

Experiments with BECs in a Painted Potential

Malcolm Boshier

Changhyun Ryu,

Paul Blackburn, Alina Blinova,

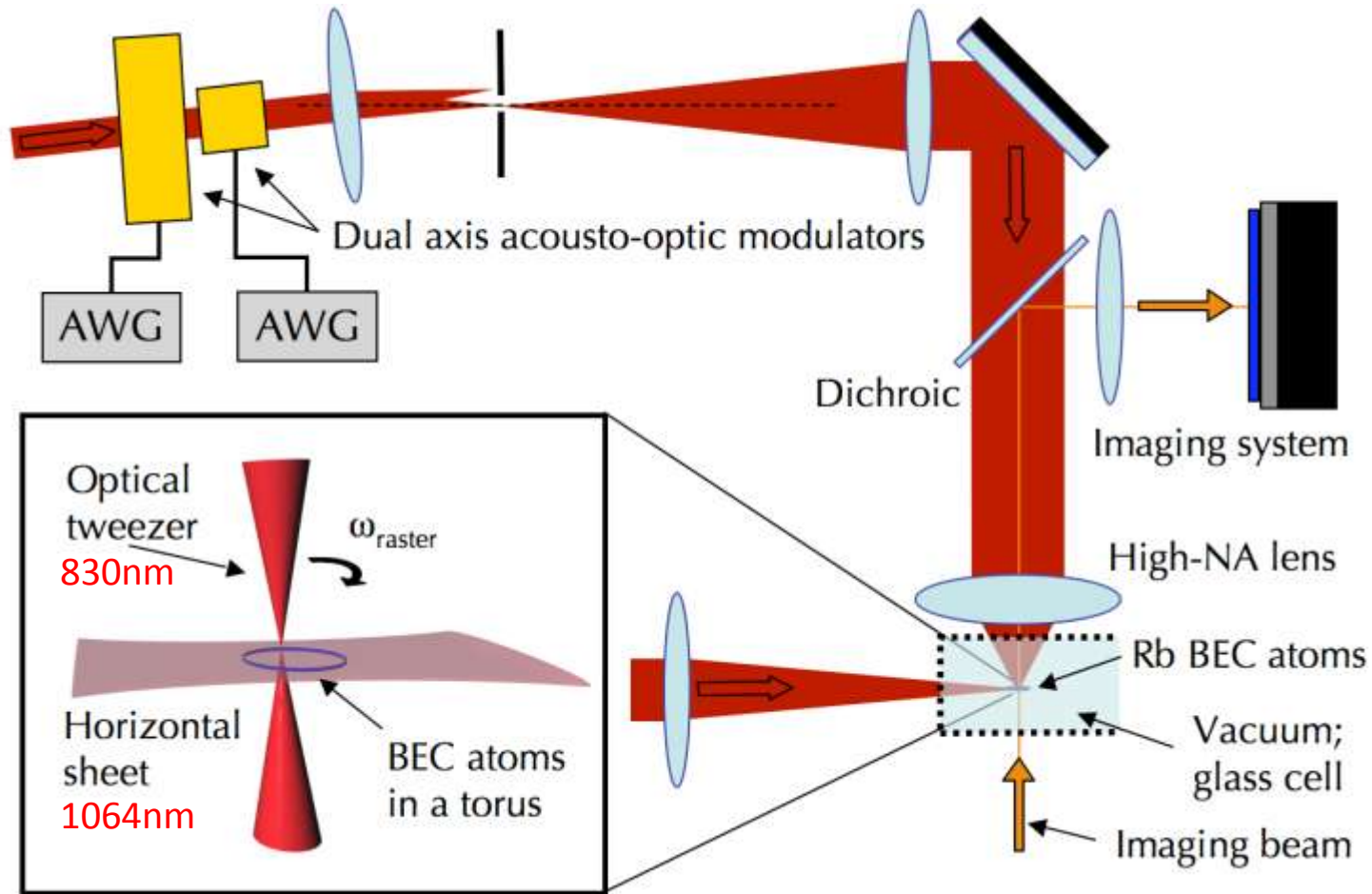
Kevin Henderson, Calum MacCormick

Outline

- **The Painted Potential**
- **Toroidal waveguides**
 - Quantized circulation and Bessel beams
 - Josephson junctions for a dc atom SQUID
- **Matter wave circuits**
 - Straight and bent waveguides
 - Y-junctions
 - Excitations at bends

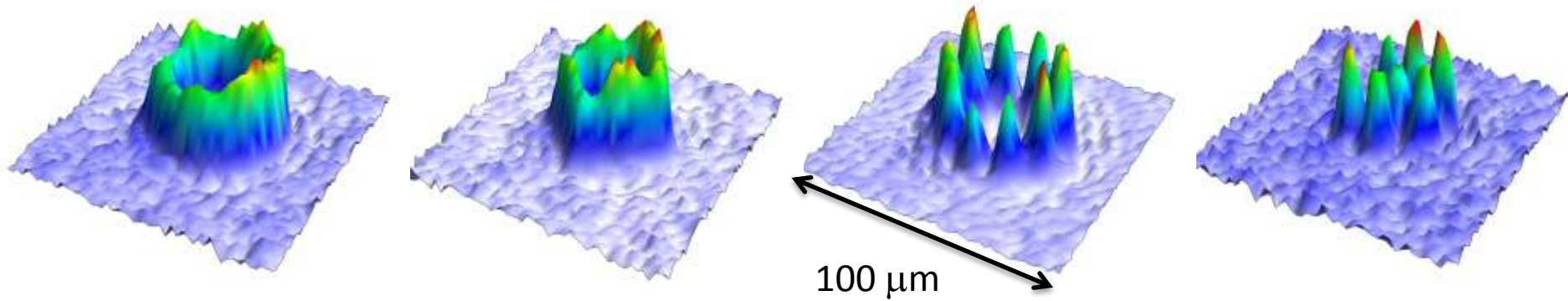
THE PAINTED POTENTIAL

The Painted Potential

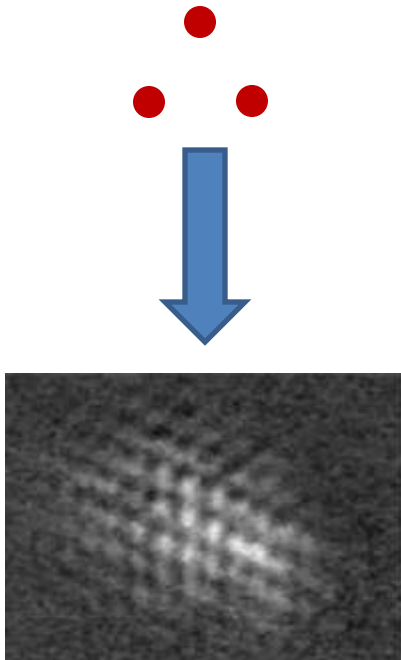


- Red-detuned (attractive) optical dipole potentials
- Typical painting frequency 10 kHz, typical trap frequency 400 Hz
- Evaporate ^{87}Rb to form BEC by lowering intensity of light sheet
- K. Henderson *et al*, New J. Phys. **11**, 043030 (2009)
- Earlier work: time-averaged barriers at UT Austin, Hannover, Weizmann

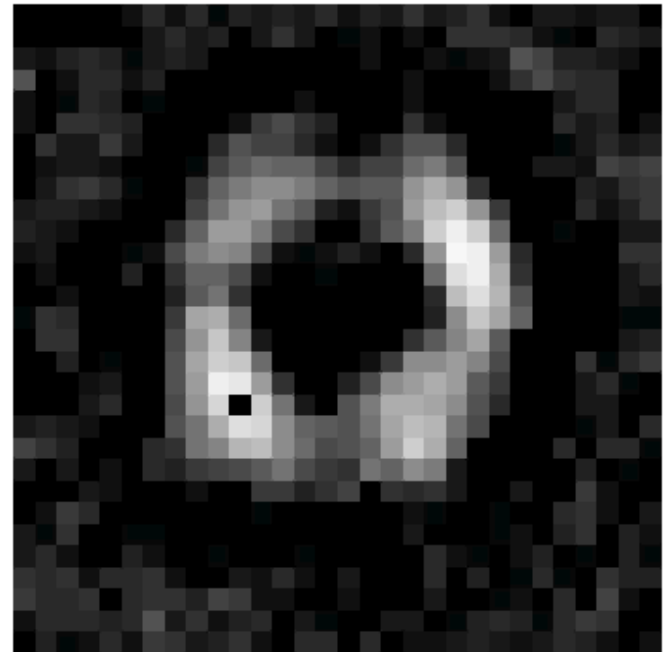
BECs in Arbitrary Shapes



Interference patterns confirm BEC.
TOF image from painted triple well:



The painted potential is dynamic

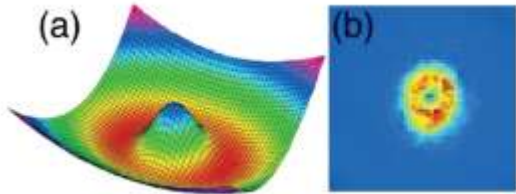


Can adjust intensity during scan to flatten potential

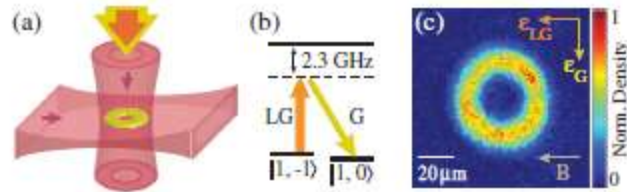
Quantized circulation and Bessel beams

TOROIDAL WAVEGUIDES

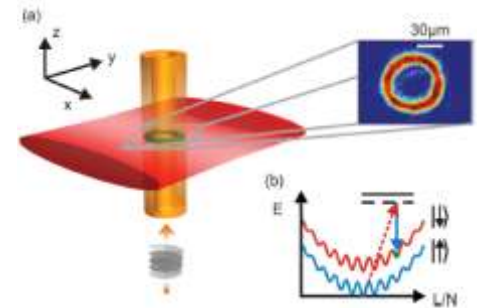
BECs in Toroidal Traps and Waveguides



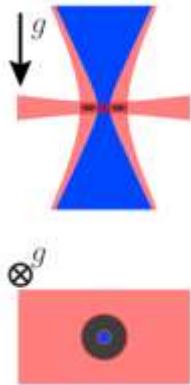
Phillips (NIST)
Plugged magnetic trap



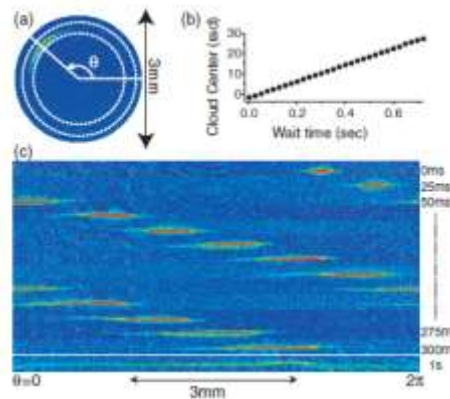
Campbell/Phillips (NIST)
LG beam + light sheet



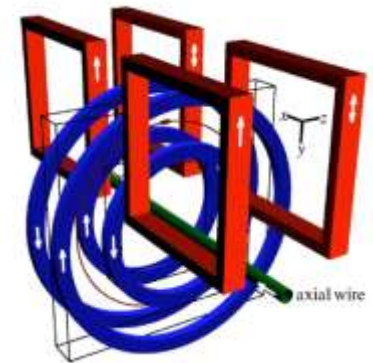
Hadzibabic (Cambridge)
LG beam + light sheet



Stamper-Kurn (Berkeley)
Two-color trap + light sheet



Stamper-Kurn (Berkeley)
Magnetic waveguide



Arnold (Strathclyde)
Magnetic waveguide

Superfluid Circulation is Quantized

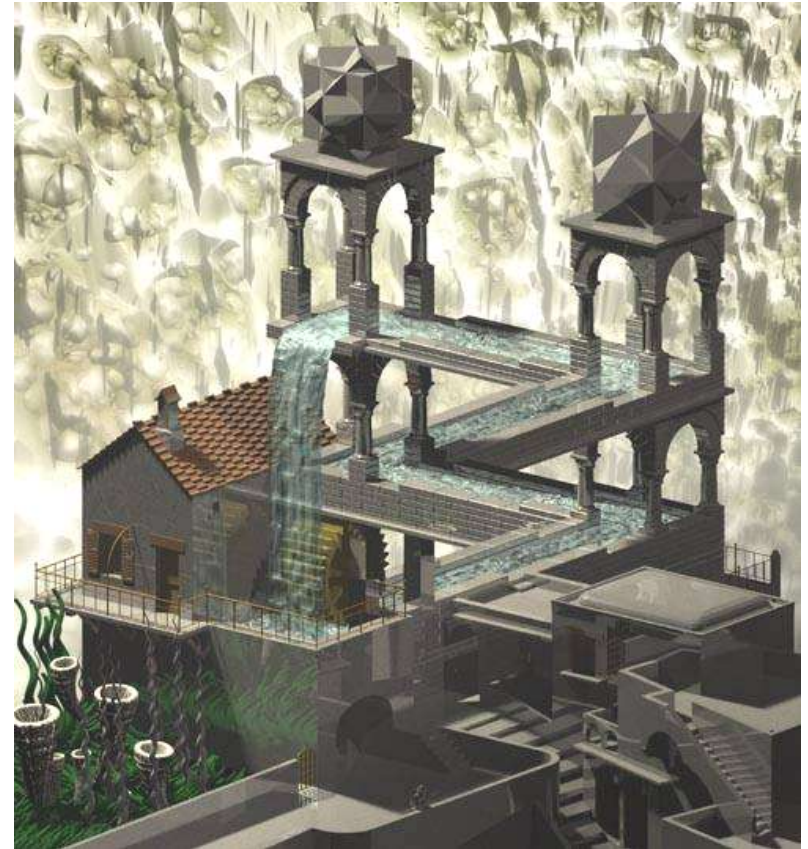
Superfluid velocity is

$$\mathbf{v} = \frac{\hbar}{m} \nabla \theta$$

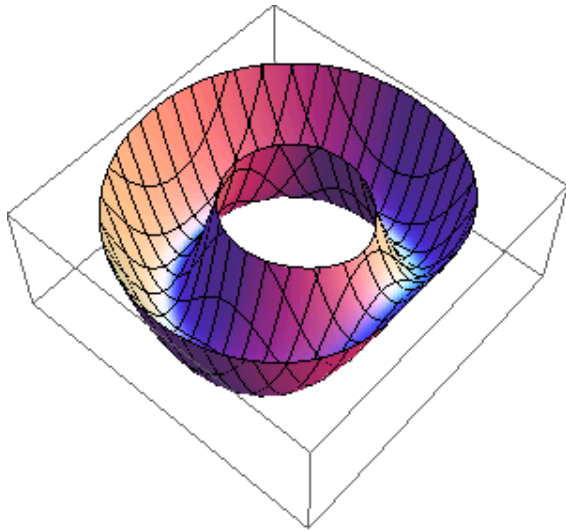
Ψ must be single-valued,
so superfluid circulation

$$\oint \mathbf{v} \cdot d\mathbf{l} = \frac{\hbar}{m} \Delta \theta = n \frac{h}{m}$$

is quantized ($n = \text{winding number}$)



