A Dynamical Theory of Superfluidity in One dimension

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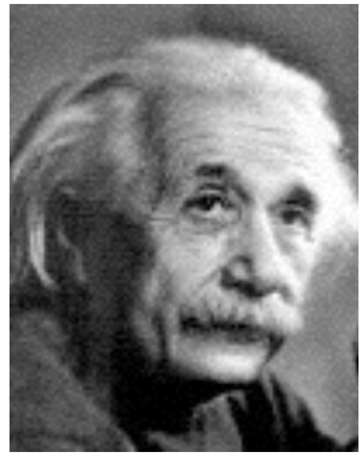
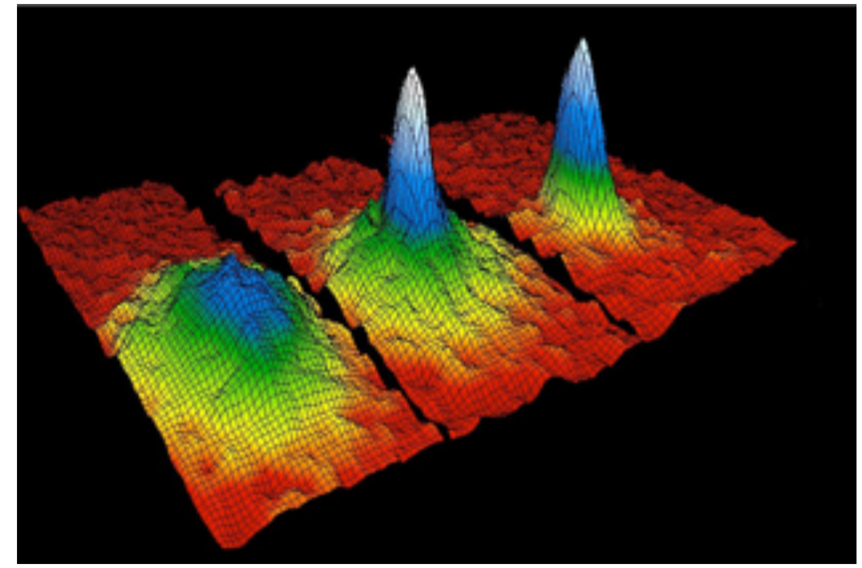
Benasque, Atomtronics 2015

Superfluid ≠ Bose-Einstein Condensate (BEC)

$$\lim_{|\mathbf{r}-\mathbf{r}'|\to+\infty}\langle\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}')\rangle = |\Psi_0|^2 \neq 0$$

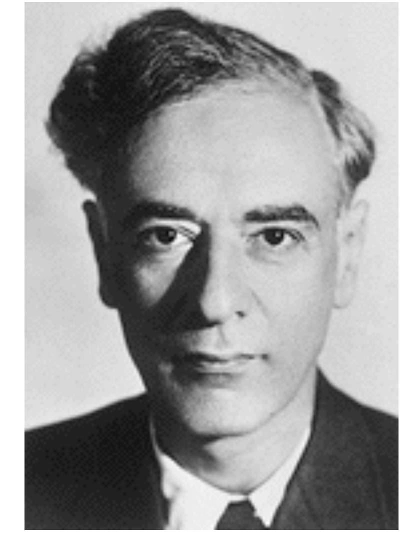
Worth a Nobel Prize (2001)



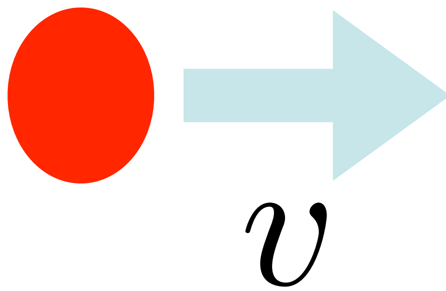

But BEC is not the same as Superfluidity!!

(but in 3D BEC and SF are intimately related...)

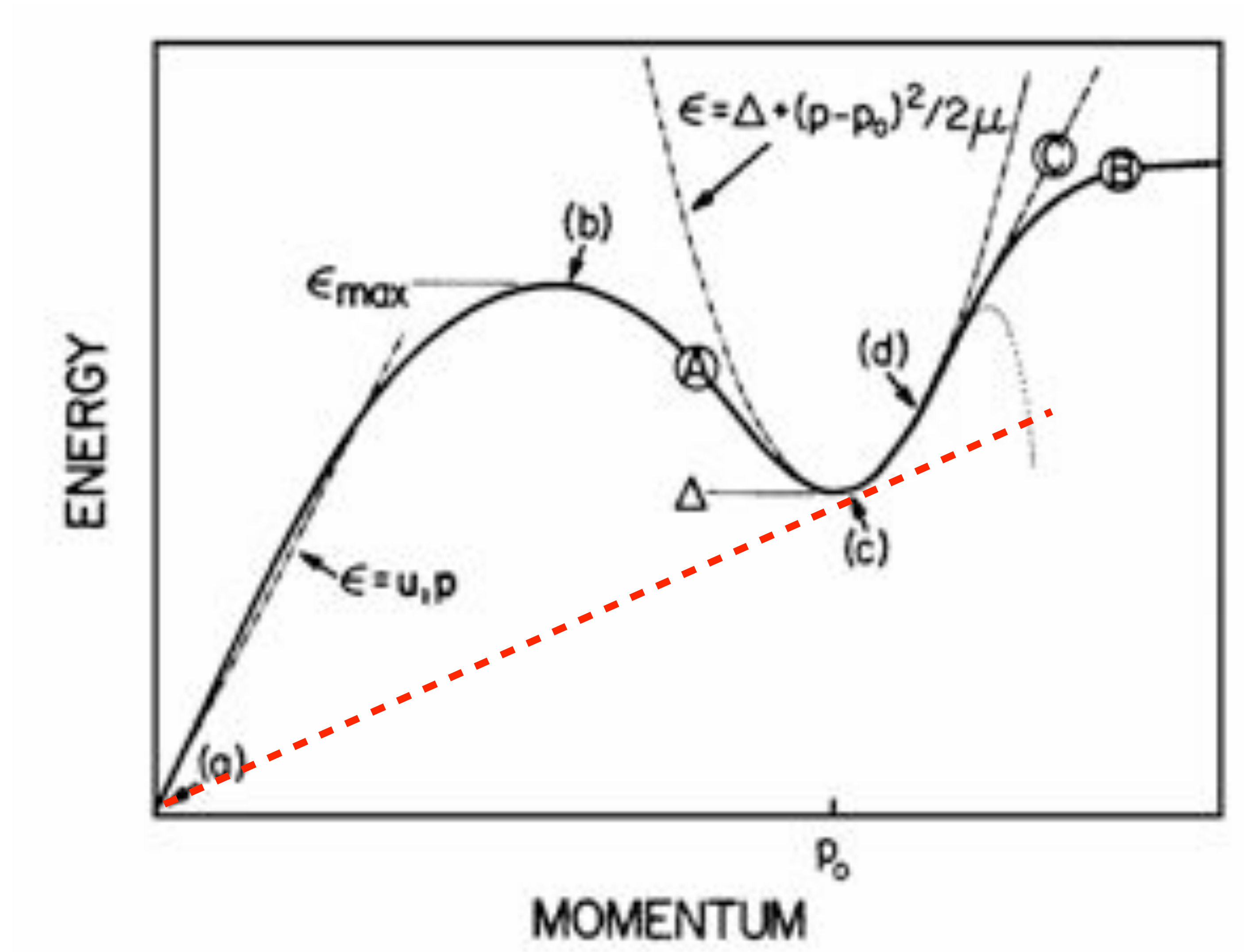
Landau's criterion



Consider a moving object:



Spectrum of liquid ^{4}He



Finite critical velocity

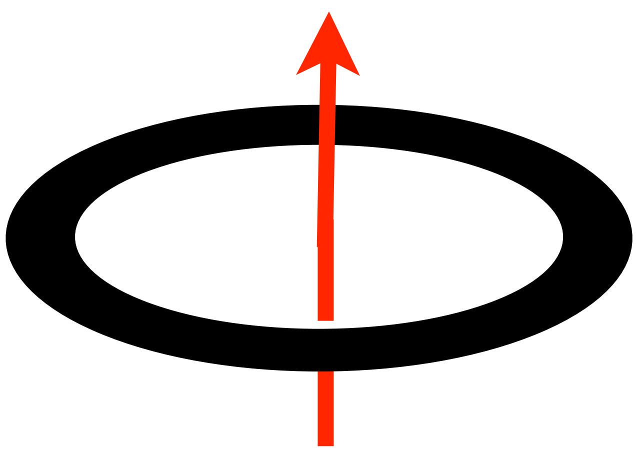
$$\min\left\{\frac{\epsilon(p)}{p}\right\} = v_{\mathrm{Landau}} > 0$$

Problem: How to define the SF properties at T > 0?

Fisher's criterion

Thermodynamics:

Superfluidity = non-vanishing Helicity Modulus



Twisted BC's

$$\hat{\Psi}(x+L,y,z) = e^{i\varphi}\hat{\Psi}(x+L,y,z)$$

$$\Upsilon(T) = \lim_{L\to+\infty} \frac{L}{S} \left(\frac{\partial^2 F(\varphi)}{\partial \varphi^2} \right) \Bigg|_{\varphi=0} \neq 0$$

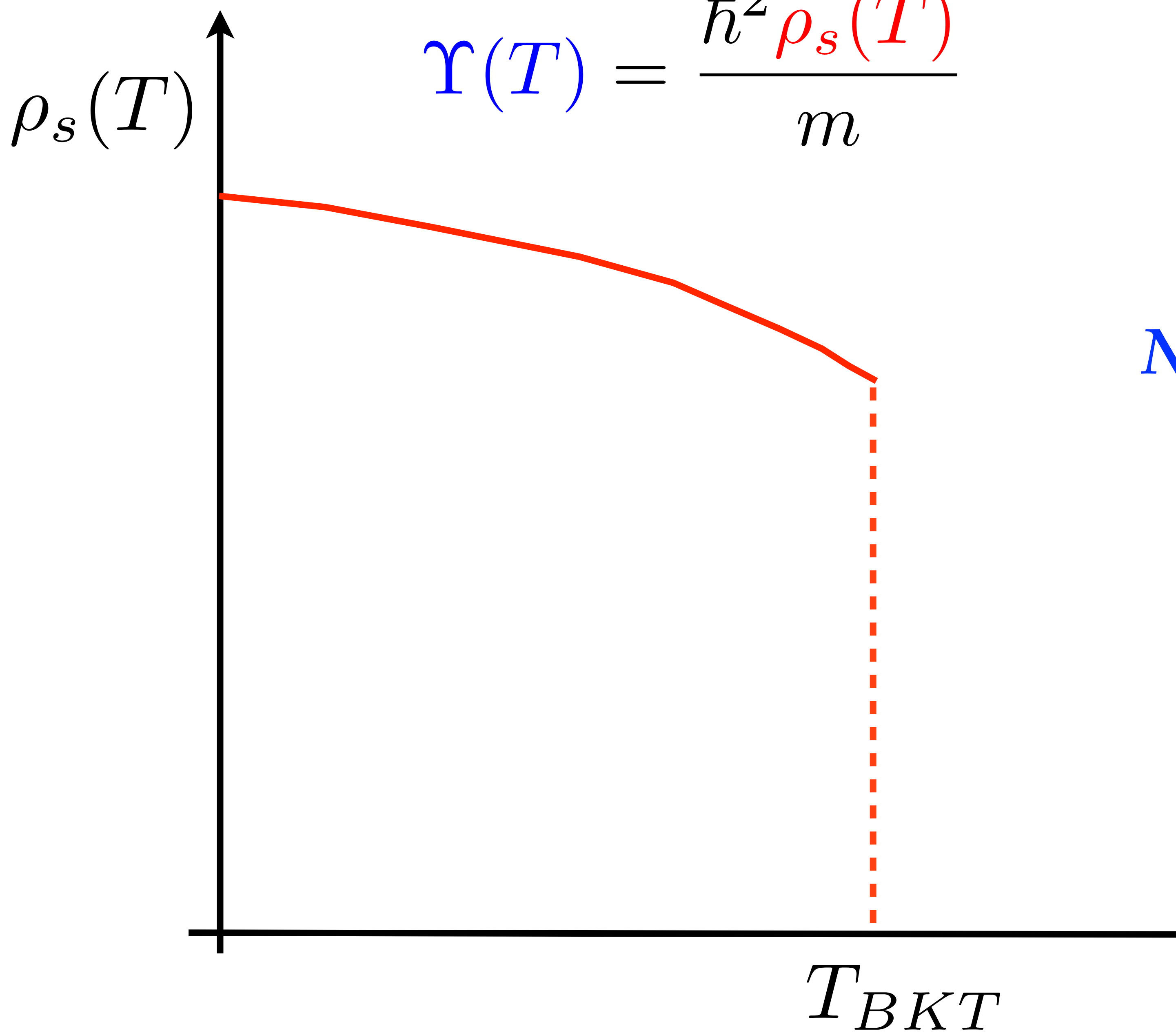
ME Fisher et al PRA (1973)

Superfluid density: $\Upsilon(T) = \frac{\hbar^2 \rho_s(T)}{m}$

Interacting Bose fluids (BEC) in 2D

Absence of BEC (T > 0) $\langle \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r}')\rangle \sim |\mathbf{r}-\mathbf{r}'|^{-\frac{1}{2K(T)}} \to 0$

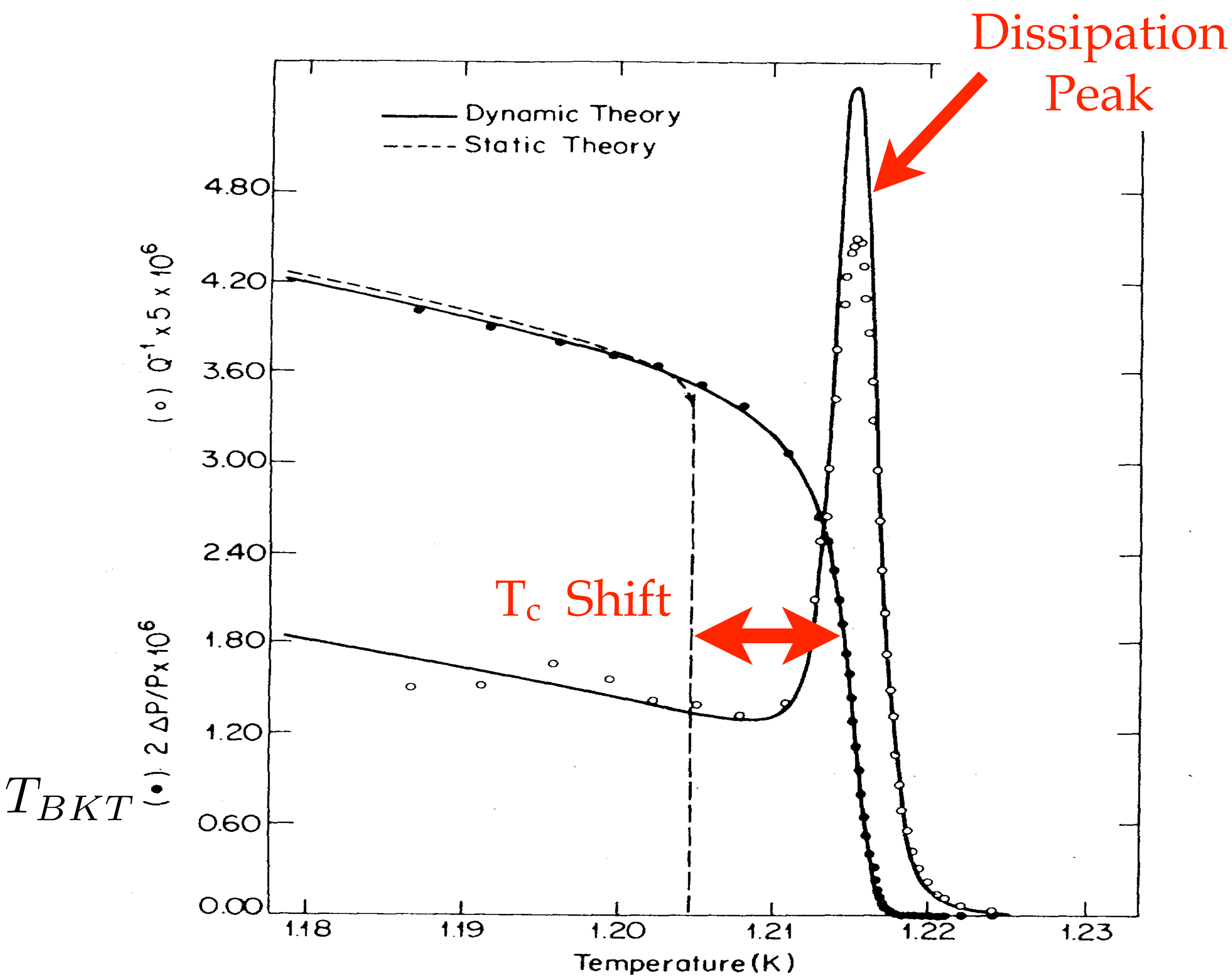
$$\Upsilon(T) = \frac{\hbar^2 \rho_s(T)}{m}$$



Nelson & Kosterlitz
PRL (1977)
Universal Jump
of the SF density

Superfluidity in 2D (Experiments)

2D ^{4}He films: Torsional oscillator measurements



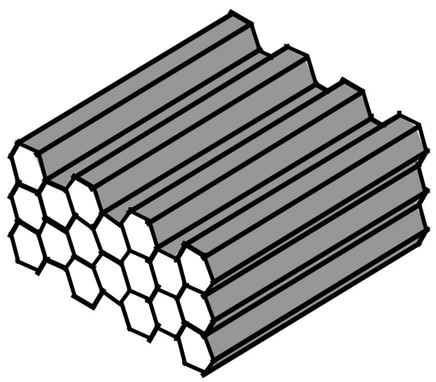
Experiment: DJ Bishop & JD Reppy PRL (1978)

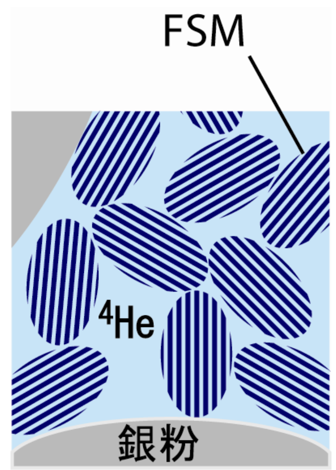
Theory: Ambegaokar, Halperin, Nelson & Siggia PRL (1978)

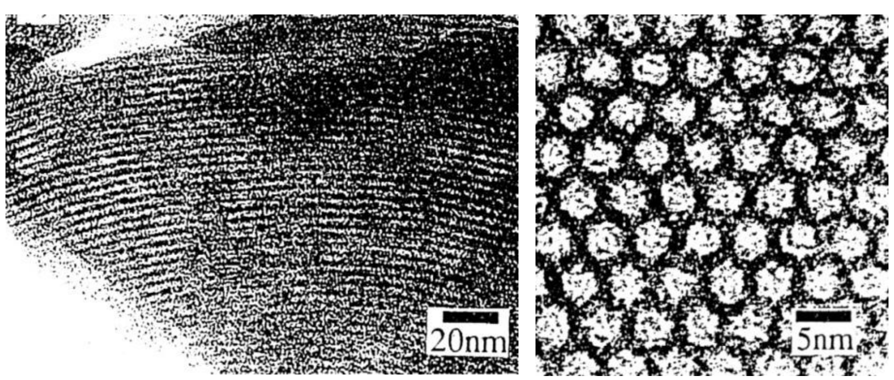
Prof. Fisher meets One dimension

FSM-16 Pore size: 2.8 nm

Brought to you by

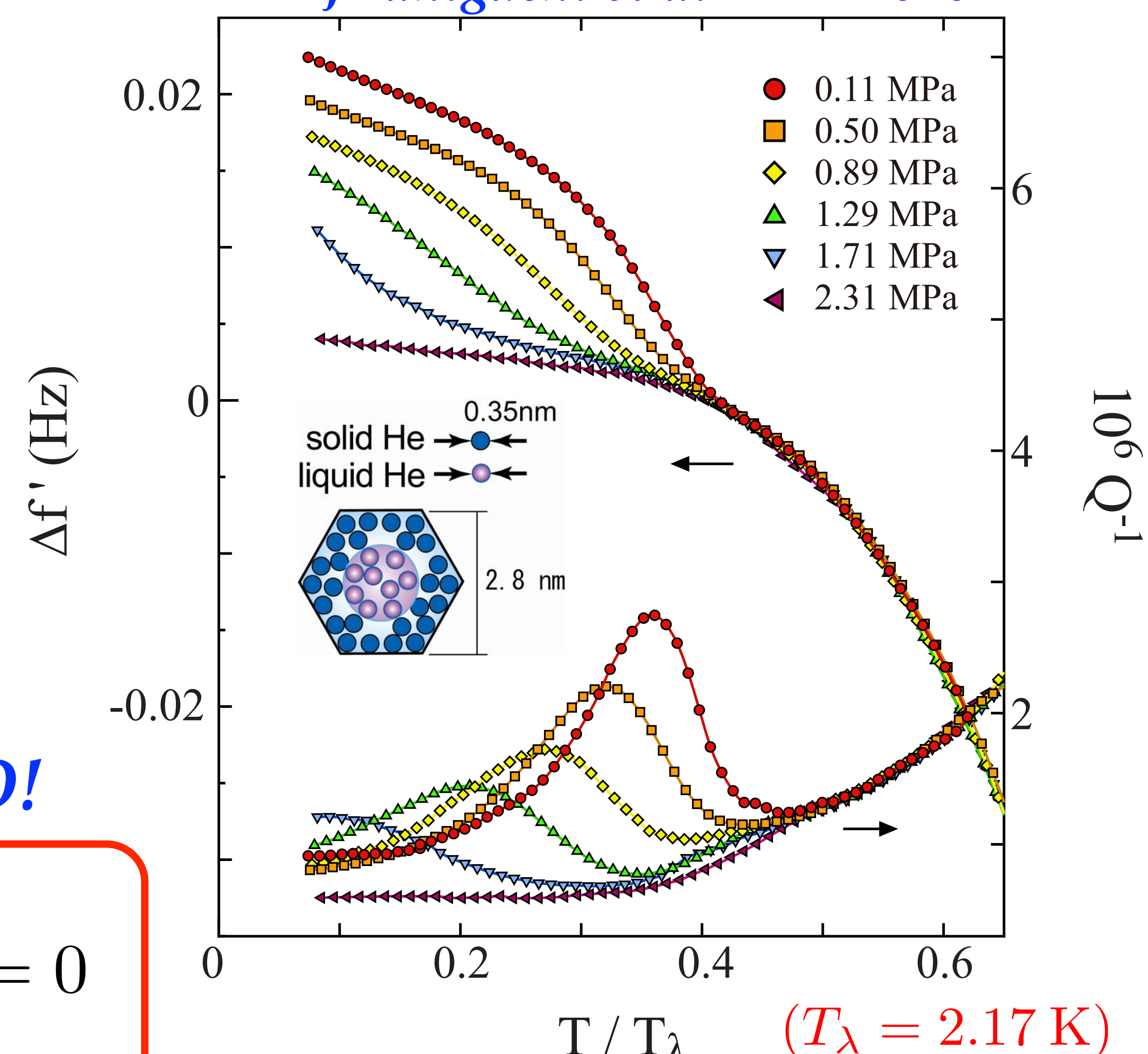






TEM of FSM16 (2.8 nm)

