Poincaré invariance in low-energy EFTs for QCD

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Taller de Altas Energias 2015, Benasque

Technische Universität München

MAX-PLANCK-GESELLSCHAFT

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- Variety of EFTs of QCD depending on the physical process
- In this talk, focus on a pair of heavy quark and anti-quark bound state (e.g, bottomonium, charmonium)

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- Limitation: not the most useful theory for transitions in quarkonium states (i.e., mass splitting)

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- Operators come with Wilson coefficients

Wilson coefficients in EFT

Wilson coefficients c_n are undetermined scalar functions in front of the series of operators in EFT (e.g., HQET/NRQCD):

$$\mathcal{L}_{NRQCD} = \sum_n \frac{c_n \mathcal{O}_n}{M^{d_n - 4}}, \quad \text{where} \quad [\mathcal{O}_n] = d_n$$

- $\mathcal{O}_n$ consists of effective DOF

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- Poincaré invariance from the UV theory is the symmetry we use here

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- Field redefinitions and constraints in pNRQCD
- Outlook (SCET)

QFT under Poincaré group

- Quantum field under the Lorentz: $\psi_a \to M(\Lambda)_{ab}\psi_b(\Lambda^{-1}x)$ with $M(\Lambda)$ a representation of the Lorentz group
- Infinitesimal form: $\delta\psi = i(a_0 h - \mathbf{a}\cdot\mathbf{p} - \boldsymbol{\theta}\cdot\mathbf{j} + \boldsymbol{\eta}\cdot\mathbf{k})\psi$
- boost generator: $\mathbf{k} = \mathbf{r}h - t\mathbf{p} \pm i\boldsymbol{\Sigma}$
- Generic quantum field under the spatial boost $\mathcal{B}$:

$$\psi_a(x) \to (e^{\mp\boldsymbol{\eta}\cdot\boldsymbol{\Sigma}})_{ab}\psi_b(\mathcal{B}^{-1}x)$$

QM under Poincaré group [Weinberg, 1995]

- A momentum eigenstate in Hilbert space $P^\mu \Psi_{p,\sigma} = p^\mu \Psi_{p,\sigma}$, transforms under Lorentz group

$$U(\Lambda)\Psi_{p,\sigma} = \sum_{\sigma'} C_{\sigma',\sigma}(\Lambda, p)\Psi_{\Lambda p,\sigma'}$$

- For fixed reference frame (or momentum) k, the *little group* W given, s.t. $Wk = k$.
- Representation of little group: $\Psi_{k,\sigma}$
- Lorentz transformation L, $L(p)k = p$, gives generic representation: $\Psi_{p,\sigma} = U(L(p))\Psi_{k,\sigma}$

$$U(\Lambda)\Psi_{p,\sigma} = \sum_{\sigma'} D_{\sigma',\sigma}(W)\Psi_{\Lambda p,\sigma'}$$

- Little group element: $W = L^{-1}(\Lambda p)\Lambda L(p)$

Induced representation [Heinonen, Hill, Solon, 2012]

Transformation of a free massive field with mass M through the induced representation

$$\psi_a(x) \to D[W(\Lambda, i\partial)]_{ab}\psi_b(\Lambda^{-1}x)$$

where D the representation of the little group, and W the little group element associated with the Lorentz transformation Λ. The transformation of the field in particular under the Lorentz boost:

$$\psi_a(x) \to \exp\left[\mp\boldsymbol{\eta}\cdot\left(\frac{\boldsymbol{\Sigma}\times\boldsymbol{\partial}}{M+\sqrt{M^2-\boldsymbol{\partial}^2}}\right)\right]_{ab}\psi_b(\mathcal{B}^{-1}x)$$

with the reference frame chosen $v = (1,0,0,0)$ (e.g., rest frame of heavy particle).

