Principles of a Matterwave Transistor

An Atomtronics Analog of a semiconductor transistor

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An Obsession for Transistors

Transistor $\Longleftrightarrow$ Gain

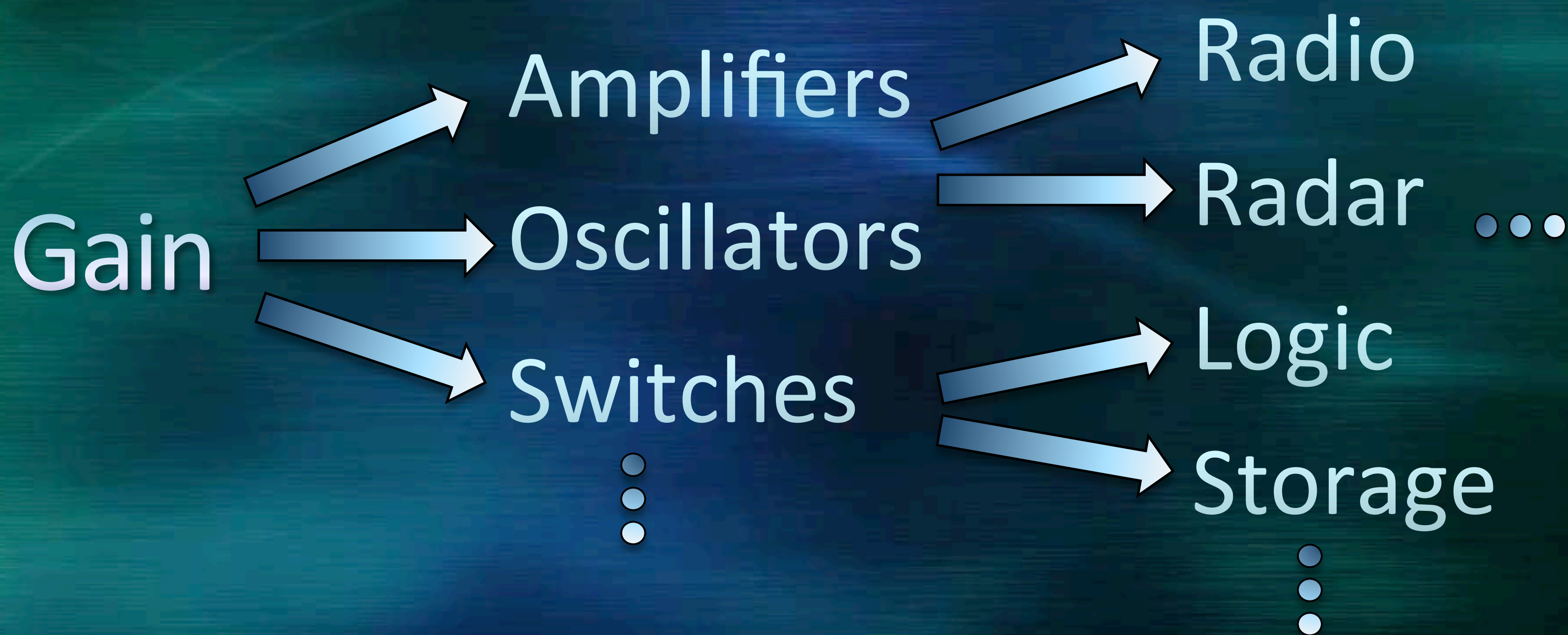
Dana's criterion:

No self-respecting scientific field should append "(tr)onics" to its name unless it offers a means to provide gain.

9 Billion Transistors

…in an iPhone can't all be wrong

The utility of transistor gain:



Of Quantum Signal Processing

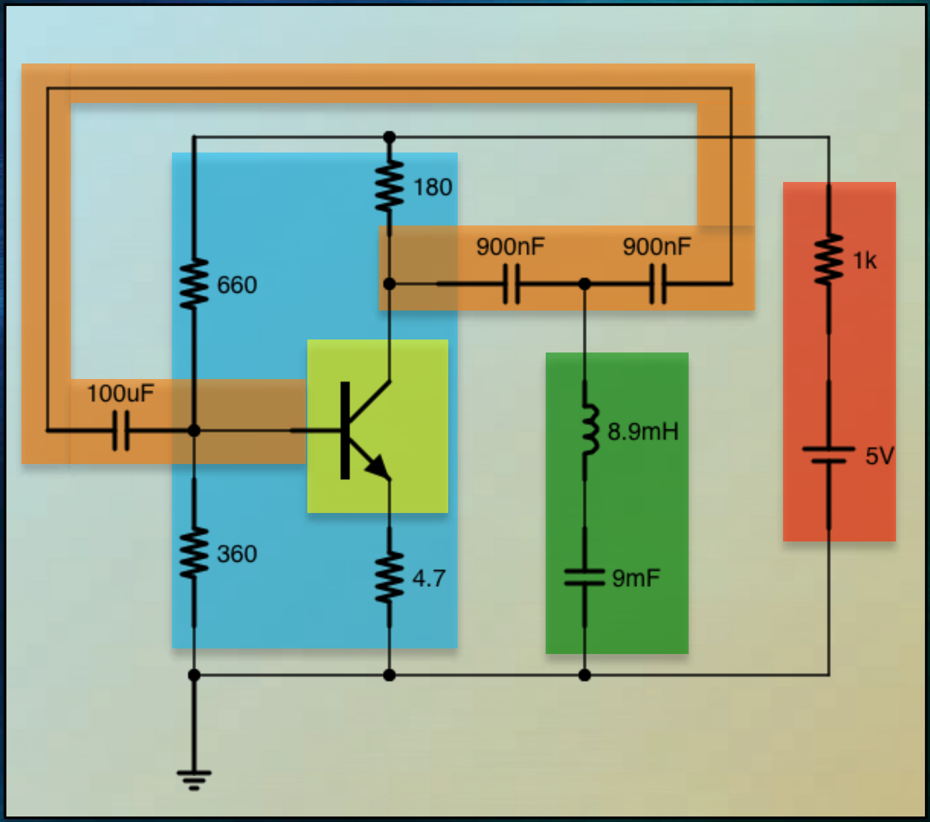
- Signal processing:
  - The generation, manipulation, and detection of the physical entities that bear information.
    - (agnostic to the information *per se)*
- Electronics:
  - A powerful paradigm for information related problem-solving.
- Atomtronics:
  - A paradigm for information problem solving in the quantum domain(?)

Overview

- Experimental status of an atomtronic transistor
- Model that predicts transistor behavior
- Results showing (non-classical) transistor oscillation.
- Atomtronics tutorial

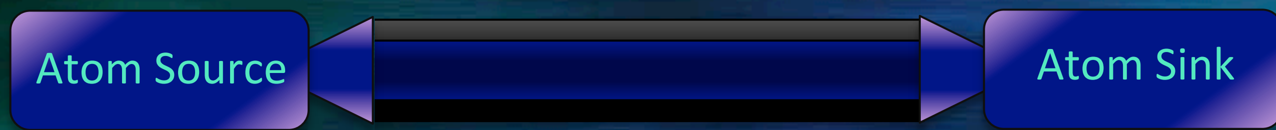
Elements of a Transistor Oscillator

- Gain element
  - BJT transistor
- Biasing
  - To establish operating point
- Tuning
  - Series L and C resonance
- Feedback
  - Positive, from Collector to Base
  - Frequency tuned
- Power supply
  - To provide energy

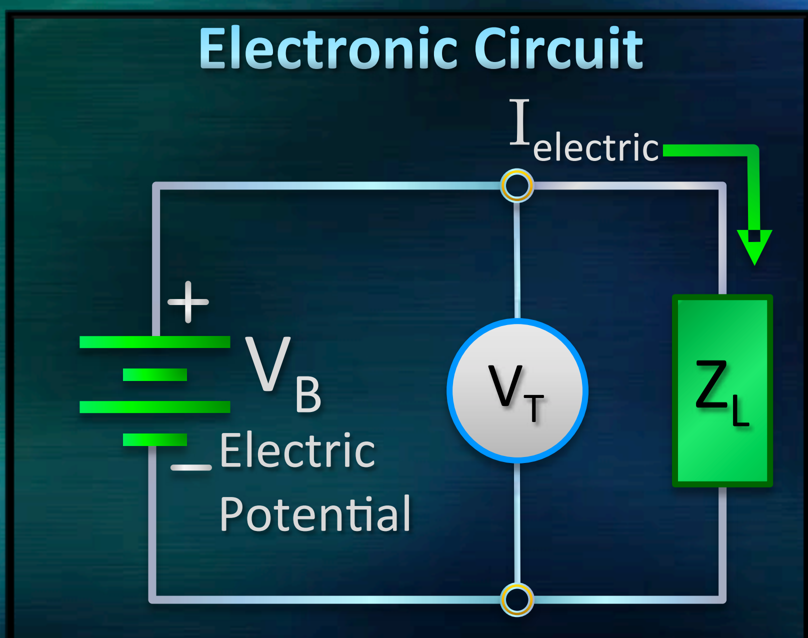


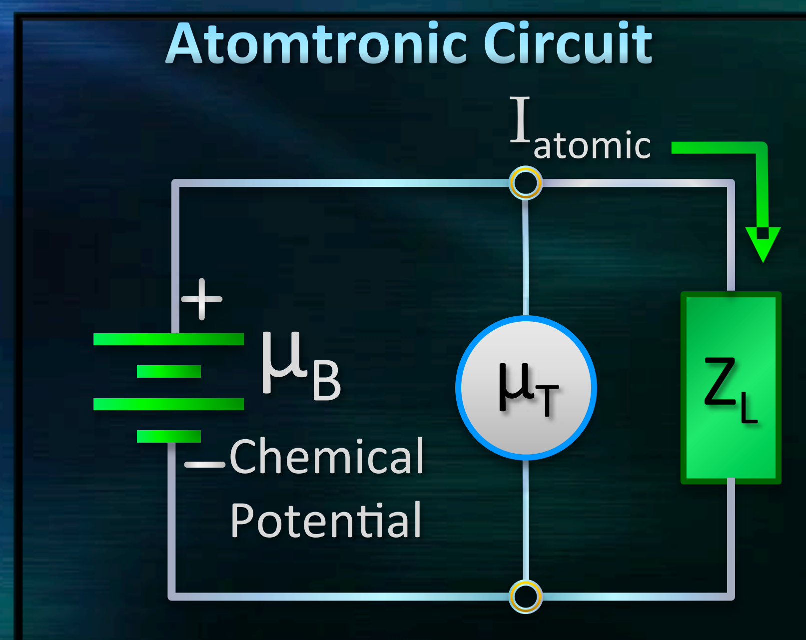
Atomtronics: A Faithful Analog of Electronics

- Atom flux substitutes for electric current



- Chemical potential substitutes for electric potential.

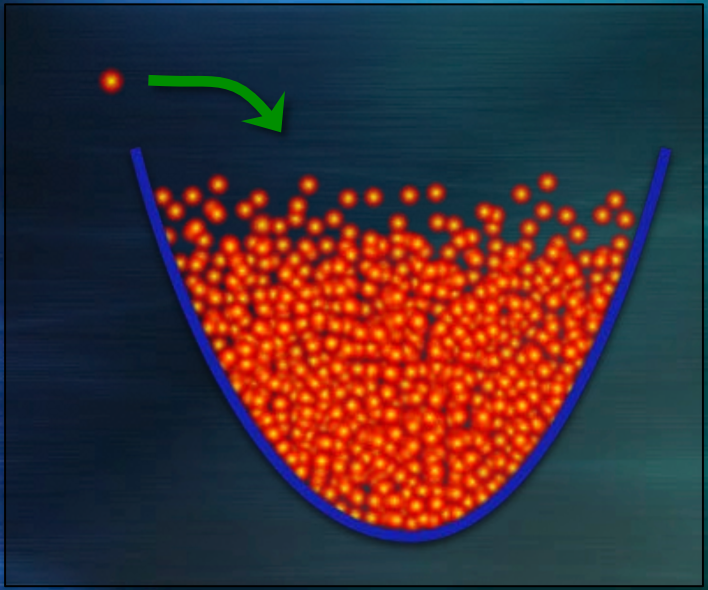




Chemical Potential

$$dU = TdS - PdV + \mu dN$$

$$\mu = \left.\frac{dU}{dN}\right|_{S,V}$$



The chemical potential is the energy required to add a single particle to the system.

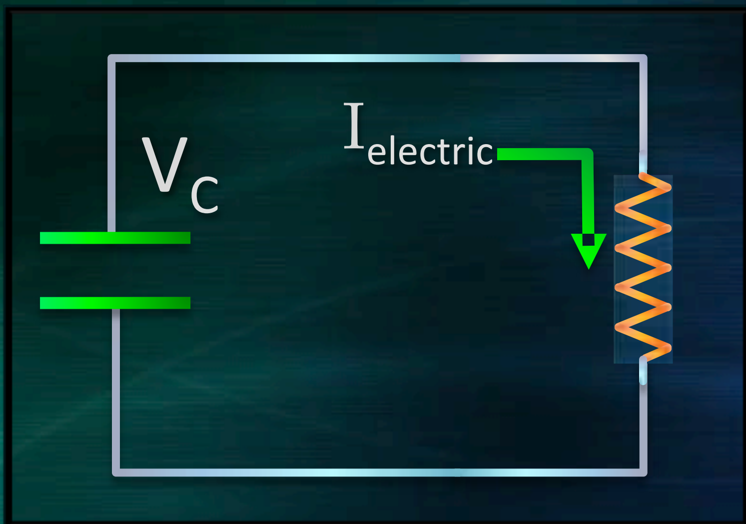
Finite temperature gas:

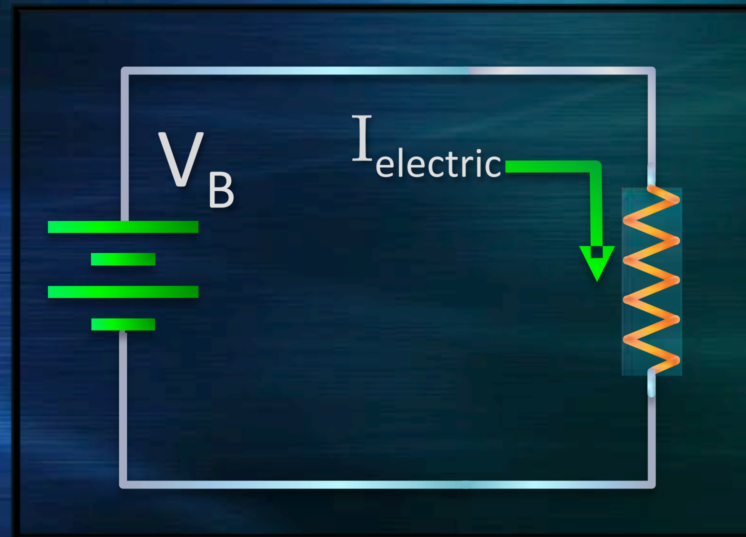
$$\mu < 0$$

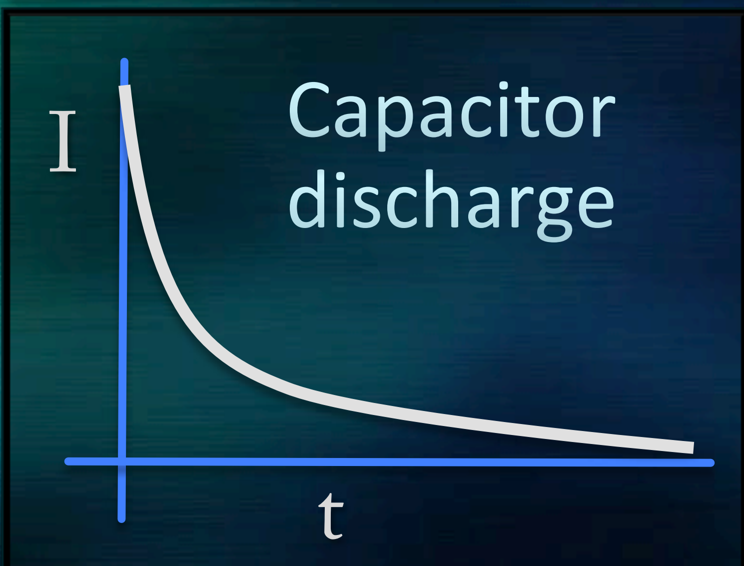
Bose condensed gas:

$$\mu > 0$$

A Circuit Seeks Thermodynamic Equilibrium





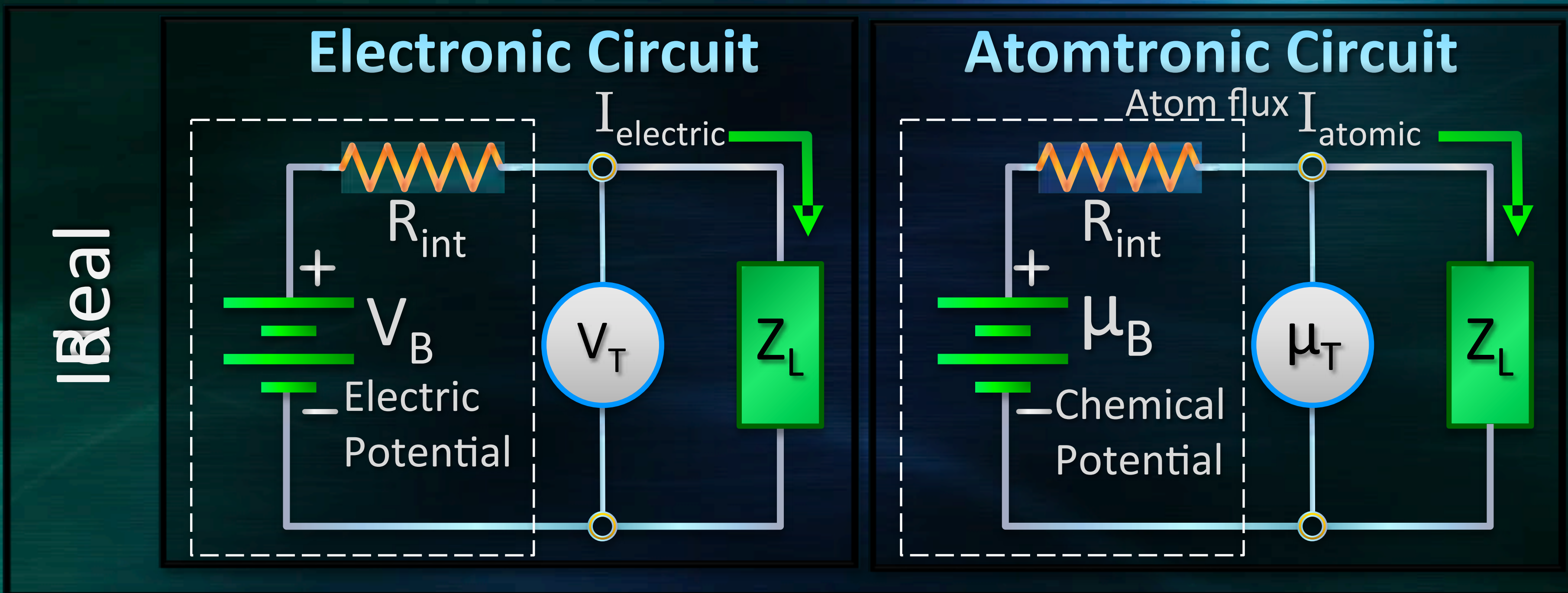


Battery operates on chemical potential difference –

Generates an electric potential

Maintains current flow

The Atomtronic Battery

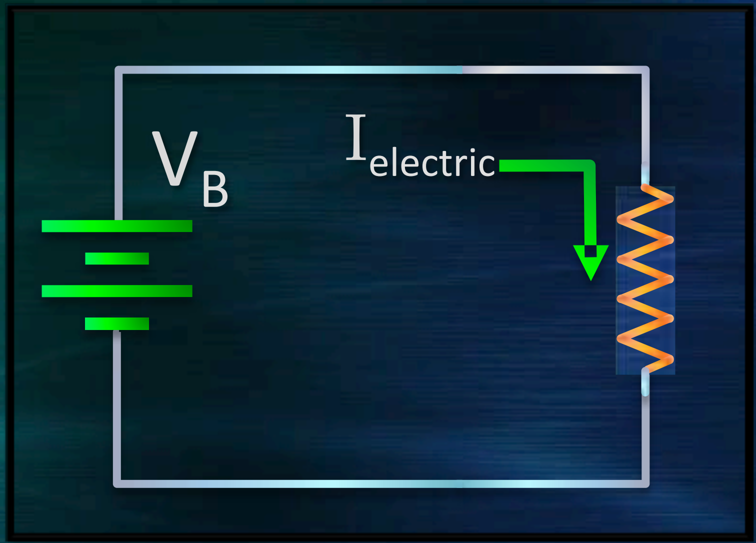


A real battery *necessarily* has internal resistance

- Maximum power delivered to a load $P_{\max} = V^2 / R_{\text{int}}$
- Heat dissipated in battery $P_d = I^2 R_{\text{int}}$
- Thermal noise $V_n = V_n\left(R_{\text{int}}, T\right)$

Expect an internal resistance in the atomtronic battery, too.

Temperature nominally uniform



$$P_D = I^2 R$$

Resistor heats up

Temperature gradient opposes ohmic current

Electronics, use a fan, don't worry about heat

Atomtronics: no fan, no environment, worry about heat.

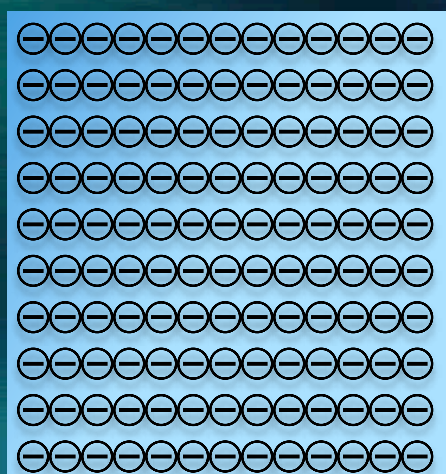
Transistor Thermodynamics

Chemical Potential & Silicon Doping

Silicon: "Perfect" semiconductor crystal

Doping: Silicon remains electrically neutral but has excess carriers:

N-type



Excess electrons

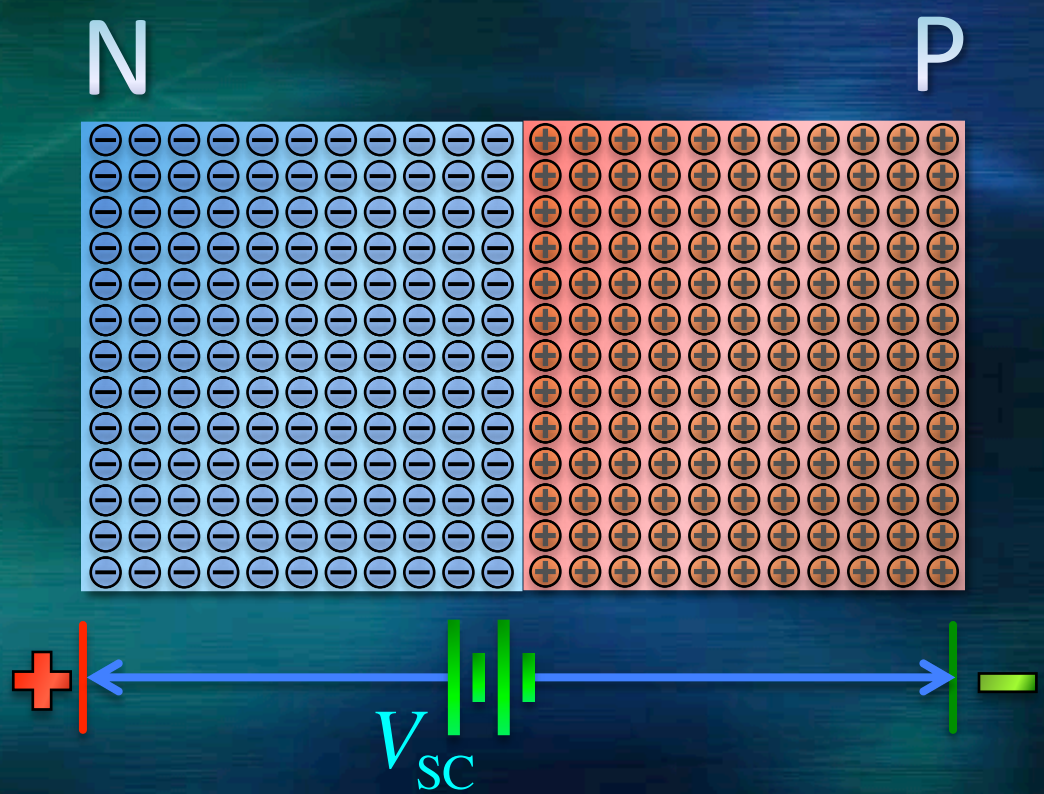
P-type

Excess holes

Semiconductor Junctions

The joining of a pair of dissimilarly doped materials introduces a chemical gradient.

In Thermodynamic equilibrium a *space-charge* electric field develops to counteract the initial chemical potential difference.

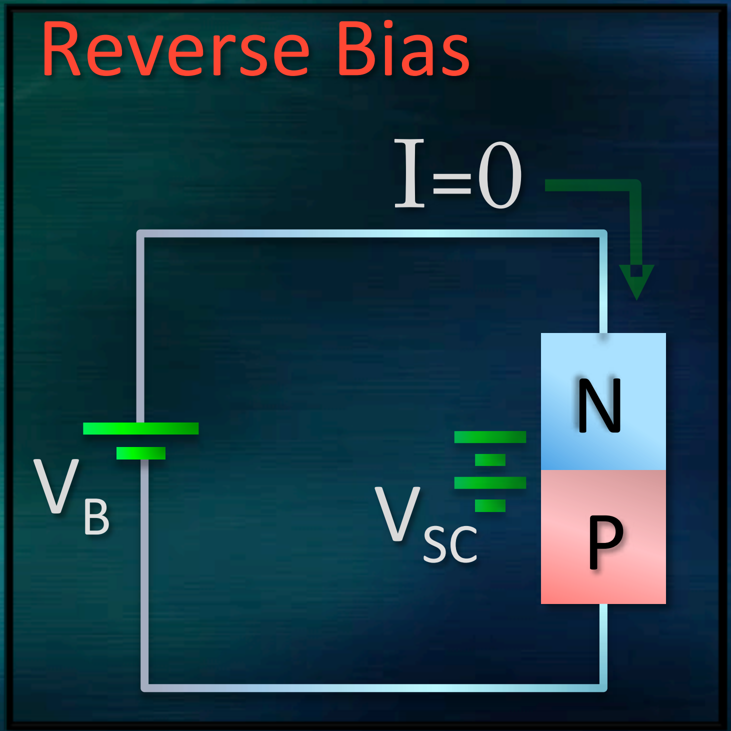


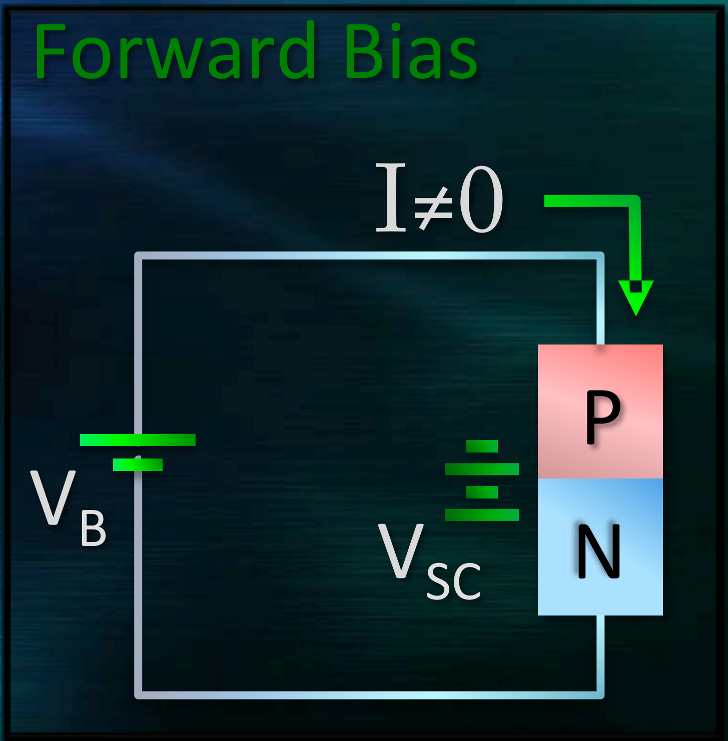
$$V_{sc} = -\Delta\mu_{\text{N-P}}$$

Thermodynamics of Diode Circuits

Reverse bias: no current flow since space-charge field balances battery (chemical) potential.

Forward bias: current flow since space-charge field does not compensate for battery potential.

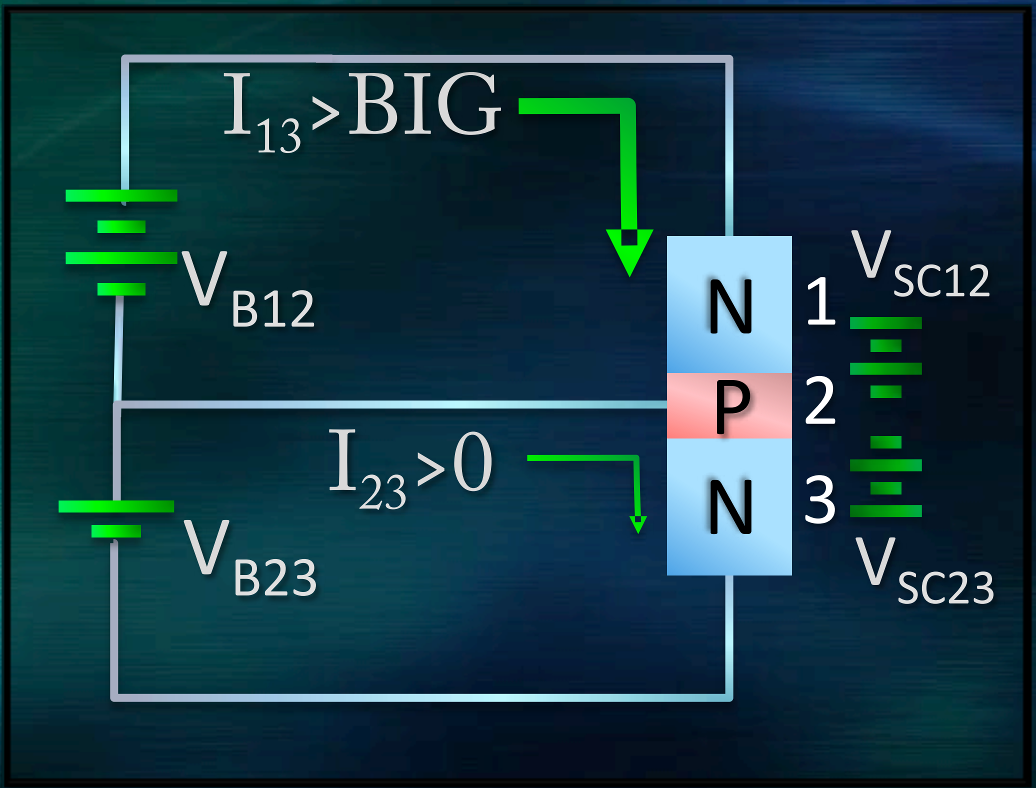


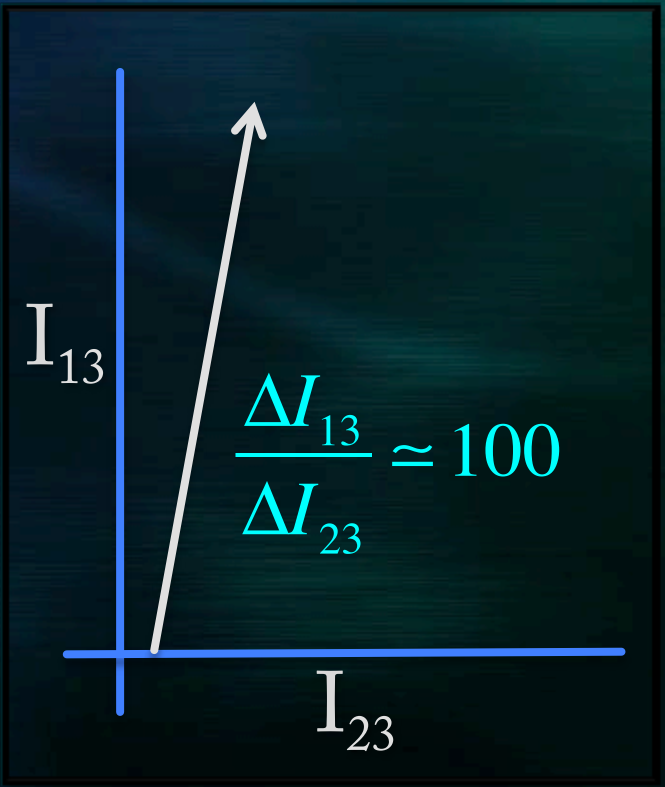


Transistor Current Gain

Applied potentials in conflict: One reverse biased, one forward biased.

