Atomtronics, Benasque, 13.05.2015

Quantum Simulation of the Quantum Hall Effect

Tobias Grass (ICFO - Barcelona)

In collaboration with:

Bruno Julia-Diaz (University Barcelona)

Maciej Lewenstein (ICFO)

Nuria Barberan (University Barcelona)

David Raventos (ICFO)

Cold atomic quantum Hall effect – Why?

Systems with well controlled Hamiltonians

Quantum Hall solver

- Hard problem: Competition between different phases

Novel quantum Hall phases

- Quantum Hall effect of bosons
- *Interacting* integer quantum Hall phases
- Ideal phases (parent Hamiltonians)

Exploring anyonic properties

- First experimental detection of fractional statistics
- Braiding of (non-Abelian) anyons
- Anyon technologies

Outline

1. Quantum Hall Physics – in general:

- Single-particle physics: Landau levels
- Many-body effects: Trial states

2. Fractional quantum Hall physics of spin-1/2 bosons:

- Abelian vs. Non-Abelian phases
- Numerical results: ambiguous
- Prospects of a quantum simulation

3. *Integer* quantum Hall physics of spin-1/2 bosons:

- Interactions are crucial
- Edge spectrum as a fingerprint

4. Anyon braiding in small systems

Outline

1. Quantum Hall Physics – in general:

- Single-particle physics: Landau levels
- Many-body effects: Trial states

2. Fractional quantum Hall physics of spin-1/2 bosons:

- Abelian vs. Non-Abelian phases
- Numerical results: ambiguous
- Prospects of a quantum simulation

3. *Integer* quantum Hall physics of spin-1/2 bosons:

- Interactions are crucial
- Edge spectrum as a fingerprint

4. Anyon braiding in small systems

Quantum Hall Systems

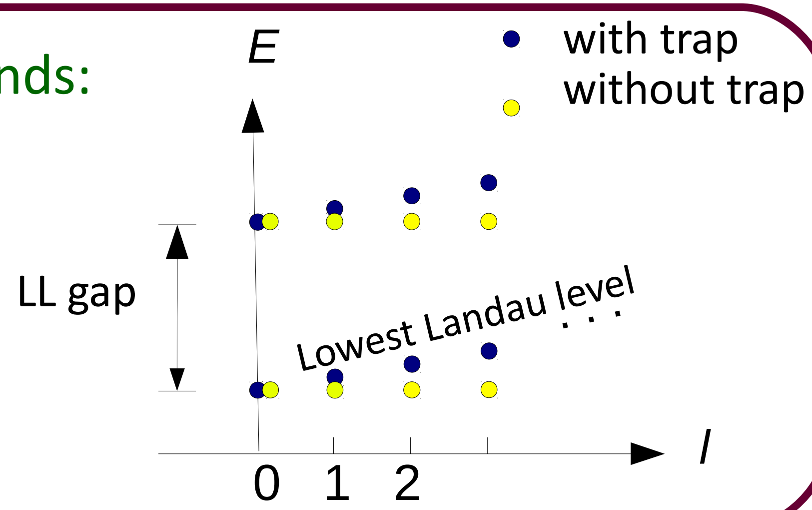
$$H = \frac{(\mathbf{p}+\mathbf{A})^2}{2M} + \frac{M}{2}\omega^2\mathbf{r}^2$$

Gauge potential:

$$\mathbf{A} = \frac{B}{2}(y, -x)$$

Trapping potential

Flat bands:



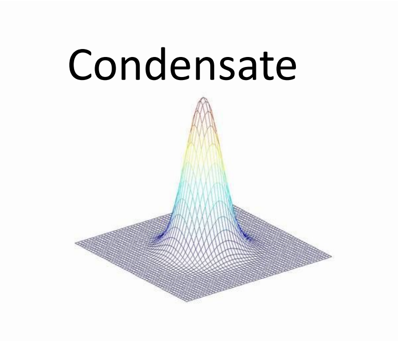
Fermions:

Integer filling:
→ effectively non-interacting
→ Integer Quantum Hall Phases

Fractional filling:
→ Gapped phases due to interactions
→ Fractional Quantum Hall Phases

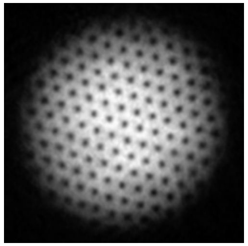
Bosons:

Condensate



→ *symmetry breaking*

Vortex lattice



→ *melting*

Interacting Quantum Hall Phases:

→ fractional
(like the fermionic ones , with or without spin)

→ integer
(no fermionic counterpart, needs spin)

Trial states: Laughlin and Halperin

Wave functions with "zeros" for all particle pairs:

- Laughlin wave function (spinless system)

$$\Psi_{\mathrm{L}}^{(q)} = \prod_{i<j} (z_i - z_j)^q \exp[-\sum_i |z_i|^2/2] \qquad \text{filling} \quad \nu = 1/q$$

$$(z = x + iy)$$

- Halperin wave function (two-component system)

$$\Psi_{\mathrm{H}}^{(lmn)} \sim \prod_{1\leq i<j\leq N_\uparrow} (z_{i\uparrow} - z_{j\uparrow})^l \prod_{1\leq i<j\leq N_\downarrow} (z_{i\downarrow} - z_{j\downarrow})^m \prod_{\substack{1\leq i\leq N_\uparrow \\ 1\leq j\leq N_\downarrow}} (z_{i\uparrow} - z_{j\downarrow})^n$$

fillings $\nu_\uparrow = \dfrac{l-n}{lm-n^2}$ and $\nu_\downarrow = \dfrac{m-n}{lm-n^2}$

Exact zero-energy solutions in contact potential!

Trial states: Pairing states

1) Divide system into k clusters.
2) Each cluster forms a Laughlin/Halperin state.
3) (Anti-)Symmetrize over all possible clusters.

Moore/Read (1991)
Read/Rezayi (1999)

→ Read-Rezayi series (spinless):

$$\Psi_{\mathrm{RR}}^{(k)} \sim \mathcal{S}[\Psi_{\mathrm{L}}^{(2)}(z_{i_1}, \ldots, z_{i_M})\Psi_{\mathrm{L}}^{(2)}(z_{i_{M+1}}, \ldots, z_{i_{2M}}) \ldots]$$

filling $\nu = k/2$

→ Non-Abelian spin singlet (NASS) series

Ardonne/Schoutens (1999)

$$\Psi_{\mathrm{NASS}}^{(k)} \sim \mathcal{S}[\Psi_{\mathrm{H}}^{(221)}(z_{i_1\uparrow}, \ldots, z_{i_M\uparrow}, z_{i_1\downarrow}, \ldots, z_{i_M\downarrow})\Psi_{\mathrm{H}}^{(221)}(z_{i_{M+1}\uparrow}, \ldots, z_{i_{2M}\uparrow}, z_{i_{M+1}\downarrow}, \ldots, z_{i_{2M}\downarrow}) \ldots]$$

filling $\nu = 2k/3$

Exact ground states for (*k+1*)-body contact interactions!

Trial states: Composite fermion states

Construction Recipe:

Jain/Kawamura (1995)
Cooper/Wilkin (1999)

1. Composite fermion = particle + *m* magnetic fluxes

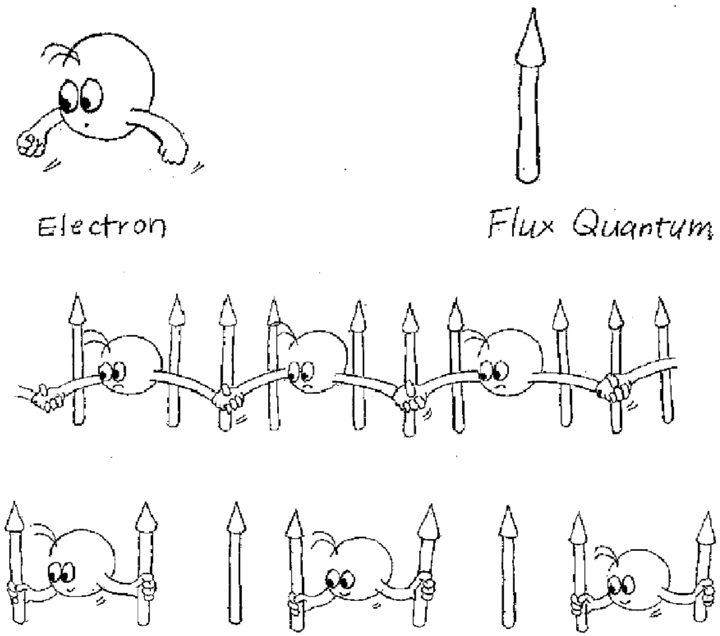
   → Jastrow factor: $J(z) = \prod_{i>j} (z_i - z_j)^m$

2. CFs fill Landau levels at modified magnetic field

   → Slater determinant ϕ of filled LLs

3. Project back into low-energy space:
   Lowest Landau level of the original system

$$\Psi_{\rm CF} = \mathcal{P}_{\rm LLL} \Phi(z) J(z)$$



(from Jain book)

Construction works for fermionic and bosonic systems with or without spin, at filling factors $\nu = \frac{n}{mn \pm 1}$ where the number *m* of attached fluxes per particle must be even for fermions or odd for bosons.