NR field under little group

- Free field under the little group:

$$\psi_a(x) \to \Big\{1 + iM\boldsymbol{\eta}\cdot\mathbf{x} - \frac{i\boldsymbol{\eta}\cdot\boldsymbol{\partial}}{2M} - \frac{i\boldsymbol{\eta}\cdot\boldsymbol{\partial}\boldsymbol{\partial}^2}{4M^3} + \frac{(\boldsymbol{\Sigma}\times\boldsymbol{\eta})\cdot\boldsymbol{\partial}}{2M}\Big[1 + \frac{\boldsymbol{\partial}^2}{4M^2}\Big] + \mathcal{O}(1/M^4)\Big\}\psi_a(\mathcal{B}^{-1}x)$$

- Interacting field inspired by the induced representation (postulate):

$$\psi_a(x) \to \Big\{1 + iM\boldsymbol{\eta}\cdot\mathbf{x} - \frac{i\boldsymbol{\eta}\cdot\boldsymbol{D}}{2M} - \frac{i\boldsymbol{\eta}\cdot\boldsymbol{D}\boldsymbol{D}^2}{4M^3} + \frac{(\boldsymbol{\Sigma}\times\boldsymbol{\eta})\cdot\boldsymbol{D}}{2M} + \frac{(\boldsymbol{\Sigma}\times\boldsymbol{\eta})\cdot\boldsymbol{D}}{2M}\frac{\boldsymbol{D}^2}{4M^2} + \mathcal{O}(g, 1/M^4)\Big\}\psi_a(\mathcal{B}^{-1}x)$$

NRQCD - Lagrangian [Caswell, Lepage, 1986]

Bilinear sector of the NRQCD Lagrangian up to $1/M^2$:

$$
\begin{aligned}
\mathcal{L}_{NRQCD} \ni\ & \psi^\dagger \Big\{ i\mathbf{D}_0 + c_1 \frac{\mathbf{D}^2}{2M} + c_2 \frac{\mathbf{D}^4}{8M^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2M} \\
& + c_D' g \frac{[\mathbf{D}\cdot, \mathbf{E}]}{8M^2} + i c_s g \frac{\boldsymbol{\sigma}\cdot[\mathbf{D}\times, \mathbf{E}]}{8M^2} \Big\} \psi \\
& + \chi^\dagger \Big\{ i\mathbf{D}_0 - c_1 \frac{\mathbf{D}^2}{2M} - c_2 \frac{\mathbf{D}^4}{8M^3} - c_F g \frac{\boldsymbol{\sigma}\cdot\mathbf{B}}{2M} \\
& + c_D' g \frac{[\mathbf{D}\cdot, \mathbf{E}]}{8M^2} + i c_s g \frac{\boldsymbol{\sigma}\cdot[\mathbf{D}\times, \mathbf{E}]}{8M^2} \Big\} \chi
\end{aligned}
$$

- ψ annihilates a heavy quark
- χ creates a heavy anti-quark

Field transformation

- Transformation under induced representation:

$$\psi(x) \rightarrow \Big\{1 + iM\boldsymbol{\eta}\cdot\mathbf{x} - \frac{i}{2M}\boldsymbol{\eta}\cdot\mathbf{D} + \frac{1}{4M}\boldsymbol{\eta}\cdot(\mathbf{D}\times\boldsymbol{\sigma}) + \mathcal{O}(g, M^{-2})\Big\}\psi(x')$$

$$\chi(x) \rightarrow \Big\{1 - iM\boldsymbol{\eta}\cdot\mathbf{x} + \frac{i}{2M}\boldsymbol{\eta}\cdot\mathbf{D} - \frac{1}{4M}\boldsymbol{\eta}\cdot(\mathbf{D}\times\boldsymbol{\sigma}) + \mathcal{O}(g, M^{-2})\Big\}\chi(x')$$

- "Left over" terms in the Lagrangian up to 1/M :

$$\delta\mathcal{L}_{2\psi} = \psi^\dagger\Big[i(1-c_1)\boldsymbol{\eta}\cdot\mathbf{D} - \frac{1}{2M}(1-c_1)\{D_0, \boldsymbol{\eta}\cdot\mathbf{D}\} + \frac{1}{4M}(1-2c_F+c_s)\boldsymbol{\eta}\cdot(g\mathbf{E}\times\boldsymbol{\sigma})\Big]\psi$$

Constraints on the Wilson coefficients in NRQCD

Constraints on the Wilson coefficients by the invariance of the Lagrangian, $\delta\mathcal{L}_{2\psi} = 0$

- $c_1 = 1, c_s = 2c_F - 1$
- Coincides with the literature [Brambilla, Gromes, Vairo, 2003]
- Relations valid at higher loop matching calculations

pNRQCD - power counting

Potential non-relativistic QCD (pNRQCD) given by integrating out the relative momentum, $Mv \sim 1/\mathrm{r}$, from NRQCD