Small forward biased current induces very large reverse bias current –despite reverse current flow





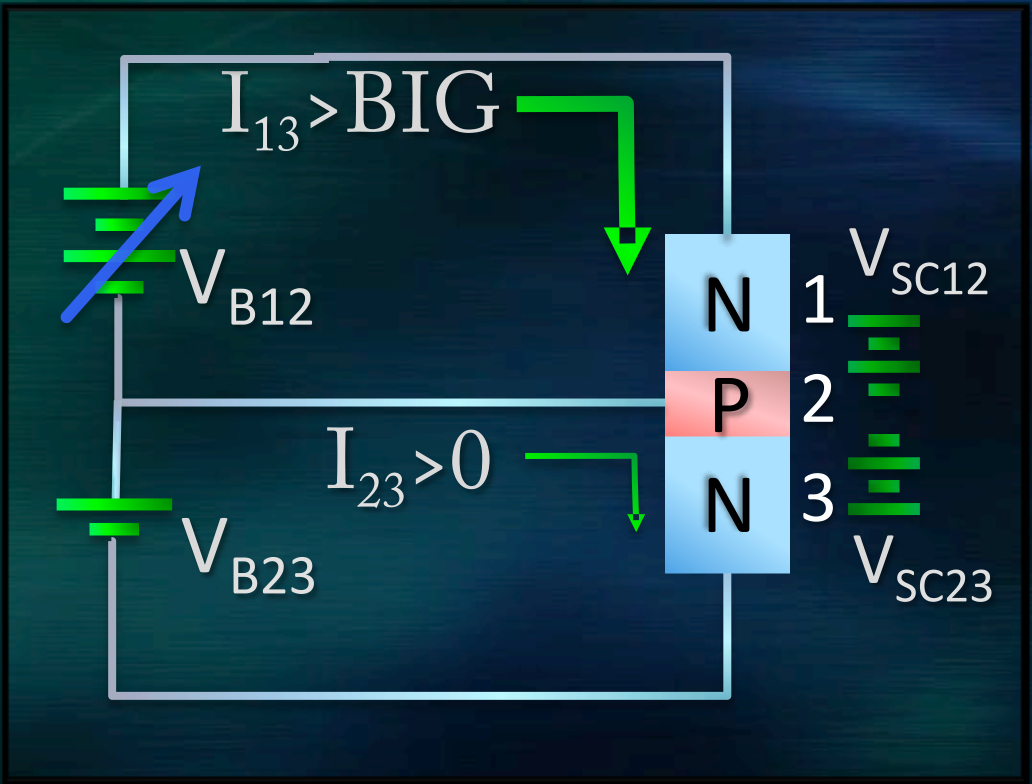
A Difficult Concept

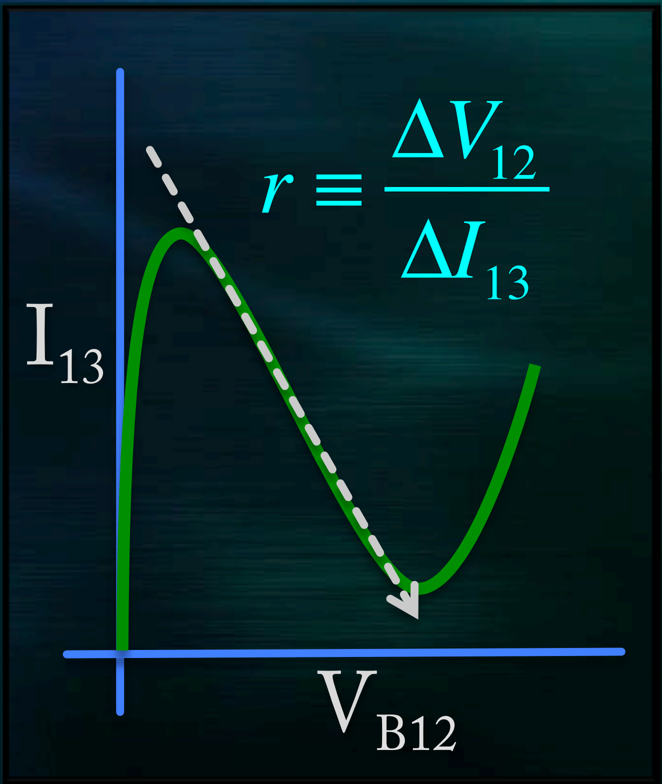
Negative Resistance

Transistor = *Transfer + Resistance*

Control current flow with V_{B12}

Presence of gain signified by *negative* transresistance.



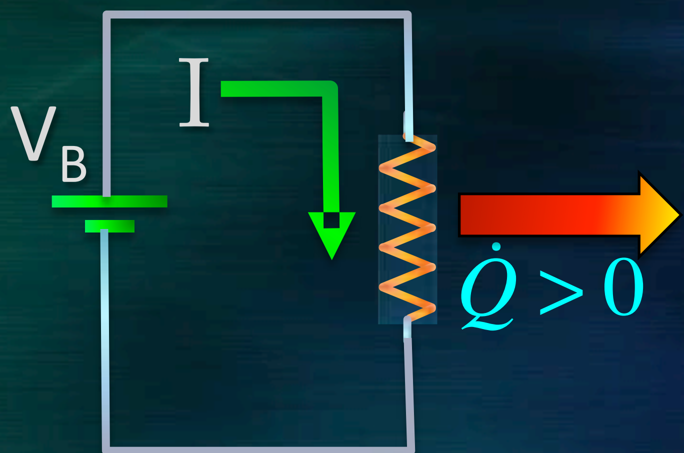


Negative Resistance = Cooling

Under transistor action the device *really does* become cooler than it would in absence of gain.

Positive Resistance

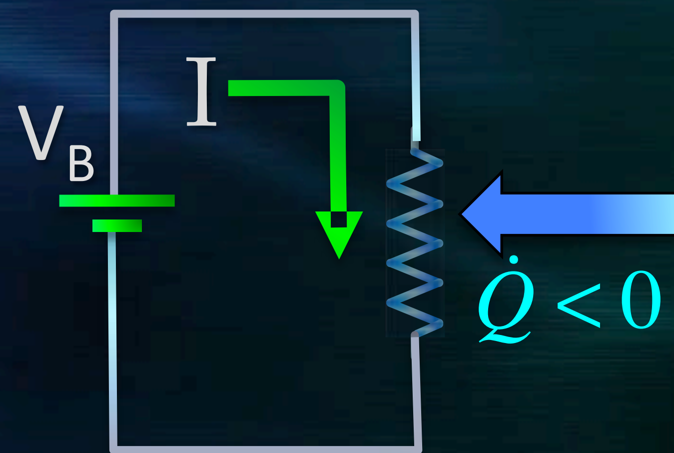
$P_D = IR > 0$



$\dot{Q} > 0$

Negative Resistance

$P_D = IR < 0$



$\dot{Q} < 0$

Gain ⟷ Negative Resistance

Tutorial Summary

- Circuit currents are largely driven by potential differences.
- Temperature/heating effects are negligible
- Transistor action is based on competing gradients
- Gain = Negative resistance

All About Atomtronics…

Comparison

- Electronics
  - Thermal energies tiny
  - Heating effects negligible
  - Electron quantum coherence negligible
  - "DC" current meaningful
  - Gain = negative resistance
- Atomtronics
  - Thermal and chemical energies comparable
  - Heating inherent contributor to dynamics
  - Atom coherence long lived
  - "DC" meaningless
  - Microwave circuit considerations (10 kHz)
  - Gain = negative resistance

A Look at Temperature, Potential, and Current Flow

Three Gedanken experiments at fixed total internal energy

- Current *opposes* (chemical) potential drop.
- Potential rises.
- Temperature falls.
- (Thermo-motive regime)

- Current follows potential drop.
- Potential falls.
- Temperature rises(!)
- (Chemo-motive regime)

- Normal (thermal) current opposes potential drop.
- Condensed component follows potential drop.
- Temperature falls/rises on the left/right.
- (Superfluid-like behavior)

Elements of a Transistor Oscillator

- Gain element
  - BJT transistor
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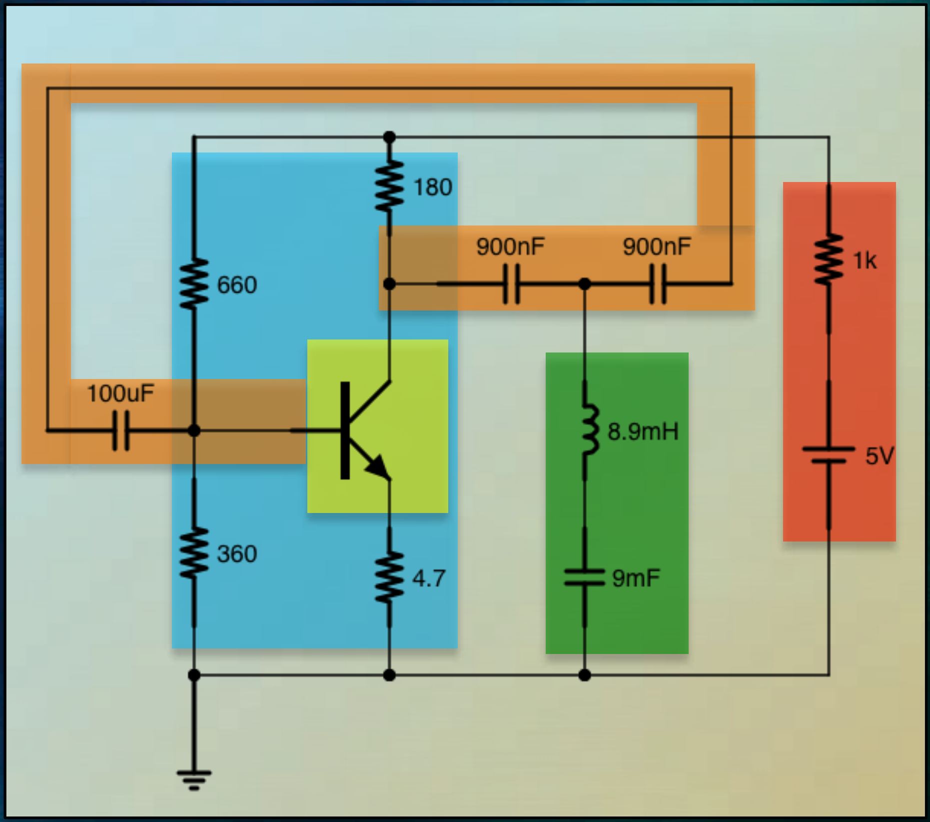
  - To establish operating point
- Tuning
  - Series L and C resonance
- Feedback
  - Positive, from Collector to Base
  - Frequency tuned
- Power supply
  - To provide energy



The Electronic Driven Oscillator

DEUTSCHES REICH

AUSGEGEBEN AM 23. JUNI 1919

REICHSPATENTAMT

PATENTSCHRIFT

— № 291604 —

KLASSE 21a GRUPPE 66

GESELLSCHAFT FÜR DRAHTLOSE TELEGRAPHIE M. B. H. IN BERLIN.

Einrichtung zur Erzeugung elektrischer Schwingungen.

Patentiert im Deutschen Reiche vom 10. April 1913 ab.

Die vorliegende Erfindung betrifft eine Einrichtung zur Erzeugung elektrischer Schwingungen, die darauf beruht, daß ein oder mehrere Schwingungskreise mit einem elektrischen Relais, dessen die Relaiswirkung ausübendes Mittel aus Kathodenstrahlen oder einem ionisierenden Gas o. dgl. besteht, so verbunden wird, daß die in den Schwingungskreisen durch irgendwelche Stöße oder andere Mittel hervorgerufenen Anfangsschwingungen dem Relais zugeführt werden und die von diesem verstärkten Ströme wieder auf die Schwingungskreise einwirken, wodurch die Amplitude der Eigenschwingung der Kreise erhöht wird. Da der wirksame Bestandteil dieser Relais im Gegensatz zu mechanisch arbeitenden keine Masse besitzt, so ist es gerade mit diesem Relais möglich, elektrische Schwingungen bis zu den höchsten Frequenzen in der angegebenen Einrichtung zu erzeugen, und da die Frequenz der Schwingungen nur durch die Eigenschwingung des Schwingungssystems bedingt ist, und im Relais andererseits keine störenden Einflüsse sich geltend machen, so erzielt man mit dieser Einrichtung zum ersten Mal ungedämpfte Schwingungen von absolut konstanter Schwingungszahl und Amplitude.

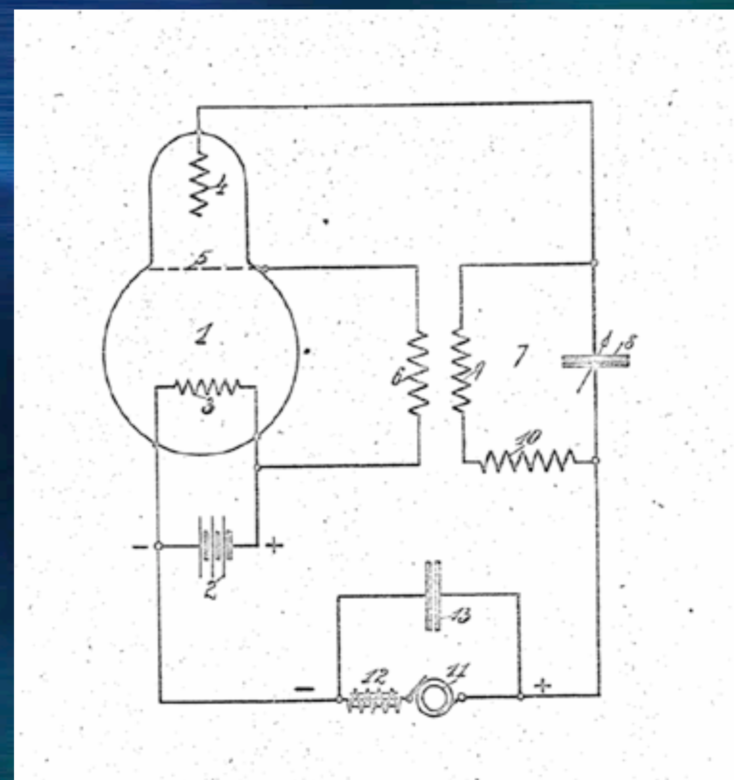
Eine z. B. für diesen Zweck sehr geeignete Form dieser Relais besteht aus einem ganz oder teilweise evakuierten oder mit Gasen oder Dämpfen unter geringem Druck gefüllten Glasgefäß, das eine erhitzte Kathode und eine oder mehrere Anoden enthält. Die von der Kathode ausgehenden Kathodenstrahlen sind entweder selbst das wirksame Agens oder bewirken eine Ionisation des Gases, so daß ein dauernder Stromweg im Relais hergestellt wird. Die zu verstärkenden Ströme können der erhitzten Kathode und einer Hilfsanode (Primärseite des Relais) zugeführt werden. Die verstärkten Ströme treten dann in dem über die Kathode und Hauptanode (Sekundärseite) geführten Stromkreis auf, wobei die vermehrte Energie dieser Ströme aus der den Dauerstrom liefernden Stromquelle entsteht.

Eine Ausführungsform der Schaltung unter Zugrundelegung eines solchen Relais ist in der Figur dargestellt.

1 ist das Kathodenstrahlrelais, das die von der Batterie 2 geheizte Kathode 3 und die Anode 4 enthält. 5 ist eine Hilfselektrode, die mit der Kathode 3 über die Kopplungsspule 6 leitend verbunden ist. 7 ist ein Schwingungskreis, der aus dem Kondensator 8 und den beiden Selbstinduktionen 9 und 10 besteht. Dieser Schwingungskreis 7 steht einerseits mit der Anode 4 und andererseits über die Stromquelle 11 mit der Kathode 3 in Verbindung. Der Schwingungskreis 7 ist also sowohl mit der Primär-, wie mit der Sekundärstrecke des Relais gekoppelt. 12 ist eine Drosselspule, die gemeinsam mit der Kapazität 13 erforderlich ist, sofern als Stromquelle 11 nicht eine Batterie, sondern eine Maschine zur Speisung des Entladungsrohres 1 benutzt wird. In diesem Fall dient die Einrichtung dazu, die störenden Wirkungen des

13

Meissner vacuum tube oscillator patent circa 1913



Coherence from Driven Oscillators

Grandfather clock

Vacuum tube oscillator (magnetron)

Klystron

Maser

Transistor oscillator

Laser

…

In contrast to a *forced oscillator* such as a playground swing under the command of a child, a *driven* oscillator incorporates a gain-induced dynamical instability that drives the system to an oscillating state.

