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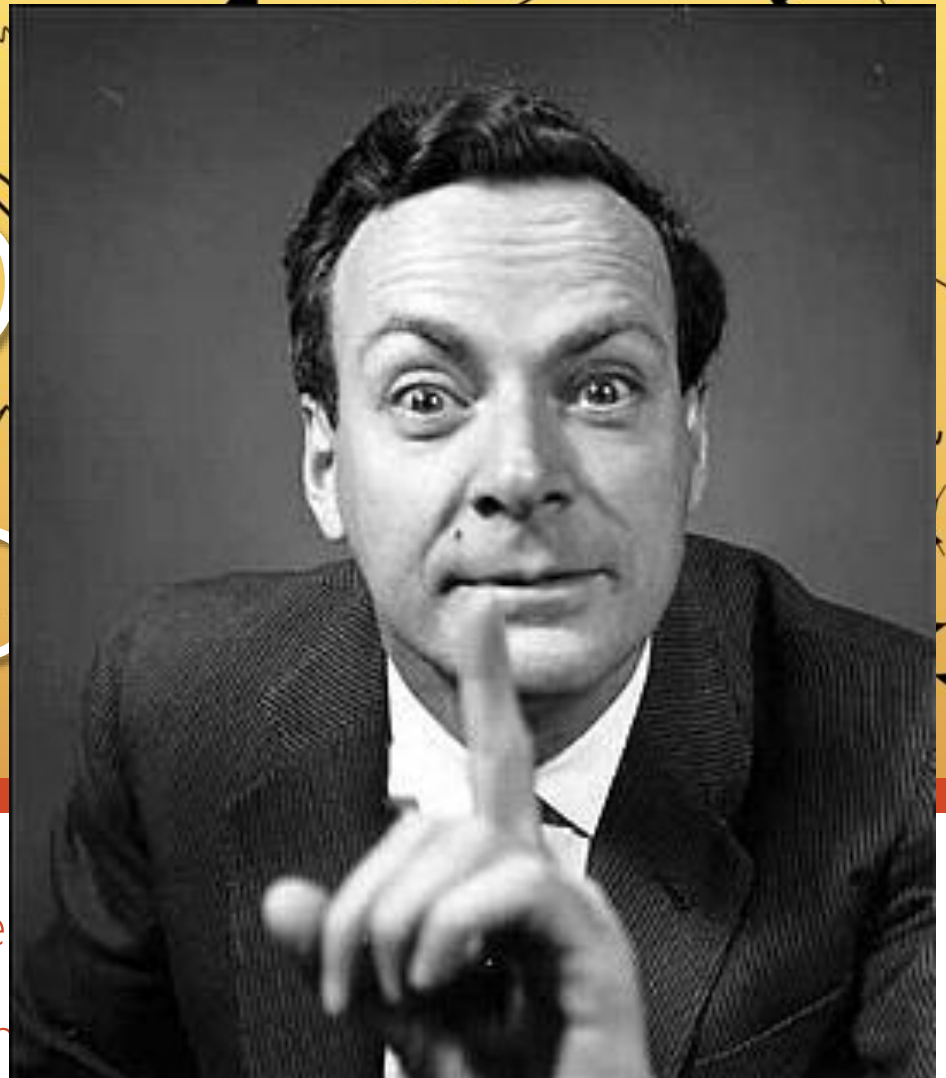
tk CENTRO DE CIENCIAS
DE **BENASQUE**
PEDRO PASCUAL

QCD: 2nd lecture

Taller de Altas Energías TAE2015, September 2015

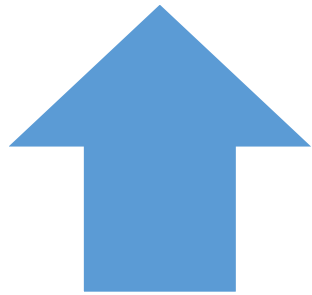
New perturbative methods

- To reach a new frontier in higher orders
- But also to better understand the



One-loop amplitudes

The classical paradigm for the calculation of one-loop diagrams was established in **1979**



Calculation of one-loop scalar integrals

Nuclear Physics B
Volume 153, 1979, Pages 365-401

Scalar one-loop integrals

G. 't Hooft, M. Veltman

Received 16 January 1979



Reduction of tensor one-loop integrals to scalar integrals

Nuclear Physics B
Volume 160, 26 Nov 1979, Pages 151-207

One-loop corrections for e^+e^- annihilation into $\mu^+\mu^-$ in the Weinberg model

G. Passarino, M. Veltman

Received 22 March 1979

Not adequate for processes beyond $2 \rightarrow 2$
(Gramm determinants + large number of Feynman diagrams)

Recursion relations and unitarity methods



Properties of the S-Matrix

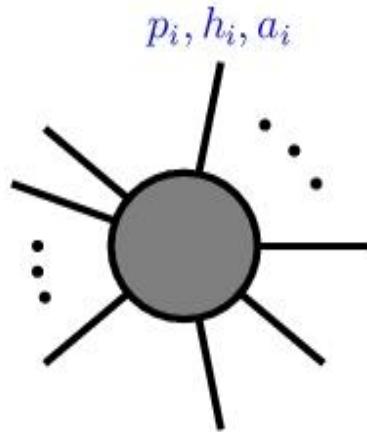
- ▶ **Analyticity:** scattering amplitudes are determined by their singularities
- ▶ **Unitarity:** the residues at singular points are products of scattering amplitudes with lower number of legs and/or less loops



Here are the words of some enthusiast: “One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane”, “... the theory of functions of complex variables plays the role not of a mathematical tool, but of a fundamental description of nature inseparable from physics”

J. Schwinger, *Particles, Sources, and Fields*, Vol.1, p.36

- ▶ **recycling:** using scattering amplitudes to calculate other scattering amplitudes



Four-dimensional spinors of definite helicity

$$|i^\pm\rangle = \frac{1}{2}(1 \pm \gamma_5)u(p_i) = v_{\mp}(p_i) , \quad \langle i^\pm| = \bar{u}_\pm(p_i) = \bar{v}_{\mp}(p_i)$$

$$p_i^2 = 0 , \quad p_i^{a\dot{a}} = k_i^\mu \sigma_\mu^{a\dot{a}} = \lambda_i^a \tilde{\lambda}_i^{\dot{a}}$$

- spinor inner products and other useful identities

$$\langle ij \rangle = \langle i^- | j^+ \rangle = \epsilon_{ab} \lambda_i^a \lambda_j^b = \sqrt{|s_{ij}|} e^{i\phi_{ij}} = -\langle ji \rangle \quad \text{holomorphic}$$

$$[ij] = [i^+ | j^-] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}_i^{\dot{a}} \tilde{\lambda}_j^{\dot{b}} = -\langle ij \rangle^* = -[ji] \quad \text{antiholomorphic}$$

$$[i|\gamma^\mu|j\rangle = \langle j|\gamma^\mu|i]$$

$$s_{ij} = (p_i + p_j)^2 = \langle ij \rangle [ji]$$

$$\not{p}_i = |i\rangle [i| + |i] \langle i| \quad \text{sum over polarizations}$$

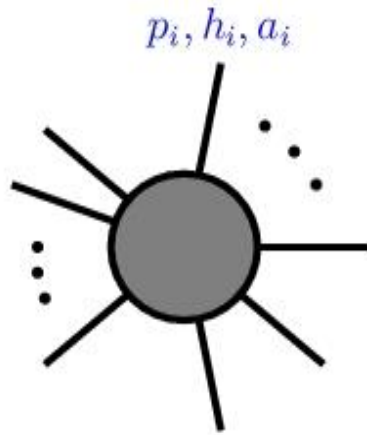
$$\not{p}_i |i^\pm\rangle = 0 \quad \text{equation of motion} \quad \langle ij \rangle = 0 = \langle ii \rangle$$

- polarization vector $\epsilon^2 = 0 , \quad \epsilon^+ \cdot \epsilon^- = 0 , \quad k \cdot \epsilon^\pm(k) = 0$

$$\epsilon_\mu^+(k, \xi) = \frac{\langle \xi | \gamma_\mu | k \rangle}{\sqrt{2} \langle \xi k \rangle}$$

$$\epsilon_\mu^-(k, \xi) = \frac{[\xi | \gamma_\mu | k \rangle}{\sqrt{2} [k \xi]}$$

- equivalent to **axial gauge** $\xi = n$
- a clever choice of the gauge momentum can simplify calculations



- spinor identities

$$\langle 1 | \gamma^\mu | 2 \rangle [3 | \gamma_\mu | 4 \rangle = 2 \langle 14 \rangle [32]$$

Fierz

$$\langle 12 \rangle \langle 34 \rangle = \langle 14 \rangle \langle 32 \rangle + \langle 13 \rangle \langle 24 \rangle$$

Shouten

Exercise: proof the Fierz and Shouten identities

Hint: divide and multiply by $\langle 23 \rangle$ and apply the Dirac identity

$$\gamma^\mu \gamma^\nu \gamma^\sigma \gamma_\mu = 4g^{\nu\sigma}$$

Exercises:

Calculate the scattering amplitudes and square amplitude for $e^+e^- \rightarrow q\bar{q}$ by using the helicity method, and compare with the traditional calculation

How many independent helicity amplitudes there are ?

$$M_{e^+e^- \rightarrow q\bar{q}} \sim [\bar{u}(p_1)\gamma^\mu v(p_2)] [\bar{v}(p_3)\gamma^\nu u(p_4)] d_{\mu\nu}(p_{12}, n)$$

$$|M|^2 \sim \text{Tr}(\not{p}_1\gamma^\mu\not{p}_2\gamma^\sigma)\text{Tr}(\not{p}_3\gamma^\nu\not{p}_4\gamma^\rho)d_{\mu\sigma}(p_{12}, n)d_{\nu\rho}(p_{12}, n)$$