J Taniguchi et al PRB 2010



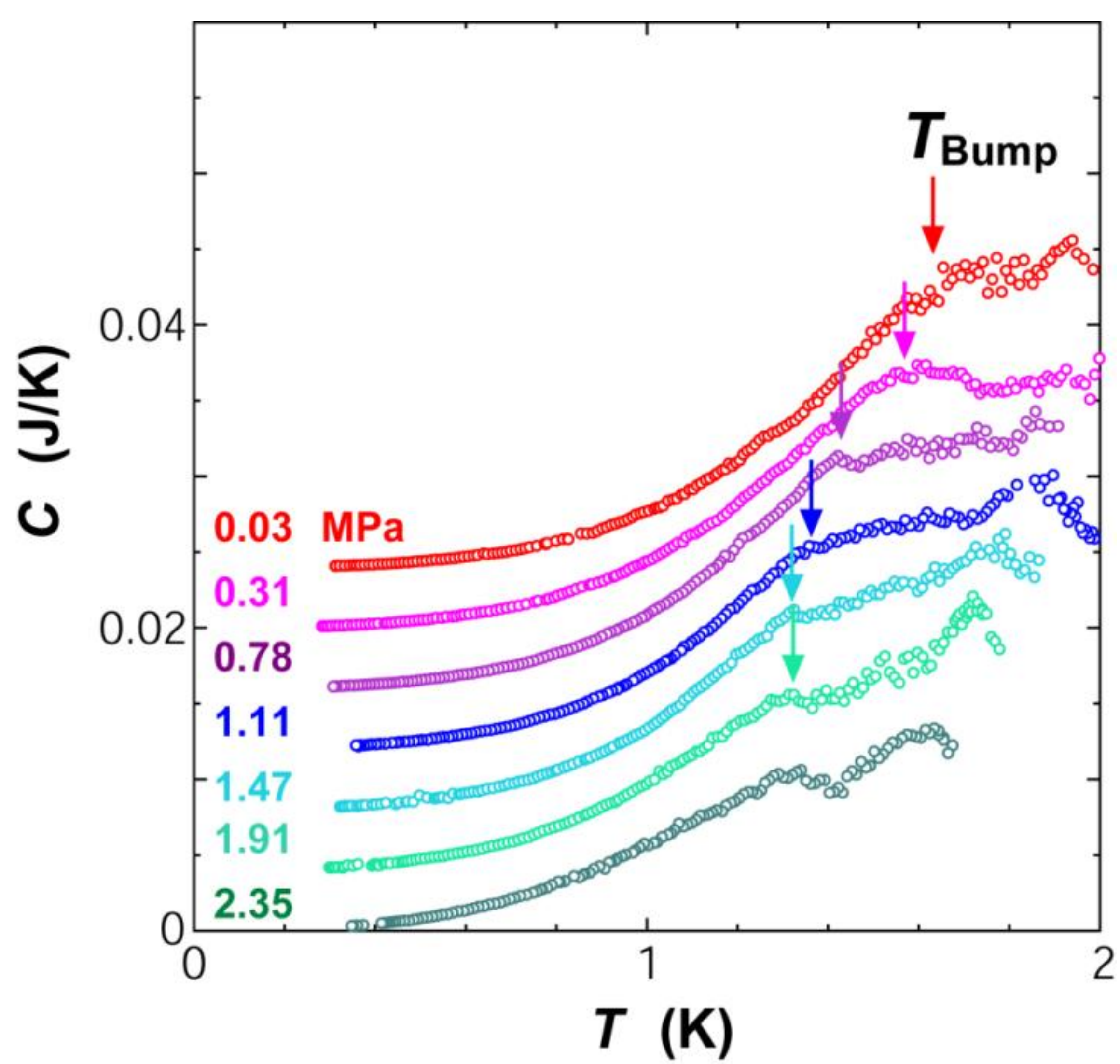
$(T_\lambda = 2.17\,\mathrm{K})$

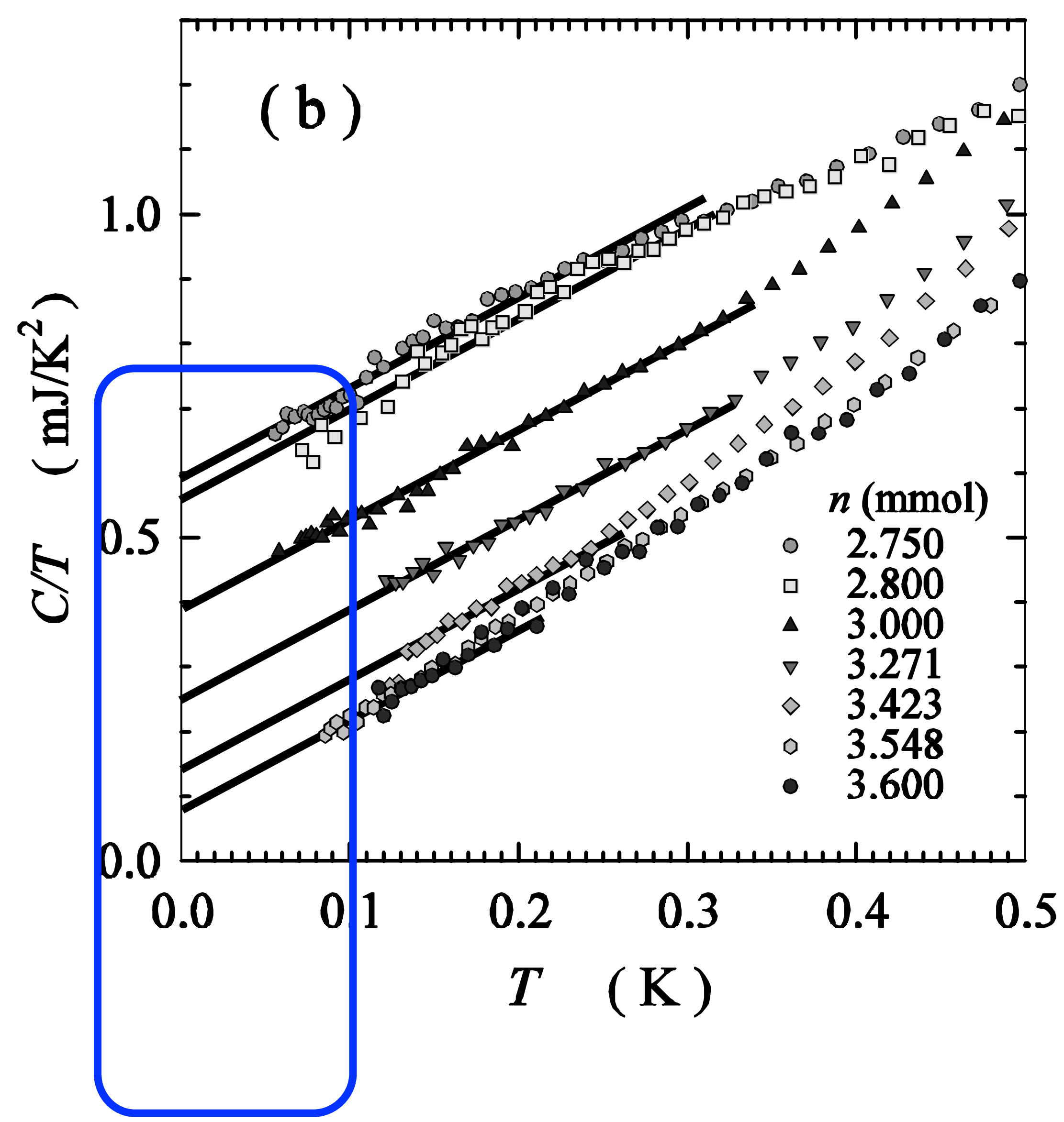
The helicity modulus vanishes in 1D!

$$\Upsilon_{1D}(T) = \lim_{L\to+\infty} L\left(\frac{\partial^2 F(\varphi)}{\partial\varphi^2}\right)\Bigg|_{\varphi=0} = 0$$

So what is the origin of this SF signal?

Specific Heat of a He-filled Nanopore

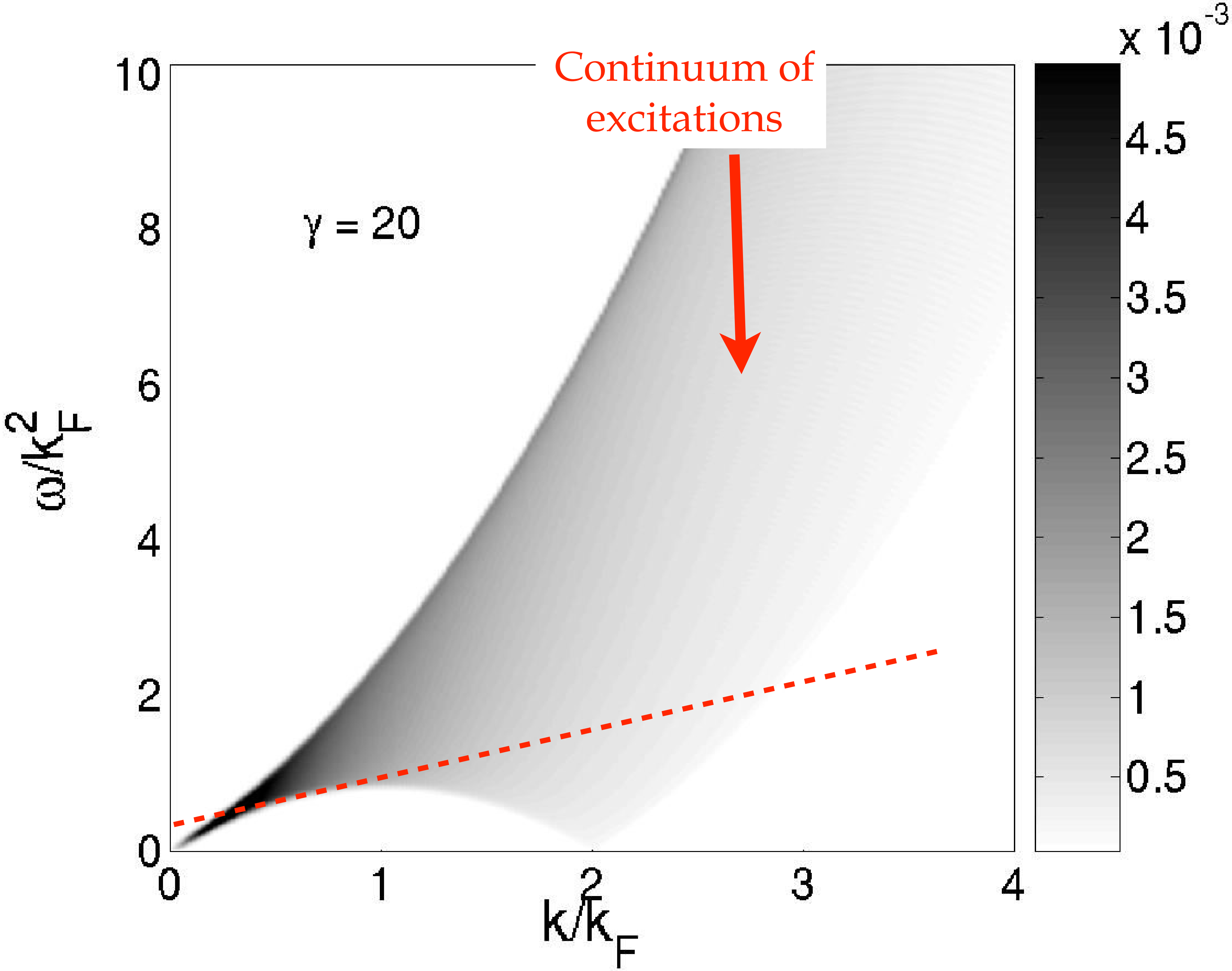




Finite intersect for C/T → 1D Phonons

Landau's criterion is violated in 1D

Support of the dynamic structure factor $S(k,\omega)$

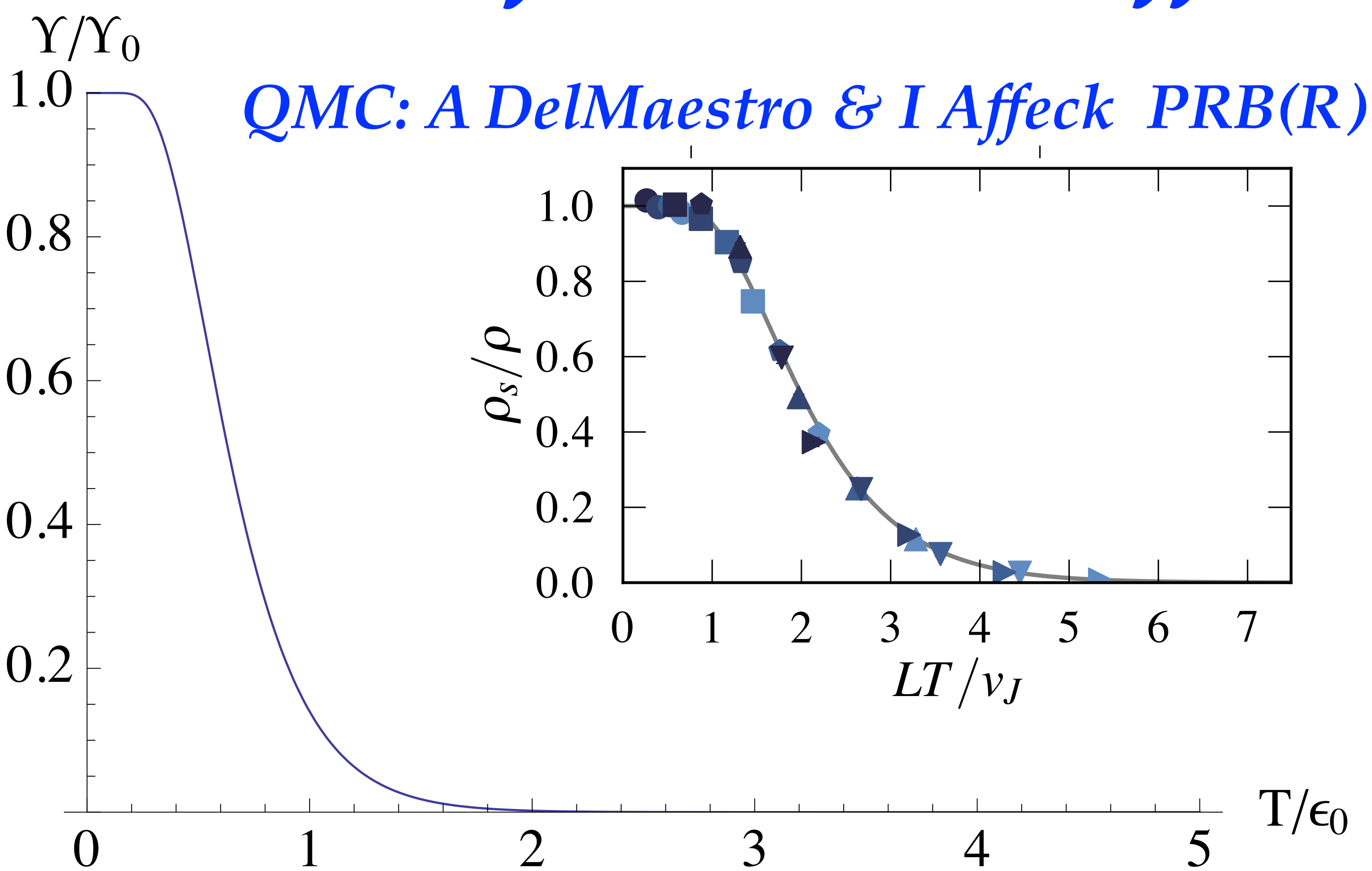


Exact result for a 1D interacting Bose gas JS Caux and P Calabrese, PRA(R) (2006)

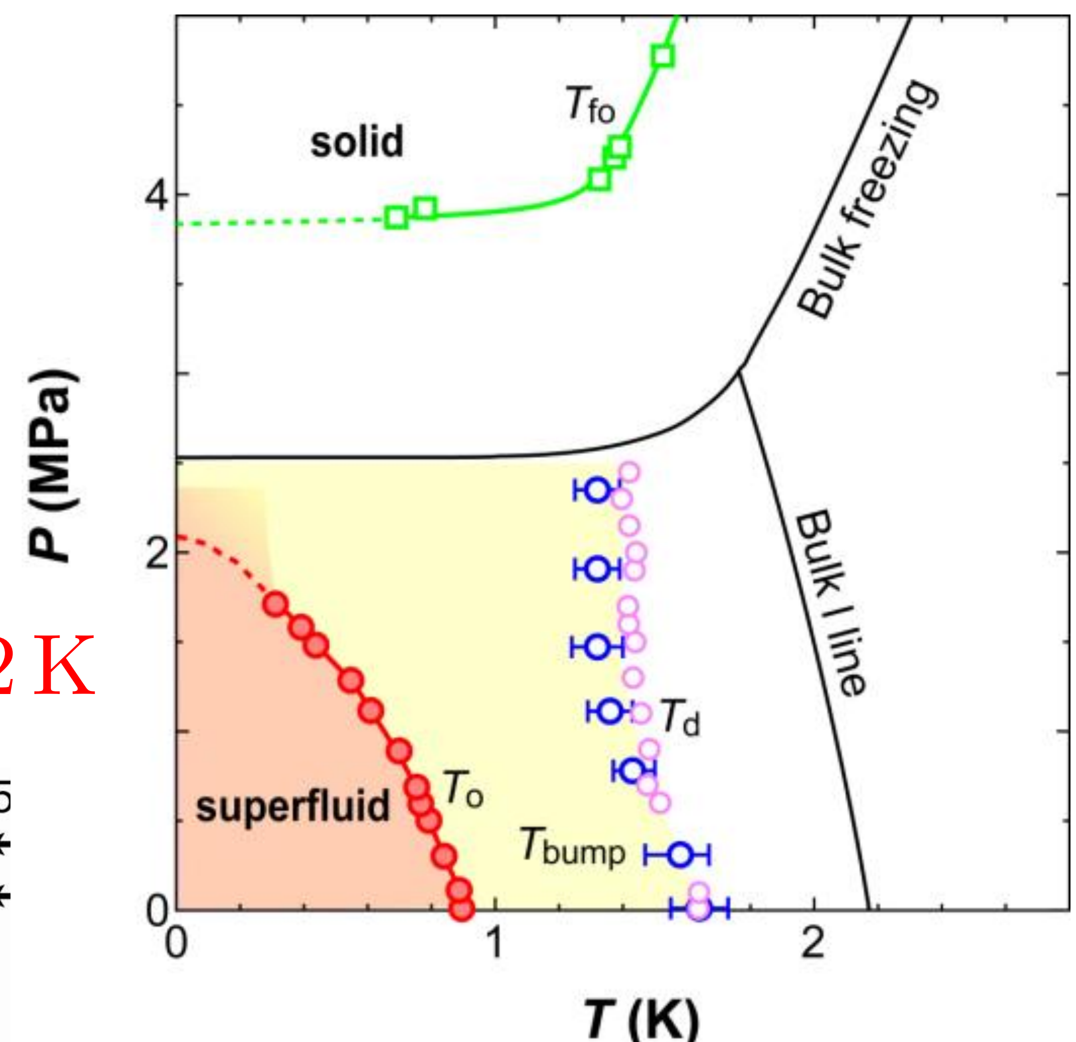
A detective story

Is it a finite size effect?