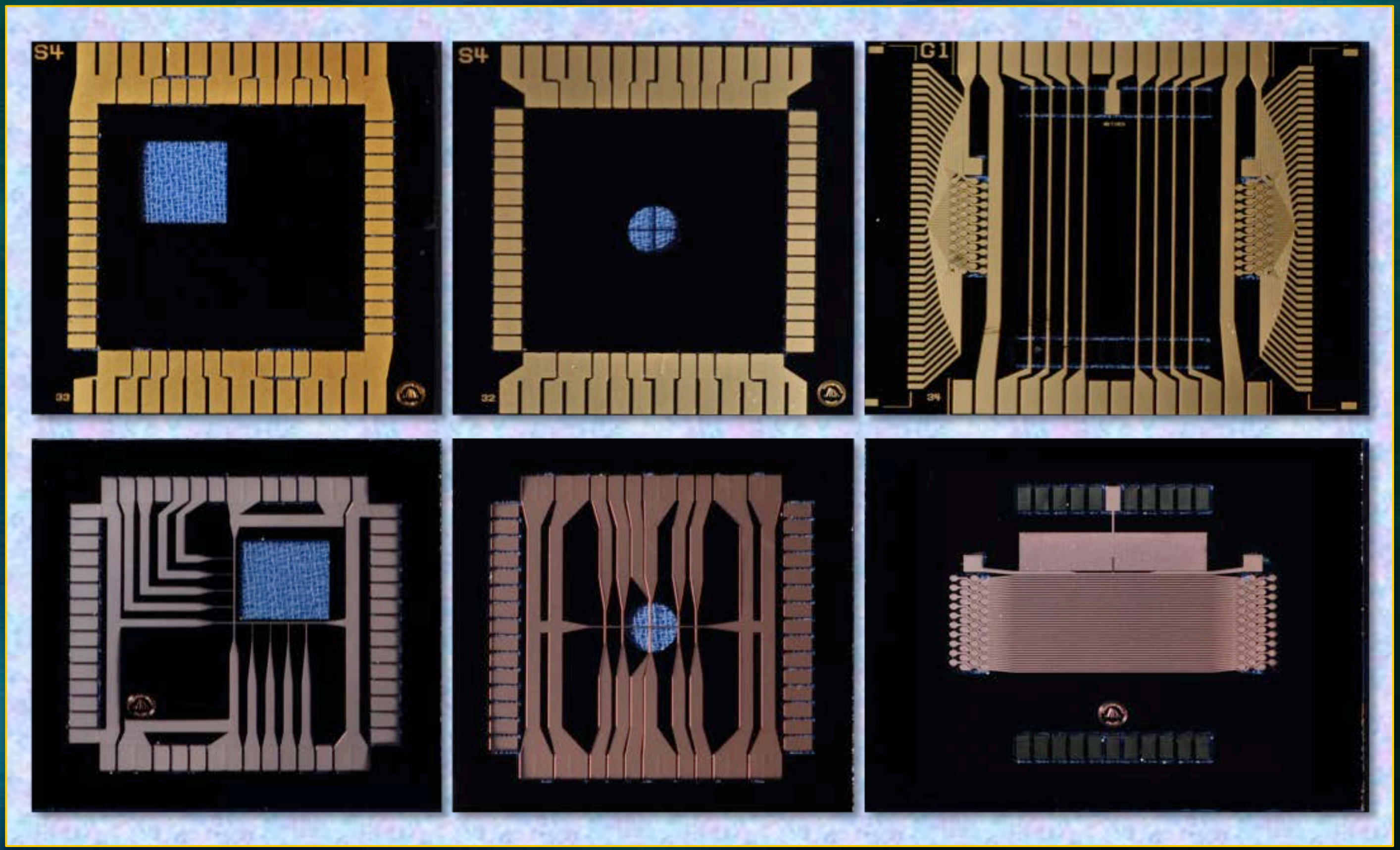
A matterwave driven oscillator has more in common with a transistor oscillator than it does with a laser.

Atom Chip for Ultracold

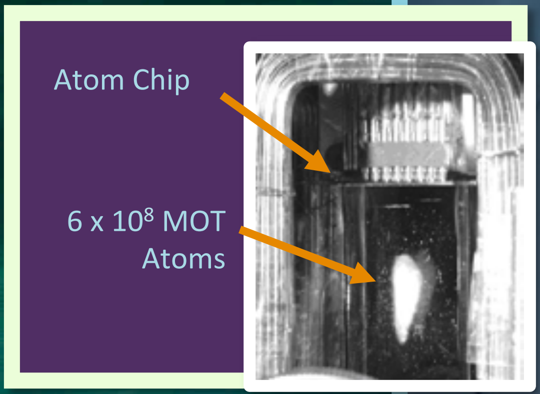
- The ultracold atom source remains constant, while *functionality is* determined by chip design.
- Below are chips for three different BEC systems (chips are ~23 mm x 23 mm).

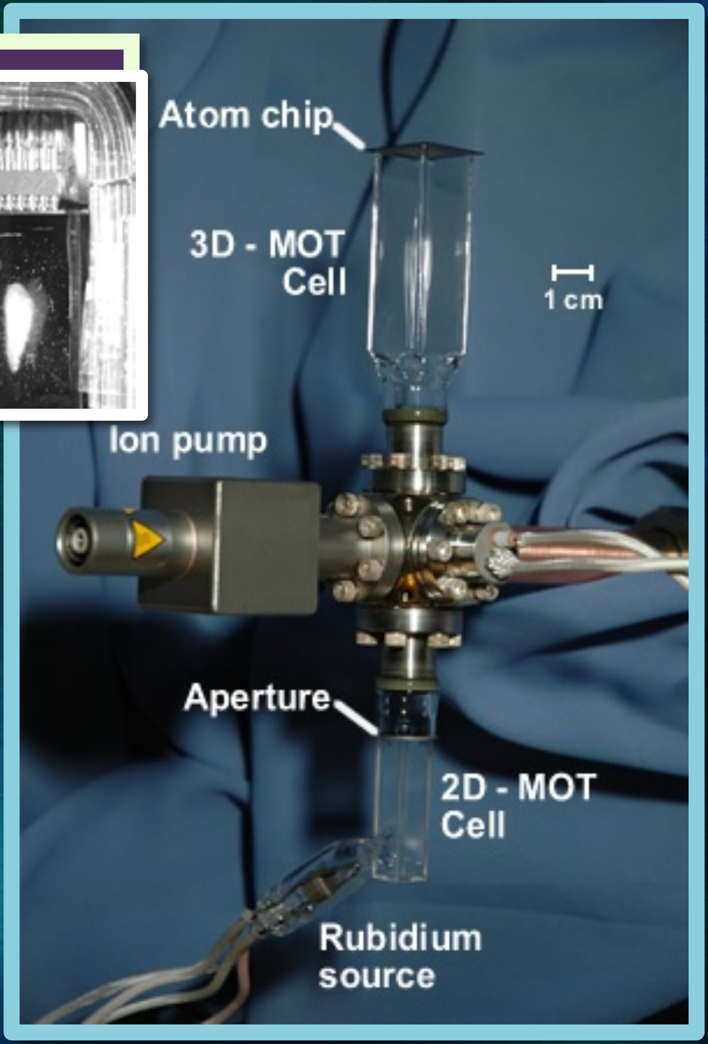
Ambient

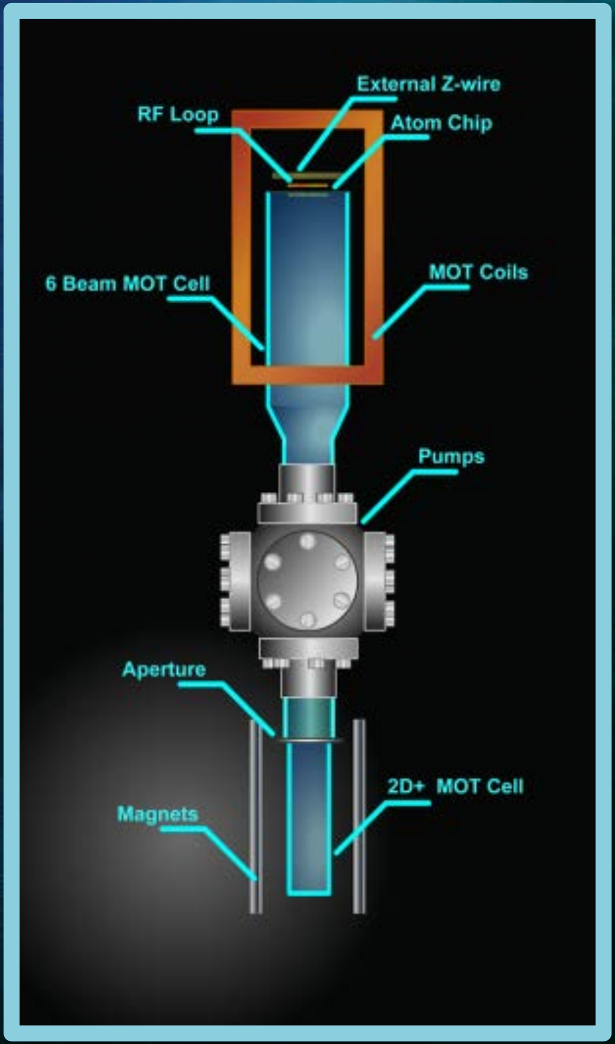
Vacuum



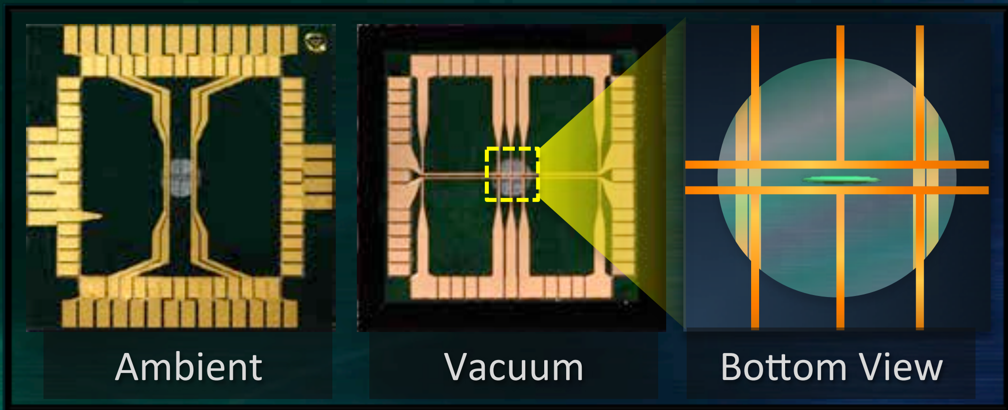
Miniature Atom Chip Vacuum Cells

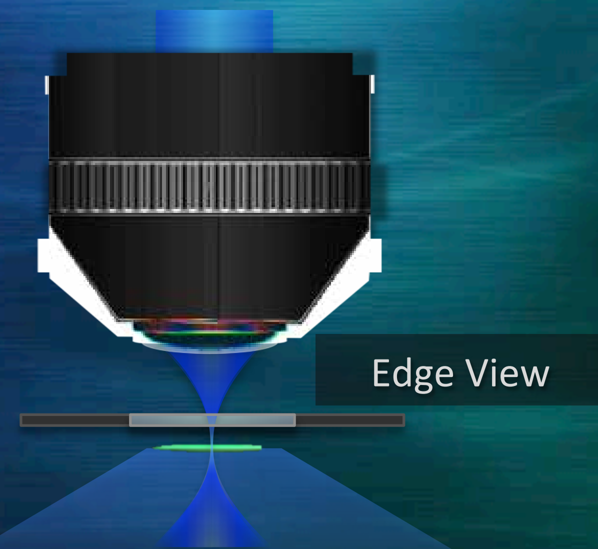




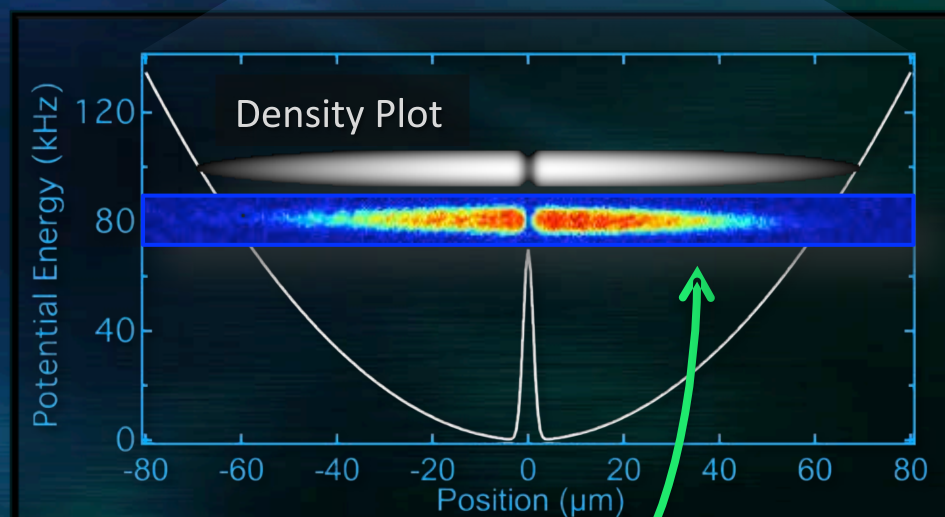


Hybrid Atom Chip Apparatus



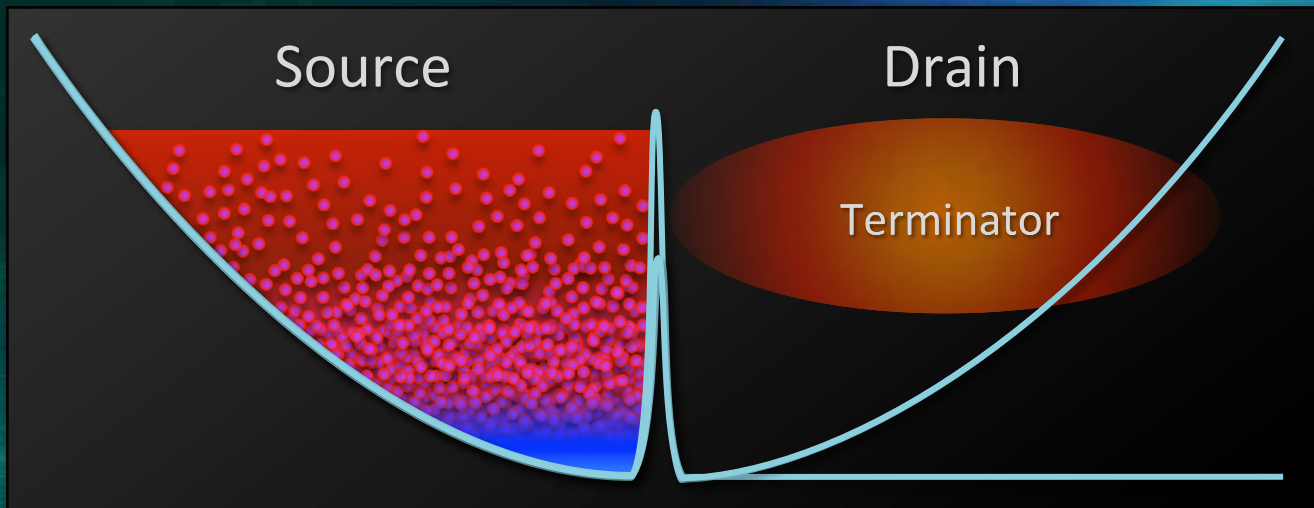


- Compound silicon & glass substrate
  - Metalized both sides
  - 60 UHV compatible electrical feed-throughs
- Window provides high NA optical access to atoms.
- Magnetic trapping provides cigar-shaped harmonic trap
- Optical projection provides barrier potential (4.2 µm e^{-2} width)



In-trap image of atoms just above critical temperature (~750 nK)

Experimental Scenario



1. Atoms loaded into "Source" (left) well.
2. Atoms prepared to predetermined T_i, μ_i.
3. Barrier is lowered, defines $t=0$.
4. "Hot" atoms begin to flow to "Drain"
5. Terminator beam effectively couples output atoms to the vacuum.