Trial states: Overview

	Spinless fermions	Spinless bosons	Two-component bosons (fully unpolarized)
Abelian Fractional Quantum Hall States	Laughlin $\nu = 1/q, \ q \text{ odd}$ CF states $\nu = \frac{n}{mn+1}, \ m \text{ even}$	Laughlin $\nu = 1/q, \ q \text{ even}$ CF states $\nu = \frac{n}{mn+1}, \ m \text{ odd}$	Halperin $\nu = \frac{2}{m+n}, \ m \text{ even}$ CF states $\nu = \frac{n}{n \pm 1} \notin \mathbb{N}$
Non-Abelian Fractional Quantum Hall States	Read-Rezayi $\nu = \frac{k}{k+2}$	Read-Rezayi $\nu = \frac{k}{2}$	NASS $\nu = \frac{2k}{3}$
Integer Quantum Hall States	trivial	✗	CF state $\nu = 2$

Trial states: Overview

	Spinless fermions	Spinless bosons	Two-component bosons (fully unpolarized)
Abelian Fractional Quantum Hall States	Laughlin $\nu = 1/q, \ q \text{ odd}$ CF states $\nu = \dfrac{n}{mn+1}, \ m \text{ even}$	Laughlin $\nu = 1/q, \ q \text{ even}$ CF states $\nu = \dfrac{n}{mn+1}, \ m \text{ odd}$	Halperin $\nu = \dfrac{2}{m+n}, \ m \text{ even}$ CF states $\nu = \dfrac{n}{n \pm 1} \notin \mathbb{N}$
Non-Abelian Fractional Quantum Hall States	Read-Rezayi $\nu = \dfrac{k}{k+2}$	Read-Rezayi $\nu = \dfrac{k}{2}$	NASS $\nu = \dfrac{2k}{3}$
Integer Quantum Hall States	trivial	✗	CF state $\nu = 2$

Outline

1. Quantum Hall Physics – in general:

- Single-particle physics: Landau levels
- Many-body effects: Trial states

2. Fractional quantum Hall physics of spin-1/2 bosons:

- Abelian vs. Non-Abelian phases
- Numerical results: ambiguous
- Prospects of a quantum simulation

3. *Integer* quantum Hall physics of spin-1/2 bosons:

- Interactions are crucial
- Edge spectrum as a fingerprint

4. Anyon braiding in small systems

The System

We now focus on:

- bosons
- pseudospin-1/2
- in the lowest Landau level
- with contact interactions:

$$H = \sum_{i<j} \left[g_{\uparrow\uparrow}\delta(z_{i\uparrow} - z_{j\uparrow}) + g_{\downarrow\downarrow}\delta(z_{i\downarrow} - z_{j\downarrow}) + g_{\uparrow\downarrow}\delta(z_{i\uparrow} - z_{j\downarrow}) + g_{\uparrow\downarrow}\delta(z_{i\downarrow} - z_{j\uparrow}) \right]$$

- SU(2)-symmetric: $g_{\uparrow\uparrow} = g_{\uparrow\downarrow} = g_{\downarrow\downarrow}$

Numerical studies on different geometries:

Disk	Torus	Sphere
· most realistic · edge effects	· Purely bulk physics · Complicated wave functions	· Purely bulk physics · Relatively simple wave func. · Shifted filling factors

NASS series on the torus?

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 031604(R) (2012)

Quantum Hall states in rapidly rotating two-component Bose gases

Shunsuke Furukawa and Masahito Ueda
Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 021603(R) (2012)

Non-Abelian spin-singlet states of two-component Bose gases in artificial gauge fields

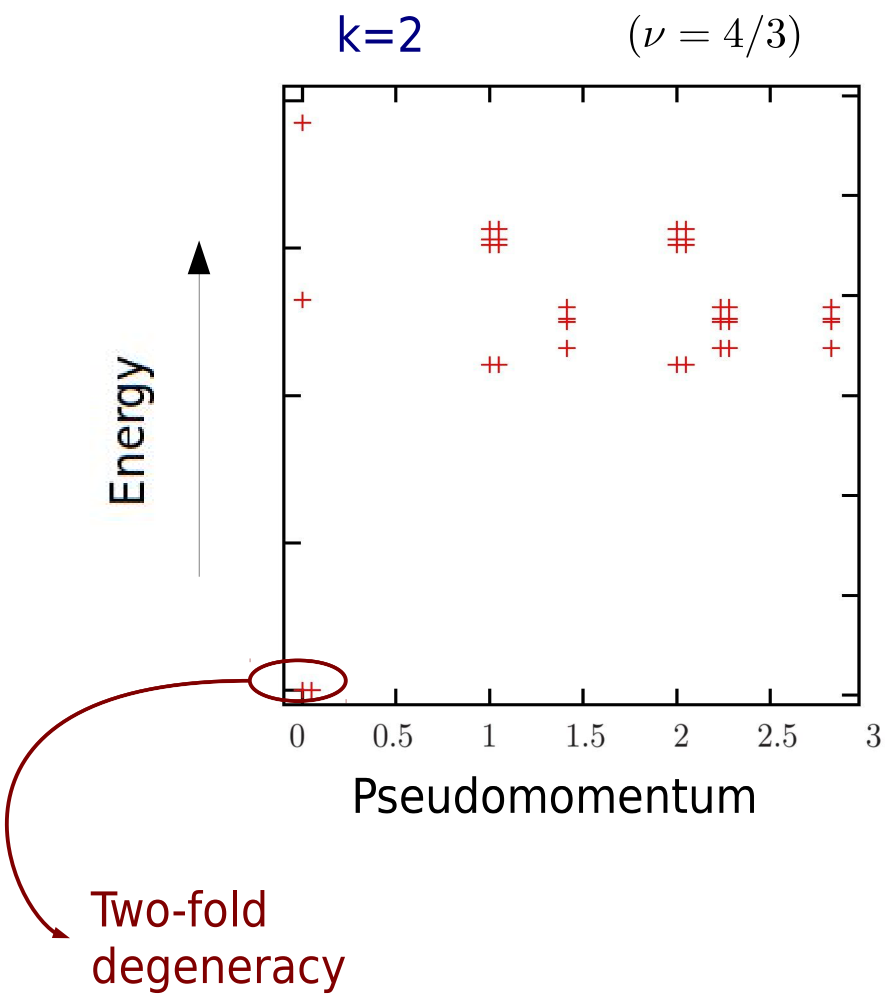
T. Graß,[1] B. Juliá-Díaz,[1] N. Barberán,[2] and M. Lewenstein[1,3]
[1]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament ECM, Facultat de Física, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*ICREA-Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

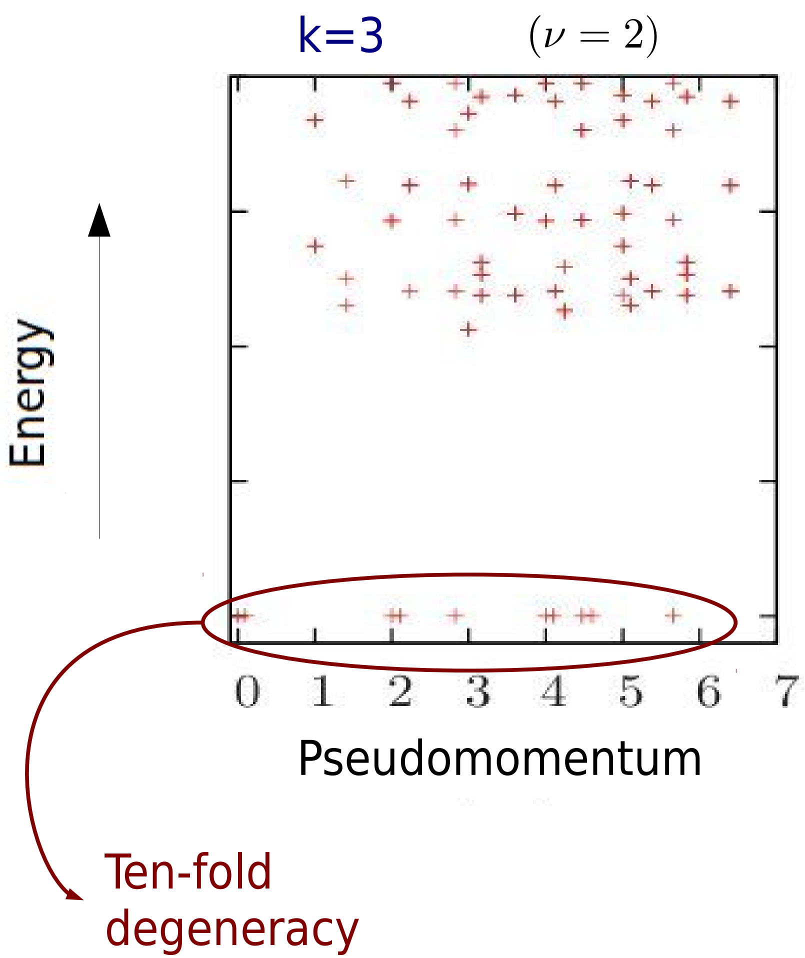
Exact diagonalization on the torus:

- Evidence of incompressible (gapped) phases at $\nu = \frac{2k}{3}$ for $k = 1, 2, 3.$
- NASS series?