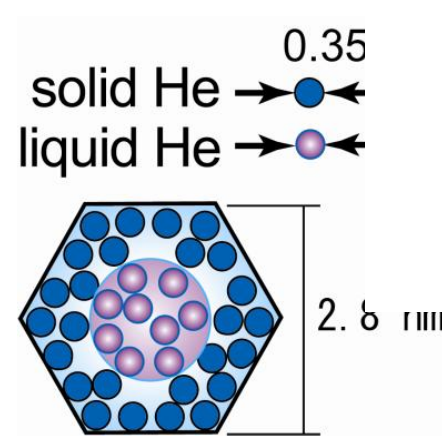
QMC: A DelMaestro & I Affeck PRB(R) (2010)



I Taniguchi et al PRB 2010



$$\frac{\epsilon_0}{k_B} = \frac{\hbar v K}{L} < \frac{\hbar^2 \pi \rho_0}{m(^4He)L} = T_{\rm onset}^{\rm max} \simeq 0.2\,{\rm K}$$



So what is the origin of the SF signal?

Laughlin's criterion

"Superfluidity?... it's like pornography
I can't define it but I know it when I see it."

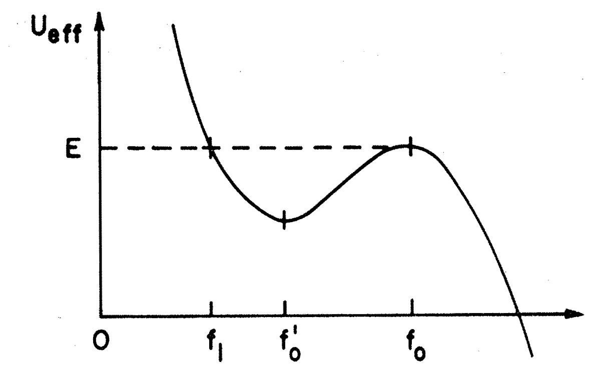
R. B. Laughlin in "Mesoscopic Protectorates",
talk at KITP (2000)

Could it be a dynamical effect? Phase Slips

Thermal Phase Slips (from GL theory)

$$\Gamma_{\mathrm{TPS}} = \Omega(T) e^{-\frac{\Delta F(T)}{k_B T}}$$

$$|T - T_c|/T_c \ll 1$$



Langer-Ambegaokar PR (1967)
McCumber-Halperin PR (1970)

Quantum Phase Slips:

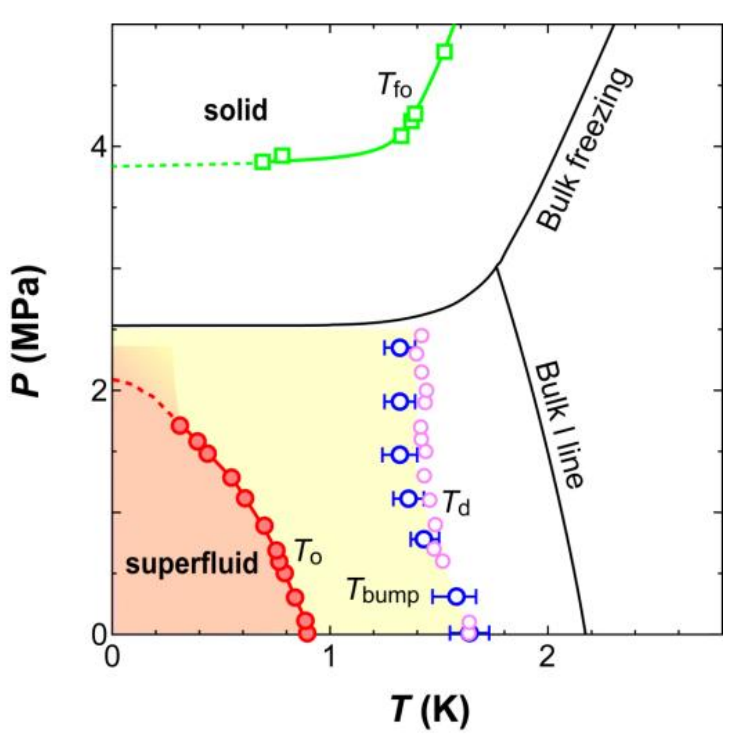
$$\rho = \rho_0 + \delta\rho$$

$$S = \int dx d\tau \left[i\rho\, \partial_\tau \theta + \cdots \right] = \int dx \left[i\rho_0 \partial_\tau \theta + \cdots \right]$$

Non-trivial Berry phase!

$$\Gamma_{\mathrm{QPS}} \sim e^{-\frac{\hbar \pi v \rho_0}{k_B T}}$$

Khlebnikov PRA (2005)



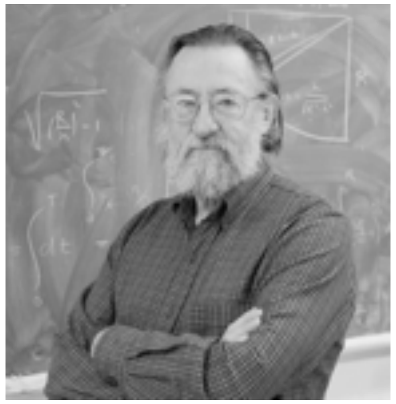
Γ_{PS} should be very small at low temperatures but it is contradicted by the experiment!

J Taniguchi et al PRB 2010

Torsional Oscillator (TO)

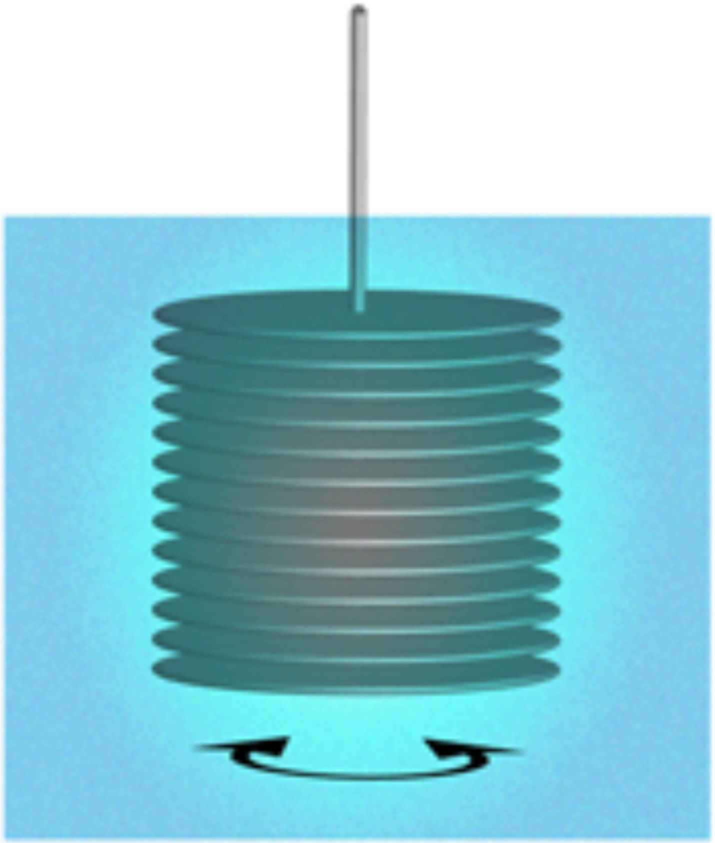
Modern torsional oscillator

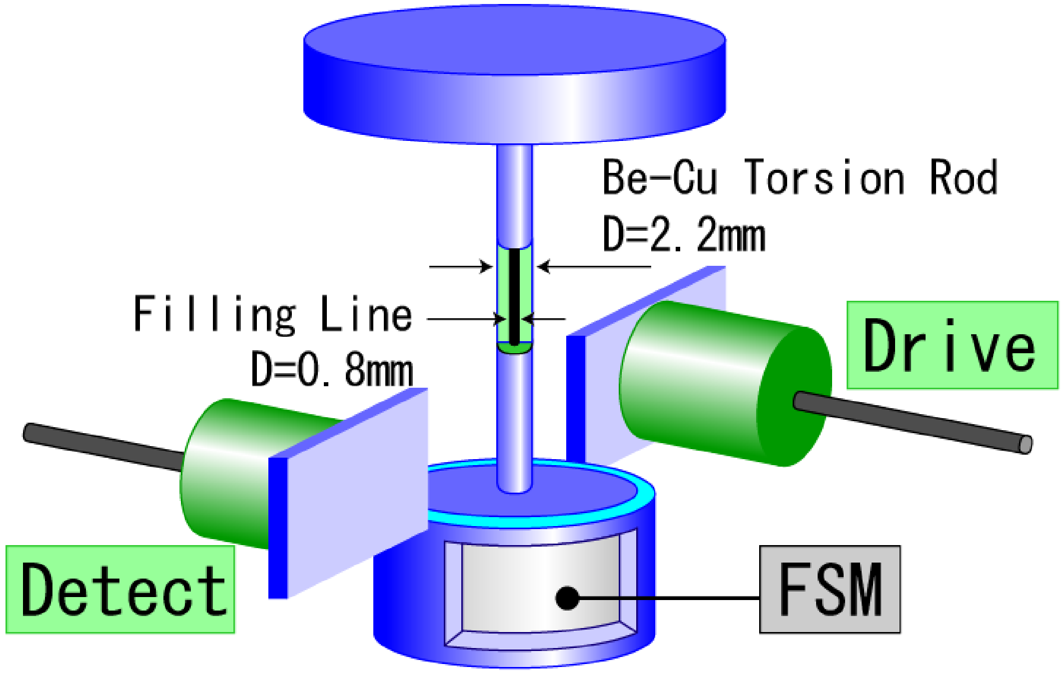
(As devised by JD Reppy)



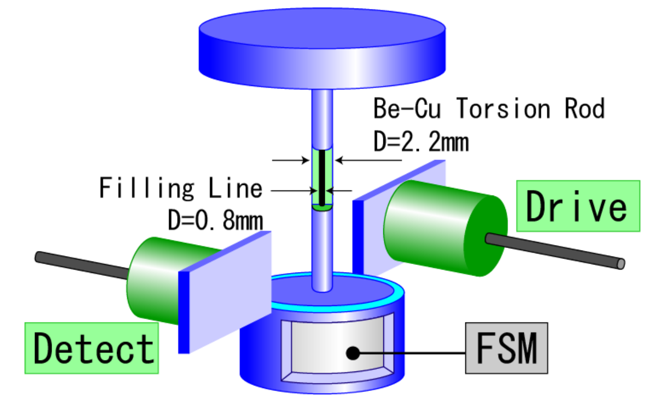
Andronikashvili's Experiment

(As suggested by Landau)





What is being probed by the TO?



Rate of change of angular momentum — *Torques*

$$\overbrace{\frac{d}{dt}\left[I_0\dot{\varphi}(t)+\mathcal{L}_z(t)\right]}^{\text{Rate of change of angular momentum}} = \overbrace{-k\varphi(t)-\eta\dot{\varphi}(t)+\tau_{\text{ext}}(t)}^{\text{Torques}}$$

Empty TO | *Normal Fluid* | *Restoring Torque* | *Friction* | *Driving Torque*

Linear response theory

$$\mathcal{L}_z(t) = \int d\mathbf{r}(\hat{\mathbf{z}}\times\mathbf{r})\cdot\langle\mathbf{\Pi}(\mathbf{r},t)\rangle$$

$$\mathbf{\Pi}(\mathbf{r}) = \frac{\hbar}{2i}\left[\Psi^\dagger(\mathbf{r})\nabla\Psi(\mathbf{r}) - \nabla\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\right]$$

TO response function

$$\chi_n(\omega;T) = \sum_{\mu,\nu}\int d\mathbf{r}d\mathbf{r}'\,(\hat{\mathbf{z}}\times\mathbf{r})_\mu(\hat{\mathbf{z}}\times\mathbf{r}')_\nu\,\chi_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega;T)$$

$$\chi_{TO}^{-1}(\omega) = \omega^2\left[I_0 - \boxed{\chi_n(\omega;T)}\right] - k + i\eta\omega.$$

$$\delta\omega(T) \Leftrightarrow \text{Re}\,\chi_n(\omega_0;T)$$

$$\delta Q^{-1}(T) \Leftrightarrow \text{Im}\,[-\chi_n(T;\omega_0)]$$

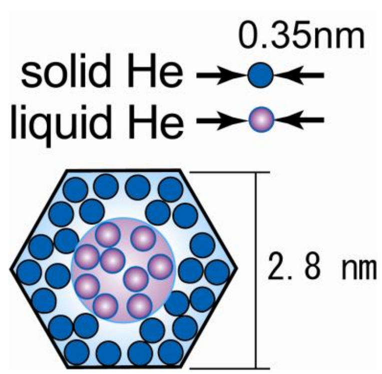
$$\omega_0 \approx 2000\text{ Hz}$$

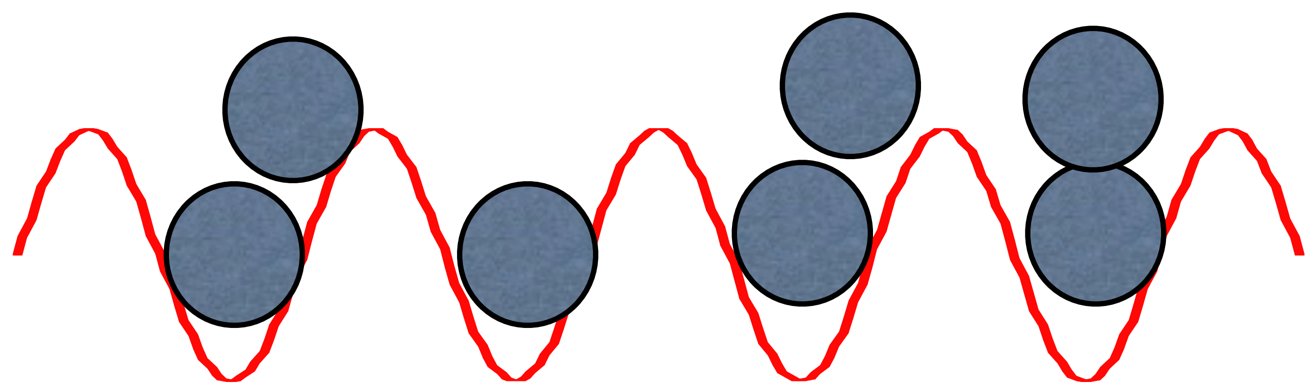
T Eggel, MAC & M Oshikawa PRL 2011

Momentum Response in d = 1

$$\chi(x,t) = -\frac{i}{\hbar}\theta(t)\,\langle[\Pi(x,t),\Pi(0,0)]\rangle$$

Model for the nano-pore potential: Periodic potential





$$H(t) = H_0 + \sum_{i=1}^{N} V_{\rm ext}(x_i - X(t))$$

$$H'(t) = H_0 + \sum_{i=1}^{N} V_{\rm ext}(x_i) - \dot{X}(t)\Pi$$

Calculation of momentum response akin to conductivity in Solid state/Mesoscopic Physics

T Eggel, MAC & M Oshikawa PRL 2011

Harmonic Fluid Description

FDM Haldane PRL (1981
MAC et al RMP (2011)

RG fixed point Hamiltonian (just phonons)

$$H_* = \sum_{q\neq 0} \hbar v |q|\, b^\dagger(q) b(q) + \cdots \frac{\hbar v}{2\pi} \int dx \left[K^{-1} \left(\partial_x \phi\right)^2 + K \left(\partial_x \theta\right)^2 \right] = \int dx\, \epsilon(x)$$

$$P = \sum_{q\neq 0} \hbar q\, b^\dagger(q) b(q) + \cdots = \frac{\hbar}{\pi} \int dx\, \partial_x \phi \partial_x \theta = \frac{1}{v^2} \int dx\, j_\epsilon(x)$$ $\propto$ *Energy current*

$$J = \frac{mvK}{\pi} \int dx\, \partial_x \theta(x,t) = \int dx\, j(x,t)$$ Particle mass current

Momentum current

(including the leading irrelevant operator)

$$\Pi = J + \frac{vK}{v_F} P$$

J and P separately conserved by the fixed-point Hamiltonian

$$[H_*, J] = [H_*, P] = 0$$

Phase slips and Memory matrix

Phase Slips (for a periodic wall potential) Leading irr. operators

$$H_{PS} = \sum_{n>0,m} \frac{\hbar v g_{nm}}{\pi a_0^2} \int dx \cos\left(2n\phi(x) + \Delta k_{nm} x\right) \qquad \Delta k_{mn} = (2n\pi\rho_0 - 2mG)$$

$$[H_{PS}, J] \neq 0 \qquad [H_{PS}, P] \neq 0$$

J and P are coupled and acquire a finite decay rate

$$\chi(\omega;T) = \mathrm{Tr}\left\{ V \left[\omega \mathbf{1} + iM(\omega;T)\right]^{-1} iM(\omega;T) \hat{\boldsymbol{\chi}}(T) \right\}$$

$$\hat{\boldsymbol{\chi}}(T) \simeq \mathrm{diag}\{\chi_{JJ}, \chi_{PP}(T)\} = -\mathrm{diag}\{\frac{M^2 v K}{\hbar\pi}, \frac{\pi (k_B T)^2}{6\hbar v^3}\} + \cdots$$

$M(\omega,T)$ *is a 2 x 2 matrix whose eigenvalues are the current decay rates (it can be evaluated perturbatively in H_{PS})*