On-shell recursion relations at tree-level: BCFW

[Britto, Cachazo, Feng, Witten]

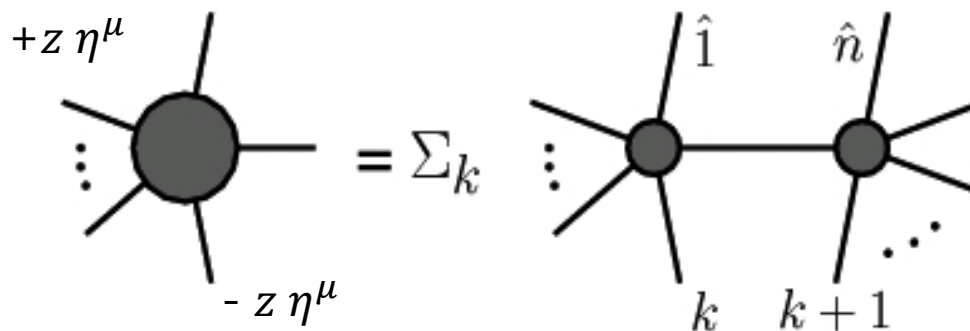


How to reconstruct scattering amplitude from its singularities

Add $z \eta^\mu$ (z complex) to the four-momentum of one external particle and subtract it on another such that the shift leaves them on-shell

$$0 = \frac{1}{2\pi i} \oint_{C \text{ at } \infty} \frac{A_n(z)}{z} = A_n(0) - \sum_{z_i} \frac{\text{Res}_{z_i}(A_n(z))}{z_i}$$

has the correct residue at any multi-particle pole



$$A_n(1, 2, \dots, n) = \sum A_L(\hat{1}, 2, \dots, -\hat{p}_{1,k}) \frac{i}{s_{1,k}} A_R(\hat{p}_{1,k}, k+1, \dots, \hat{n})$$

- Diagrammatic proof [Draggiotis, Kleiss, Lazopoulos, Papadopoulos]
- Compact analytical results, although colour dressed Berends-Giele (off-shell recursion) might be more efficient numerically [Duhr, Höche, Maltoni]

holomorphic shift $(-, +)$ is not a safe shift)

$$\hat{p}_i^\mu = p_i^\mu + \frac{z}{2}[i|\gamma^\mu|j\rangle \quad |\hat{i}\rangle = |i\rangle + z|j\rangle \quad |\hat{i}] = |i]$$

$$\hat{p}_j^\mu = p_j^\mu - \frac{z}{2}[i|\gamma^\mu|j\rangle \quad |\hat{j}\rangle = |j\rangle \quad |\hat{j}] = |j] - z|i]$$

anti-holomorphic shift $(i \leftrightarrow j)$

z determined setting on-shell the intermediate momenta

$$\hat{p}_{1,k}^\mu = p_{1,k}^\mu + \frac{z}{2}[i|\gamma^\mu|j\rangle, \quad \hat{p}_{1,k}^2 = 0, \quad z = -\frac{s_{1,k}}{[i|p_{1,k}|j\rangle}$$

- ✓ use only on-shell amplitudes
- ✓ rather compact expressions
- ✗ generates spurious poles at
while physical IR divergences at



$$[i|p_{1,k}|j\rangle$$

$$s_{i,j} = (p_i + p_{i+1} + \dots + p_j)^2$$

Exercises:

Proof by induction that the Maximal Helicity Violating (MHV) amplitude for gluons is given by the expression

$$A_n(1^+, \dots, i^\pm, \dots, n) = 0$$

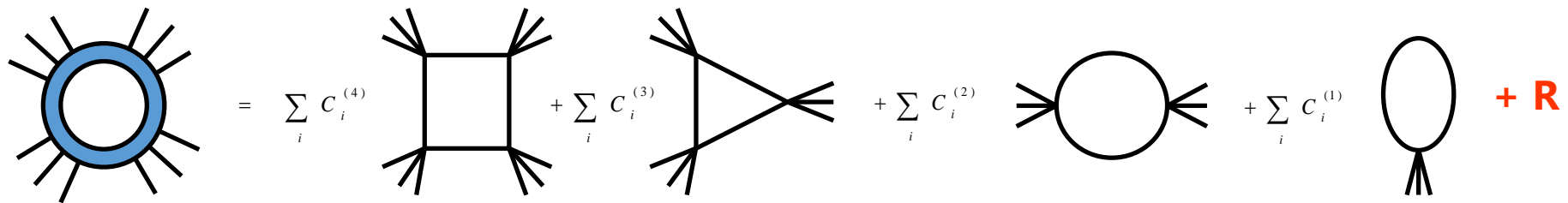
$$A_n^{\text{MHV}}(1^+, \dots, i^-, \dots, j^-, \dots, n^+) = i \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle (n-1)n \rangle \langle n1 \rangle}$$

Calculate by using BCFW the six-gluon amplitude

$$A_6(1^+, 2^+, 3^+, 4^-, 5^-, 6^-) = \frac{i}{\langle 2|1 + 6|5 \rangle} \left(\frac{\langle 6|1 + 2|3 \rangle^3}{\langle 61 \rangle \langle 12 \rangle [34] [45] s_{126}} + \frac{\langle 4|5 + 6|1 \rangle^3}{\langle 23 \rangle \langle 34 \rangle [56] [61] s_{561}} \right)$$

Generalized Unitarity: the one-loop basis

A dimensionally regulated n-point one-loop integral (scattering amplitude) is a linear combination of boxes, triangles, bubbles and tadpoles with rational coefficients

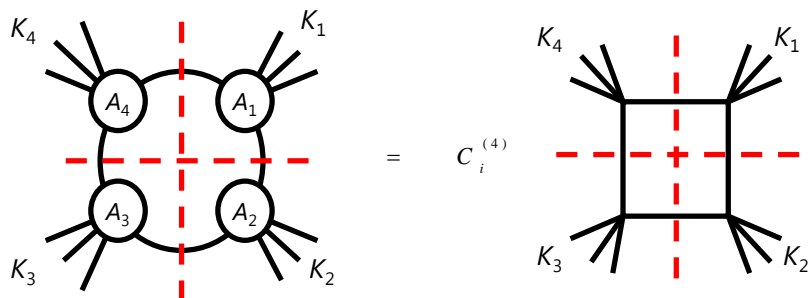


The diagram shows a sun-like diagram (a circle with many radial lines) on the left, followed by an equals sign. To the right of the equals sign are four terms: a sum over i of $C_i^{(4)}$ multiplied by a box diagram (a square with four external lines), plus a sum over i of $C_i^{(3)}$ multiplied by a triangle diagram (a triangle with three external lines), plus a sum over i of $C_i^{(2)}$ multiplied by a bubble diagram (a circle with two external lines), plus a sum over i of $C_i^{(1)}$ multiplied by a tadpole diagram (an oval with one external line). To the right of the tadpole diagram is a red plus sign followed by the letter R .

- Pentagons and higher n-point functions can be reduced to lower point integrals and higher dimensional polygons that only contribute at $O(\epsilon)$ [Bern, Dixon, Kosower]
- The task is reduced to determining the coefficients: by applying multiple cuts at both sides of the equation [Brito, Cachazo, Feng]
- R is a finite piece that is entirely rational: can not be detected by four-dimensional cuts

Generalized Unitarity

Quadruple cut



The discontinuity across the leading singularity is unique

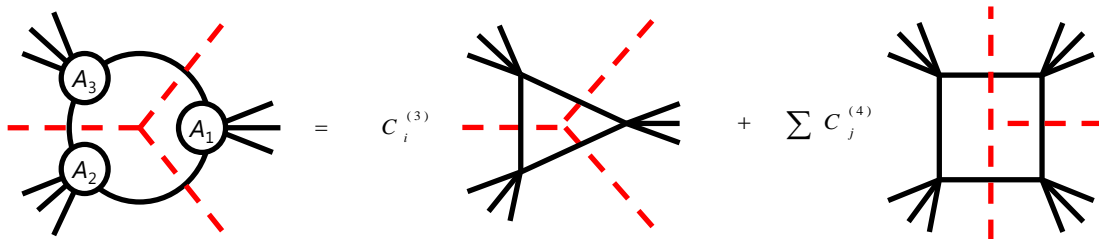
$$C_i^{(4)} = A_1 \times A_2 \times A_3 \times A_4$$

