Optimal persistent currents for interacting bosons on a ring with gauge field

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Atomtronics @ Benasque - 7.5.2015

M.Cominotti, D. Rossini, M. Rizzi, F. Hekking, A. Minguzzi, PRL 113, 025301 (2014)

M.Cominotti, et al., EPJ ST 224, 519 (2014) // D. Aghamalyan, et al., NJP 17 045023 (2015)

OUTLINE

- Introduction
- Definition of the problem
- Analytical & numerical treatment
- Conclusions & open problems

Persistent currents in condensed matter

Introduction

multiply connected geometry

+

U(1) gauge potential

↓

Aharonov-Bohm effect $\Omega = 2\pi\Phi/\Phi_0$

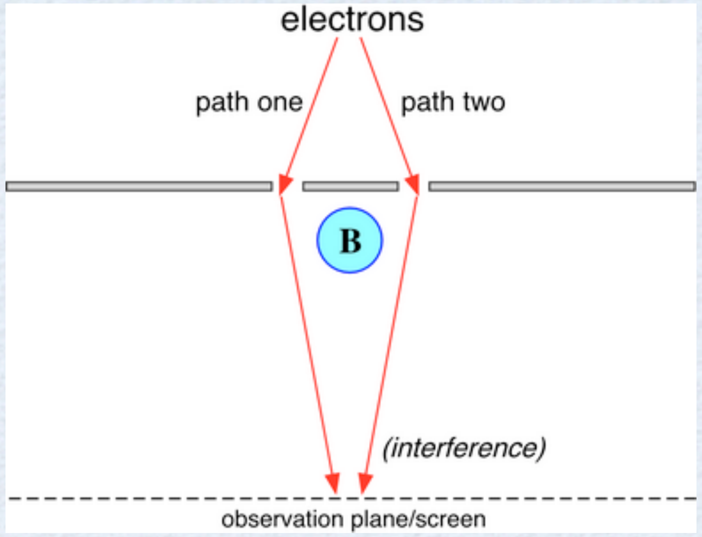
+

macroscopic quantum coherence

$$\vec{\nabla} \times \vec{A} = \vec{B}$$

$$\Phi = \oint \vec{A} \cdot \mathrm{d}\vec{l}$$

$$\Phi_0 = h/e$$



→

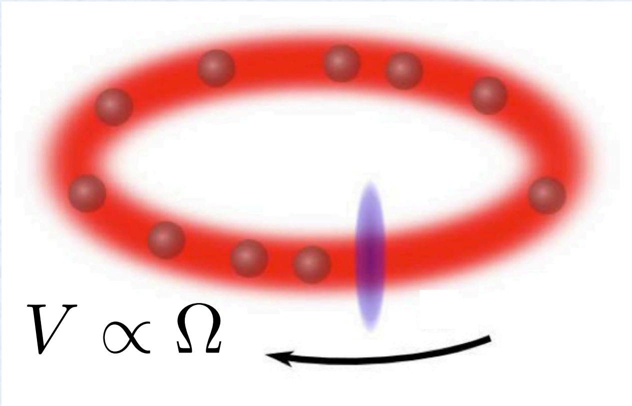
persistent current

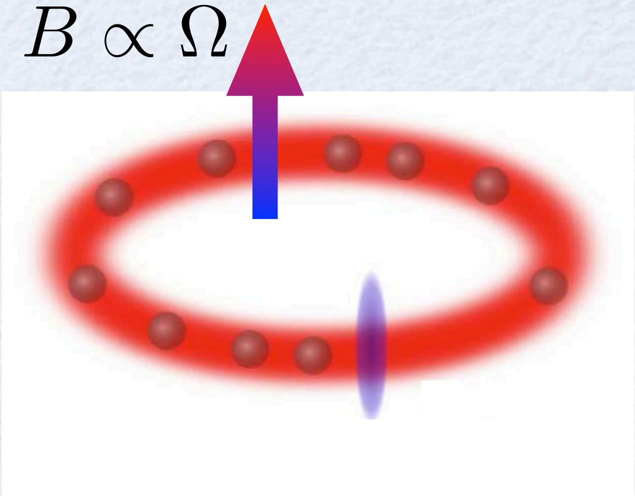
$$I(\Omega) = -\frac{1}{2\pi\hbar}\frac{\partial E(\Omega)}{\partial \Omega}$$

Bloch, PRB 2, 109 (1970)

equivalence with rotation

$$p_x \longrightarrow \left(p_x - \tfrac{2\pi\hbar}{L}\Omega\right)$$





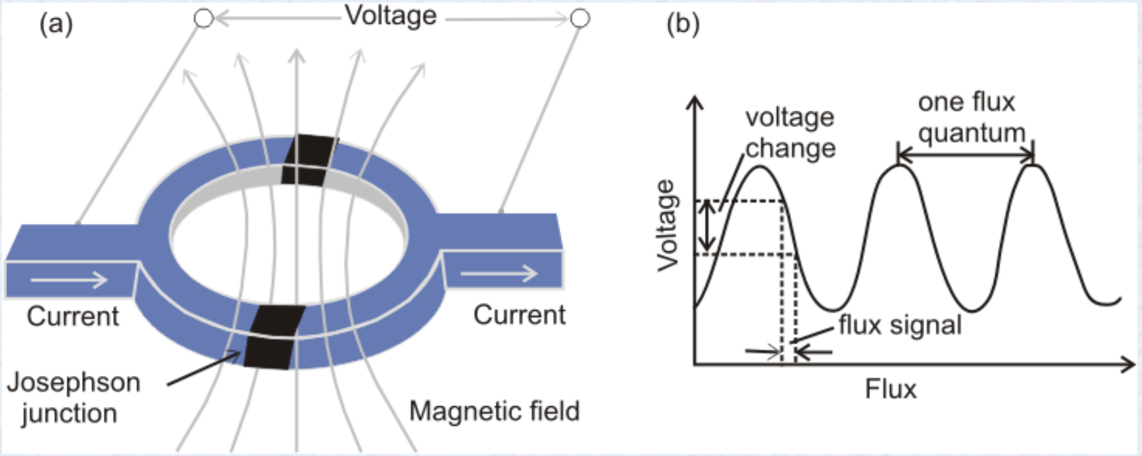
Persistent currents in condensed matter

Introduction

- bulk superconductors

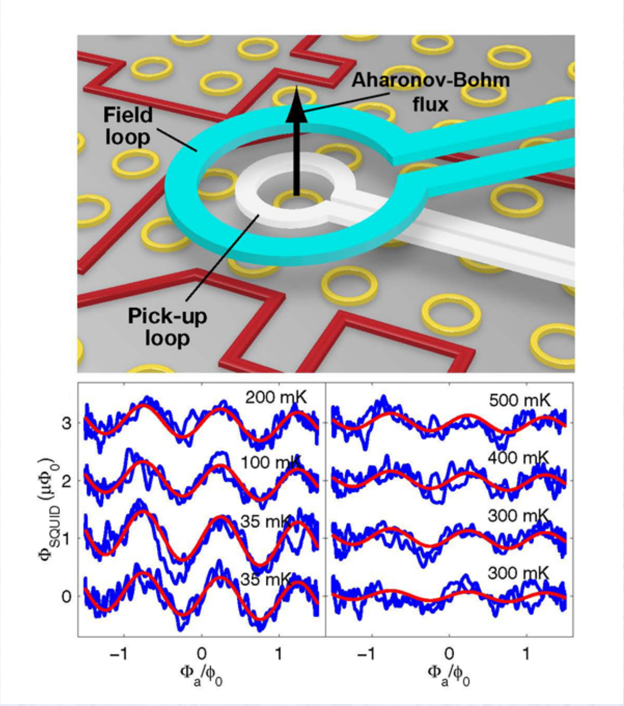
B. S. Deaver and W. M. Fairbank, PRL 7, 43 (1961)
N. Byers and C. N. Yang, PRL 7, 46 (1961)
L. Onsager, PRL 7, 50 (1961)

- SQUID = superconducting quantum interference device



- normal metallic rings

L. P. Levy, et al., PRL 64, 2074 (1990)
D. Mailly, et al., PRL 70, 2020 (1993)
H. Bluhm et al., PRL 102, 136802 (2009)
A. C. Bleszynski-Jayich, et al., Science 326, 272 (2009)

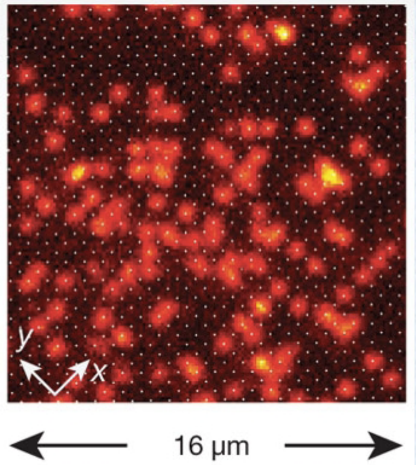


? Effects of interactions & barrier/impurities & statistics ?
… go beyond “natural” … quantum engineering!

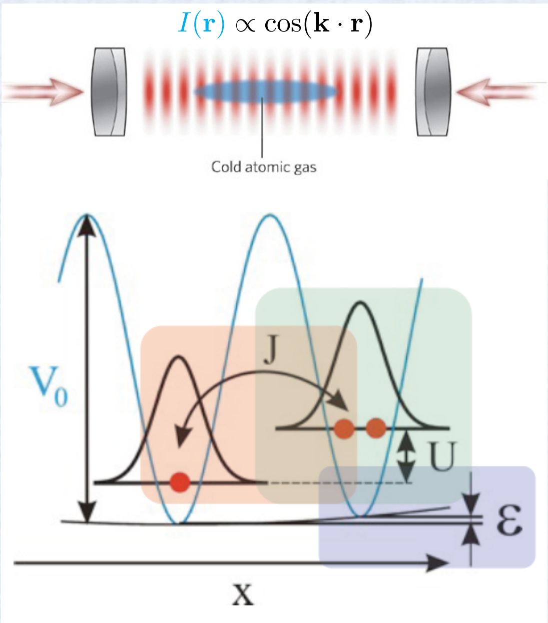
Ultracold atoms: a quantum engineering platform

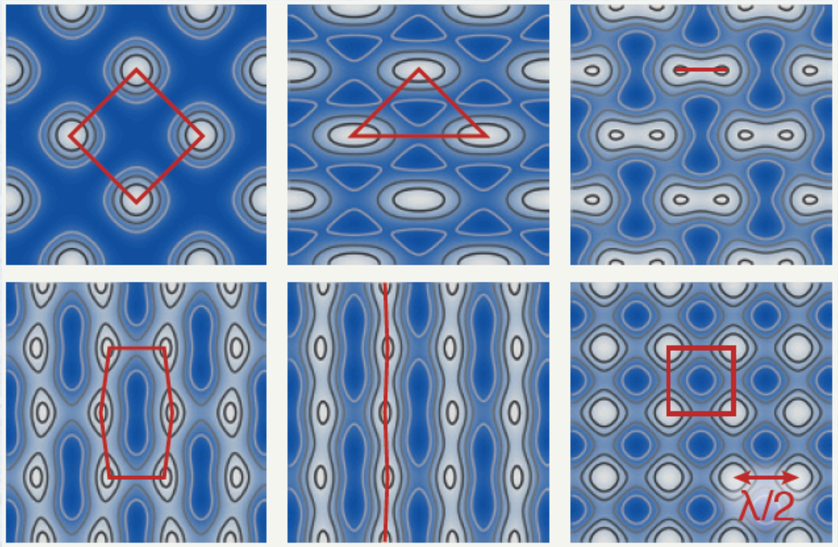
Introduction

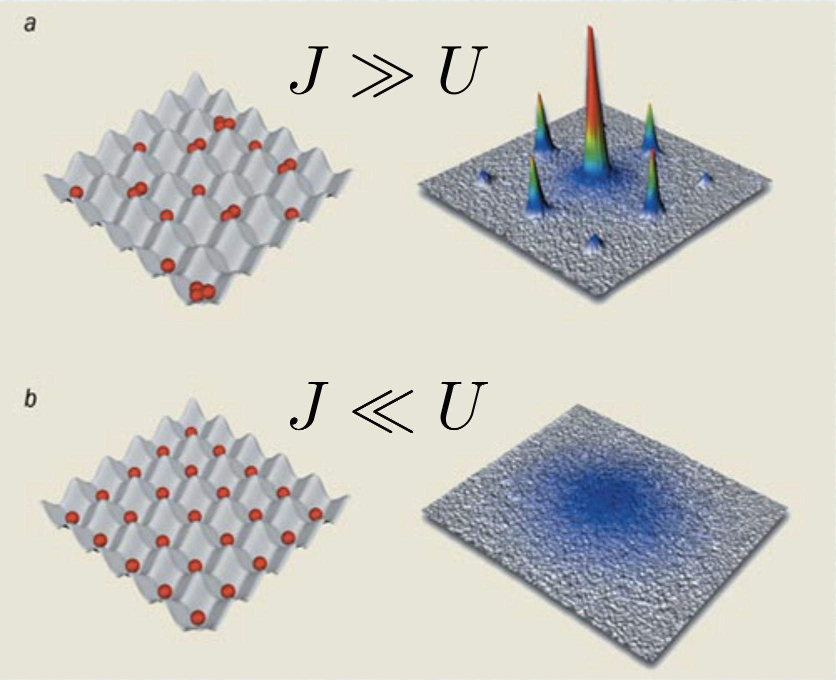
- isolated neutral quantum systems (long coherence times)
- high tunability of microscopic parameters (also interactions!)
- access to many microscopic observables



$$\mathcal{H} = -J \sum_{<i,j>} b_i^\dagger b_j + \sum_i \epsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i(\hat{n}_i - 1)$$





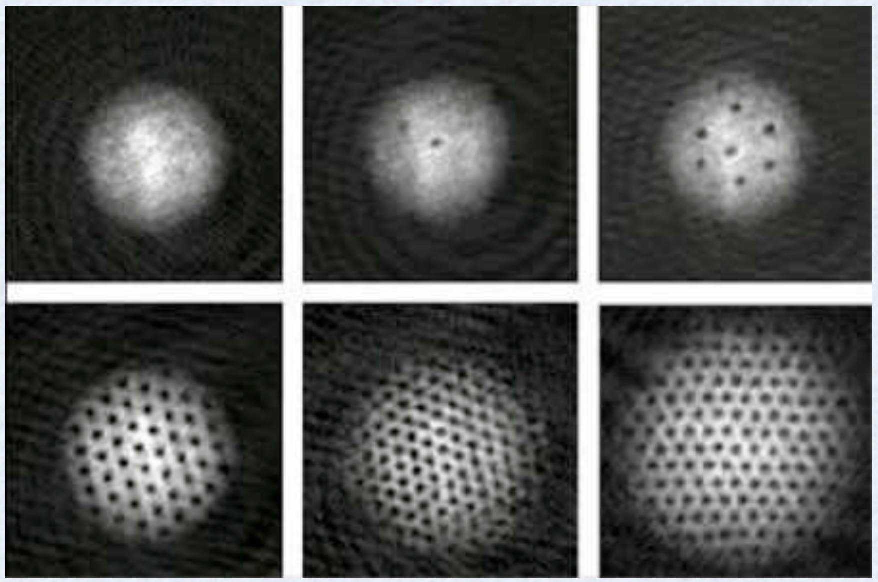


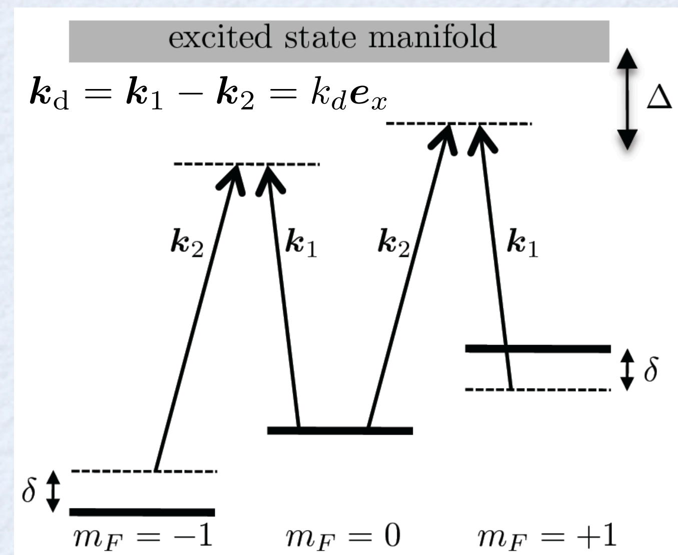
M. Lewenstein, et al., Adv Phys 56, 243–379 (2007).
I. Bloch, J. Dalibard, W. Zwerger, RMP 80, 885 (2008)
J. Dalibard, F. Gerbier, G. Juzeliunas, and P. Öhberg, RMP 83, 1523 (2011)

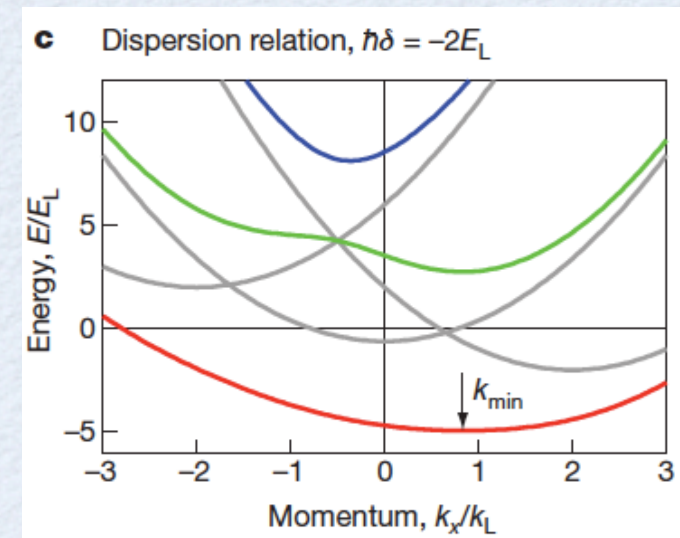
Ultracold atoms: a quantum engineering platform