Creating BEC in a Rotating Trap



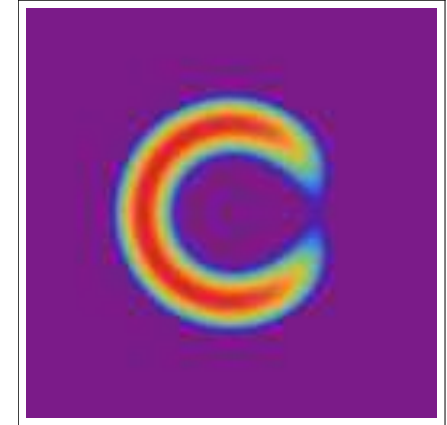
toroidal trapping potential

Direct condensation of atoms into a rotating trap with a high potential barrier

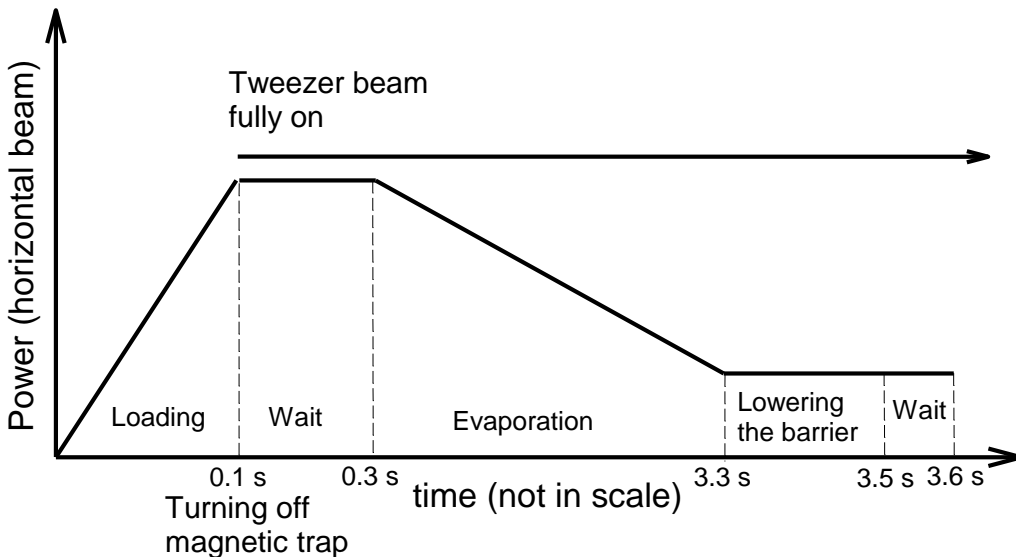
BEC size: 40,000 atoms

Diameter: 19 μm

$n = 1$ rotation frequency ~ 1.5 Hz



calculated ground state 2D density



- Turn on the horizontal beam and tweezer beam
- Turn off the magnetic trap
- Wait for 200ms
- Lower the horizontal beam power for the evaporative cooling
- Lower the barrier for 200ms
- Wait for 100ms and turn off the trap for TOF images

Measuring rotation velocity

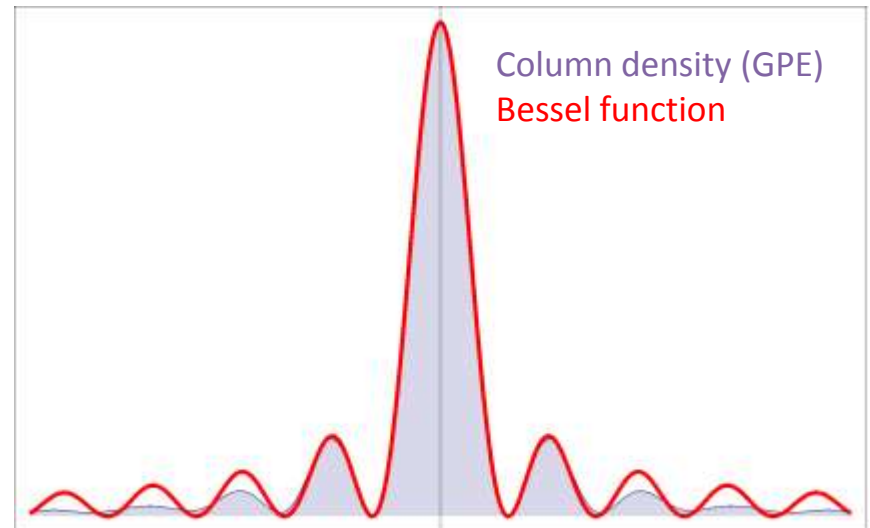
- Centrifugal effects are small, so look at BEC after free expansion
- Classical: central hole develops with asymptotic radius proportional to velocity
- Quantum prediction: wavefunction at long times is proportional to initial momentum space wavefunction:

$$\psi(\mathbf{r}, t) = A \exp(i m r^2 / 2 \hbar t) \frac{\Phi(m \mathbf{r} / \hbar t, t = 0)}{\hbar t / m}$$

- For a thin torus $\psi(r, \phi) = \delta(r - R) \exp(-i n \phi)$ the momentum space wavefunction is a Bessel function:

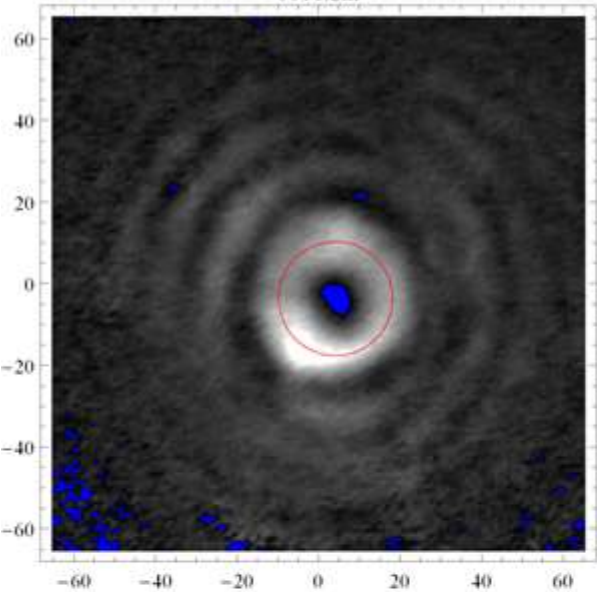
$$\Phi(k, 0) = J_n(kR)$$

- GPE simulations show that the effects of finite width, interactions, and the third dimension are small

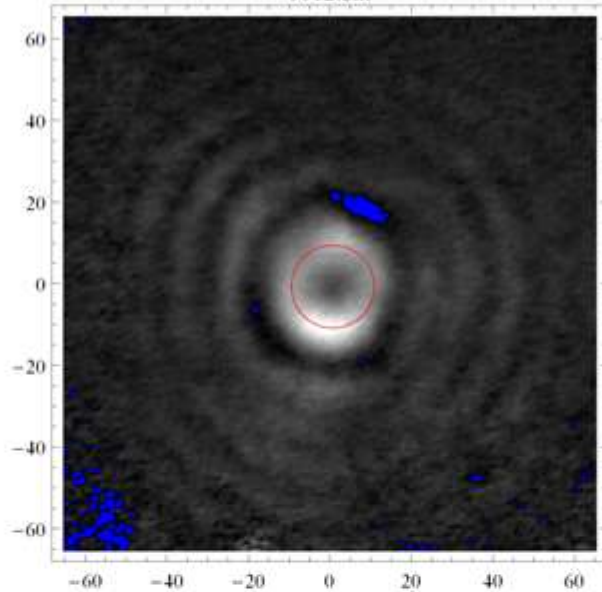


Successive TOF Images (2 Hz rotation)