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- Coordinates: $\mathbf{R} \equiv (\mathbf{x}_1 + \mathbf{x}_2)/2$, $\mathbf{r} \equiv \mathbf{x}_1 - \mathbf{x}_2$, t
- Power counting:

$$\begin{aligned} \boldsymbol{\nabla}_r, \frac{1}{r} &\sim Mv \\ \partial_0, \boldsymbol{\nabla}_R, A_\mu &\sim Mv^2 \\ \mathbf{E}, \mathbf{B} &\sim M^2 v^4 \end{aligned}$$

pNRQCD - field contents

I Quark-antiquark color singlet S and octet O^a configuration

II 2×2 spin matrix S_{ij}, O^a_{ij}, for quark spin i and antiquark spin j

III Coordinate dependence of the fields: $S = S(t, \mathbf{R}, \mathbf{r})$, $O^a = O^a(t, \mathbf{R}, \mathbf{r})$

IV Multipole expanded gluon fields $A^a_\mu(t, \mathbf{R})$

Writing S and O^a as 3×3 matrices in color space

$$S \to \frac{1}{\sqrt{3}} S\, I_3, \quad O^a \to O = \sqrt{2} O^a T^a$$

so that $\mathrm{Tr}[S^\dagger S] = S^\dagger S$, $\mathrm{Tr}[O^\dagger O] = O^{a\dagger} O^a$, $\mathrm{Tr}[S^\dagger \mathbf{E} O] = S^\dagger \mathbf{E}^a O^a$.

pNRQCD Lagrangian

Schematic form of the Lagrangian up to quadratic order in singlet and octet fields:

$$\mathcal{L}_{pNRQCD} \ni \int d^3x \mathrm{Tr} \left\{ S^\dagger \mathcal{K}_{SS} S + O^\dagger \mathcal{K}_{OO} O + [S^\dagger \mathcal{K}_{SO} O + \mathrm{H.C.}] \right\} - \frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu}(\mathbf{R}, t)$$

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- Integration over relative distance r
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- $\mathcal{K}$ includes charge conjuate terms
- F^2: ultrasoft gluons
- We focus on the $S^\dagger \mathcal{K}_{SS} S$ for the simplicity in this talk

$S^\dagger \mathcal{K}_{SS} S$ [Brambilla, Gromes, Vairo, 2003]

$$
\begin{aligned}
S^\dagger \mathcal{K}_{SS} S = S^\dagger \Big(& i\partial_0 + \frac{1}{2M}\left\{ c_s^{(1,-2)}, \boldsymbol{\nabla}_r^2 \right\} + \frac{c_s^{(1,0)}}{4M}\boldsymbol{\nabla}_R^2 - V_S^{(0)} \\
& - \frac{V_S^{(1)}}{M} + \frac{V_{p^2Sa}}{8M^2}\boldsymbol{\nabla}_R^2 + \frac{1}{2M^2}\left\{ \boldsymbol{\nabla}_r^2, V_{p^2Sb} \right\} + \frac{V_{L^2Sa}}{4M^2r^2}(\mathbf{r}\times\boldsymbol{\nabla}_R)^2 \\
& + \frac{V_{L^2Sb}}{4M^2r^2}(\mathbf{r}\times\boldsymbol{\nabla}_r)^2 - \frac{V_{S_{12}S}}{M^2r^2}\Big(3(\mathbf{r}\cdot\boldsymbol{\sigma}^{(1)})(\mathbf{r}\cdot\boldsymbol{\sigma}^{(2)}) - \mathbf{r}^2\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} \Big) \\
& - \frac{V_{S^2S}}{4M^2}\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} + \frac{iV_{LSSa}}{4M^2}(\mathbf{r}\times\boldsymbol{\nabla}_R)\cdot(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}) \\
& + \frac{V_{LSSb}}{4M^2}(\mathbf{r}\times\boldsymbol{\nabla}_r)\cdot(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}) \Big) S
\end{aligned}
$$

- (a,b): a denotes order of $1/M$, b denotes order of r
- $\boldsymbol{\sigma}^{(1/2)}$: spin matrix for quark/antiquark

Symmetries for the field transformation

We construct the most generalised form of the spatial boost for the singlet field with following criteria