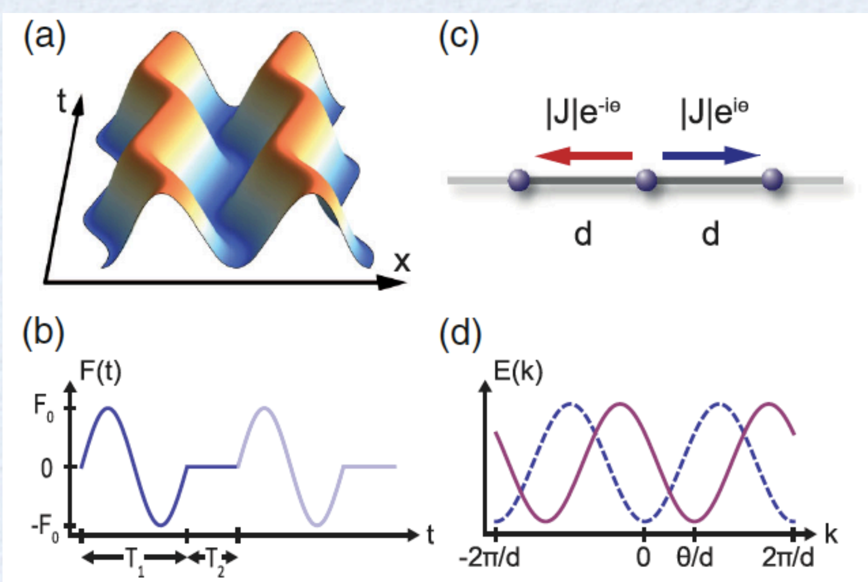
Introduction

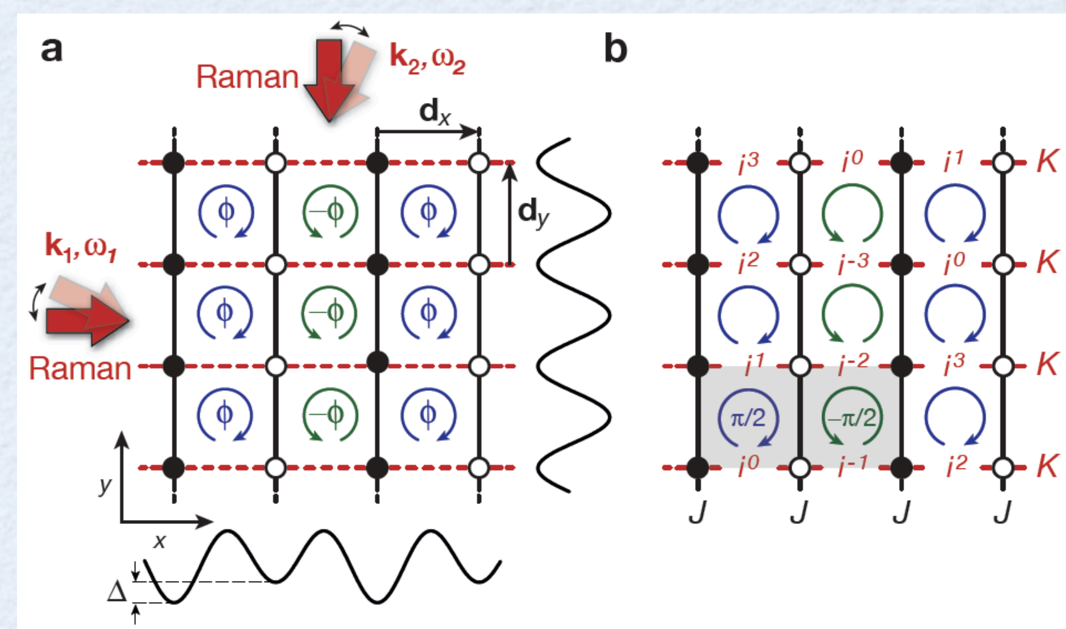
- possibility of inducing **artificial gauge potentials**
  (by rotation / adiabatic Berry phase / shaking / Raman hopping / ...)







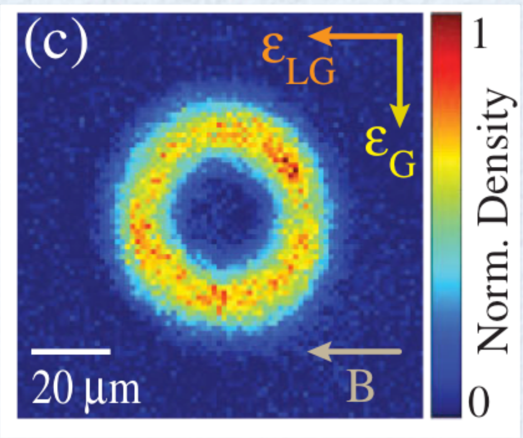


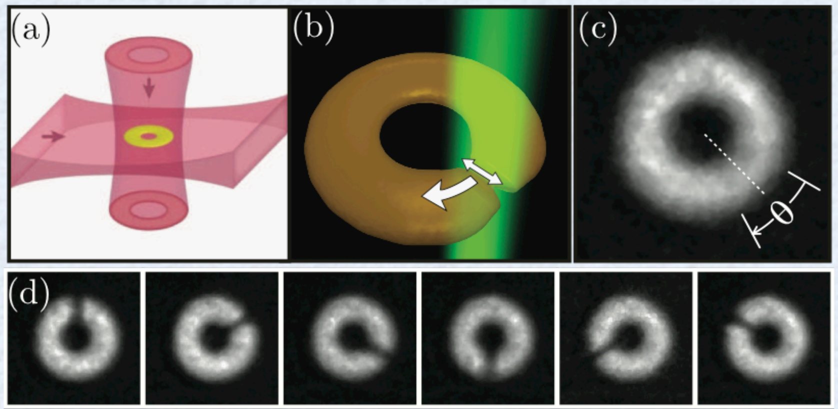


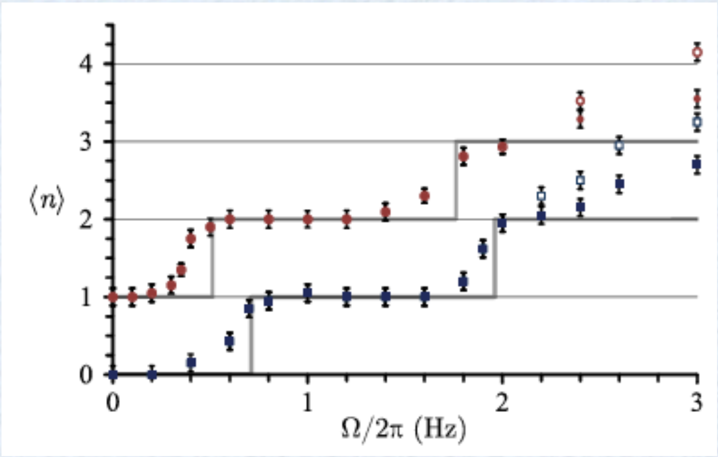
J. Dalibard, F. Gerbier, G. Juzeliunas, and P. Öhberg, RMP 83, 1523 (2011)
N. Goldman, G. Juzeliunas, P. Öhberg, and I.B. Spielman, arXiv:1308.6533

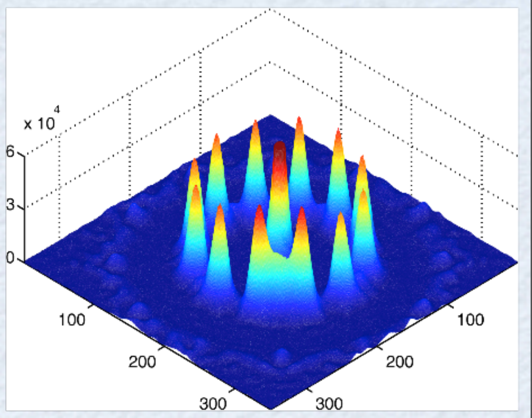
Cold atoms in ring traps

Introduction









Ramanathan et al., PRL 106, 130401 (2011); Wright et al., PRL 110, 025302 (2013); Moulder et al., PRA 86, 013629 (2012) ; Beattie, et al., PRL 110, 025301 (2013);

Amico et al., Sci. Rep. 4, 4298 (2014)

Talks by R. Dumke & D. Aghamalyan

- achieved results:
  - persistent currents flowing for up to 40s !
  - quantization of flux via TOF imaging
  - observation of instabilities in multi-species setups

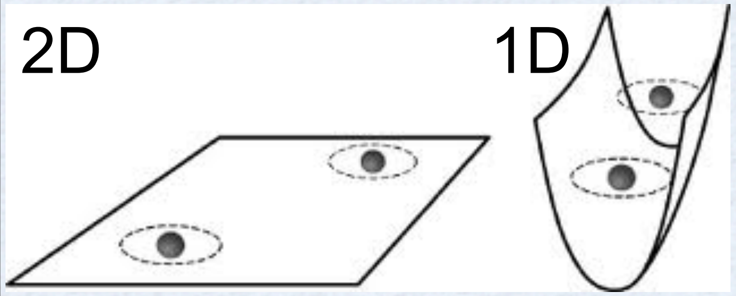
& many more references from the previous days here in Benasque :)

- applications:
  - quantum info [atomic qubit]
  - high-precision measurements [interferometry]
  - **studying regimes inaccessible to CMP :)**

Richness & oddness of a 1D scenario

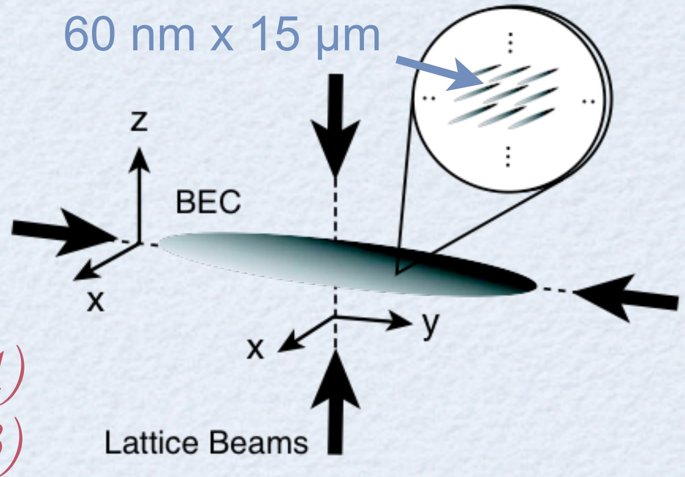
Introduction

- obtained by strong transverse confinement and / or optical lattice



$$\hbar\omega_\perp \gg k_B T,\ \mu$$

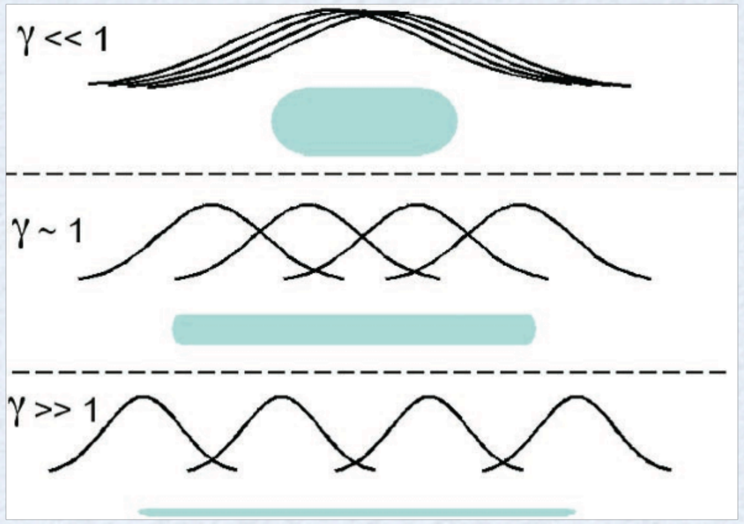
$$\Psi_B(\vec{r}_1,\ldots,\vec{r}_N) = \psi_B^{1D}(x_1,\ldots,x_N)\prod_{i=1}^{N}\phi_0(\vec{r}_i^{\perp})$$



Greiner et al., PRL 87, 160405 (2001)
Moritz et al., PRL 91, 250402 (2003)

- Interaction growth with diluteness !

$$\gamma = \frac{E_{int}}{E_{kin}} = \frac{gn}{\hbar^2 n^2/m} = \frac{gm}{\hbar^2 n} \qquad n = \frac{N}{L}$$



- Fermionization of hard-core bosons

Paredes, et al., Nature 429, 6989 (2004);
Kinoshita et al., Nature 440, 900 (2006);

- Quantum fluctuations are crucial
  (only quasi-long range order)

$$\langle\hat{\Psi}^\dagger(x)\hat{\Psi}(x')\rangle \simeq \frac{1}{|x-x'|^{1/2K}}$$

- **lots of analytics & numerics at hand :)**

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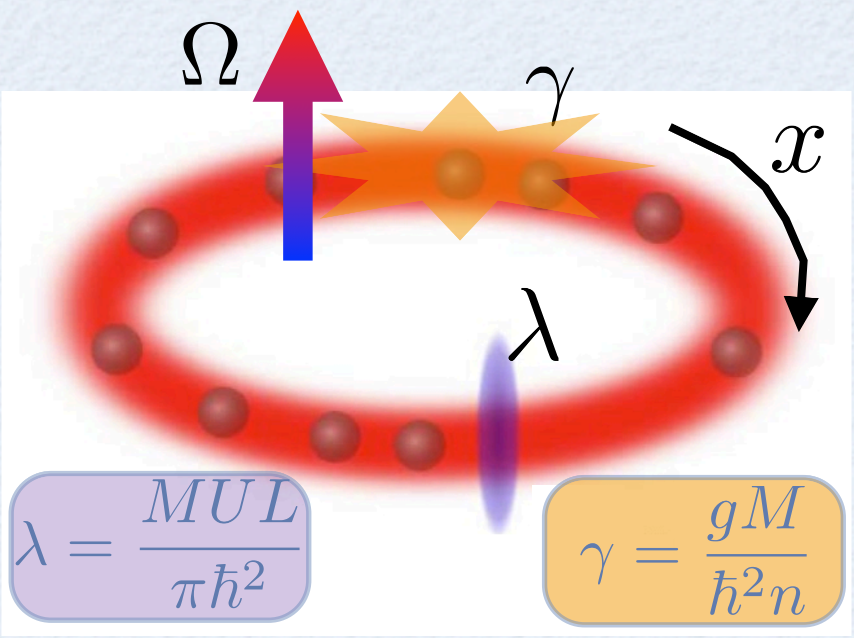
The system Hamiltonian

Definition

$$\mathcal{H} = \sum_{j=1}^{N} \left[\frac{\hbar^2}{2M} \left(-i \frac{\partial}{\partial x_j} - \frac{2\pi\Omega}{L} \right)^2 + U\,\delta(x_j) + g \sum_{l<j}^{N} \delta(x_l - x_j) \right]$$



- rotating frame <=> magnetic field
- ultracold bosons (T=0)
- 1D regime (no vortex instability)
- mesoscopic sizes (no TL, for now)

TARGET: Persistent current $I(\Omega) = -\frac{1}{2\pi\hbar}\frac{\partial E(\Omega)}{\partial \Omega}$ in all regimes of γ & λ

Bloch, PRB 2, 109 (1970)

for interacting fermions … *Loss, PRL 69, 343 (1992); Mueller-Gröeling et al., EPL 22, 193 (1993)*

and BEC-BCS crossover … *A. Spuntarelli, P. Pieri, and G. C. Strinati, PRL 99, 040401 (2007)*

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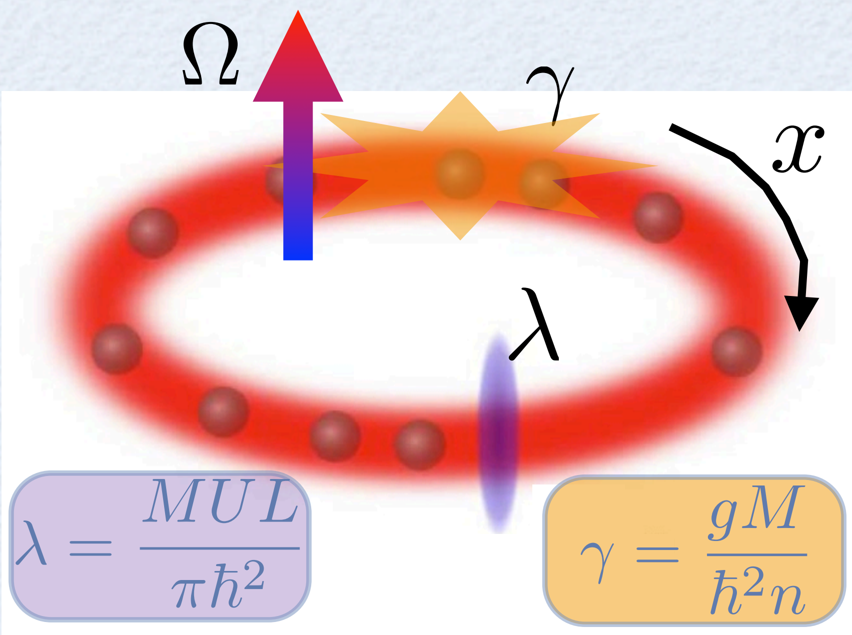

