Effective dynamics of the BEC confinded in ring-shaped optical lattices

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Phys. Rev. A **88**, 063627 (2013).

Atomtronics, Benasque, Spain
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Outline

Introduction and Motivations

Part 1:

Ring with a weak link: Realization, Model, Mapping to a qubit

Part 2:

Two coupled rings:

Realization, Model, Mapping to a qubit and MQST

Part 3:

Single- and two- qubit gates and state readout

Conclusions

Why Atomtronic devices?

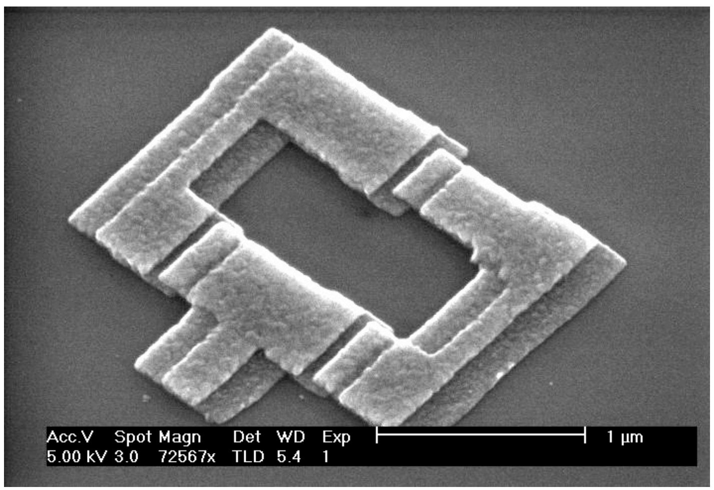
- Neutrality of currents: reduced decoherence
- The flexibility on the statistics
- Tunable interactions, dimensionality and disorder

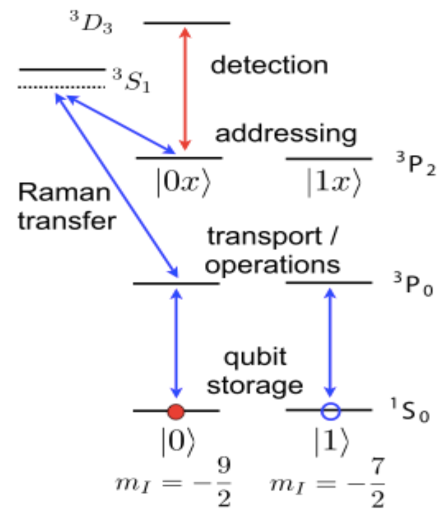
Seaman et al., Atomtronics: Ultracold-atom analogs of electronic devices. Physical Review A, 75(2):023615, 2007.

Motivation

Bringing together the advantages of

Josephson junctions → Fast quantum gates (nanoseconds)

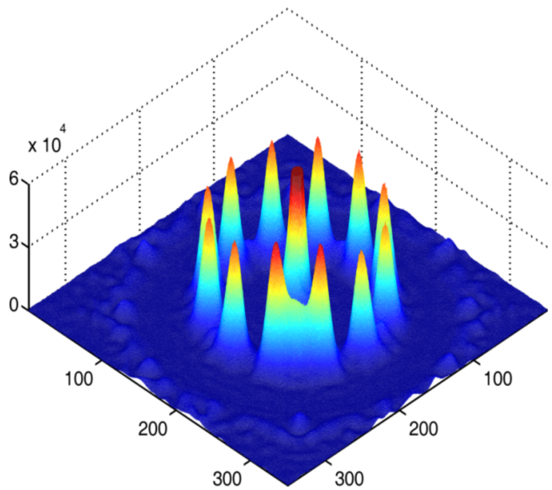


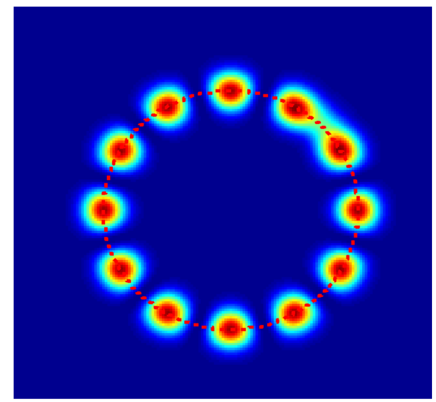


Cold atoms→ Long coherence times (minutes)