- Right behaviors under C, P, T

$$\mathbf{k} \xrightarrow{P} -\mathbf{k}, \quad \mathbf{k} \xrightarrow{C} -\sigma_2 \mathbf{k}^* \sigma_2, \qquad \mathbf{k} \xrightarrow{T} \sigma_2 \mathbf{k} \sigma_2$$

- Proper order of truncation: include up to $1/M$ and r
- Same criteria apply to both singlet and octet fields

Generalised(!) boost transformation of singlet

$$S'(t,\boldsymbol{R},\boldsymbol{r}) = \Bigg(1 - 2iM\boldsymbol{\eta}\cdot\boldsymbol{R} + \frac{ik_D^{(1,0)}}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\left\{k_{a'}^{(1,0)}\boldsymbol{\eta}\cdot\boldsymbol{r},\boldsymbol{\nabla}_R\cdot\boldsymbol{\nabla}_r\right\}$$

$$+ \frac{i}{4M}\left\{k_{a''}^{(1,0)}\boldsymbol{r}\cdot\boldsymbol{\nabla}_R,\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\right\} + \frac{i}{4M}\left\{k_{a'''}^{(1,0)}\boldsymbol{r}\cdot,\boldsymbol{\nabla}_r(\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R)\right\} + \frac{i}{4M}\left\{\frac{k_b^{(1,0)}}{r^2}(\boldsymbol{\eta}\cdot\right.$$

$$- \frac{k_c^{(1,0)}}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right) + \frac{k_{d'}^{(1,0)}}{8Mr^2}(\boldsymbol{\eta}\cdot\boldsymbol{r}\times\boldsymbol{\nabla}_R)\left(\boldsymbol{r}\cdot\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)$$

$$- \frac{k_{d''}^{(1,0)}}{8Mr^2}\left(\boldsymbol{\eta}\cdot\boldsymbol{r}\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)(\boldsymbol{r}\cdot\boldsymbol{\nabla}_R) - \frac{1}{8M}\left\{k_a^{(1,-1)},\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2}\right.\right.$$

$$+ \frac{1}{8M}\left\{\frac{k_{b'}^{(1,-1)}}{r^2}\left(\boldsymbol{r}\cdot\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)(\boldsymbol{\eta}\times\boldsymbol{r})\cdot,\boldsymbol{\nabla}_r\right\}$$

$$- \frac{1}{8M}\left\{\frac{k_{b''}^{(1,-1)}}{r^2}\left(\boldsymbol{\eta}\cdot\boldsymbol{r}\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)\boldsymbol{r}\cdot,\boldsymbol{\nabla}_r\right\}\Bigg)S(t',\boldsymbol{R}',\boldsymbol{r}')$$

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- Take field redefinition by unitary transformation!

Field redefinition by unitary transformation

Define new singlet via $S = \mathcal{U}_S \widetilde{S}$

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Define new singlet via $S = \mathcal{U}_S \widetilde{S}$

- $\mathcal{U}_S$: unitary operator for singlet
- Natural choice for unitary operator: exponential function with operators on the exponents
- Exponents are to be anti-hermitian
- What about the order of $1/M$ expansion on the exponents?

Boost transformation of the "new" field

Boost transformation of the new field $\widetilde{S}$ determines the order of $1/M$ expansions of the unitary operator:

$$\begin{aligned}\widetilde{S}' =& \mathcal{U}_S'^{\dagger} S' = \mathcal{U}_S'^{\dagger}(1 - \boldsymbol{\eta}\cdot\boldsymbol{k})\mathcal{U}_S\widetilde{S} \\ =& [1 - \mathcal{U}_S^{\dagger}(i\boldsymbol{\eta}\cdot\boldsymbol{k})\mathcal{U}_S + (\delta\mathcal{U}_S^{\dagger})\mathcal{U}_S]\widetilde{S} \\ =& (1 - i\boldsymbol{\eta}\cdot\boldsymbol{k} - [2iM\boldsymbol{\eta}\cdot\boldsymbol{R}, \ln\mathcal{U}_S] + \mathcal{O}(1/M^2))\widetilde{S}\end{aligned}$$

- $\mathcal{U}_S'^{\dagger} = \mathcal{U}_S + \delta\mathcal{U}_S$ assumed

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- $\mathcal{U}_S'^{\dagger} = \mathcal{U}_S + \delta\mathcal{U}_S$ assumed
- Exponents of the unitary operator to be in the order of $1/M^2$

