Cabibbo-Kobayashi-Maskawa matrix first row unitarity

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Introduction

- Standard Model describes strong, weak and electromagnetic interactions through exchange of gauge bosons.
- Cabibbo-Kobayashi-Maskawa matrix (CKM) contains the coupling between quarks and W boson.

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \longrightarrow \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Wolfenstein parametrization

$$\lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} \qquad A\lambda^2 = \lambda \left| \frac{V_{cb}}{V_{us}} \right|$$

$$V_{ub}^* = A\lambda^3(\rho + i\eta)$$

- Any deviation from unitarity would indicate the existence of new physics $\longrightarrow$ compute every element with precision

Introduction

- Constraints on the scale of new physics.
- Study of the first row of the matrix and $|V_{us}|$
- Determination of $|V_{us}|$ using semileptonic decays: one-loop corrections in ChPT to reduce error.

Methods to obtain V_{us}

- Extraction of $|V_{us}|$ using semileptonic kaon decays.
- Extraction of $|V_{us}|$ using leptonic kaon decays.
- Extraction of $|V_{us}|$ using hadronic tau decays.

All of these methods need an experimental input and a theoretical input that describes the non-perturbative physics (hadronization) that describes the process.

Methods to obtain V_{us}

Extraction of $|V_{us}|$ using semileptonic kaon decays.

$K \to \pi \ell \nu$

$$\Gamma(K \to \pi \ell \nu) \propto |V_{us}|^2 \, |f_+(0)|^2$$

Experimental lattice

Form factor is defined as

$$\langle \pi(k')| \, \bar{s}\gamma^\mu u \, |K(k)\rangle = (p+p')^\mu f_+(q^2) + (p-p')^\mu f_-(q^2) \quad \text{donde} \quad q \equiv p - p'$$

using the average of FLAG (hep-lat 1310.8555)

$f_+(0) = 0.9634(32)$ for $N_f = 2+1$ we obtain

$|V_{us}| = 0.2247(7)$ $\longrightarrow$ 0.31%

the theoretical error dominates the calculation of $f_+(0)$

Methods to obtain V_{us}

Extraction of $|V_{us}|$ using leptonic kaon decays.

$K \to \ell\nu$ y $\pi \to \ell\nu$

$$\frac{\Gamma(K \to l\nu)}{\Gamma(\pi \to l\nu)} \propto \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2}{f_\pi^2}$$

(Marciano 2005)

experimental

lattice

Where f_K and f_π are the decay constants of kaon and pion.

FLAG average is $f_K/f_\pi = 1.192(5)$ so, we obtain

$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2316(12)$ using $|V_{ud}| = 0.97417(21)$

(nucl-ex 1411.5987)

$|V_{us}| = 0.2256(11)$ $\longrightarrow$ 0.48%

the theoretical error dominates the calculation of f_K/f_π

Methods to obtain V_{us}

Extraction of $|V_{us}|$ using hadronic tau decays.

Experimentally one can distinguish between a hadron decays states with and without strangeness

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau hadrones)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = R_{\tau, sin\, s} + R_{\tau, con\, s}$$

experimental

we can obtain $|V_{us}|$ from the subtraction

$$\delta R_\tau \equiv \frac{R_{\tau, sin\, s}}{|V_{ud}|^2} - \frac{R_{\tau, con\, s}}{|V_{us}|^2}$$

theorical

this quantity vanishes in the SU(3) limit

we obtain

$$|V_{us}| = 0.2173(22) \longrightarrow 1.01\%$$

In this case the error is dominated by the experimental error.

Test of the SM: unitarity of CKM matrix

Study of the unitarity of the CKM matrix

$$|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$|V_{ub}| = 4.15(49) \times 10^{-3}$ is negligible

$|V_{ud}| = 0.97417(21)$ nuclear beta decay

(nucl-exp 1411.5987)

we study the unitarity with the values of $|V_{us}|$ obtained by leptonic and semileptonic kaon decays.

Tests of the SM: Lattice QCD

- Tool that can calculate the theoretical inputs more precisely.
- Discretization QCD on a spacetime lattice

  (a lattice spacing)
- Statistical and systematic errors (discretization, extrapolation physical masses, FV ....)
- Calculations are performed by using:

  $N_f = 2+1$ $\longrightarrow$ They have included the effects of vacuum polarization from quarks up, down and strange. Quarks up and down are degenerate.

