Thermalization & localization in extended quantum systems

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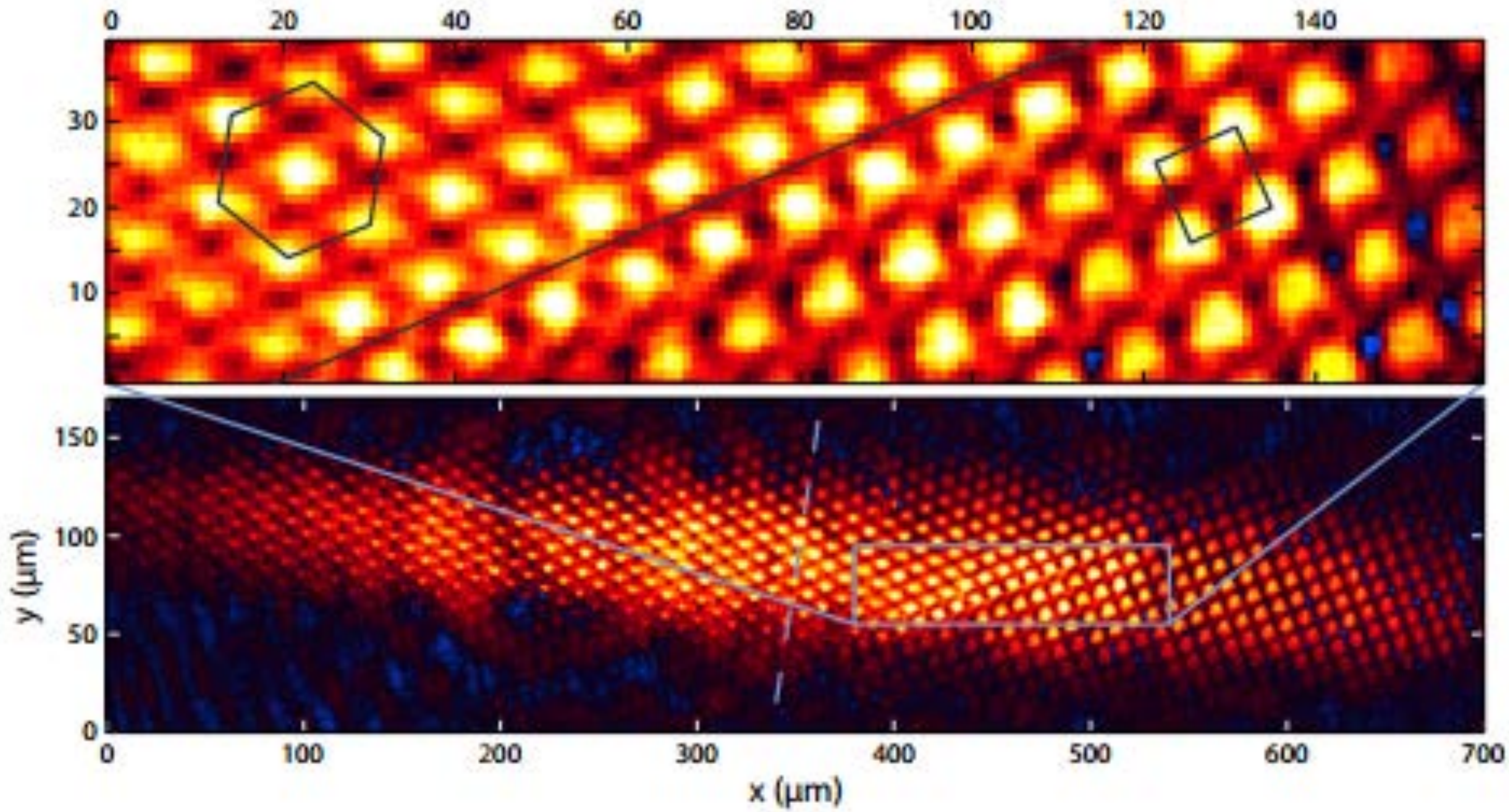
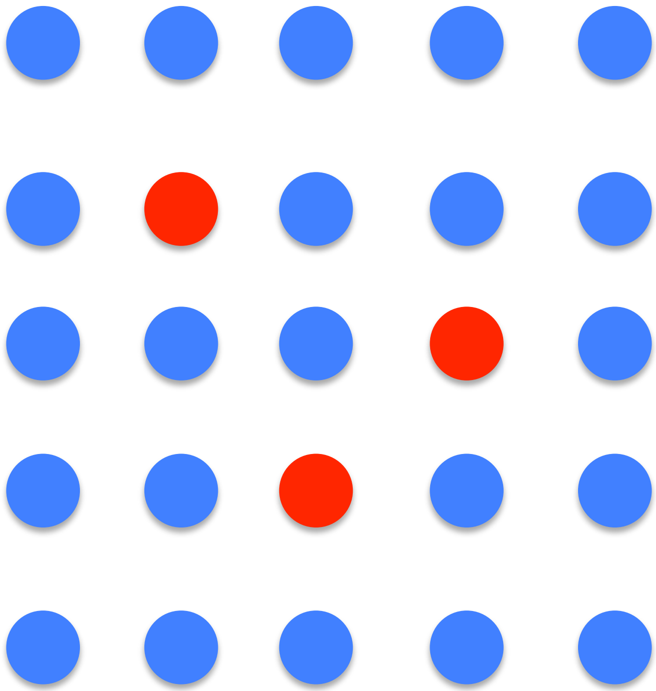
1. Statistical mechanics of eigenstates:
   - Eigenstate Thermalization Hypothesis (ETH)
   - ETH violations due to disorder (Anderson localization)
2. Consequences for the propagation of information in large-scale quantum systems