Unitary operator for singlet

Motivated by [Brambilla, Gromes, Vairo, 2003]

$$\begin{aligned}
\mathcal{U}_S = \exp\Bigg[& -\frac{1}{4M^2}\left\{q_{a'}^{(1,0)}\boldsymbol{r}\cdot\boldsymbol{\nabla}_R, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\right\} - \frac{1}{4M^2}\left\{q_{a''}^{(1,0)}\boldsymbol{r}\cdot\boldsymbol{\nabla}_R, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\right\} \\
& - \frac{1}{4M^2}\left\{q_{a'''}^{(1,0)}\boldsymbol{r}\cdot, \boldsymbol{\nabla}_r\right\}\boldsymbol{\nabla}_R^2 - \frac{1}{4M^2}\left\{\frac{q_b^{(1,0)}}{r^2}(\boldsymbol{r}\cdot\boldsymbol{\nabla}_R)^2\boldsymbol{r}\cdot, \boldsymbol{\nabla}_r\right\} \\
& + \frac{iq_{d''}^{(1,0)}}{8M^2r^2}\left(\boldsymbol{r}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)(\boldsymbol{r}\cdot\boldsymbol{\nabla}_R) \\
& + \frac{i}{8M^2}\left\{q_a^{(1,-1)}, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right\} \\
& - \frac{i}{8M^2}\left\{\frac{q_{b'}^{(1,-1)}}{r^2}\left(\boldsymbol{r}\cdot\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)(\boldsymbol{r}\times\boldsymbol{\nabla}_R)\cdot, \boldsymbol{\nabla}_r\right\} \\
& + \frac{i}{8M^2}\left\{q_{b''}^{(1,-1)}\left(\boldsymbol{r}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)\boldsymbol{r}\cdot, \boldsymbol{\nabla}_r\right\}\Bigg]
\end{aligned}$$

Shift in boost coefficients

Plugging this into $[2iM\boldsymbol{\eta}\cdot\boldsymbol{R}, \ln\mathcal{U}_S]$ from the boost transformation of $\widetilde{S}$, we observe the following shifts in boost coefficients

$$
\begin{aligned}
&k_{a'}^{(1,0)} \to k_{a'}^{(1,0)} - 2q_{a'}^{(1,0)} - 2q_{a''}^{(1,0)} \\
&k_{a''}^{(1,0)} \to k_{a''}^{(1,0)} - 2q_{a''}^{(1,0)} - 2q_{a'}^{(1,0)} \\
&k_{a'''}^{(1,0)} \to k_{a'''}^{(1,0)} - 4q_{a'''}^{(1,0)} \\
&k_{b}^{(1,0)} \to k_{b}^{(1,0)} - 4q_{b}^{(1,0)} \\
&\dots
\end{aligned}
$$

- q's are freely chosen

Shift in boost coefficients

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k_{a'''}^{(1,0)} &\to k_{a'''}^{(1,0)} - 4q_{a'''}^{(1,0)} \\
k_{b}^{(1,0)} &\to k_{b}^{(1,0)} - 4q_{b}^{(1,0)} \\
&\dots
\end{aligned}
$$

- q's are freely chosen
- Eliminate as many terms as possible to simplify the generalised expression of the boost

Intermezzo

After dropping tilde notation on singlet

$$
\begin{aligned}
S' = \Bigg(& 1 - 2iM\boldsymbol{\eta}\cdot\boldsymbol{R} + \frac{ik_D^{(1,0)}}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\left\{k_{a'}^{(1,0)}\boldsymbol{\eta}\cdot\boldsymbol{r}, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\right\} \\
& - \frac{k_c^{(1,0)}}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right) + \frac{k_{d'}^{(1,0)}}{8Mr^2}(\boldsymbol{\eta}\cdot\boldsymbol{r}\times\boldsymbol{\nabla}_R)\left(\boldsymbol{r}\cdot\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right) \\
& - \frac{k_{d''}^{(1,0)}}{8Mr^2}\left(\boldsymbol{\eta}\cdot\boldsymbol{r}\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)(\boldsymbol{r}\cdot\boldsymbol{\nabla}_R) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\Bigg) \\
& \times S(t,\boldsymbol{R}',\boldsymbol{r}') \equiv (1 - i\boldsymbol{\eta}\cdot\boldsymbol{k})S
\end{aligned}
$$