  $N_f = 2+1+1$ $\longrightarrow$ They have included the effects from quark charm

Tests of the SM: Lattice QCD

- With lattice QCD simulation we can obtain $f_+(0)$ and f_K/f_π for different values of a and the quarks masses .
- Extrapolation to continuous limit, $a \to 0$
- Extrapolation to physical masses of the quarks up and down.

Tests of the SM: theoretical inputs

We use the results compiled and averaged by FLAG

(Flavour Lattice Averaging Group) (hep-lat 1310.8555)

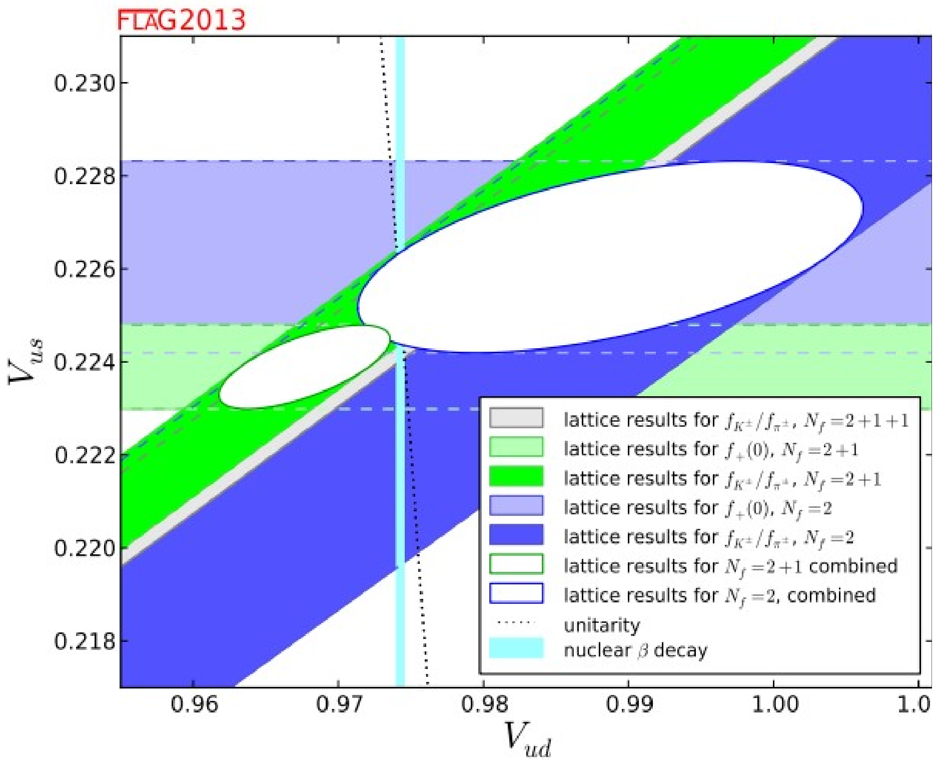
Colaboración	N_f	a partir de	$\|V_{us}\|$
HPQCD 13A	2+1+1	$f_{K^\pm}/f_{\pi^\pm}$	0.2255(5)(3)
MILC 13A	2+1+1	$f_{K^\pm}/f_{\pi^\pm}$	0.2249(6)(7)
RBC/UKQCD 13	2+1	$f_+(0)$	0.2237(7)(7)
MILC 12	2+1	$f_+(0)$	0.2238(7)(8)
MILC 10	2+1	$f_{K^\pm}/f_{\pi^\pm}$	0.2249(5)(9)
RBC/UKQCD 10A	2+1	$f_{K^\pm}/f_{\pi^\pm}$	0.2246(22)(25)
BMW 10	2+1	$f_{K^\pm}/f_{\pi^\pm}$	0.2259(13)(12)
HPQCD/UKQCD 07	2+1	$f_{K^\pm}/f_{\pi^\pm}$	0.2264(5)(13)

	$f_+(0)$	$f_{K^\pm}/f_{\pi^\pm}$	$\|V_{us}\|$	$\|V_{ud}\|$
$N_f = 2+1+1$	0.9611(47)	1.194(5)	0.2251(10)	0.97434(22)
$N_f = 2+1$	0.9634(32)	1.197(4)	0.2247(7)	0.97447(18)