Part 1:
Ring optical lattice with a weak link

The lattice potential realized in our experiment





The potential profile.
Parameters: N=12, R=88 μm, and σ ≈ 5.2 μm.

$$I(\rho,\varphi) = I_0 e^{-\frac{(\rho-R)^2}{2\sigma^2}} \cos(0.5N\phi)^2 + 0.5 I_0 e^{-\frac{(\rho\cos\varphi-5/7R)^2}{2\sigma^2}} e^{-\frac{(\rho\sin\varphi+5/7R)^2}{2\sigma^2}}$$

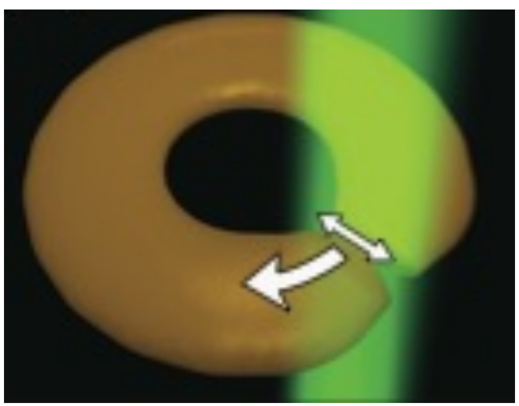
Bose-Hubbard model for the ring lattice

$$H = -\sum_{k=1}^{N} t_k \left(e^{i\Phi/N} a_k^{+} a_{k+1} + h.c.\right) + \frac{U}{2}\sum_{k=1}^{N} \hat{n}_k(\hat{n}_k - 1) \qquad \hat{n}_k = a_k^{+} a_k$$

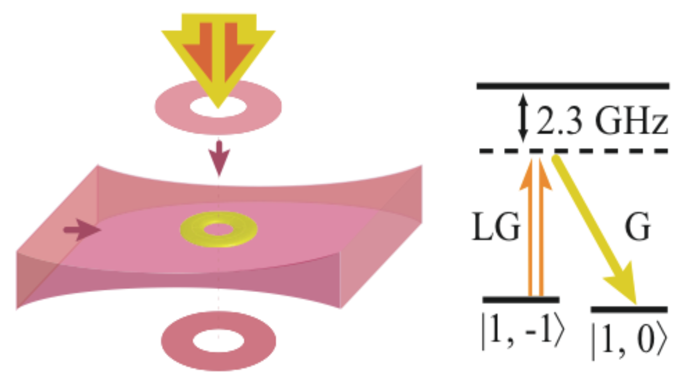
Site dependent hopping element: $t_k = t, \forall k = 0 \ldots. N-2$

Weak link: $t_{N-1} = t' < t$

Flux generated by the artificial gauge field: $\Phi / N = \int_{\vec{x}_i}^{\vec{x}_{i+1}} \vec{A}(\vec{x}) \bullet d\vec{x}$



Stirring with a blue-detuned laser



Phase imprinting via Raman transition

Wright et al., Phys. Rev. lett, 110(2):025302, 2013.

Ramanathan et al., Phys. Rev. lett., 106(13), 2011.

Mapping to a qubit

Gauge transformation → Twisted boundary conditions

$$a_k \rightarrow a_k \exp\left(ik\Phi / N\right) \qquad \Psi(x_0,\ldots,x_n+L,\ldots,x_{N-1}) = e^{i\Phi}\Psi(x_1,\ldots,x_n,\ldots,x_N)$$

Quantum phase model $\quad a_k \rightarrow \sqrt{\langle n\rangle}e^{i\phi_k} \quad n_i \approx \langle n\rangle$

$$H_{QP} = \sum_{i=0}^{N-2}\left[Un_i^2 - J\cos(\phi_{i+1}-\phi_i)\right] + \left[Un_{N-1}^2 - J\cos(\phi_0 - \phi_{N-1} - \Phi)\right]$$

$$J \approx t\langle n\rangle \qquad J' \approx t'\langle n\rangle$$

Partition function can be written as a path integral

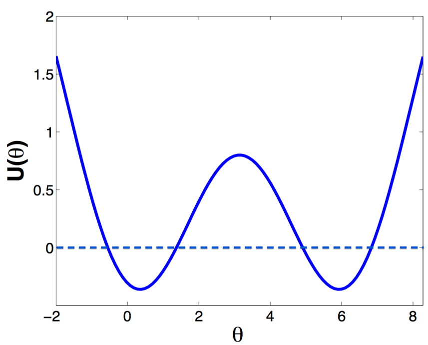
$$Z = Tr\left[e^{-\beta H_{QP}}\right] = \int \prod_i D\phi_i(\tau)\exp\left[-S\{\phi\}\right]$$

