Quantum Field Theory
TAE 2015

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Overview

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- Why is QFT the framework for the standard model which summarizes our present understanding of the physics at the smallest distances ?
- What are the theoretical reasons to doubt on the completeness of such understanding ?
- Why do we find ultraviolet divergences in QFT ?

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- Instead of considering a systematic treatment of the extension of what you have seen, including the renormalization of non-abelian gauge theories in the presence of the Higgs mechanism, which would require a whole set of lectures I will concentrate on some conceptual questions which are in my opinion essential to understand the present and future role of QFT in particle physics.

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- We will not see any detailed calculation. I hope this will be partially covered in other lectures in this TAE.

Introduction

First talk

- Free field theory
- Interactions: perturbation theory
- Diagrammatic representation

Second talk

- dimensional analysis and power counting $\rightarrow$ ultraviolet divergences
- Origin and interpretation
- Renormalization. Symmetries.

Third talk

- Effective field theories (EFT)
- Naturalness. Fine-tunning. Beyond Standard Model (BSM)
- Reduction of couplings. Approximate symmetries

- Free Relativistic field theory

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- Lagrangian formulation

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- Theory of a real scalar field (ϕ)

Lagrangian: $\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{m^2}{2}\phi^2$

Field equation: $\partial_t^2 \phi = \vec{\nabla}^2 \phi - m^2 \phi$

- Momentum space: $\phi(t,\vec{x}) \,=\, \int d^3p\, e^{i\vec{p}.\vec{x}}\, \tilde{\phi}_{\vec{p}}(t)$

  Field equation: $\partial_t^2 \tilde{\phi}_{\vec{p}} \,=\, -\omega_{\vec{p}}^2 \tilde{\phi}_{\vec{p}} \qquad \omega_{\vec{p}}^2 \,=\, \vec{p}^{\,2} + m^2$

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- Any free RQFT is a theory of free relativistic particles (different particles, spin).

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- QFT cannot be solved: approximation to the solution of the interacting theory.

- LSZ reduction formula

$$\int \prod_{i=1}^{m} d^4x_i e^{-ip_ix_i} \int \prod_{j=1}^{n} d^4y_j e^{iq_jy_j} < 0|T\{\phi(y_1)...\phi(y_n)\phi(x_1)...\phi(x_m)\}|0 >$$

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- Interaction picture: free fields ϕ_I

$$< 0|T\{\phi(y_1)...\phi(y_n)\phi(x_1)...\phi(x_m)\}|0 > =$$

$$\frac{< 0|T\{\phi_I(y_1)...\phi_I(y_n)\phi_I(x_1)...\phi_I(x_m)e^{\left[-i\int d^4x\,\mathcal{H}_I(x)\right]}\}|0 >}{< 0|T\{e^{\left[-i\int d^4x\,\mathcal{H}_I(x)\right]}\}|0 >}$$

First part: Interactions - perturbation theory

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the sum includes all possible pairings of N spacetime points

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- All one needs is the vacuum expectation value of the time ordered product of two free fields (Feynman propagator)

$$< 0|T\{\phi_I(x)\phi_I(y)\}0 > = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)}\, \frac{i}{k^2 - m^2 + i\epsilon}$$

limit when $x \to y$ is not well defined (product of local operators)

Diagramatic representation

spacetime

Each contribution to the vacuum expectation value of the time ordered product of fields can be represented by a two dimensional diagram whose points represent spacetime points:

- $\phi_I(y_1)...\phi_I(y_n)\phi_I(x_1)...\phi_I(x_m) \rightarrow (n+m)$ external lines ending at points in the diagram (vertices)
- $\mathcal{H}_I(x) \rightarrow$ vertex with as many lines as fields in $\mathcal{H}_I(x)$
- $< 0|T\{\phi_I(x)\phi_I(y)\}0 > \rightarrow$ internal line connecting two vertices.

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momentum space - Feynman rules

- External lines with momenta $q_1, ..., q_n, p_1, ..., p_m$
- Internal lines with (integrated) momentum $\rightarrow$ Propagator in momentum space.
- Integral of product of plane waves at each vertex $\rightarrow$ Momentum conservation

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- Loop expansion: number of undetermined momenta
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  number of loops (L) proportional to the number of vertices (V)

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- Subtlety: one can ignore all corrections to the external lines $\rightarrow$

  amputated diagrams

  Interaction on external lines $\rightarrow$ change in position of the pole and residue in the propagator $\rightarrow$ mass renormalization and renormalization factor Z in LSZ.