Tests of the SM: Results

With experimental averages (hep-ph 1005.2323)

$$|V_{us}|\, f_+(0) = 0.2163(5), \qquad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_\pi} = 0.2758(5)$$



For $N_f = 2+1$

$$|V_u|^2 = 0.9993(5) \text{ usando } f_+(0)$$

$$|V_u|^2 = 1.0000(6) \text{ usando } f_K/f_\pi$$

average value

$$|V_u|^2 = 0.987(10)$$

For $N_f = 2+1+1$

$$|V_u|^2 = 0.9998(7) \text{ usando } f_{K^\pm}/f_{\pi^\pm}$$

Semileptonics decays: new results

FNAL/MILC $N_f = 2+1+1$

$$f_+(0) = 0.9704(24)(23) = 0.9704(32)$$

Statistical

systematic

We can reduce the error:

- Statistical: increasing the number of simulations, decrease the value a , considering more quark masses and more different values of a .
- Systematic: dominated by finite volume simulations, in the real world the volume is infinity. We need compute the finite volume correction using ChPT.

These corrections are important to reduce the theoretical (~0.34%), to be equal to experimental error (~0.2%).

Chiral perturbation theory (ChPT)

It is the efective field theory of QCD at very low energies

$$E < \Lambda_{QCD} \sim 1GeV$$

- Concept of effective theory.
- Chiral symmetry of QCD.
- Relevant degrees of freedom: meson.
- Lagrangian and quantities compute with ChPT are organized in increasing powers of momentum.
- We can include analytically discretization and finite volume corrections.

2 loops in ChPT

$$f_+(0) = 1 + f_2(a) + f_4(a=0) + (m_\pi^2 - m_K^2)^2 \left[C_6 + C_2 a^2\right]$$

We have to compute the contrinutions to $f_2(a)$ using ChPT .

Chiral perturbation theory (ChPT)

At lowest order, the most general effective Lagrangian

$$\mathcal{L}_2 = \frac{f^2}{4}\left\langle D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U \right\rangle$$

$$U(\phi) = u(\phi)^2 = exp\{i\sqrt{2}\Phi/f\}$$

$$D_\mu U = \partial_\mu U + iU\ell_\mu$$

$$\Phi(x) \equiv \frac{\vec{\lambda}}{\sqrt{2}}\vec{\phi} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

Developing exponential,

$$\frac{e}{\sqrt{2}sen\theta_w}W_\mu^\dagger \frac{1}{f^2}\left\langle -\frac{1}{3}T_+\partial^\mu\Phi\Phi\Phi\Phi + T_+\Phi\partial^\mu\Phi\Phi\Phi - T_+\Phi\Phi\partial^\mu\Phi\Phi + \frac{1}{3}T_+\Phi\Phi\Phi\partial^\mu\Phi \right\rangle$$

$$\frac{1}{f^2}\left\langle -\partial_\mu\Phi\Phi\Phi\partial^\mu\Phi + \Phi\partial_\mu\Phi\Phi\partial^\mu\Phi \right\rangle$$

$$\frac{e}{\sqrt{2}sen\theta_w}W_\mu^\dagger \frac{1}{f^2}\left\langle T_+\partial^\mu\Phi\Phi - T_+\Phi\partial^\mu\Phi \right\rangle$$

Finite volume correction twisted boundary condition

$$\psi(x_k + L) = e^{i\theta_K}\psi(x_k); \quad k = 1, 2, 3$$ L Lattice length

for these boundary conditions we have the dispersion relation

$$E^2 = m^2 + (\vec{p}_F + \Delta\theta/L)^2$$

where $\Delta\theta$ is the difference between the twisting angles of the two valence quarks. Changing the angles we can achieve arbitrary momentum and then fit the external momentum to $q^2 = 0$

These boundary conditions are based on replacing the infinite volume integral by a sum over the 3 spacial momentum and a integral over the remaining dimension

$$\int \frac{d^d k_M}{(2\pi)^2} \to \int_V \frac{d^d k}{(2\pi)^d} \equiv \int \frac{d^{d-3}k}{(2\pi)^{d-3}} \frac{1}{L^3} \sum_{\substack{\vec{n} \in \mathbb{Z}^3 \\ \vec{k} = (2\pi\vec{n} + \vec{\theta}_M)/L}}$$