All the phases ϕ_i except $\vartheta = \phi_{N-1} - \phi_0$ can be integrated out in the harmonic approximation : $\cos(\phi) \approx 1 - \phi^2/2$

An effective action in the low temperature limit

$$S_{eff} = \int_0^\beta d\tau \left[\frac{1}{2U}\dot{\theta}^2 + V_{eff}(\theta) \right] - \frac{J}{2U(N-1)} \int_0^\beta \int_0^\beta d\tau\, d\tau' \theta(\tau)\theta(\tau')G(\tau-\tau')$$

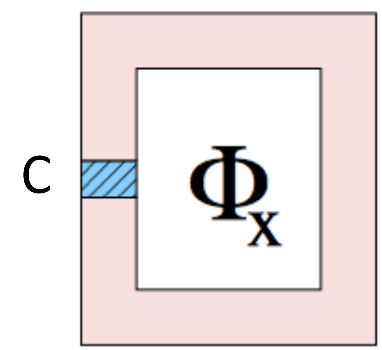
When J'(N-1)/J>1 a double-well potential is obtained



$$V_{eff}(\theta) = \frac{J}{(N-1)}(\theta - \Phi)^2 - J'\cos\theta$$

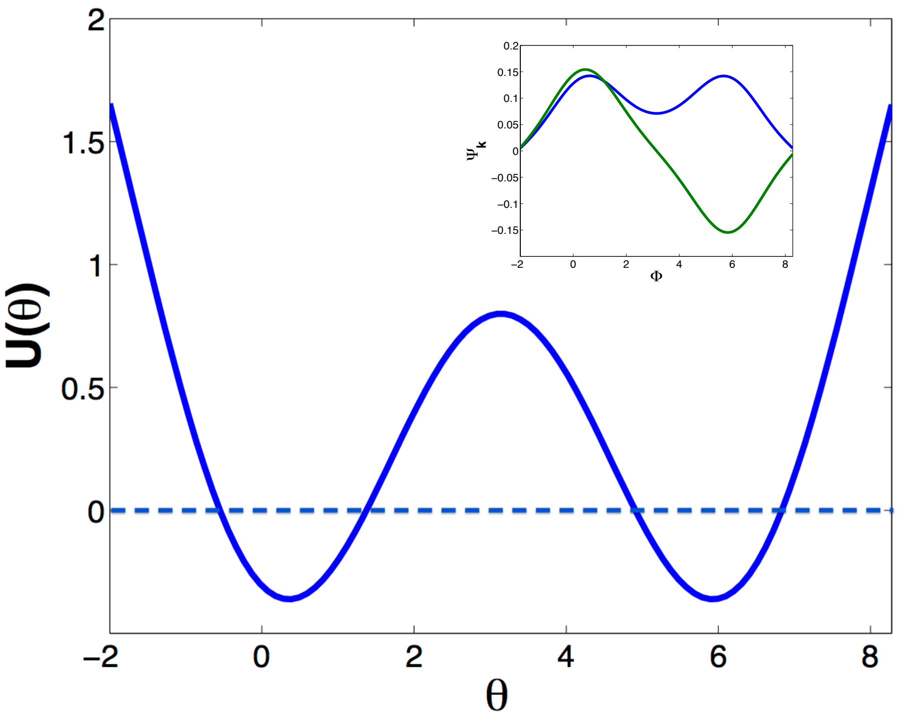
When N>>1 and T → 0, the second term is negligible,

the effective Hamiltonian is $H_{eff} = \frac{1}{2U^{-1}} \frac{d^2}{d\theta^2} + V_{eff}(\theta)$