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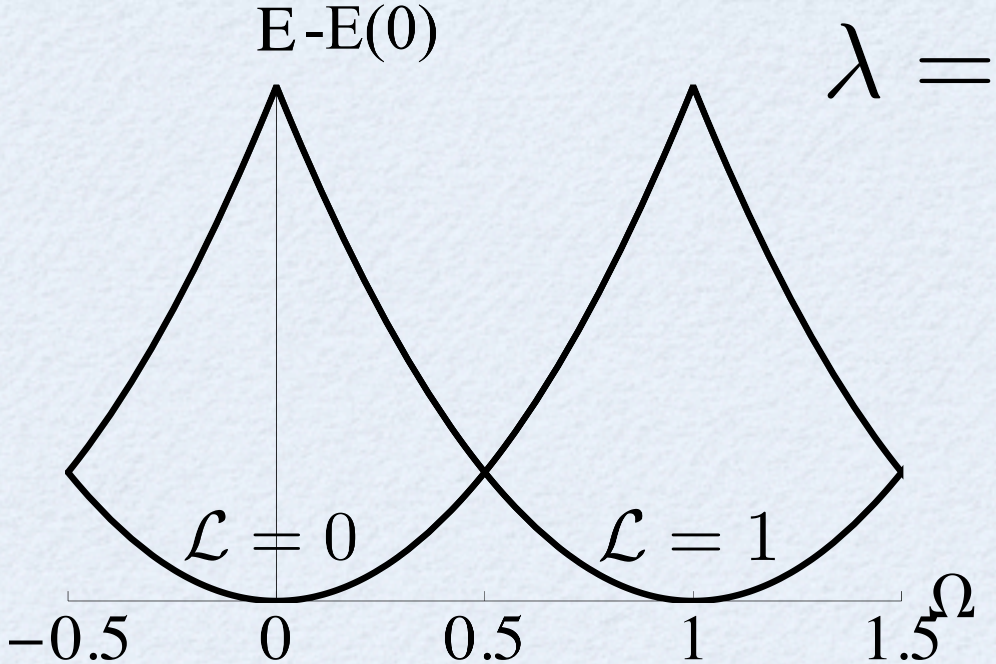
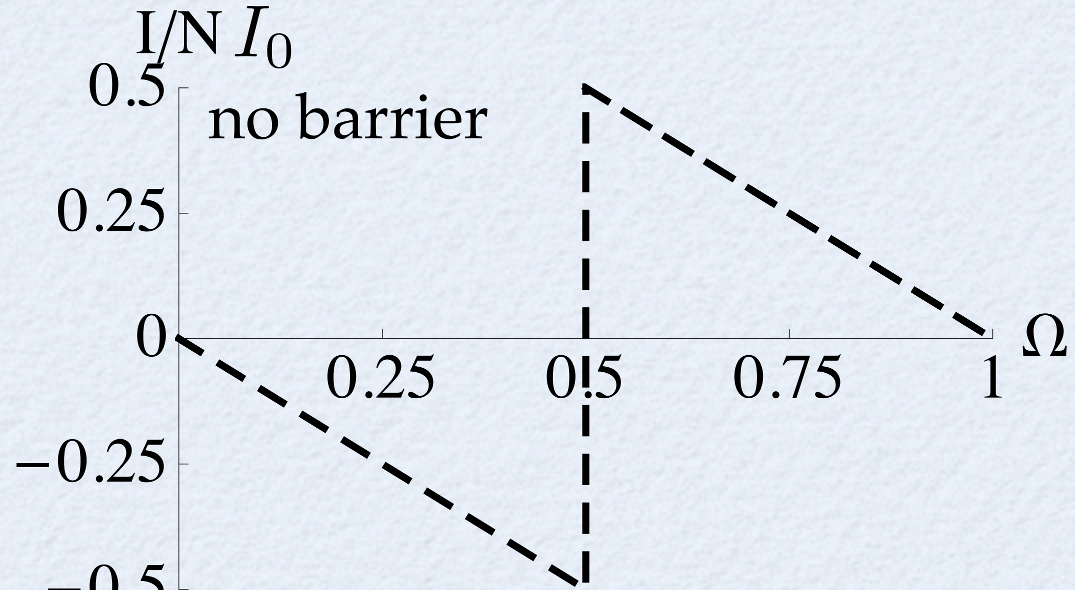
Absence of a barrier/defect

Setup



$\lambda = 0$



Rotational Invariance $[\mathcal{L}, \mathcal{H}] = 0$

↓

flux-independent "internal" energy

$$\partial_\Omega \langle \mathcal{H}_\gamma(\Omega) \rangle = 0$$

↓

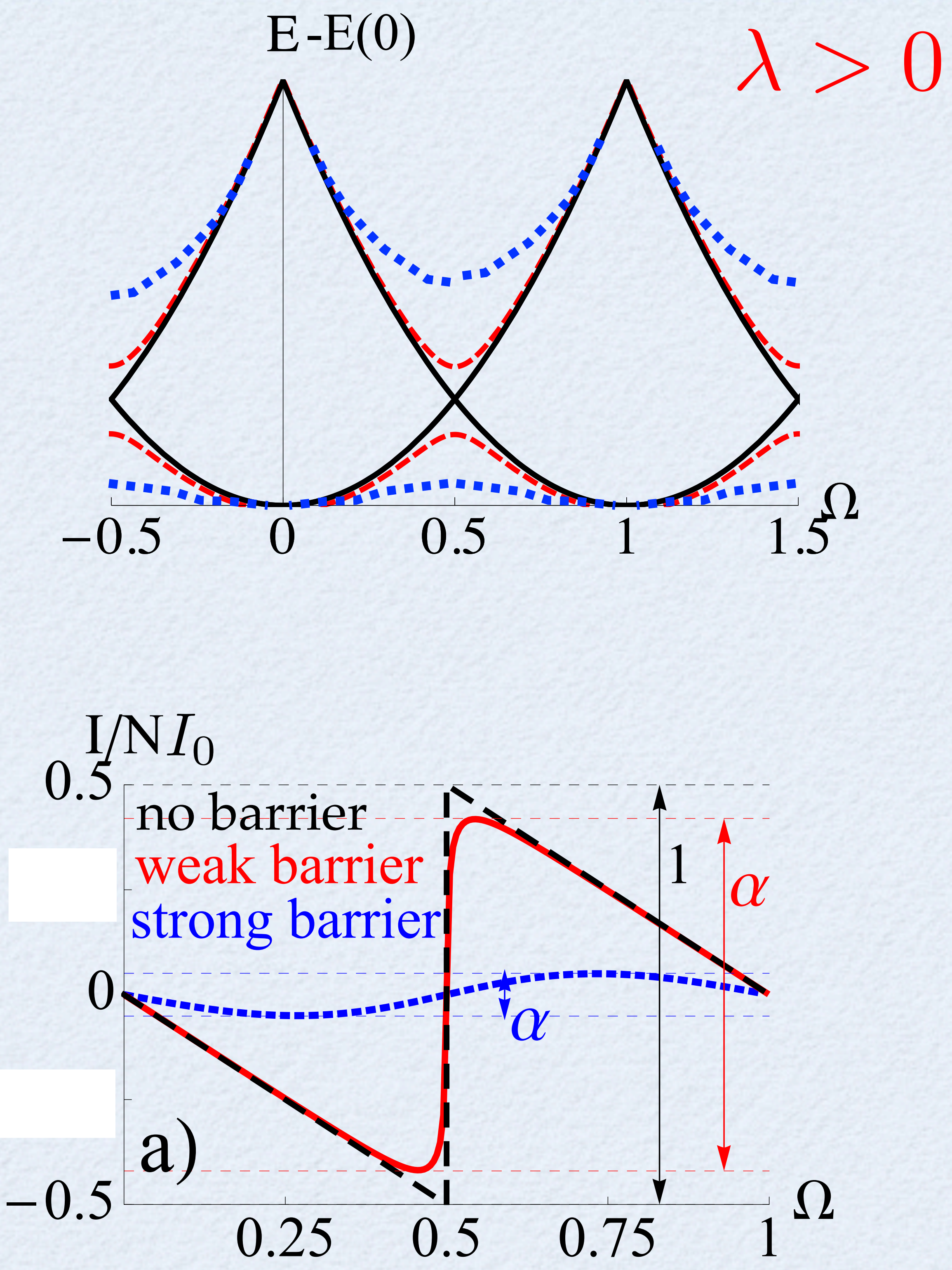
interaction-independent current

$$\partial_\gamma I(\Omega) = 0$$

↓

sawtooth amplitude $I_0 = \frac{2\pi\hbar}{mL^2}$

Presence of a barrier/defect



gap opening due to U(1) breaking

↓

flux- dependent "internal" energy

$$\partial_\Omega \langle \mathcal{H}_\gamma(\Omega) \rangle \neq 0$$

↓

interaction- dependent current

$$\partial_\gamma I(\Omega) \neq 0$$

↓

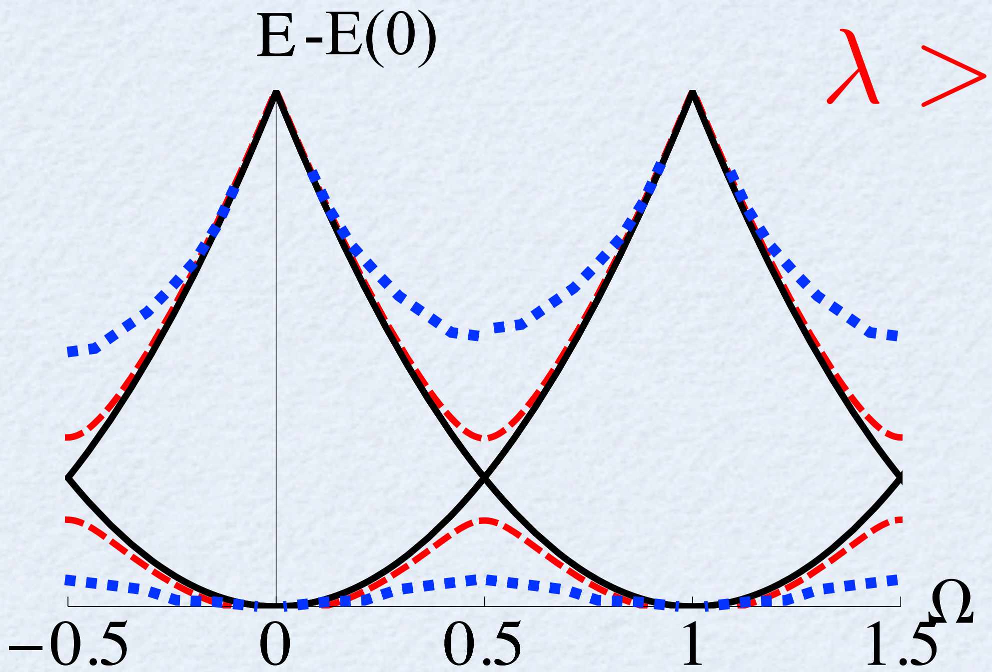
relative amplitude

$$\alpha(\lambda,\gamma) = I_{\max}/N I_0$$

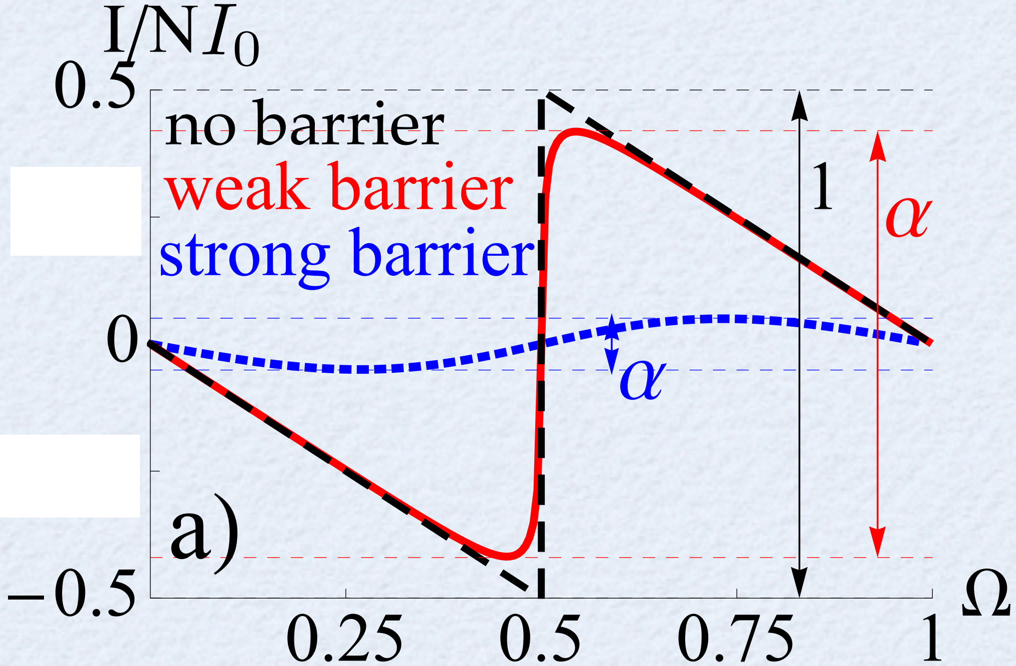
$$I_0 = \frac{2\pi\hbar}{mL^2}$$

Presence of a barrier/defect

Setup



$\lambda > 0$



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$$I_0 = \frac{2\pi\hbar}{mL^2}$$

HERE: adiabatic raising of barrier & focus on stationary regime

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Single-particle regimes

Analytic

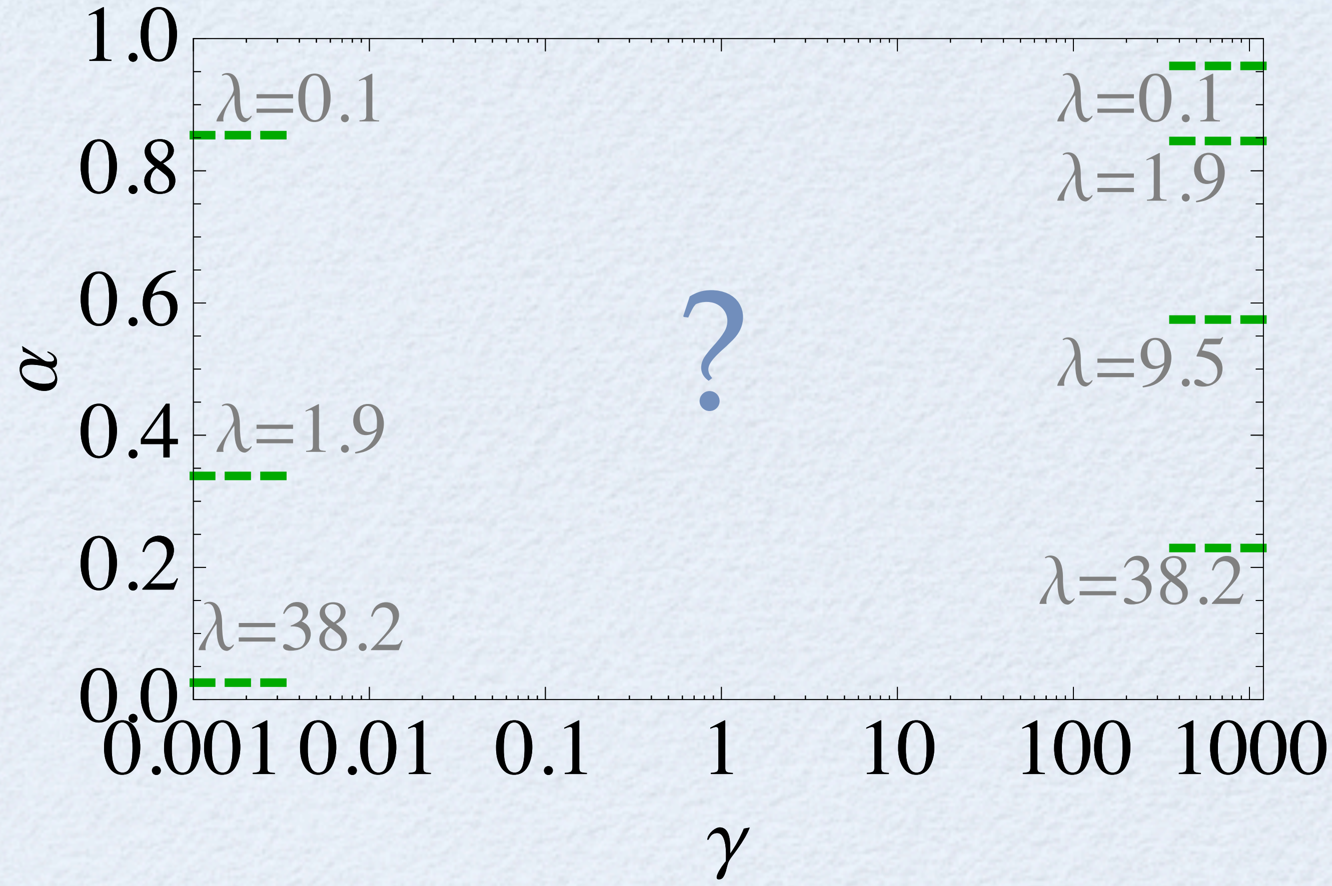
✓ plane waves + twisted b.c. + cusp @ barrier

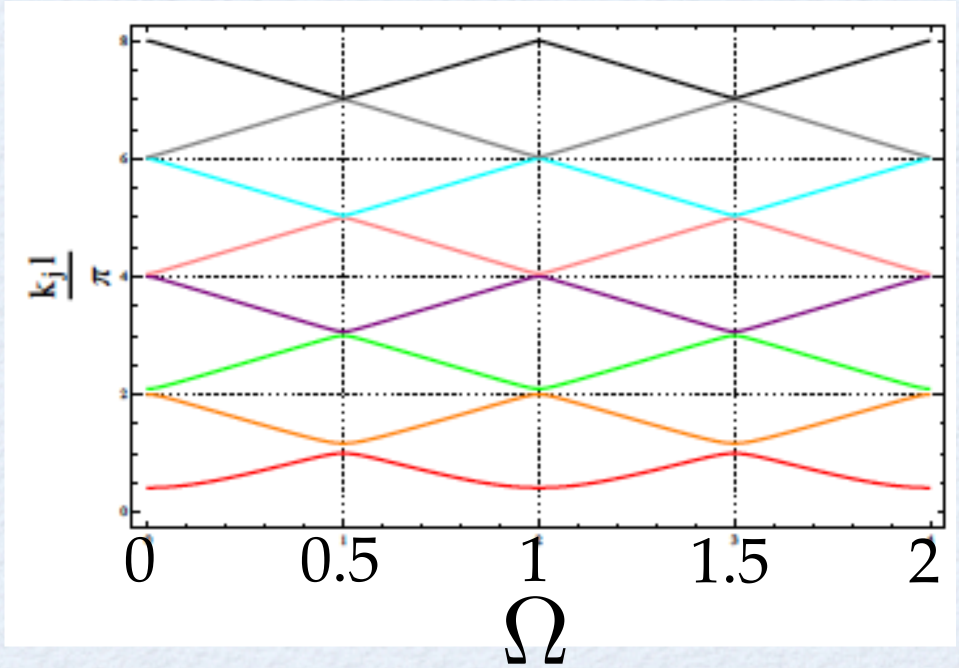
$$k_n = \pm\lambda \frac{\pi}{L} \frac{\sin(k_n L)}{\cos(2\pi\Omega) \mp \cos(k_n L)}$$