Finite volume correction twisted boundary condition

$$A(m_1^2) = \frac{1}{i}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_1^2}$$

$$B(m_1^2,\, m_2^2,\, q^2) = \frac{1}{i}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2 - m_1^2)((k-q)^2 - m_2^2)}$$

$$B_\mu(m_1^2,\, m_2^2,\, q^2) = \frac{1}{i}\int \frac{d^4k}{(2\pi)^4}\frac{k_\mu}{(k^2 - m_1^2)((k-q)^2 - m_2^2)} = q_\mu B_1(m_1^2,\, m_2^2,\, q^2)$$

$$A^V(m_N^2, n) = (-1)^n \sum_{\vec{l}\in\mathbb{Z}^3}\int \frac{d\lambda}{\Gamma(n)}\frac{\lambda^{n-3}}{(4\pi)^2} e^{-\lambda m^2} e^{iL^2\vec{l}^2/(4\lambda) - i\vec{l}\cdot\vec{\theta}}$$

$$B^V(m_1^2,\, m_2^2,\, n_1,\, n_2, q) = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)}\int_0^1 dx(1-x)^{n_1-1}x^{n_2-1}A^V(\tilde{m}^2,\, n_1+n_2)$$

$$\tilde{m}^2 = (1-x)m_1^2 + xm_2^2 - x(1-x)q^2$$

Lorentz invariance break

$$\int_V \frac{d^dk}{(2\pi)^2}\frac{k^\mu}{k^2 - m^2} \neq 0$$

Finite volume correction twisted boundary condition

The form factors are different for finite volume.

For infinite volume;

$$\langle \pi^-(p') | V_\mu^{13} | \bar{K}^0(p) \rangle = f_+(q^2)(p_\mu + p'_\mu) + f_-(q^2)(p_\mu - p'_\mu)$$

$$V_\mu^{13} = \bar{u}\gamma_\mu s \quad q = p - p'.$$

and for finite volume;

$$\langle \pi^-(p') | V_\mu^{13} | \bar{K}^0(p) \rangle = f_+^{FV}(q^2)(p_\mu + p'_\mu) + f_-^{FV}(q^2)(p_\mu - p'_\mu) + h_\mu$$

new contribution h_μ and f_+ and f_- take the values f_+^{FV} and f_-^{FV} respectively.

Computation of the form factor using ChPT to one loop

We study the decay $K^0 \to \pi^+ \ell^- \nu$ using ChPT to one loop with infinite volume.

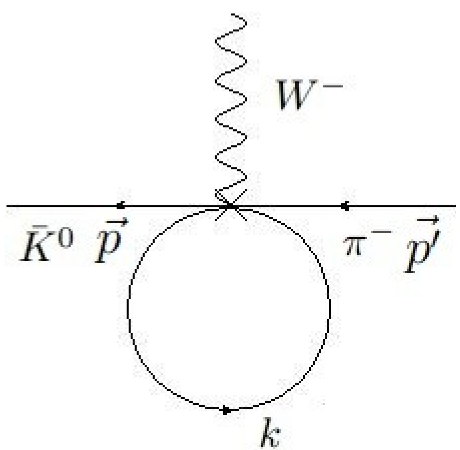


diagrama tipo 1

$$\langle \pi^-(p') | V_\mu^{13} | \bar{K}^0(p) \rangle = \frac{-i^2}{f^2}\left(-\frac{1}{6}\right)\{-p\left[4\bar{A}(m_{\pi^+}) + \bar{A}(m_{\pi^0}) + 8\bar{A}(m_{K^0}) + 8\bar{A}(m_{K^+}) + 9\bar{A}(m_\eta)\right]$$

$$-p'\left[8\bar{A}(m_{K^+}) + 7\bar{A}(m_{\pi^0}) + 4\bar{A}(m_{K^0}) + 8\bar{A}(m_{\pi^+}) + 3\bar{A}(m_\eta)\right]\} =$$

$$= \frac{1}{f^2}\left\{(p+p')\left[\bar{A}(m_{\pi^+}) + \frac{2}{3}\bar{A}(m_{\pi^0}) + \bar{A}(m_{K^0}) + \frac{4}{3}\bar{A}(m_{K^+}) + \bar{A}(m_\eta)\right]\right.$$

$$\left.+(p-p')\left[-\frac{1}{3}\bar{A}(m_{\pi^+}) - \frac{1}{2}\bar{A}(m_{\pi^0}) + \frac{1}{3}\bar{A}(m_{K^0}) + \frac{1}{2}\bar{A}(m_\eta)\right]\right\}$$

With $\bar{A}(m^2) = -\frac{m^2}{16\pi^2} ln(m^2)$ the convergent part of the integral.