0
20
40
60
80
100
120
140
30
20
10
150
100
50
0
y (µm)
0
100
200
300
400
500
600
700
x (µm)

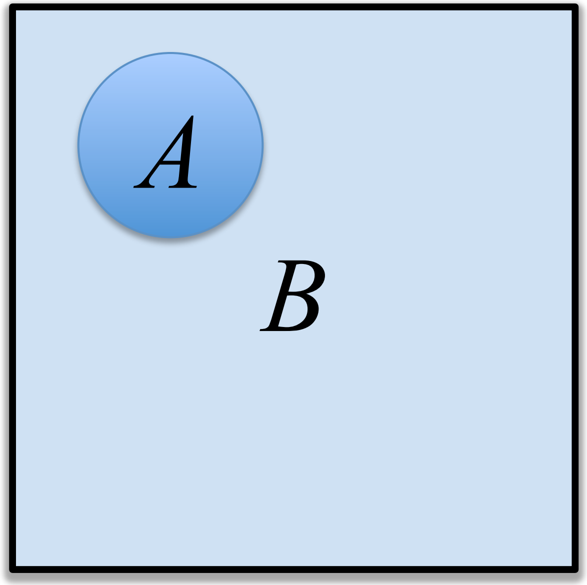


$$np + ns \leftrightarrow ns + np$$

$$\sum_{ij} a_i^{\dagger} a_j J(\vec{r_i}, \vec{r_j})$$

Dipole interaction:

$$J_{ij} = \frac{\vec{\mu_i} \cdot \vec{\mu_j} - 3(\vec{\mu_i} \cdot \vec{n})(\vec{\mu_j} \cdot \vec{n})}{4\pi\epsilon_0 R^3}$$



