

# Spin and particle conductance of a strongly interacting Fermi gas

Atomtrons in Benasque, May 2015

**Theory :**  
Ch. Grenier

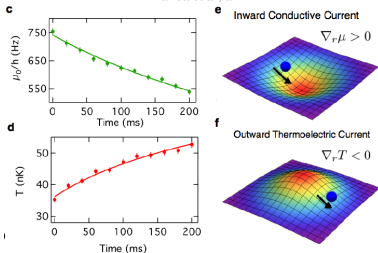
**Experiments :**  
M. Lebrat S. Krinner  
D. Husmann S. Haüsler  
S. Nakajima J.P Brantut  
T. Esslinger



# Cold atom transport : a few examples

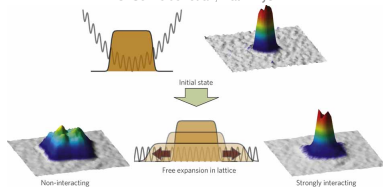
## Boson thermoelectricity (Chicago, 2013)

E. Hazlett *et al.*, arxiv



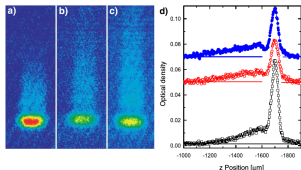
## Interactions (LMU, 2012)

U. Schneider *et al.*, Nat. Phys.



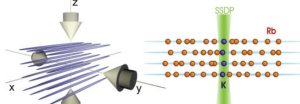
## Transport with Tonks gas (Cambridge, 2009)

S. Palzer *et al.*, Phys. Rev. Lett.

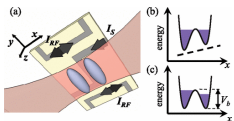


## Impurities (LENS, 2012)

J. Catani *et al.* Phys. Rev. A

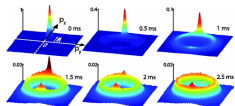


## Even closer to mesoscopic physics



### Josephson junction w. ultracold bosons

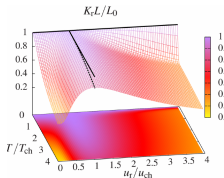
L. J. Leblanc *et al*, Phys. Rev. Lett. (2011)



### Disorder - Anderson localization

J. Billy *et al* Nature (2008), G. Roati *et al*, Nature (2008)

F. Jendrzejewski *et al*, Phys. Rev. Lett (2012)



### Violation of Wiedemann-Franz law

M. Filippone *et al* (2014)



### Engineered transmissions w. optical lattices

M. Bruderer, W. Belzig, Phys. Rev. A (2012)

# Motivation

## Transport with cold atoms ?

- New regimes : High temperature, attractive interactions, exotic lattices
- Atomtronics : Electronics with new particles
- Address fundamental questions of transport
- Input from mesoscopic physics to cold atoms

## Theory challenges :

- Interactions : controlled, but non perturbative
- Strong out of equilibrium situations
- Transport in a isolated system : Thermalization, dissipation

## Some extra motivation ...

### Questions :

- I. How close cold atom systems are from mesoscopic ones ?  
Differences, advantages, drawbacks ... ?
  
- II. What do we learn by adding a cold atom flavour to mesoscopic physics ?

# Outline

1 **Mesoscopic structures made of cold atoms**

2 **Quantized conductance and interactions**

# Outline

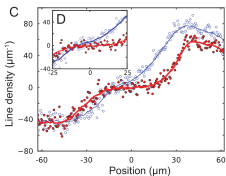
1 **Mesoscopic structures made of cold atoms**

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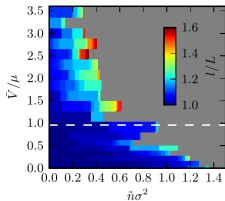




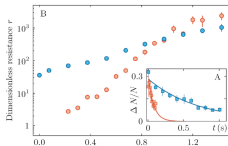
# ... To play which games ?