- 5 coefficients to be determined

Intermezzo

After dropping tilde notation on singlet

$$S' = \Bigg(1 - 2iM\boldsymbol{\eta}\cdot\boldsymbol{R} + \frac{ik_D^{(1,0)}}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\left\{k_{a'}^{(1,0)}\boldsymbol{\eta}\cdot\boldsymbol{r}, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\right\}$$

$$- \frac{k_c^{(1,0)}}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right) + \frac{k_{d'}^{(1,0)}}{8Mr^2}\left(\boldsymbol{\eta}\cdot\boldsymbol{r}\times\boldsymbol{\nabla}_R\right)\left(\boldsymbol{r}\cdot\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\right)$$

$$- \frac{k_{d''}^{(1,0)}}{8Mr^2}\left(\boldsymbol{\eta}\cdot\boldsymbol{r}\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right)\right)\left(\boldsymbol{r}\cdot\boldsymbol{\nabla}_R\right) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\Bigg)$$

$$\times S(t, \boldsymbol{R}', \boldsymbol{r}') \equiv (1 - i\boldsymbol{\eta}\cdot\boldsymbol{k})S$$

- 5 coefficients to be determined
- Any other constraints to impose?

1st constraint: commutation relation

- Commutation relation between boost generators $\boldsymbol{k}$

$$[1 - i\boldsymbol{\xi} \cdot \boldsymbol{k}, 1 - i\boldsymbol{\eta} \cdot \boldsymbol{k}]S \overset{!}{=} i(\boldsymbol{\xi} \times \boldsymbol{\eta}) \cdot \boldsymbol{j}S$$

fixes coefficients: $k_{a'}^{(1,0)} = k_{c}^{(1,0)} = 1$ and $k_{d'}^{(1,0)} = k_{d''}^{(1,0)} = 0$

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- Boost simplified further:

$$S' = \Bigg(1 - 2iM\boldsymbol{\eta} \cdot \boldsymbol{R} + \frac{ik_D^{(1,0)}}{4M}\boldsymbol{\eta} \cdot \boldsymbol{\nabla}_R + \frac{i}{4M}\{\boldsymbol{\eta} \cdot \boldsymbol{r}, \boldsymbol{\nabla}_r \cdot \boldsymbol{\nabla}_R\} - \frac{1}{8M}\boldsymbol{\eta} \cdot \boldsymbol{\nabla}_R \times \left(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}\right) - \frac{1}{4M}\boldsymbol{\eta} \cdot \boldsymbol{\nabla}_r \times \left(\boldsymbol{\sigma}^{(1)} - \boldsymbol{\sigma}^{(2)}\right)\Bigg) S$$

1st constraint: commutation relation

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$$S' = \Bigg(1 - 2iM\boldsymbol{\eta}\cdot\boldsymbol{R} + \frac{ik_D^{(1,0)}}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\{\boldsymbol{\eta}\cdot\boldsymbol{r}, \boldsymbol{\nabla}_r\cdot\boldsymbol{\nabla}_R\} - \frac{1}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times\left(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}\right) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times\left(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)}\right)\Bigg)S$$

- $k_D^{(1,0)}$ remains to be constrained

2nd constraint: Lorentz invariance

- After taking the boost transformation upon the Lagrangian

$$\begin{aligned}\delta\mathcal{L}_{2S} &= S^\dagger\Bigg(i\left(1-c_S^{(1,0)}\right)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R - \frac{1}{2M}\left(k_D^{(1,0)}-c_S^{(1,0)}\right)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\partial_0 \\ &\quad -\frac{i}{M}\left(V_{p^2Sa}+V_{L^2Sa}+\frac{1}{2}V_S^{(0)}\right)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R \\ &\quad +\frac{i}{Mr^2}\left(V_{L^2Sa}+\frac{r}{2}\partial_r V_S^{(0)}\right)(\boldsymbol{\eta}\cdot\mathbf{r})(\mathbf{r}\cdot\boldsymbol{\nabla}_R) \\ &\quad +\frac{1}{2M}\left(V_{LSSa}+\frac{1}{2r}\partial_r V_S^{(0)}\right)\boldsymbol{\eta}\cdot(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\times\mathbf{r}\Bigg)S\end{aligned}$$

is to vanish up to total derivatives.