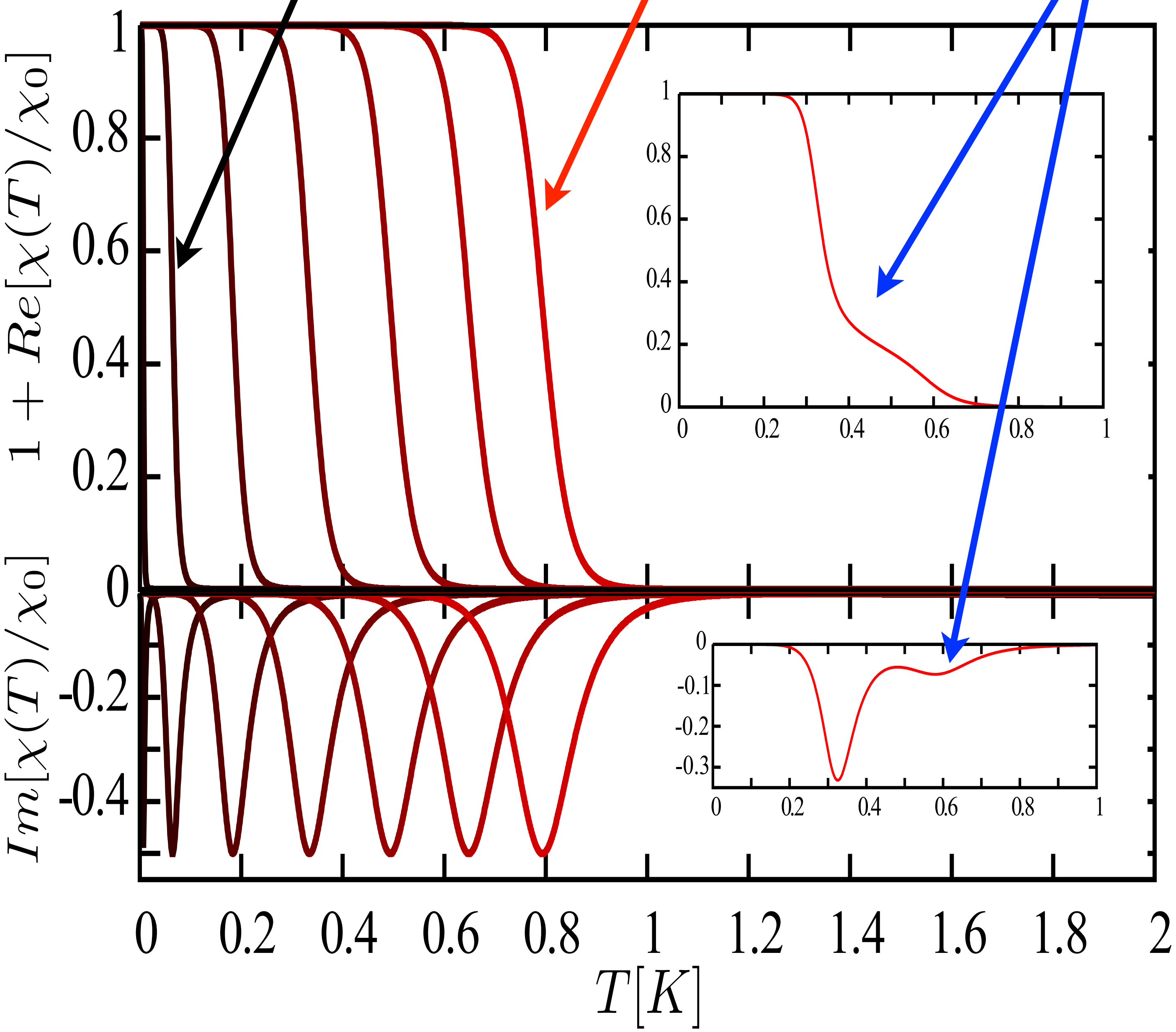
Results

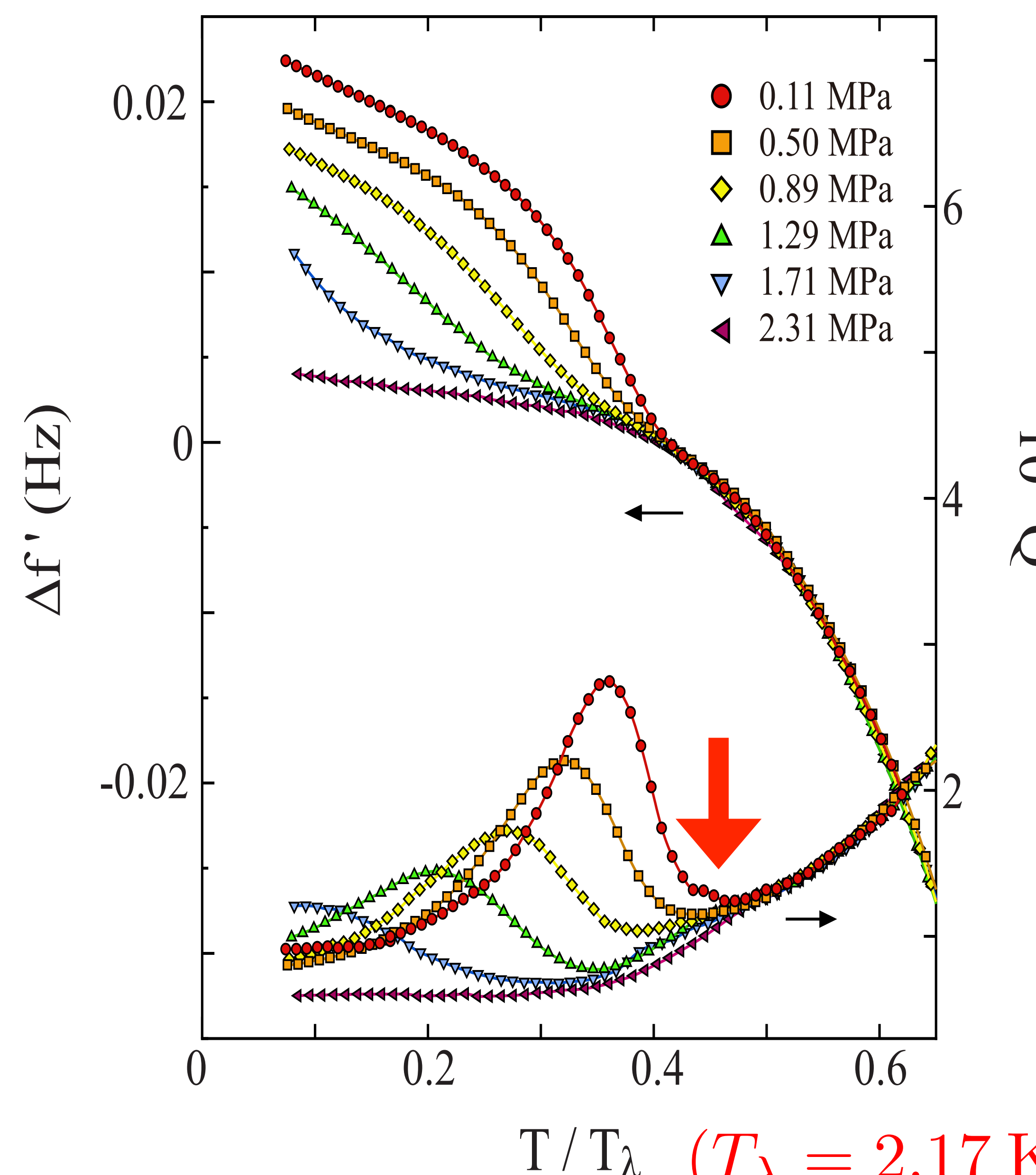
Luttinger parameter dependence (Compressibility ∝ K)

$K = 3.2$ $K = 10.2$

Double onset/peak structure

J Taniguchi et al PRB 2010



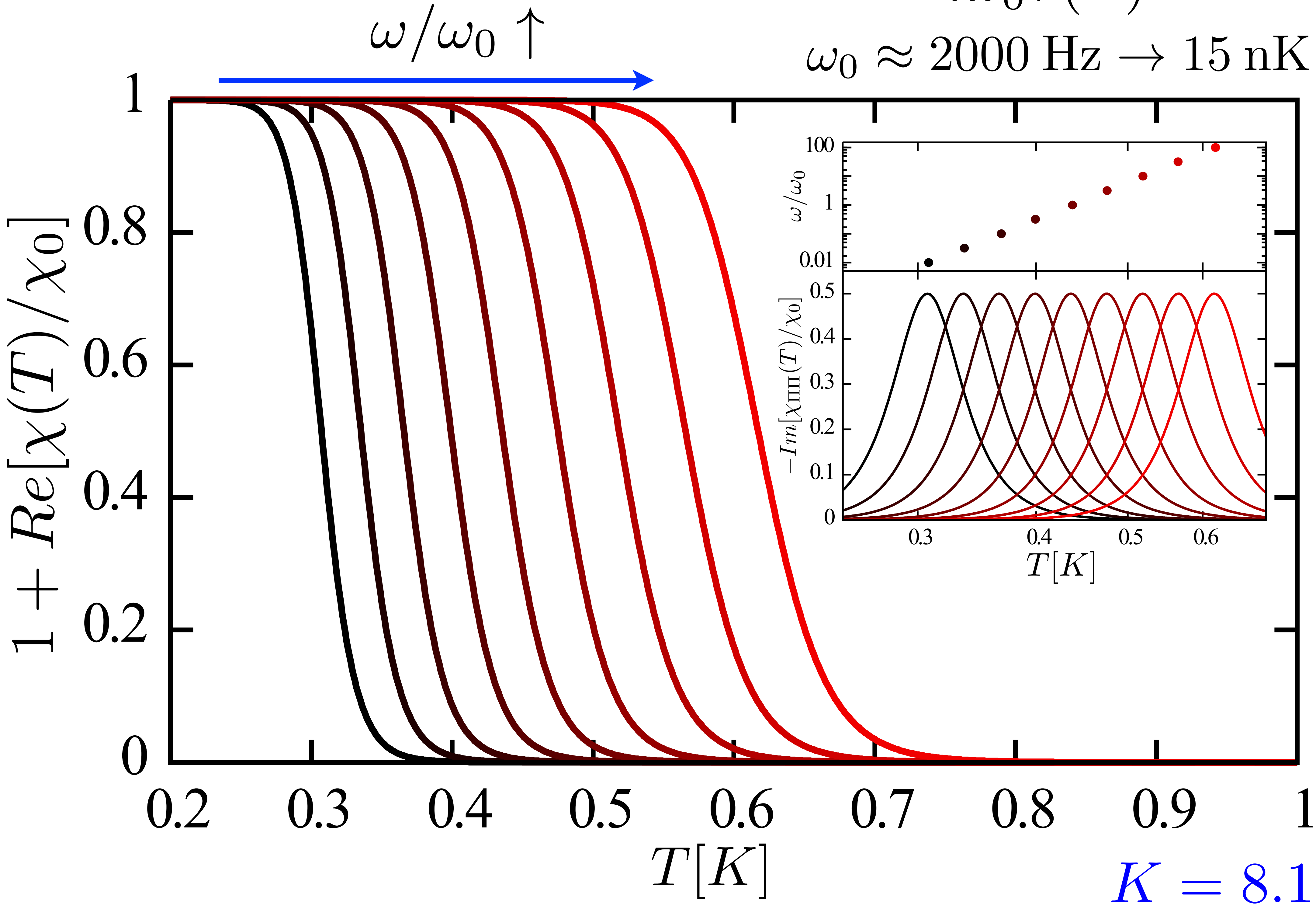


$(T_\lambda = 2.17\,\mathrm{K})$

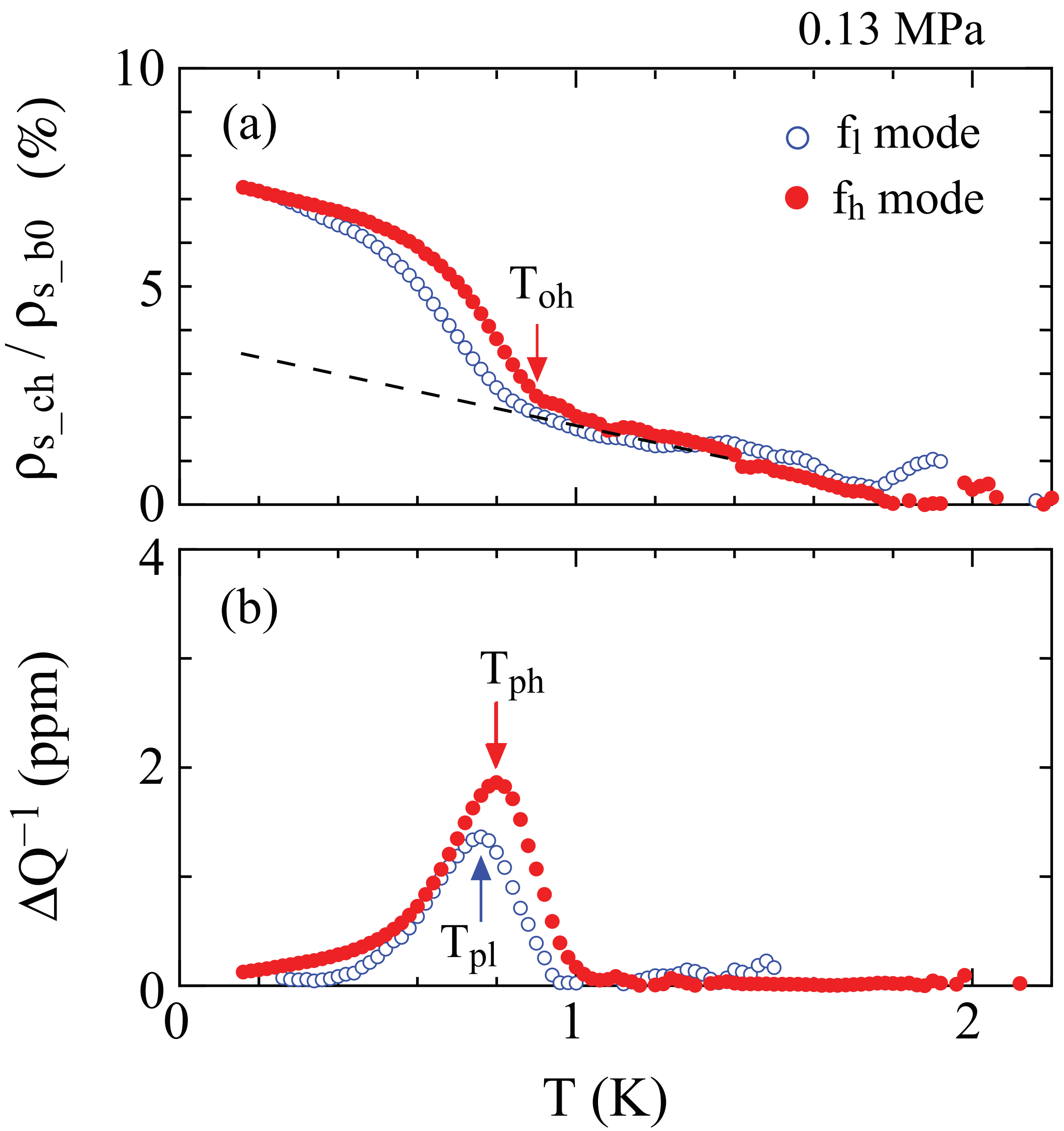
Prediction: Frequency dependence

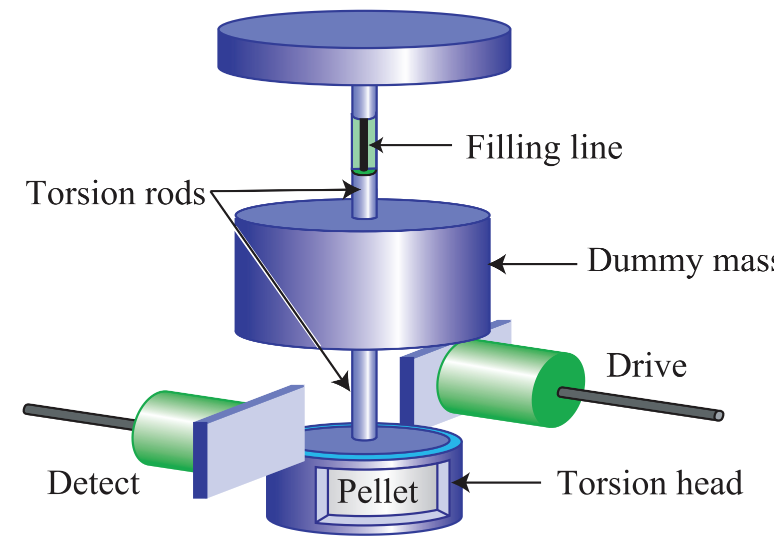
Simplified model $\chi(\omega_0;T) \approx -\frac{|\hat{\chi}(T)|}{1 - i\omega_0\tau(T)}$ $\tau(T_{\mathrm{onset}}) \sim \frac{1}{\omega_0}$

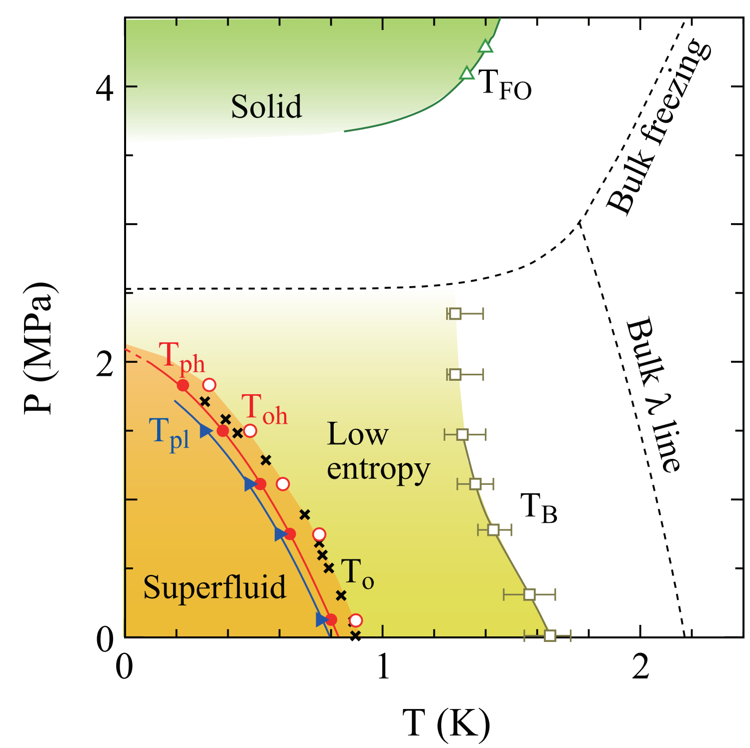
$\omega_0 \approx 2000\ \mathrm{Hz} \rightarrow 15\ \mathrm{nK}$



Experiment: Frequency dependence







$f_l = 500\,\mathrm{Hz}$ $\quad$ $f_h = 2000\,\mathrm{Hz}$

J Taniguchi et al PRB 2013

Conclusions (part I)

- *The helicity modulus in 1D vanishes*
- *Superfluidity is a dynamical effect in 1D*
- *Importance of Phase slips*
- *Importance of coupling between particle and energy currents*

Quantum Quenches:

From the generalized Gibbs Ensemble to Prethermalization

Miguel A. Cazalilla NTHU, Taiwan.

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What Quadratic Hamiltonians can teach us about non-equilibrium

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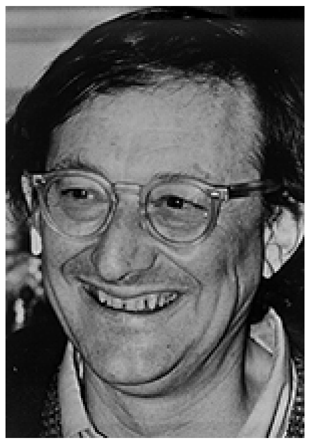
Why to bother with “boring” quadratic Hamiltonians?

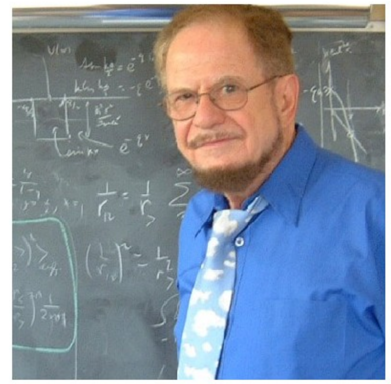
RG Fixed-point Hamiltonians at Equilibrium

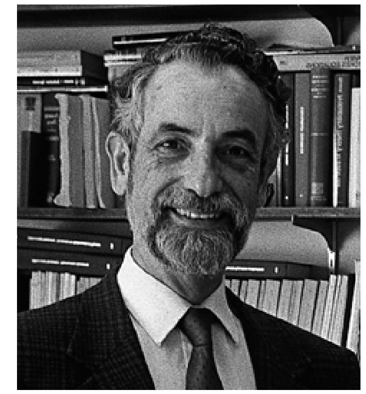
The Luttinger Model

Luttinger



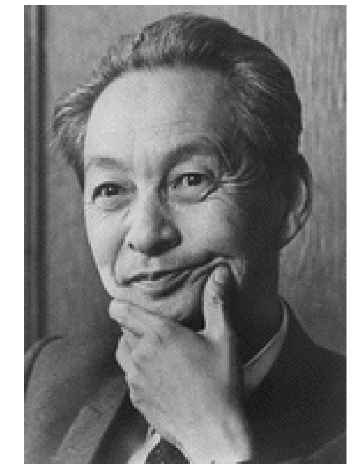
Mattis & Lieb



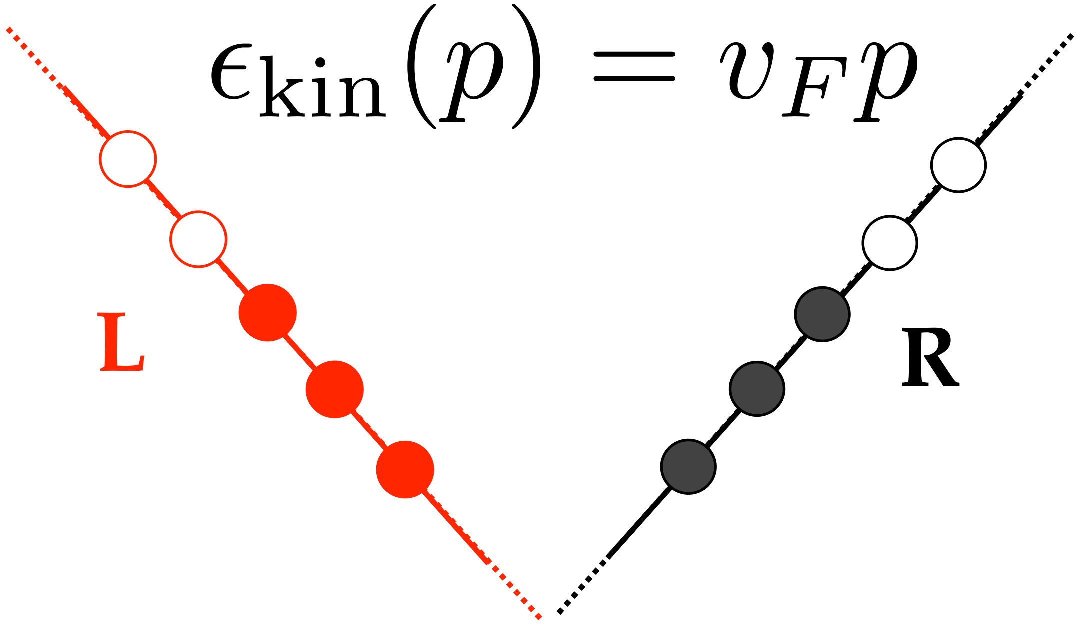


[J. Math. Phys. (1965)]

朝永



$$\epsilon_{\text{kin}}(p) = v_F p$$



"Infinite story hotel" $|0\rangle_{\text{Dirac}}$

Quasi-particle: Tomonaga bosons

$$H_{\text{LM}} = \sum_{q \neq 0} \hbar v |q| b^\dagger(q) b(q)$$

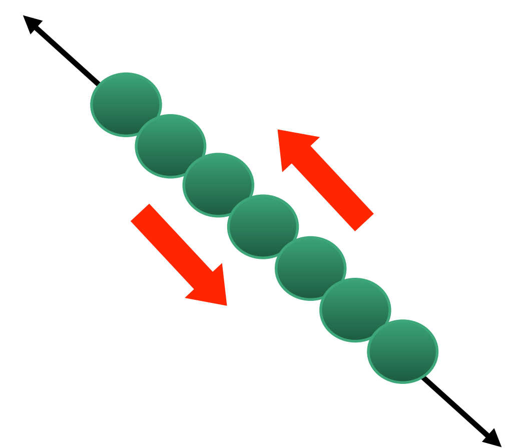
'Anomalous' commutation relations

$$[\rho_R(q), \rho_R(-q')] = \frac{qL}{2\pi} \delta_{q,q'}$$

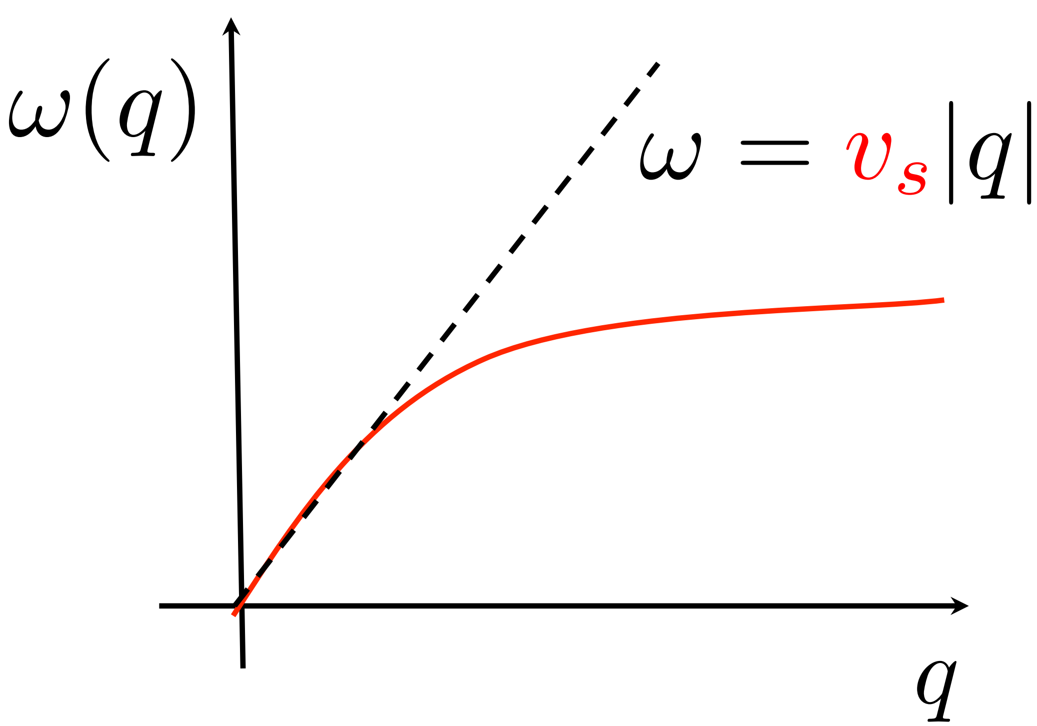
One Dimension: The Tomonaga-Luttinger Liquid