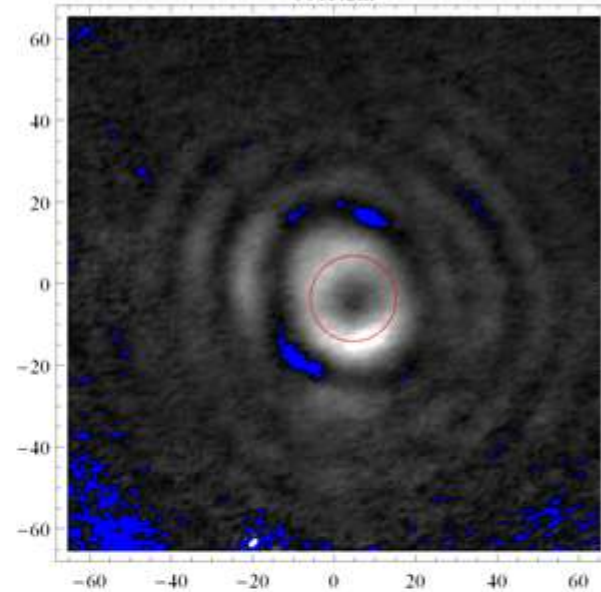
0011 bin



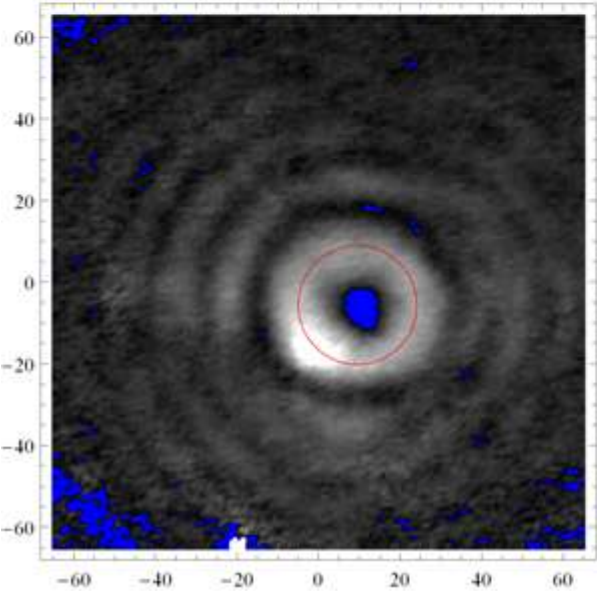
0012 bin



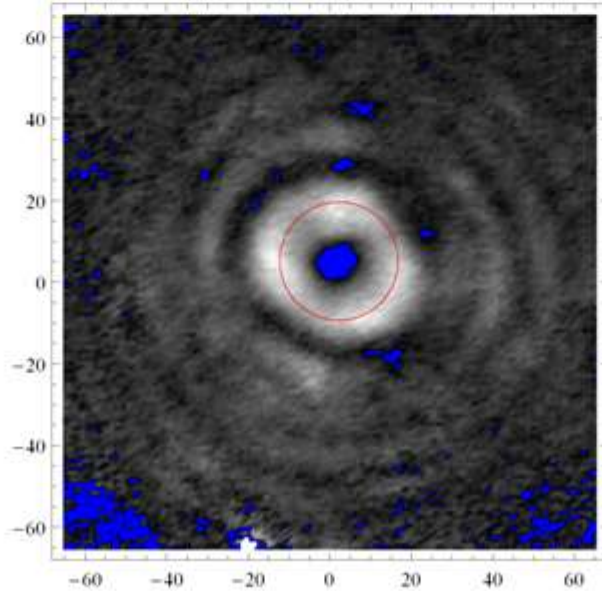
0013 bin



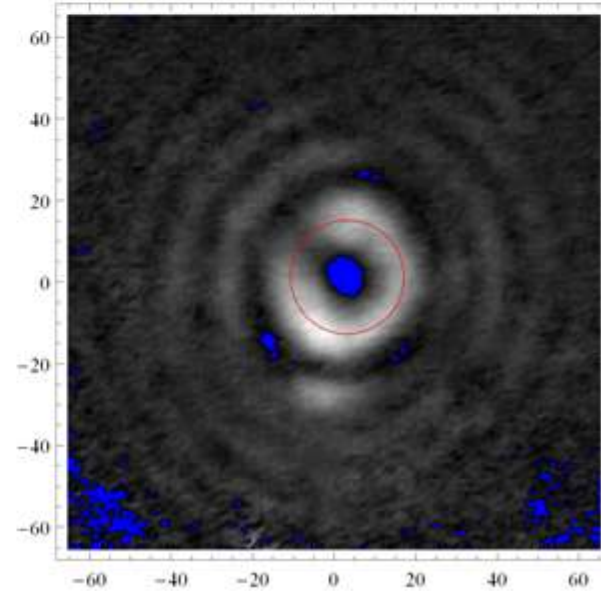
0014 bin



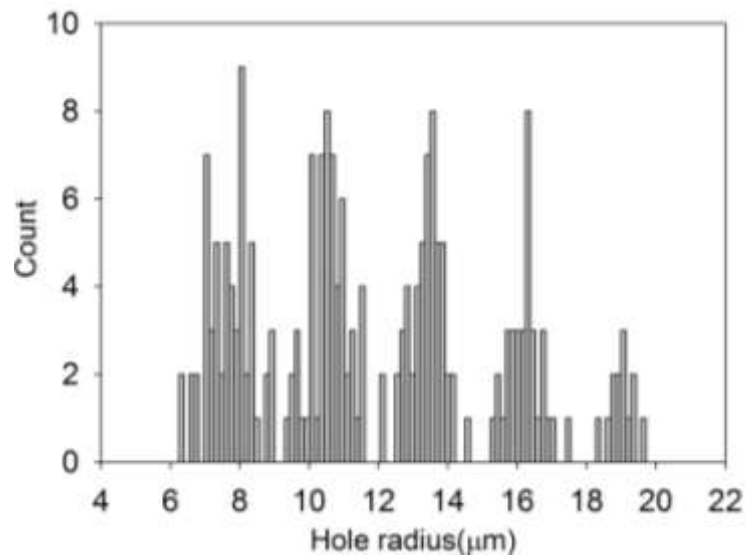
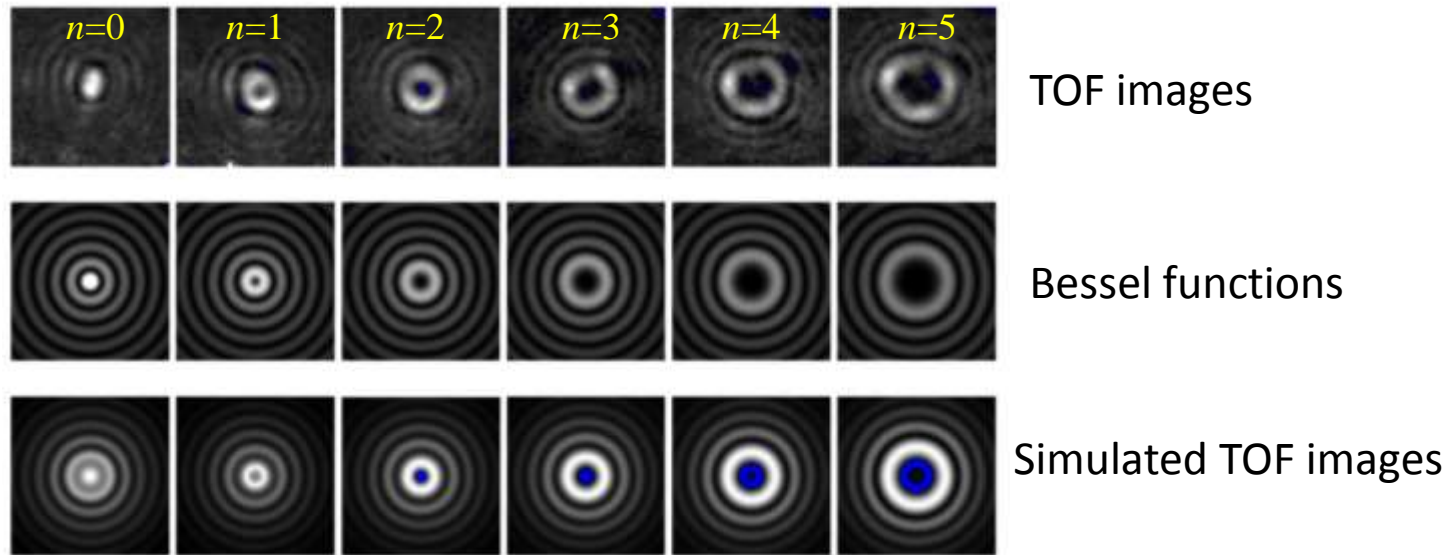
0015 bin



0016 bin



Quantization of circulation of a BEC in a toroidal trap



Control of Quantized Circulation

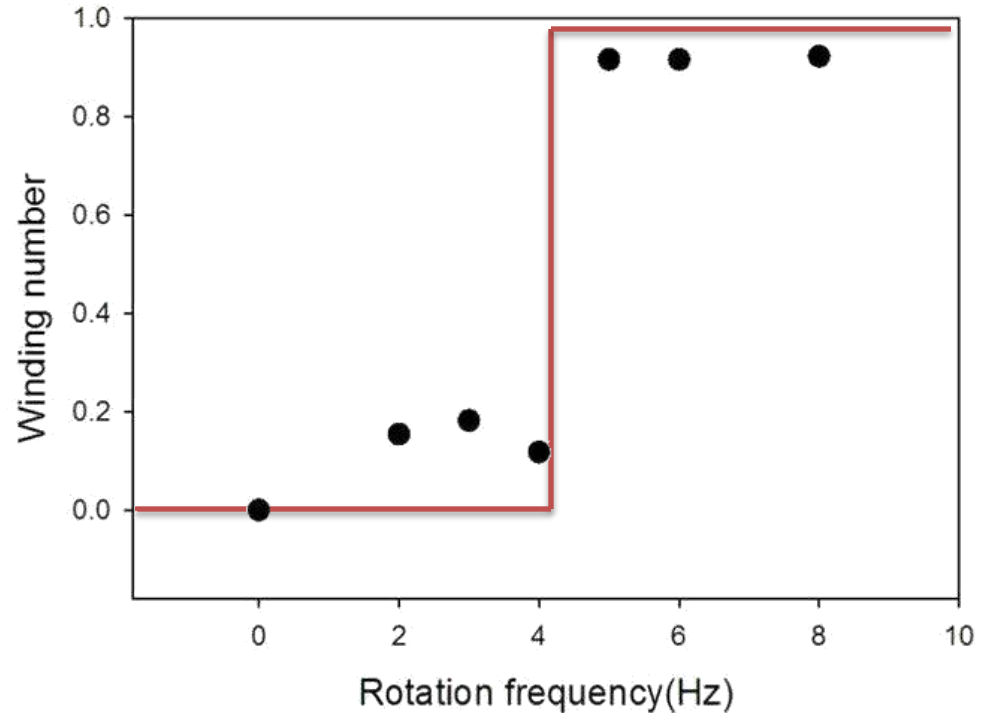
System should find energy minimum of Hamiltonian in rotating frame

$$H_{\text{eff}} = H_0 - \omega \cdot L$$

$$E = \frac{(n\hbar)^2}{2mr_0^2} - \omega n\hbar$$

$$= \frac{\hbar\omega_0}{2} \left[\left(n - \frac{\omega}{\omega_0} \right)^2 - \left(\frac{\omega}{\omega_0} \right)^2 \right]$$

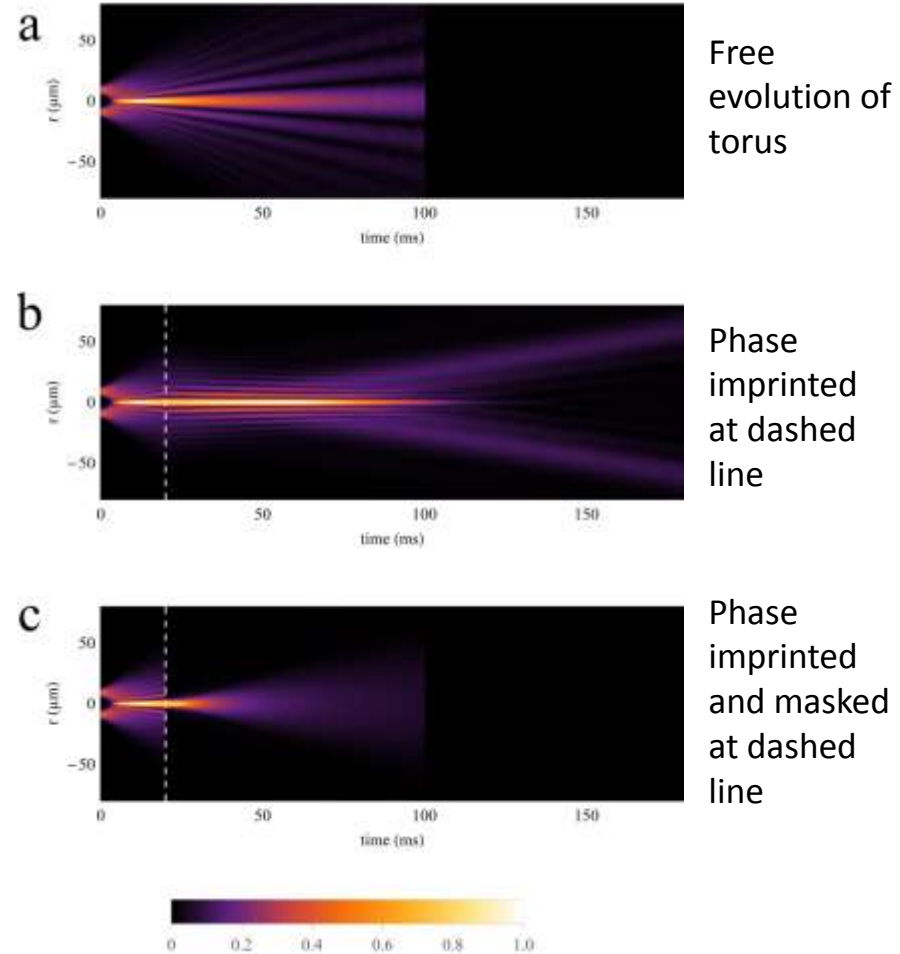
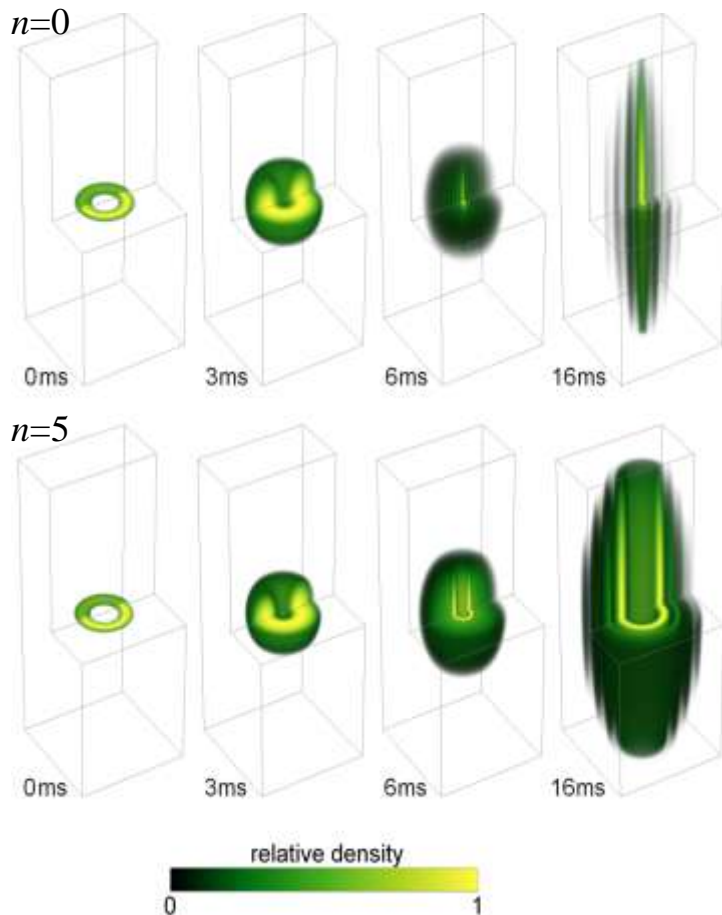
$$\left(\omega_0 = \frac{\hbar}{mr_0^2} \approx 2\pi \times 8\text{Hz} \right)$$



Energy E is minimized when winding number n is nearest integer to ω/ω_0

Matter Wave Bessel Beams

Since $\nabla^2 J_n(\rho) e^{-in\phi} = -J_n(\rho) e^{-in\phi}$ plane waves with Bessel function amplitude distribution satisfy the wave equation and are “diffraction free” (Durnin et. al, PRL 1987)
Free evolution of a rotating toroidal BEC produces a Bessel-like beam.

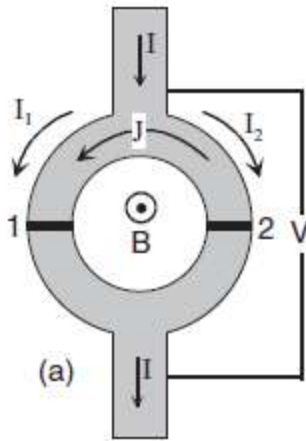


Josephson Junctions for a dc Atom SQUID

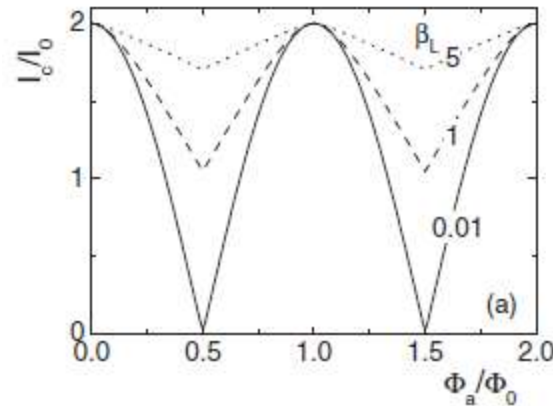
TOROIDAL WAVEGUIDES WITH TUNNEL JUNCTIONS

Superconducting Quantum Interference Device (SQUID)

Essential ingredients: multiply-connected geometry with quantized flux and Josephson Junctions



Josephson Junctions on a ring



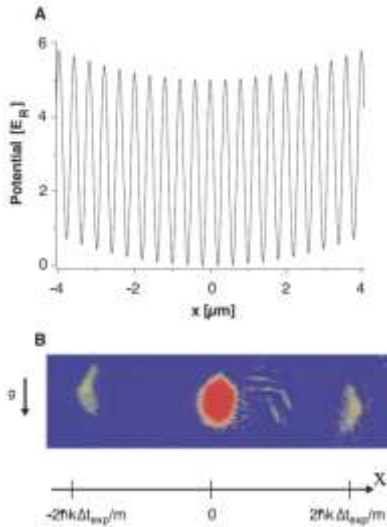
$$I_c = 2I_0 \cdot \left| \cos\left(\pi \frac{\phi_a}{\phi_0}\right) \right|$$

Basis for the most sensitive magnetometer

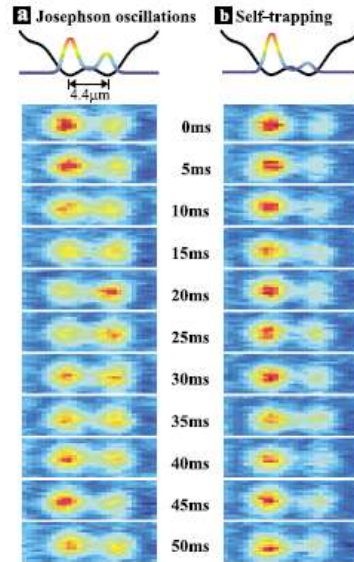
Atom SQUID: a BEC analog of a SQUID

Quantized circulation makes critical current a periodic function of rotation frequency, so the atom SQUID may be a path to a compact rotation sensor

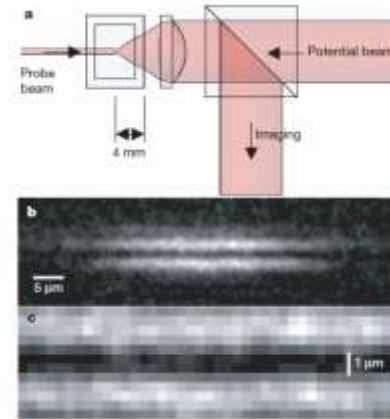
Josephson Junctions and Weak Links for BECs