Computation of the form factor using ChPT to one loop

We can obtain

$$f_+(q^2) = \frac{1}{f^2}\left[\bar{A}(m_{\pi^+}) + \frac{2}{3}\bar{A}(m_{\pi^0}) + \bar{A}(m_{K^0}) + \frac{4}{3}\bar{A}(m_{K^+}) + \bar{A}(m_\eta)\right]$$

$$f_-(q^2) = \frac{1}{f^2}\left[-\frac{1}{3}\bar{A}(m_{\pi^+}) - \frac{1}{2}\bar{A}(m_{\pi^0}) + \frac{1}{3}\bar{A}(m_{K^0}) + \frac{1}{2}\bar{A}(m_\eta)\right]$$

for finite volume

$$f_+^{FV}(q^2) = \frac{1}{f^2}\left[A^V(m_{\pi^+}) + \frac{2}{3}A^V(m_{\pi^0}) + A^V(m_{K^0}) + \frac{4}{3}A^V(m_{K^+}) + A^V(m_\eta)\right]$$

$$f_-^{FV}(q^2) = \frac{1}{f^2}\left[-\frac{1}{3}A^V(m_{\pi^+}) - \frac{1}{2}A^V(m_{\pi^0}) + \frac{1}{3}A^V(m_{K^0}) + \frac{1}{2}A^V(m_\eta)\right]$$

Computation of the form factor using ChPT to one loop

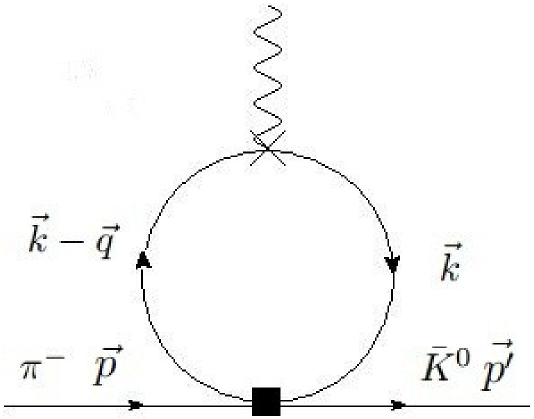


diagrama tipo 2

Considering the derivatives on the external lines

$$f_-(q^2) = \frac{m_{\bar{K}^0}^2 + m_{\pi^-}^2 - q^2}{f^2}\Big\{-\frac{3}{4}\bar{B}(m_\eta^2, m_{K^+}^2, q^2) + \frac{3}{2}\bar{B}_1(m_\eta^2, m_{K^+}^2, q^2)$$

$$+\frac{3}{4}\bar{B}(m_{\pi^0}^2, m_{K^+}^2, q^2) - \frac{3}{2}\bar{B}_1(m_{\pi^0}^2, m_{K^+}^2, q^2) - \frac{1}{2}\bar{B}(m_{K^0}^2, m_{\pi^+}^2, q^2) + \bar{B}_1(m_{K^0}^2, m_{\pi^+}^2, q^2)\Big\}$$

$$B(m_1^2, m_2^2, q^2) = \frac{1}{i}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2 - m_1^2)((k-q)^2 - m_2^2)}$$

$$B_1(m_1^2, m_2^2, q^2) = \frac{1}{2q^2}\left(A(m_2^2) - A(m_1^2) + (m_1^2 - m_2^2 + q^2)B(m_1^2, m_2^2, q^2)\right)$$