NASS series on the torus?

Spectra of *(k+1)*-body contact interaction

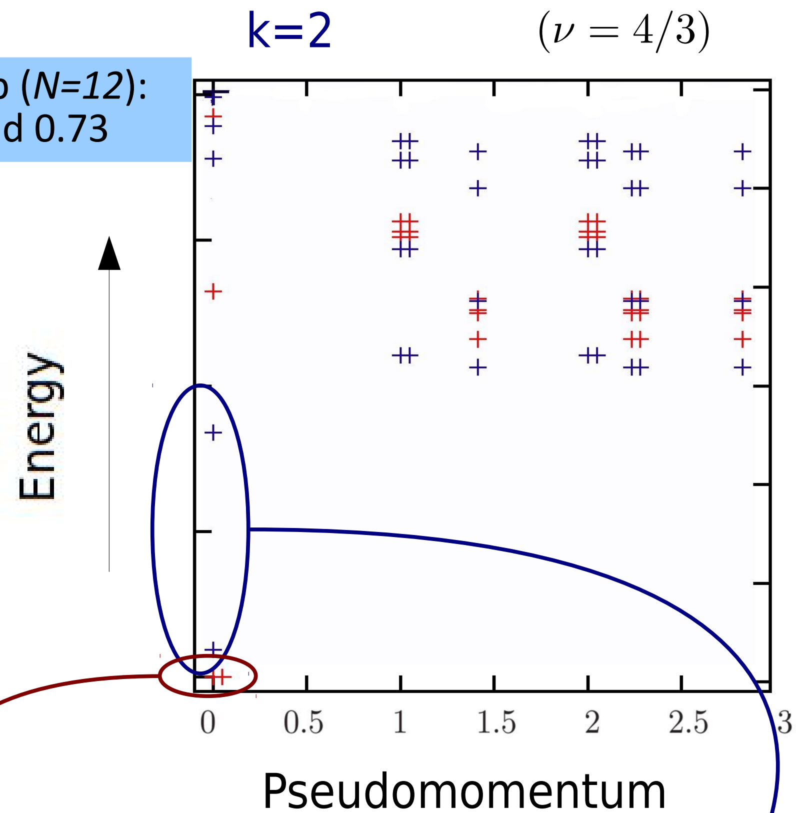




NASS series on the torus?

Spectra of *(k+1)*-body contact interaction
versus
Spectra of two-body contact interaction

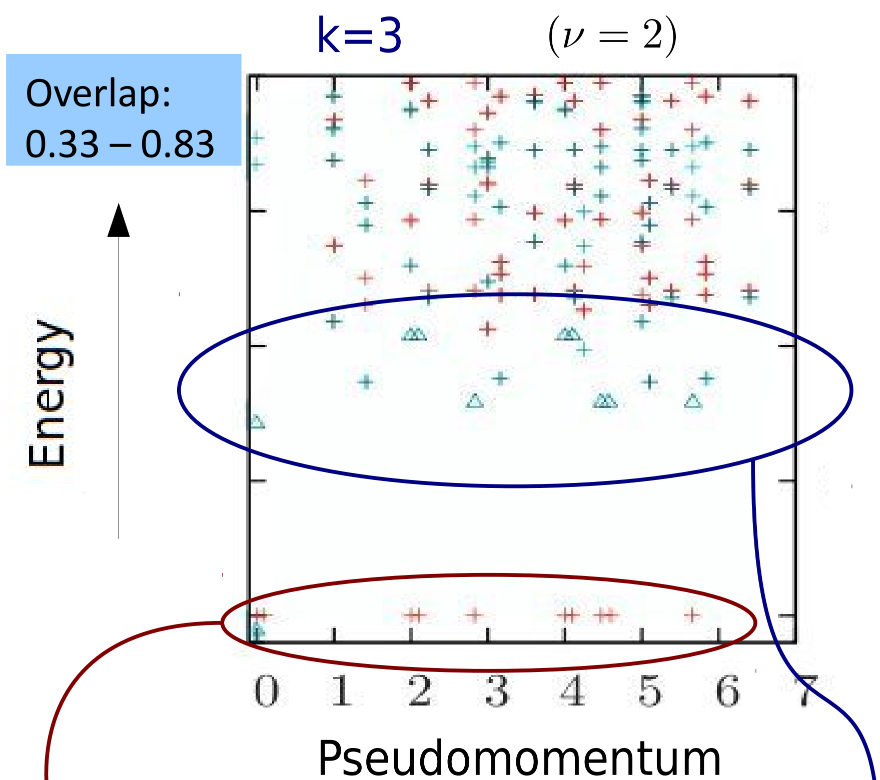
Overlap (*N=12*): 0.89 and 0.73



Two-fold degeneracy

Split two-fold degeneracy

Overlap: 0.33 – 0.83



Ten-fold degeneracy

mixing

CF states on the sphere?

ED on torus:

NASS phase at $\nu=4/3$

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 031604(R) (2012)

Quantum Hall states in rapidly rotating two-component Bose gases

Shunsuke Furukawa and Masahito Ueda

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 021603(R) (2012)

Non-Abelian spin-singlet states of two-component Bose gases in artificial gauge fields

T. Graß,[1] B. Juliá-Díaz,[1] N. Barberán,[2] and M. Lewenstein[1,3]
[1]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament ECM, Facultat de Física, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*ICREA-Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

Different picture on the sphere!

PHYSICAL REVIEW B **87**, 245123 (2013)

Quantum Hall effect of two-component bosons at fractional and integral fillings

Ying-Hai Wu and Jainendra K. Jain
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

CF states on the sphere?

Overlaps on the sphere:

- with NASS state: 0.918
- with CF state: 0.985

for *N=12* at filling *ν=4/3.*

BUT: Filling factor is biased on the sphere.

$$\nu = \frac{N}{N_{\mathrm{V}}} + \delta$$

Direct competition between NASS and CF is not possible on the sphere.
(Neither on small disks!)

PHYSICAL REVIEW B **87**, 245123 (2013)

Quantum Hall effect of two-component bosons at fractional and integral fillings

Ying-Hai Wu and Jainendra K. Jain

Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

CF states on the sphere?

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 031604(R) (2012)

Quantum Hall states in rapidly rota

Shunsuke Furukawa an
Department of Physics, University of Tokyo, 7-3-1

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 021603(R) (2012)

Non-Abelian spin-singlet states of two-component Bose gases in artificial gauge fields

T. Graß,[1] B. Juliá-Díaz,[1] N. Barberán,[2] and M. Lewenstein[1,3]
[1]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament ECM, Facultat de Física, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*ICREA-Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

?

PHYSICAL REVIEW B **87**, 245123 (2013)

Quantum Hall effect of two-component bosons at fractional and integral fillings

Ying-Hai Wu and Jainendra K. Jain
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

CF states on the sphere?

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 031604(R) (2012)

Quantum Hall states in rapidly rota

Shunsuke Furukawa an

Department of Physics, University of Tokyo, 7-3-1

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **86**, 021603(R) (2012)

Non-Abelian spin-singlet states of two-component Bose gases in artificial gauge fields

T. Graß,[1] B. Juliá-Díaz,[1] N. Barberán,[2] and M. Lewenstein[1,3]
[1]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament ECM, Facultat de Física, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*ICREA-Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

?

To solve the competition, one might study:

- Overlaps on the torus
- System on a disk → Quantum simulation!

PHYSICAL REVIEW B **87**, 245123 (2013)

Quantum Hall effect of two-component bosons at fractional and integral fillings

Ying-Hai Wu and Jainendra K. Jain
Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA

Cold atom quantum simulation

All ingredients of the Hamiltonian are available:

- ✓ Synthetic magnetic fields
- ✓ 2-body contact potential

But how could a quantum simulation distinguish between different states?