Isolated quantum system undergoing unitary time evolution

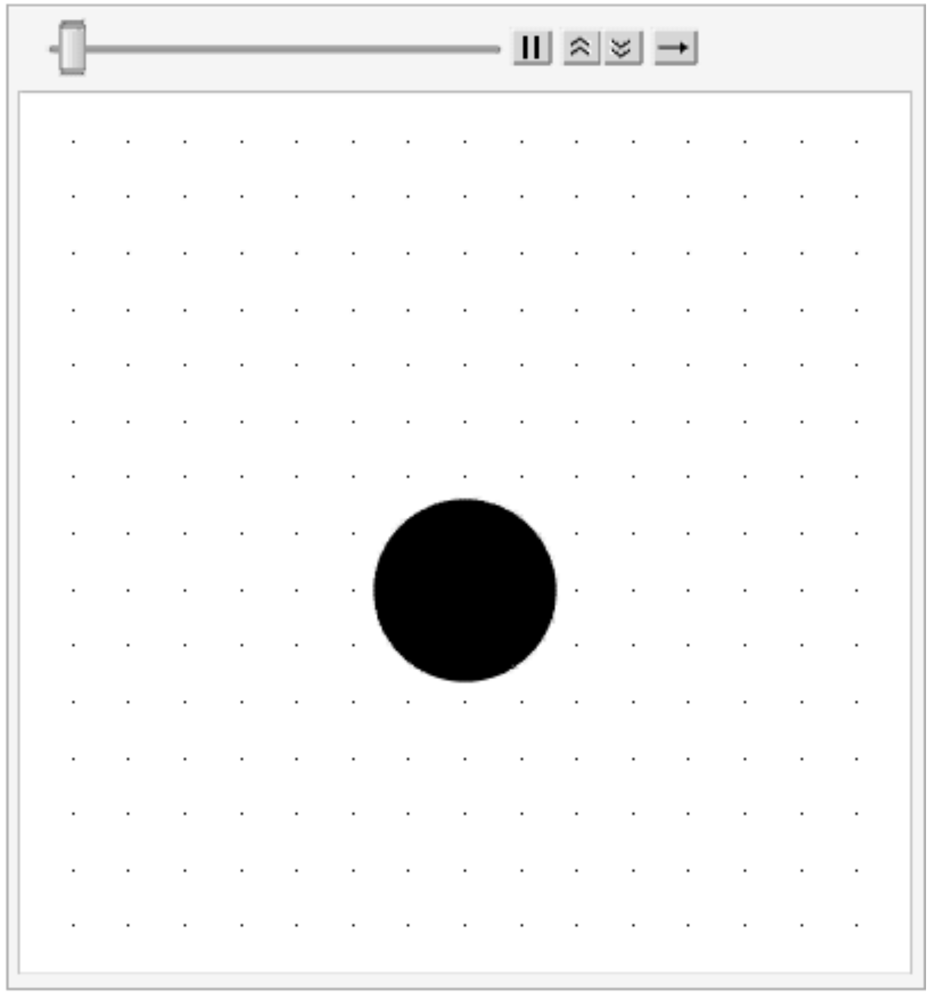
Thermalization in such a closed system implies that for any subsystem A the rest of the system acts as a reservoir B