Typical parameters

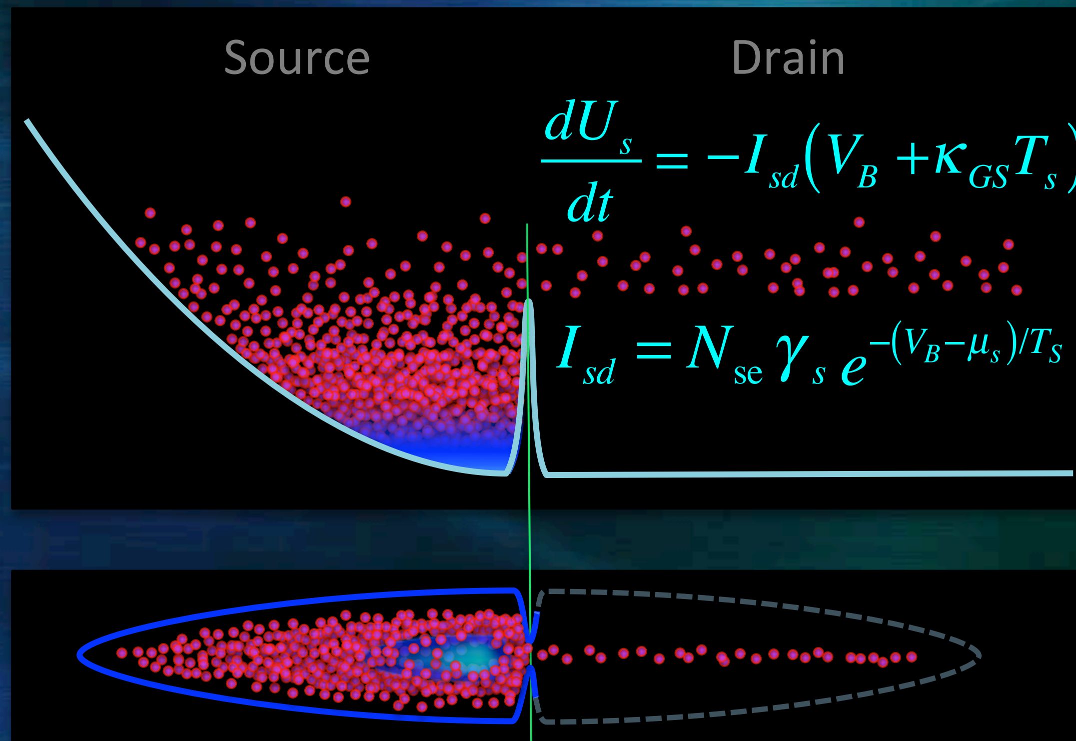
Atom number: 10-40 x 10^3

Energies:

- Trap
  - Longitudinal frequency 120 Hz
  - Radial frequency 1.2 kHz
  - Barrier height 20 - 40 kHz
- Temperature 5 – 15 kHz
- Chemical potential 0 – 8 kHz.

Single Barrier Dynamics

- Atom flux
  - Current flow depends on
    - Source thermal atom number
    - Effective collision rate
    - Boltzmann factor
- Source well cooling
  - Mean energy per lost particle
    - Barrier height
    - Temperature
    - Excess energy factor (κ≈1 to 4)



Expect:

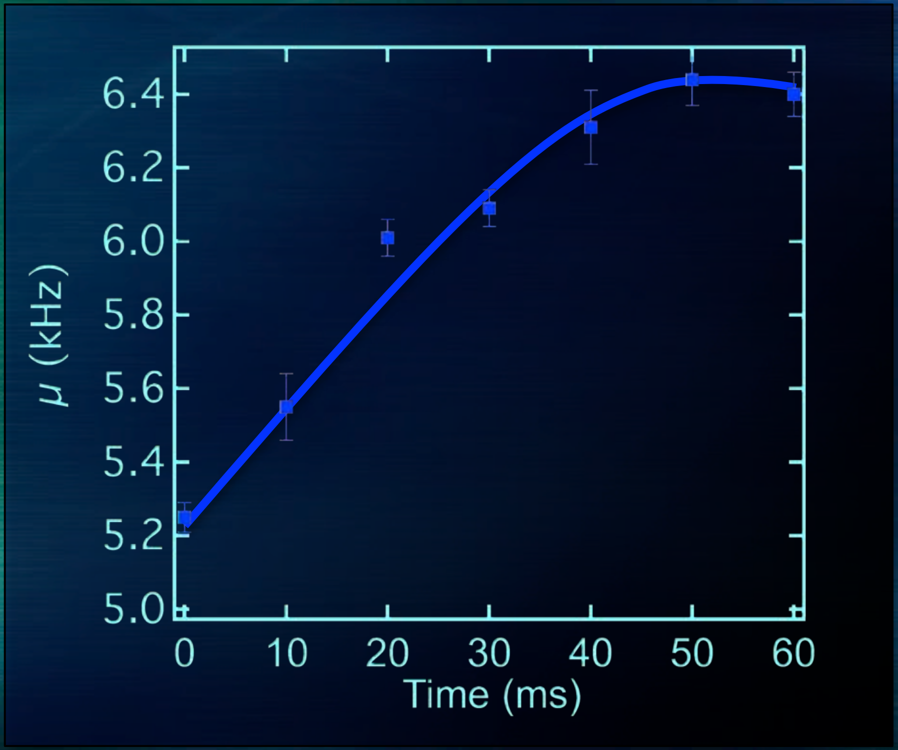
- Evaporative cooling causes Source temperature to decrease, chemical potential to increase.

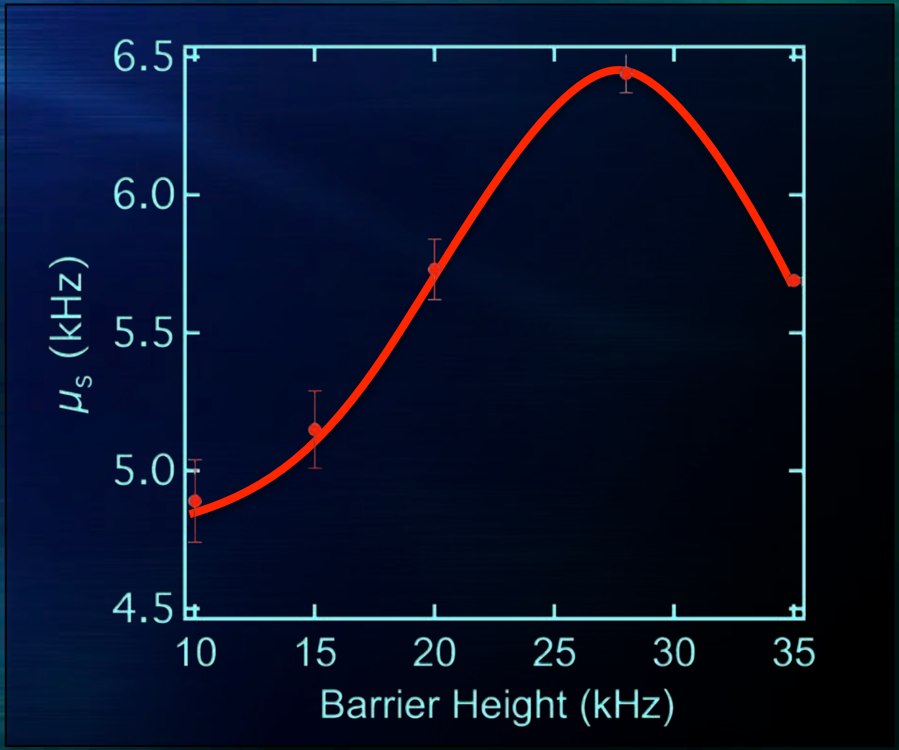
Time of flight measurements will tell…

The Dynamical Behavior of μ

- As atoms emerge from the Source the chemical potential rises…

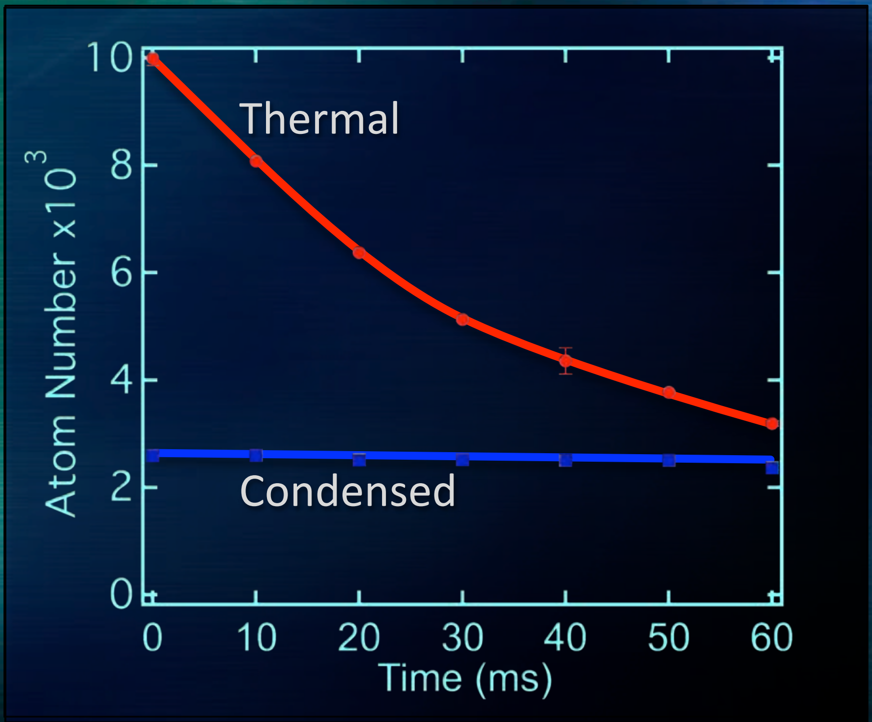
  …then reaches a steady state.
- Steady state chemical potential depends on barrier height.
- Chemical potential is a maximum for a barrier height $V_b \cong 30$kHz

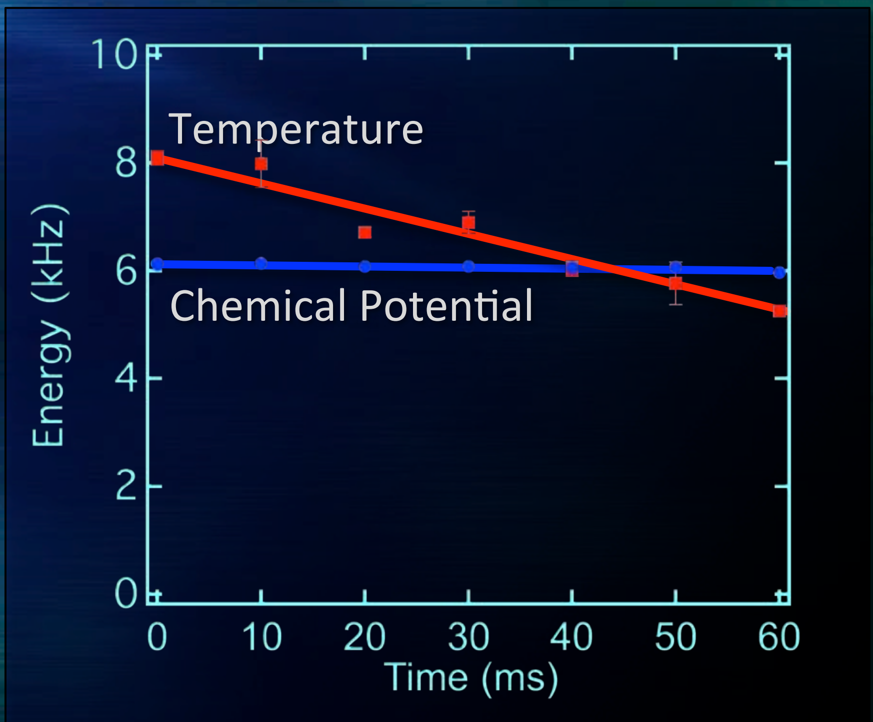




Acting as a Battery

- Choose a barrier height ($V_b \cong 30$ kHz)
- Prepare atoms with $\mu_i = \mu_s(30\ \text{kHz})$
- Number of condensed atoms stays constant
- Even as thermal number reduces dramatically.
- μ is constant —like a battery— while temperature drops
- Deduce that a heating mechanism precludes condensate growth.





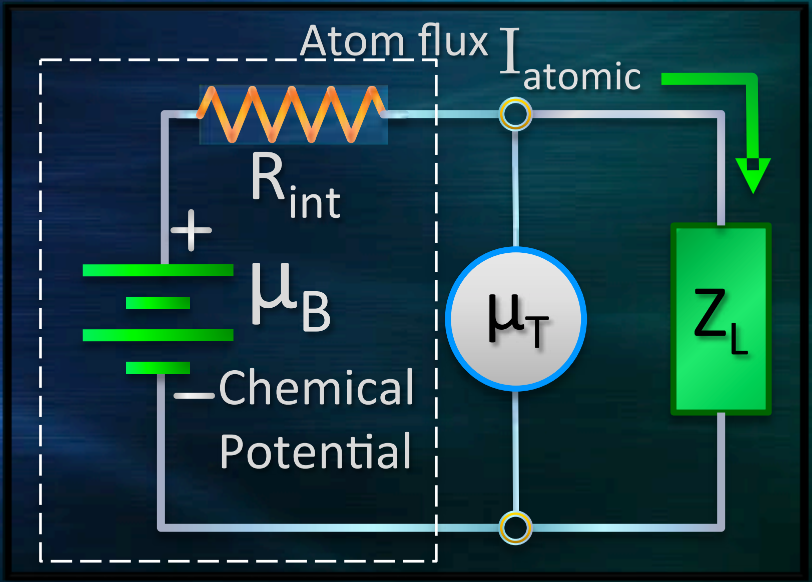
The Battery Model Analog

- The thermo-motive regime:
  - Current flow is *against* the potential difference.
  - Potential *increases.*
  - (Cooling).

Internal resistance is *negative*

- The chemo-motive regime:
  - Current flow is *with* the potential difference.
  - Potential *decreases*.
  - Heating.

Internal resistance is *positive*

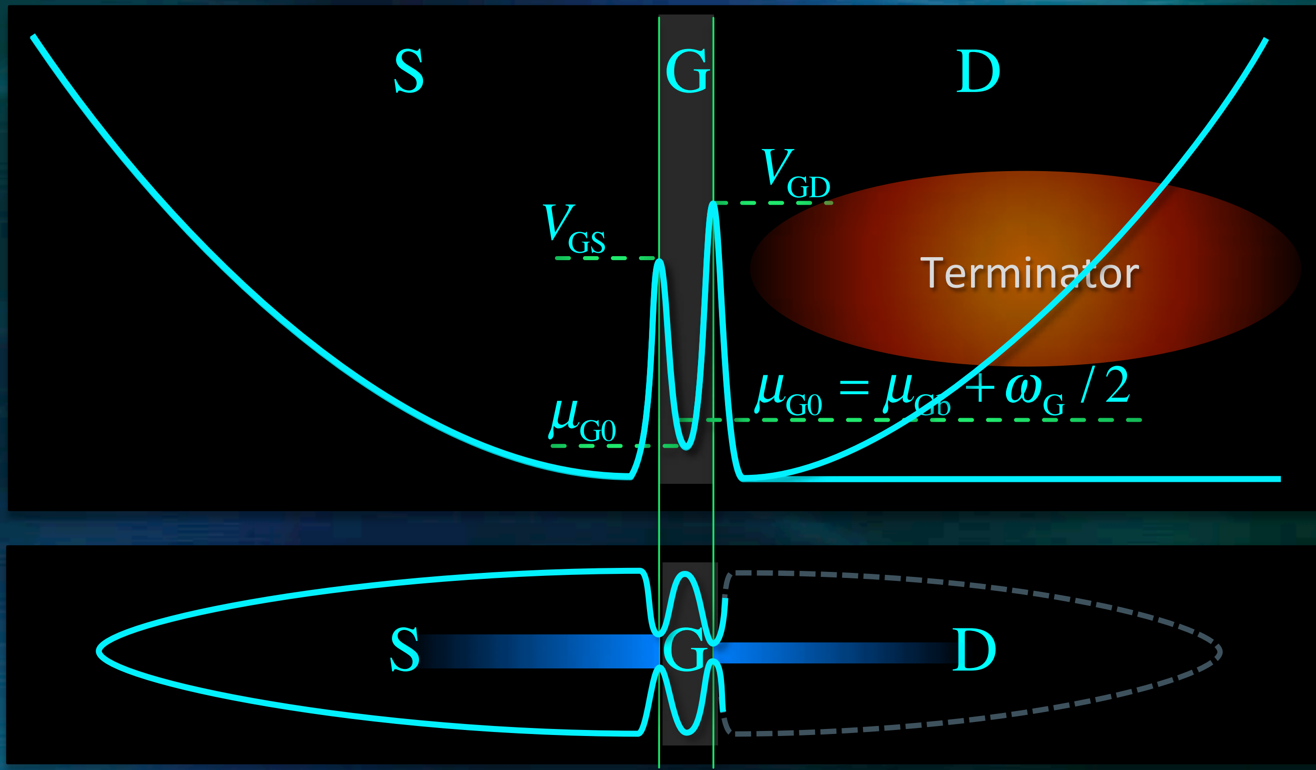


To Remember:

- ❖ A battery that supplies *thermal (hot)* atoms exhibits *negative* internal resistance
- ❖ A battery that supplies *condensed (cold)* atoms exhibits *positive* internal resistance.