Computation of the form factor using ChPT to one loop

For finite volume

$$f_-^{FV}(q^2) = \frac{m^2_{\bar{K}^0} + m^2_{\pi^-} - q^2}{f^2}\left\{-\frac{3}{4}B^V(m^2_\eta, m^2_{K^+}, q^2) + \frac{3}{2}B_1^V(m^2_\eta, m^2_{K^+}, q^2)\right.$$

$$\left.+\frac{3}{4}B^V(m^2_{\pi^0}, m^2_{K^+}, q^2) - \frac{3}{2}B_1^V(m^2_{\pi^0}, m^2_{K^+}, q^2) - \frac{1}{2}B^V(m^2_{K^0}, m^2_{\pi^+}, q^2) + B_1^V(m^2_{K^0}, m^2_{\pi^+}, q^2)\right\}$$

$$h_\mu(q^2) = \frac{1}{f^2}(m^2_{\bar{K}^0} + m^2_{\pi^-} - q^2)\left\{\frac{3}{2}B_2^V(m^2_\eta, m^2_{K^+}, q^2) - \frac{3}{2}B_2^V(m^2_{\pi^0}, m^2_{K^+}, q^2)\right.$$

$$\left.+B_2^V(m^2_{K^0}, m^2_{\pi^+}, q^2)\right\}$$

Computation of the form factor using ChPT to one loop

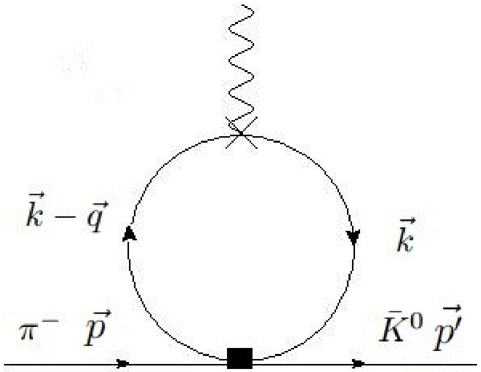


diagrama tipo 2

Considering the derivatives on the internal lines

$$f_-(q^2) = \frac{1}{f^2}\Big\{(m_K^2+m_\pi^2-q^2)\Big[\frac{3}{2}\bar{B}_1(m_{K^+}^2,\, m_\eta^2,\, q^2) - \frac{3}{4}\bar{B}(m_{K^+}^2,\, m_\eta^2,\, q^2) - \frac{3}{2}\bar{B}_1(m_{K^+}^2,\, m_{\pi^0}^2,\, q^2)$$

$$+\frac{3}{4}\bar{B}(m_{K^+}^2,\, m_{\pi^0}^2,\, q^2) + \bar{B}_1(m_{\pi^+}^2,\, m_{K^0}^2,\, q^2) - \frac{1}{2}\bar{B}(m_{\pi^+}^2,\, m_{K^0}^2,\, q^2)\Big] + \frac{3}{4}\bar{A}(m_\eta^2)$$

$$-\frac{3}{4}\bar{A}(m_{\pi^0}^2) + \frac{1}{2}\bar{A}(m_{K^0}^2) - \frac{1}{2}\bar{A}(m_{\pi^+}^2) + (m_\pi^2 - m_K^2)\left[\bar{B}(m_{K^+}^2,\, m_\eta^2,\, q^2) - 2\bar{B}_1(m_{K^+}^2,\, m_\eta^2,\, q^2)\right]\Big\}$$

For finite volume

$$f_-^{FV}(q^2) = \frac{1}{f^2}\Big\{(m_K^2+m_\pi^2-q^2)\Big[\frac{3}{2}B_1^V(m_{K^+}^2,\, m_\eta^2,\, q^2) - \frac{3}{4}B^V(m_{K^+}^2,\, m_\eta^2,\, q^2) - \frac{3}{2}B_1^V(m_{K^+}^2,\, m_{\pi^0}^2,\, q^2)$$

$$+\frac{3}{4}B^V(m_{K^+}^2,\, m_{\pi^0}^2,\, q^2) + B_1^V(m_{\pi^+}^2,\, m_{K^0}^2,\, q^2) - \frac{1}{2}B^V(m_{\pi^+}^2,\, m_{K^0}^2,\, q^2)\Big] + \frac{3}{4}A^V(m_\eta^2)$$