Eigenstate Thermalization Hypothesis

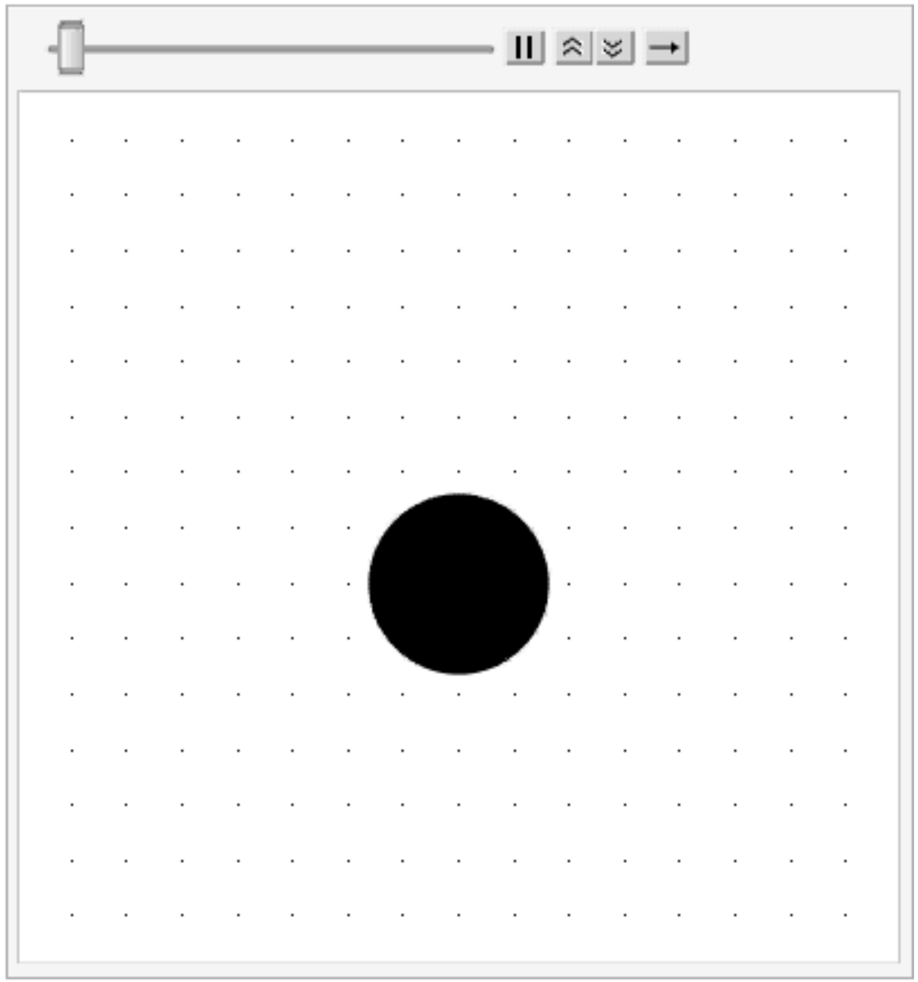
All many-body eigenstates are thermal –
atypical out-of-equilibrium initial states will become typical states in the limit of infinite time

Testing ETH -
obtain many-body eigenstates of the system's Hamiltonian from exact diagonalization, extrapolate to the thermodynamic limit