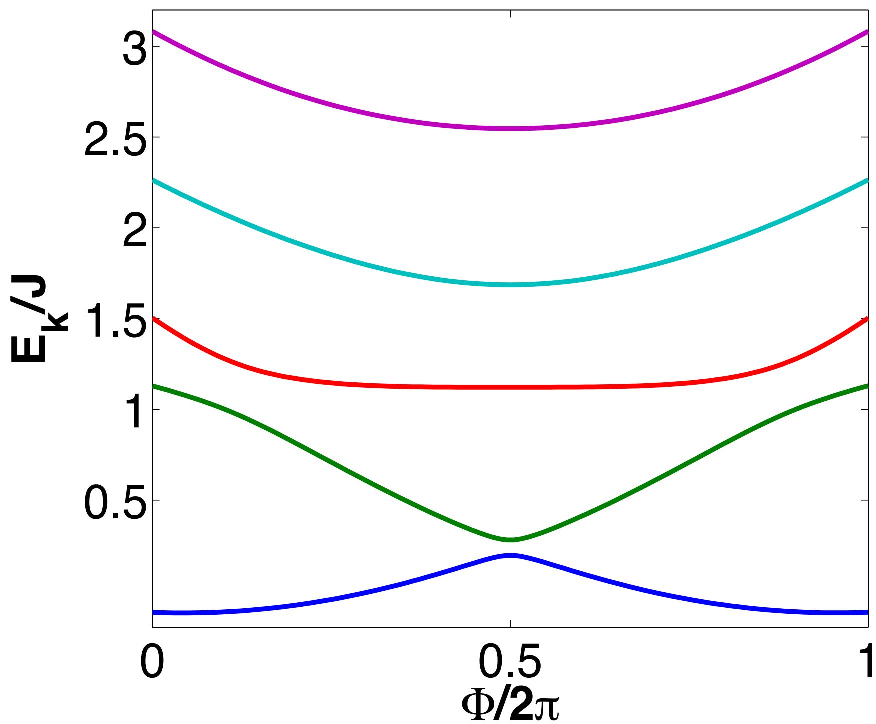
Correnspondences with rf-SQUID: U^{-1} → C, (N-1)/J → L

AQUID system based on the clockwise and anti-clockwise currents



Double well for AQUID.
Parameters: J'(N-1)/J = 16, $\Phi = \pi$.

$$|\Psi\rangle_G = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right) \quad |\Psi\rangle_E = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$$

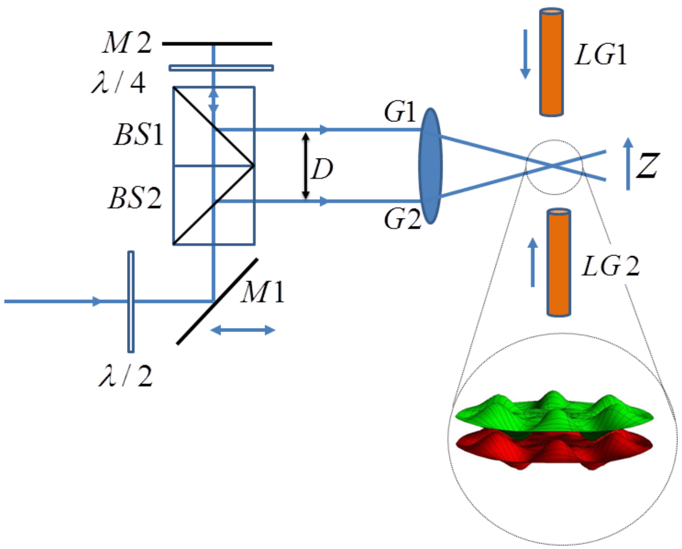


Energy spectrum.
Same parameters with U=2. Due to avoided crossing near $\Phi=\pi$, $I_0=-I_1$.

The current in the k-th energy state is given by: $$I_k \propto \frac{dE_k(\Phi)}{d\Phi}$$

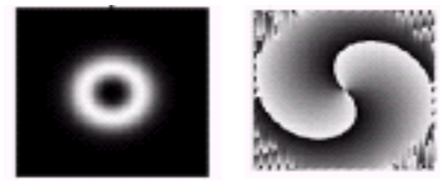
Part 2:
Two coupled rings

Experimental realization of two coupled rings with tunable tunneling



$$E(r,\varphi) = E_0 f_{pl}(r) e^{il\varphi} e^{i(\omega t - kz)}$$

$$f_{pl}(r) = (-1)^p \sqrt{\frac{2p!}{\pi(p+l)!}} \xi^l L_p^l(\xi^2) e^{-\xi^2}, \xi = \sqrt{2} r / r_0$$