Luttinger *Mattis* *Lieb* *朝永* *Luther* *Emery* *Peschel* *Haldane*

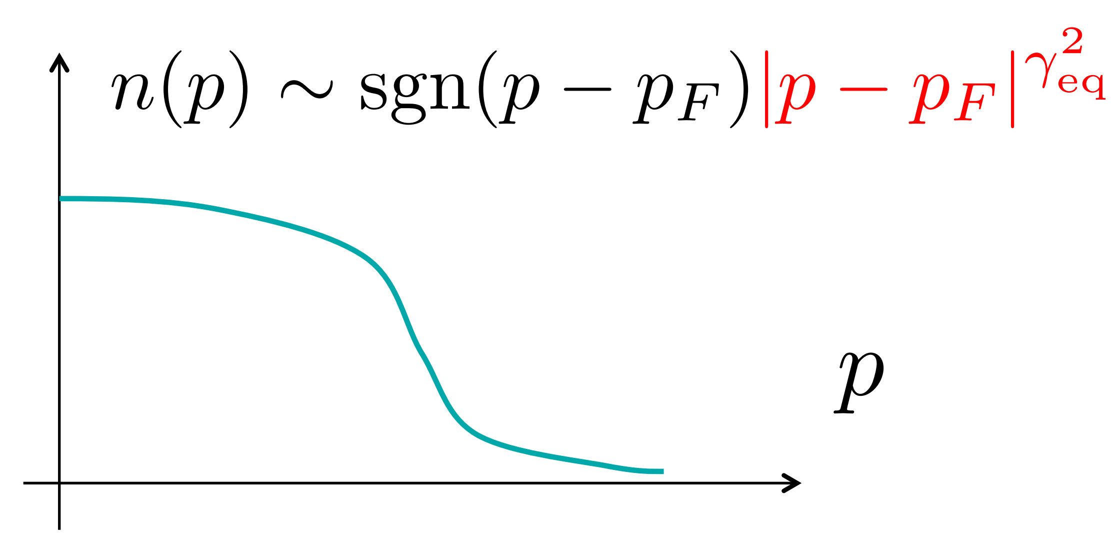
(There are more, but I simply couldn't fit in every one…)



Collective modes exhaust the low-energy spectrum

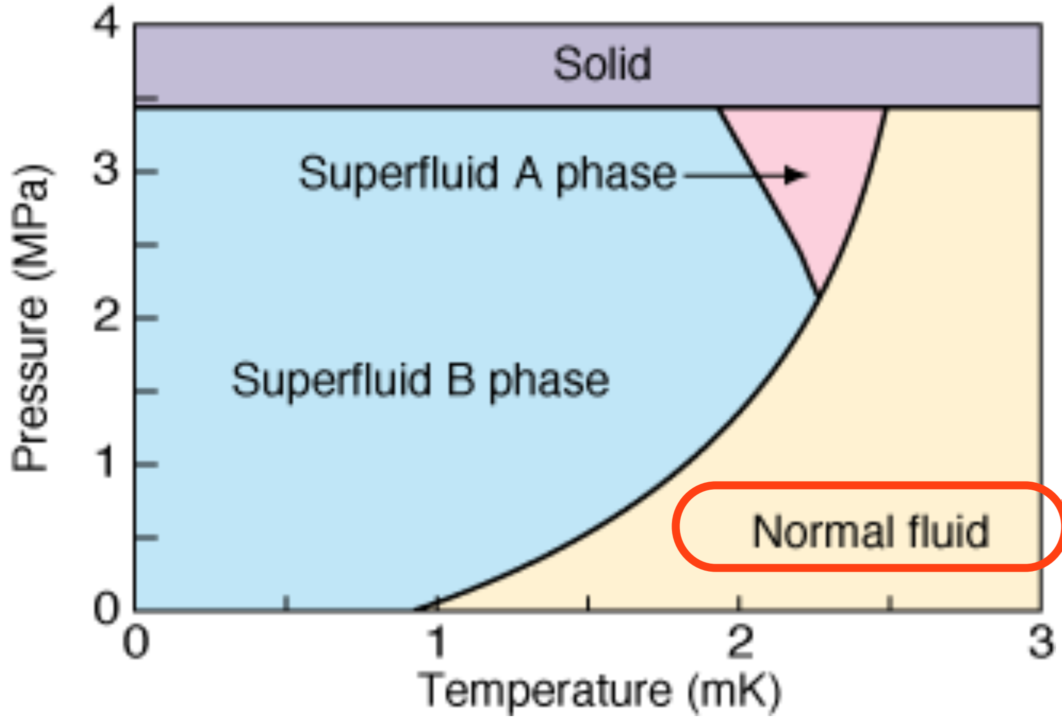


Power-law Momentum distribution

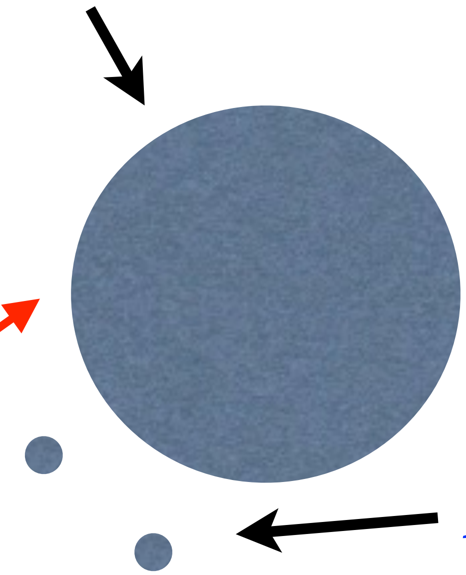


Fermi Liquid Theory

Helium 3 Phase diagram



Fermi Surface (FS)



Landau quasi-particles: form a dilute gas

Quasi-particle occupation

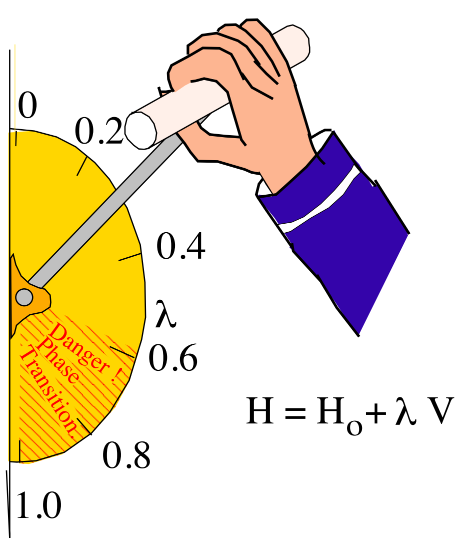
$$n_\sigma(\boldsymbol{p})$$

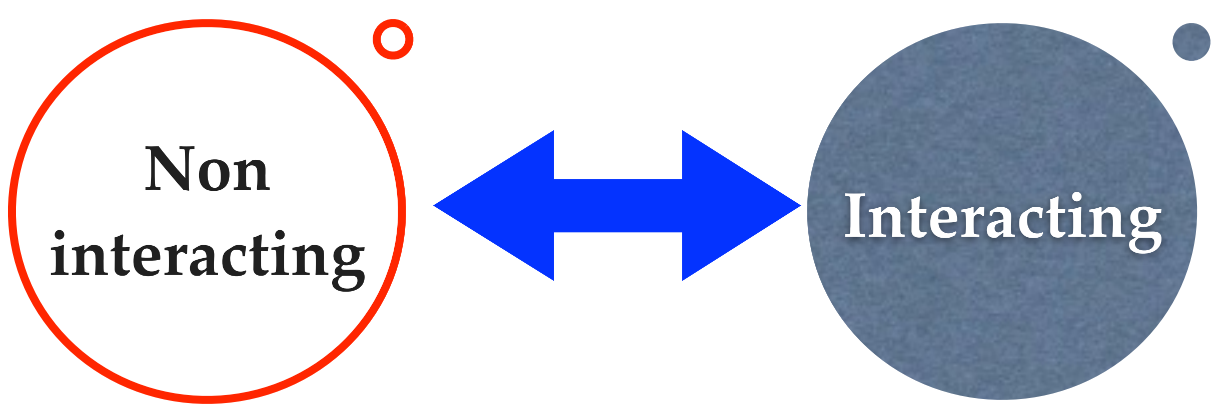
is a good quantum number

Quasi-particle scattering rate:

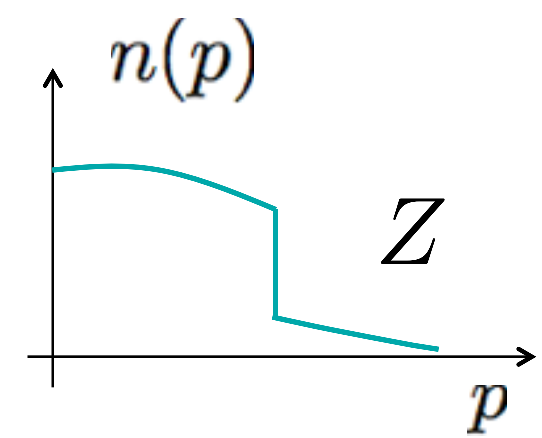
$$\Gamma_{\rm QP} \sim \epsilon^2 + \pi^2 T^2$$

Adiabatic Continuity





Momentum distrib.



Landau Fermi Liquid

Quasi-particle Hamiltonian

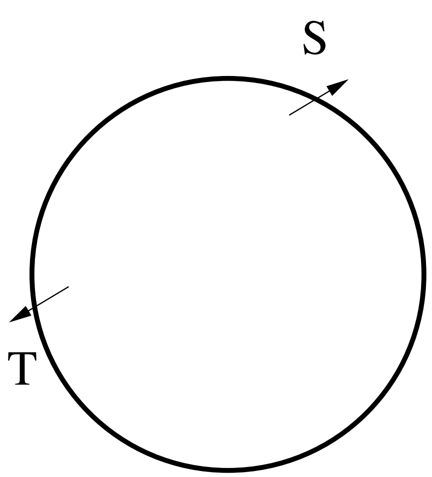
Forward scattering interactions

$$H_{FL} = \sum_{\boldsymbol{k}} \epsilon(\boldsymbol{k}) q^\dagger_{\boldsymbol{k}} q_{\boldsymbol{k}} + \frac{1}{2V} \sum_{\boldsymbol{kpp}} F_{\boldsymbol{pk}}(\boldsymbol{q}) q^\dagger_{\boldsymbol{k+q}} q^\dagger_{\boldsymbol{p-q}} q_{\boldsymbol{p}} q_{\boldsymbol{k}}$$

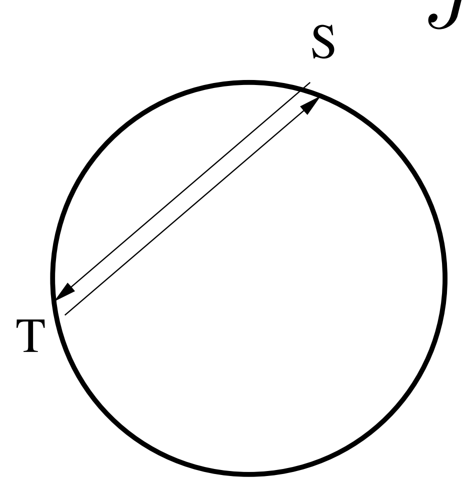
Single Quasi-particle energy

Bosonization of the Fermi Surface

Haldane, Houghton & Marston
Castro Neto & Fradkin,
Kim & Wen & Lee, …



Forward



Exchange

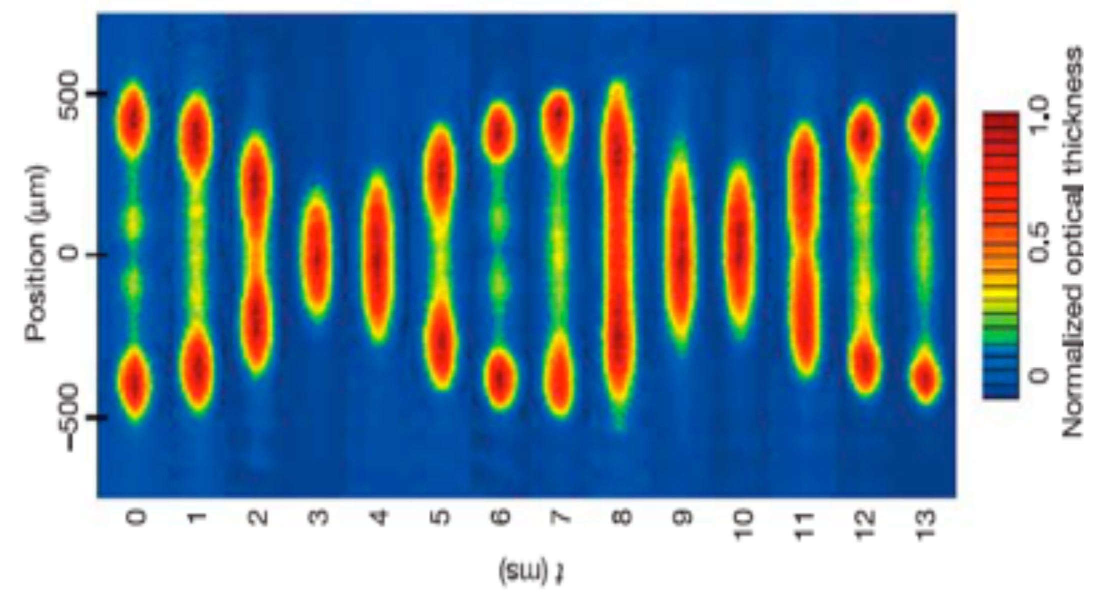
$$J_{\boldsymbol{S}}(\boldsymbol{q}) \sim \sum_{\boldsymbol{k} \in \boldsymbol{S}} q^\dagger_{\boldsymbol{k+q}} q_{\boldsymbol{k}}$$

$$\left[J_{\boldsymbol{S}(\boldsymbol{q}),J_{\boldsymbol{T}}(\boldsymbol{p})}\right] = \delta_{\boldsymbol{S},\boldsymbol{T}} \delta_{\boldsymbol{p+q},\boldsymbol{0}}\, \hat{\boldsymbol{n}}_{\boldsymbol{S}} \cdot \boldsymbol{q}$$

$$H = \frac{1}{2} \sum_{\boldsymbol{S},\boldsymbol{T}} \left[\frac{v_F}{\Omega} \delta_{\boldsymbol{S},\boldsymbol{T}} + \frac{F_{\boldsymbol{S},\boldsymbol{T}}(\boldsymbol{q})}{V} \right] J_{\boldsymbol{T}}(\boldsymbol{q})$$

Eigenmodes $$H = \sum_{l,\boldsymbol{q}} \omega(\boldsymbol{q}) \alpha^\dagger(\boldsymbol{q}) \alpha_l(\boldsymbol{q})$$

What can we learn about Non-Equilibrium from Quadratic models?



T Tinoshita et al Nature (2006)

Sudden Quantum Quenches

State prep — ρ_0 — $t = 0$

Unitary evolution — $e^{-iHt/\hbar}$

Measurement — $\rho(t)$ — $t > 0$

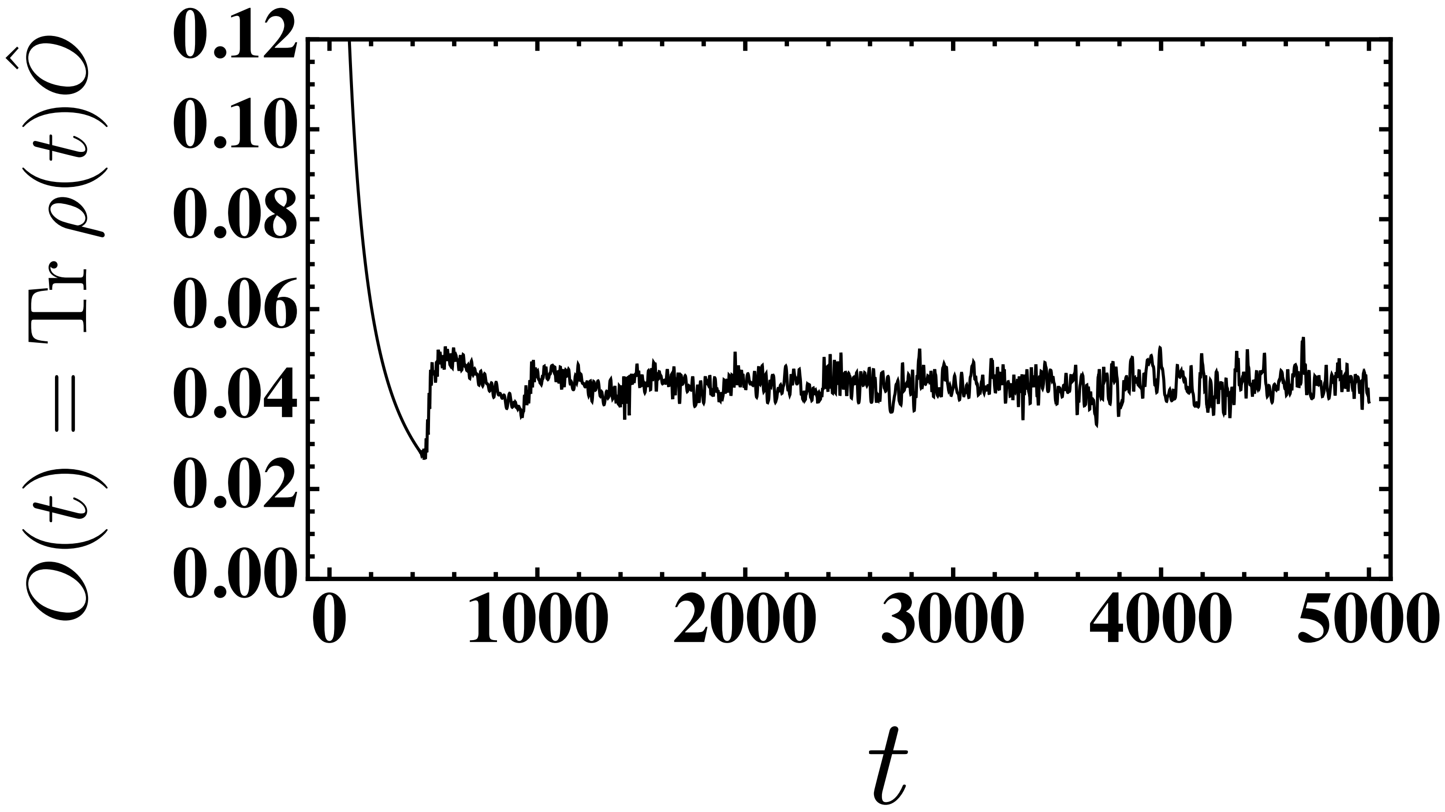
Some Important Questions

Does the system reach a steady state?

$$\overline{O} = \lim_{T\to+\infty} \frac{1}{T} \int_{t_0}^{t_0+T} dt\, O(t)$$

$$O(t) = \mathrm{Tr}\, \rho(t)\hat{O}$$

$$\hat{O} = n(\mathbf{r}), \Psi^\dagger(\mathbf{r})\Psi(\mathbf{0}), \ldots$$

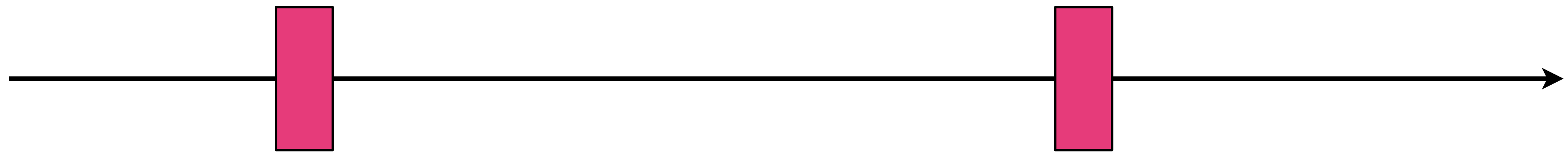


If so, what are its properties? Does it thermalize?