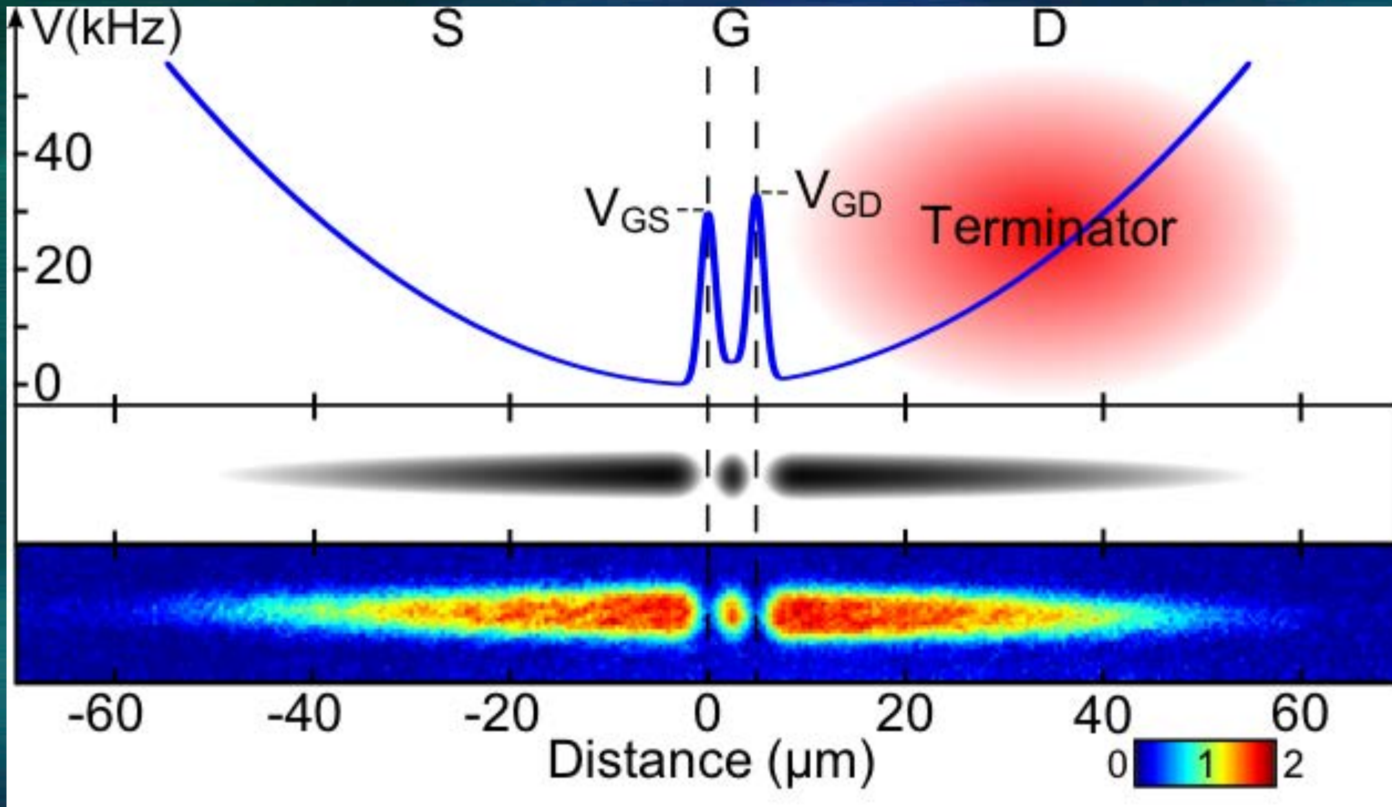
Atomtronic Transistor

- A triple-well structure
- Electronic FET nomenclature:
  - Source
  - Gate
  - Drain
- Barriers and biasing is defined by magnetic and optical fields
- Terminator beam couples output flux to the impedance of vacuum
- Gate is biased at a few kHz.

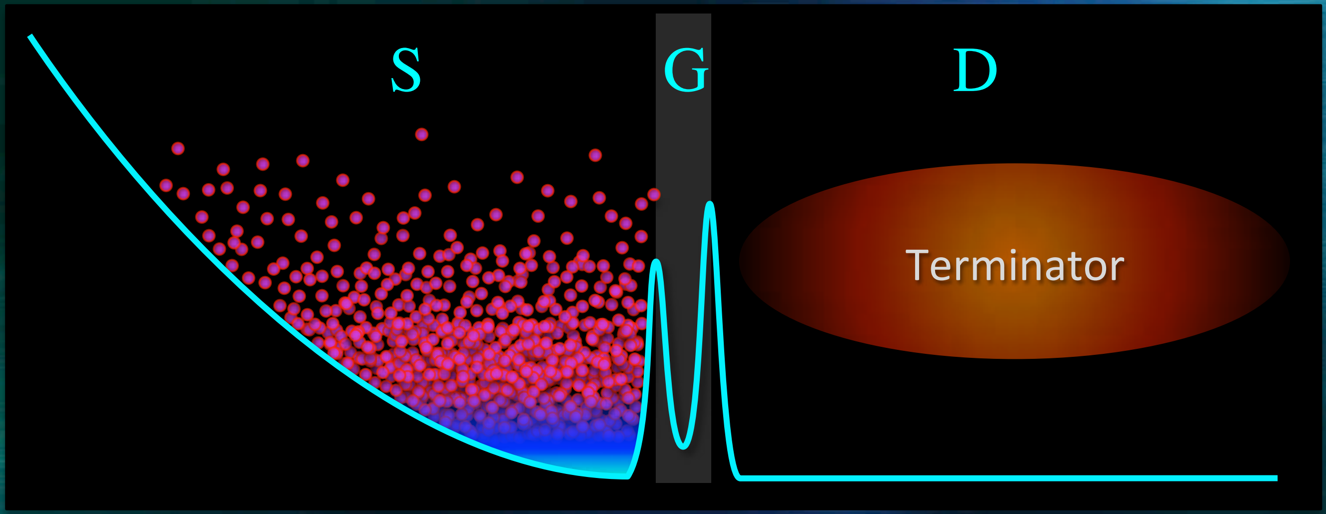


Matterwave Transistor Realization

Experiments similar to battery: uses an AO modulator driven by two independent frequency sources to generate two barriers.

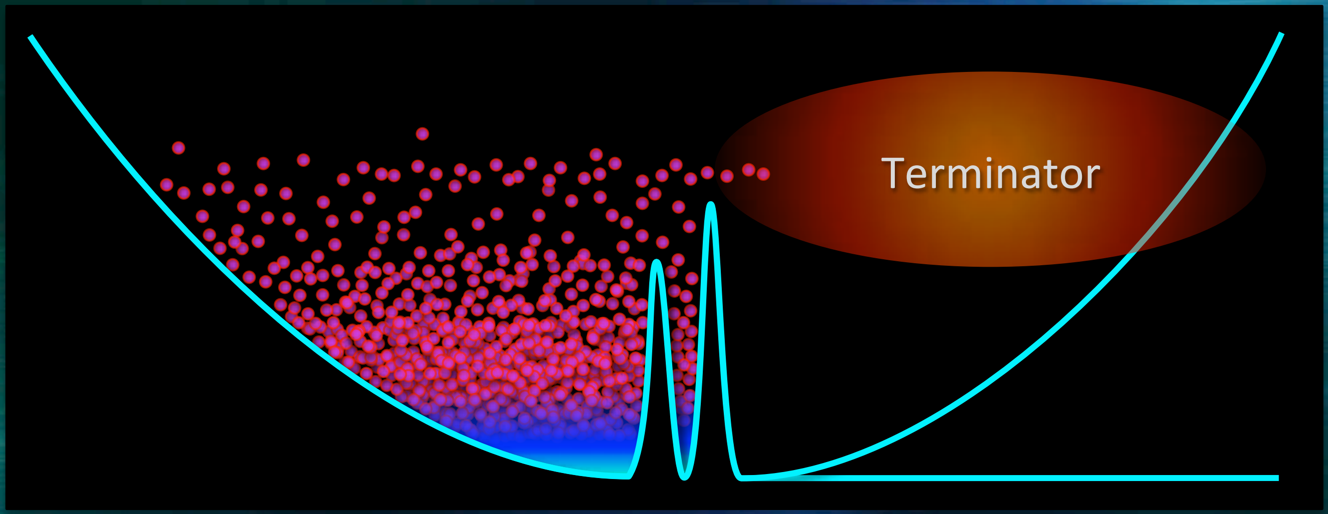


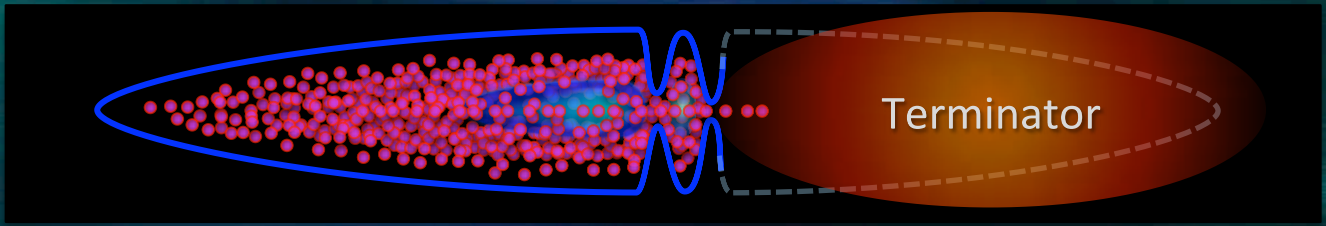
Oscillator Experiments



1. Atoms prepared in Source well to predetermined T_s, μ_s.
2. Terminator beam effectively couples output atoms to the vacuum.
3. Gate well is empty
4. Barriers are lowered, defines *t=0*.
5. Let current flow for a time τ
6. Absorption image
   - In-trap
   - Time-of-flight (TOF)
7. Time of flight data
   - $N_{Condensed}$, $N_{thermal}$, μ, T

In-Trap Absorption Imaging





- System is allowed to evolve for a predetermined time.
- Terminator beam is extinguished
- Wait ~5ms — atoms reach classical turning point.
- Snap image

In-Trap Absorption Image

$\omega_{\perp} = 2\pi \times 1.2\text{kHz}$

$\omega_{G} = 2\pi \times 0.8\text{kHz}$

$V_{GS} = 30\text{kHz}$

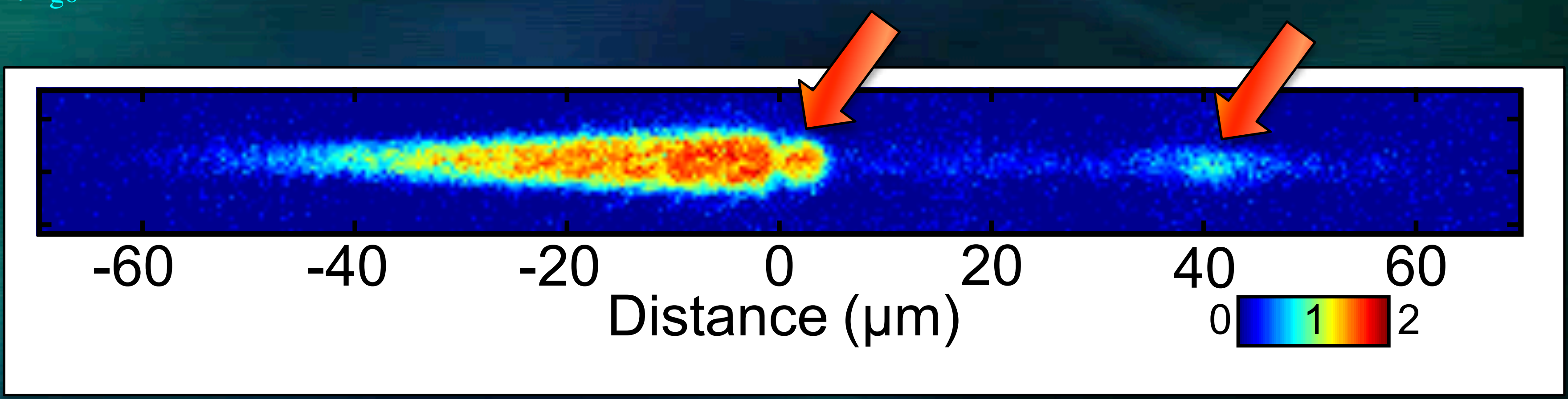
$V_{GD} = 33\text{kHz}$

$T_{s} = 5\text{kHz}$

$\mu_{s} = 5\text{kHz}$

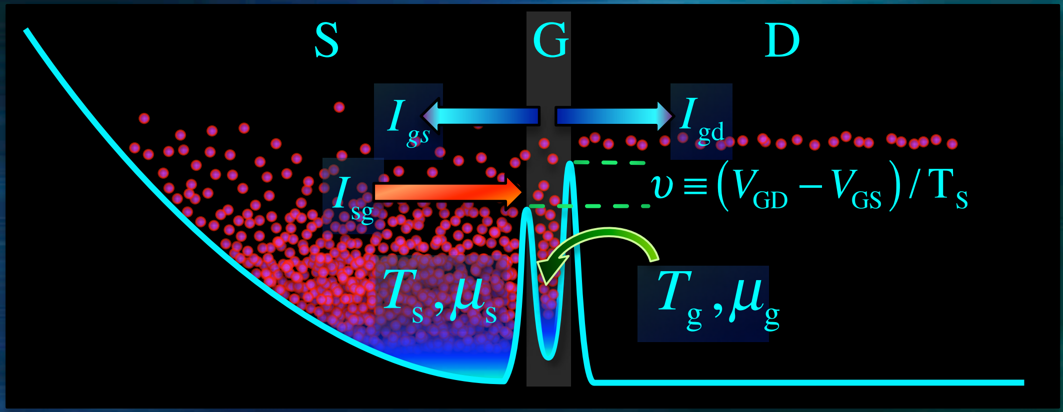
$\mu_{g0} = 3\text{kHz}$

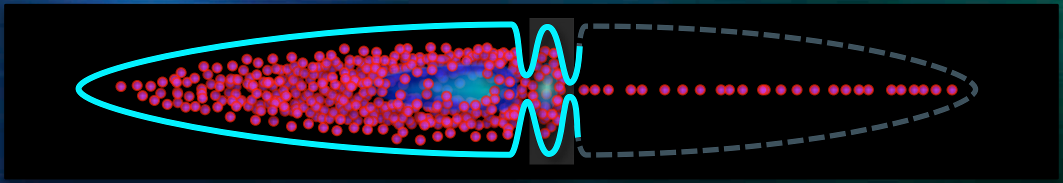
- Drain atoms are seen at classical turning point. They are in nearly a single transverse mode.
- Initially empty gate has a healthy population.



A Semi-Classical Model of Oscillator Behavior

- Assume Source and Gate have well-defined temperature and chemical potential.
- Treat Source as a reservoir with quasi-static properties.