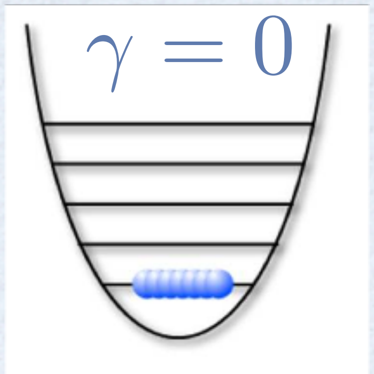
$$\varepsilon_n = \hbar^2 k_n^2 / 2m$$

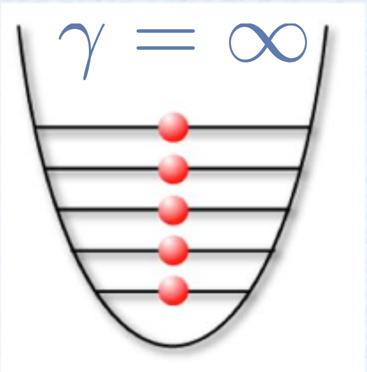
low-lying k's are most affected !

$$\alpha(\lambda, \gamma = 0) < \alpha(\lambda, \gamma = \infty) \quad \forall\lambda$$









$E = N\varepsilon_0$

$E = \sum_{n=0}^{N-1} \varepsilon_n$

Weakly interacting regime

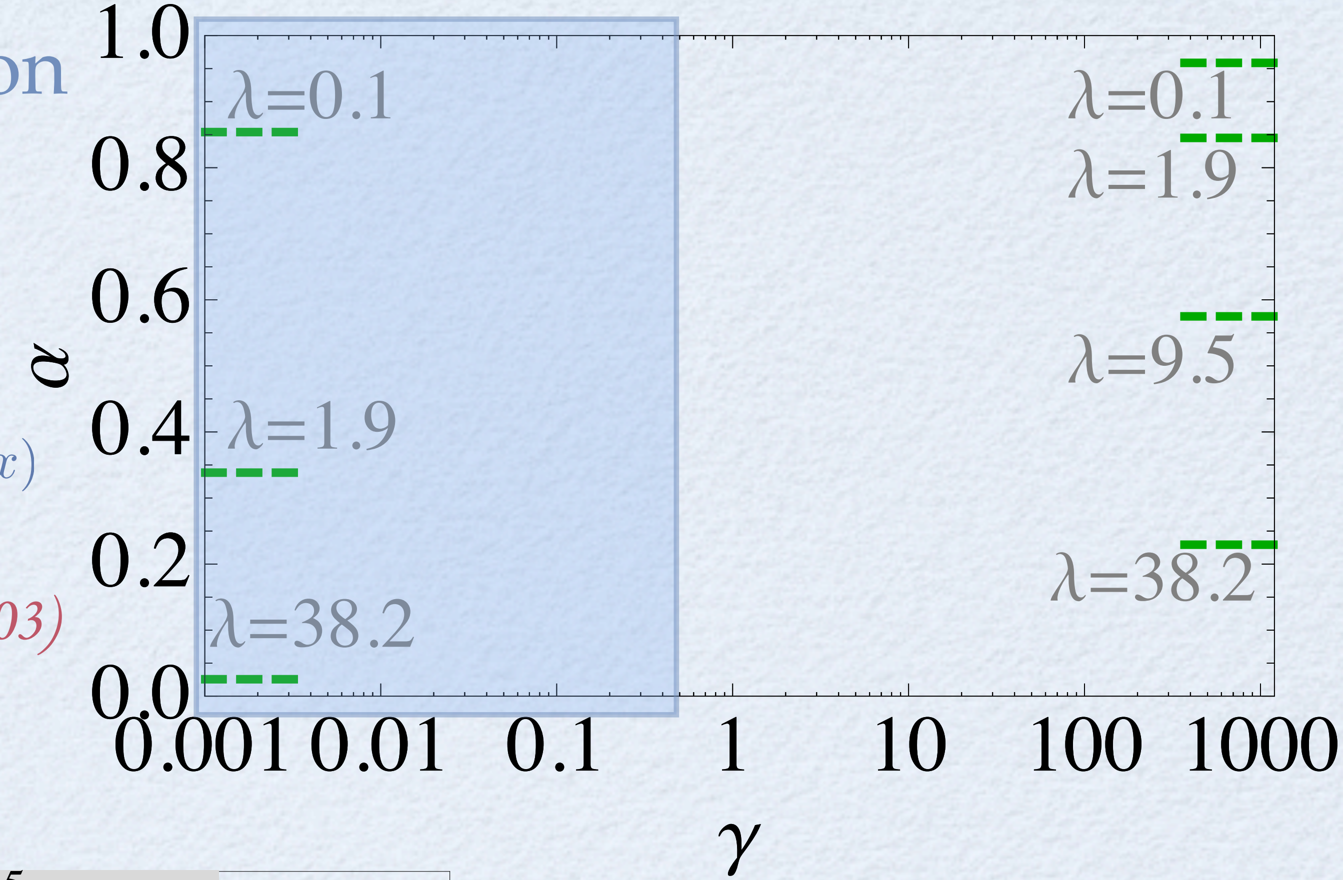
Analytic

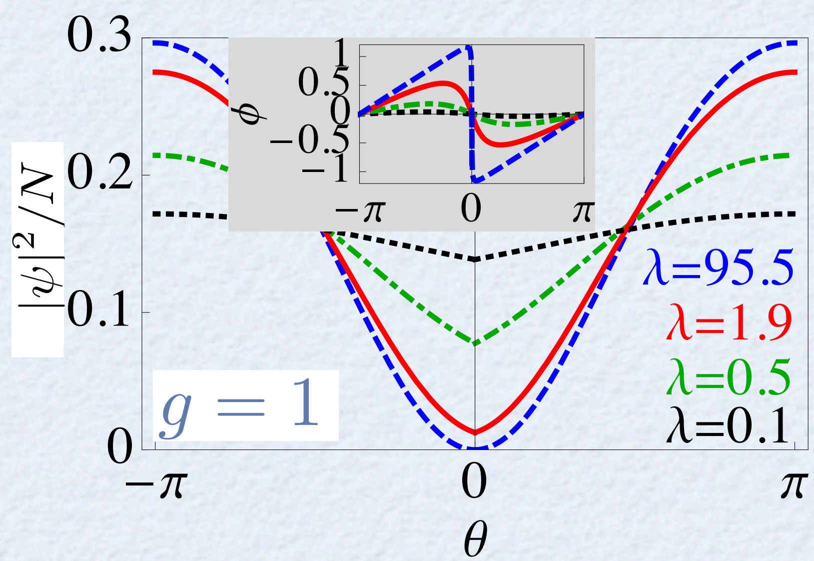
✓ mean-field: Gross-Pitaevskii equation

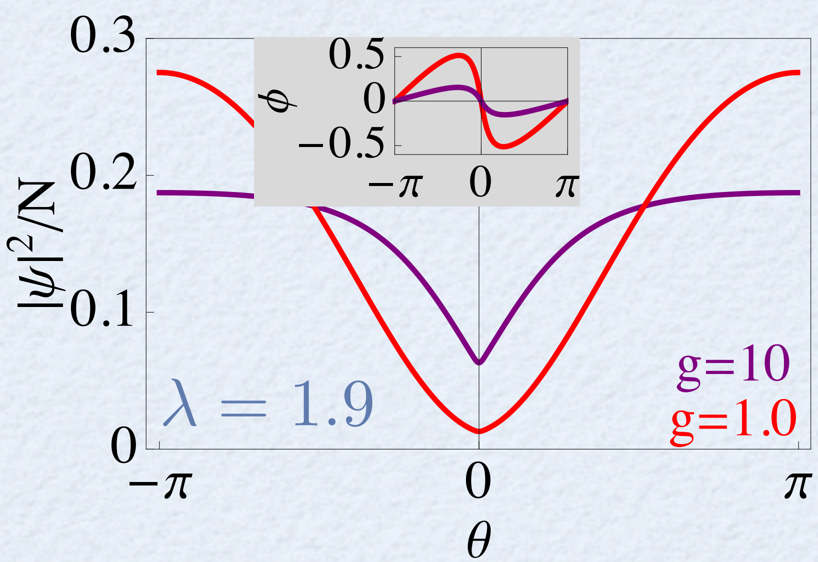
$$\langle \hat{\psi}(x) \rangle = \Psi(x) = |\Psi(x)|\, e^{i\phi(x)}$$

$$\frac{\hbar^2}{2M}\left[\left(-i\partial_x - \frac{2\pi}{L}\Omega\right)^2 + \lambda\delta(x) + \tilde{g}|\Psi(x)|^2\right]\Psi(x) = \mu\Psi(x)$$

Pitaevskii & Stringari, Bose-Einstein Cond., Oxford (2003)







healing length

$$\xi = \hbar/\sqrt{2mn_0 g}$$

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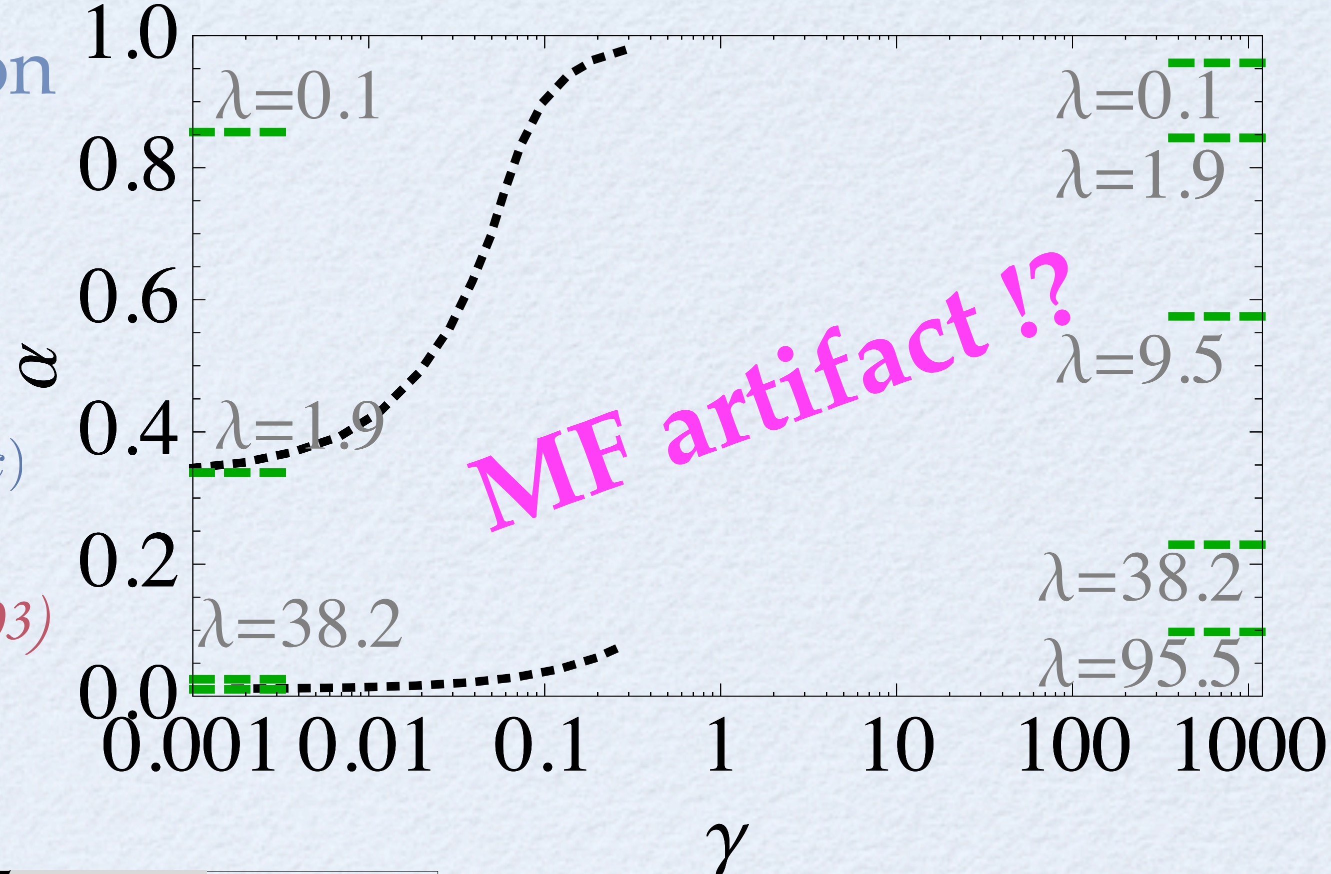
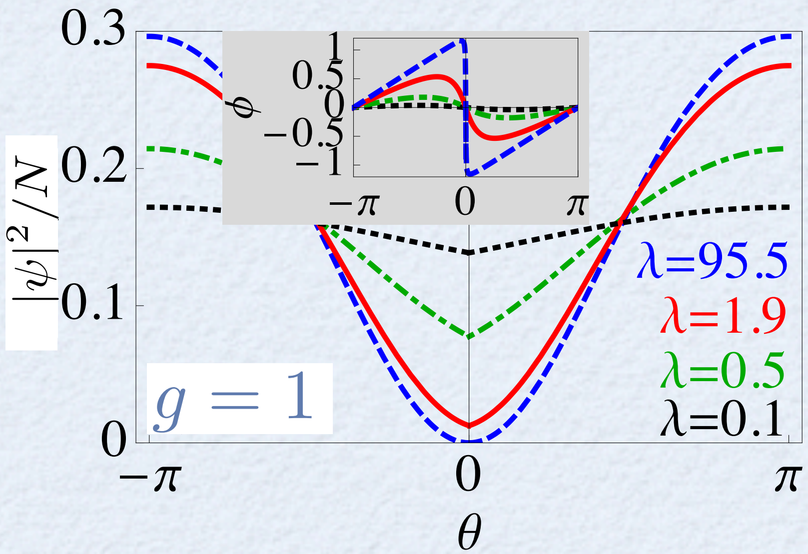
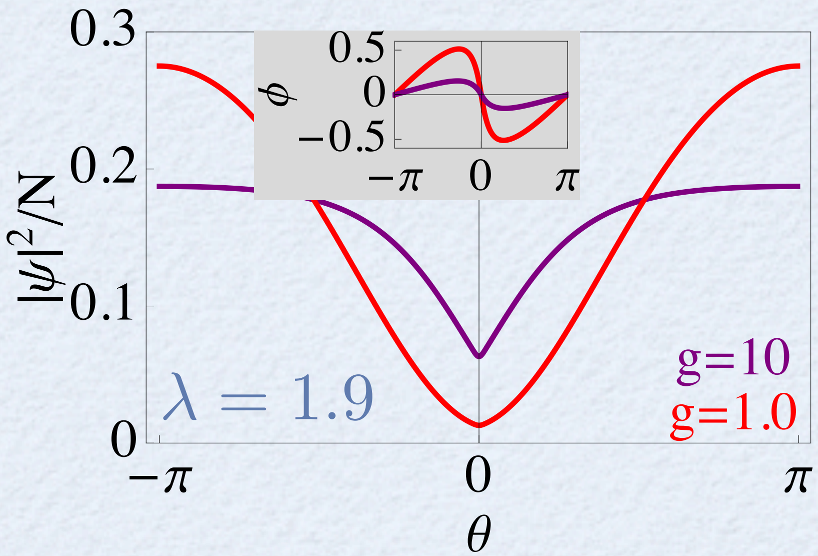






healing length

$$\xi = \hbar/\sqrt{2mn_0 g}$$

deeper density hole → cheaper phase-slip → lower current !