Cold atom quantum simulation

All ingredients of the Hamiltonian are available:

- ✔ Synthetic magnetic fields
- ✔ 2-body contact potential

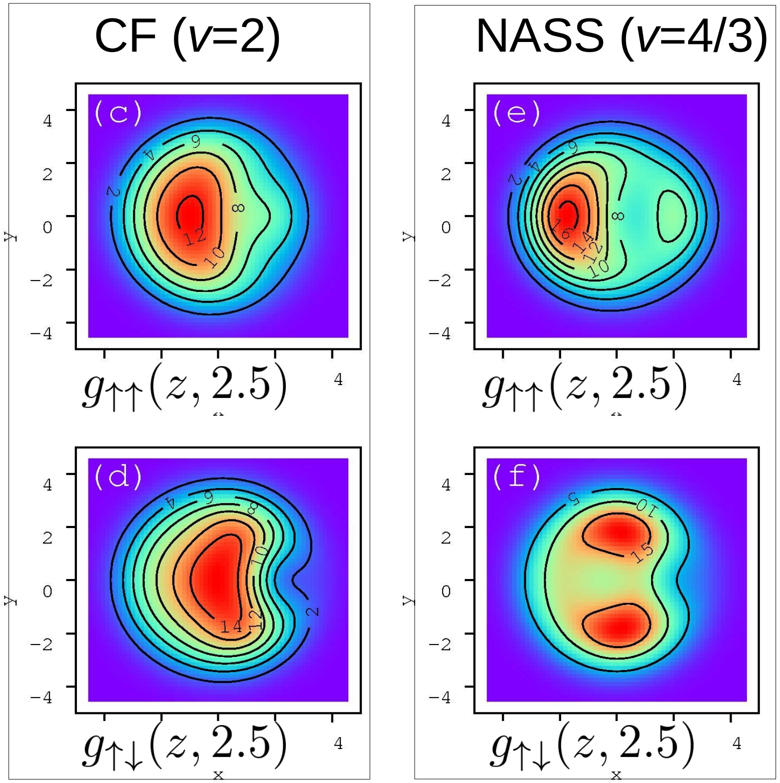
But how could a quantum simulation distinguish between different states?

Example:

$N_\uparrow = 4$

$N_\downarrow = 4$

$L = 16$



Correlation functions: $C(z_1, z_2) = \langle \Psi | \hat{\psi}^\dagger(z_1) \hat{\psi}^\dagger(z_2) \hat{\psi}(z_1) \hat{\psi}(z_2) | \Psi \rangle$

Cold atom quantum simulation

All ingredients of the Hamiltonian are available:

- ✓ Synthetic magnetic fields
- ✓ 2-body contact potential

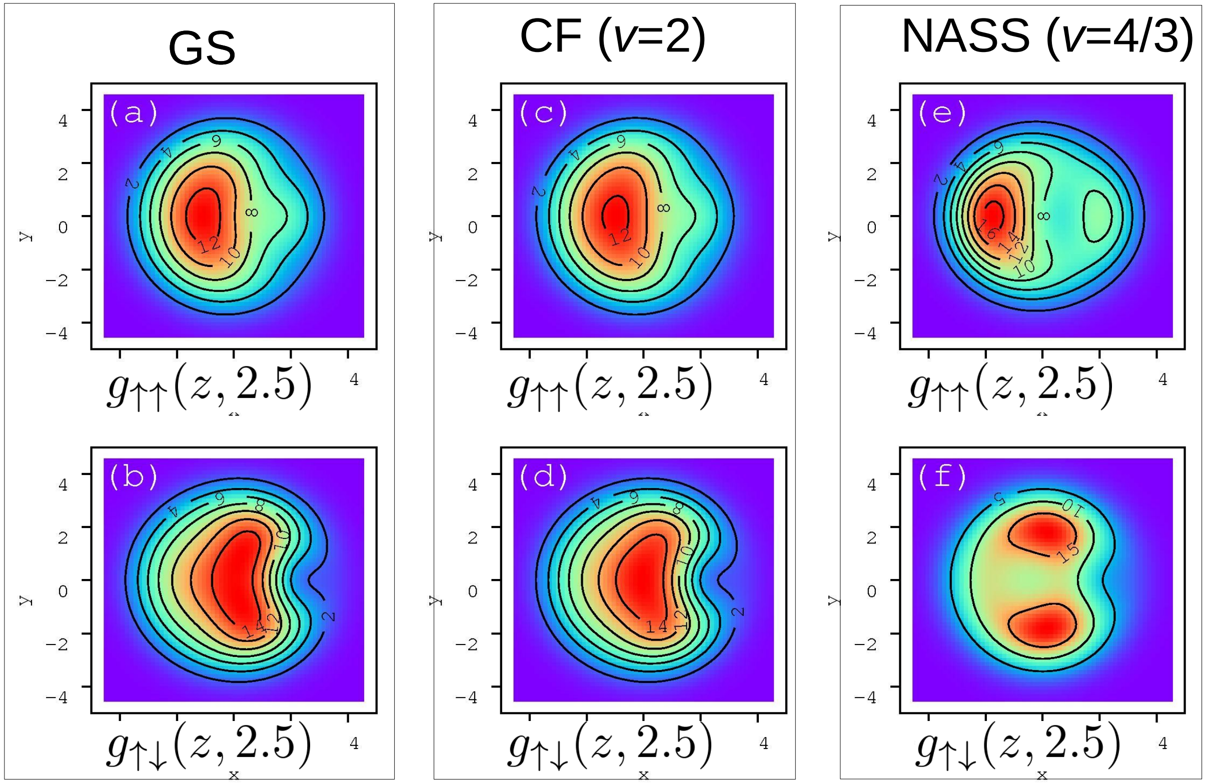
But how could a quantum simulation distinguish between different states?

Example:

$N_\uparrow = 4$

$N_\downarrow = 4$

$L = 16$



Correlation functions: $C(z_1, z_2) = \langle \Psi | \hat{\psi}^\dagger(z_1) \hat{\psi}^\dagger(z_2) \hat{\psi}(z_1) \hat{\psi}(z_2) | \Psi \rangle$

Outline

1. Quantum Hall Physics – in general:

- Single-particle physics: Landau levels
- Many-body effects: Trial states

2. Fractional quantum Hall physics of spin-1/2 bosons:

- Abelian vs. Non-Abelian phases
- Numerical results: ambiguous
- Prospects of a quantum simulation

3. *Integer* quantum Hall physics of spin-1/2 bosons:

- Interactions are crucial
- Edge spectrum as a fingerprint

4. Anyon braiding in small systems

What happens at $\nu=2$?

PRL **110,** 046801 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

Integer Quantum Hall Effect for Bosons

T. Senthil[1] and Michael Levin[2]

[1]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Department of Physics, Condensed Matter Theory Center, University of Maryland, College Park, Maryland 20742, USA*

Effective field theory: Possibility of an interacting integer quantum Hall effect for two-component bosons at *ν=2.*

PRL **111,** 090401 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Integer Quantum Hall State in Two-Component Bose Gases in a Synthetic Magnetic Field

Shunsuke Furukawa and Masahito Ueda

Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku,

PHYSICAL REVIEW B **87**, 245123 (2013)

Quantum Hall effect of two-component bosons at fractional and integral fillings

Ying-Hai Wu and Jainendra K. Jain

Pennsylvania State University, University Park, Pennsylvania 16802, USA

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **88**, 161106(R) (2013)

Microscopic model for the boson integer quantum Hall effect

N. Regnault[1,2] and T. Senthil[3]

[1]*Department of Physics, Princeton University, Princeton, New*
[2]*Laboratoire Pierre Aigrain, ENS and CNRS, 24 rue Lhomond,*
[3]*Department of Physics, Massachusetts Institute of Technology, Cambrid*

PHYSICAL REVIEW B **89**, 045114 (2014)

Quantum Hall phases of two-component bosons

T. Graß,[1] D. Raventós,[2] M. Lewenstein,[1,3] and B. Juliá-Díaz[1,2]

[1]*ICFO–Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*ICREA–Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

What happens at ν=2 ?

PRL **110,** 046801 (2013) PHYSICAL REVIEW LETTERS week ending 25 JANUARY 2013

Integer Quantum Hall Effect for Bosons

T. Senthil[1] and Michael Levin[2]

[1]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Department of Physics, Condensed Matter Theory Center, University of Maryland, College Park, Maryland 20742, USA*

Effective field theory: Possibility of an interacting integer quantum Hall effect for two-component bosons at ν=2.

PRL **111,** 090401 (2013) PHYSICAL REVIEW LETTERS week ending 30 AUGUST 2013

Integer Quantum Hall State in Two-Component Bose Gases in a Synthetic Magnetic Field

Shunsuke Furukawa and Masahito Ueda

Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku,

PHYSICAL REVIEW B **87**, 245123 (2013)

Quantum Hall effect of two-component bosons at fractional and integral fillings

Ying-Hai Wu and Jainendra K. Jain

Pennsylvania State University, University Park, Pennsylvania 16802, USA

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **88**, 161106(R) (2013)

Microscopic model for the boson integer quantum Hall effect

N. Regnault[1,2] and T. Senthil[3]

[1]*Department of Physics, Princeton University, Princeton, New*
[2]*Laboratoire Pierre Aigrain, ENS and CNRS, 24 rue Lhomond,*
[3]*Department of Physics, Massachusetts Institute of Technology, Cambrid*

PHYSICAL REVIEW B **89**, 045114 (2014)

Quantum Hall phases of two-component bosons

T. Graß,[1] D. Raventós,[2] M. Lewenstein,[1,3] and B. Juliá-Díaz[1,2]

[1]*ICFO–Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Barcelona, Spain*
[2]*Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 08028 Barcelona, Spain*
[3]*ICREA–Institució Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*

What happens at ν=2 ?