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- Feynman propagator for a Dirac field

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First part: Diagramatic representation

- Higher spin particles $\rightarrow$ Generalized Feynman propagator and LSZ formula.

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- Gauge theories : local dynamical symmetry.
  spin one massless particles → free vector field theory
  spin one massive particles → Higgs mechanism

  manifest relativistic invariance → new ingredients
  (gauge fixing, ghosts, BRST symmetry)

"Auxiliary" fields → extension of diagrammatic representation

For diagrams with loops ($L > 0$) integration over momenta can lead to divergences.

- Infrared divergences when the QFT contains massless particles (like the photon in QED).

  Origin: In the perturbative calculation one is not considering observables :
  - massless particles can have energies smaller than the precision in the energy determination
  - combination of collinear massless particles is indistinguishable from a single particle

  Obervables, instead of transitions among states with a given number of particles, are free of infrared divergences.

Second part: Divergences

Ultraviolet divergences from the limit of infinite momenta.

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- Any QFT will have a limited domain of validity fixed by the necessity to include additional degrees of freedom or to go beyond the QFT framework.
- Extending the integration to arbitrarily large momenta (which is the origin of ultraviolet divergences) one is using QFT beyond its domain of validity.

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- Primitive divergences ; diagrams with $D \geq 0$

Expression of D in terms of the number of external lines case by case

Second part: Power counting

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- $n = 4 \rightarrow \boxed{D = 4 - E}$. Primitive divergences:
  $E = 0$ ("irrelevant") quartic divergent contribution.
  $E = 2$ quadratic divergent contribution to the self-energy of the field.
  $E = 4$ logarithmic divergent correction to the vertex.

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- $n > 4$ Infinite number of primitive divergences to be reabsorved in the perturbative expression of observables $\rightarrow$ Infinite number of measurements $\rightarrow$ predictive power lost $\rightarrow$ non renormalizable theory.
- Simple dimensional argument : $[\lambda_n] = M^{4-n}$
  Higher orders in perturbative expansion $\rightarrow$ momentum integral with a higher dimension of mass (superficial degree of divergence).
  A necessery condition to have a finite number of primitive divergences (renormalizable theory) is the absence of couplings with a negative dimension of mass

Second part: Power counting

QED

QED

- $\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^\mu)(\partial_\nu A^\nu) + \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi$
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- Dimensional analysis: $[A_\mu] = M$, $[\Psi] = M^{3/2} \rightarrow [e] = M^0$
- Symmetries modify relation between D and UV divergences:

QED

- $\boxed{E_f = 0}$ $\rightarrow E_\gamma$ even (charge conjugation symmetry).

  $E_\gamma = 4 \rightarrow D = 0$ but correction to the vertex is not divergent
  $E_\gamma = 2 \rightarrow D = 2$ but correction to the photon self-energy is logarithmically divergent (gauge symmetry)

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- $\boxed{E_f = 2}$

  $E_\gamma = 0 \to D = 1$ but logarithmic divergence in the fermion self-energy (quiral symmetry in the limit $m \to 0$)
  $E_\gamma = 1 \to D = 0$ logarithmic divergence.

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- To summarize one has a logarithmic primitive divergence in the photon and fermion self-energies and in the vertex correction.

Second part: Renormalization

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Infinities $\leftrightarrow$ predictions from QFT

- Infinities in expressions of observables in terms of parameters of the theory.
- Relations between observables are free of infinities.
- One needs a regularization to make sense of the perturbative expansion and to reabsorb infinities in a redefinition of parameters (renormalization)

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- Higher derivative terms in the lagrangian, Discrete space-time (lattice regularization), Product of fields at different points (point splitting) ...

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- Search for a physical regularization may be a guide principle to look for an ultraviolet completion of QFT (?)

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- Evaluate the integral in terms of Euler Gamma function and use their properties to isolate the divergences that appear as poles in the limit $\epsilon \to 0$.