Strongly interacting regime

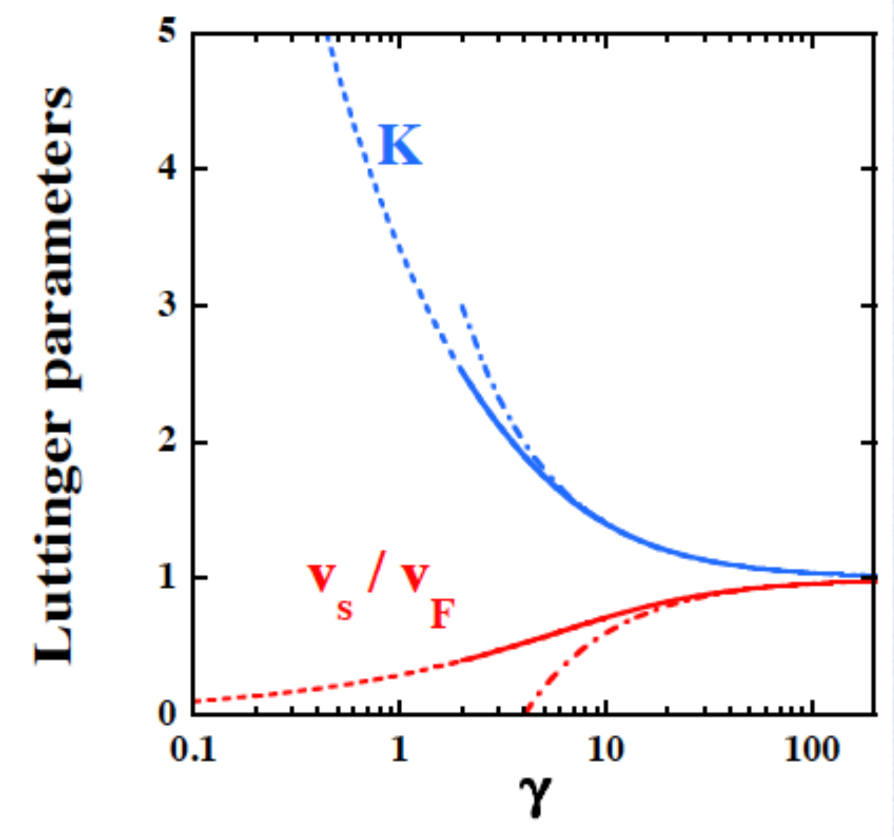
✓ effective field theory: Luttinger liquid

$$\psi(x) = \sqrt{\rho(x)}e^{i\phi(x)} \qquad \omega(k) \simeq \hbar v_s |k|$$

$$\rho(x) \simeq (n_0 + \partial_x\theta(x)/\pi)\sum_{l\in\mathbb{Z}} e^{2il(\theta(x)+\pi n_0 x)} \qquad n_0 = N/L$$

$$[\partial_x\theta(x), \phi(x')] = i\pi\delta(x-x')$$

Cazalilla, J. Phys. B: At. Mol. Opt. Phys. 37, S1 (2004)

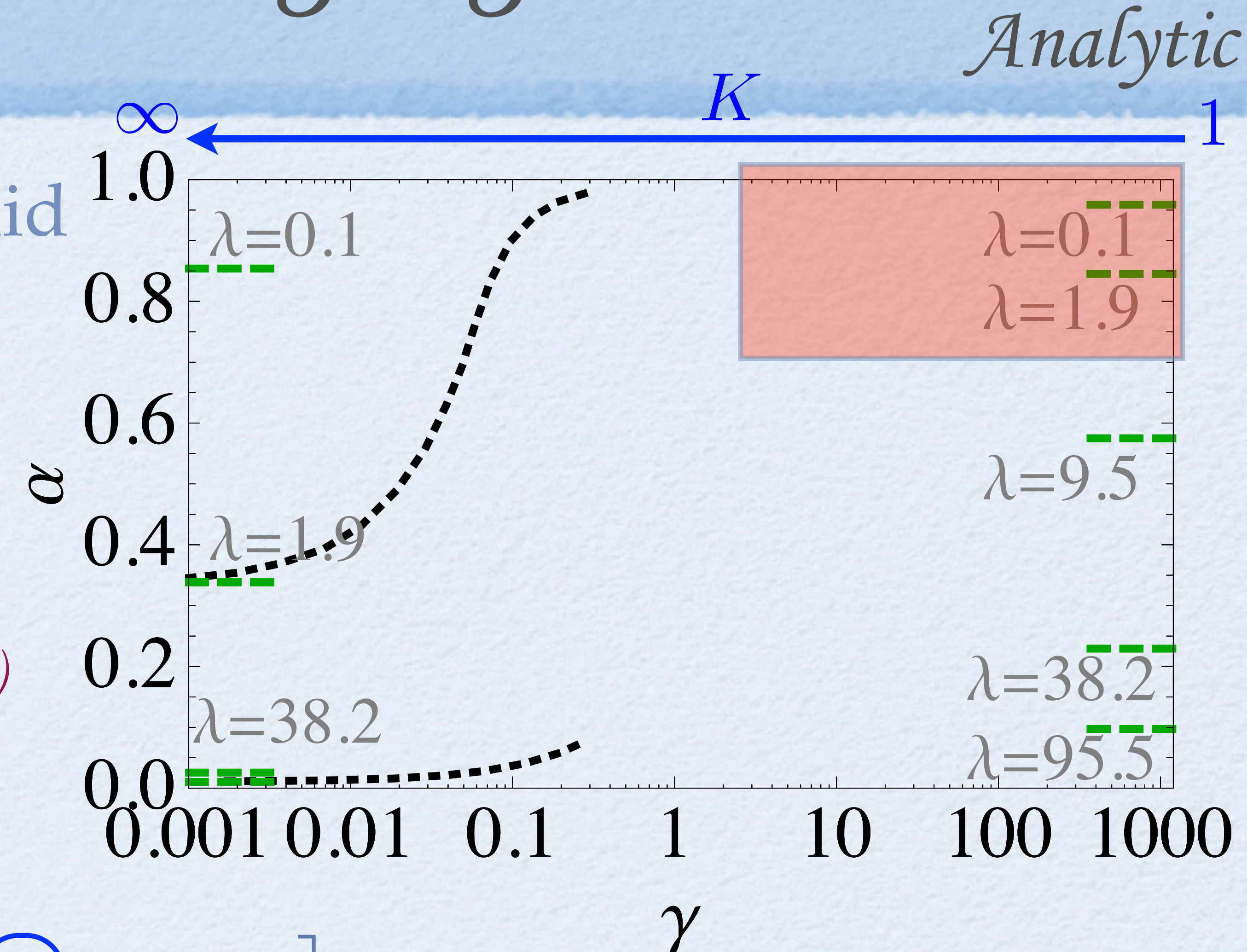


✓ presence of gauge field

$$\mathcal{H}_{LL} = \frac{\hbar v_s}{2\pi}\int_0^L \mathrm{d}x \left[K\left(\partial_x\phi(x) - \frac{2\pi}{L}\Omega\right)^2 + \frac{1}{K}(\partial_x\theta(x))^2\right]$$

$$\langle\hat{\Psi}^\dagger(x)\hat{\Psi}(x')\rangle \simeq |x-x'|^{-1/2K}$$

Strongly interacting regime



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✓ weak barrier ~ backscattering term → average over density fluct.

$$\mathcal{H}_J = E_0(J-\Omega)^2 + n_0 U_{\text{eff}} \sum_J |J+1\rangle\langle J| + h.c.$$

$$U_{\text{eff}} = U_0\langle e^{\pm i2\delta\theta(0)}\rangle \simeq U_0(d/L)^K$$

Strongly interacting regime

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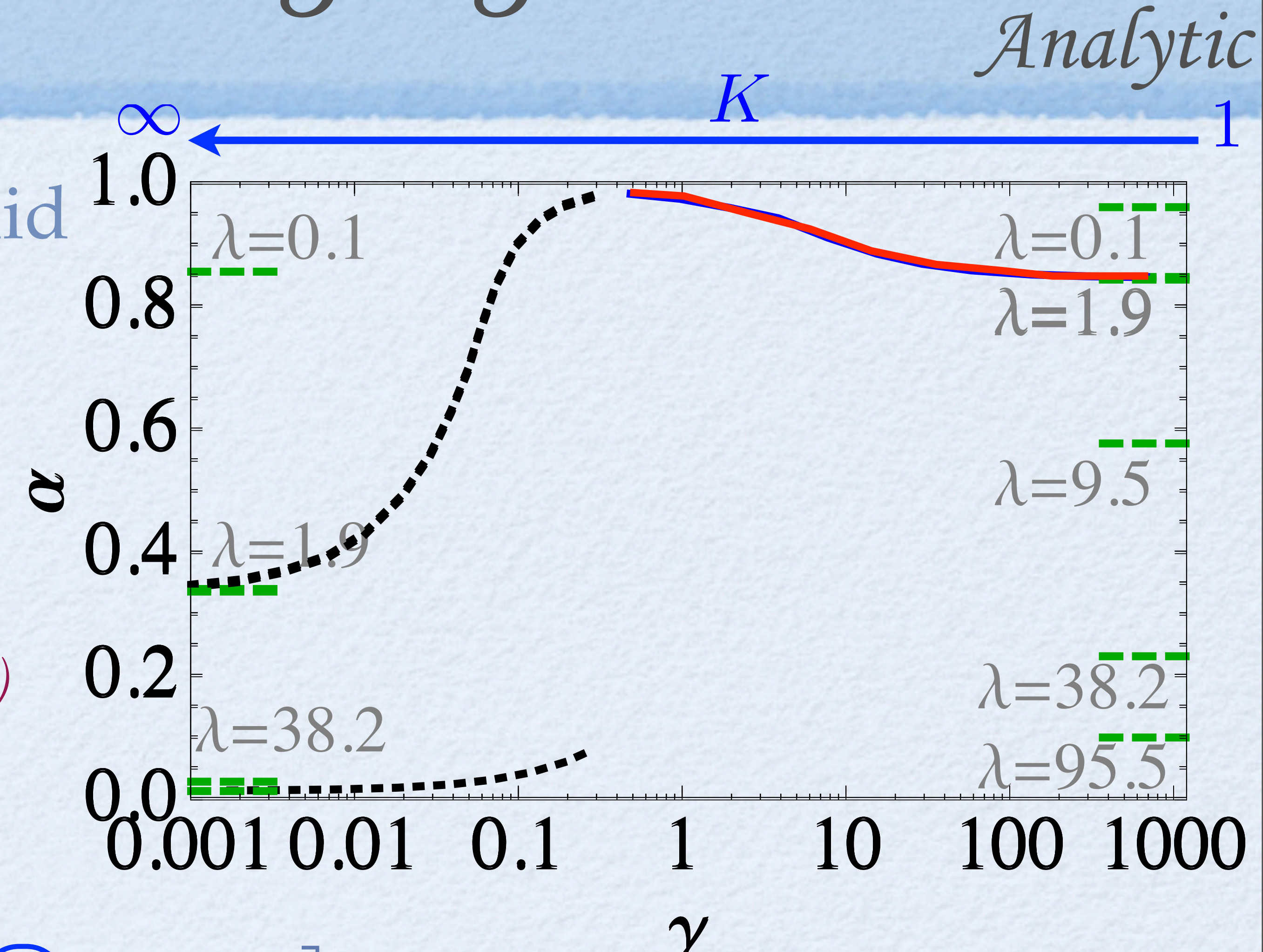

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$$U_{\rm eff} = U_0\langle e^{\pm i2\delta\theta(0)}\rangle \simeq U_0(d/L)^K$$

weaker γ
higher K → stronger $\delta\theta(x)$ → more screening → higher current

Strongly interacting regime

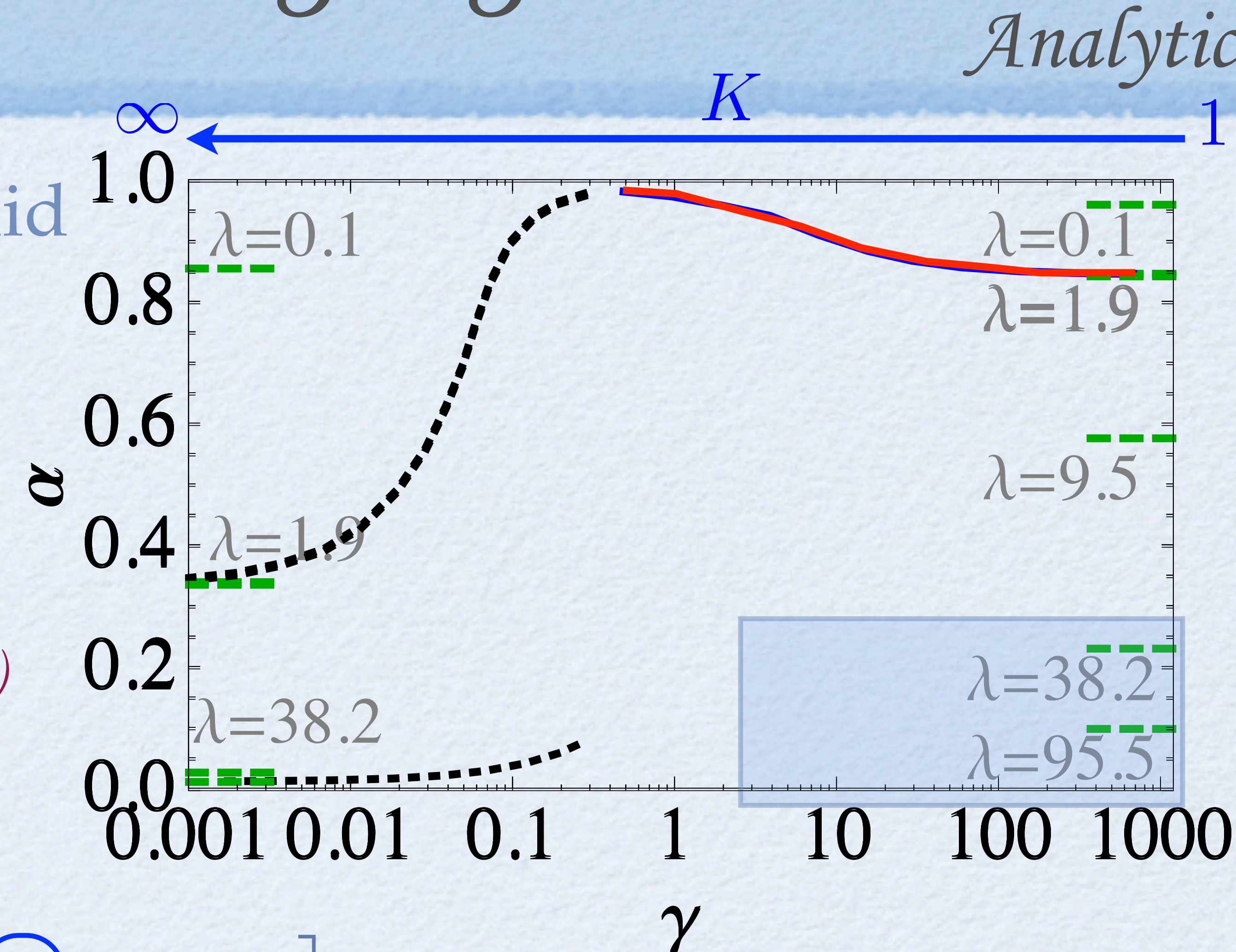
✓ effective field theory: Luttinger liquid

$$\psi(x) = \sqrt{\rho(x)} e^{i\phi(x)}$$

$$\rho(x) \simeq (n_0 + \partial_x \theta(x)/\pi) \sum_{l \in \mathbb{Z}} e^{2il(\theta(x)+\pi n_0 x)}$$

$$[\partial_x \theta(x), \phi(x')] = i\pi\delta(x - x')$$

Cazalilla, J. Phys. B: At. Mol. Opt. Phys. 37, S1 (2004)



✓ presence of gauge field

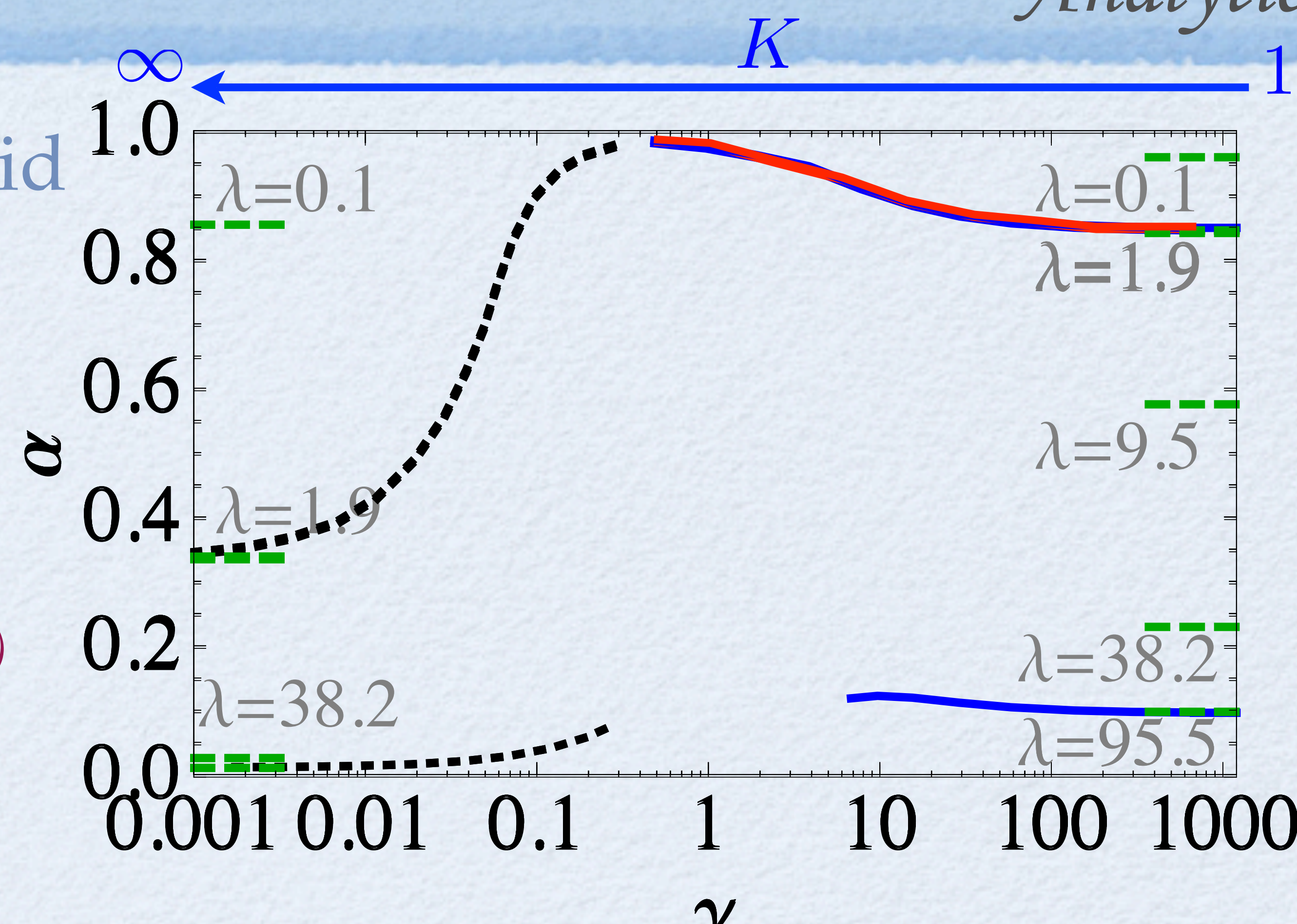
$$\mathcal{H}_{LL} = \frac{\hbar v_s}{2\pi} \int_0^L \mathrm{d}x \left[K \left(\partial_x \phi(x) - \frac{2\pi}{L}\Omega \right)^2 + \frac{1}{K} (\partial_x \theta(x))^2 \right] - 2\, t\, n_0 \cos[\phi(L) - \phi(0) + 2\pi\Omega]$$

✓ strong barrier ~ weak link tunnelling → average over phase fluct.

$$E(\Omega) = -2\, t_{\text{eff}}\, n_0 \cos(2\pi\Omega)$$

$$t_{\text{eff}} = t \langle \cos[\delta\phi(L) - \delta\phi(0)] \rangle \simeq t(d/L)^{1/K}$$