Four on-shell constraints

➔ freeze the loop momenta



Triple cut



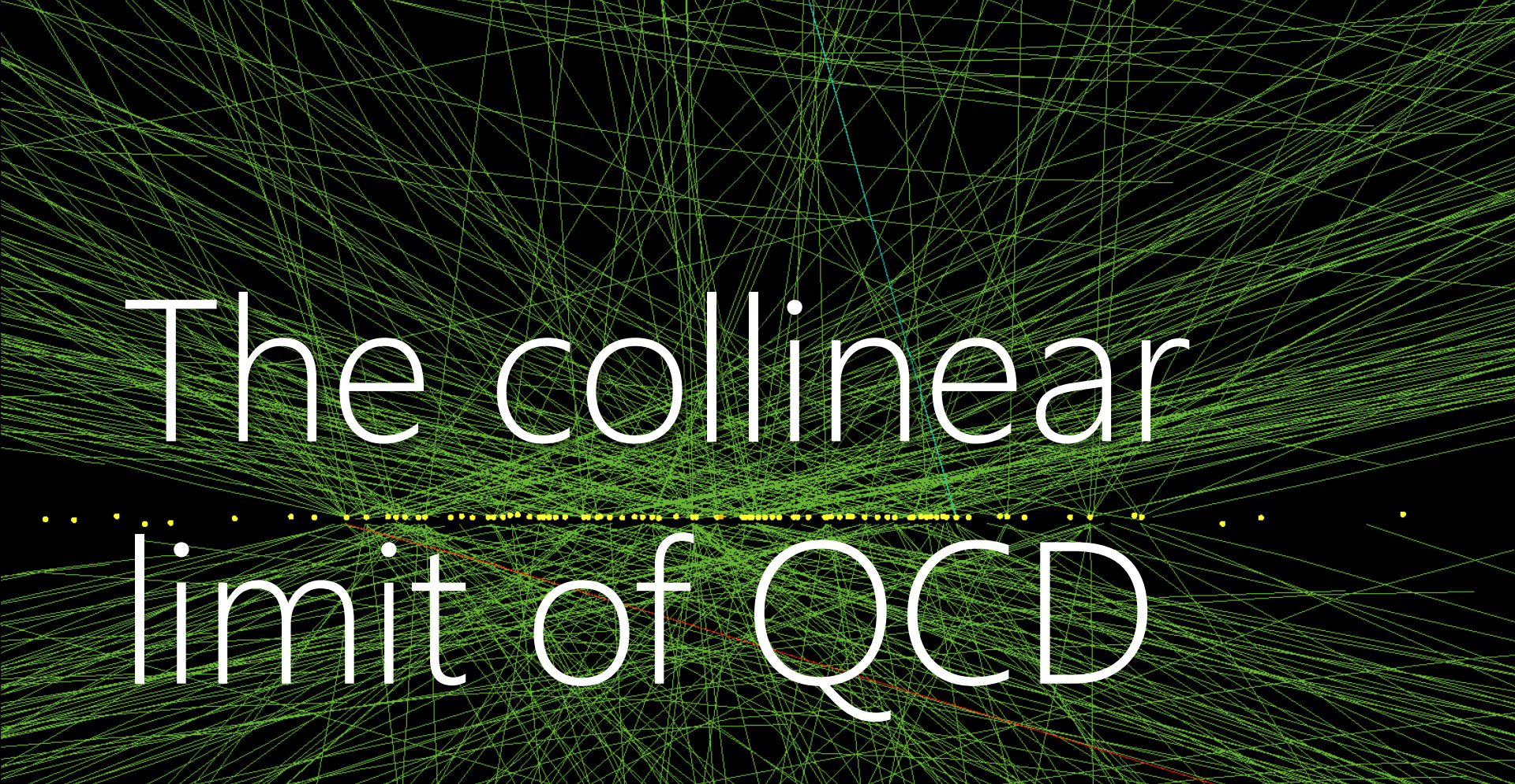
Only three on-shell constraints ➔ one free component of the loop momentum

And so on for **double and single cuts**

- **OPP** [Ossola, Pittau, Papadopoulos]: a systematic way to extract the coefficients

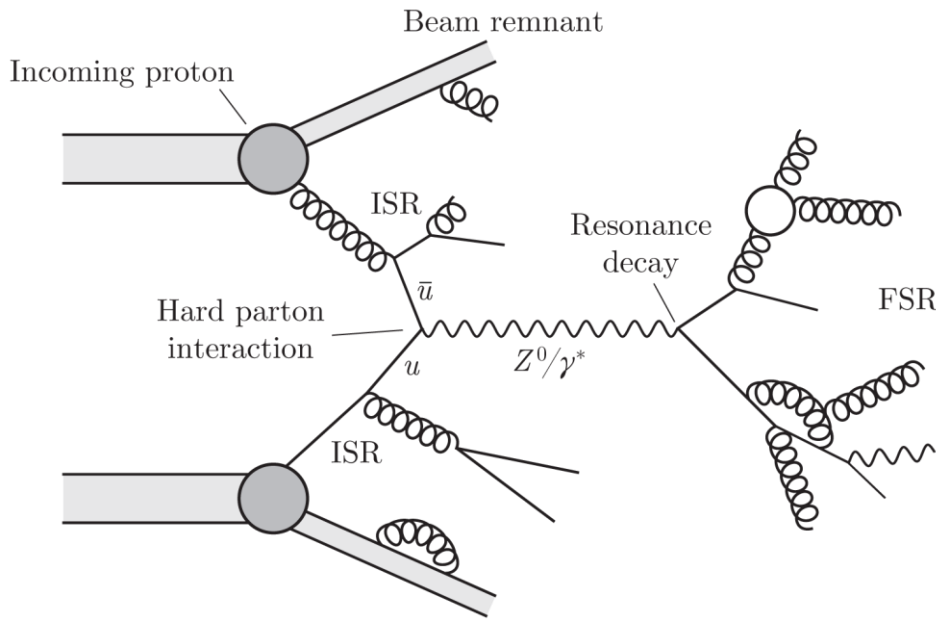
Rational terms

d-dimensional cuts, recursion relations (BCFW), Feynman rules ...



The collinear limit of QCD

Factorization in hadronic collisions



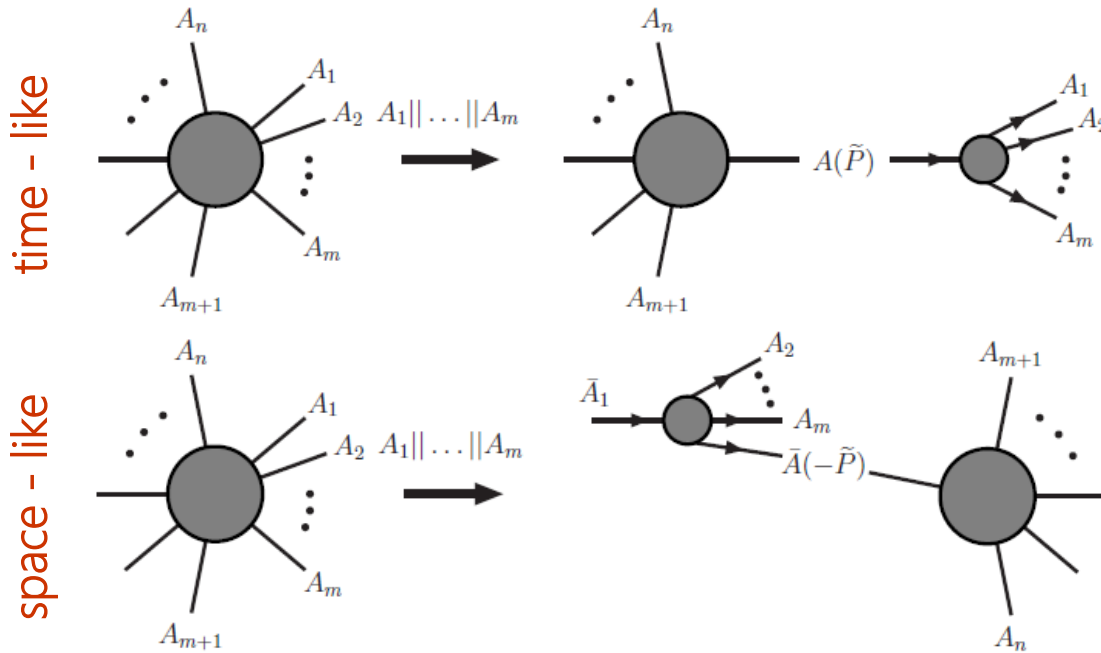
- Factorize physics into **long distance** (hadronic $\sim M_{\text{had}}$), and **short distance** (partonic $Q \gg M_{\text{had}}$),
- factorization violation is power suppressed $\sim \mathcal{O}(M_{\text{had}}/Q)^q$

$$\sigma = \sum_{a,b} \int dx_1 dx_2 f_a(x_1, \mu_F) f_b(x_2, \mu_F) \hat{\sigma}_{ab}(x_1 p_A, x_2 p_B; \mu_F, \mu_R) + \mathcal{O}\left(\frac{1}{Q}\right)$$

Parton densities PDF (points to f_a, f_b)
Hard scattering cross-section (points to $\hat{\sigma}_{ab}$)
Factorization and renormalization scales (points to μ_F, μ_R)
Partonic cms energy $\hat{s} = x_1 x_2 s$ (points to x_1, x_2)
Higher twist (points to $\mathcal{O}(1/Q)$)

Collinear factorization at tree-level

- Momenta p_1, \dots, p_m of m partons become parallel
- Sub-energies $s_{ij} = (p_i + p_j)^2$ of the same order and vanish simultaneously
- Leading behaviour $\left(\frac{1}{\sqrt{s_{1,m}}}\right)^{m-1}$