Intensity Phase

$$I(r,\varphi,z) = 4E_0^2 (f_{pl} \cos^2(k_{LG} z) + \cos^2(k_G z) + 2 f_{pl} \cos(k_{LG} z) \cos(k_G z) \cos(\varphi l))$$

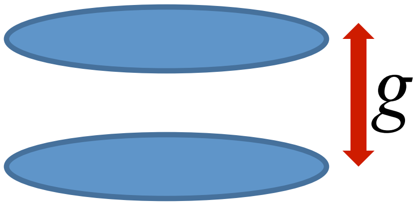
The distance between rings is given by $$d = \frac{2\pi}{k_G} = \frac{\lambda f}{D}$$

Tunneling matrix element $$t \propto \frac{1}{\sqrt{d}} e^{-d}$$

D is easily controllable in this setup!

Amico et al., *PRL*, *95*(6), 063201(2005). Li et al, Optics Express Vol 16, No 8, 5465 (2008)

Bose-Hubbard ladder model for two coupled rings without impurity

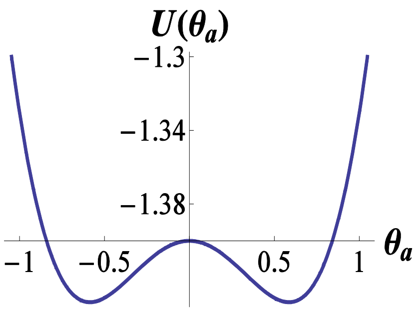


$$H = H_a + H_b + H_{\text{int}}$$

$$H_a = -t\sum_{i=1}^{N}(e^{i\Phi_a/N}a_i^+ a_{i+1} + h.c) + \frac{U}{2}\sum_{i=1}^{N}\hat{n}_i^a(\hat{n}_i^a - 1) - \mu_a\sum_{i=1}^{N}\hat{n}_i^a$$

$$H_b = -t\sum_{i=1}^{N}(e^{i\Phi_b/N}b_i^+ b_{i+1} + h.c) + \frac{U}{2}\sum_{i=1}^{N}\hat{n}_i^b(\hat{n}_i^b - 1) - \mu_b\sum_{i=1}^{N}\hat{n}_i^b$$

$$H_{\text{int}} = -g\sum_{i=1}^{N}(a_i^+ b_i + b_i^+ a_i) \qquad \hat{n}_i^a = a_i^+ a_i \qquad n_i^b = b_i^+ b_i$$



Rings are pierced with different "fluxes"

$$\Phi_a / N = \int_{\vec{x}_i}^{\vec{x}_{i+1}} \vec{A}(\vec{x}) \bullet d\vec{x} \qquad \Phi_b / N = \int_{\vec{x}_i}^{\vec{x}_{i+1}} \vec{B}(\vec{x}) \bullet d\vec{x}$$

The parameters are J'/J = 0.8 and $\Phi=\Phi_a - \Phi_b = \pi$.

In the limit N>>1 , double-well potential is obtained for $\vartheta_a = -\vartheta_b$

$$U(\theta_a,\theta_b) = -J\cos\theta_a - J\cos\theta_b - J'\cos\left[\theta_a - \theta_b - (\Phi_a - \Phi_b)\right]$$

Effective time dynamics and MQST

$$\frac{\partial z}{\partial t} = -\sqrt{1-z^2}\,\sin\Theta$$

$$\frac{\partial \Theta}{\partial t} = \Delta + \Lambda z + \frac{z}{\sqrt{1-z^2}}\cos\Theta$$