Torus:
Grass, Julia-Diaz, Barberan, Lewenstein (PRA, 2012)
Regnault & Senthil (PRB, 2013)
Furukawa & Ueda (PRL, 2013)

→ no NASS phase
→ unique, gapped GS

Sphere:
Furukawa & Ueda (PRL, 2013)
Wu & Jain (PRB, 2013)

Entanglement spectra:
→ edge physics of iIQHE

Overlap: with CF state
0.888 (N=14)

Disk:
Wu & Jain (PRB, 2013)
Grass, Raventos, Julia-Diaz, Lewenstein (PRB, 2014)

Edge spectrum agrees with IQH theory.

Overlap with CF state:
0.970 (N=8, L=16)

Edge spectrum at ν=2

Effective edge Hamiltonian of singlet state

[J.E. Moore, F.D.M. Haldane, PRB **55** 7818 (1997)]

$$H_{\mathrm{edge}} \propto v_s\Big(S_z^2 + \sum_l l b_l^\dagger b_l\Big) + v_c \sum_l l c_l^\dagger c_l$$

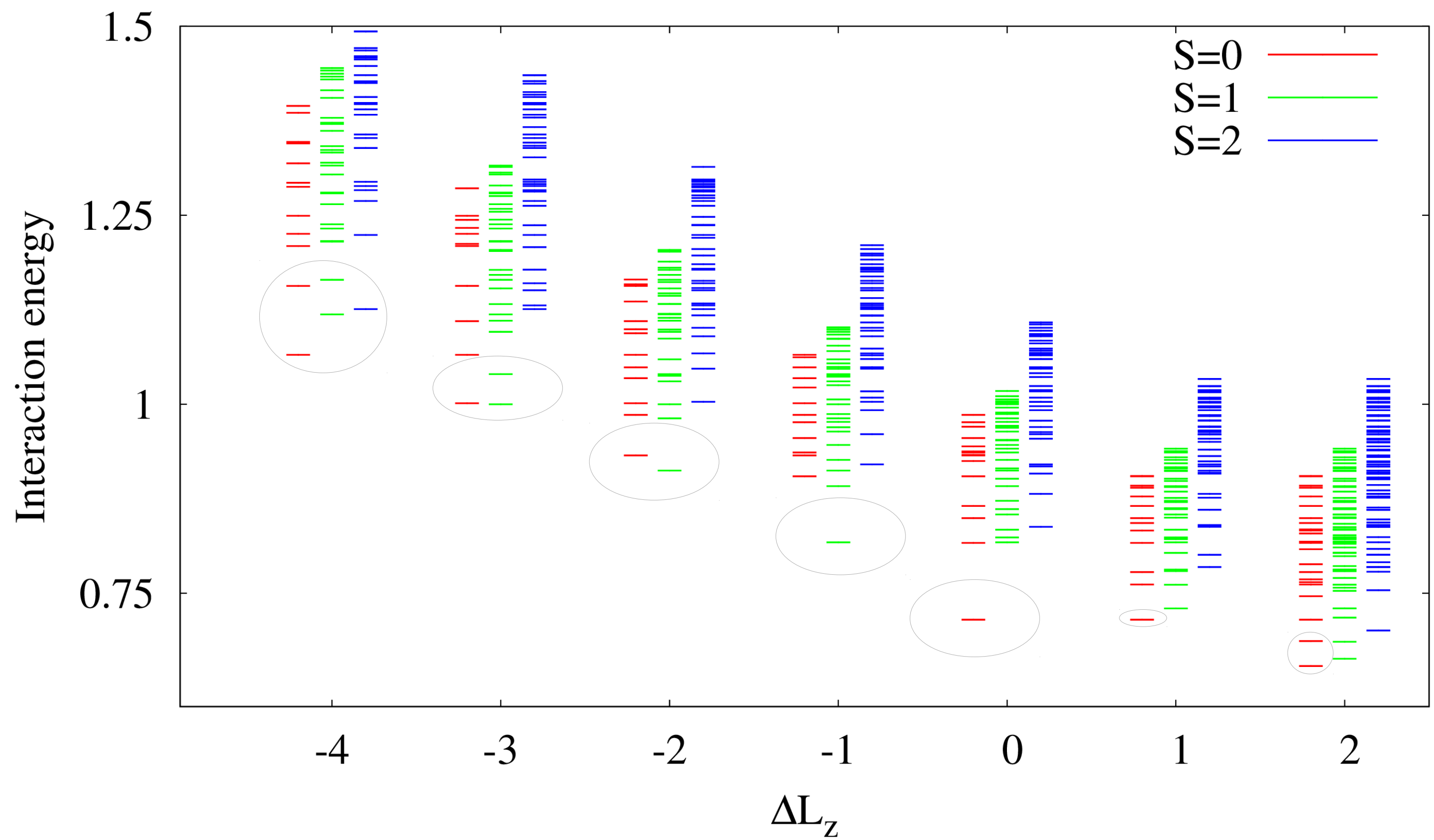
TABLE I. Number of modes of H_{edge} with $v_s < 0$ and $v_c > 0$.

ΔL_z	-4	-3	-2	-1	$+1$	$+2$	$+3$	$+4$
Number of singlets	2	1	1	0	1	2	3	5
Number of triplets	2	2	1	1	0	0	0	0
Number of quintets	1	0	0	0	0	0	0	0

Numerical results on a disk

[T. Grass *et al.*, PRB **89** 045114 (2014)]

$$N_\uparrow = 4$$
$$N_\downarrow = 4$$
$$L_z = 16 + \Delta L_z$$



Edge spectrum at *ν=2*

Effective edge Hamiltonian of singlet state

[J.E. Moore, F.D.M. Haldane, PRB **55** 7818 (1997)]

$$H_{\text{edge}} \propto v_s\left(S_z^2 + \sum_l l b_l^\dagger b_l\right) + v_c \sum_l l c_l^\dagger c_l$$

TABLE I. Number of modes of H_{edge} with $v_s < 0$ and $v_c > 0$.

ΔL_z	-4	-3	-2	-1	$+1$	$+2$	$+3$	$+4$
Number of singlets	2	1	1	0	1	2	3	5
Number of triplets	2	2	1	1	0	0	0	0
Number of quintets	1	0	0	0	0	0	0	0