2nd constraint: Lorentz invariance

- After taking the boost transformation upon the Lagrangian

$$
\begin{aligned}
\delta\mathcal{L}_{2S} \quad = \quad & S^\dagger\Big(i\Big(1-c_S^{(1,0)}\Big)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R - \frac{1}{2M}\Big(k_D^{(1,0)} - c_S^{(1,0)}\Big)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\partial_0 \\
& -\frac{i}{M}\Big(V_{p^2Sa} + V_{L^2Sa} + \frac{1}{2}V_S^{(0)}\Big)\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R \\
& +\frac{i}{Mr^2}\Big(V_{L^2Sa} + \frac{r}{2}\partial_r V_S^{(0)}\Big)(\boldsymbol{\eta}\cdot\mathbf{r})(\mathbf{r}\cdot\boldsymbol{\nabla}_R) \\
& +\frac{1}{2M}\Big(V_{LSSa} + \frac{1}{2r}\partial_r V_S^{(0)}\Big)\boldsymbol{\eta}\cdot(\boldsymbol{\sigma}^{(1)} - \boldsymbol{\sigma}^{(2)})\times\mathbf{r}\Big)S
\end{aligned}
$$

is to vanish up to total derivatives.

- Constraints on the Wilson coefficients are found:

$$
\begin{aligned}
k_D^{(1,0)} \quad &= \quad c_S^{(1,0)} = 1, \quad V_{p^2Sa} + V_{L^2Sa} + \frac{1}{2}V_S^{(0)} = 0, \\
V_{L^2Sa} \quad &= \quad -\frac{r}{2}\partial_r V_S^{(0)}, \quad V_{LSSa} = -\frac{1}{2r}\partial_r V_S^{(0)}
\end{aligned}
$$

Coda

Finalised version of the boost after free parameters are chosen

$$S'(t,\mathbf{R},\mathbf{r}) = \Big(1 - 2iM\boldsymbol{\eta}\cdot\mathbf{R} + \frac{i}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R + \frac{i}{4M}\{\boldsymbol{\eta}\cdot\mathbf{r},\boldsymbol{\nabla}_R\cdot\boldsymbol{\nabla}_r\}$$
$$-\frac{1}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})$$
$$+\mathcal{O}(1/M^3)\Big)S(t',\mathbf{R}',\mathbf{r}')$$

- But why choose the free parameters as was shown?

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- This matches to the one from *induced representation* from Wigner's little group formalism

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$$-\frac{1}{8M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_R\times(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}) - \frac{1}{4M}\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\times(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})$$
$$+\mathcal{O}(1/M^3)\Big)S(t',\mathbf{R}',\mathbf{r}')$$

- But why choose the free parameters as was shown?
- This matches to the one from *induced representation* from Wigner's little group formalism
- Bottomline: boost from induced representation is rather a particular choice in the non-interacting theory

Epilogue: another application

Soft collinear effective theory (SCET)

- B meson decay to light and energetic hadrons
- Different power counting scheme in EFT expansion: $\lambda = \Lambda_{QCD}/E$ instead of $1/M$
- DOF: collinear quarks and gluons, soft gluons
- Transformation of the field with similar scheme

Summary and Outlook

Summary

- Lagrangian structure of NRQCD (bilinear sector)
- Boost transformations in NRQCD and constraints on the Wilson coefficients
- Lagrangian structure of pNRQCD (bilinear sector)
- Boost transformations in pNRQCD and constraints on the Wilson coefficients

Outlook

- SCET (work in progress)
- Dark matter candidates (bottom-up approach in EFT)

Thank you for your attention.

Little group element

- $L(p)$ a rotation in the plane of $k/M \equiv w$ and $p/M \equiv v$:

$$L(w,v)^{\mu}_{\nu} = g^{\mu}_{\nu} - \frac{1}{1+v\cdot w}(w^{\mu}w_{\nu} + v^{\mu}v_{\nu}) + w^{\mu}v_{\nu} - v^{\mu}w_{\nu}$$

- For $\Lambda = \mathcal{B}(\eta)$, $\mathcal{B}(\eta)v = v + \eta$, and explicit form of the boost:

$$\mathcal{B}(\eta)^{\mu}_{\nu} = g^{\mu}_{\nu} - (v^{\mu}\eta_{\nu} - \eta^{\mu}v_{\nu}) + \mathcal{O}(\eta^2)$$