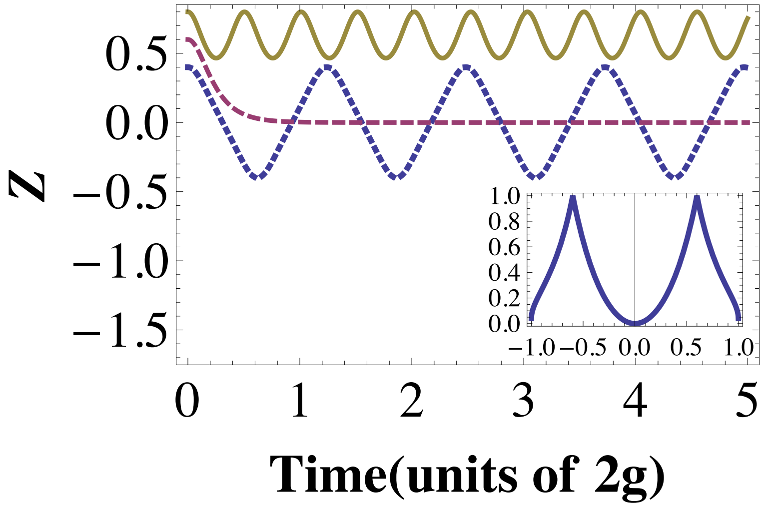
$$z(t) = (N_b - N_a)/(N_b + N_a)$$

$$\Theta(t) = \theta_a - \theta_b$$

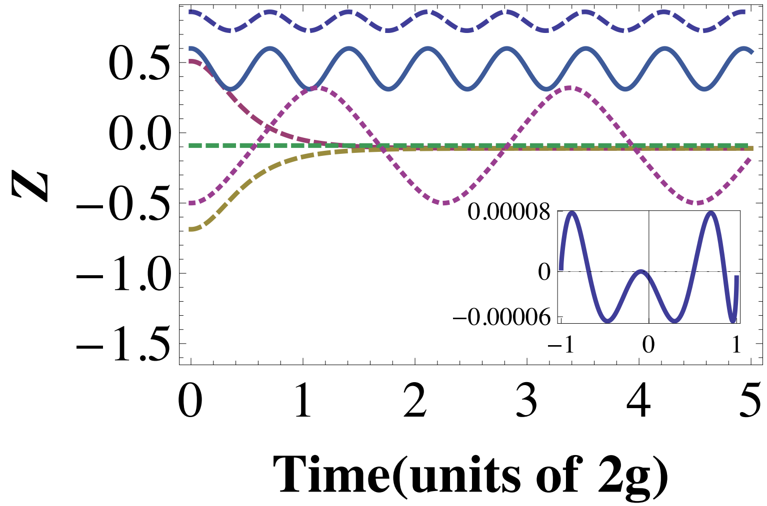
$$2g\tau/\hbar \to \tau$$

$$\Delta = \left(2k(\cos\Phi_a - \cos\Phi_b) + \mu_b - \mu_a\right)/2g$$

$$\Lambda = U(N_a + N_b)/2Ng$$



Here Δ=0, Θ_0=0 and Λ=10.