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- $\Psi \; = \; Z_\Psi{}^{1/2}\Psi_R \qquad A^\mu \; = \; Z_A{}^{1/2}A^\mu_R \qquad m_0 \; = \; Z_m m \qquad e_0 \; = \; Z_e e$

$$\begin{aligned}\mathcal{L} \;\; = \;\; & Z_2\,\bar\Psi_R i\not\partial\Psi_R - Z_0\, m\bar\Psi_R\Psi_R + Z_1\, e\bar\Psi_R\gamma_\mu\Psi_R A^\mu_R \\ & -Z_3\frac{1}{4}(\partial^\mu A^\nu_R - \partial^\nu A^\mu_R)(\partial_\mu A_{R\nu} - \partial_\nu A_{R\mu})\end{aligned}$$

$$Z_2 \; = \; Z_\Psi \qquad Z_0 \; = \; Z_m Z_\Psi \qquad Z_1 \; = \; Z_e Z_\psi Z_A^{1/2} \qquad Z_3 \; = \; Z_A$$

- The original lagrangian can be decomposed as a sum of a renormalized lagrangian ($\mathcal{L}_R$)

$$\mathcal{L}_R = \bar{\Psi}_R i\partial\!\!\!/ \Psi_R - m\bar{\Psi}_R\Psi_R + e\bar{\Psi}_R\gamma_\mu\Psi_R A_R^\mu - \frac{1}{4}(\partial^\mu A_R^\nu - \partial^\nu A_R^\mu)(\partial_\mu A_{R\nu} - \partial_\nu A_{R\mu})$$

and a counterterms lagrangian ($\delta\mathcal{L}$) where $\delta Z_i = Z_i - 1$

$$\delta\mathcal{L} = \delta Z_2\,\bar{\Psi}_R i\partial\!\!\!/ \Psi_R - \delta Z_0\, m\bar{\Psi}_R\Psi_R + \delta Z_1\, e\bar{\Psi}_R\gamma_\mu\Psi_R A_R^\mu - \delta Z_3\,\frac{1}{4}(\partial^\mu A_R^\nu - \partial^\nu A_R^\mu)(\partial_\mu A_{R\nu} - \partial_\nu A_{R\mu})$$

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- Different renormalized fields and parameters
- Different expressions of observables in terms of renormalized parameters
- Relations among observables that one obtains after eliminating the renormalized parameters are independent of the renormalization scheme (physical content of QFT)

- Example of equivalence of renormalization schemes: different choices for the dimensional scale μ introduced in dimensional regularization lead to different renormalized couplings $e_R(\mu)$ in order to reproduce the same theory:

$$\boxed{\mu \frac{de_R(\mu)}{d\mu} = \beta(e_R(\mu))}$$

β function: expansion in powers of the renormalized coupling.

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- Quantum corrections to the photon propagator in the scattering of electrons leads to the introduction of an effective coupling

$$\frac{e_0^2}{1-\Pi(q^2)} = \frac{e^2 Z_3^{-1}}{1-\Pi(q^2)} = \frac{e^2}{1-[\Pi(q^2)-\Pi(0)]} \doteq e^2(Q^2)$$

  q is the transferred momentum in the scattering; $Q^2 = -q^2 > 0$

- Dependence of the effective coupling $e(Q^2)$ on the transferred momentum is just the dependence of the renormalized coupling on the square of the renormalization scale.
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- Relations among couplings compatible with the renormalization group not associated to a symmetry (reduction of couplings).

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- QED with a cutoff in (Wick rotated) momentum integral:
  Simple relation between renormalization constants is lost.
  Additional counterterms required.

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- Appropriate fermion field content $\leftrightarrow$ Consistency condition in order to be able to reabsorb all the divergences into a redefinition of fields and parameters.

Third part: Effective field theory (EFT)

From renormalizable QFT to EFT

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- UV divergences $\leftrightarrow$ domain of validity of QFT $\leftrightarrow$ EFT as an approximation to the dependence on the details of the more fundamental theory.

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Simplest example

- Renormalizable QFT with two real scalar fields with masses m, M ($m \ll M$).
  Effects of the heavy particles on processes involving only light particles with energies $E \ll M$?

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- Local interactions $\leftrightarrow$ limitation to very small distances of the violations of the conservation of energy-momentum by the uncertainty principle.

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- A consequence of the reinterpretation of QFT as an effective field theory which results from the integration of the "high energy" degrees of fredom is that the action has an infinite number of terms going beyond renormalizable QFT.