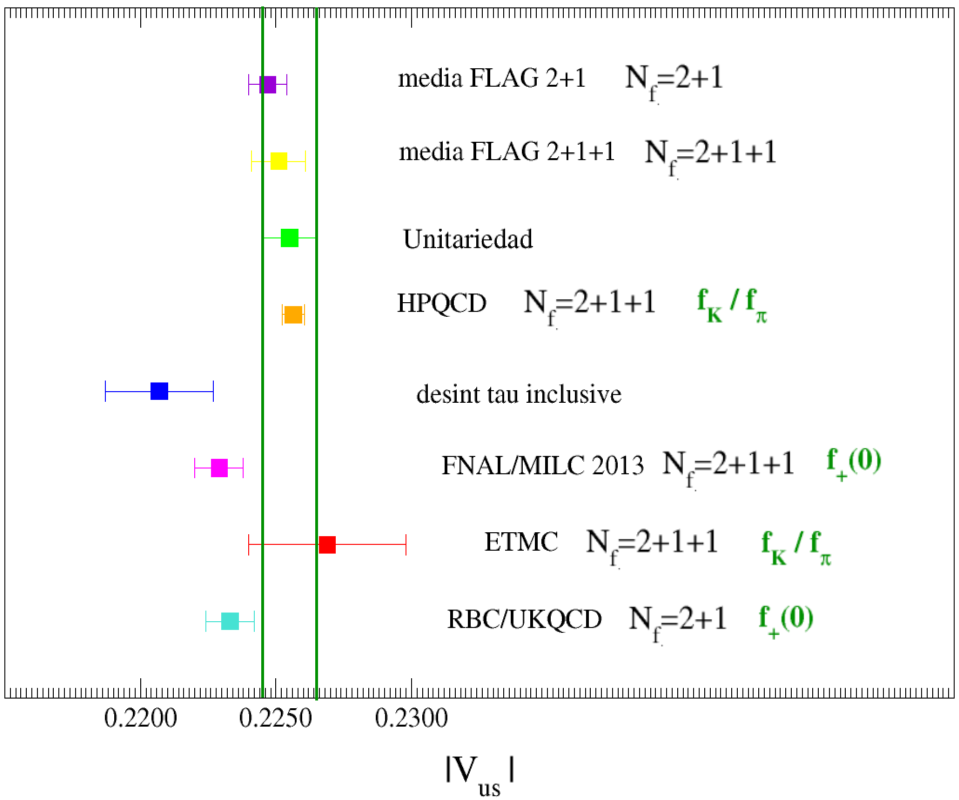
$$-\frac{3}{4}A^V(m_{\pi^0}^2) + \frac{1}{2}A^V(m_{K^0}^2) - \frac{1}{2}A^V(m_{\pi^+}^2) + (m_\pi^2 - m_K^2)\left[B^V(m_{K^+}^2,\, m_\eta^2,\, q^2) - 2B_1^V(m_{K^+}^2,\, m_\eta^2,\, q^2)\right]\Big\}$$

$$h_\mu(q^2) = \frac{1}{f^2}\Big\{(m_K^2+m_\pi^2-q^2)\Big[\frac{3}{2}B_2^{V_\mu}(m_{K^+}^2,\, m_\eta^2,\, q^2) - \frac{3}{2}B_2^{V_\mu}(m_{K^+}^2,\, m_{\pi^0}^2,\, q^2) + B_2^{V_\mu}(m_{\pi^+}^2,\, m_{K^0}^2,\, q^2)\Big]$$

$$-A^{V_\mu}(m_{K^0}^2) + A^{V_\mu}(m_{\pi^+}^2) + (m_\pi^2 - m_K^2)\left[-\frac{2}{3}B_2^{V_\mu}(m_{K^+}^2,\, m_\eta^2,\, q^2)\right]$$

Latest results

(nucl-exp 1411.5987)



Using $|V_{ud}| = 0.97417(21)$

and neglecting $|V_{ub}| \approx 10^{-3}$
we obtain:

ETMC $|V_{us}|^2+|V_{ud}|^2 = 1.0005(13)$

HPQCD $|V_{us}|^2+|V_{ud}|^2 = 0.9999(4)$

RBC/UKQCD $|V_{us}|^2 + |V_{ud}|^2 = 0.9989(5)$

FNAL/MILC $|V_{us}|^2 + |V_{ud}|^2 = 0.9987(5)$

The leptonic results are agree with unitarity. There are tensions between semileptonic results and unitarity, and between leptonic and semileptonic results.

Conclusion

- We have analyzed different methods for compute $|V_{us}|$. Best results are provided by leptonic and semileptonic decay methods.
- Check the unitarity of the CKM matrix

  FNAL/MILC

  $|V_{us}| = 0.22290(90)$ Tension with unitarity $\sim 2.8\sigma$

- Finite volume correction are need to reduce the error at the semileptonic method and reject (confirm) tension with unitarity.