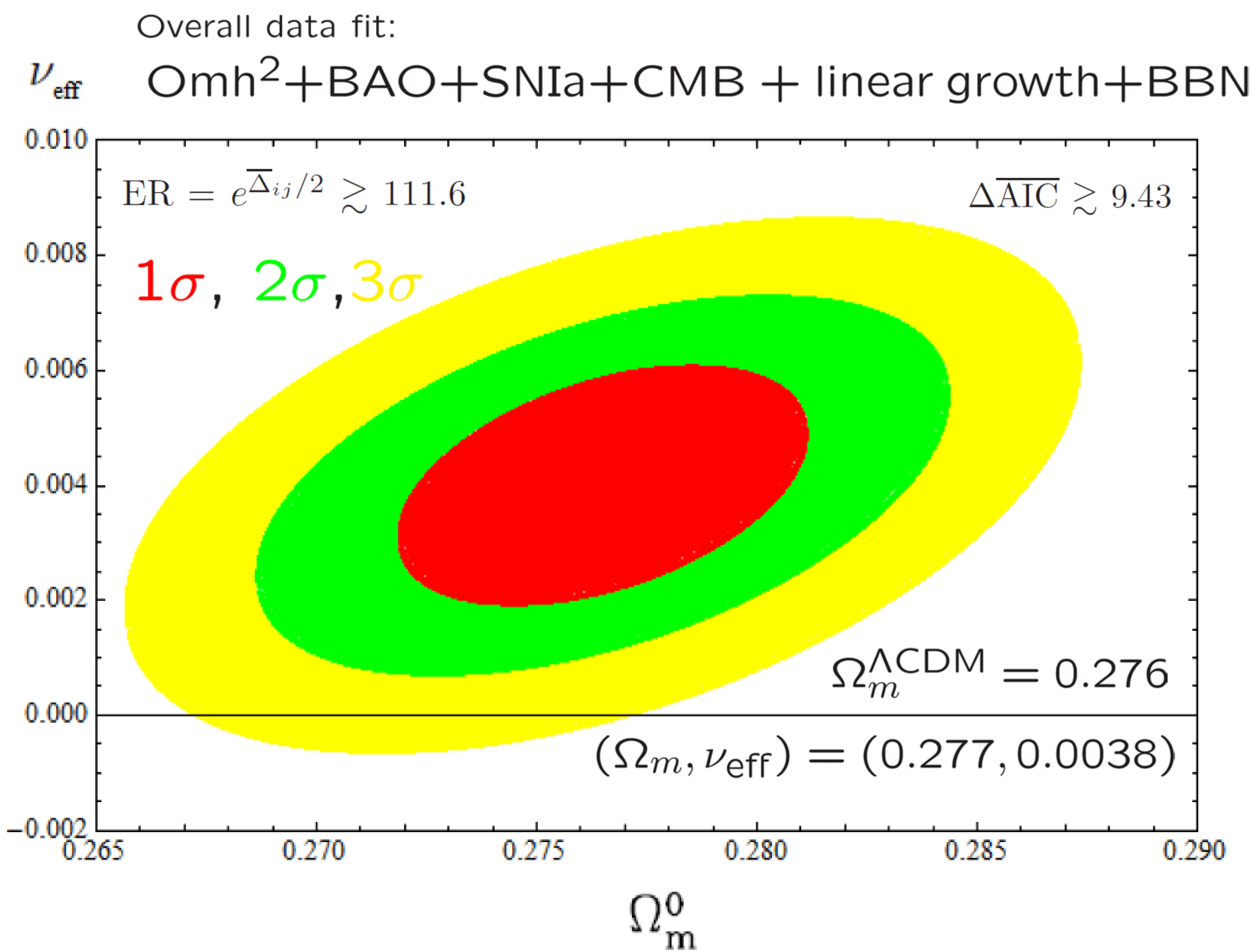
Solution of the cosmological equations:

$(E \equiv H/H_0)$

$$E^2(a) = 1 + \frac{\Omega_m}{\xi}\left[-1 + a^{-4\xi'}\left(a + \frac{\xi\,\Omega_r}{\xi'\Omega_m}\right)^{\frac{\xi'}{1-\alpha}}\right]$$

$$\xi = \frac{1-\nu}{1-\alpha} \equiv 1 - \nu_{\rm eff}\,, \quad \xi' = \frac{1-\nu}{1-\frac{4}{3}\alpha} \equiv 1 - \nu'_{\rm eff}$$

$$G(a) = G_0\, a^{4(1-\xi')} \simeq G_0(1 + 4\nu'_{\rm eff}\,\ln\, a)$$



Overall data fit:
ν_{eff}
Omh^2+BAO+SNIa+CMB + linear growth+BBN
$ER = e^{\overline{\Delta}_{ij}/2} \gtrsim 111.6$
$\Delta\overline{AIC} \gtrsim 9.43$
1σ, 2σ, 3σ
$\Omega_m^{\Lambda CDM} = 0.276$
$(\Omega_m, \nu_{eff}) = (0.277, 0.0038)$
0.010
0.008
0.006
0.004
0.002
0.000
−0.002
0.265
0.270
0.275
0.280
0.285
0.290
Ω_m^0

Joan Solà (TAE 2015)