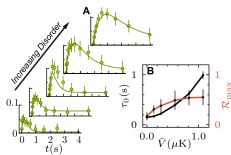
**Ballistic vs. diffusive**



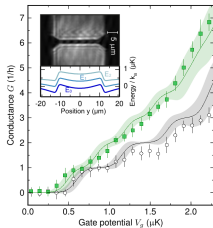
**Superfluid and disorder**



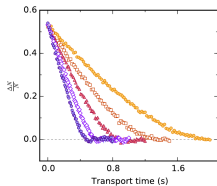
**Superfluid vs. normal**



**Thermoelectricity**



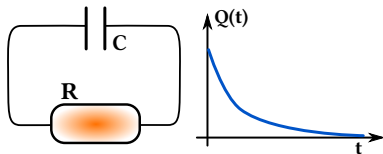
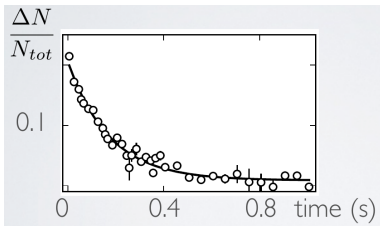
**Quantized conductance**



**Nonlinear SF**

## By the way

### How is conductance measured ?



Discharge of a capacitor  $\Rightarrow$  typical timescale  $\tau = RC$

i. **Linear response** :

$$I_N = \frac{d\Delta N}{dt} = -G\Delta\mu \Leftrightarrow I = G \cdot V$$

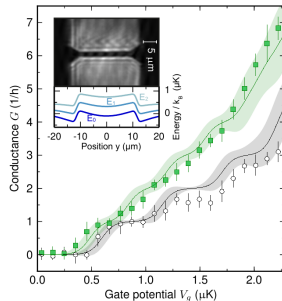
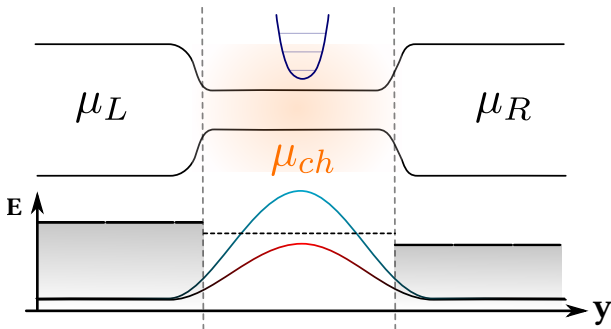
ii. **Thermodynamics** :

$$\Delta N(t) = \kappa \cdot \Delta\mu(t) \Leftrightarrow Q = C \cdot V$$

iii. **Imbalance** :

$$\Delta N(t) = \Delta N_0 \exp[-t/\tau], \tau = \kappa/G$$

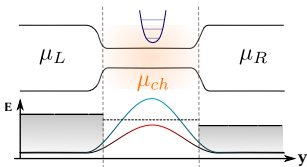
# Quantized conductance



Two ways to tune transport :

- At fixed  $\mu_{ch}$ , vary confinement
- At fixed confinement, vary  $\mu_{ch}$  (gate potential)

## Where does it come from ?

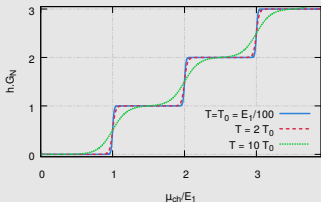


- velocity :  $v_n(\epsilon) = \frac{\hbar k_n}{m} = \sqrt{\frac{2(\epsilon - \epsilon_n)}{m}}$
- d.o.s. :  $g_n(\epsilon) = \frac{1}{2\pi} \frac{dk_n}{d\epsilon} = \frac{1}{2\pi\hbar v_n(\epsilon)}$
- current :  $I = \sum \int_{\epsilon_F}^{\epsilon_F + \Delta\mu} d\epsilon v_n(\epsilon) g_n(\epsilon) T_n(\epsilon)$

## Quantized conductance

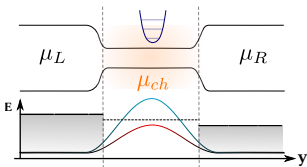
$$G_N = \frac{1}{h} \sum_n f(E_n - \mu_{ch})$$

- **Requires** : - Elastic scattering  
- Good mode matching
- **OK for** : - Ballistic systems  
- Fermi liquids



What for spin ? What happens when interactions are switched on ?