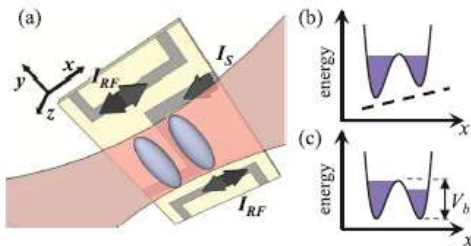
Inguscio (Florence)
Optical lattice JJ



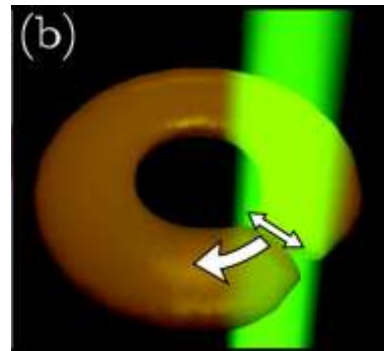
Oberthaler (Heidelberg)
Optical lattice JJ



Steinhauer (Technion)
Blue detuned laser JJ



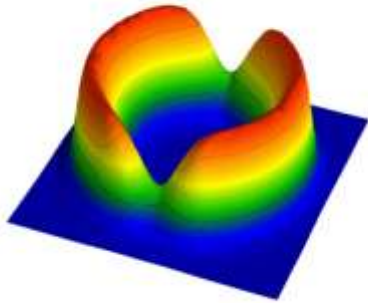
Thywissen (Toronto)
RF dressed magnetic trap JJ



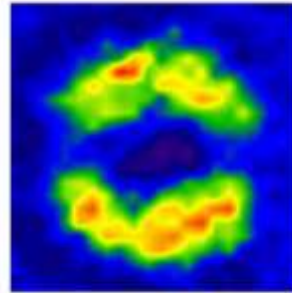
Campbell/Phillips (NIST)
Blue-detuned laser weak link

BEC analog of a dc SQUID

Create symmetric junctions on a toroidal trap for a BEC by reducing intensity of painting beam



8 μm diameter
Barrier width: 2 μm
Depth: 75nK
Barrier height: 44nK



Number: 1,000-8,000
Chemical potential: 29nK-58nK

$$1 \quad z = (N_1 - N_2) / N$$

$$2 \quad \phi = \phi_1 - \phi_2$$

Josephson equations ($z \ll 1$)

$$I = \dot{z} = I_c \sin \phi \quad \dot{\phi} = -\frac{\Delta\mu}{\hbar}$$

$$I_c = 2E_J / \hbar N$$

$$\Delta\mu = \hbar\omega_c(z - z_0)$$

$$\omega_c = E_c / 2\hbar N$$

Fantoni (1999)

Bergeman (2006)

Josephson Plasma Oscillation

$$\phi = \phi_0 \sin\left(\sqrt{I_c \omega_c} t\right)$$

DC Josephson Effect

$$I_0 = I_c \sin \phi_0$$

AC Josephson Effect

$$I = -I_c \sin\left(\frac{\Delta\mu}{\hbar} t\right)$$

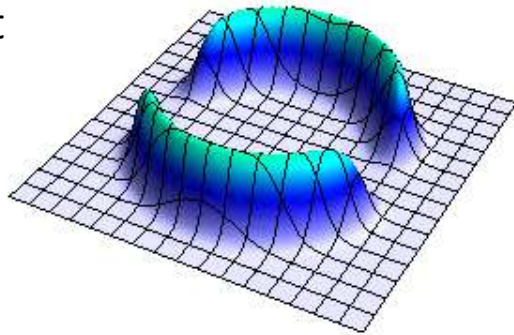
Observation of oscillations in number is difficult because of the small amplitude

Observation of Josephson Effects

- Giovanazzi, Smerzi, and Fantoni PRL **84**, 4521(2000)
- A bias current can be generated by moving junctions relative to atoms $\dot{z}_0 = I_{bias} = 4f$
- Measurement of critical currents can be used to observe Josephson effects

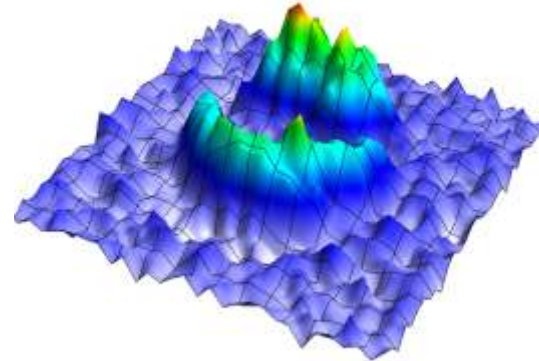
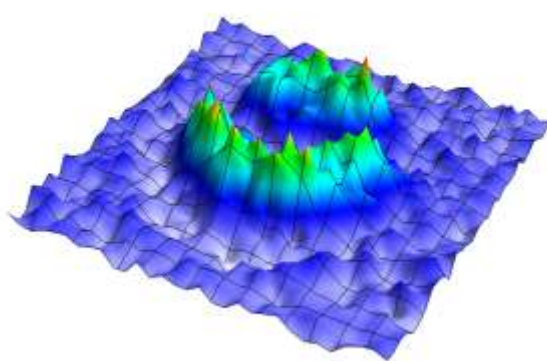
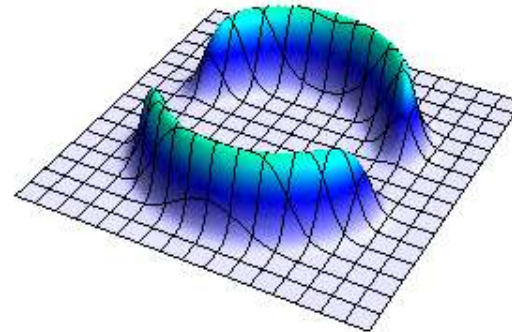
DC Josephson effect
Tunneling

$$I_{bias} < I_c$$
$$\Delta\mu = 0$$



AC Josephson effect
No tunneling

$$I_{bias} > I_c$$
$$\Delta\mu \neq 0$$

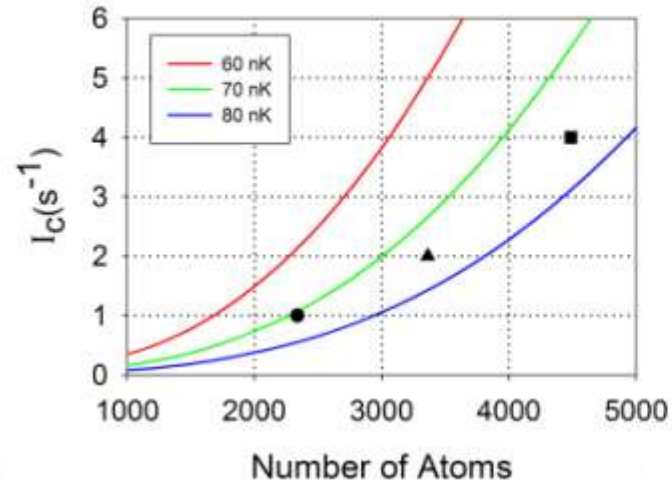


In the experiment we reduce oscillations by accelerating the barrier gradually

Experimental Results

$$I_c = 2E_J / \hbar N$$

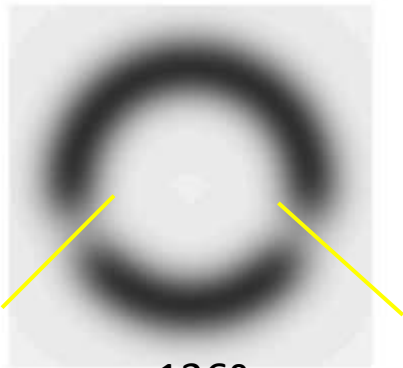
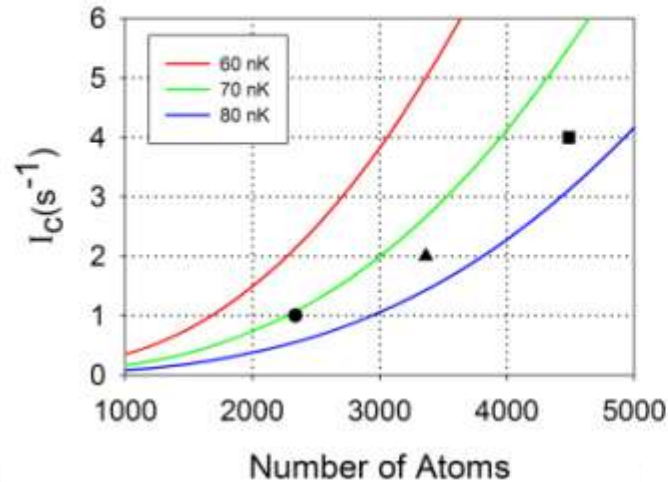
Critical current can be changed by changing atom number at fixed barrier height



Experimental Results

$$I_c = 2E_J / \hbar N$$

Critical current can be changed by changing atom number at fixed barrier height



136°

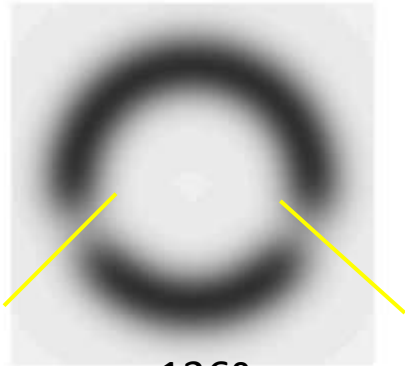
No tunneling: $z = 0$

Tunneling: $z = 0.245$

Experimental Results

$$I_c = 2E_J / \hbar N$$

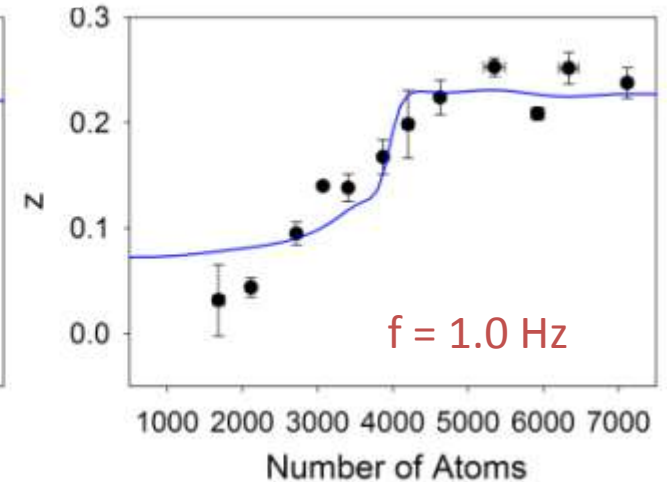
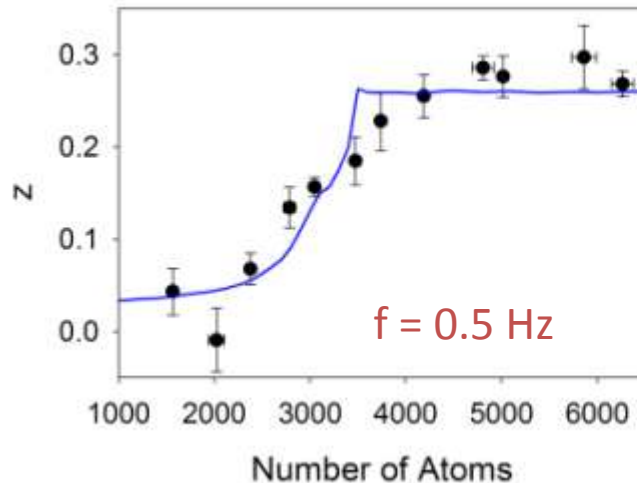
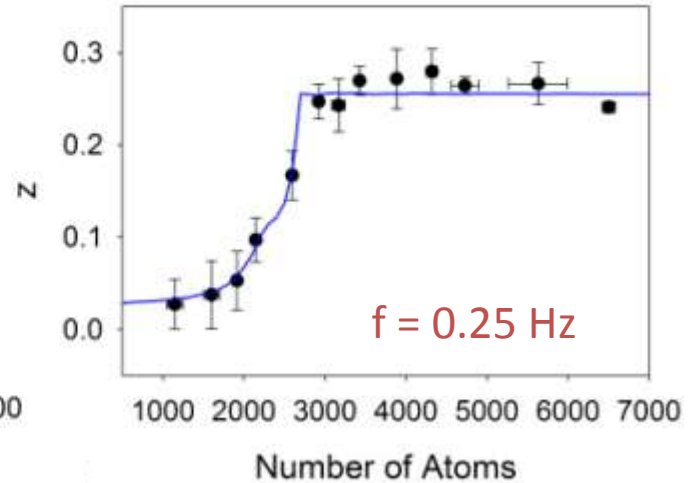
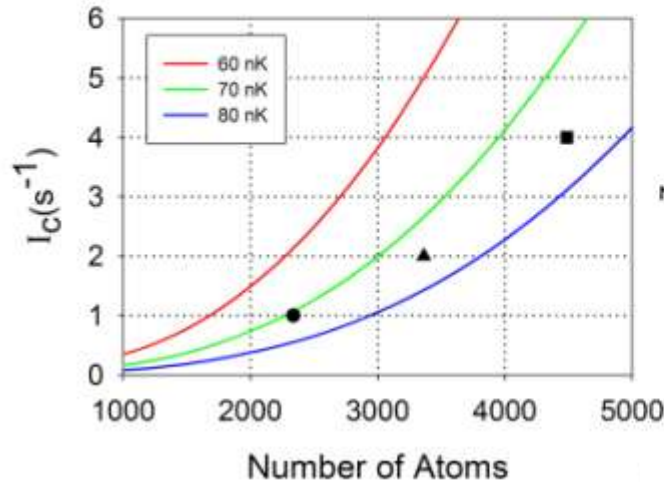
Critical current can be changed by changing atom number at fixed barrier height