$$\bar{O} = \mathrm{Tr}\, \rho_{\mathrm{steady}}\hat{O},$$

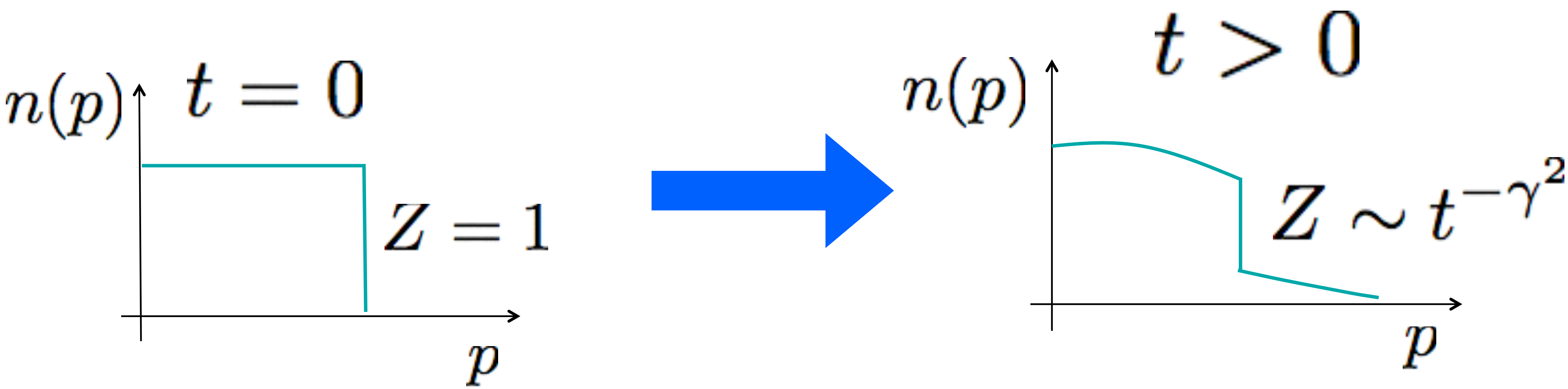
$$\rho_{\mathrm{steady}} \propto e^{-H/T_{\mathrm{eff}}} \;?$$

Quantum Quench in the LM



$$H_{\rm int} = 0 \;\Rightarrow\; H_{\rm int} \neq 0$$

Momentum distribution at time t :



MAC Phys Rev Lett (2006)

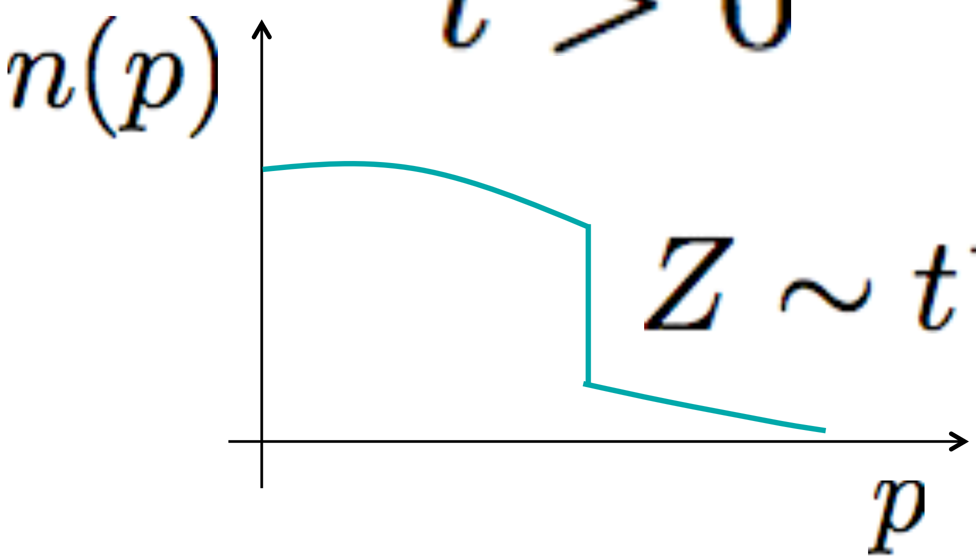
A Iucci & MAC Phys Rev A (2009)

Does this work lattice models?

Model: XXZ + NN int

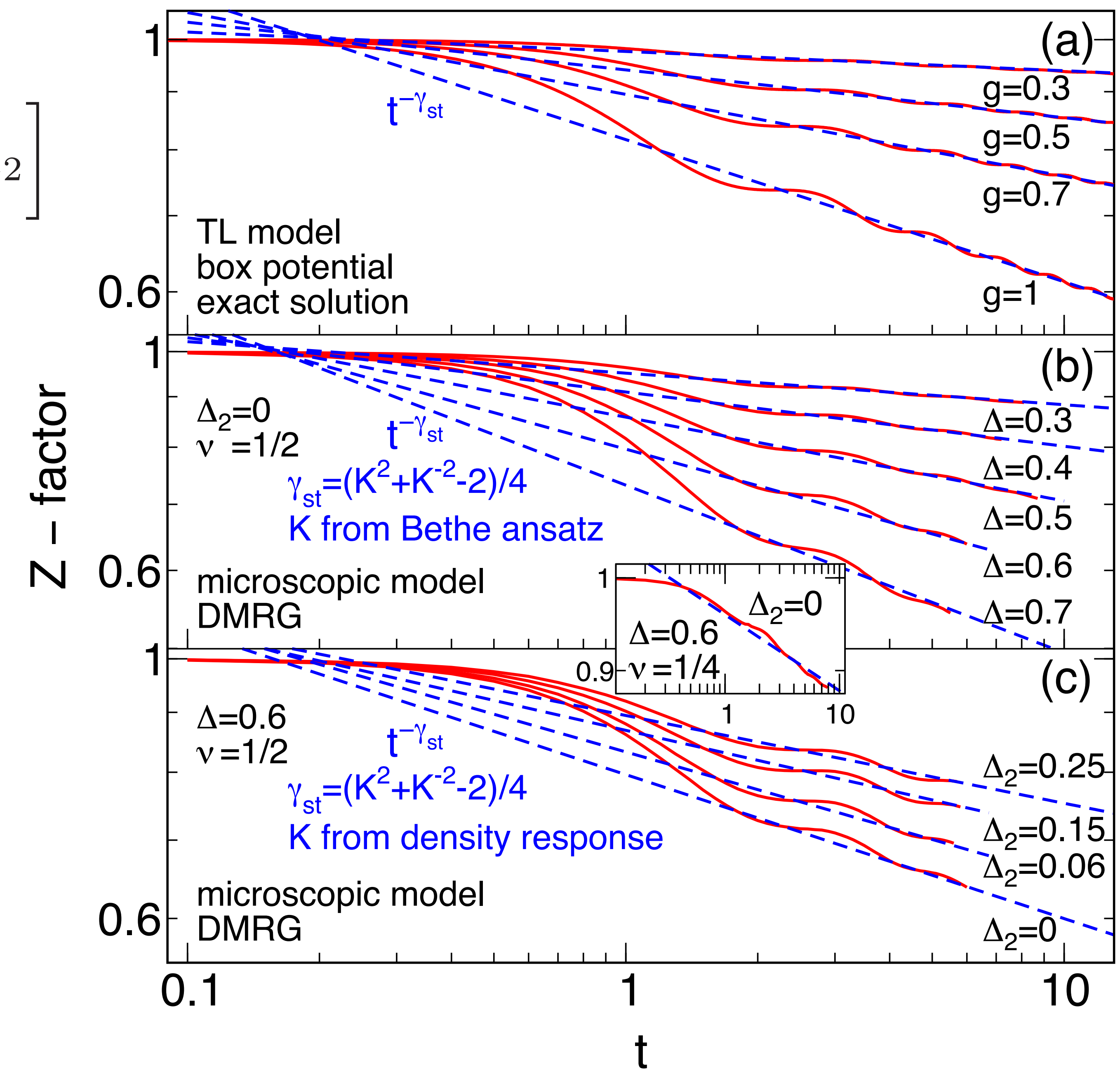
$$H = \sum_j \left[\frac{1}{2} c_j^\dagger c_{j+1} + \text{H.c.} + \Delta n_j n_{j+1} + \Delta_2 n_j n_{j+2} \right]$$

$$t > 0$$

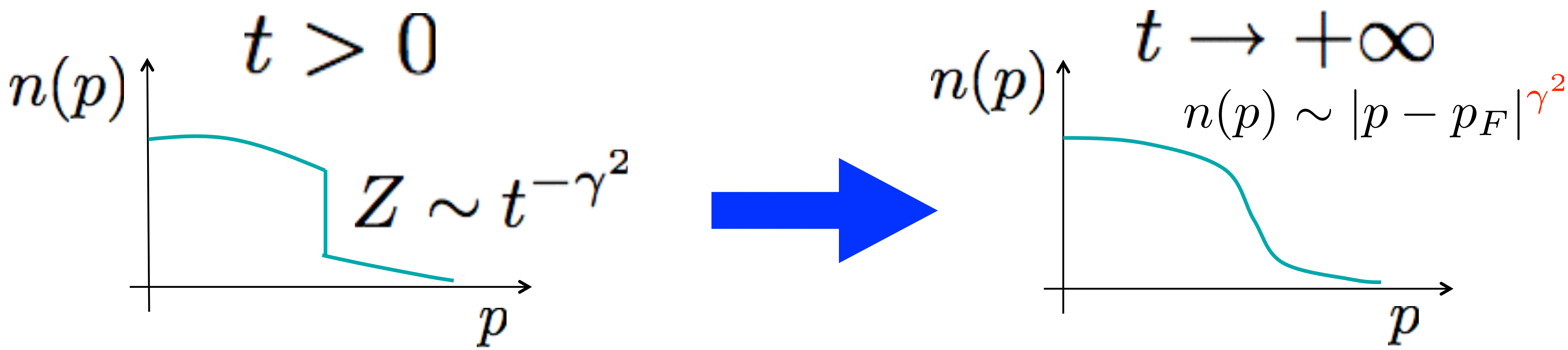


$$Z \sim t^{-\gamma^2}$$

C Karrasch et al PRL (2012)



Where does the system go?



Non-equilibrium exponent : $\gamma > \gamma_{eq}$

The system does not thermalize! Why?

Infinite number of conserved quantities!!

$$[H, I(q)] = 0 \qquad I(q) = b^\dagger(q) b(q)$$

The GGE Conjecture

M Rigol, B Dunjko, V Yurovsky, and M Olshanii PRL (2007)

Apply the Maximum Entropy Principle

[E.T. Jaynes, PR (1957)]

$$\bar{O} = \lim_{t\to+\infty} \langle \Psi(t)|\hat{O}|\Psi(t)\rangle = \mathrm{Tr}\rho_{GGE}\,\hat{O},$$

$$\rho_{GGE} = \frac{e^{\sum_k \lambda_k I(k)}}{Z_{GGE}}, \quad \langle I(k)\rangle_{GGE} = \langle \Psi(t=0)|I(k)|\Psi(t=0)\rangle$$

Need Integrals of Motion $[H, I(k)] = 0$

Luttinger Model Integrals of Motion $I(k) = b^\dagger(k)b(k)$

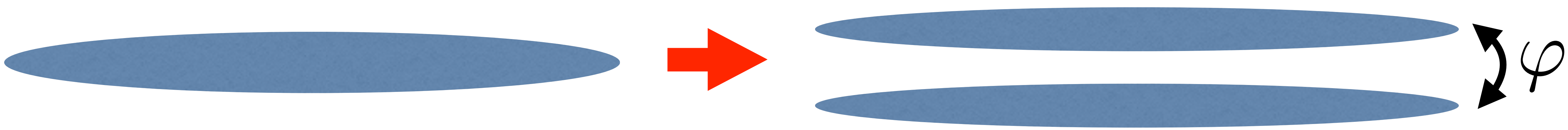
But only O(N) integrals are needed!

Why?

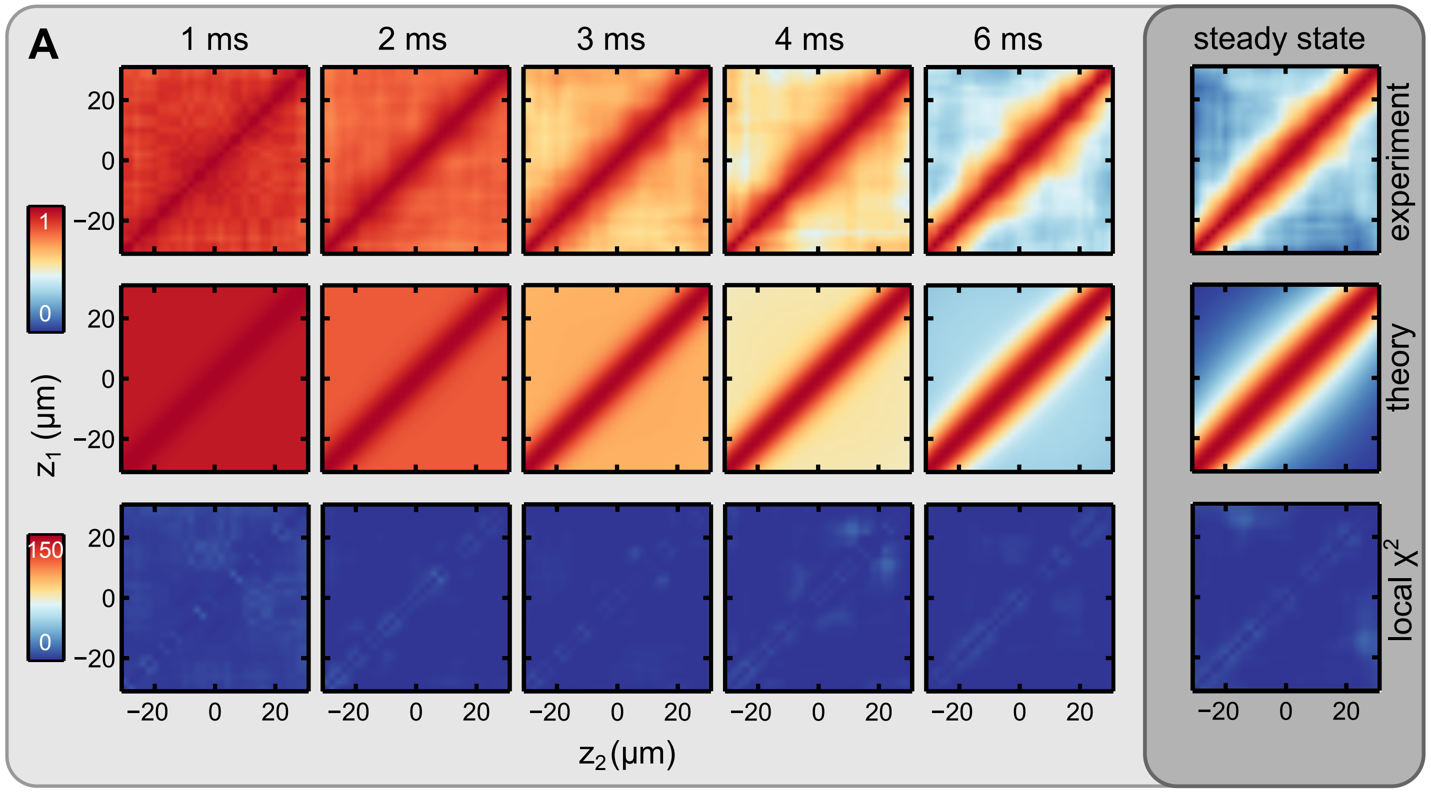
MAC, A Iucci, MC Chung PRE (2012)

Experimental Observation of GGE

Sudden Splitting of a 1D Bose gas

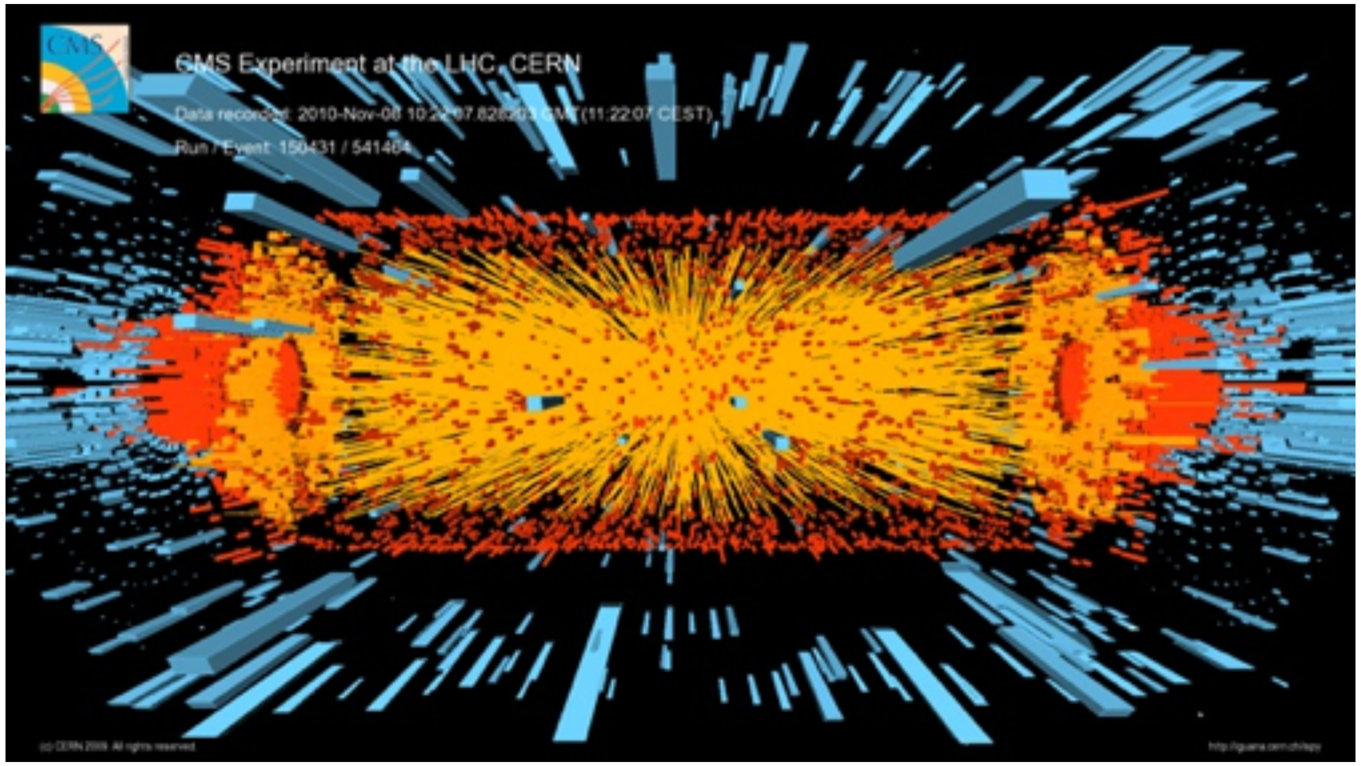


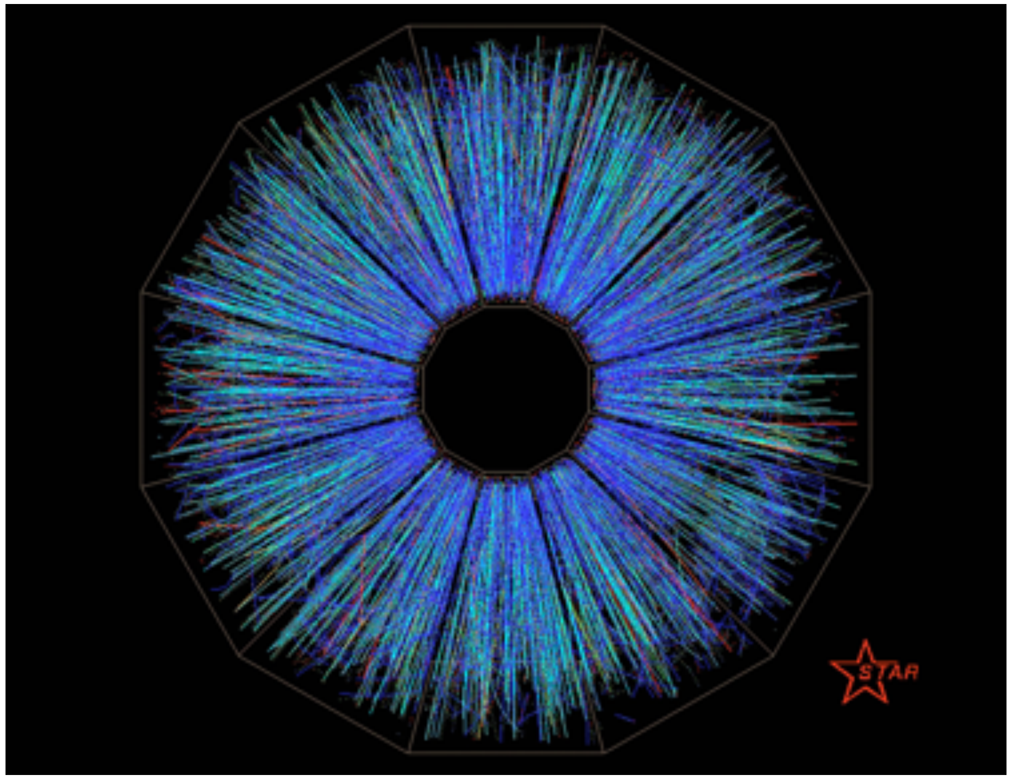
Dynamics of the relative phase $C(x,x') = \langle e^{i\varphi(x)} e^{-i\varphi(x')} \rangle$



T Langren et al arxiv:1411.7185 (2014)

Pre-thermalization in d > 1?