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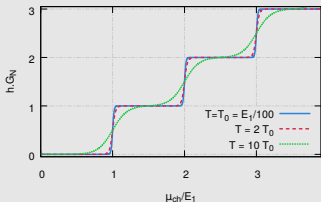


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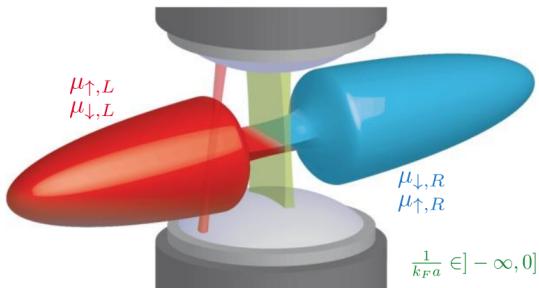
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# Outline

1 Mesoscopic structures made of cold atoms

2 **Quantized conductance and interactions**

## Interacting spin and particle transport



- Attractive interactions : BCS side of the resonance
- Particle and spin transport
- **Particle** : Deviation from quantization ?
- **Spin** : Spin drag ? Spin gap ?



## At low interaction strength ...

→ The two species are independent :

$$I_{\uparrow} = -G_{\uparrow} \Delta\mu_{\uparrow}$$

$$I_{\downarrow} = -G_{\downarrow} \Delta\mu_{\downarrow}$$

**Equivalent picture** : Particle/spin :

$$I_N = -G_N \Delta\mu$$

$$I_S = -G_S \Delta b$$

$$\mu = \frac{\mu_{\uparrow} + \mu_{\downarrow}}{2} \text{ and } b = \frac{\mu_{\uparrow} - \mu_{\downarrow}}{2}$$

**At low interaction strength, particle and spin have the same conductance...  
What when  $a$  increases ?**

## Spin drag physics

In the up/down picture :

$$\begin{aligned}I_{\uparrow} &= -G_{\uparrow}\Delta\mu_{\uparrow} - \Gamma\Delta\mu_{\downarrow} \\I_{\downarrow} &= -\Gamma\Delta\mu_{\uparrow} - G_{\downarrow}\Delta\mu_{\downarrow}\end{aligned}$$

$\Gamma/G_{\uparrow}$  = fraction of carried down spin

For particle and spin conductance :

$$I_N = -G_N\Delta\mu \quad \text{with} \quad G_N = G_{\uparrow} + G_{\downarrow} + 2\Gamma$$

$$I_S = -G_S\Delta b \quad \text{with} \quad G_S = G_{\uparrow} + G_{\downarrow} - 2\Gamma$$

**In a single channel**, with the Hamiltonian :

$$\mathcal{H} = \sum_{k\sigma} \varepsilon_{k,\sigma} c_{k,\sigma}^{\dagger} c_{k,\sigma} + \frac{U}{2} \sum_{k,k',q,\sigma} c_{k+q,\sigma}^{\dagger} c_{k'-q,-\sigma}^{\dagger} c_{k',-\sigma} c_{k,\sigma}.$$

→ Simple estimate of spin drag (eqn. of motion + 2nd order RPA) :

$$\Gamma \simeq \frac{4\gamma^2}{9h} \left[ \frac{T}{T_F} + \frac{3}{2} \left( \frac{T}{T_F} \right)^2 \right] \text{Ln}, \gamma \equiv \text{int. parameter}$$

## Including thermodynamics

Uncoupled spin and particle transport :

$$I_N = \frac{d\Delta N}{dt} = -G_N \Delta \mu$$

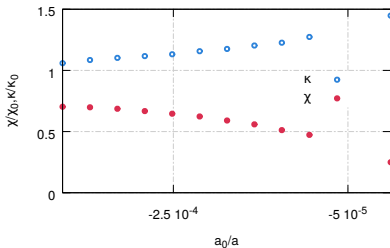
$$I_S = \frac{d\Delta M}{dt} = -G_S \Delta b$$