136 μ m

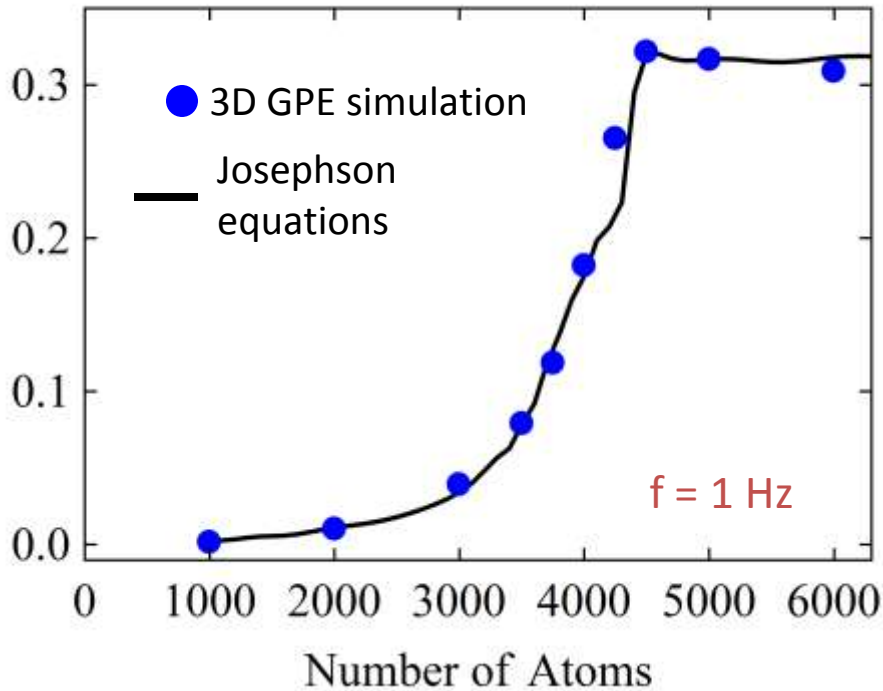
No tunneling: $z = 0$

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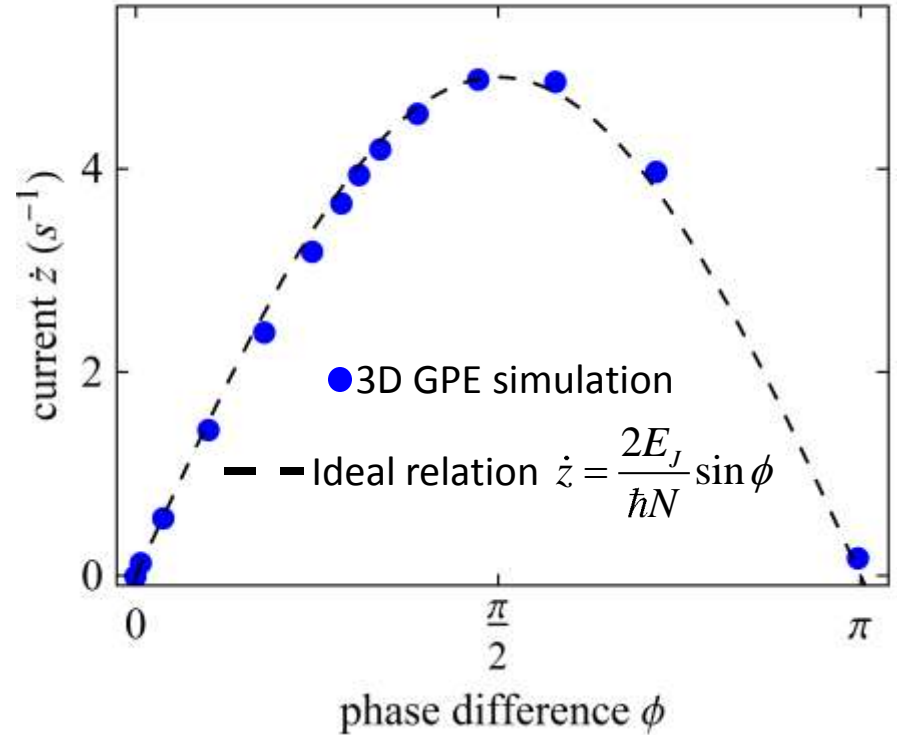


Solid line is best fit of Josephson equations to data (free parameters are trap depth, and initial and final values of z)

Current-Phase Relation



Josephson equations agree with GPE
(and experiment)



GPE says current-phase relation is
ideal sinusoid for our parameters

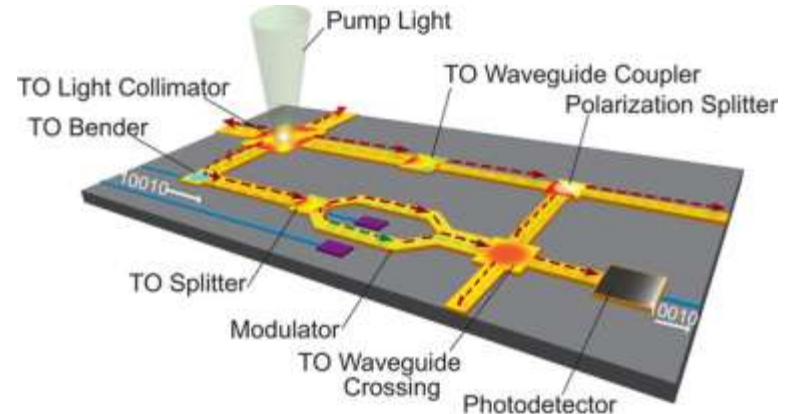
Measured critical currents and dynamic behavior are in good agreement with the simple Josephson equations for a tunnel junction with the ideal sinusoidal current-phase relation expected for the parameters of the experiment.

C. Ryu, P.W. Blackburn, A.A. Blinova, and M.G. Boshier, PRL **111**, 205301 (2013)
[see also F. Jendrzejewski, PRL **113**, 045305 (2014)]

MATTER WAVE CIRCUITS

The Goal: An Integrated Coherent Matter Wave Circuit

The de Broglie wave analog of an integrated optical circuit - a single device in which coherent de Broglie waves are created and then launched into waveguides where they can be switched, divided, recombined, and detected.



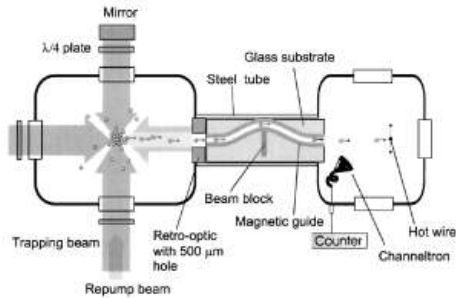
[*Light: Science & Applications* (2012) **1**, e38]

Motivation

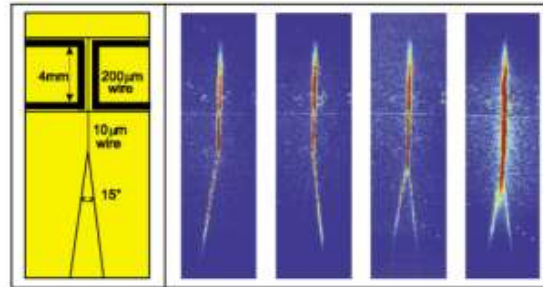
- Guided atom interferometers
- Precisely controlled delivery of atoms – the “atom laser pointer”
- Wiring up circuits of atomtronic transistors, diodes, batteries, ...
- Creating complex atomic superfluid circuits

Circuit Elements for Guided Cold Thermal Atoms

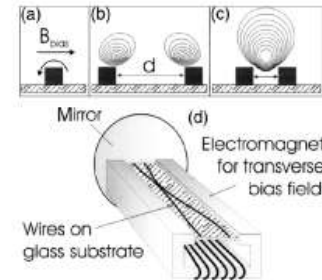
Magnetic Potentials



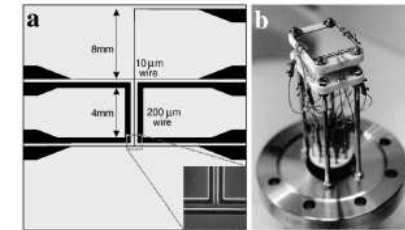
Waveguide Bends
Anderson (1999)



Y-junctions
Schmiedmayer (2000)

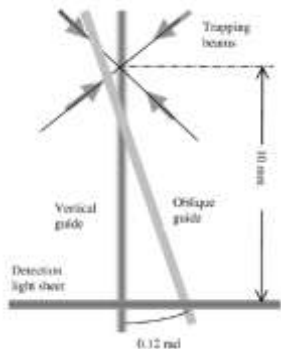


Anderson (2000)

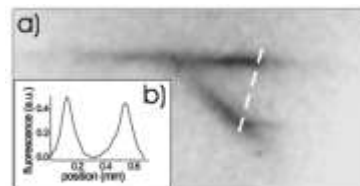
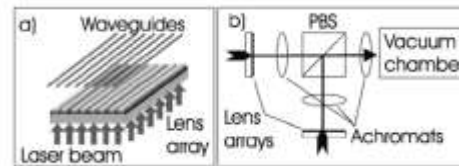


Atom Chip
Schmiedmayer (2000)

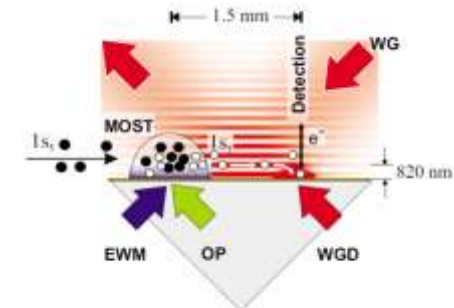
Optical Dipole Potentials



Crossed beam Y-junction
Pruvost (2000)



Microlens Arrays
Birkel (2002)

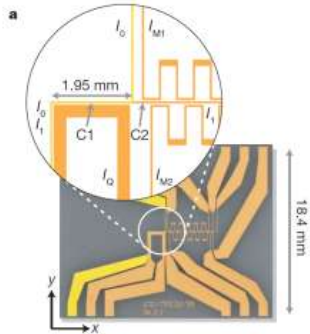


Guiding atoms out of surface MOT
Mlynek (2003)

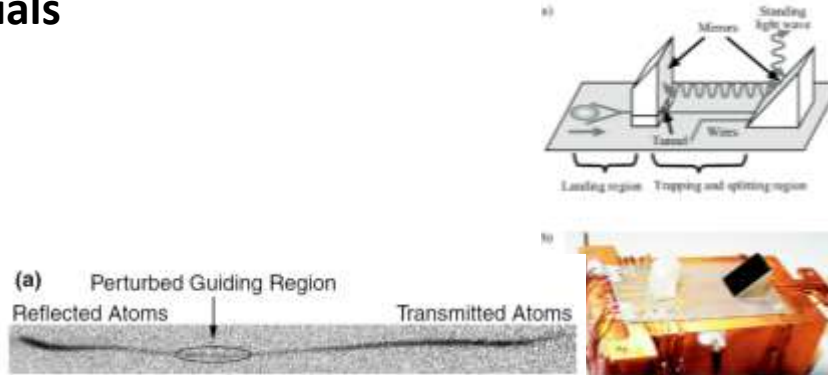
Limited source coherence and multimode propagation

Circuit Elements for Guided BECs

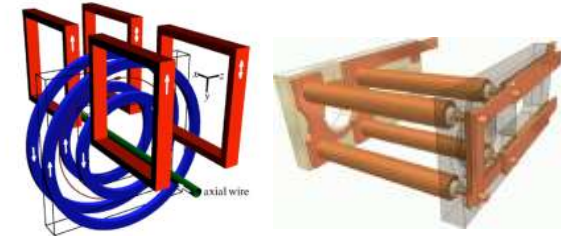
Magnetic Potentials



BEC on a Chip
Hansch/Reichel (2001)



BEC propagation in linear waveguides on atom chips
Ketterle (2002) Anderson (2005)

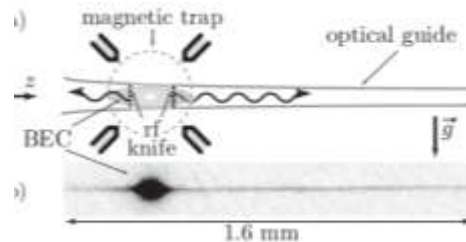


Propagation in macroscopic waveguides
Stamper-Kurn (2005) Sackett (2006)
Arnold (2005)

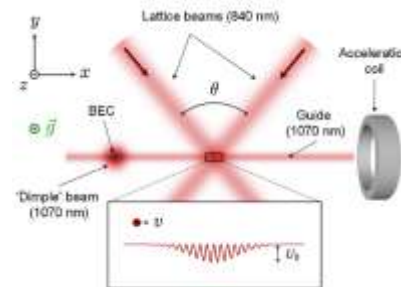
Optical Dipole Potentials



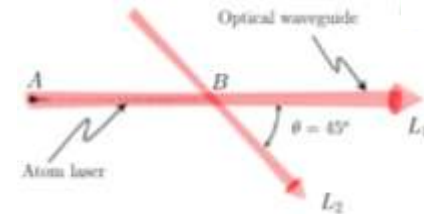
Stationary BEC in Optical Guide
Ketterle (1998), Sengstock (2001)



Guided Atom Laser
Guéry-Odelin (2006)



Bragg Reflector
Guéry-Odelin (2011)

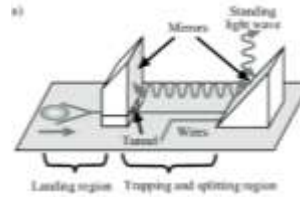
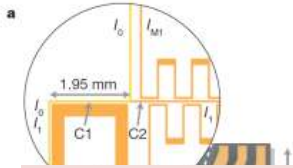


Crossed beam Y-junction
Guéry-Odelin (2012)

[See also talks by Garraway and von Klitzing]

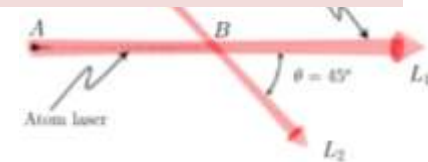
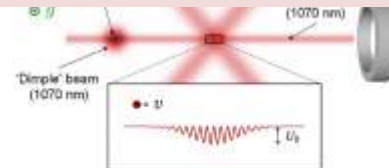
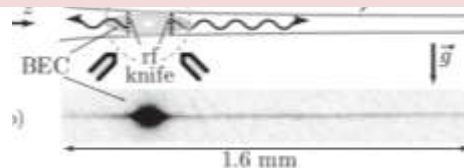
Circuit Elements for Guided BECs

Magnetic Potentials



Guided BEC Circuit To-Do List

- Bends connecting straight waveguides, preferably single-mode
- Coherent waveguide splitters, preferably single mode
- Arbitrary waveguide circuit geometries, preferably dynamic



Stationary BEC in Optical Guide
Ketterle (1998), Sengstock (2001)