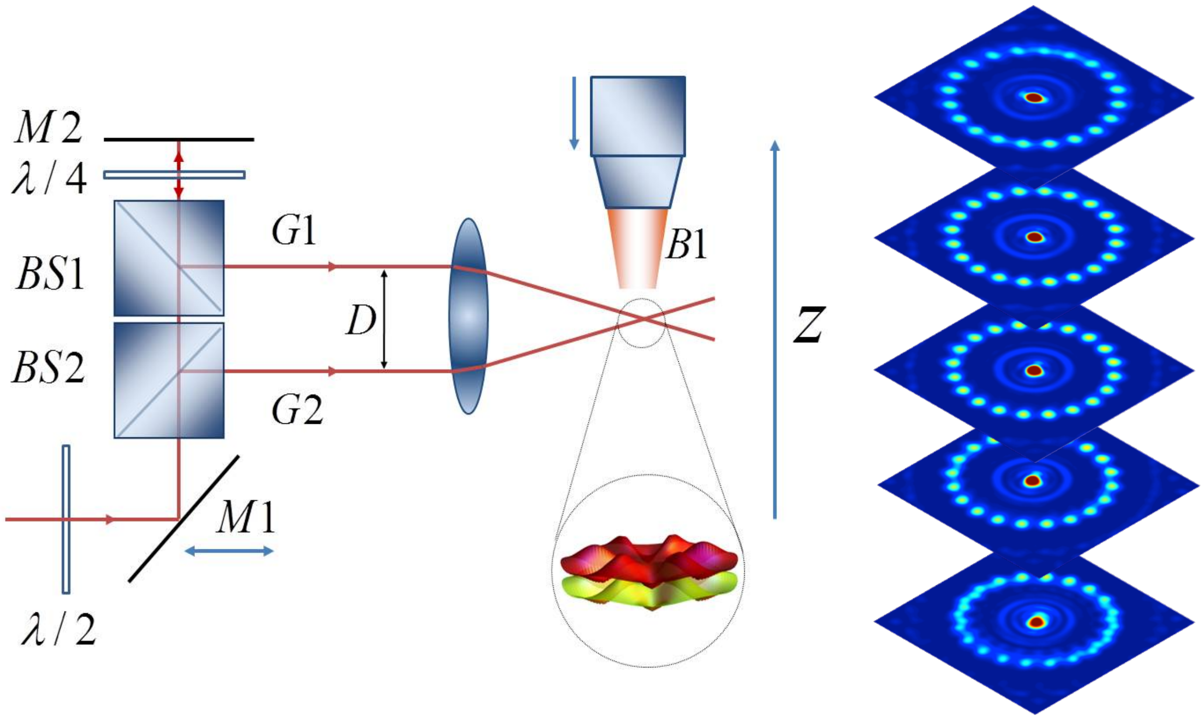


Here Δ=1, Θ_0=0 and Λ=10.

S.Raghavan et. al, Phys,Rev A,1998

Part 3:
Quantum gates and state readout

Experimental realization of interacting rings and AQUIDs



+500 μm (5.6*R)

+200 μm (2.2*R)

Z= 0

-200 μm (-2.2*R)

-500 μm (-5.6*R)

Effect of the axial translation ΔR/R = 0.0097×z.

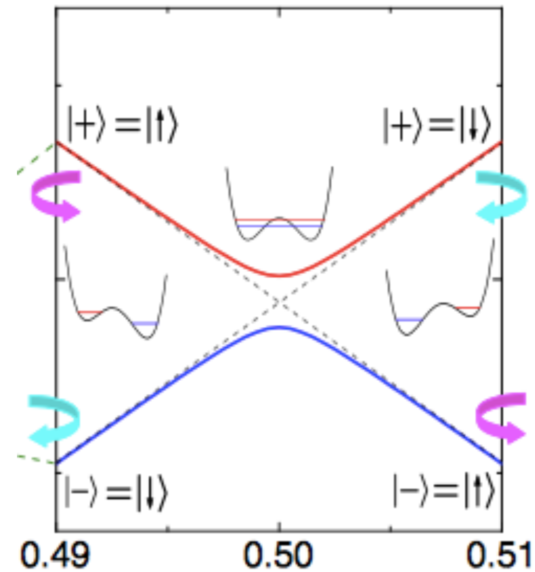
Single qubit gates

Hamiltonian in the two level basis takes form:

$$H \simeq \varepsilon\sigma_z + \frac{J'(\Phi-\pi)}{\delta}\langle\theta\rangle_{01}\sigma_x$$

$$\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$$
$$\sigma_x = |1\rangle\langle 0| - |0\rangle\langle 1|$$

$$\delta = \frac{J'(N-1)}{J} > 1$$



WKB estimate for the energy gap is given by:

$$\varepsilon \simeq \frac{2\sqrt{UJ'}}{\pi}\sqrt{\left(1-\frac{1}{\delta}\right)}e^{-12\sqrt{J'/U}(1-1/\delta)^{3/2}}$$

Phase gate $$U_z(\beta) = exp(i\varepsilon\tau\sigma_z) = \begin{pmatrix} e^{i\varepsilon\tau} & 0 \\ 0 & e^{-i\varepsilon\tau} \end{pmatrix}$$

NOT gate $$U_x(\beta) = exp(i\alpha\tau\sigma_x) = \begin{pmatrix} \cos\alpha & i\sin\alpha \\ i\sin\alpha & \cos\alpha \end{pmatrix}$$

Two-qubit gates

In the limit J''<<J' and $\Phi_a = \Phi_b = \Phi$ the Hamiltonian of coupled rings takes form:

$$H = J' \left[\sum_{\alpha=a,b} H_\alpha + \frac{J''}{J'} \frac{(\theta_a - \theta_b)^2}{2} \right]$$