On the thermodynamics side :

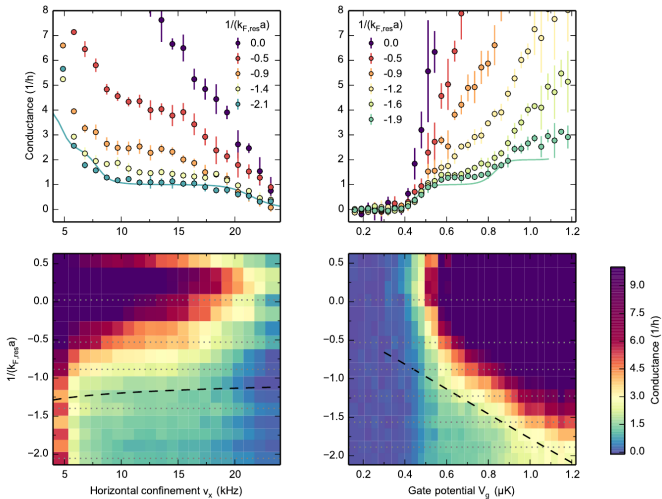
$$\Delta N = \kappa \Delta \mu$$

$$\Delta M = \chi \Delta b$$

compressibility, susceptibility : ENS data on polarized gas (+ trap averaging) :



# Results - Particle conductance



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## Normal phase :

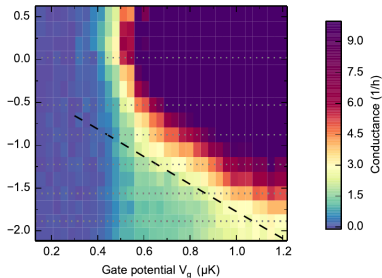
- Quantized conductance ( $\approx$  Particle number conservation in the Hamiltonian)
- Modified width of the plateaux, and transition slopes  $\equiv$  mean-field physics

## Fluctuation regime :

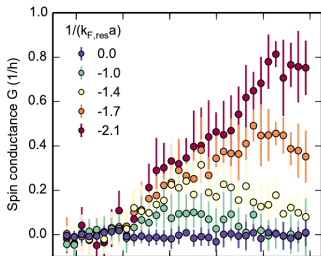
- Still quantized
  - Renormalized height
- Entrance to the superfluid phase

## Superfluid transport :

- No quantization anymore
- Enhanced particle transport

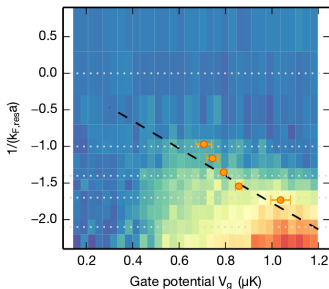


## Results - Spin conductance



### $G_S$ vs. $V_g$ :

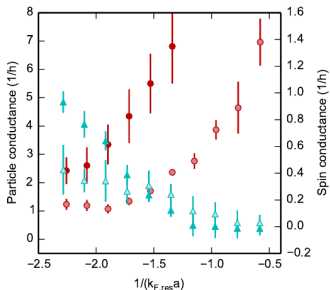
- Strong suppression at large gate
- Small increase at small  $V_g$
- Competition between channel opening and increasing gap



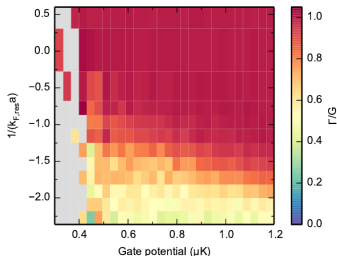
### $G_S$ vs. $1/k_{F} a$ :

- Spin transport already suppressed for weak interactions
- No spin transport at all for unitary
- Max.  $\simeq$  matches modified BCS expression

# Results - Comparison spin-particle transport



- **Low interactions** : same effect of  $V_g$  on  $G_S$  and  $G_N$
- **Strong interactions** :  $G_N \nearrow \nearrow$   $G_S \searrow \searrow$  : superfluidity, spin gap