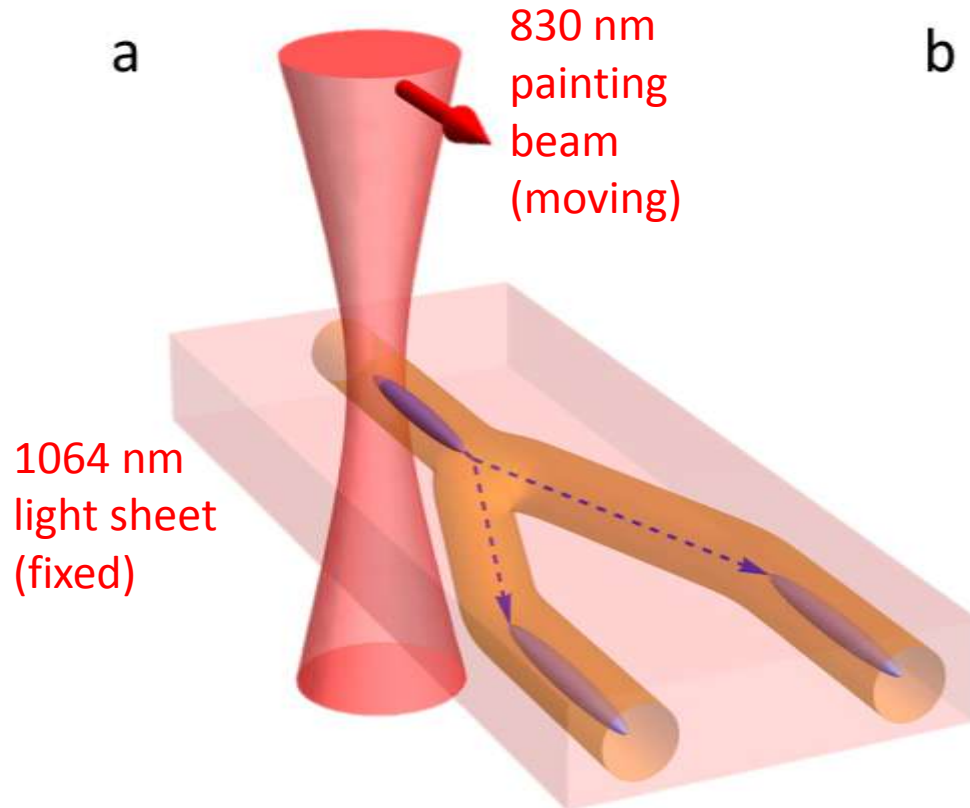
Guided Atom Laser
Guéry-Odelin (2006)

Bragg Reflector
Guéry-Odelin (2011)

Crossed beam Y-junction
Guéry-Odelin (2012)

[See also talks by Garraway and von Klitzing]

Painting Complex Matter Waveguides



Red detuned for ^{87}Rb BEC

Beam waist = $2.15 \mu\text{m}$

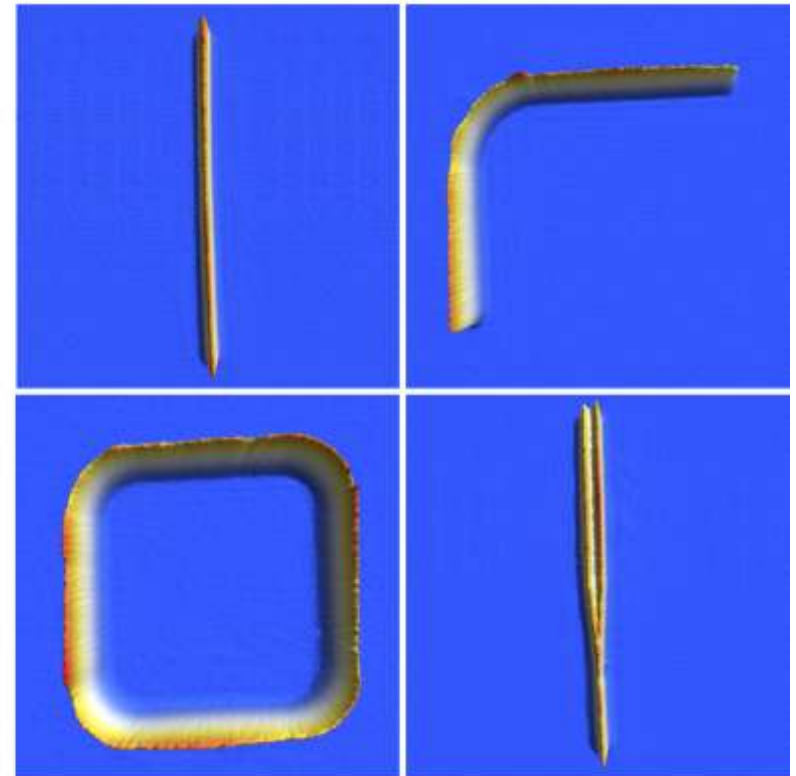
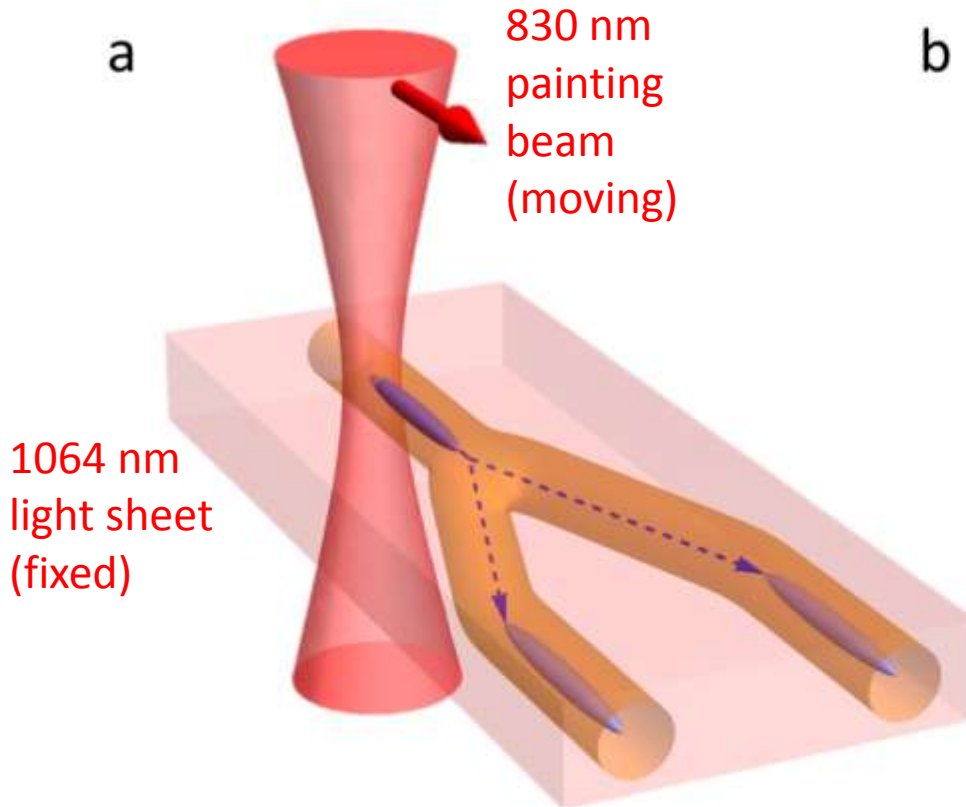
Waveguide trap frequency = 1705 Hz

Waveguide trap depth = $1.39 \mu\text{K}$

Painting frequency = 10 kHz

K. Henderson *et al*, New J. Phys. **11**, 043030 (2009)

Painting Complex Matter Waveguides



Painted optical dipole potentials
(image dimensions $\sim 100\mu\text{m} \times 100\mu\text{m}$)

Complexity: 100x100 addressable spots

Red detuned for ^{87}Rb BEC

Beam waist = $2.15 \mu\text{m}$

Waveguide trap frequency = 1705 Hz

Waveguide trap depth = $1.39 \mu\text{K}$

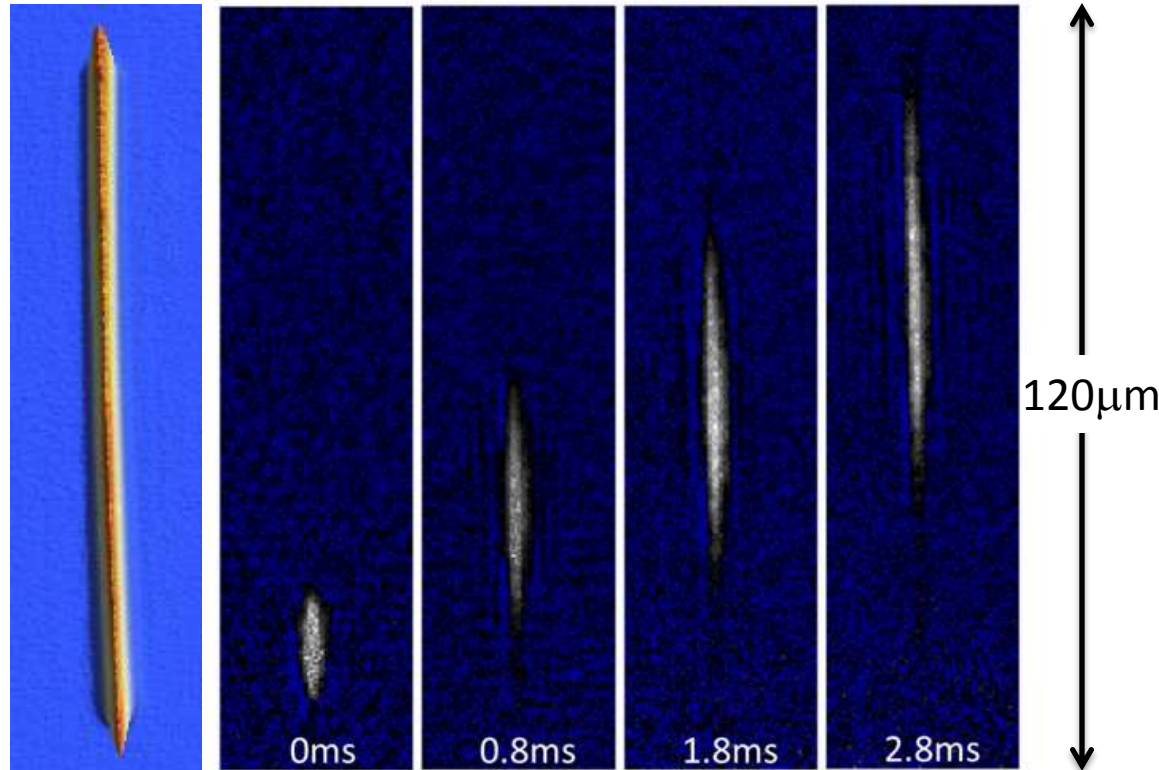
Painting frequency = 10 kHz

K. Henderson *et al*, New J. Phys. **11**, 043030 (2009)

Ryu & Boshier arXiv:1410:8814

Matter Wave Source : Launching a BEC into the Waveguide

- In situ absorption images
- Atom number ~ 4000
- Experimental sequence:
 1. Create BEC in short waveguide
 2. Switch to circuit potential with slope at BEC location
 3. Allow BEC to accelerate for 1.3ms, to $v = 19$ mm/s
 4. Switch to flat circuit potential
 5. Image
- Timing is relative to end of launch
- We have tuned velocity up to 25 mm/s

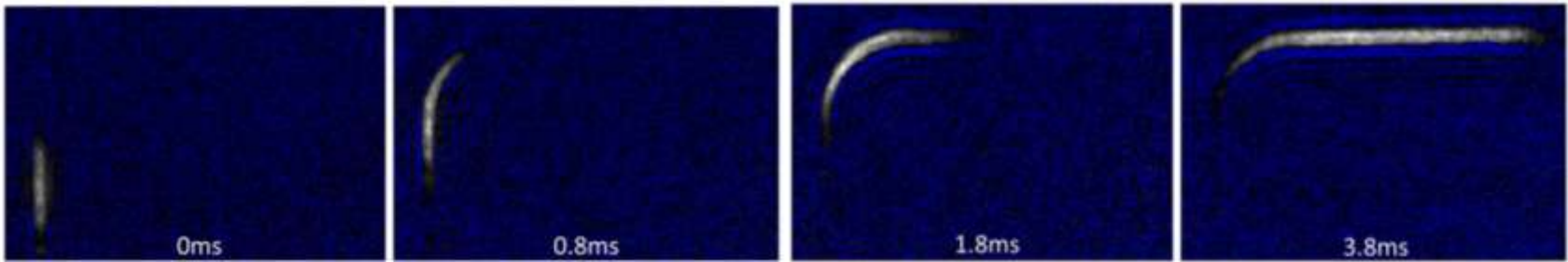


Waveguide Bends

L-Bend

Bend radius = $18.6 \mu\text{m}$

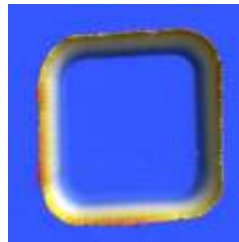
$v = 19 \text{ mm/s}$



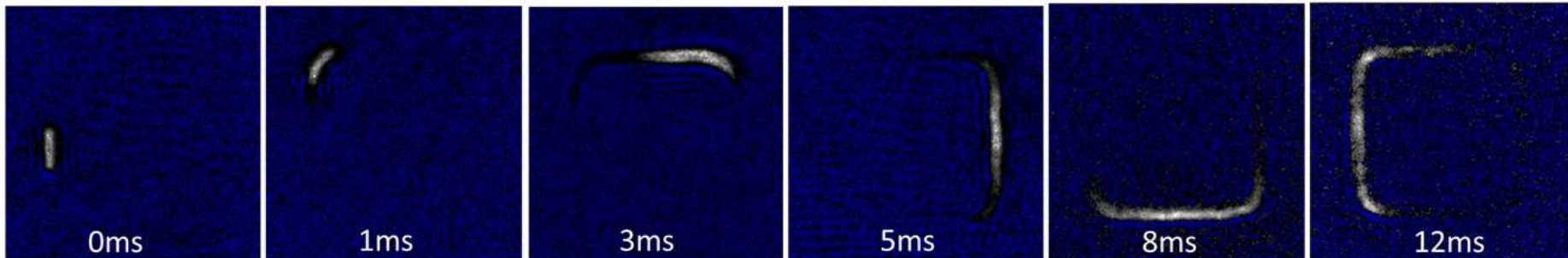
Closed waveguide with circular corners

Bend radius = $9.3 \mu\text{m}$

$v = 19 \text{ mm/s}$



← $70 \mu\text{m}$ →



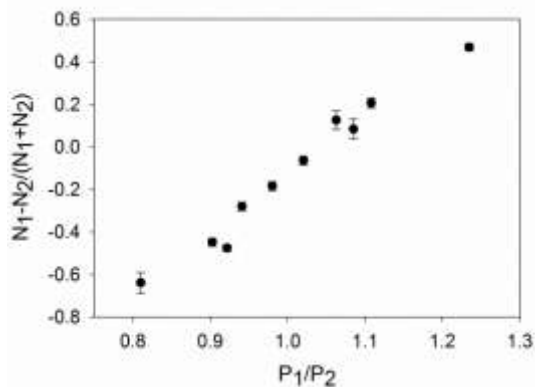
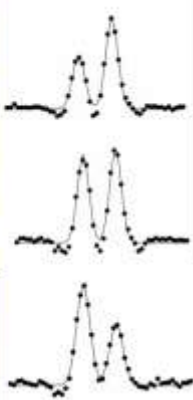
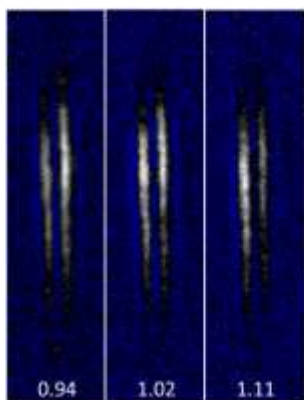
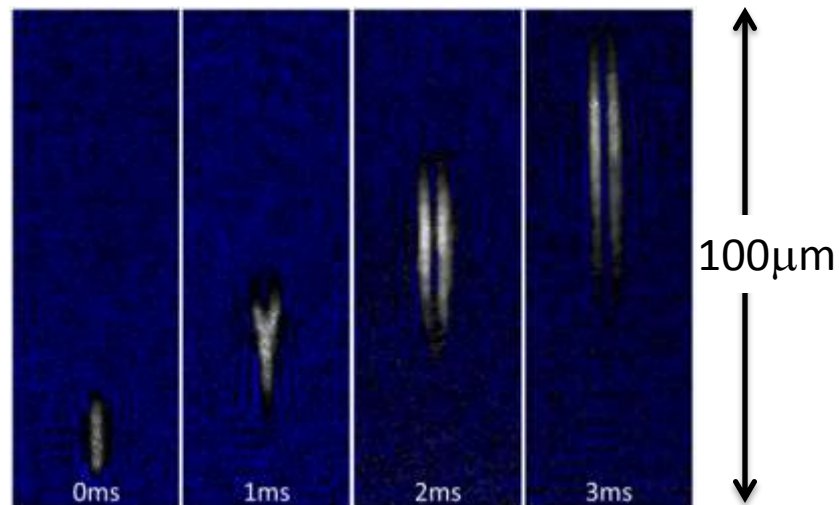
Waveguide Y-junction