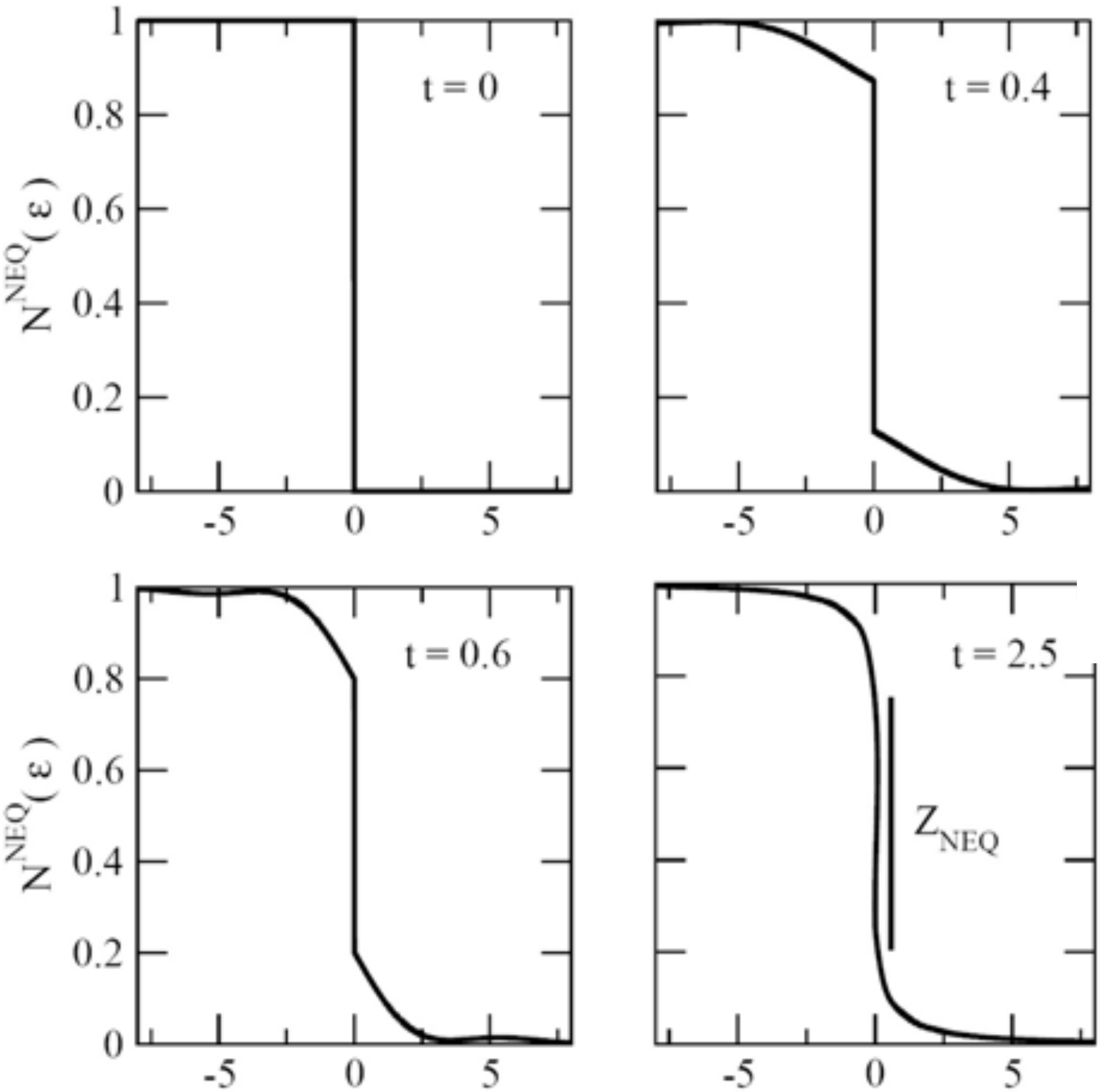
Prethermalization [...] describes the very rapid establishment of [..] a kinetic temperature ***based*** *on average kinetic energy [...] the occupation numbers of individual momentum modes still show strong deviations from the late-time Bose-Einstein or Fermi-Dirac* **distribution.**

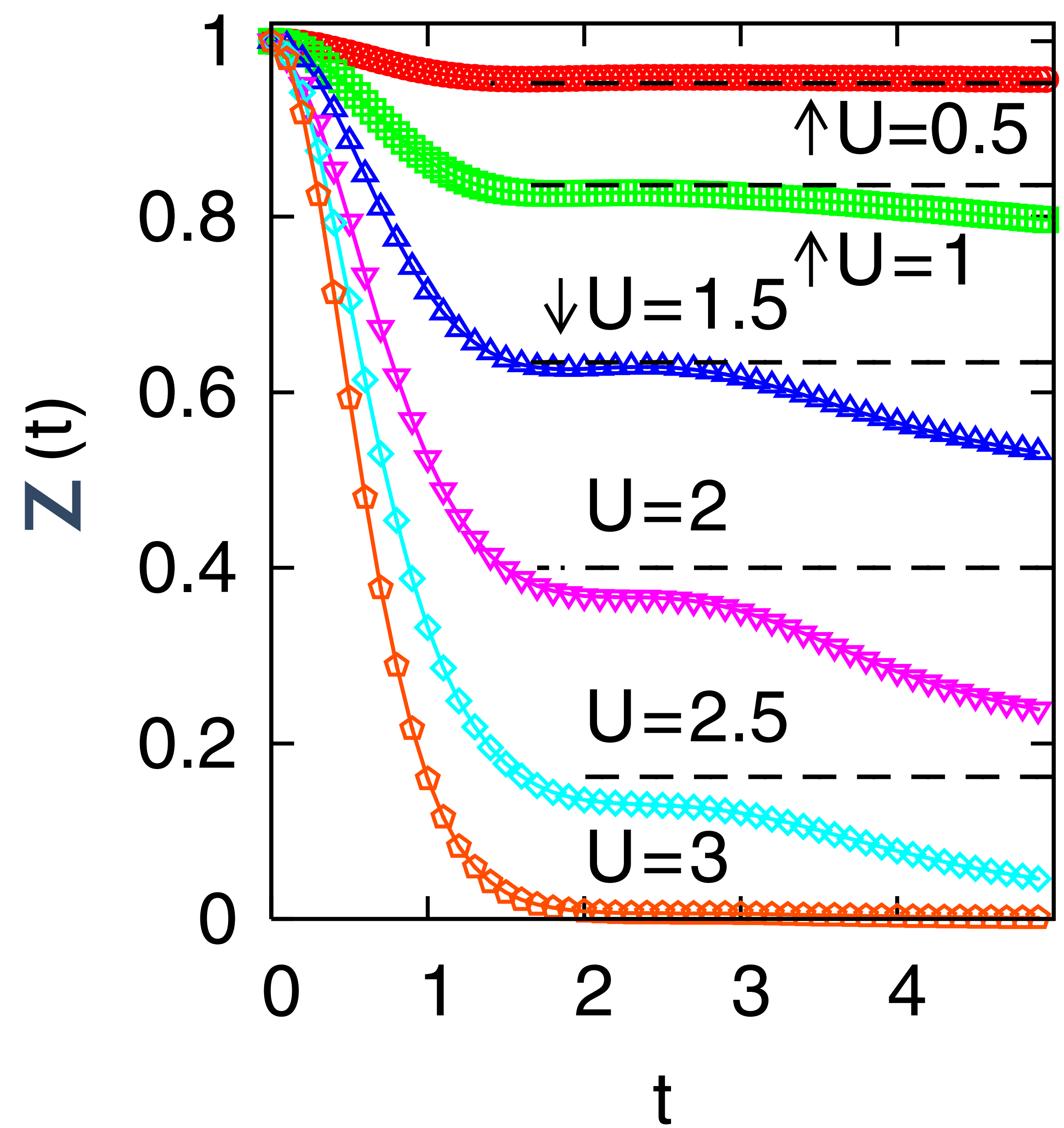
J Berges et al Phys Rev Lett 2004

Prethermalization in the Hubbard Model

$$H = \sum_{k,\sigma} \epsilon(k) c_k^\dagger c_k + \frac{U}{V}\theta(t) \sum_{kpq} c^\dagger_{k+q\uparrow} c^\dagger_{p-q\downarrow} c_{p\downarrow} c_{k\uparrow}$$

M Eckstein, M Kollar, &P Werner PRL (2009)





$$1 - Z_{\text{neq}} = 2(1 - Z_{\text{eq}})$$

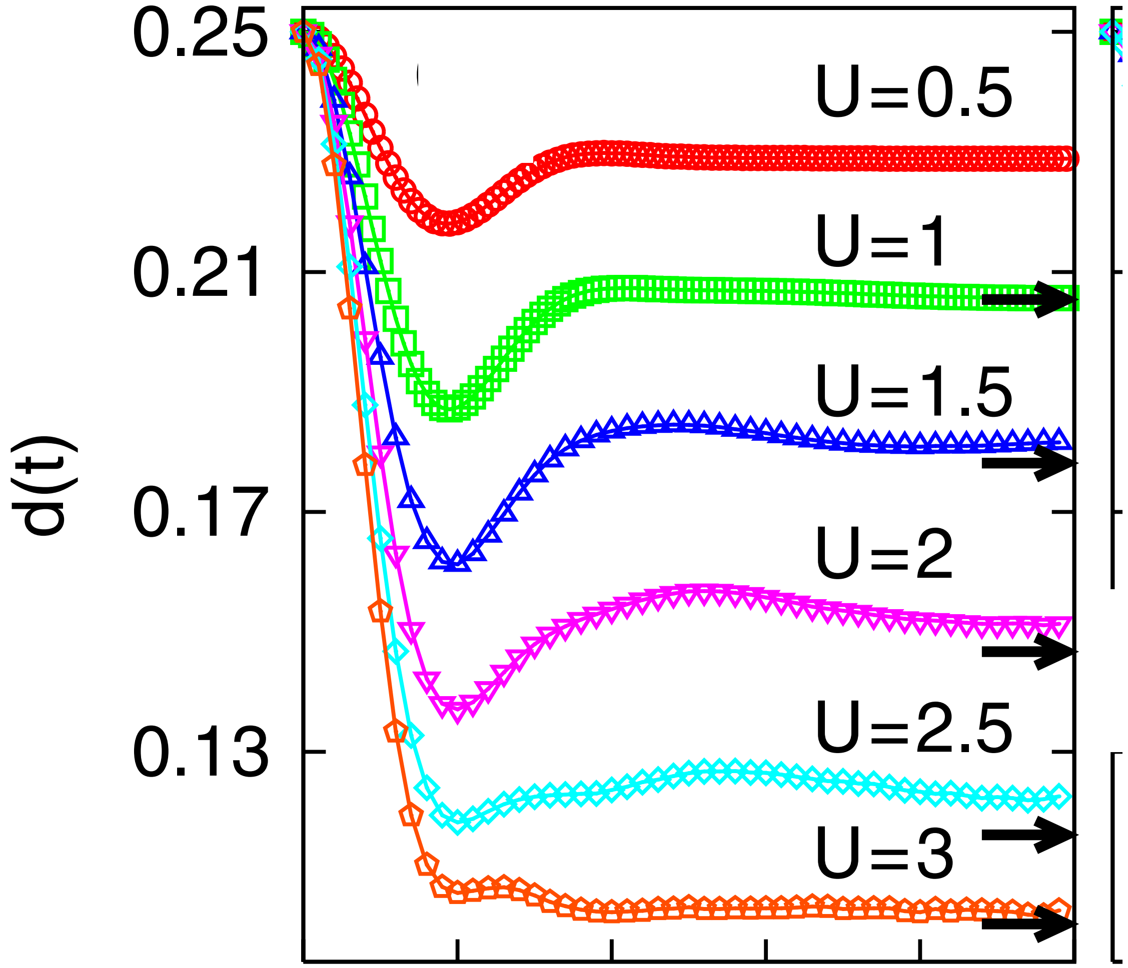
M Moeckel & S Kehrein PRL (2006)

Prethermalization in the Hubbard Model

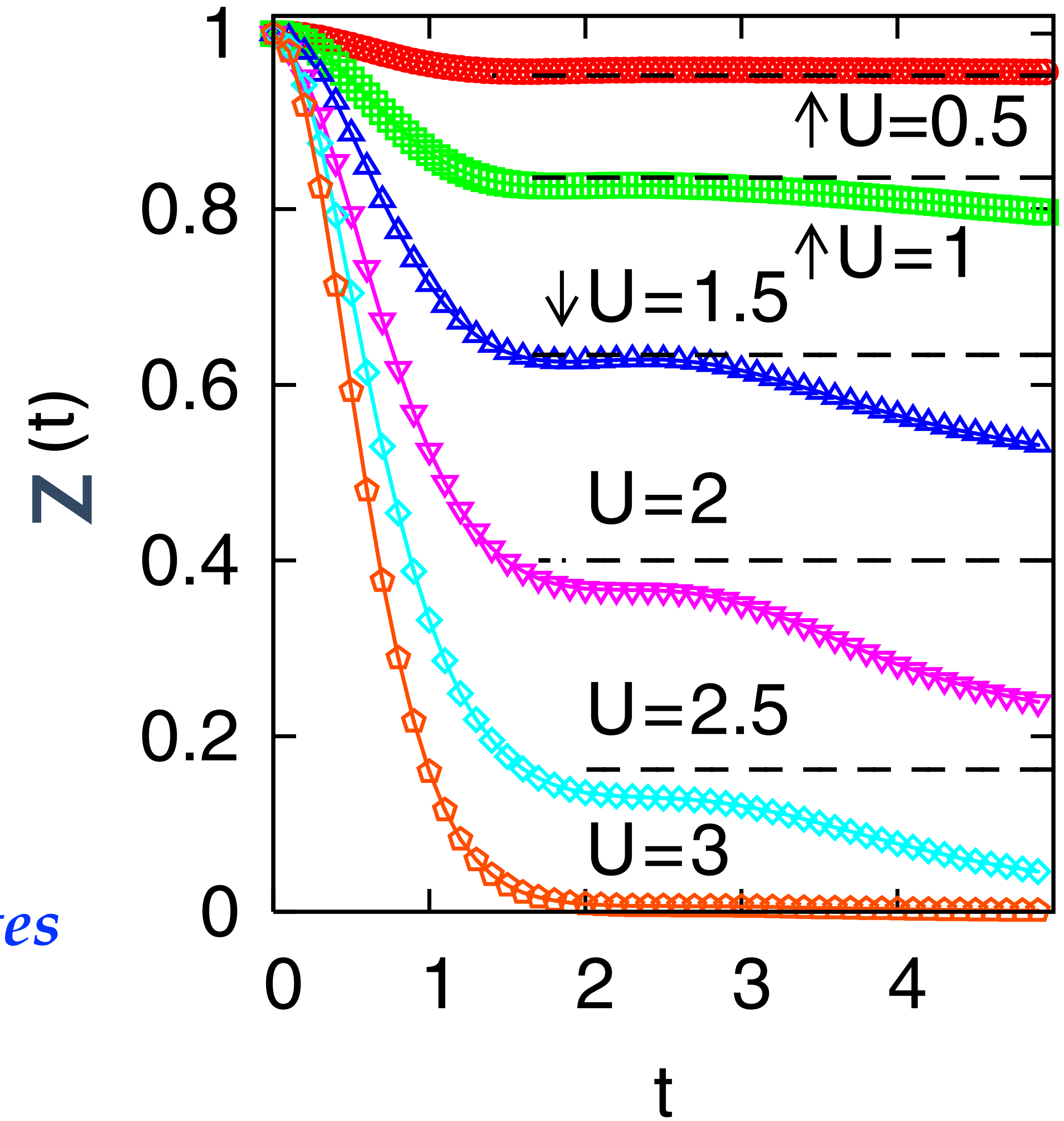
(in infinite dimensions)

Double Occupancy

M Eckstein, M Kollar, &P Werner PRL (2009)



Discontinuity at k_F

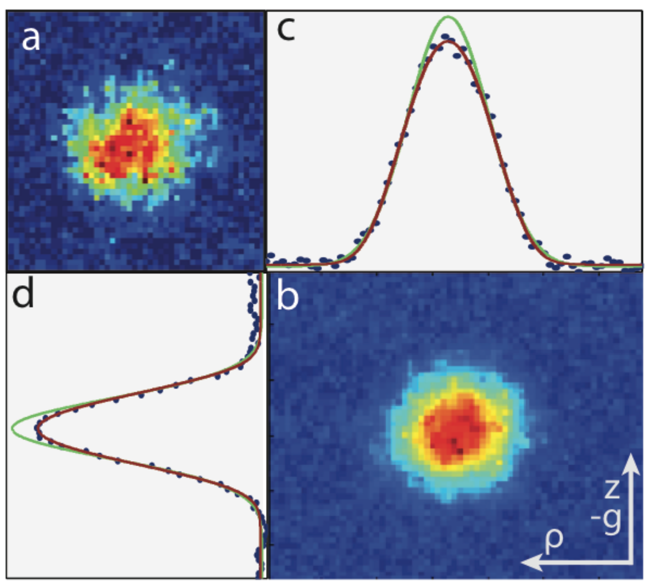


Kinetic energy rapidly equilibrates

$$d(t) = Un_{i\uparrow}(t)n_{i\downarrow}(t)$$

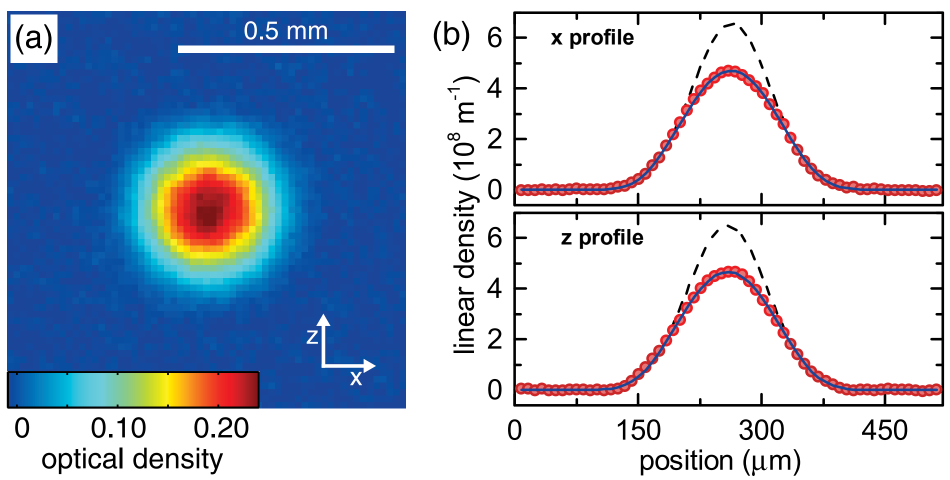
$$E = \langle K(t) \rangle + UNd(t) = \text{const.}$$

Pre-thermalization in a 2D Fermi gas with long range interactions



Degenerate ^{161}Dy Fermi gas

M Lu et al Phys Rev Lett (2012)

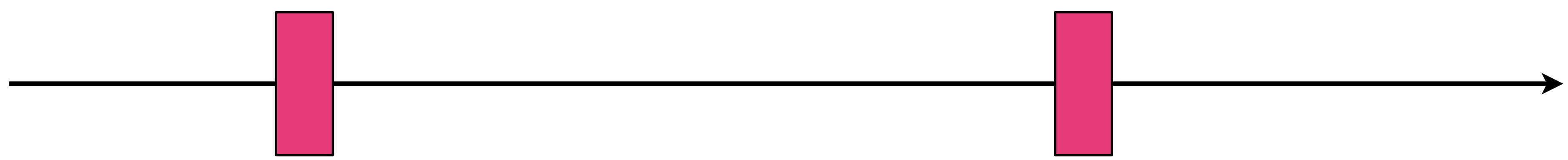


Degenerate ^{167}Er Fermi gas

K Aikawa et al Phys Rev Lett (2014)

Quench in a 2D interacting Fermi Gas

N Nessi, A Iucci & MAC, Phys. Rev. Lett (2014)



$$H_{\rm int} = 0 \quad \Rightarrow \quad H_{\rm int} \neq 0$$

Hamiltonian for t ≤ 0

$$H_0 = \sum_{\boldsymbol{k}} \epsilon(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}}$$

Hamiltonian for t > 0

$$H = H_0 + H_{\rm int} = \sum_{\boldsymbol{k}} \epsilon(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}} + \frac{1}{V} \sum_{\boldsymbol{kpq}} f(q) c^\dagger_{\boldsymbol{k}+\boldsymbol{q}} c^\dagger_{\boldsymbol{p}-\boldsymbol{q}} c_{\boldsymbol{p}} c_{\boldsymbol{k}}$$

Long-range (non-singular) interaction $q_c^{-1} \gg k_F^{-1}$

$$f(q) = f_0 F(q) \quad F(q \gg q_c) \sim e^{-q/q_c} \quad F(q=0) = \text{const.}$$

Pre-thermalization, perturbative? YES!