Particle Flux

$$I_{gd} = \gamma_g N_{ge} \exp\left[-\left(V_{GD} - \mu_g\right) / T_g\right]$$

$$I_{sg} = \gamma_s N_{se} \exp\left[-\left(V_{GS} - \mu_s\right) / T_s\right] \quad I_{gs} = \gamma_g N_{ge} \exp\left[-\left(V_{GS} - \mu_g\right) / T_g\right]$$

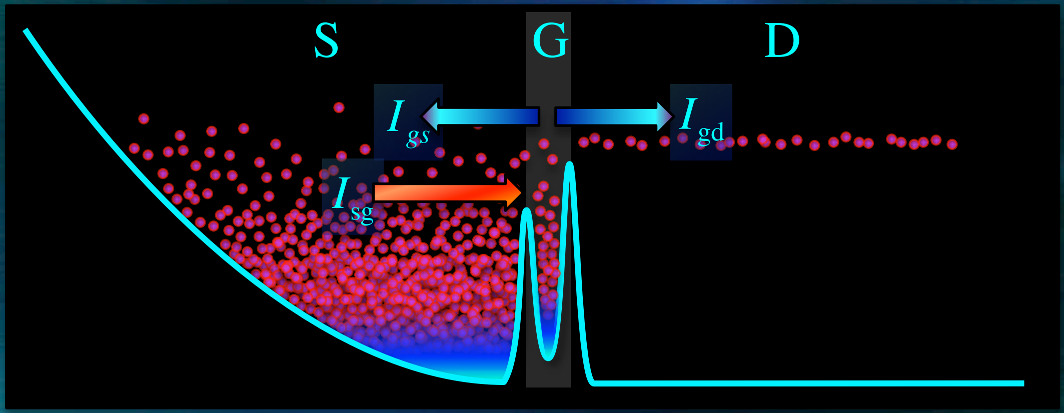
Feedback is key: $\upsilon \equiv \left(V_{GD} - V_{GS}\right) / T_S$

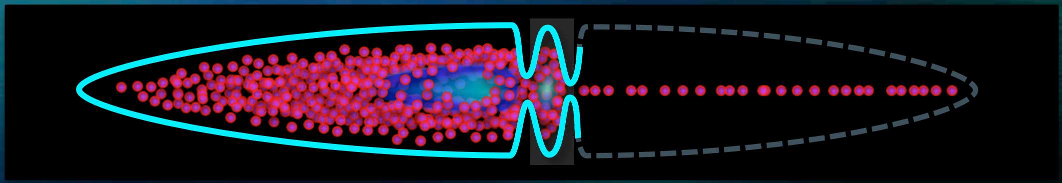
Steady-State Circuit Behavior

Kirchhoff's Current Law $I_{sg} = I_{gs} + I_{gd}$

Kirchhoff's "Voltage Law"

$$I_{sg}\left(V_{GS} + \kappa_{GS} T_s\right) = I_{gs}\left(V_{GS} + \kappa_{GS} T_g\right) + I_{gd}\left(V_{GD} + \kappa_{GD} T_g\right)$$



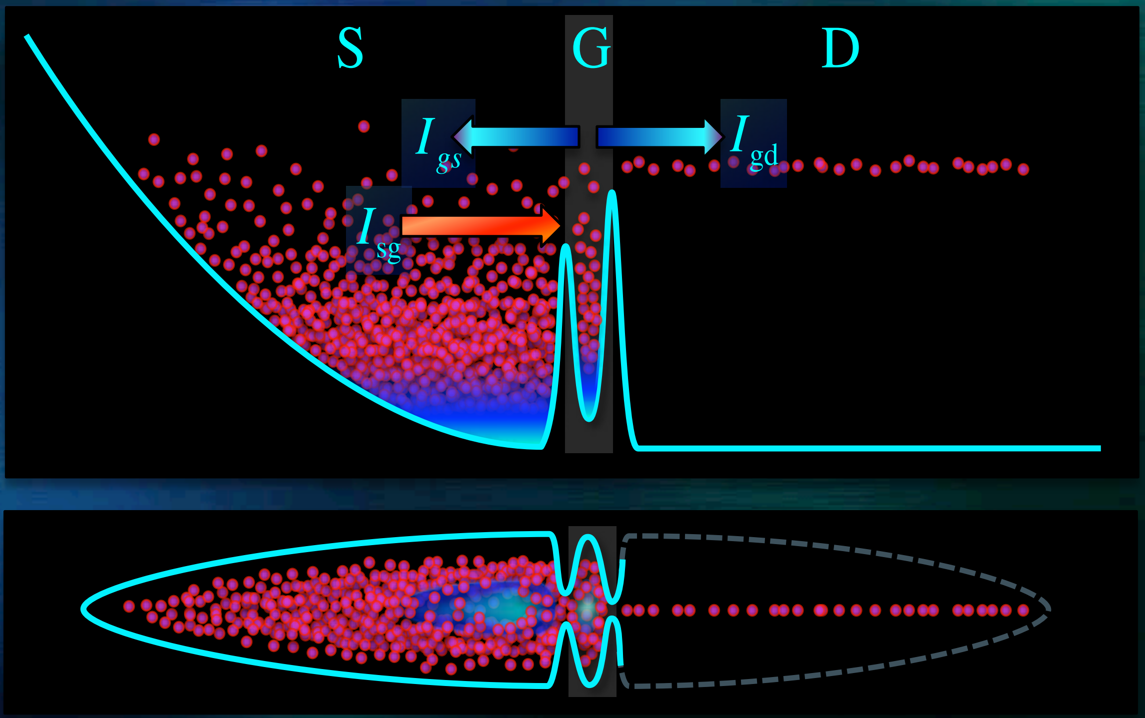


Non-Intuitive Predictions

Within a range of values of the feedback

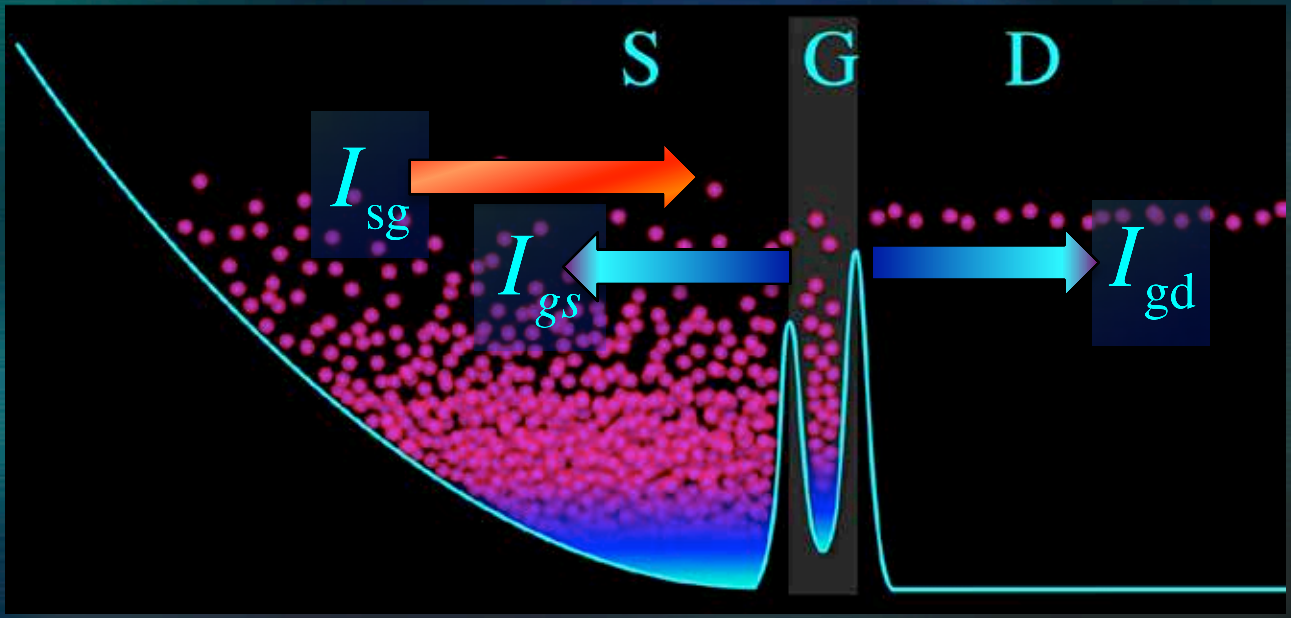
1. The Gate atoms are *colder* than the Source atoms
   (positive temperature drop)
2. The potential drop from the Source to the Gate is *negative*, i.e.,
   (Junction is *reversed biased)*

- Current is dominated by thermo-motive flow.
- Atoms emitted into drain are *colder* than they would be without the Gate-Source barrier.



Insight to Intriguing Behavior

- High energy Gate-Drain flux efficiently cools (via evaporation) the small population of Gate atoms.
- Gate potential rises
  - Increasing atom flux out of the Gate,
  - Inducing *reverse potential* bias across Source-Gate Junction
  - Decreasing net current flow into Gate.
- Feedback from Gate to Source cools the Source well.
- Output from Gate into Drain is colder than the input from Source into Gate.



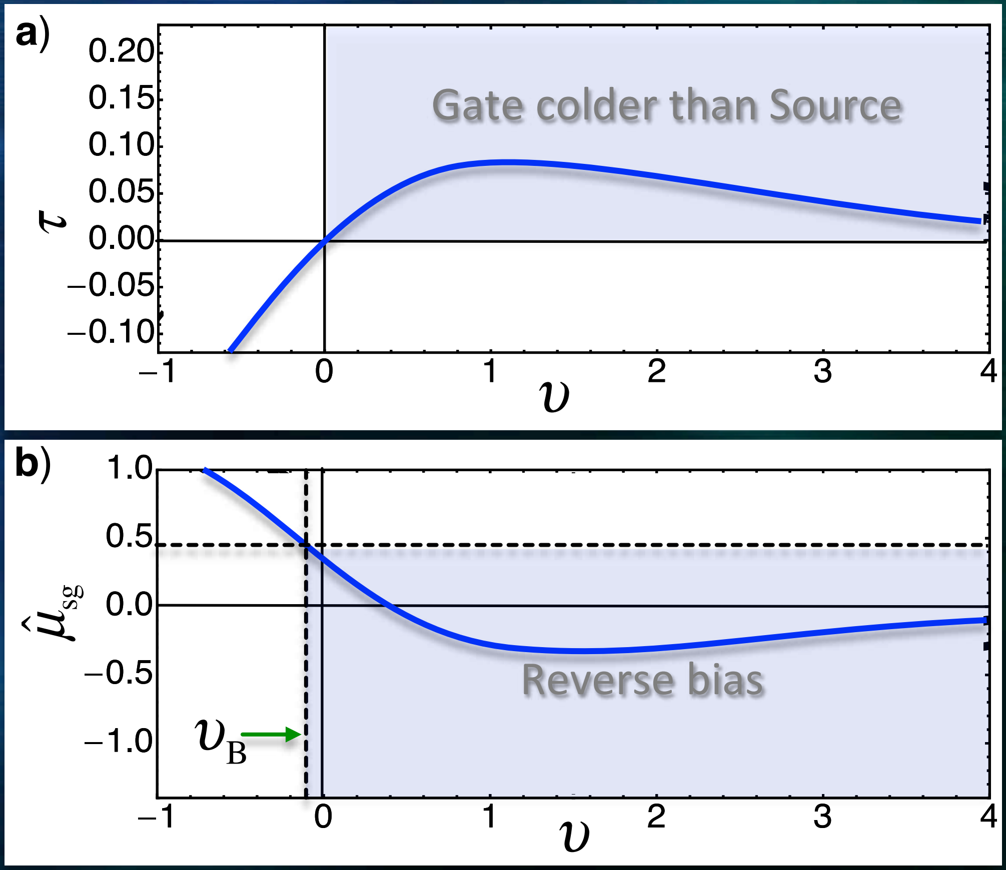
Threshold for (Non-Classical) Oscillation

Normalized temperature drop $\tau \equiv (T_s - T_g)/T_s$

Feedback parameter $\upsilon \equiv (V_{GD} - V_{GS})/T_s$

Normalized Source-Gate potential drop $\hat{\mu}_{sg} = (\mu_s - \mu_g)/T_s$

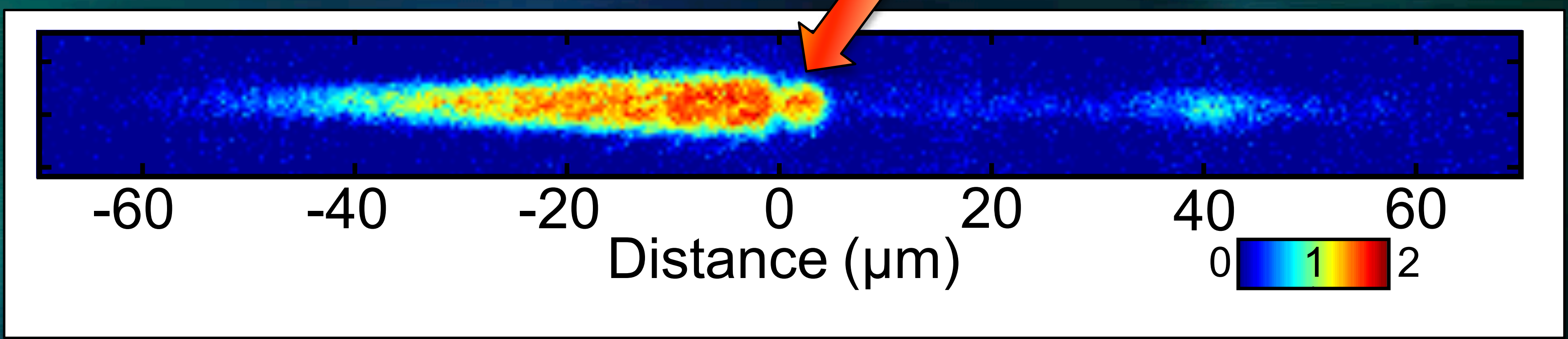
- Asymptotic state as v becomes large agrees with thermal equilibrium result.
- Positive temperature drop and reverse bias occur near zero feedback.
  - (Reverse bias region offset from zero accounts for Gate bias).
- Instability arises from population inversion in the initially empty gate.
- Non-classical oscillation threshold corresponds to formation of a condensate in the Gate.



In-Trap Absorption Image

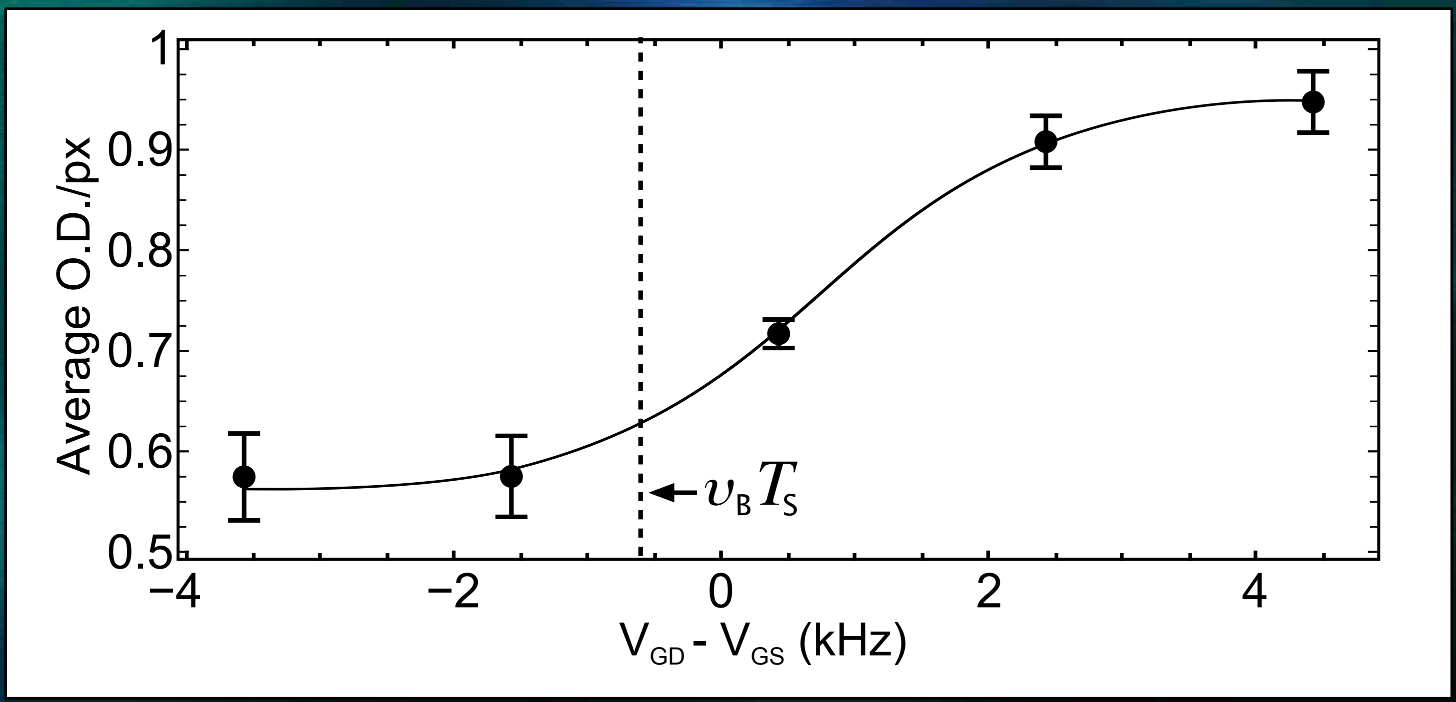
$$\omega_\perp = 2\pi \times 1.2\text{kHz}$$
$$\omega_\text{G} = 2\pi \times 0.8\text{kHz}$$
$$V_\text{GS} = 30\text{kHz}$$
$$V_\text{GD} = 33\text{kHz}$$
$$\text{T}_\text{s} = 5\text{kHz}$$
$$\mu_\text{s} = 5\text{kHz}$$
$$\mu_\text{g0} = 3\text{kHz}$$

- Drain atoms are seen at classical turning point. The are in nearly a single transverse mode.
- Initially empty gate has a healthy population.
- Given large Gate bias, the Source-Gate potential drop must be negative.