Collinear limit

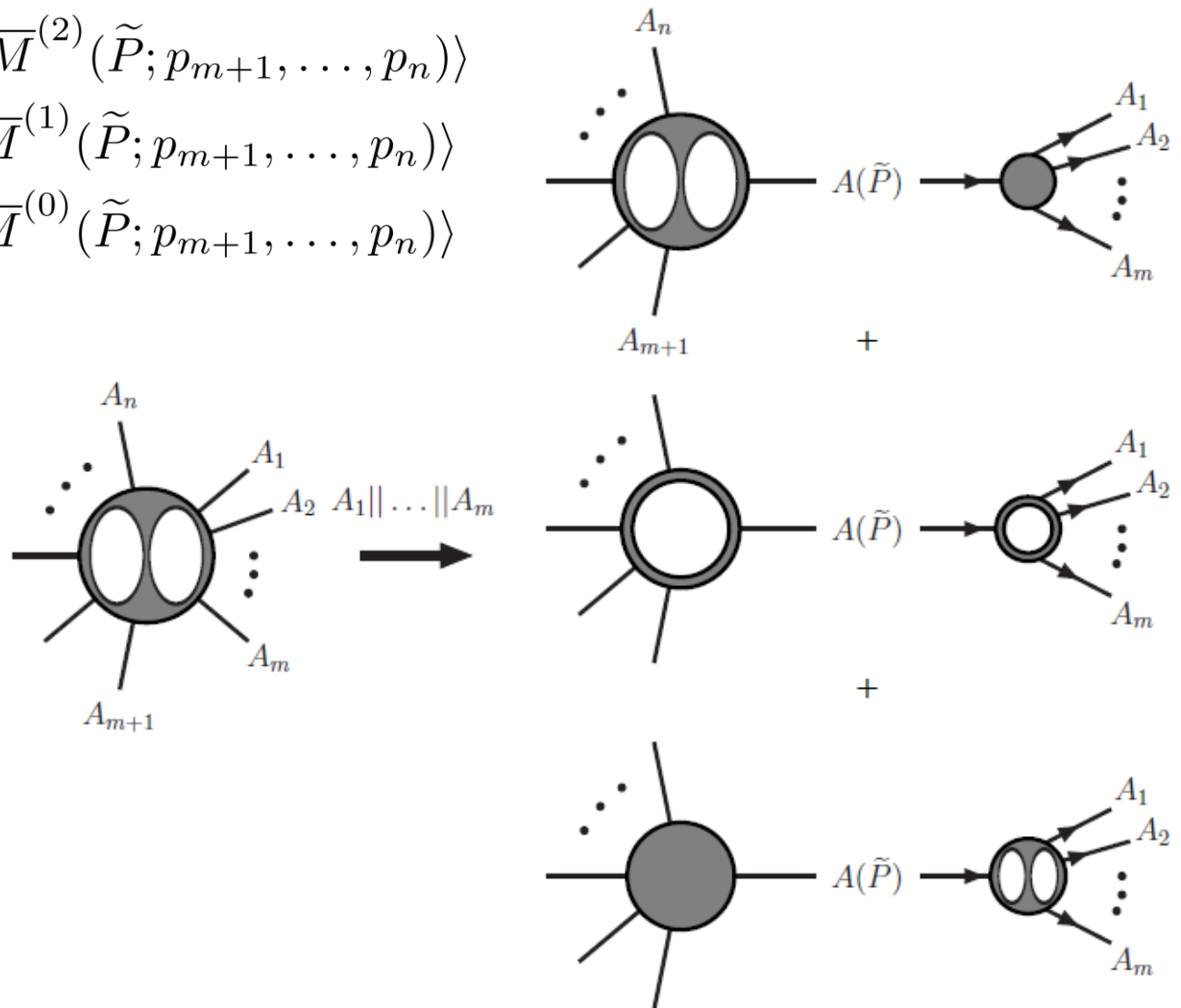
- Most singular behaviour captured by **universal** (process independent) factorisation properties
- **Splitting matrix** depends on the collinear partons only.
- Space-like and time-like related by crossing

$$|M^{(0)}(p_1, \dots, p_n)\rangle$$

$$= \mathbf{Sp}^{(0)}(p_1, \dots, p_m; \tilde{P}) |\overline{M}^{(0)}(\tilde{P}; p_{m+1}, \dots, p_n)\rangle + \mathcal{O}((\sqrt{s})^{3-m})$$

At two loops

$$\begin{aligned}
 & |M^{(2)}(p_1, \dots, p_n)\rangle \\
 & \simeq \mathbf{Sp}^{(0)}(p_1, \dots, p_m; \tilde{P}) |\overline{M}^{(2)}(\tilde{P}; p_{m+1}, \dots, p_n)\rangle \\
 & + \mathbf{Sp}^{(1)}(p_1, \dots, p_m; \tilde{P}) |\overline{M}^{(1)}(\tilde{P}; p_{m+1}, \dots, p_n)\rangle \\
 & + \mathbf{Sp}^{(2)}(p_1, \dots, p_m; \tilde{P}) |\overline{M}^{(0)}(\tilde{P}; p_{m+1}, \dots, p_n)\rangle
 \end{aligned}$$



The collinear projection

- The projection over the collinear limit is obtained by setting the parent parton at on-shell momenta

$$\tilde{P}^\mu = p_{1,m}^\mu - \frac{s_{1,m} n^\mu}{2n \cdot \tilde{P}}$$

\tilde{P}^μ : collinear direction

n^μ : describes how the collinear limit is approached $\tilde{P}^2 = 0, n^2 = 0$

$z_i = \frac{n \cdot p_i}{n \cdot \tilde{P}}$: longitudinal momentum fraction, $\sum z_i = 1$

- Work in the axial gauge (physical polarizations): only diagrams where the parent parton emitted and absorbed collinear radiation

$$\frac{1}{\not{p}_{12}} = \frac{1}{s_{12}} \not{p}_{12} = \frac{1}{s_{12}} \left(\tilde{\not{P}} + \frac{s_{12}}{2n \cdot \tilde{P}} \not{n} \right) \simeq \frac{1}{s_{12}} u(\tilde{P}) \bar{u}(\tilde{P}) + \dots$$

$$d_{\mu\nu}(p_{12}, n) = d_{\mu\nu}(\tilde{P}, n) + \dots = \epsilon_\mu(\tilde{P}) \epsilon_\nu^*(\tilde{P}) + \dots$$

Splitting functions

The square of the splitting amplitude, summed over final-state colours and spins, and averaged over colours and spins of the parent parton, defines the m -parton splitting function

$$\langle P_{a_1 \dots a_m}^{(0)} \rangle = \left(\frac{S_{1,m}}{2\mu^{2\epsilon}} \right)^{m-1} \frac{1}{|\mathbf{Sp}_{a_1 \dots a_m}^{(0)}|}$$

Which is a generalization of the customary (i.e. with $m = 2$) **Altarelli-Parisi** splitting function

- Probability to emit further radiation with given longitudinal momenta, from the leading singular behavior
- Universal (process independent): e+e-, DIS or hadron collisions

Exercise:

Calculate the splitting functions for the collinear processes $q \rightarrow qg$, $g \rightarrow q\bar{q}$ and $g \rightarrow gg$ by using the helicity method

Hint:

$$\mathbf{Sp}_{q \rightarrow q_1 g_2}^{(0)} = \mathbf{T}^a \frac{1}{s_{12}} \bar{u}(p_1) \not{\epsilon}(p_2) v(\tilde{P})$$

$$P_{q \rightarrow q_1 g_2}^{(0)} = C_F \frac{1+z^2}{1-z} \quad z = z_1 = \frac{n \cdot p_1}{n \cdot \tilde{P}} \quad z_2 = 1 - z$$

Relevance of the collinear limit in QCD

- ◉ evaluate IR finite cross-sections ▶ subtraction terms
- ◉ IR properties of amplitudes exploited to compute logarithmic enhanced perturbative terms ▶ resummations
- ◉ improve physics content of Monte Carlo event generators ▶ parton showers
- ◉ Evolution of PDF's and fragmentation functions
- ◉ beyond QCD: hints on the structure of highly symmetric gauge theories (e.g. N=4 super-Yang-Mills)
- ◉ Factorization theorems: pQCD for hard processes


Altarelli, Parisi, Berends, Giele, Mangano, Parke ...

A dark background with intricate, glowing white particle tracks. The tracks are complex, with many overlapping loops and spirals, resembling a particle detector's output. The most prominent feature is a large, dense spiral on the right side of the image.

PDF

Parton distribution functions

- Non-perturbative input determined from global fits to collider data, scale evolution from pQCD (NNLO)
- Vast choice: e.g. <http://hepdata.cedar.ac.uk/pdfs>



The Durham HepData Project

REACTION DATABASE • DATA REVIEWS • **PDF PLOTTER** • ABOUT HEPDATA • SUBMITTING DATA

HepData Compilation of Parton Distribution Functions

On-line Unpolarized Parton Distribution Calculator with Graphical Display.

Unpolarized Parton Distributions

Access the parton distribution code, on-line calculation and graphical display of the distributions, from CTEQ, GRV, MRST/MSTW, Alekhin, ZEUS, H1, HERAPDF, BBG and NNPDF.

- CTEQ fortran code and grids
- CTEQ-Jefferson Lab (CJ) the CJ12 PDF sets
- GRV/GJR fortran code and grids
- MRST fortran code and grids, C++ code
- MSTW fortran, C++ and Mathematica codes + grids etc.
- ALEKHIN fortran, C++, Mathematica code, and grids
- ZEUS ZEUS 2002 PDFs, ZEUS 2005 jet fit PDFs
- HERAPDF Combined H1/ZEUS page, HERAPDF1.0 paper
- H1 H1 2000
- BBG BBG06_NS
- NNPDF Non Singlet PDF code - hep-ph/0701127

Polarized Parton Distributions

Currently available parametrizations

LSS2001 E.Leader, A.V.Sidorov and D.B.Stamenov, Eur.Phys.J.C23 (2002) 479

Online PDF plotting and calculation

$xf(x, Q^2) \nu x$

Using the form below you can calculate, in real time, values of $xf(x, Q^2)$ for any of the PDFs from the different groups. You can also generate and compare plots of $xf(x, Q^2)$ νx at any Q^2 for up to 4 different parton types or PDF sets.