Strongly interacting regime



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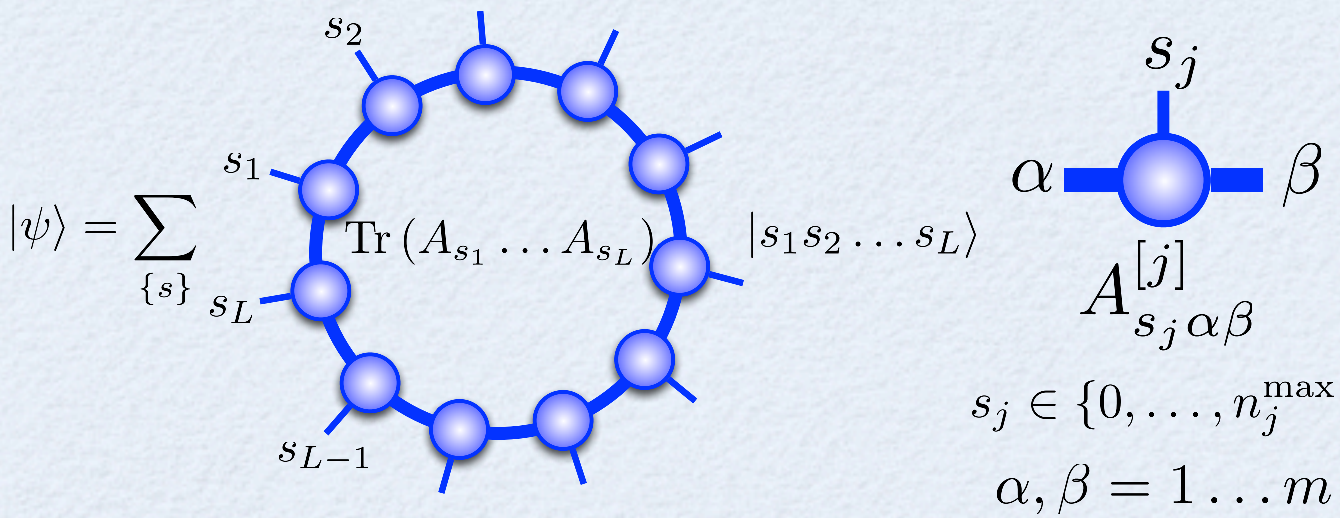
higher K → weaker $\delta\phi(x)$ → more coherence → higher current

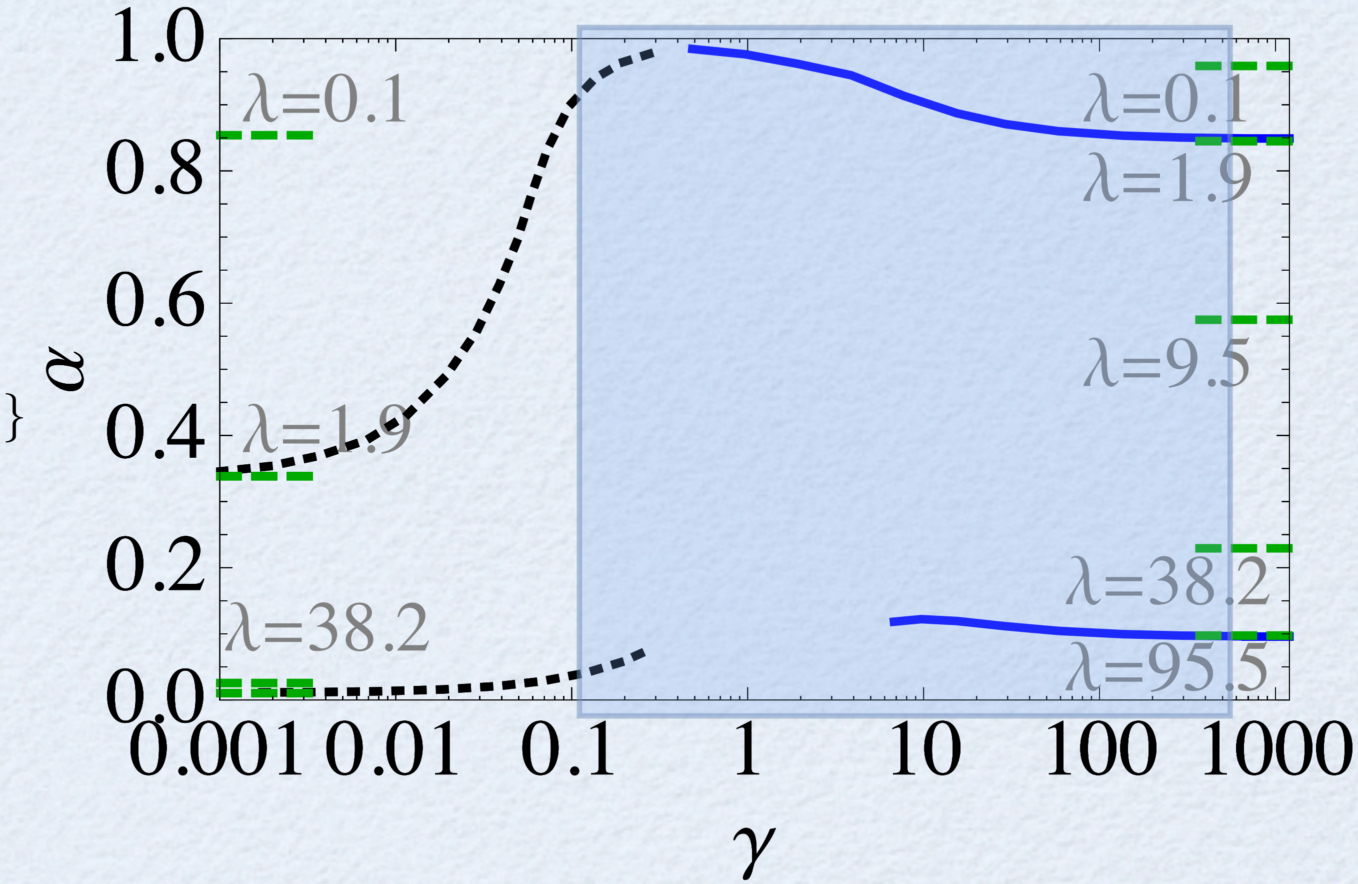
MPS variational ansatz

Numeric



$s_j \in \{0, \ldots, n_j^{\max}\}$

$\alpha, \beta = 1 \ldots m$



Verstraete, et al, PRL 93, 227205 (2004);
Schollwock, Ann. Phys. 326, 96 (2011);

$O(Ldm^2)$ vs. $O(d^L)$ parameters

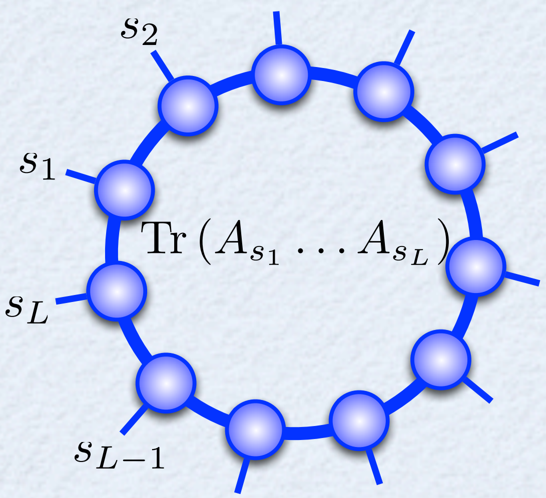
Bose-Hubbard-Peierls model @ low filling (here <n> ~ 0.15 …)

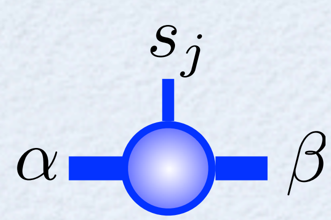
$$\mathcal{H}_{\rm lat} = -t_{\rm BH}\sum_{j=1}^{N_s}\left(e^{-\frac{2\pi i\Omega}{N_s}} b_j^\dagger b_{j+1} + \text{H.c.}\right) + \frac{U_{\rm BH}}{2}\sum_{j=1}^{N_s} n_j(n_j-1) + \sum_j \left(\lambda_{\rm BH}\,\delta_{j,1} n_j - \mu\, n_j\right)$$

Pippan, et al. PRB 81, 081103(R) (2010); Rossini, et al., J. Stat. Mech., P05021 (2011); Weyrauch, Rakov, arXiv:1303.1333

MPS variational ansatz

$$|\psi\rangle = \sum_{\{s\}} \mathrm{Tr}\left(A_{s_1} \ldots A_{s_L}\right) |s_1 s_2 \ldots s_L\rangle$$





$A^{[j]}_{s_j \alpha\beta}$

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Verstraete, et al, PRL 93, 227205 (2004);
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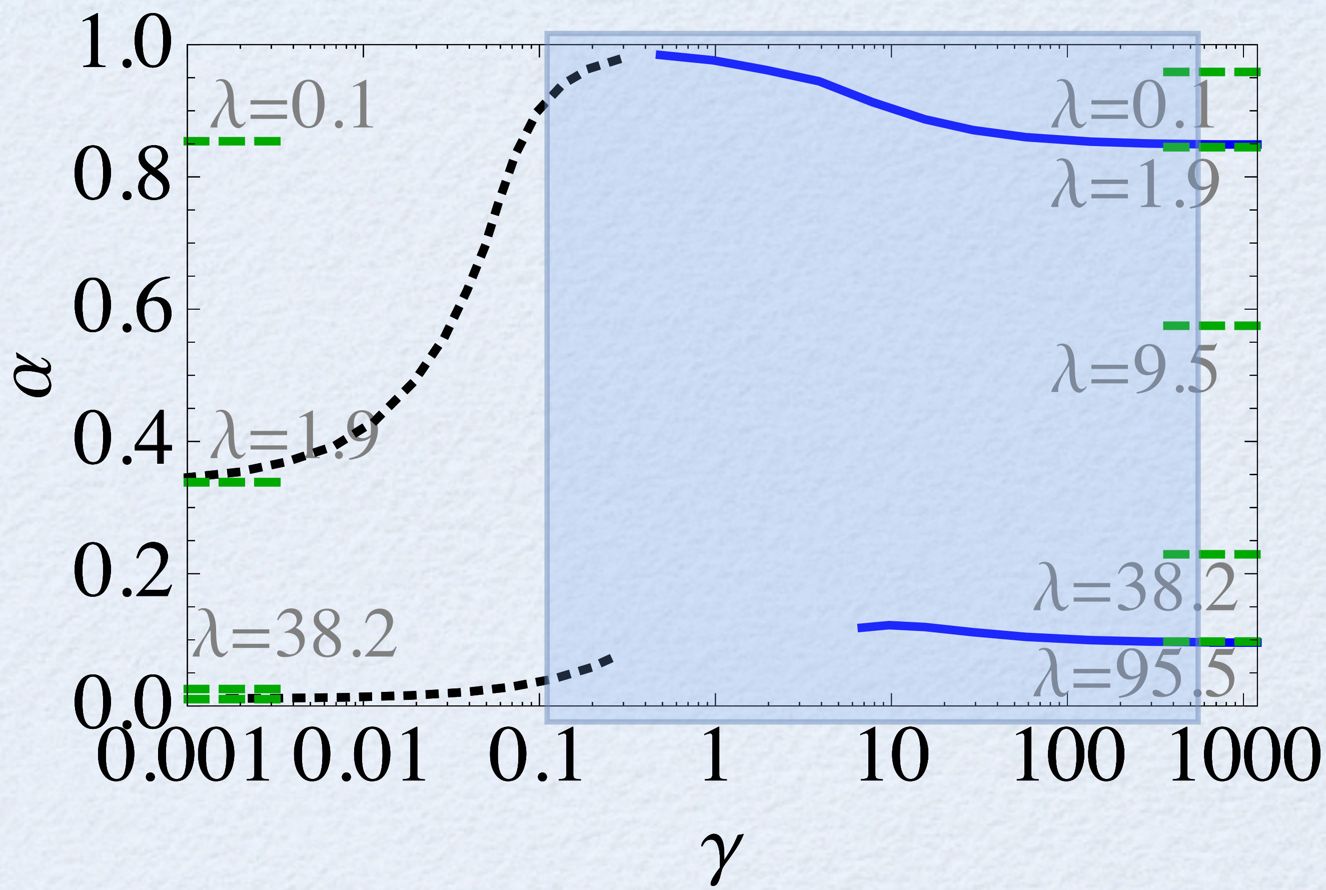

◉ absence of an isometric gauge
==> generalized eigenvalue problem

presence of an explicit loop →

◉ less agile number conservation ...

◉ some tricks for long chains: $O(pm^3)$ vs. $O(m^5)$?
keep p eivals/eivecs of transfer matrix ...
... p often scales like $O(m)$:(

Pippan, et al. PRB 81, 081103(R) (2010);
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MPS variational ansatz

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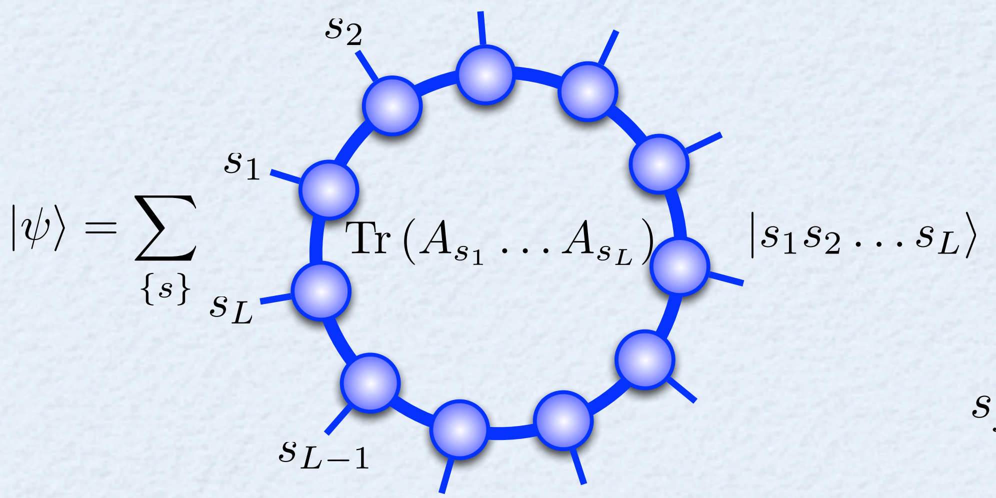