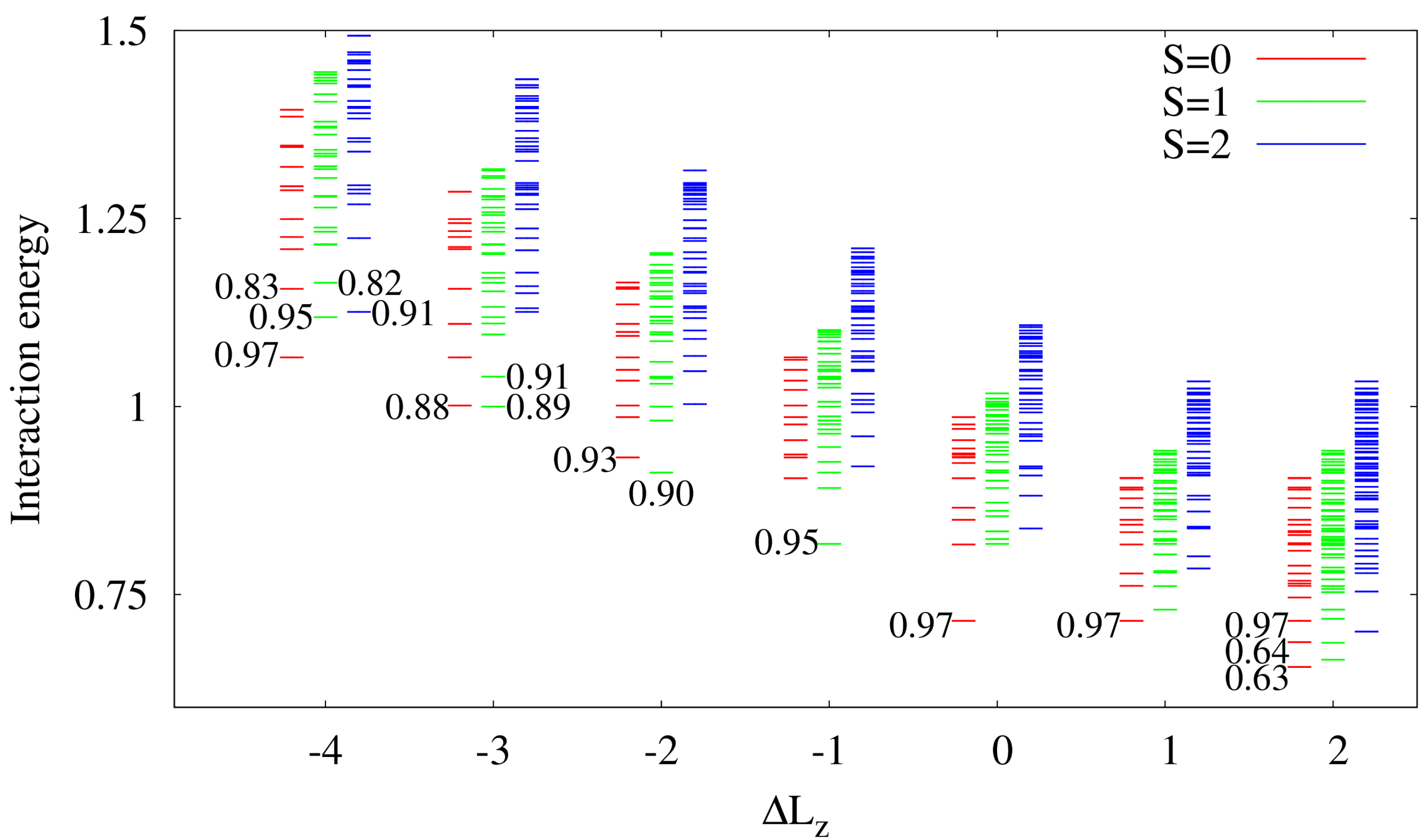
Numerical results on a disk

[T. Grass *et al.*, PRB **89** 045114 (2014)]

$$N_\uparrow = 4$$
$$N_\downarrow = 4$$
$$L_z = 16 + \Delta L_z$$

Explicit construction of edge states provides the same counting, and good overlaps:

- Backward states:
  Excite CFs in flux-reversed LLs
- Forward states:
  Symmetric polynomial



Outline

1. Quantum Hall Physics – in general:

- Single-particle physics: Landau levels
- Many-body effects: Trial states

2. Fractional quantum Hall physics of spin-1/2 bosons:

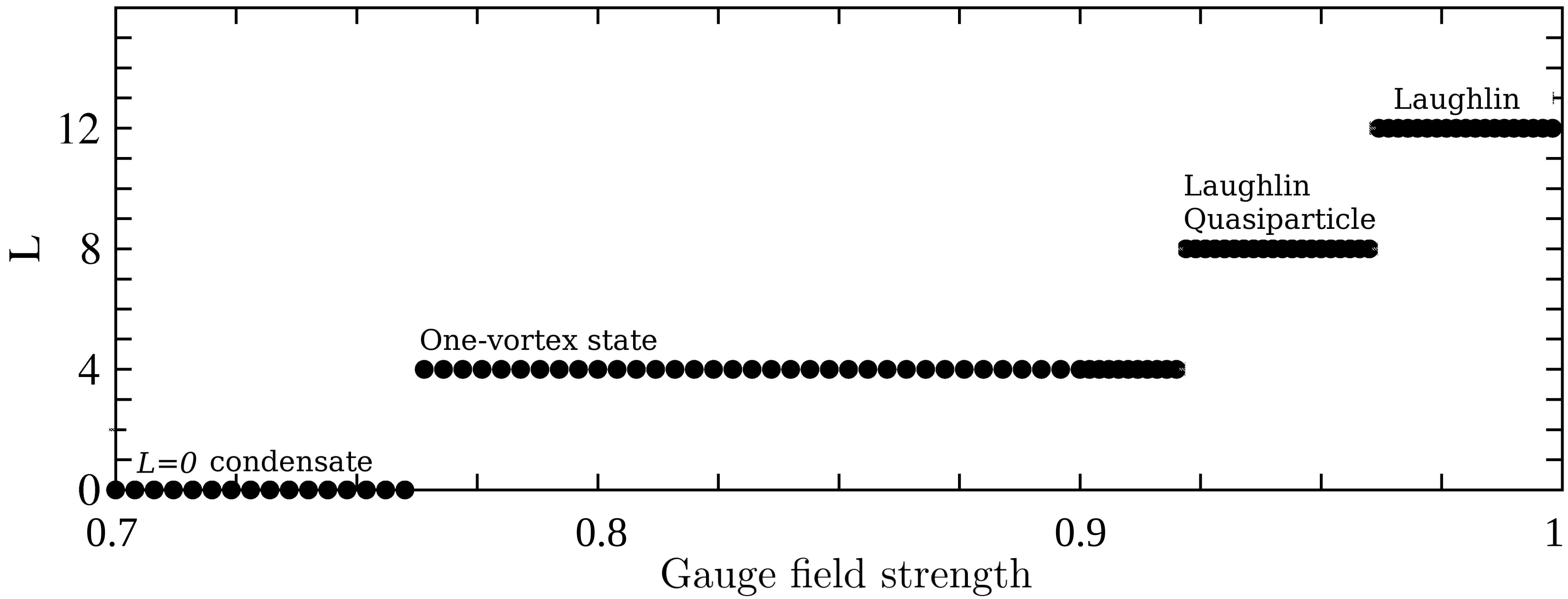
- Abelian vs. Non-Abelian phases
- Numerical results: ambiguous
- Prospects of a quantum simulation

3. *Integer* quantum Hall physics of spin-1/2 bosons:

- Interactions are crucial
- Edge spectrum as a fingerprint

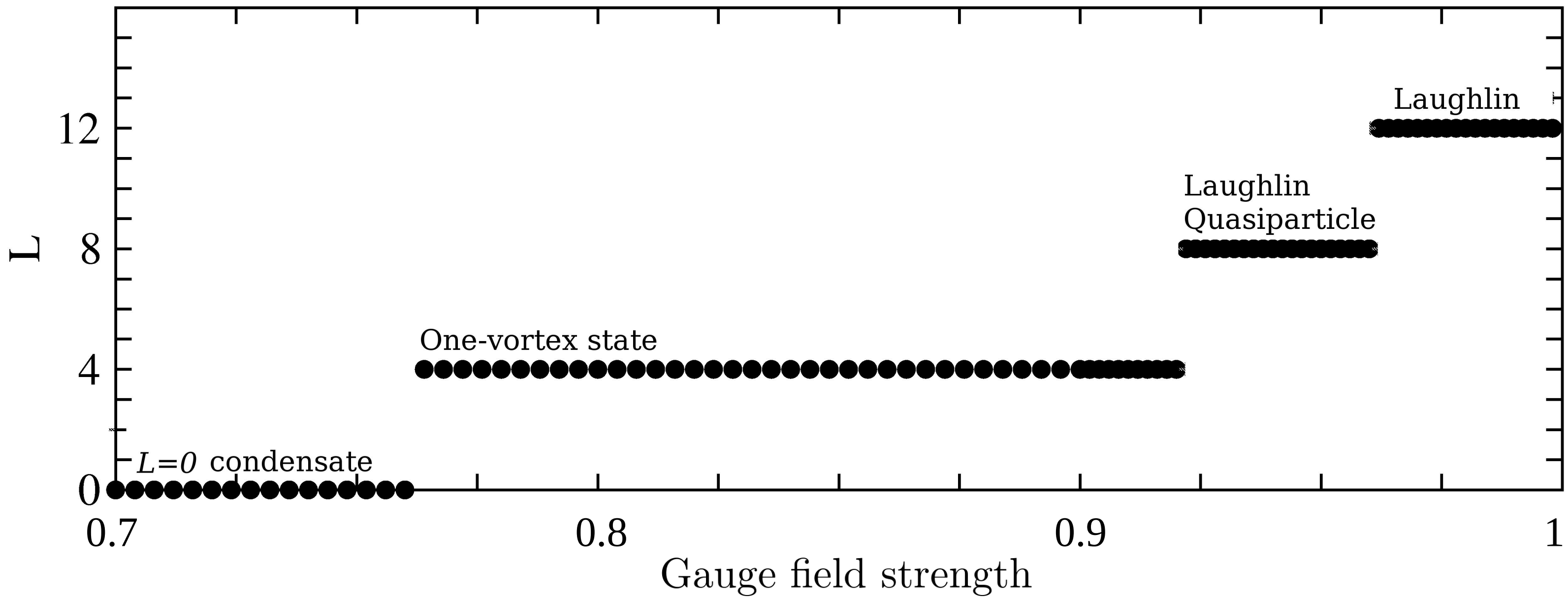
4. Anyon braiding in small systems

One-component Bose gas



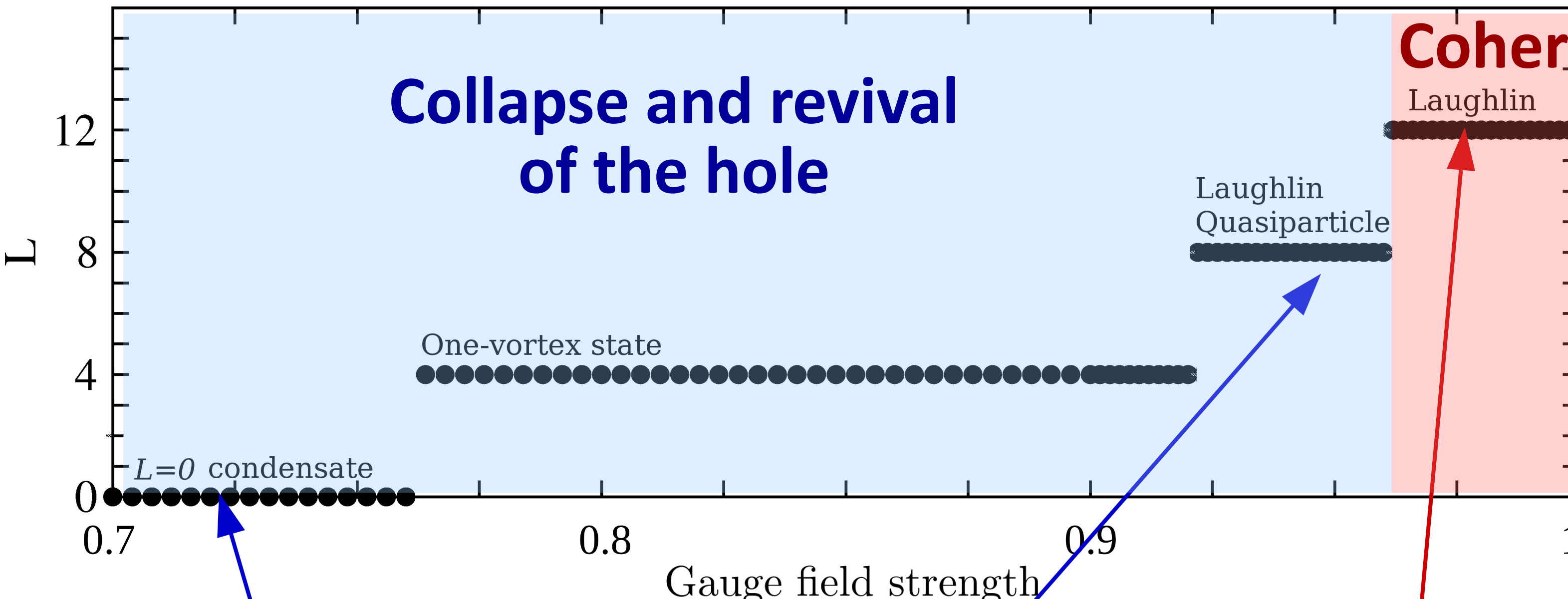
One-component Bose gas

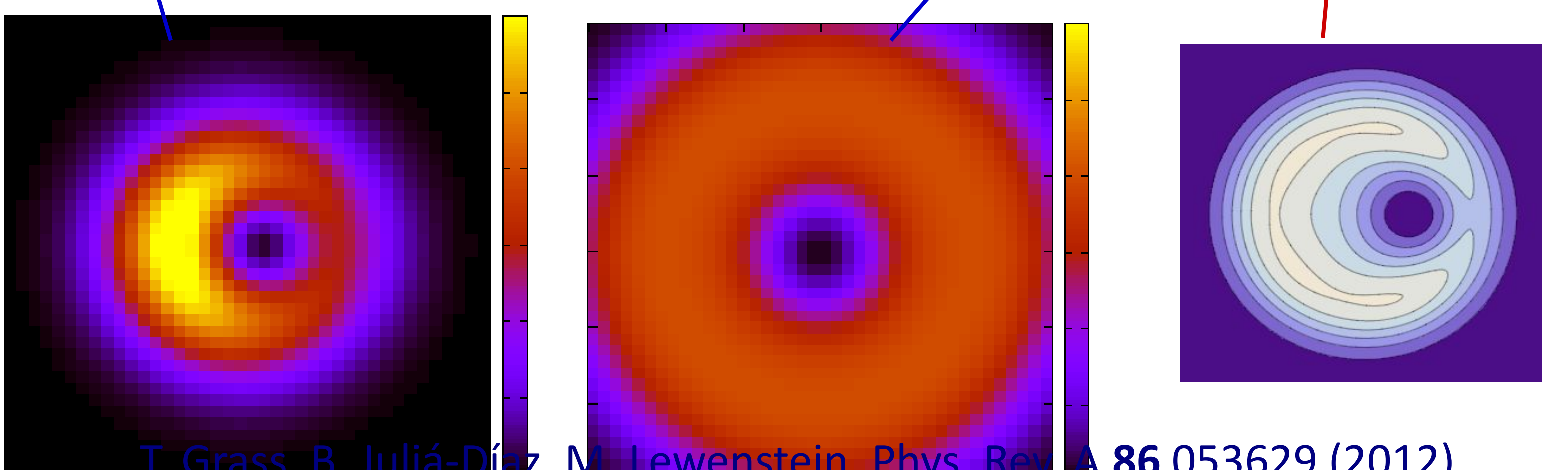
Pierce holes $\Psi \to \prod_i (z_i - \xi)\Psi$ for laser potential $V_\mathrm{L} \propto \delta(\xi)$.



One-component Bose gas

Pierce holes $\Psi \to \prod_i (z_i - \xi)\Psi$ for laser potential $V_{\mathrm{L}} \propto \delta(\xi)$.



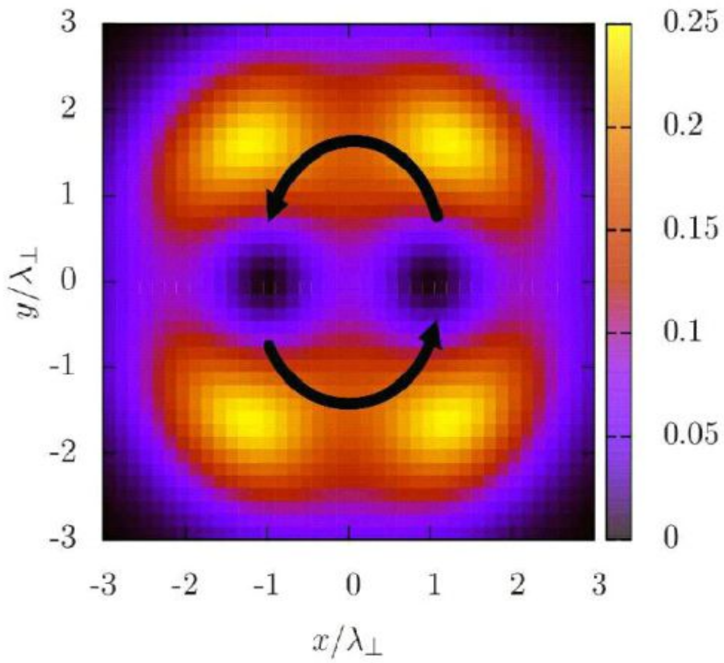


T. Grass, B. Juliá-Díaz, M. Lewenstein, Phys. Rev. A **86** 053629 (2012)

Quasiholes in FQH states

Holes in fractional quantum Hall systems:

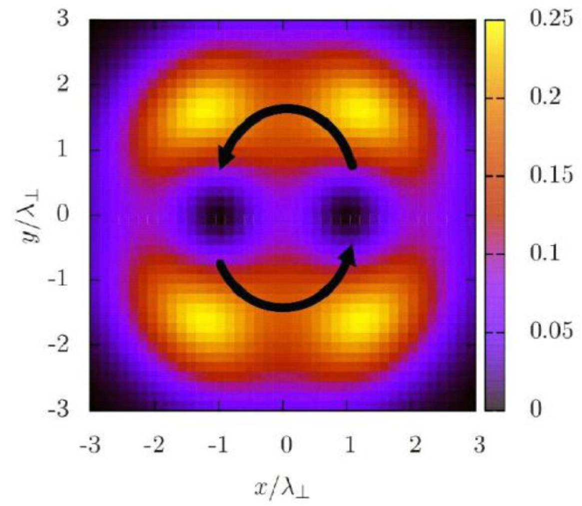
- Candidates for anyonic quasi-particle excitations
- Fractional quantum statistics
- Atomic samples:
  - *Creation by a laser beam*
  - *Control of the position*
  - *Ramsey interferometry*