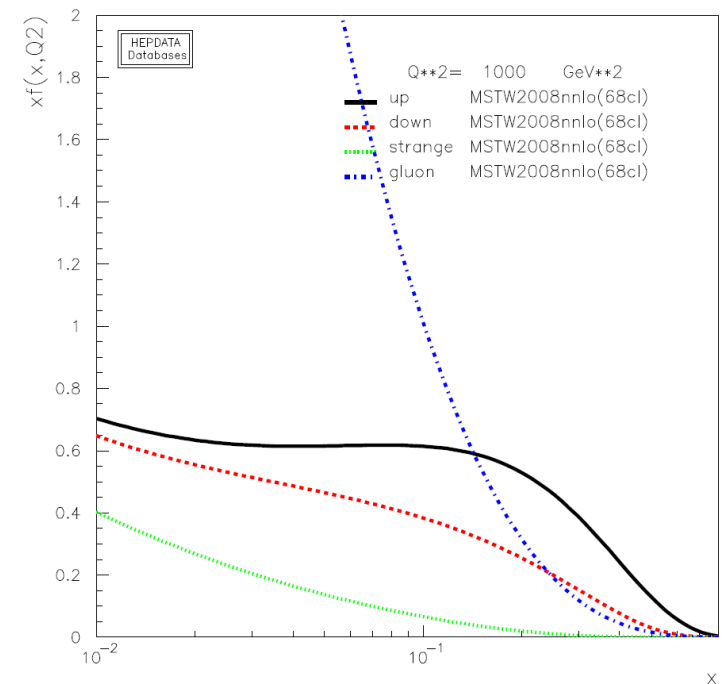
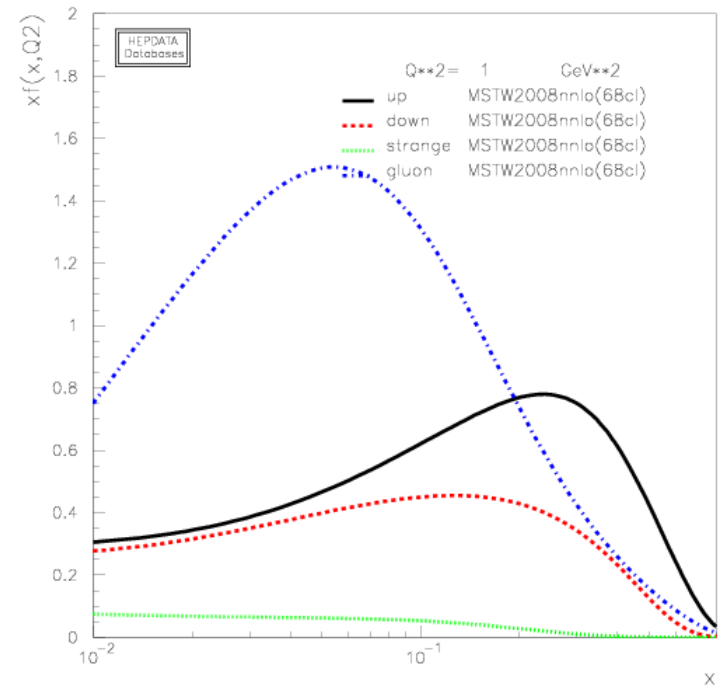
Select:	Parton	Group	Set
<input checked="" type="checkbox"/>	up	MSTW-nnlo	MSTW2008nnlo
<input checked="" type="checkbox"/>	down	MSTW-nnlo	MSTW2008nnlo
<input checked="" type="checkbox"/>	strange	MSTW-nnlo	MSTW2008nnlo
<input checked="" type="checkbox"/>	gluon	MSTW-nnlo	MSTW2008nnlo

Xmin = 0.01 Xmax = 0.8 Xinc = 0.01
 Q2 = 1 GeV²
 x axis: lin log
 y axis: lin log, ymin= 0.0 ymax = 2.0
 Output as: numbers or plot (line width = 10) as ratio

Make the Plot **add sets** **remove sets**

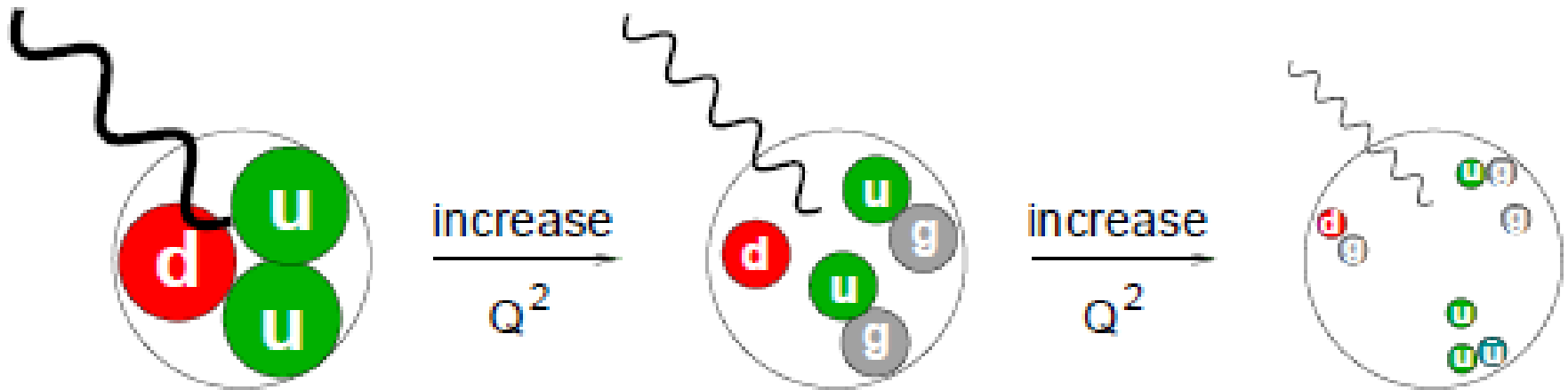
Change to plotting versus Q²

Change to Error Set plotting

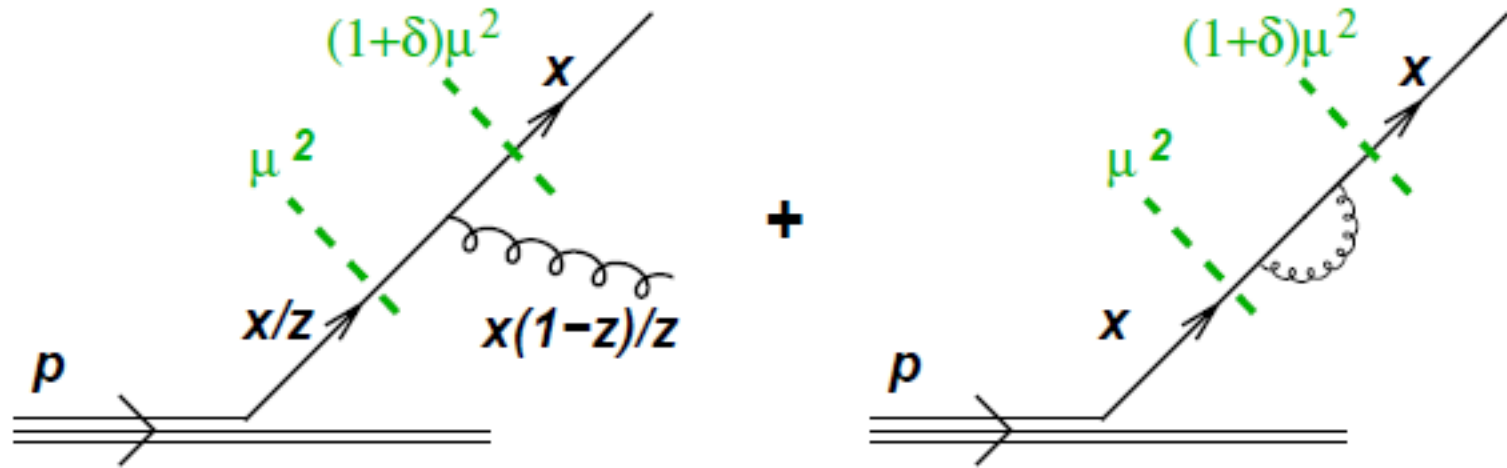


- Maximum of up and down at $x = 1/3$: three quarks sharing the proton momentum
- up = 2 x down
- Gluon evolves faster: color charge $C_A = 3$ versus quark color charge $C_F = 4/3$

looking inside de proton



DGLAP evolution



$$\frac{\partial q(x, \mu^2)}{\partial \log \mu^2} = \frac{\alpha_S}{2\pi} \int_x^1 \frac{dz}{z} P_{q \rightarrow qg}(z) q(x/z, \mu^2)$$

DGLAP flavour structure

The proton contains both quarks and gluons: DGLAP is a **matrix in flavour space**

$$\frac{\partial}{\partial \log \mu^2} \begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{q \rightarrow qg} & P_{g \rightarrow q\bar{q}} \\ P_{q \rightarrow gq} & P_{g \rightarrow gg} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

spanning over all flavours and anti-flavours

$$P_{q \rightarrow qg} = C_F \left(\frac{1+z^2}{1-z} \right)_+$$

$$P_{q \rightarrow gq} = C_F \frac{1+(1-z)^2}{z}$$

$$P_{g \rightarrow q\bar{q}} = T_R [z^2 + (1-z)^2]$$

$$P_{g \rightarrow gg} = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \delta(1-z)b_0$$

with the plus-prescription $\int_0^1 dz [g(z)]_+ f(z) = \int_0^1 dz g(z) [f(z) - f(1)]$

PDFs: strategy in a nutshell

- Make an **ansatz** for the functional form of the PDFs at some fixed value low scale $Q_0^2 \sim 1 \text{ GeV}^2$
e.g. in MRST/MSTW

$$x u_V = A_u x^{\eta_1} (1-x)^{\eta_2} (1 + \epsilon_u \sqrt{x} + \gamma_u x) \quad u_V = u - \bar{u}$$

$$x d_V = A_d x^{\eta_3} (1-x)^{\eta_4} (1 + \epsilon_d \sqrt{x} + \gamma_d x) \quad d_V = d - \bar{d}$$

$$x g = A_g x^{-\lambda_g} (1-x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g x)$$

Note: **NNPDF** use neural networks and does not need such explicit functional form

- Collect data at various (x, Q^2) from different experiments (e.g. DIS), use DGLAP equation to evolve down to Q_0^2 and fit parameters, including α_s