Y-junction

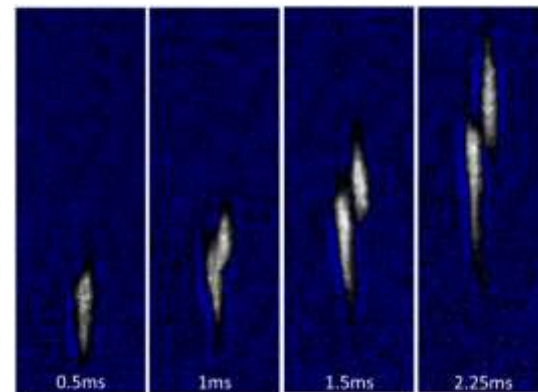
Arm separation = $3.7 \mu\text{m}$

$v = 21 \text{ mm/s}$

50/50 beam splitter



Tunable splitting ratio



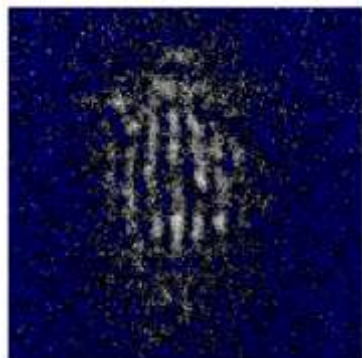
Switching by jumping splitting ratio during BEC transit

Phase Coherence

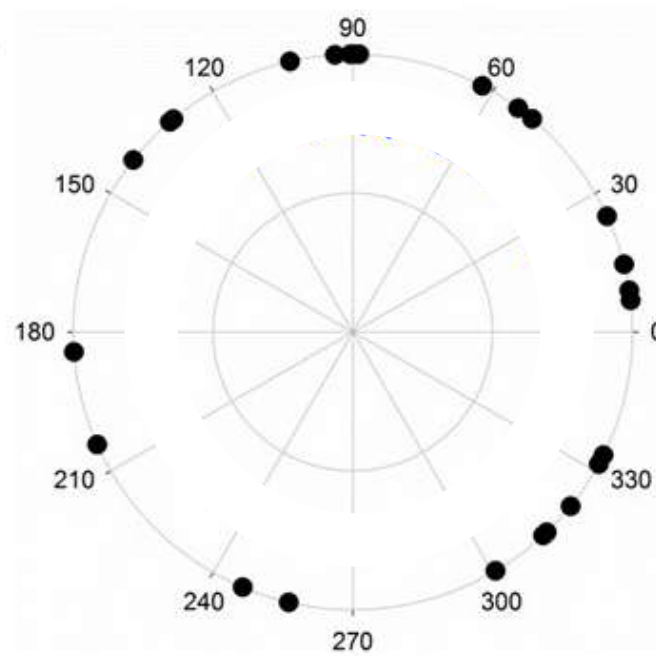
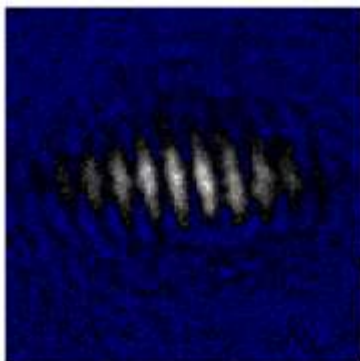


BECs
created
 $3.7 \mu\text{m}$
apart

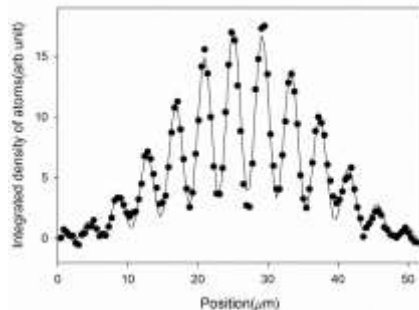
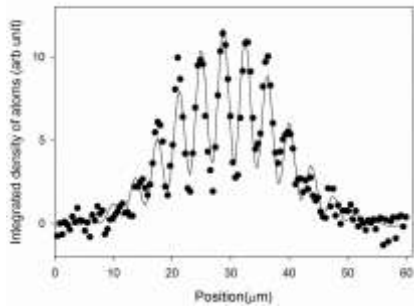
2.5ms TOF



3.0 ms TOF



Black = separate BECs



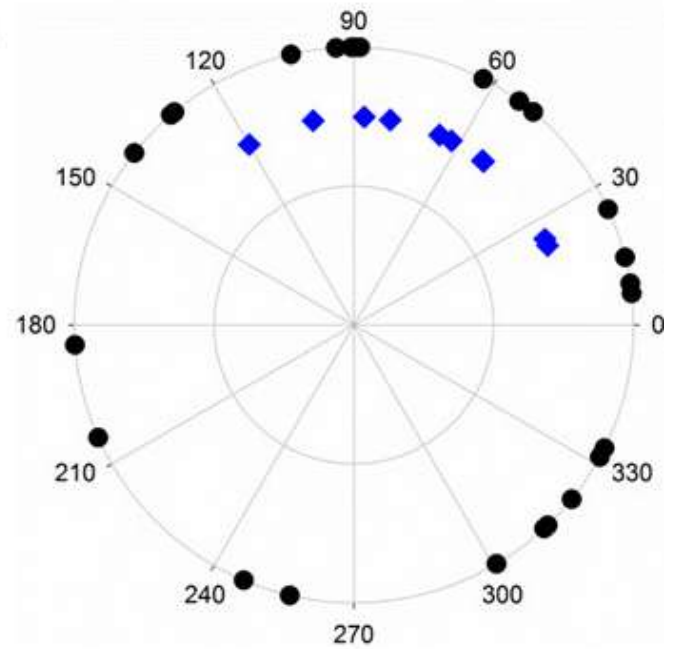
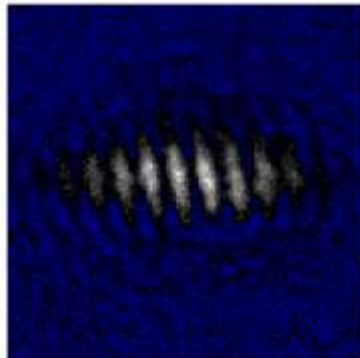
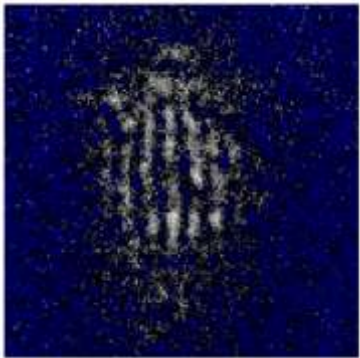
Phase Coherence



BECs
created
 $3.7 \mu\text{m}$
apart

2.5ms TOF

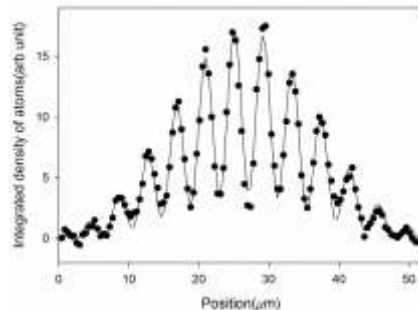
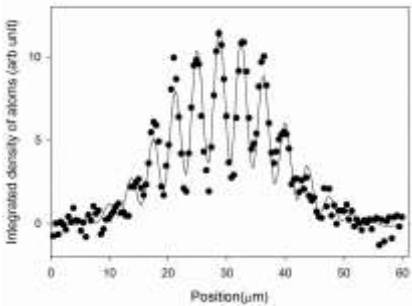
3.0 ms TOF



Black = separate BECs
Blue = split BEC

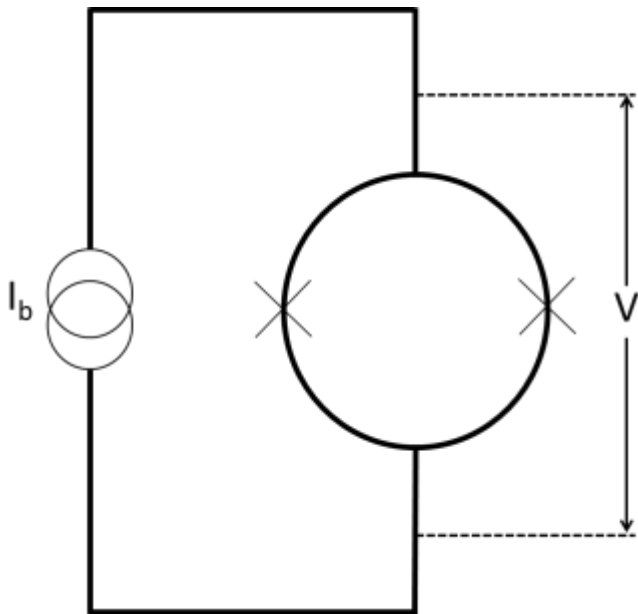
Standard deviation of phase
difference after Y-junction = 29.7°

The phase coherence is
established in the splitting process

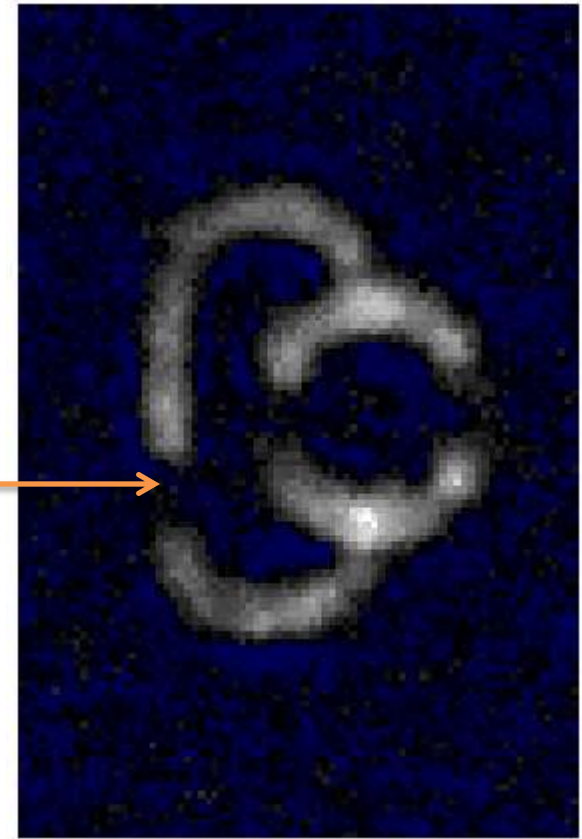


Superfluid Atom Circuits

- Painted potential waveguides support superfluid flow.
- May be possible to add leads to a dc atom SQUID and drive current through it