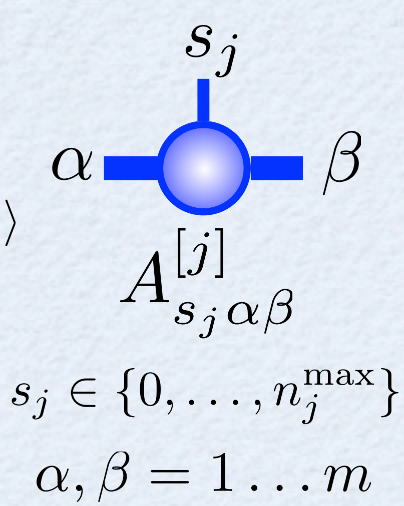

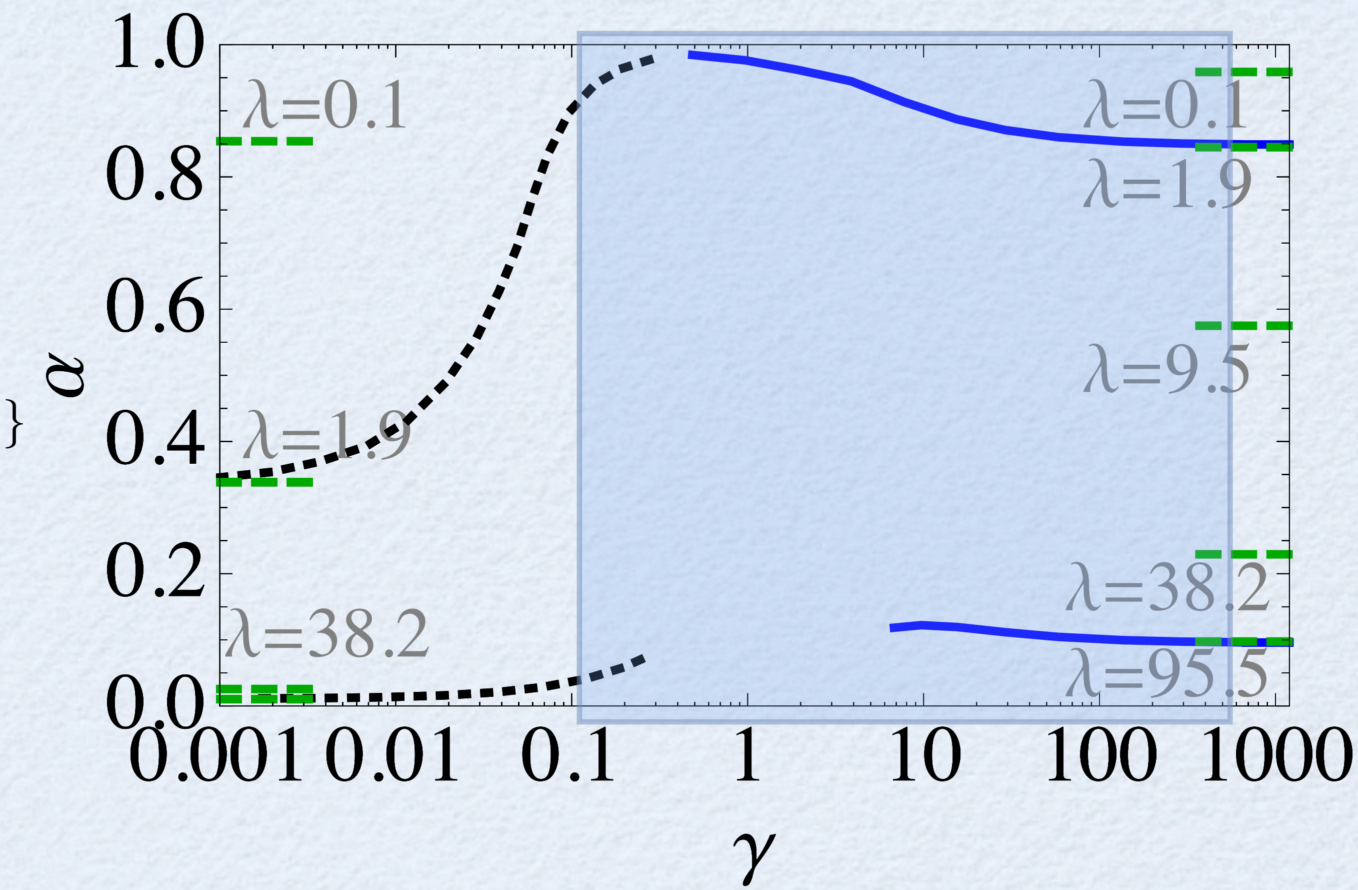

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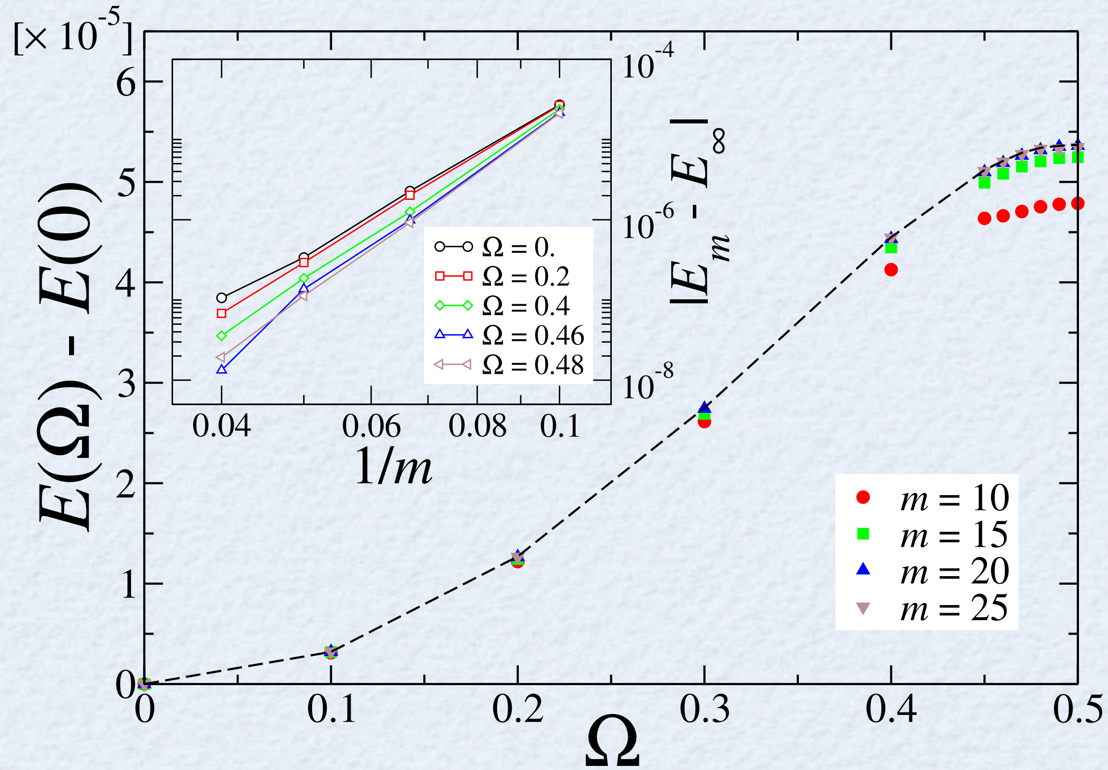

- absence of an isometric gauge
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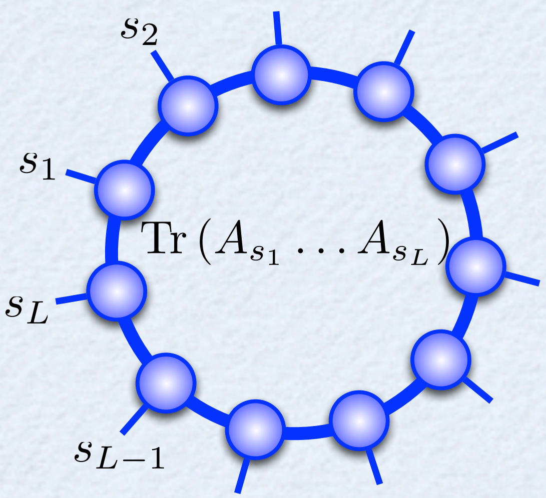
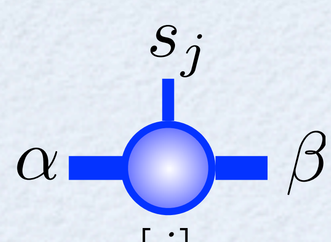
MPS variational ansatz

Numeric

$$|\psi\rangle = \sum_{\{s\}} \mathrm{Tr}\left(A_{s_1}\dots A_{s_L}\right)|s_1 s_2 \dots s_L\rangle$$





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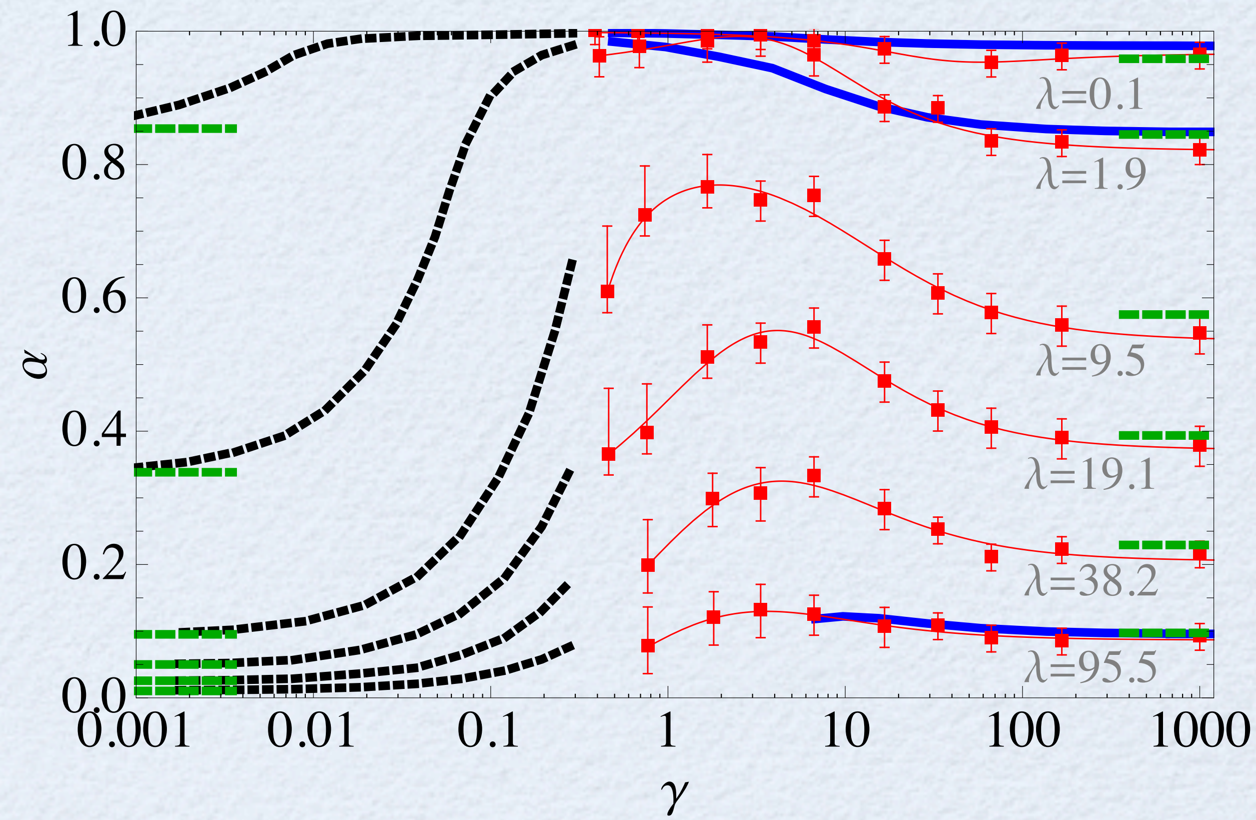

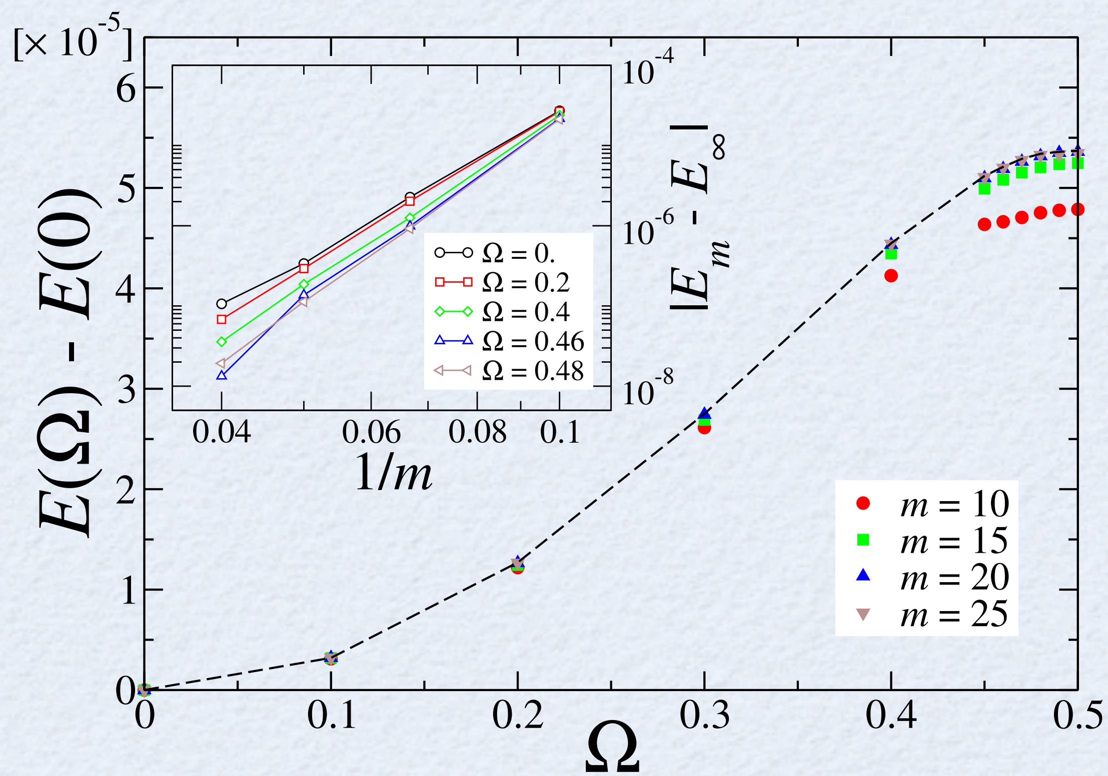

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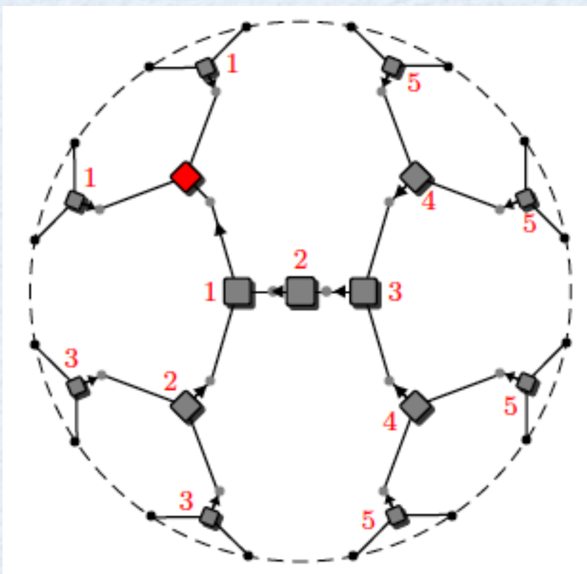
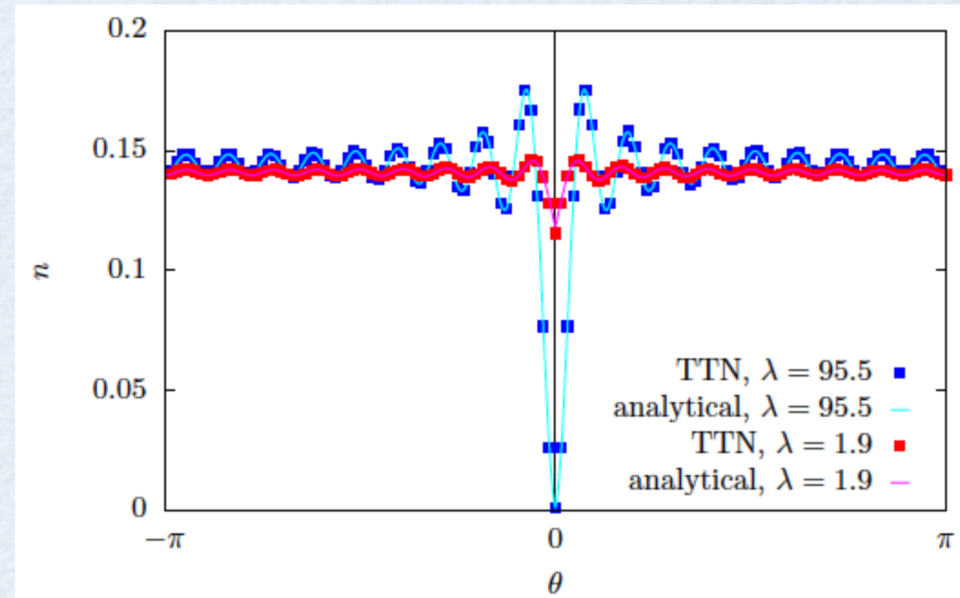
TTN variational ansatz

Numeric



binary **T**ree **T**ensor **N**etwork

Shi, Duan, Vidal, PRA 74, 022320 (2006);
A. J. Ferris, PRB 87, 125139 (2013)



absence of explicit loops

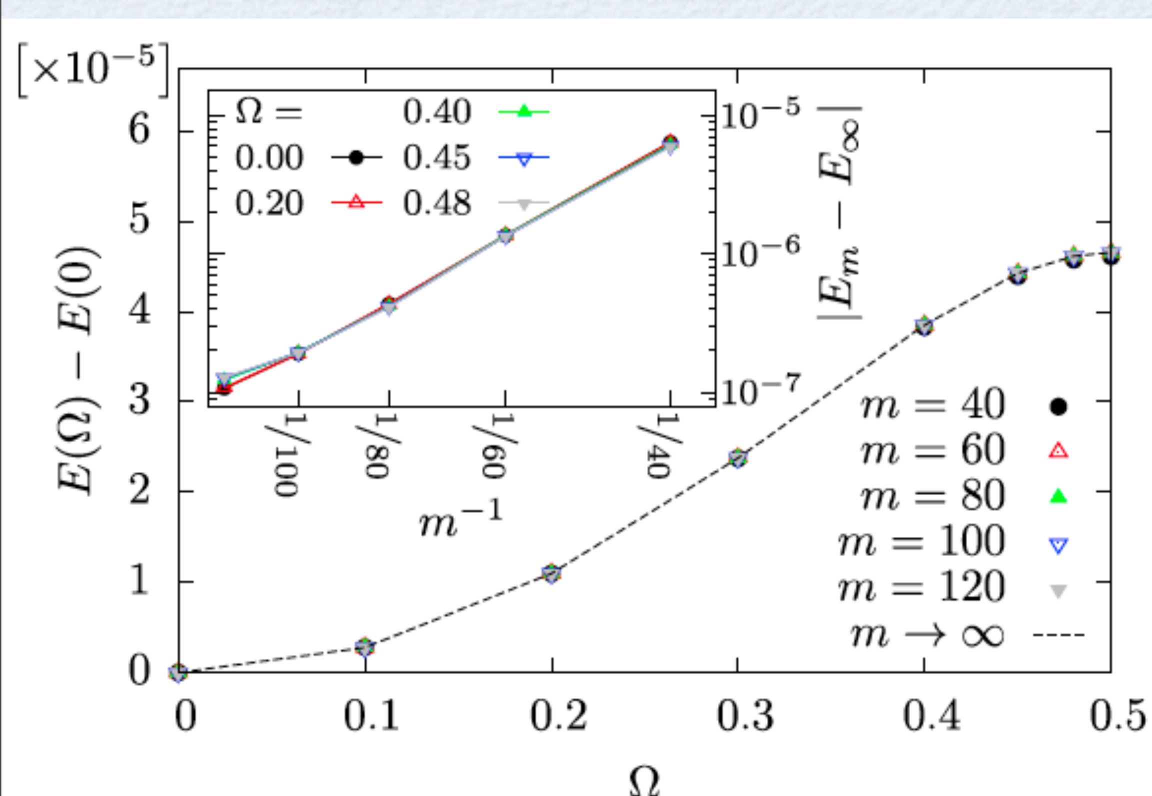
✓ possibility of an isometric gauge
==> standard eigenvalue problem

✓ symmetries implemented as usual !!