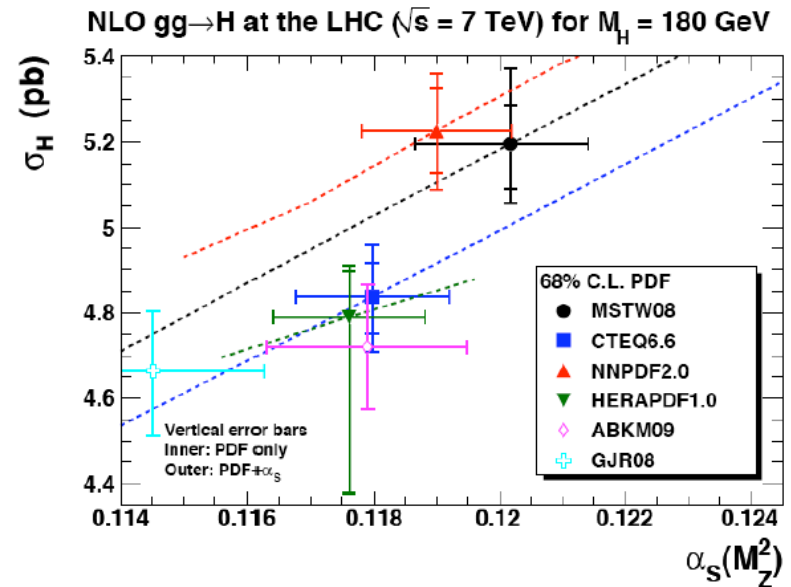
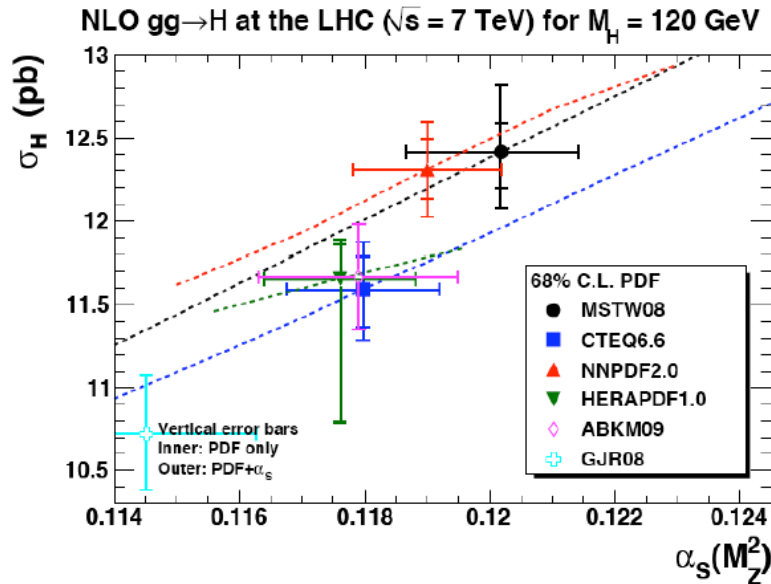
- Ensure **sum rules**
(Gottfried, momentum, ...):
$$\int dx x \sum_i f_i(x, Q^2) = 1$$

Parton distribution functions

■ Differences are due to different:

Data sets in fits, parameterization of starting distributions, order of pQCD evolution, power law contributions, nuclear target corrections, resummation corrections ($\ln 1/x, \dots$), treatment of heavy quarks, strong coupling, choice of factorization and renormalization scales.

■ at least 5-10% uncertainty in theoretical predictions



A visualization of particle jets, showing a central point from which numerous yellow lines radiate outwards, representing the paths of particles. The background is dark blue with some red and orange streaks. The word "Jets" is written in large white letters across the center.

Jets

What's a jet



- a bunch of energetic and collimated particles
- 60% of LHC papers use jets [Salam, Soyez]

Why and how do we see jets?

Gluon emission

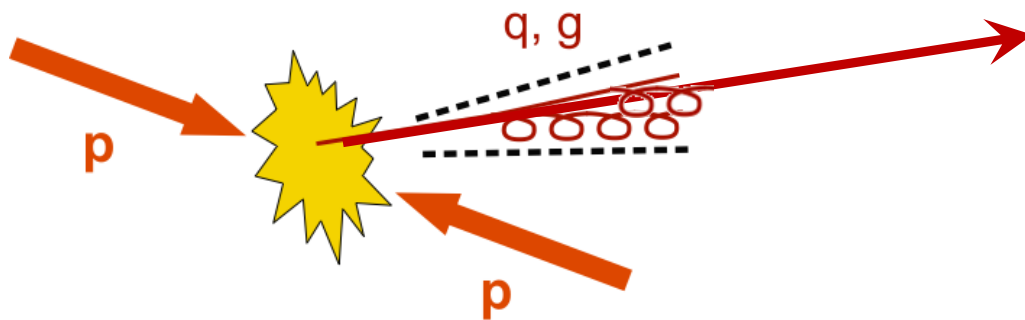
$$\int \alpha_s \frac{dE}{E} \frac{d\theta}{\theta} \gg 1$$

higher probability at small angle
(collinear) and small energy (soft)

Non-perturbative
transition to hadrons

$$\alpha_s \sim 1 \quad \Lambda_{\text{QCD}} \sim 200\text{MeV}$$

Parton level



$\{j_k\}$

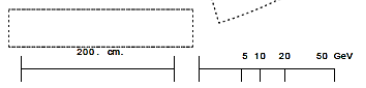
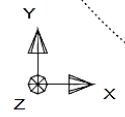
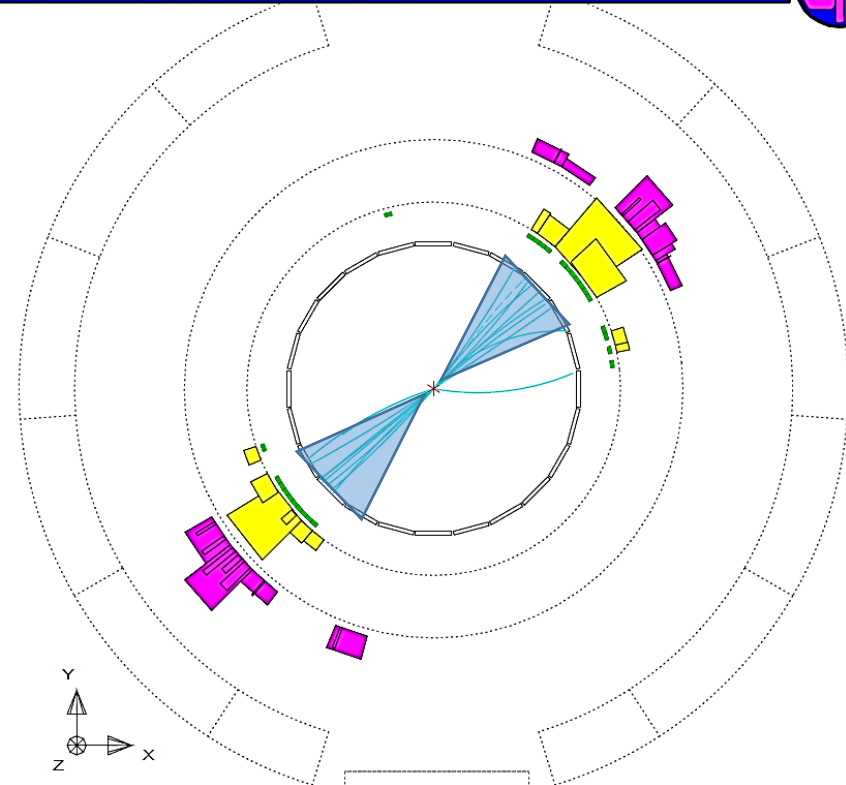
jets

jet definition
←

$\{p_i\}$

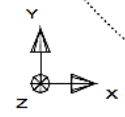
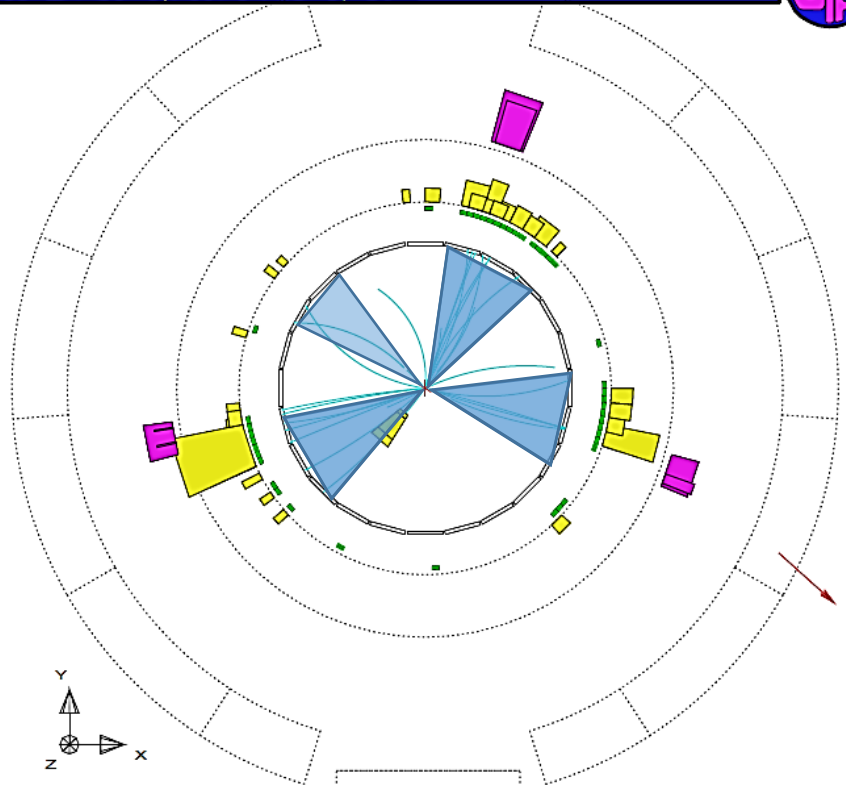
final-state
4-momenta

Run: event 4093 : 1000 Date 930527 Time 20716 Ctrk(N= 39 Sump= 73.3) Ecal(N= 25 SunE= 32.6) Hcal(N=22 SunE= 22.6)
 Ebeam 45.658 Evis 99.9 Emiss -8.6 Vix (-0.07, 0.06, -0.80) Muon(N= 0) Sec Vix(N= 3) Fdel(N= 0 SunE= 0.0)
 Bz=4.350 Thrust=0.9873 Aplan=0.0017 Oblat=0.0248 Spher=0.0073