- Little group element for the infinitesimal boost:

$$W(\mathcal{B}(\eta),p) = 1 + \frac{i}{2}\left[\frac{1}{M+v\cdot p}(\eta^{\alpha}p_{\perp}^{\beta} - p_{\perp}^{\alpha}\eta^{\beta})\mathcal{J}_{\alpha\beta}\right] + \mathcal{O}(\eta^2)$$

where $p_{\perp}^{\beta} \equiv p^{\beta} - (v\cdot p)v^{\beta}$ and $(\mathcal{J}^{\alpha\beta})_{\mu\nu} = i(g^{\alpha}_{\mu}g^{\beta}_{\nu} - g^{\beta}_{\mu}g^{\alpha}_{\nu})$

Unitary operator: octet-octet

- Unitary operator for the octet - octet sector

$$\mathcal{U}_o = \exp\left[\frac{i}{2M}(-i\mathbf{D}_R\cdot(\tilde{\mathbf{k}}_{oo}^{(0,2)}+\tilde{\mathbf{k}}_{oo}^{(1,-1)}+\tilde{\mathbf{k}}_{oo}^{(1,0)})+h.c.\right]$$

- Boost on the octet after the field redefinition:

$$\begin{aligned}\widetilde{O}' = &\Big(1 - i\boldsymbol{\eta}\cdot\mathbf{k} - \frac{i}{8}\tilde{k}_a^{(0,2)}(\mathbf{r}\cdot g\mathbf{E})(\mathbf{r}\cdot\boldsymbol{\eta}) - \frac{i}{8}\tilde{k}_b^{(0,2)}\mathbf{r}^2(\boldsymbol{\eta}\cdot g\mathbf{E}) \\ &-\frac{i}{2M}\{\tilde{k}_{a'}^{(1,0)}\boldsymbol{\eta}\cdot\mathbf{r},\boldsymbol{\nabla}_r\cdot\mathbf{D}_R\} - \frac{i}{2M}\{\tilde{k}_{a''}^{(1,0)}\mathbf{r}\cdot\boldsymbol{D}_R,\boldsymbol{\eta}\cdot\boldsymbol{\nabla}_r\} \\ &+\cdots+\mathcal{O}\left(\frac{1}{M^2}\right)\Big)\widetilde{O}\end{aligned}$$

- Similar shift in the parameters

Constraints: octet-octet sector (1/2)

Octet - octet sector includes:

$$\begin{aligned}\mathcal{L}_{2O} \ni\; & O^\dagger\Big(\frac{i}{8M}V^{(1,0)}_{OOa}(r)\{\boldsymbol{\nabla}_r\cdot,\mathbf{r}\times g\mathbf{B}\}\\ & +\frac{1}{16M^2}\{(\boldsymbol{\nabla}_r\cdot\mathbf{D}_R),V^{(2,0)}_{OOc'}(r)(\mathbf{r}\cdot g\mathbf{E})\}\\ & +\frac{1}{16M^2}\{\boldsymbol{\nabla}^i_r\mathbf{D}^j_R,V^{(2,0)}_{OOc''}(r)\mathbf{r}^j g\mathbf{E}^i\}\\ & +\frac{1}{16M^2}\{\boldsymbol{\nabla}^i_r\mathbf{D}^j_R,V^{(2,0)}_{OOc'''}(r)\mathbf{r}^i g\mathbf{E}^j\}\\ & +\frac{1}{16M^2}\{\boldsymbol{\nabla}^i_r\mathbf{D}^j_R,\frac{V^{(2,0)}_{OOd}(r)}{r^2}\mathbf{r}^i\mathbf{r}^j(\mathbf{r}\cdot\mathbf{E})\}\Big)O+C.C.\end{aligned}$$

Constraints: octet-octet sector (2/2)

Constraints under the little group:

$$
\begin{aligned}
V^{(1,0)}_{OOa} + V^{(2,0)}_{OOc'} &= 0 \\
V^{(1,0)}_{OOa} - V^{(2,0)}_{OOc''} &= 2 \\
rV^{(2,0)}_{OOc'''} &= 0 \\
V^{(2,0)}_{OOd} &= 0 \qquad (1)
\end{aligned}
$$

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