Oscillator Turn-On

- Gate optical density as a function of feedback
- As feedback is increased, circuit exhibits a classic "S" shaped turn-on behavior.
- BEC Threshold appears approximately as predicted.



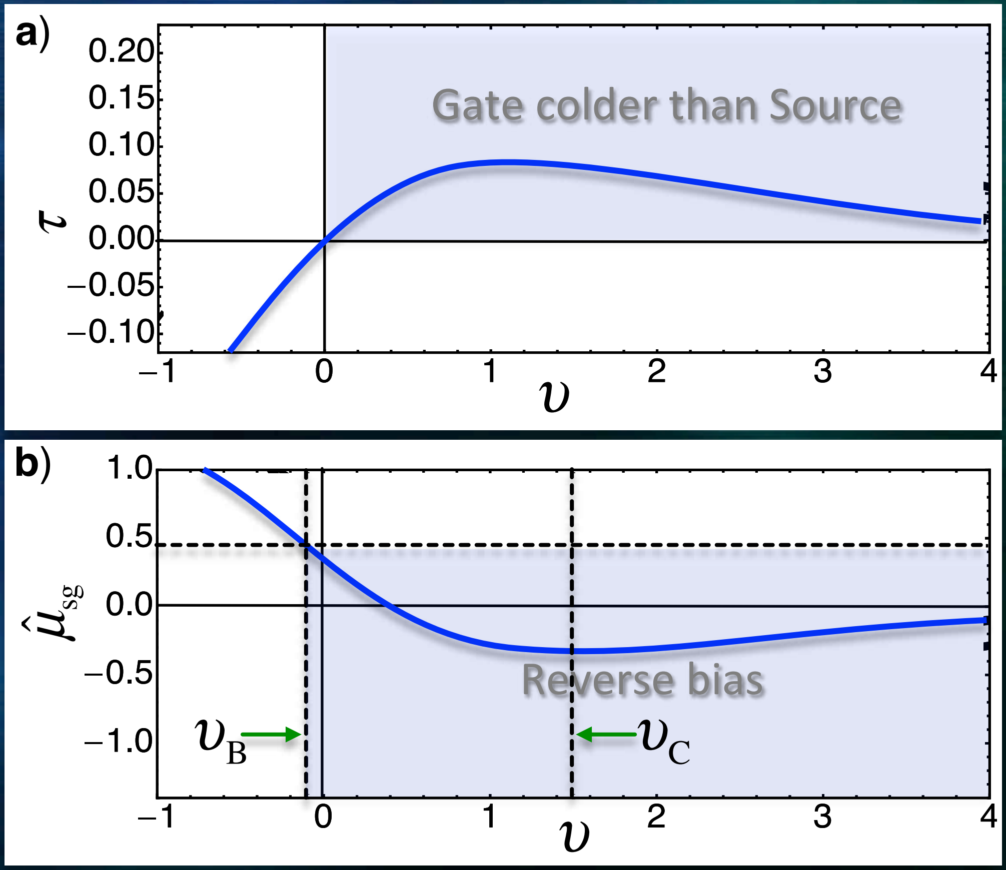
Classical Oscillation Threshold

Normalized temperature drop $\tau \equiv (T_s - T_g)/T_s$

Feedback parameter $\upsilon \equiv (V_{GD} - V_{GS})/T_s$

Normalized Source-Gate potential drop $\hat{\mu}_{sg} = (\mu_s - \mu_g)/T_s$

- Asymptotic state as v becomes large agrees with thermal equilibrium result.
- Positive temperature drop and reverse bias occur near zero feedback.
  - (Reverse bias region offset from zero accounts for Gate bias).
- Non-classical oscillation threshold corresponds to formation of a condensate.
- The classical oscillation threshold occurs where the *transresistance* becomes negative.



Resistance & *Trans*resistance

Gate resistance:

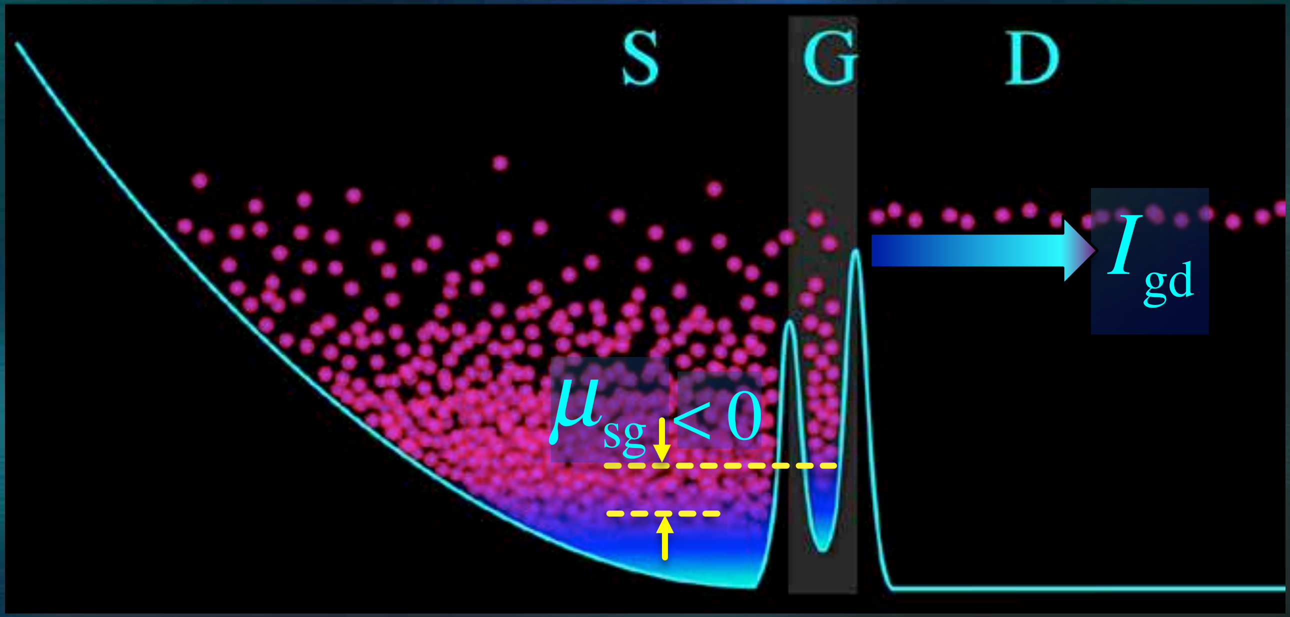
$$R_g \equiv \frac{\mu_{sg}}{I_{gd}} < 0$$

Negative gate resistance implies Source cooling

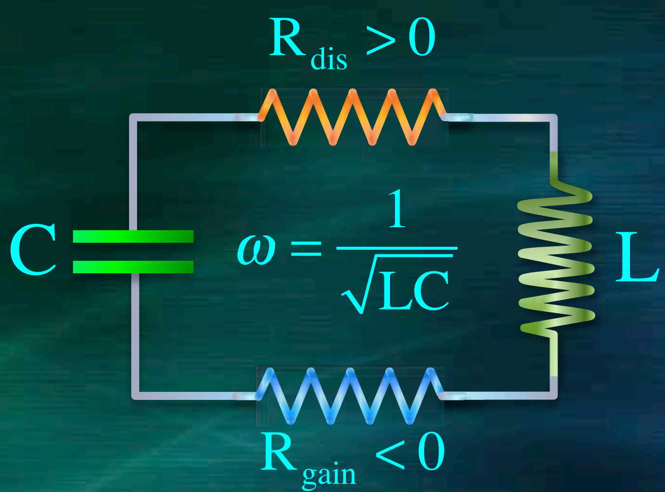
Transresistance

$$r_g \equiv \frac{d\mu_{sg}}{dI_{gd}} = \frac{\partial \mu_{sg}}{\partial T_s}\left(\frac{\partial I_{gd}}{\partial T_s}\right)^{-1}$$

Negative transresistance resistance implies Gain



Negative Transresistance = Gain

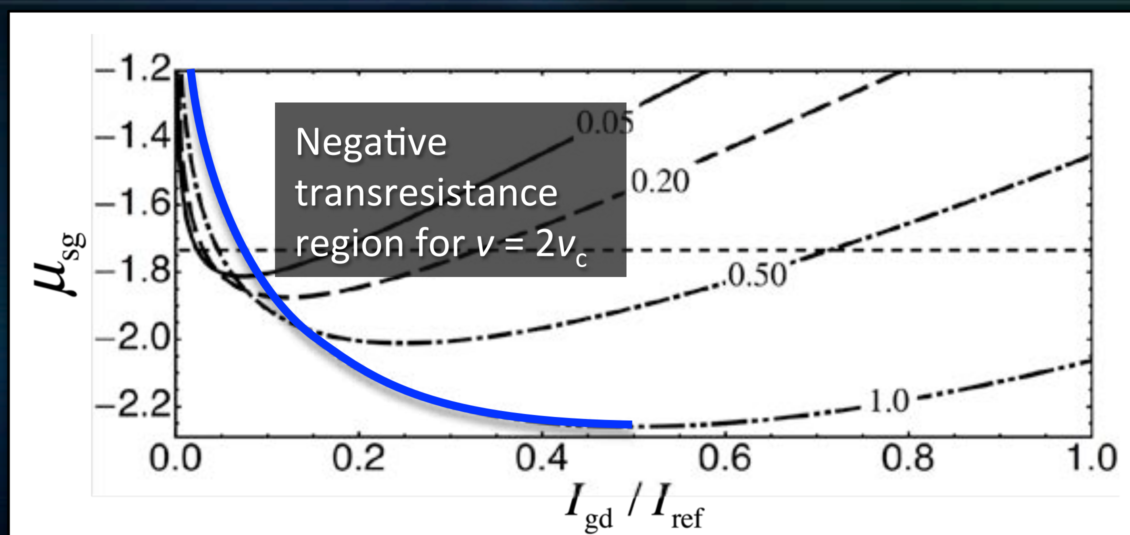


The classic driven oscillator:

- Any physical oscillator, such as an LC circuit, is subject to dissipation.
- Incorporate a negative resistance to compensate loss.
- Achieve sustained oscillation

Feedback above the classical threshold values gives rise to negative transresistance.

Here are plotted transresistance curves for 4 values of feedback, indicated as fraction above threshold.



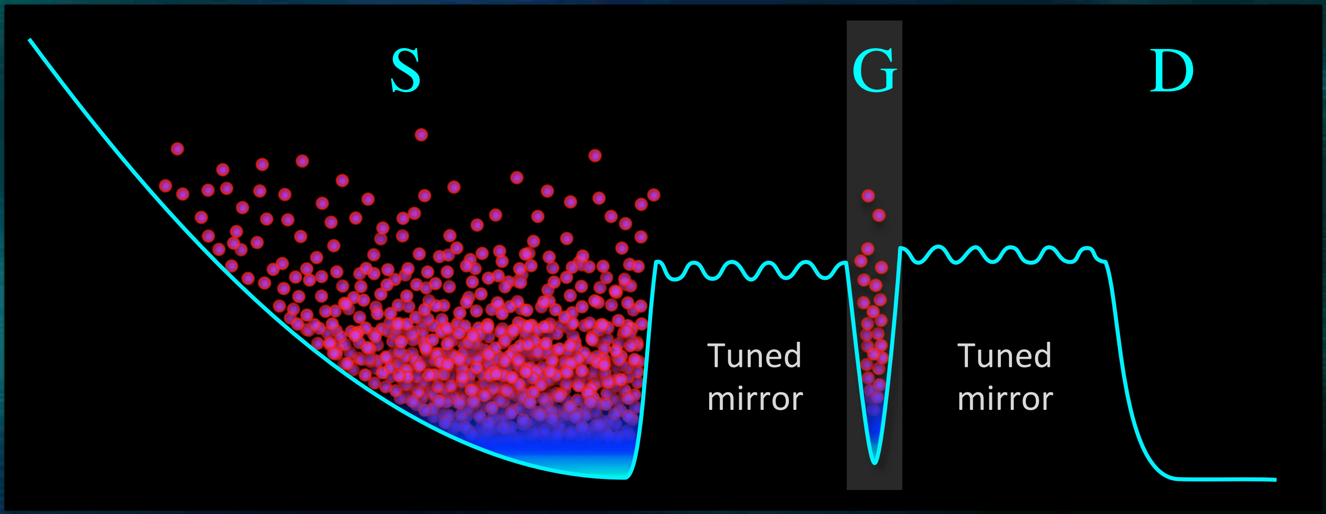
Summary

- Triple-well atomic potential exhibits transistor behavior
- Gate oscillation corresponds to the formation of a BEC (non-classical effect).
- Gate acquires colder temperature, high chemical potential then Source.
- Barrier (junction) height difference (feedback) is a critical parameter.
- Negative resistance = gain for sufficiently high feedback parameter.

Towards a "classical" atom driven oscillator

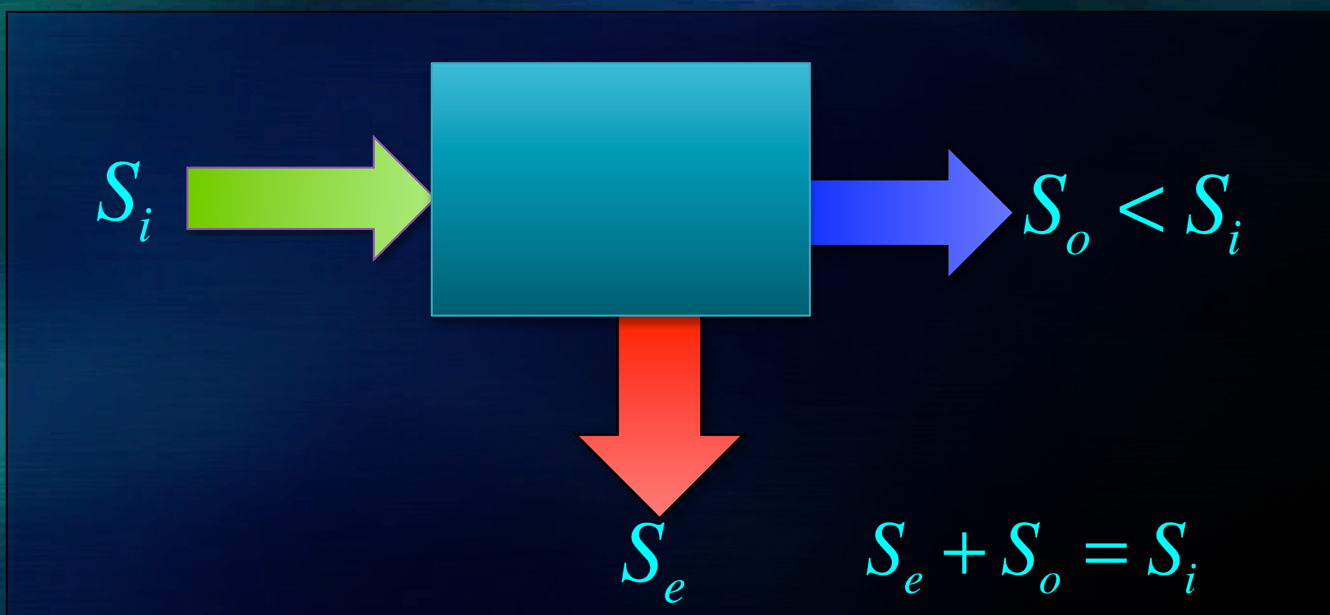
- Looking to utilize negative resistance to maintain high-Q dipole oscillation.
- Considering ways to assess temporal coherence (e.g. with a Mach-Zehnder interferometer with delay.
  - (not easy).
- Want to add frequency selectivity to circuit's input and output lines.

Potentials we'd like to implement



Atomtronics, Signal Processing and Entropy

- The ultracold regime enables substantial control over the physical domain of the system
  - i.e. reservoir coupling, dissipation, etc.
- As non-equilibrium systems, atomtronic circuits force one to consider the whole of entropy flow and conservation.
- Highlights the connection between information processing, entropy, and energy.



Thank you!