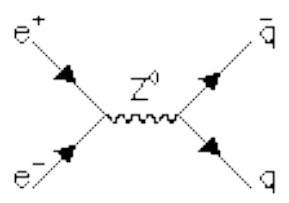


Centre of screen is (0.0000, 0.0000, 0.0000)

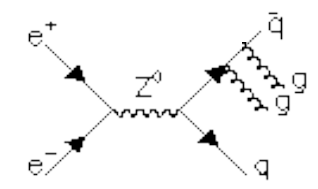
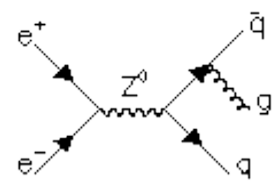
Run: event 2542: 63750 Date 911014 Time 35925 Ctrk(N= 28 Sump= 42.1) Ecal(N= 42 SunE= 59.8) Hcal(N= 8 SunE= 12.7)
 Ebeam 45.609 Evis 86.2 Emiss 5.0 Vix (-0.05, 0.12, -0.90) Muon(N= 1) Sec Vix(N= 0) Fdel(N= 2 SunE= 0.0)
 Bz=4.350 Thrust=0.8223 Aplan=0.0120 Oblat=0.3338 Spher=0.2463



Centre of screen is (0.0000, 0.0000, 0.0000)



■ Clearly a two-jet event



- Three- or four-jet event ?
- Depends on the jet resolution parameter

The k_T jet algorithm at hadron colliders

[Catani, Dokshitzer, Seymour, Webber, 93]

[Ellis, Soper, 93]

- Define distance among particles: e.g. $d_{ij} = (p_i + p_j)^2$
- Is this distance smaller than a resolution parameter? Combine into the same jet recursively
- At hadron colliders there are beams, introduce also "beam distance":
$$d_{iB} = p_{Ti}^2 = 2E_i^2(1 - \cos \theta_{iB})$$
- Preference to use longitudinal invariant variables: transverse momenta (p_T), rapidity (Δy) and azimuthal angle (ϕ)

Kinematics

Longitudinal variables

- Rapidity:

$$y = \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right)$$

- Pseudo-rapidity:

$$\eta = -\log(\tan(\theta/2))$$

Transverse plane

- Azimuthal angle

$$\phi$$

- Transverse momentum

$$p_T = \sqrt{p_x^2 + p_y^2}$$

- Transverse mass

$$m_T = \sqrt{p_T^2 + m^2}$$

$$p^\mu = (m_T \cosh(y), p_T \cos(\phi), p_T \sin(\phi), m_T \sinh(y))$$

Exercises:

1. Show that $\eta = y$ for massless particles
2. Show that $\Delta y = y_i - y_j$ is invariant under boost

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Inclusive k_T

$$d_{ij} = \min(p_{Ti}^2, p_{Tj}^2) \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{Ti}^2 \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

- Compute the smallest distance d_{ij} or d_{iB}
- If d_{ij} , cluster i and j together
- If d_{iB} , call i a jet and remove from the list of particles
- Repeat until no particle is left
- Two parameters: R and minimal transverse momentum $p_{Ti} > p_{T,min}$

The anti- k_T jet algorithm

[Cacciari, Salam, Soyez 08]

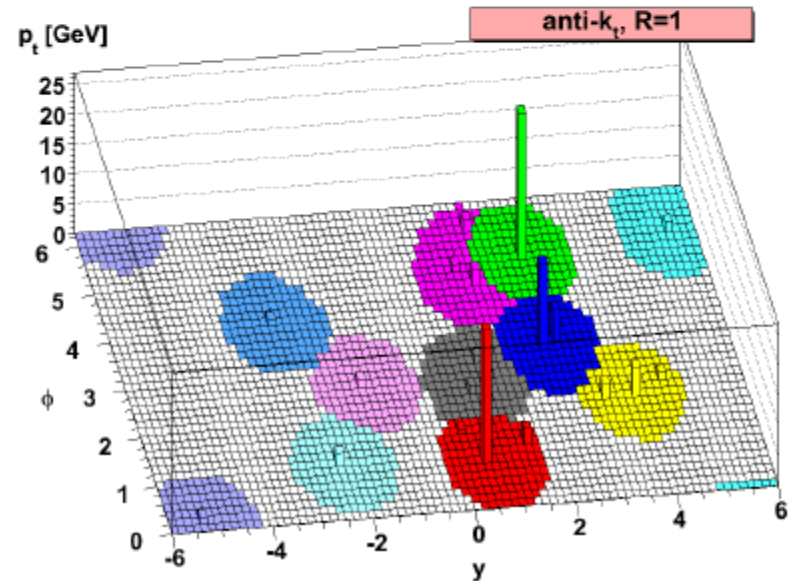
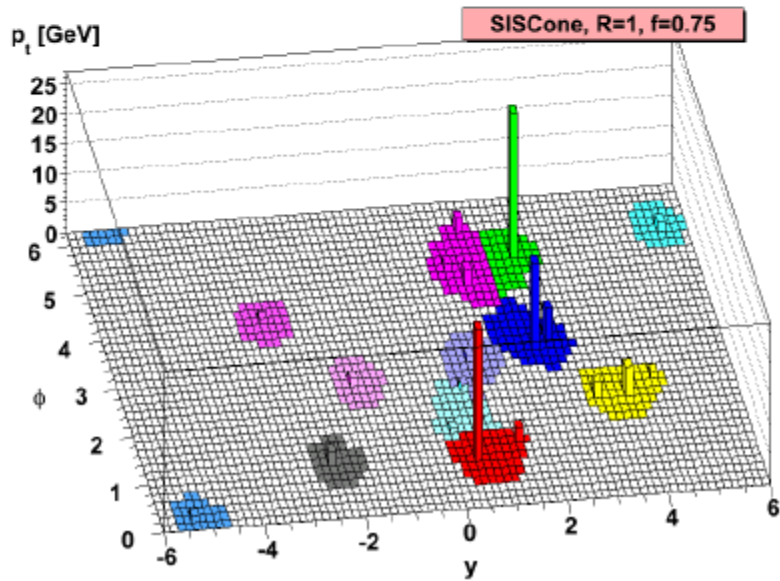
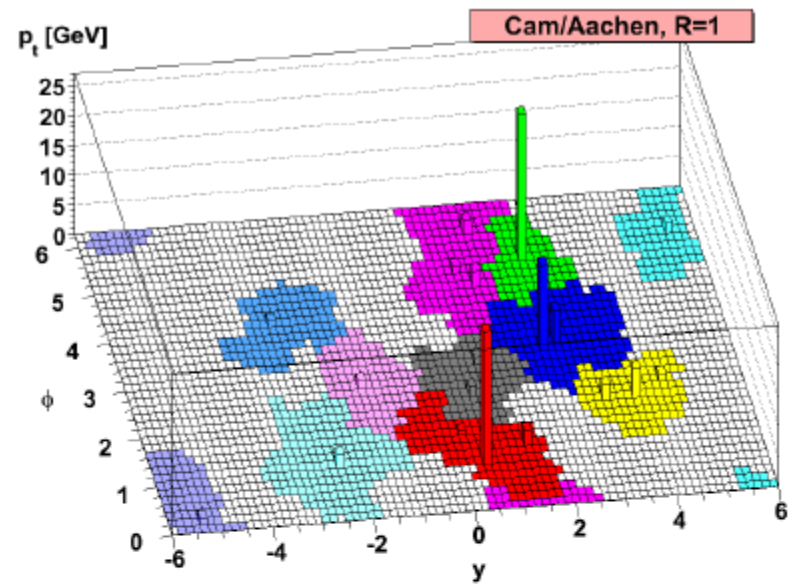
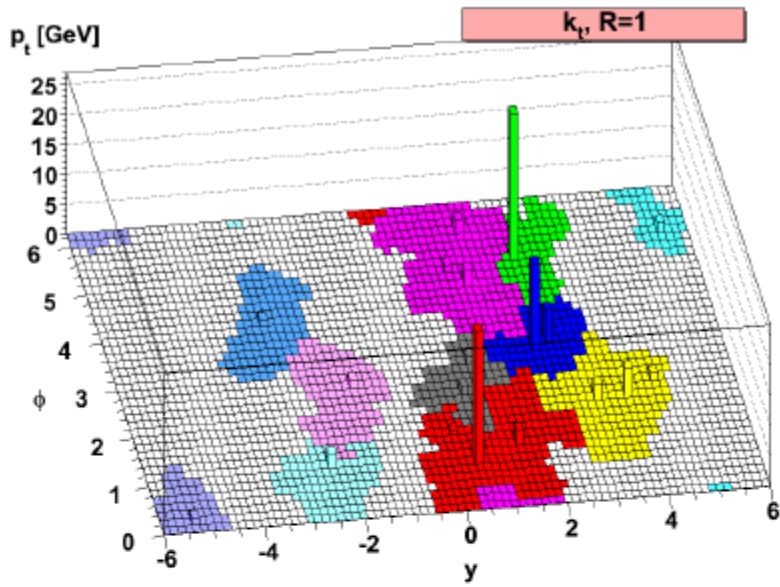
- k_T has a physical meaning: the stronger the divergence between a pair of particles, the more likely it is they should be associated with each other
- However, ATLAS and CMS have adopted anti- k_T as default

anti- k_T

$$d_{ij} = \min(p_{Ti}^{-2}, p_{Tj}^{-2}) \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = p_{Ti}^{-2} \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

- Clusters hardest particles first
- IRC safe, and cone-shaped jets
- Easier to get jet energy scale right
- CAMBRIDGE/AACHEN: $d_{ij} = \Delta R_{ij} / R^2$