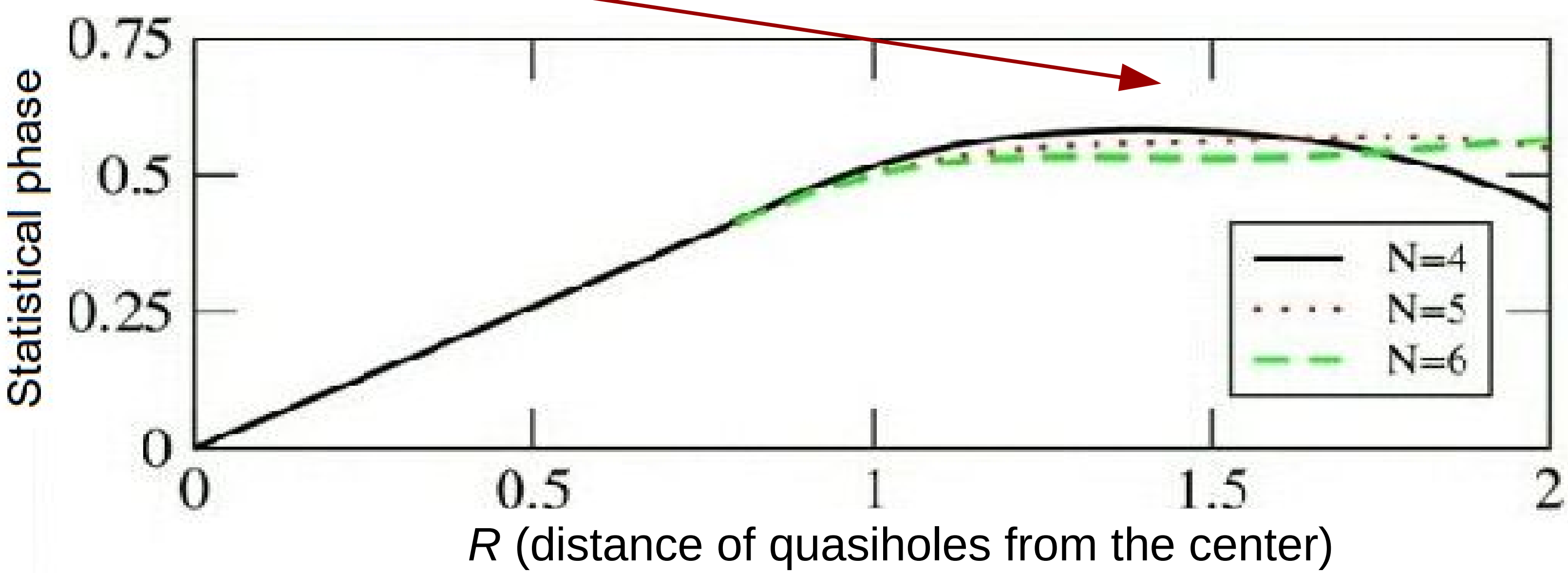
B. Paredes, P. Fedichev, J. I. Cirac, and P. Zoller. PRL **87**, 010402 (2001)
B. Juliá-Díaz, T. Grass, N. Barberán, M. Lewenstein, NJP **14**, 055003 (2012)
T. Grass, B. Juliá-Díaz, M. Lewenstein, PRA **89**, 013623 (2014)

Anyon braiding

Interchange two quasiholes and observe the phase difference: $\Psi \to \mathrm{e}^{i\varphi}\Psi$



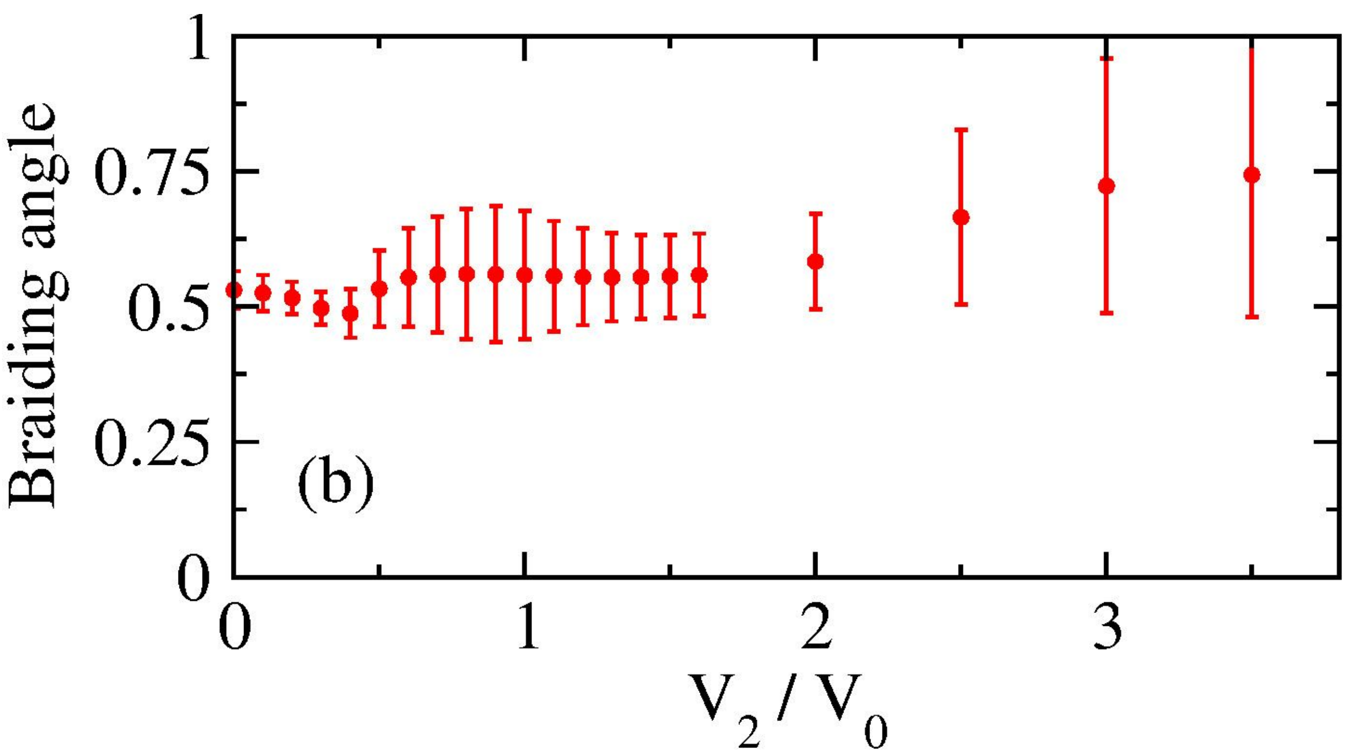
Fractional statistical phase becomes robust even in small Laughlin systems

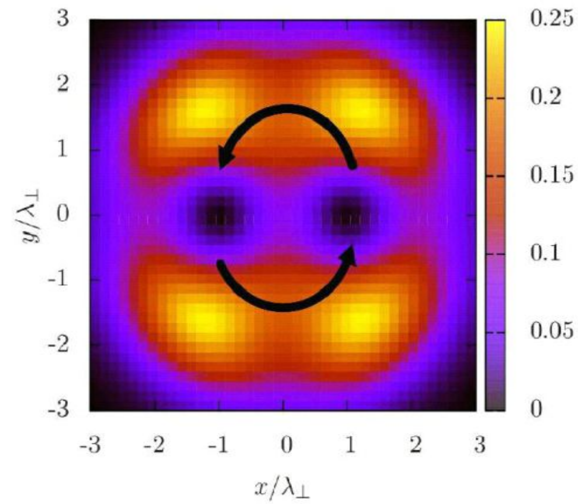


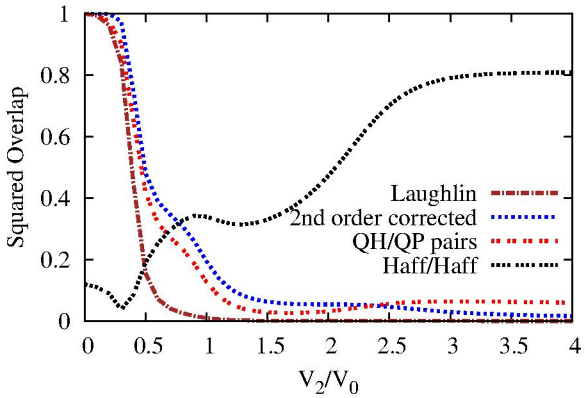
B. Juliá-Díaz, T. Grass, N. Barberán, M. Lewenstein, NJP **14**, 055003 (2012)

Anyon braiding

To test the robustness of braiding phase, we switch on an additional, tunable d-wave interaction:







T. Grass, B. Juliá-Díaz, M. Lewenstein, PRA **89**, 013623 (2014)

Outline

1. Quantum Hall Physics – in general:

- Single-particle physics: Landau levels
- Many-body effects: Trial states

2. Fractional quantum Hall physics of spin-1/2 bosons:

- Abelian vs. Non-Abelian phases
- Numerical results: ambiguous
- Prospects of a quantum simulation

3. *Integer* quantum Hall physics of spin-1/2 bosons:

- Interactions are crucial
- Edge spectrum as a fingerprint

4. Anyon braiding in small systems

The people

David Raventós

Bruno Juliá-Díaz

Maciej Lewenstein

Nuria Barberán