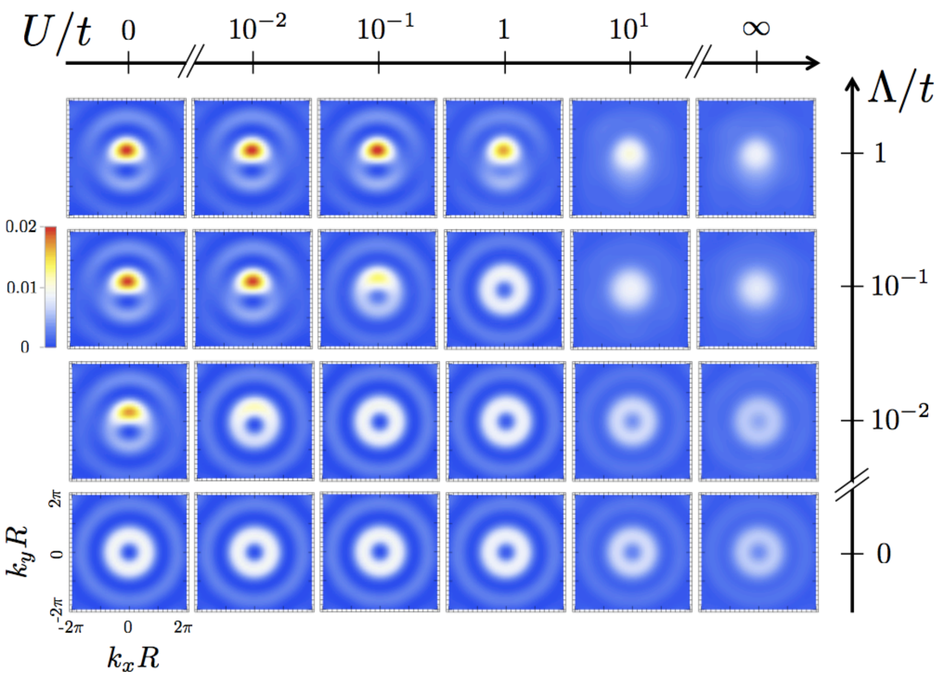
$$H = H_a + H_b + \frac{J''}{J'} \sigma_x^1 \sigma_x^2 \langle\theta\rangle_{01}^2$$

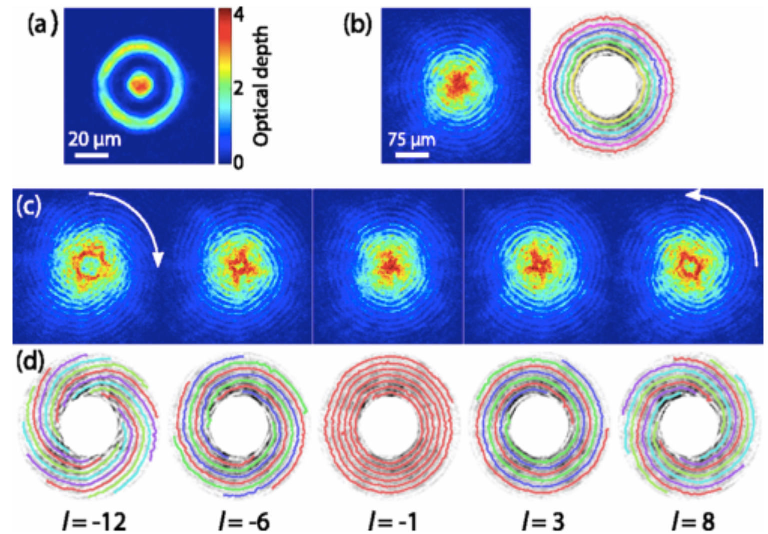
$$H_\alpha = \epsilon \sigma_z^\alpha + \left(\frac{\Phi - \pi}{\delta} + \frac{J''\pi}{J'}\right) \langle\theta\rangle_{01} \sigma_x^\alpha$$

By choosing ε=0 and Φ=π-(δ J''π)/J':

$$U(\tau) = exp[-i \frac{J''}{J'} \sigma_x^1 \sigma_x^2 \tau]$$

AQUID state readout





It is possible to see signatures of the superposition states by studying TOF

The chirality of the spiral like interferogram determines direction of the current

Aghamalyan *et al New J. Phys.* 17, 045023 (2015)

Eckel, et al. Physical Review *X,* 4, 031052, (2014)

Conclusions

1. Single ring with an impurity implements the AQUID
2. Two-coupled rings setup realizes a qubit and MQST
3. Qubit-qubit interactions can be realized
4. Atomic qubits have long coherence times
5. Gates can be implemented with AQUIDs