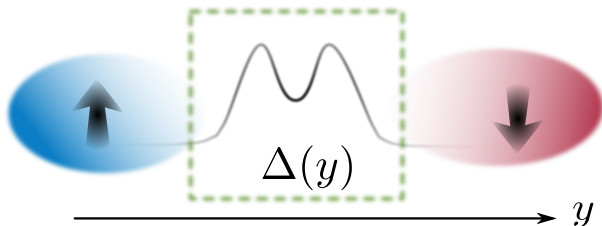


**No net polarization** :  $G_{\uparrow} = G_{\downarrow} \equiv G$

$$G_{N,S} = 2G - 2\Gamma \quad : \quad \Gamma/G = \frac{G_N - G_S}{G_N + G_S}$$

$\Gamma/G$  : fraction of  $-\sigma$  carried by  $\sigma$

## Results - Spin : interpretation-1

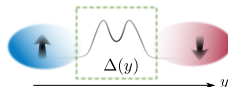


- ① Space dependent gap
- ② Only Bogoliubov QP can transport spin : spin transport  $\equiv$  excitations
- ③ Polarization  $\rightarrow$  QP generation in reservoirs



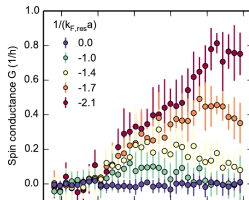
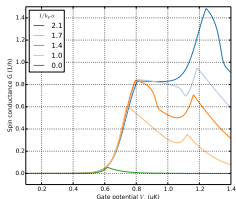
## Results - Spin : interpretation-2

- Spin transport  $\equiv$  scattering on a space dependent potential
- Landauer-like transport with Bogoliubov QP



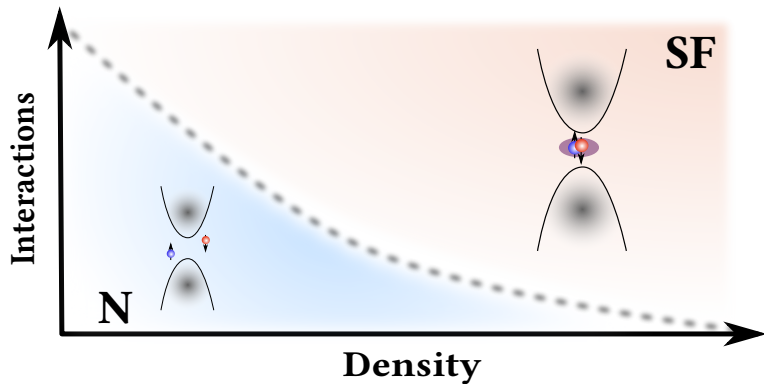
### Transmission :

$$\mathcal{T}(\varepsilon, V_g) = \left| \exp \left[ -\frac{\sqrt{2m}}{\hbar} \int dy \sqrt{(\Delta(y) - \varepsilon)} \right] \right|^2, \quad \Delta(y) \propto E_F(y) \exp \left[ \frac{\pi}{2k_F(y)a} \right]$$



$$G_S = \frac{1}{h} \sum_n \int_{\Delta_{res}}^{+\infty} d\varepsilon \frac{\mathcal{T}(\varepsilon, V_g) \vartheta(\varepsilon - (E_n - V_g))}{4 \cosh^2 \left[ \frac{\varepsilon - b}{2k_B T} \right]}$$

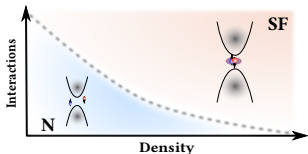
# Summary



# Summary-Conclusion

## In a nutshell :

- Effects of interactions on quantized conductance
- Investigation of spin drag/spin gap physics
- Qualitative features captured by mean field physics
- Spin and particle transport are complementary



## What's next ?

- For mesoscopic lovers : attractive equivalent of the 0.7 anomaly
- Investigation of polarization/interaction competition ?
- Continue exploring transport along the BEC-BCS crossover : thermal transport ?