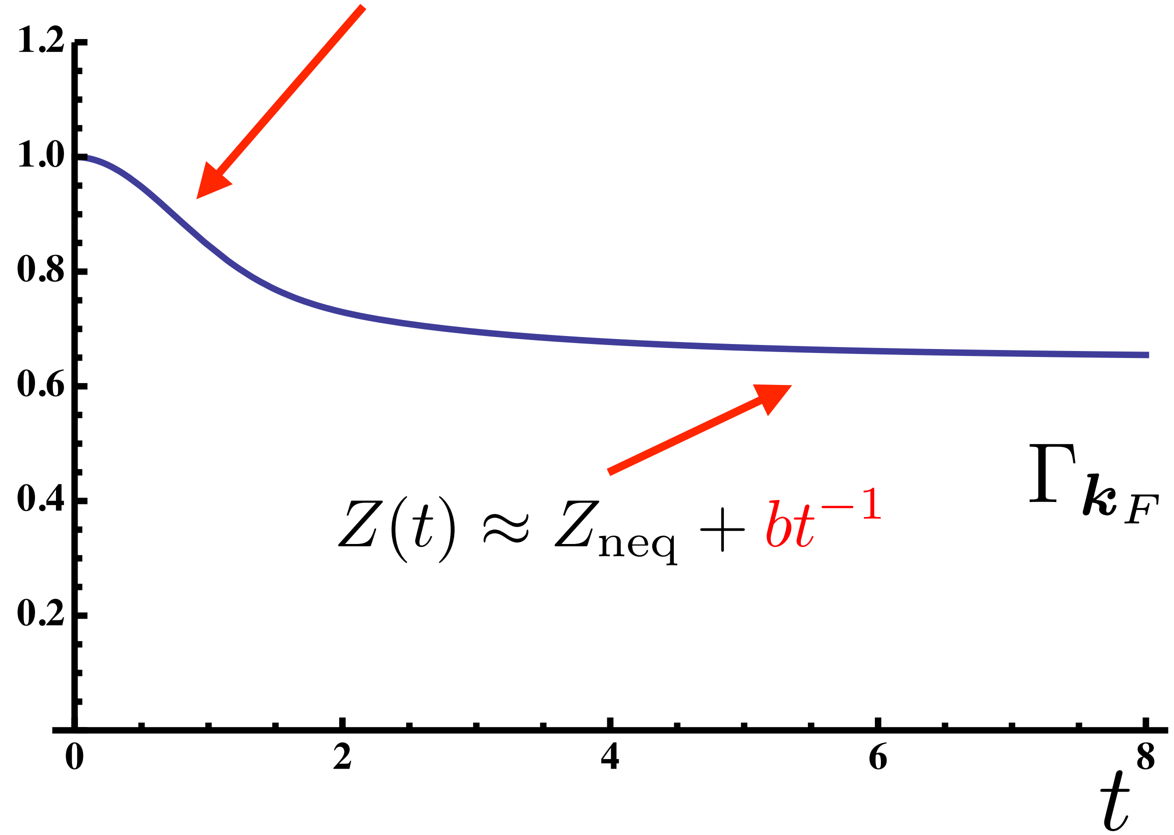
Perturbation theory valid for

$$t \ll \tau_{coll} \sim [f_0^3 (N(0))^{-2}]^{-1}$$

M Moeckel & S Kehrein PRL (2006)
M Eckstein, M Kollar &P Werner PRL (2009)
M Stark & M Kollar, arxiv:1308161

$Z(t)$

$$Z(t) \approx 1 - at^2$$



$$Z(t) \approx Z_{\rm neq} + bt^{-1}$$

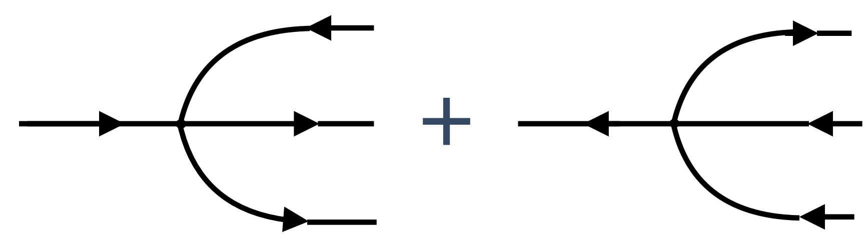
$$1 - Z_{\rm neq} = 2(1 - Z_{\rm eq})$$

Perturbation theory $[n(k), \rho_0] = 0 \Rightarrow O(f_0^2)$

$$Z(t) = 1 - 4 \int \frac{dE}{2\pi} \frac{\Gamma_{\boldsymbol{k}_F}(E)}{E^2} \sin^2\left(\frac{Et}{2}\right)$$

(Off-shell) scattering rate

$\Gamma_{\boldsymbol{k}_F}(E) =$



$= O(f_0^2)$

Fermi Liquid theory (Luttinger, PR 1963)

$$\Gamma_{\mathbf{k}}(E) \sim E^2$$

PT tells us there is a pre-thermalization plateau, but WHY?

Making an Interacting Gas Exactly Solvable

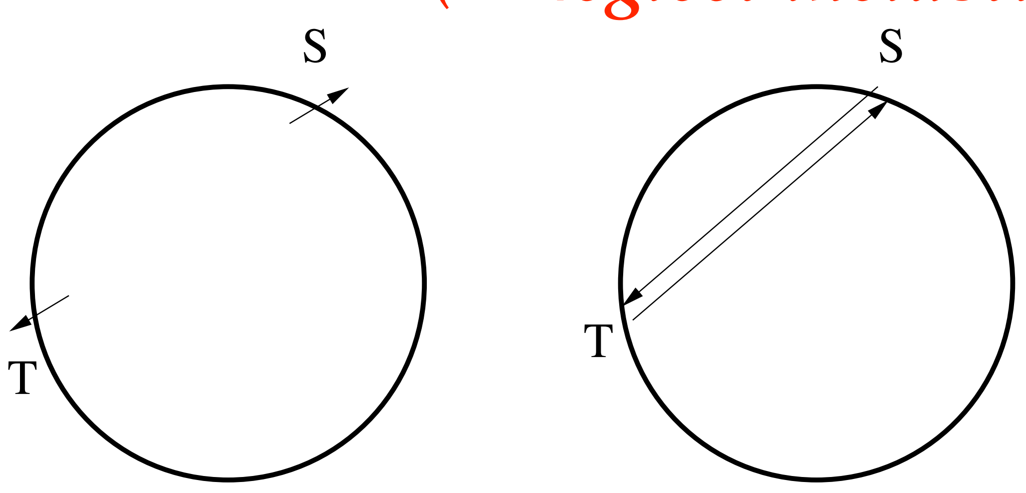
Hamiltonian for t > 0

Contains inelastic processes

$$H = H_0 + H_{\text{int}} = \sum_{\boldsymbol{k}} \epsilon(\boldsymbol{k}) c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}} + \frac{1}{V} \sum_{\boldsymbol{kpq}} f(q) c^\dagger_{\boldsymbol{k+q}} c^\dagger_{\boldsymbol{p-q}} c_{\boldsymbol{p}} c_{\boldsymbol{k}}$$

Fermi-liquid-like truncation of the bare Hamiltonian

(= Neglect inelastic processes)



Forward Exchange

FS Bosonization $J_{\boldsymbol{S}}(\boldsymbol{q}) \sim \sum_{\boldsymbol{k} \in \boldsymbol{S}} c^\dagger_{\boldsymbol{k+q}} c_{\boldsymbol{k}}$

$$H = \frac{1}{2} \sum_{\boldsymbol{S},\boldsymbol{T},\boldsymbol{q}} J_{\boldsymbol{S}}(\boldsymbol{q}) \left(\frac{v_F}{\Omega} \delta_{\boldsymbol{S},\boldsymbol{T}} + \frac{f(q)}{V} \right) J_{\boldsymbol{T}}(-\boldsymbol{q})$$

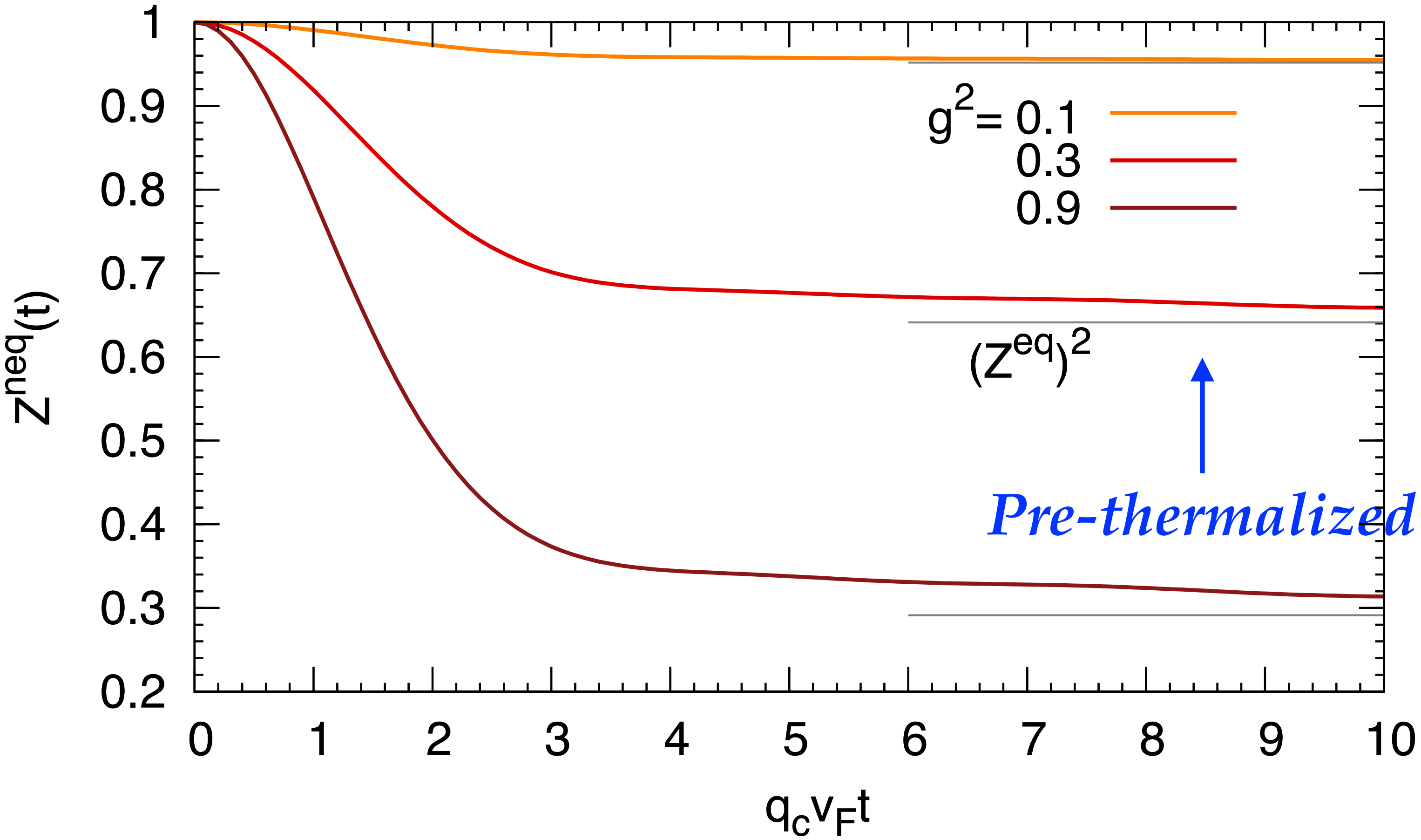
$$[J_{\boldsymbol{S}}(\boldsymbol{q}), J_{\boldsymbol{T}}(\boldsymbol{p})] = \delta_{\boldsymbol{S},\boldsymbol{T}} \delta_{\boldsymbol{q+p}} \Omega \, \hat{\boldsymbol{n}}_{\boldsymbol{S}} \cdot \boldsymbol{q}$$

Eigenmodes $H = \sum_{l,\boldsymbol{q}} \omega(\boldsymbol{q}) \alpha^\dagger(\boldsymbol{q}) \alpha_l(\boldsymbol{q})$

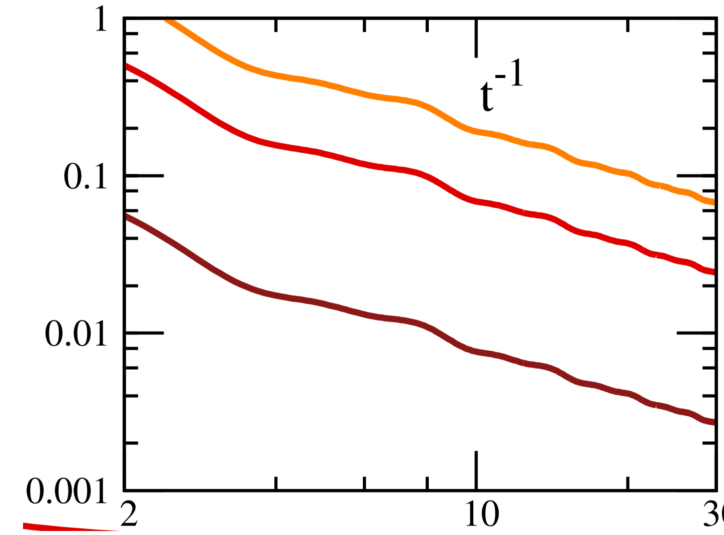
Houghton, Kwon & Marston Adv. in Phys. (2000)
+Haldane, Castro Neto & Fradkin,
Kim & Wen & Lee, …

Interaction quench in a 2D Fermi Gas

N Nessi, A Iucci, and MAC, arXiv:1401.1986



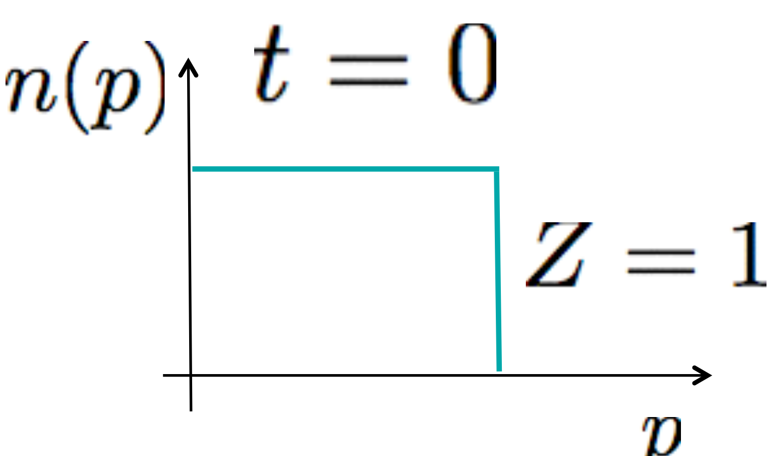
$$Z^{\mathbf{eq}} = 1 + O(g^2)$$

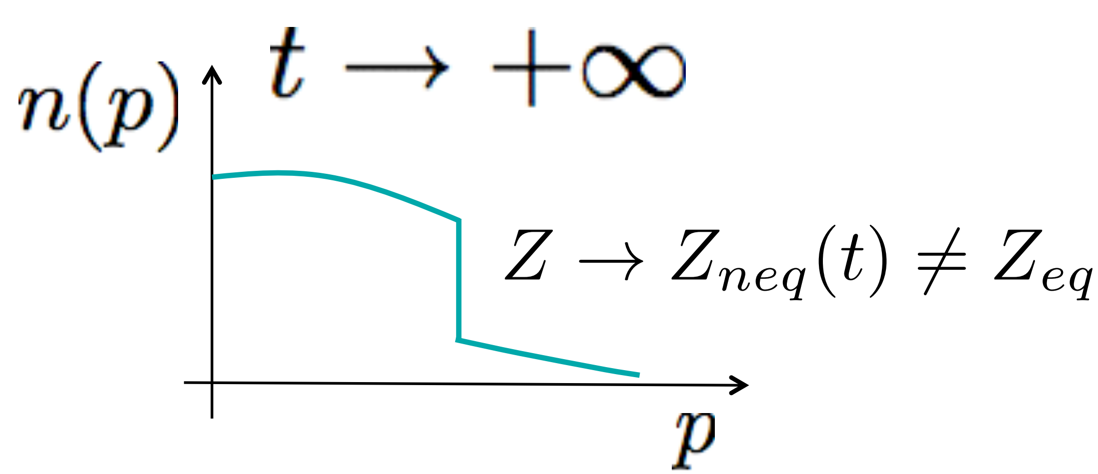


Kinetic Energy rapidly equilibrates

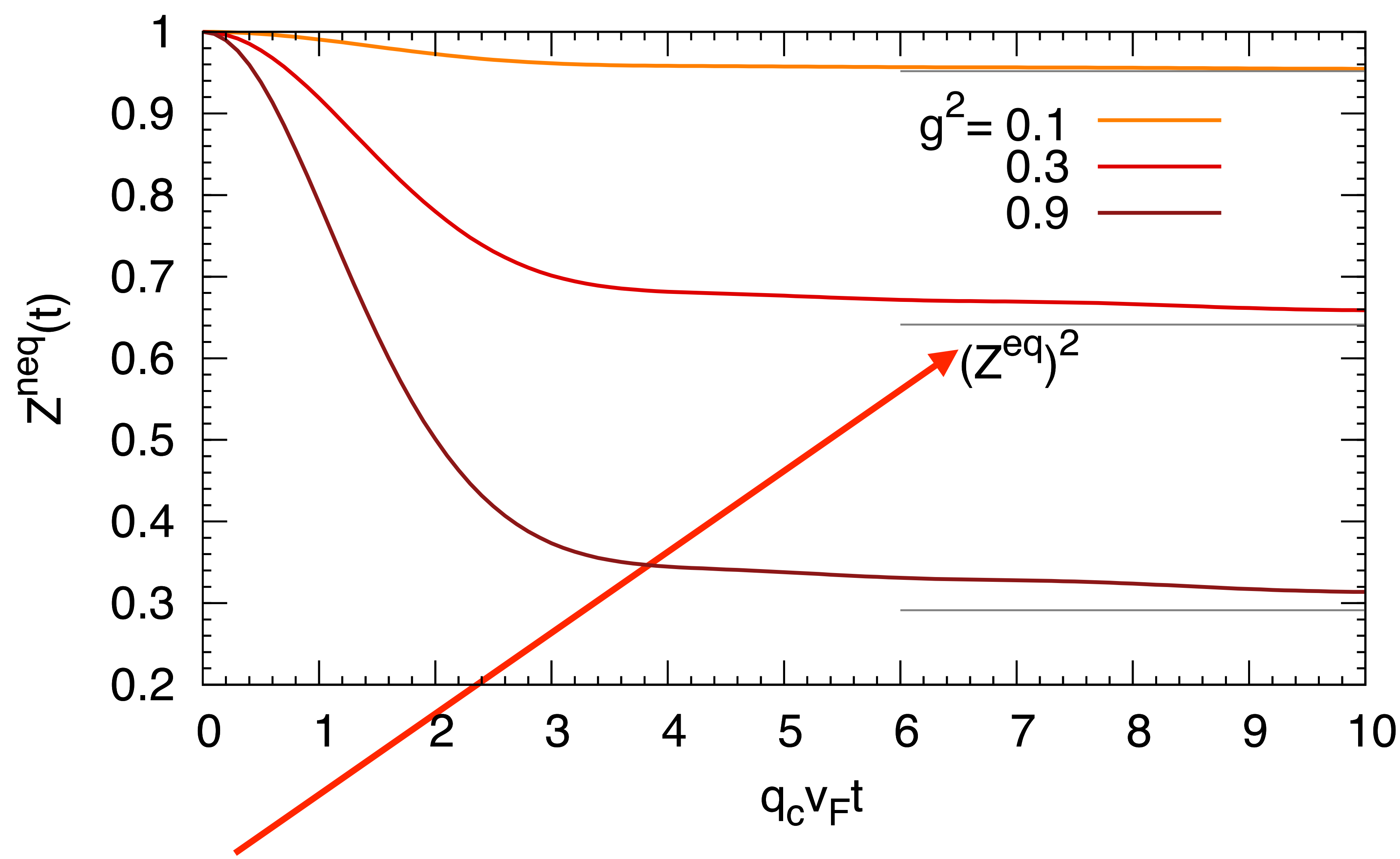
$$\langle K(t)\rangle \to \langle \Psi_0|(H - E_{\rm gs})|\Psi_0\rangle + O(g^3)$$

Momentum distribution





Prethermalized State = GGE



How do we describe the pre-thermalized state?

Eigenmodes $H = \sum_{l,\boldsymbol{q}} \omega(\boldsymbol{q})\alpha^{\dagger}(\boldsymbol{q})\alpha_l(\boldsymbol{q})$

Generalized Gibbs Ensemble

$$\rho_{\mathrm{GGE}} = \frac{1}{Z_{\mathrm{GGE}}} \exp\left[\sum_{l,\boldsymbol{q}} \lambda_l(\boldsymbol{q}) I_l(\boldsymbol{q})\right]$$

$$I_l(\boldsymbol{q}) = \alpha_l^{\dagger}(\boldsymbol{q})\alpha_l(\boldsymbol{q})$$

Fermi Surface eigenmodes

Conclusions (part II)

- *Generally speaking, systems that can be described in terms of quadratic Hamiltonians of Bosonic or Fermionic elementary excitations thermalize to a Generalized Gibbs Ensemble (GGE).*
- *Close to the fixed point, interacting Fermions in 1D exhibit very slow relaxation dynamics following a quantum quench. At $T = 0$, the discontinuity at the Fermi energy vanishes as a power law.*
- *Even systems that eventually do thermalize can exhibit an intermediate regime known as pre-thermalization. The system dynamics may be describable for short times by a quadratic Hamiltonian, and therefore the pre-thermal state will be described by the GGE.*