Propagation of 1 excitation in a lattice

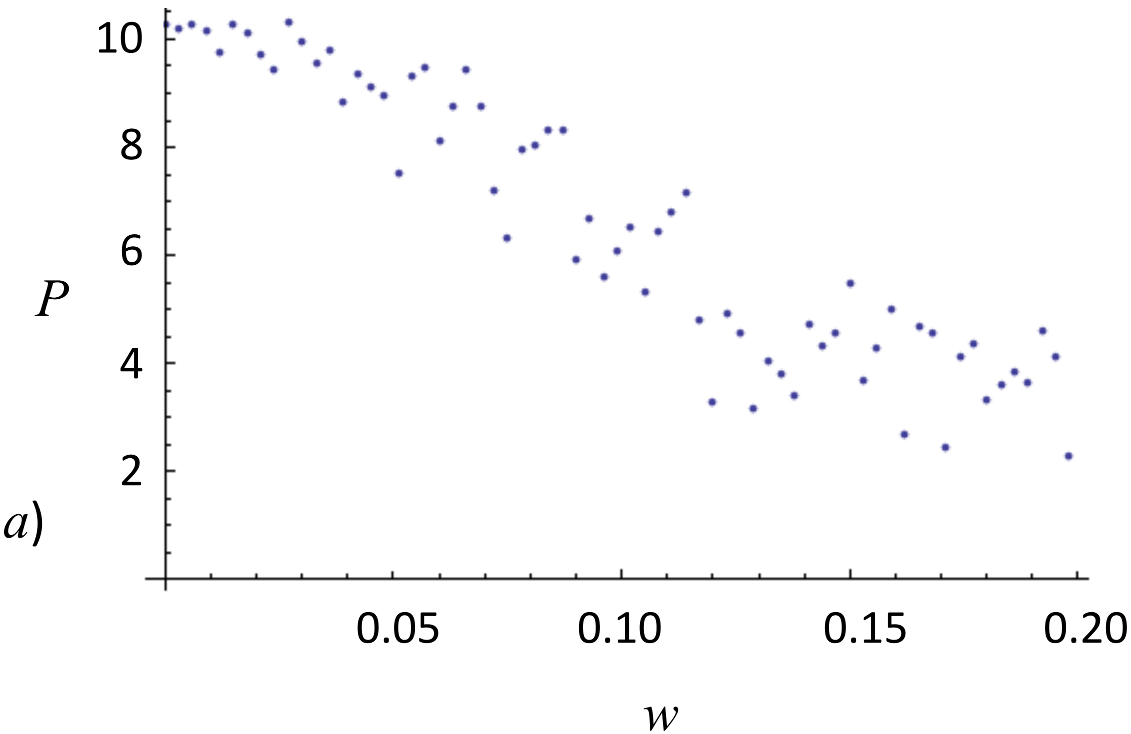


Single particle localization



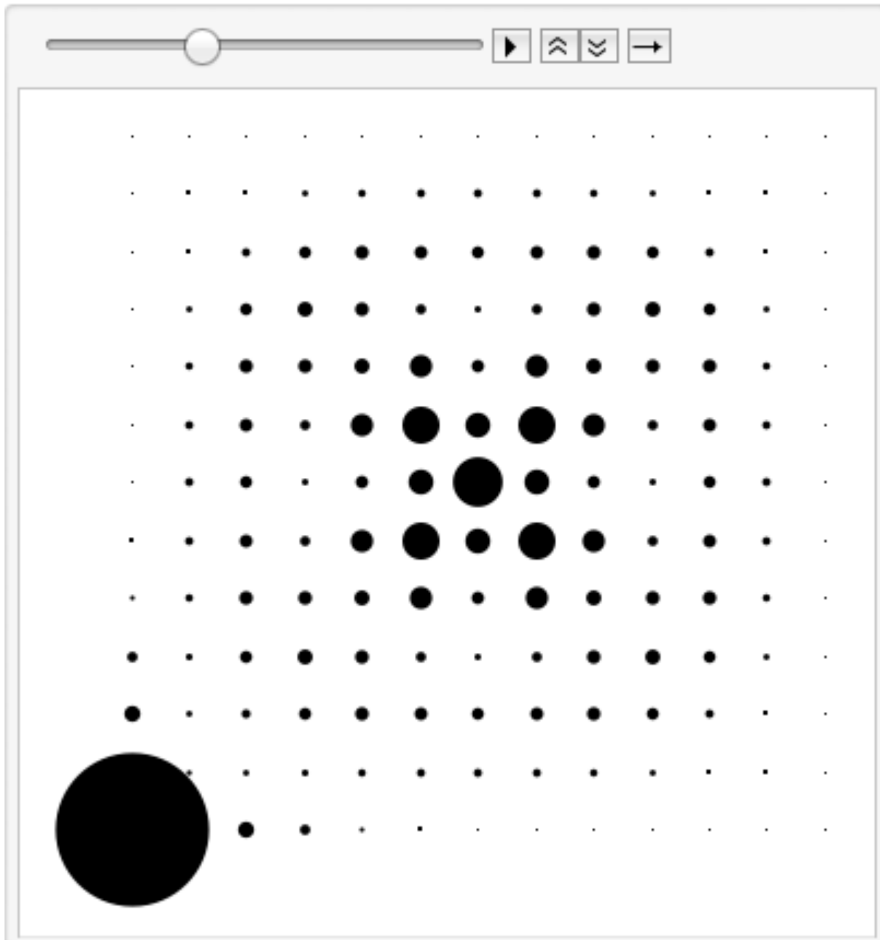
$$P = \left(\frac{\sum_i |\psi_i|^4}{\left(\sum_i |\psi_i^2|\right)^2} \right)^{-1}$$

where:
P : Participation length
(in units of lattice spacing a)
w : Randomness



- Propagation exponentially decreasing, akin to Anderson localization
- Basis of extended and localized eigenstates

With interactions



(Most obvious) consequences of ETH violation for quantum information processing

- Scalability is based on the creation of large networks
- Disorder in the network is inevitable
- Small amounts of disorder limits the propagation speed of information
- Above a certain threshold, information is localized - determines the effective size of the system