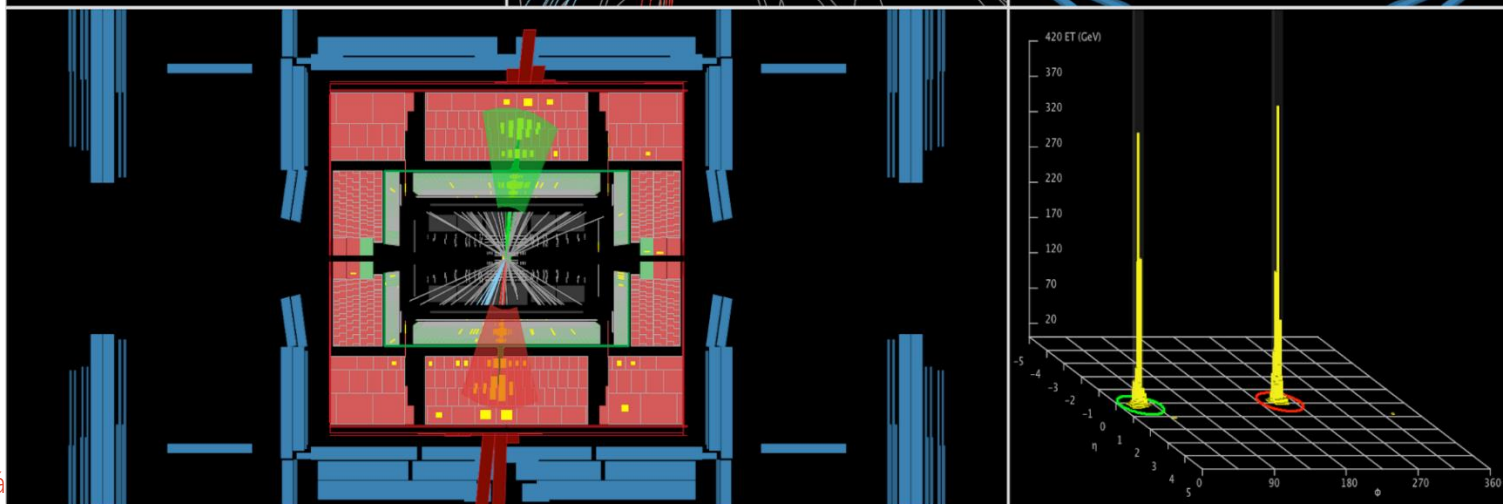
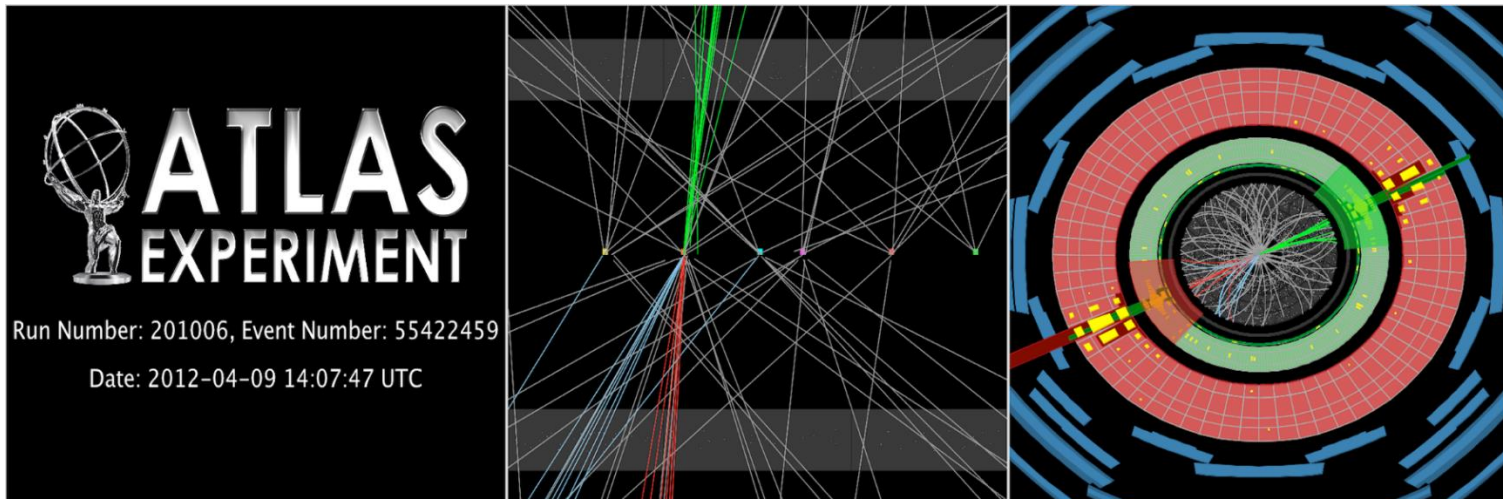
- For the first time ever, a hadron collider carries out measurements that can be consistently compared with theoretical (perturbative QCD) calculations
- Cones extensively used at Tevatron are not IRC safe



High mass central di-jet event

A track p_T cut of 0.5 GeV has been applied for the display.

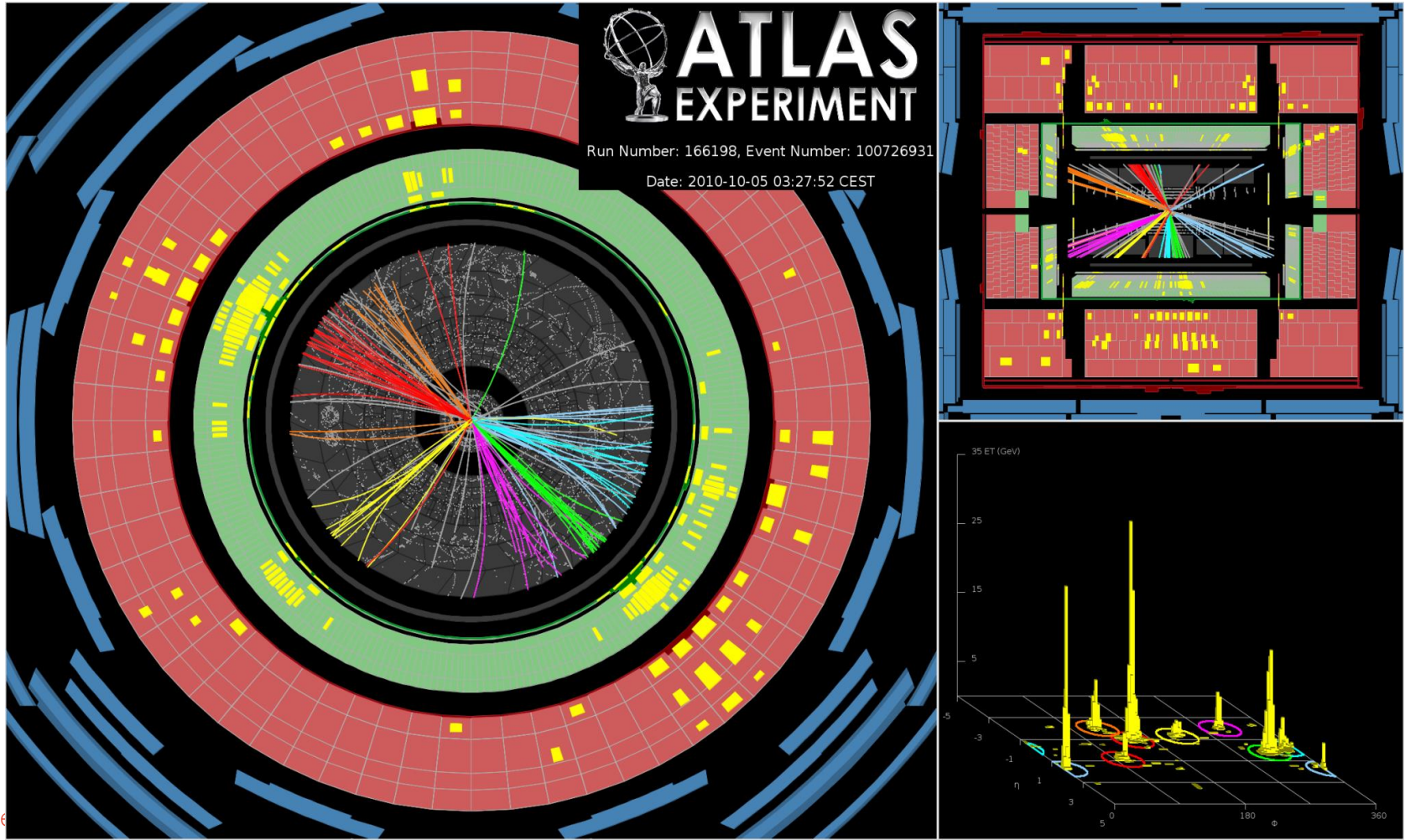
- 1st jet (ordered by p_T): $p_T = 1.96$ TeV, $\eta = -0.07$, $\varphi = -2.68$
- 2nd jet: $p_T = 1.65$ TeV, $\eta = 0.17$, $\varphi = 0.48$
- Missing $E_T = 318$ GeV, $\varphi = 0.43$
- Sum $E_T = 3.81$ TeV



A high jet multiplicity event

counting jets with p_T greater than 60 GeV: this event has eight

- 1st jet (ordered by p_T): $p_T = 290$ GeV, $\eta = -0.9$, $\varphi = 2.7$
- 2nd jet: $p_T = 220$ GeV, $\eta = 0.3$, $\varphi = -0.7$
- missing $E_T = 21$ GeV, $\varphi = -1.9$
- sum $E_T = 890$ GeV



Display of a **semi-leptonic top quark pair** event

at high invariant mass (714 GeV)

The top quark boosts lead the decay products to be collimated, albeit still distinguishable using standard reconstruction algorithms.