Emulate
current
source with
high moving
barrier



BEC filling a painted circuit

Excitation of Fast Wavepackets at Waveguide Bends

Circular bends

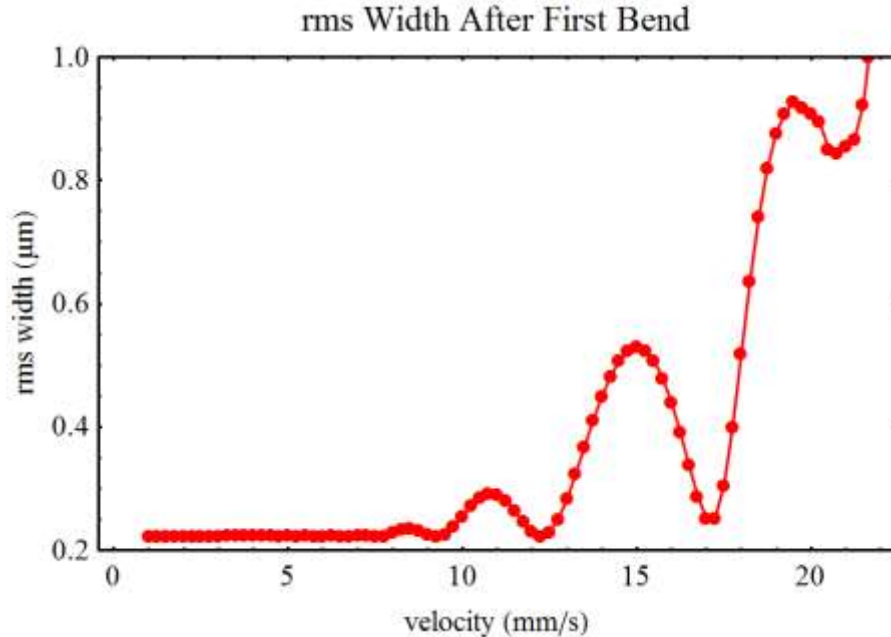
No interactions

$v = 12 \text{ mm/s}$

Guide freq. = 1.2 kHz

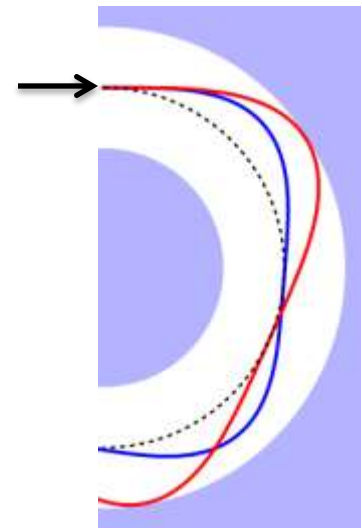
Guide depth = 940 nK

Contour = 470 nK

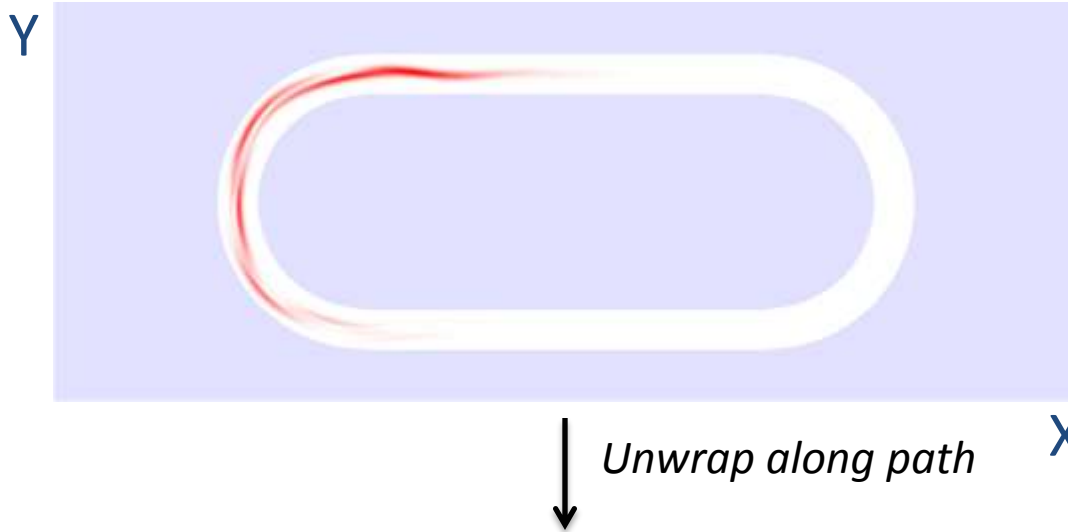


It is known that a classical description works well [Bromley & Esry, PRA **69**, 053620 (2004)].

Excitation is minimized when a particle completes an integer number of oscillations inside the bend



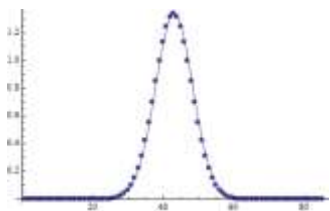
Quantifying Excitation



↓ Unwrap along path X



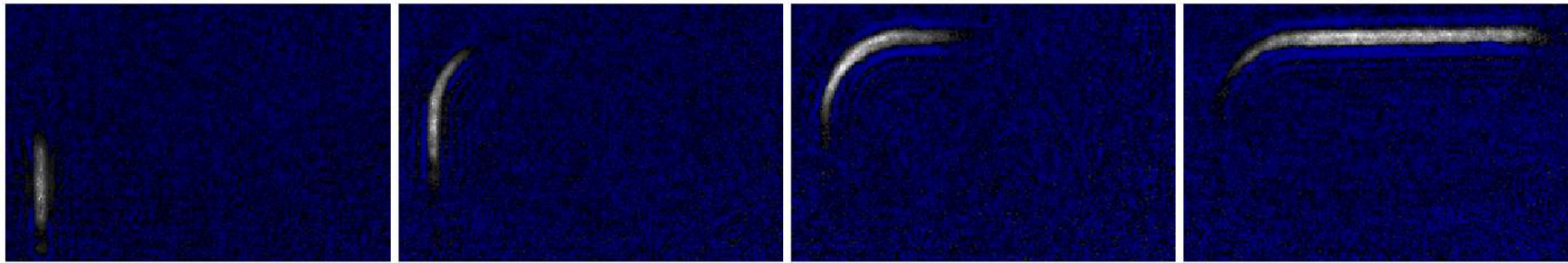
↓ Project and average



$\psi_0(q_{\perp})$

$$p = \int dq_{\parallel} \left| \int \psi(q_{\parallel}, q_{\perp}) \psi_0(q_{\perp}) dq_{\perp} \right|^2 = 0.42$$

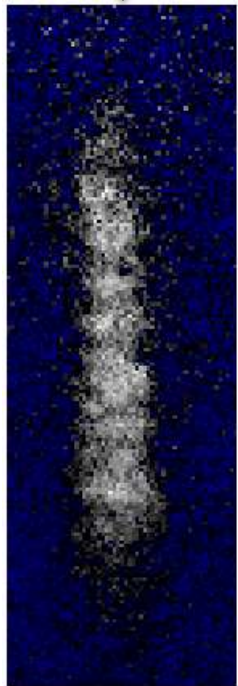
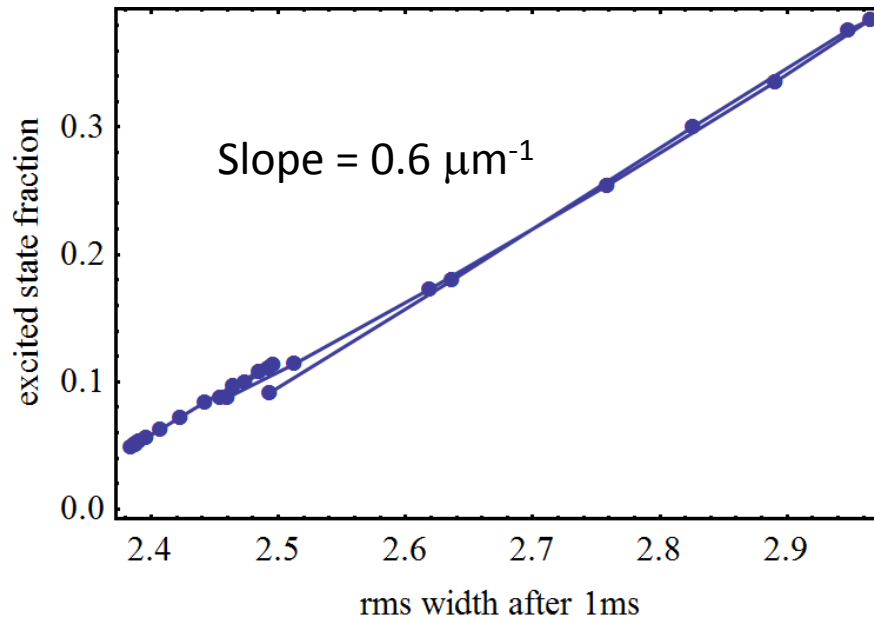
Almost Single-Mode Propagation Around a Bend



1ms TOF

1ms TOF

Relation between excited state fraction and TOF rms width from GPE simulations for different bend radii



rms = 2.68(15) μm



rms = 2.82(6) μm

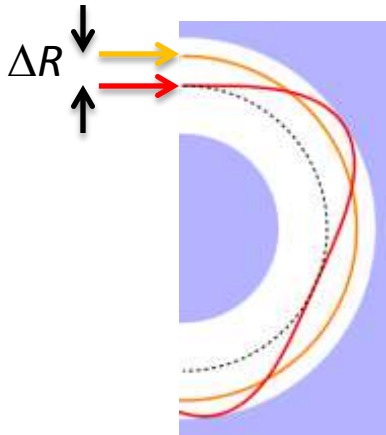
Relative ground state population after bend = 92(9)%

Designing Bends to Reduce Excitation

Two alternatives to tuning velocity to bend radius, both compatible with the painted potential

Offset circular bends

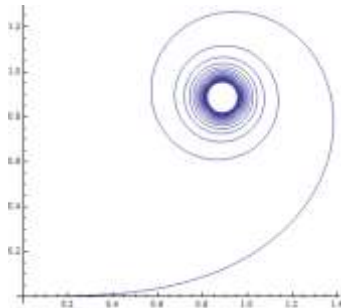
$$m \omega^2 \Delta R = m v^2 / (R + \Delta R)$$



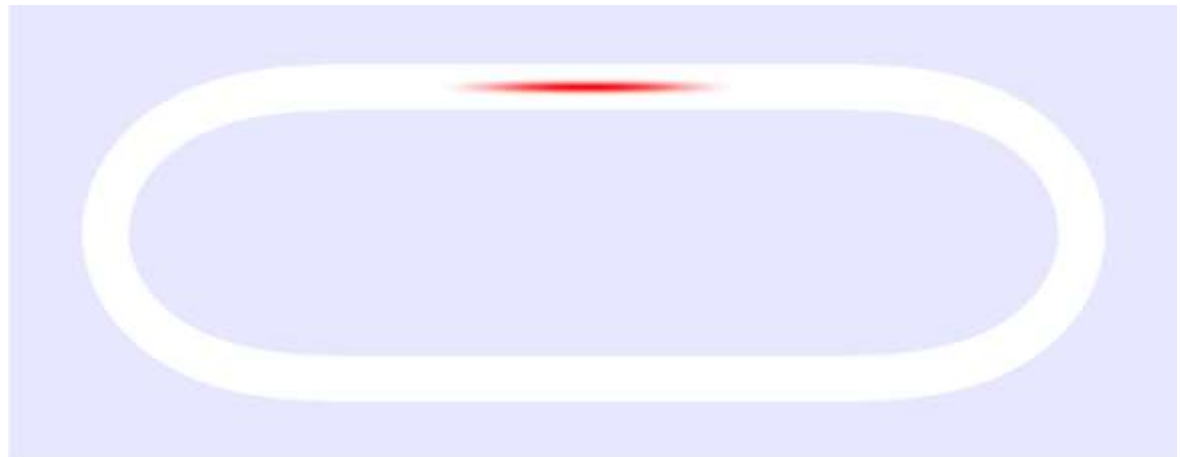
Euler spiral bends

Curvature

$$\kappa(s) = s/a^2$$

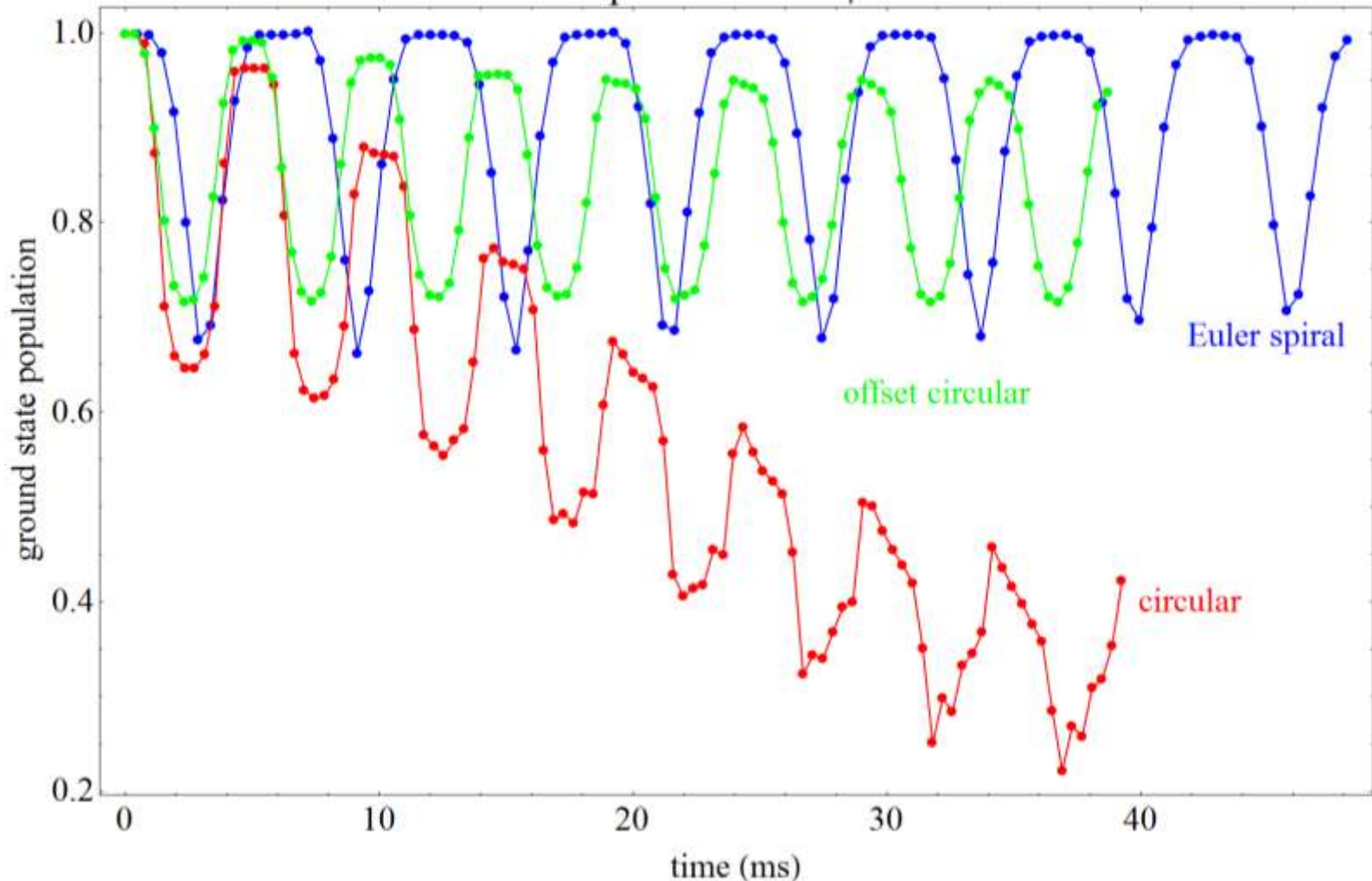


(Cornu spiral or clothoid)



Comparison of the Three Bend Types

Evolution of excited state fraction over four laps of the stadium



Euler curve bend is best, returning to almost 100% occupation of ground state in the straight sections of the stadium.

Summary

- **Toroidal waveguides**
 - Quantized circulation
 - Bessel beams
 - Josephson junctions for atom SQUID
- **Matter wave circuits**
 - Launching into straight and curved waveguides
 - Phase-coherent splitting at Y-junction
 - Excitation at bends