Third part: Power counting in EFT

- In the lagrangian of an EFT one has, together with the terms of a renormalizable QFT, terms $\mathcal{L}_i$ of mass dimension $d_i > 4$ which will have coefficients proportional to $(1/\Lambda)^{d_i-4}$ where Λ is a scale with dimension of mass which characterizes the physics beyond the EFT.

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- Observables in terms of a finite number of renormalized parameters which will include the dimensionless coefficients of the new terms $\mathcal{L}_i$ in the lagrangian.

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Renormalizable QFT $\leftrightarrow$ Particular case where the zero order term of the expansion in powers of $(1/\Lambda)$ is sufficient for the required accuracy.
This is the case when $\Lambda \gg E$ unless we have a very precise determination of observables $\rightarrow$ Special role played by renormalizable QFT.

But EFT, which is a nonrenormalizable QFT in the traditional sense, does have a predictive power.

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- Physics behind the effective field theory is nonperturbative.

Third part: Examples of EFT in particle physics

- Interaction of photons with a macroscopic current distribution at energies much smaller than the mass of the electron:
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- Weak interactions at energies $E < M_W$
  Violations to decoupling: not all the effects of the top quark are suppressed by powers of (E/m_t) !! $(y_t \propto m_t)$
  EFT takes into account effect of heavy quarks in weak processes for light particles including the effect of strong interactions.

- EFT for light meson physics
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- Gravity on macroscopic scales
  electromagnetic interaction $\rightarrow$ gravitational interaction
  electromagnetic field $\rightarrow$ metric
  gauge invariance $\rightarrow$ general covariance
  $v = M_P$ (Planck mass $\leftrightarrow$ Newtonian coupling) $\qquad \Lambda = m_e$.

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  - Observables are independent of renormalization scheme:
    Fine tuning in dimensional regularization appears in relation between the mass of the scalar and the renormalized parameters.

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  - Extension of SM with a symmetry relating bosons and fermions (supersymmetry):
    Scalar fields in EFT consistent with naturalness.

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    - Reduction of couplings (?) in EFTSM: RGE for the 59 parameters are known but one needs to include operators which are not relevant in determination of observables.

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- Another alternative to UV completion: deviation from the standard notion of locality.

Third part: Incomplete list of topics not covered

- Relation between statistical mechanics and QFT; applications of QFT to phase transitions and critical phenomena.
- Non-perturbative effects in QFT.
- Lattice field theory.
- Relation between classical gravity on a manifold with boundary and quantum field theory on such boundary.
- QFT in higher dimensional spacetimes.
- Study of geometrical and topological properties of manifolds by formulating appropriate QFT on a manifold with a nontrivial topology.

Exercises

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- Calculate the renormalized lagrangian and counterterms at one loop in QED using a momentum cutoff in the Wick rotated euclidean momentum integrations as a regularization. Compare the results with those of dimensional regularization.

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3. Simplest example of reduction of couplings in an EFT:

- Identify the general structure of the renormalization group equations for the EFT with a real scalar field Φ and a discrete symmetry under the transformation $\Phi \to -\Phi$ at one loop. Look for relations among renormalized parameters valid at all scales.

4. Example of an EFT derived from a renormalizable QFT:

- Starting from a theory with two real scalar fields (ϕ, ξ) with a lagrangian

$$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{1}{2}\partial^\mu\xi\partial_\mu\xi - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\xi^2 - \frac{\lambda_1}{4!}\phi^4 - \frac{\lambda_2}{4!}\xi^4 - \frac{\lambda_3}{4}\phi^2\xi^2 \tag{1}$$

determine the EFT with a real scalar field ϕ that one obtains when one has a hierarchy of masses $m \ll M$ and one considers observables at energies $E \ll M$ so that one can use an expansion in powers of (E/M).
The solution can be found in [4].

5. Example of Lorentz invariance violation as a physical regulator:

- Consider a theory with a real scalar field ϕ and a lagrangian

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\vec{\nabla}\phi.\vec{\nabla}\phi + \phi K(\vec{\nabla}^2)\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 \qquad (2)$$

where K is a real function of one real variable.
Calculate observables at one loop with this lagrangian. Is it posible to choose the function K such that one does not find any divergence ?

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6. EFTSM at order $(1/\Lambda)^2$:

- Write (with the SM fields) all the operators of dimension 6 that are Lorentz scalars with products of fields and derivatives of fields.
Eliminate those operators that vanish as a consequence of the SM field equations.
The solution can be found in [5].

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