✓ computational cost $O(m^4)$ for obc / pbc :)

✓ fight entanglement clusterization by high m

M. Gerster, MR, et al. PRB 90, 125154 (2014)



OUTLINE

- Introduction
- Definition of the problem
- Analytical & numerical treatment
- Conclusions & open problems

Take-Home message

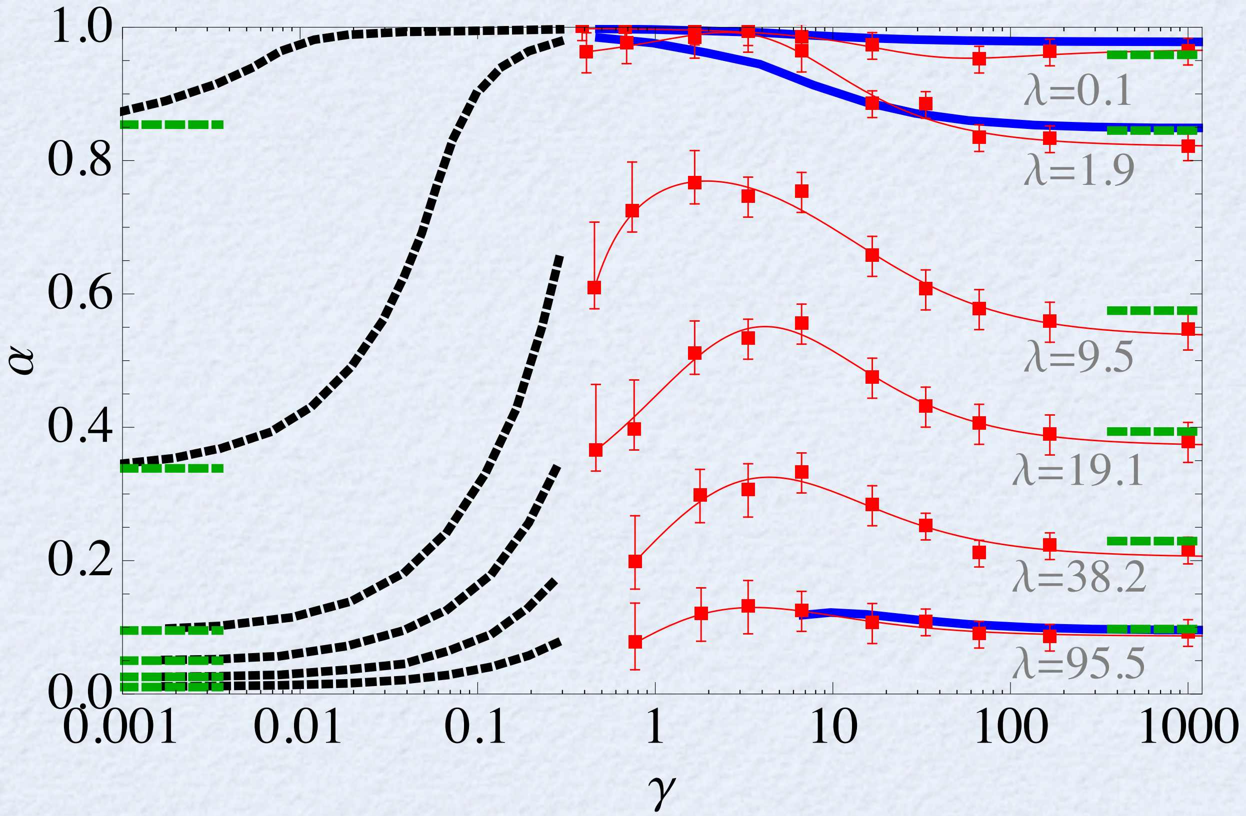
Conclusion

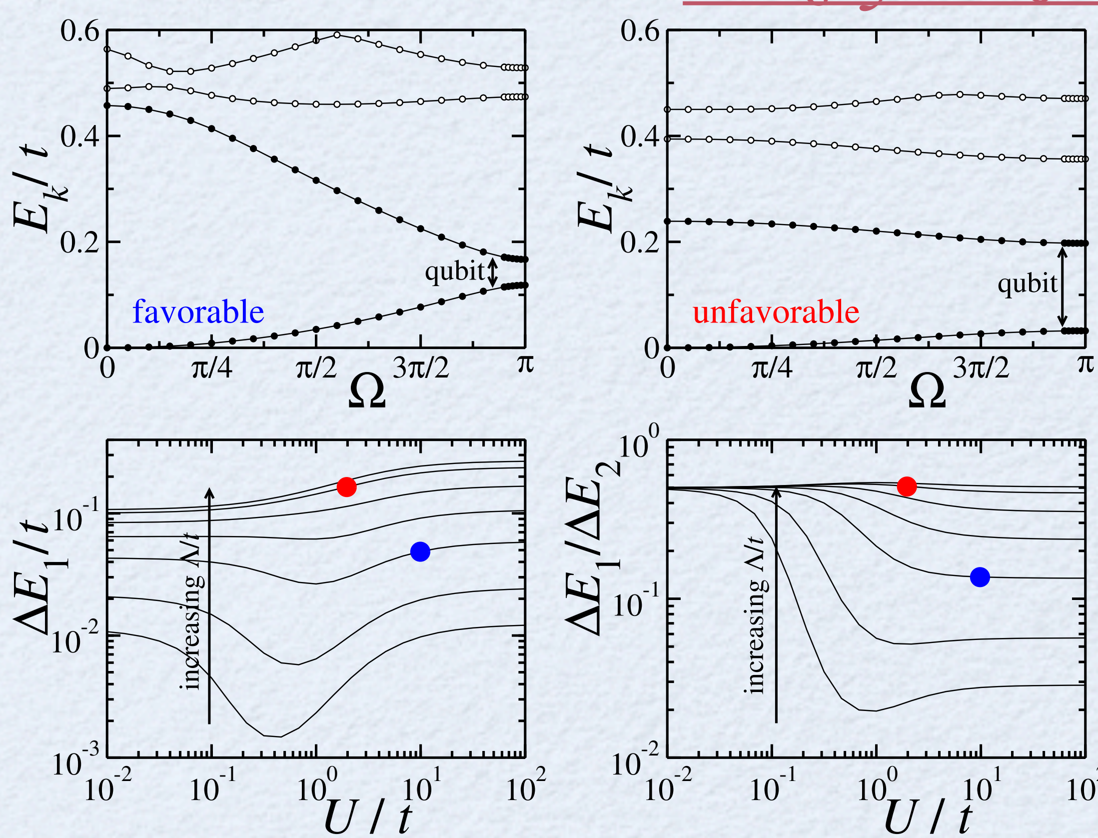
screening by interactions *vs.* quantum phase fluctuations

OPTIMAL REGIME:

- least influence by defects at $\gamma \simeq 1$ [also for scaling $I(L)$]
- best quantum state manipulation at $\gamma \gg 1$ and $\lambda \simeq L$

Talk by D.Rossini

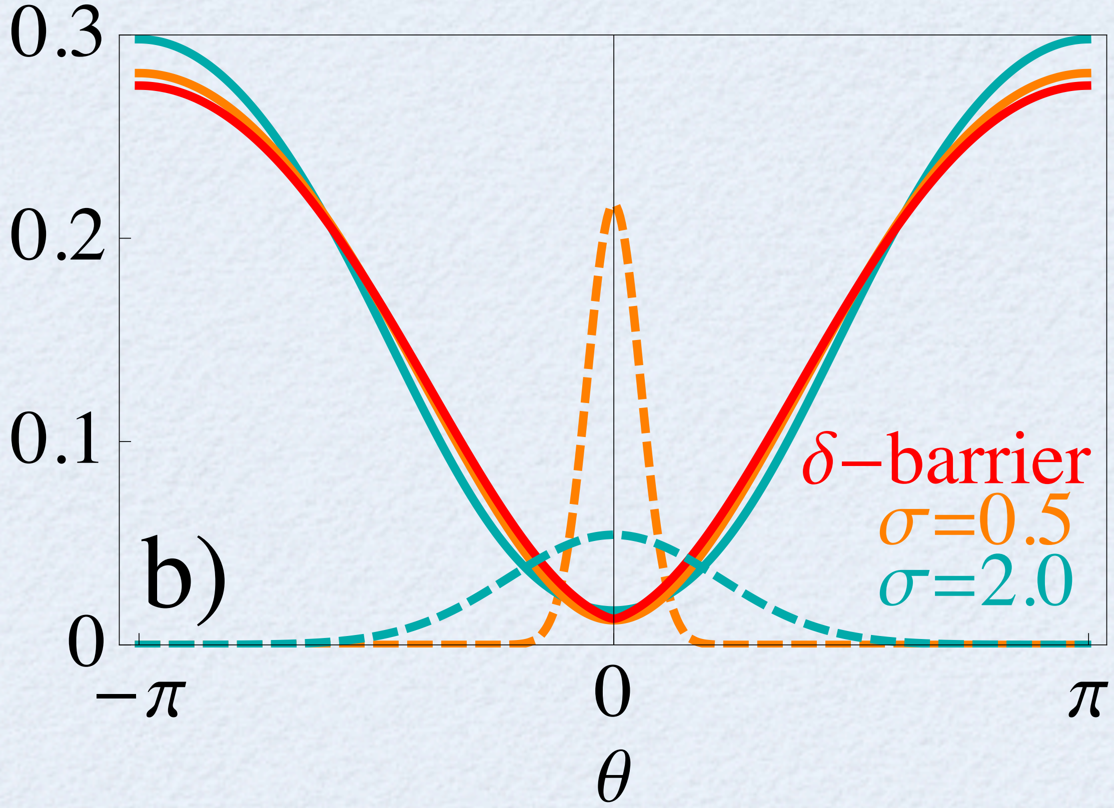


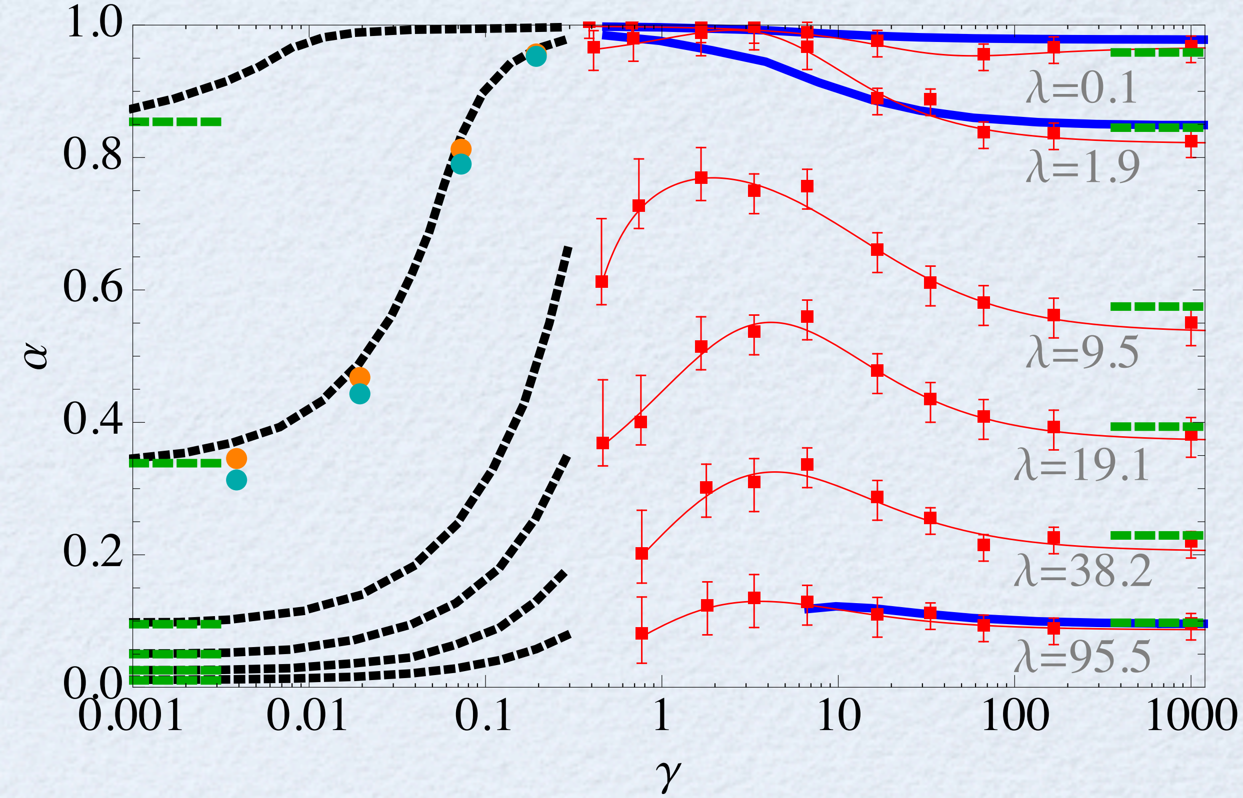


PRL 113, 025301 (2014) // EPJ ST 224, 519 (2014) // NJP 17 045023 (2015) // PRB 90, 125154 (2014)

Vicinity to experiments

Solution





✓ gaussian barriers (closer to experiments) only weakly affect results !

✓ further smearing by thermal fluctuations above $K_{\mathrm{B}}T \simeq NE_0 = \dfrac{\pi\hbar^2 n_0}{MR}$

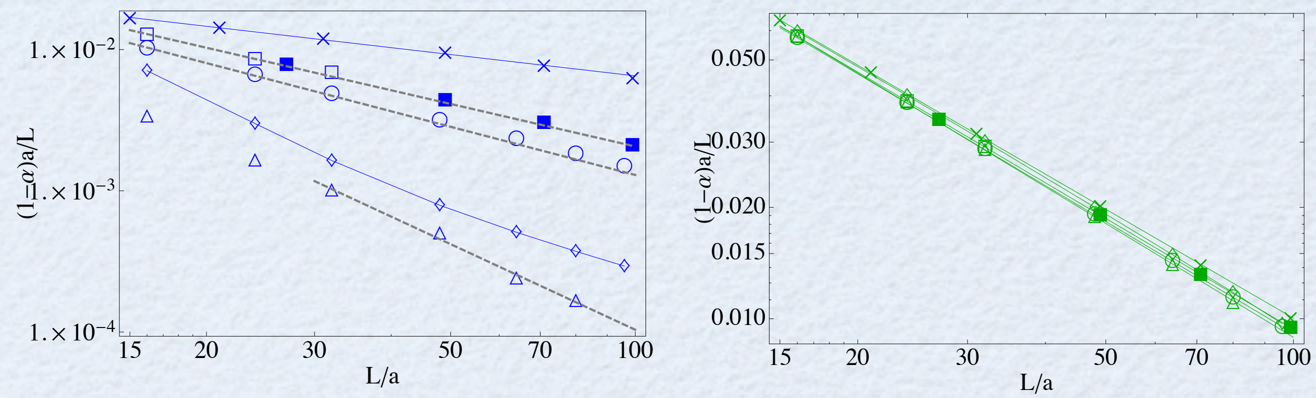
$n_0 \simeq 0.15$ $R \simeq 5\mu m$ $^{87}\mathrm{Rb}$ $K_{\mathrm{B}}T \simeq 550\mathrm{Hz} \simeq 25\mathrm{nK}$

not dramatic but should be taken into account in further studies

Further aspects

Conclusion

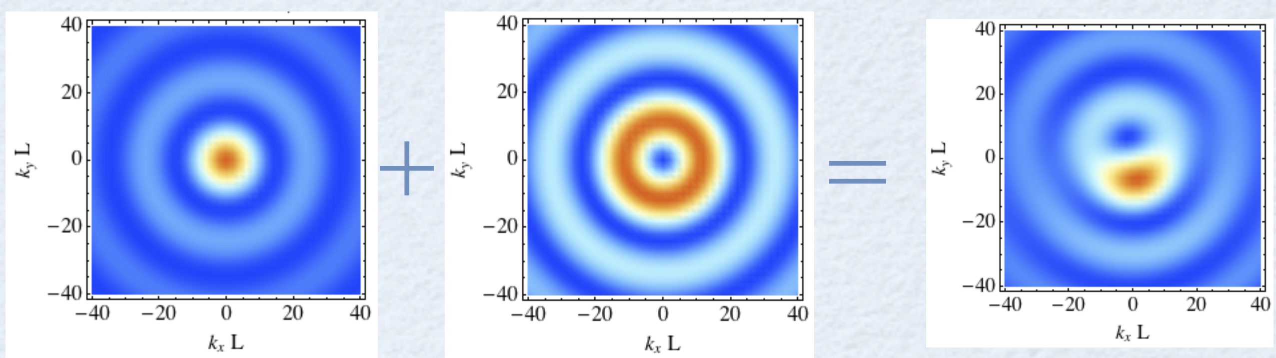
- scaling of currents with ring size



M.Cominotti, et al., EPJ ST 224, 519 (2014)

- superpositions in time-of-flight momentum distributions

$$n(\mathbf{k}) = \int \mathrm{d}\mathbf{x} \int \mathrm{d}\mathbf{x}'\ e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\rho_1(\mathbf{x},\mathbf{x}')$$



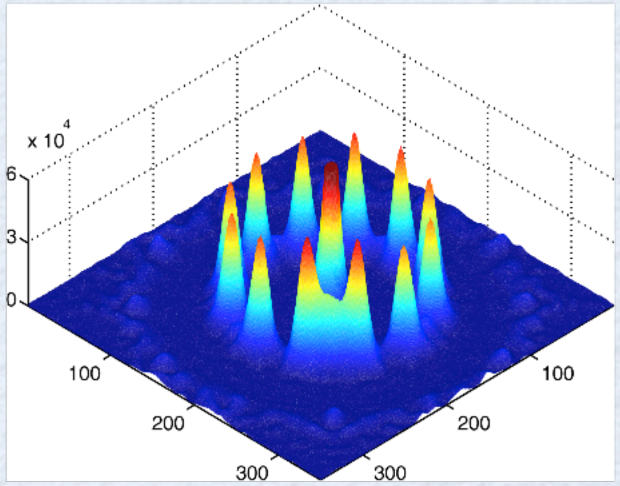
D. Aghamalyan, et al., NJP 17 045023 (2015)

- possible use as a qubit !?

Talk by D. Rossini

- actual implementation in mesoscopic lattices

Talks by R. Dumke & D. Aghamalyan



L. Amico et al., Sci. Rep. 4, 4298 (2014)

Interesting open questions

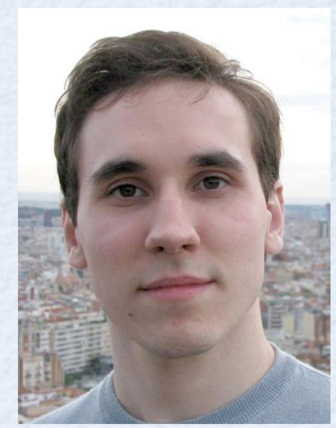
Conclusion

- optimality in lifetime?
- barrier intensity / speed quench
- finite temperature / entropy effects (relevant even in cold atoms)
- fermionic Dirac dispersion: many-body paramagnetic response?
- multi-species behaviour: Spin Drag? Andreev-Bashkin?
- finite temperature & multiple impurity effects?
- feedbacks & collaborations welcome :)

PRL 113, 025301 (2014) // EPJ ST 224, 519 (2014) // NJP 17 045023 (2015) // PRB 90, 125154 (2014)

Thanks to ...

Anna Minguzzi
LPMMC, Grenoble, FR



Marco Cominotti
LPMMC, Grenoble, FR

Frank Hekking
LPMMC, Grenoble, FR

Davide Rossini
SNS, Pisa, IT

... all of you for your attention !

PRL 113, 025301 (2014) // EPJ ST 224, 519 (2014) // NJP 17 045023 (2015) // PRB 90, 125154 (2014)