# Optomechanics based on embedded quantum emitters

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QSI, Benasque, July 2014

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- Quantum limited control
- Quantum signatures
- Quantum applications

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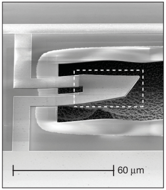


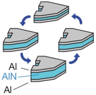


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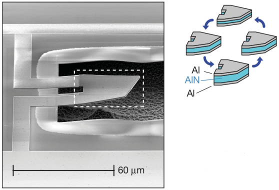


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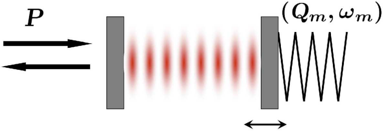
## Cavity optomechanics

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- Parametric “three-wave mixing” realisation
- Optical inputs and outputs $\rightarrow$ shot-noise limited photodetection

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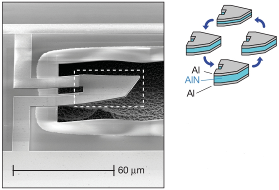


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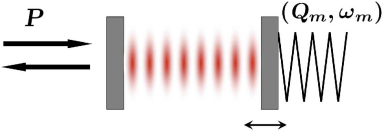
Cavity optomechanics

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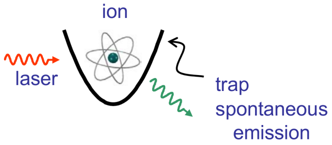
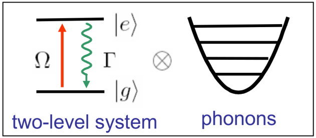
Optical resonances + Electron-phonon interactions $\rightarrow$ embedded quantum-emitter optomechanics

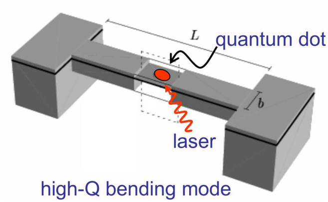
# Laser cooling mechanical resonators

**Trapped ion**




Sideband cooling via
embedded quantum dot

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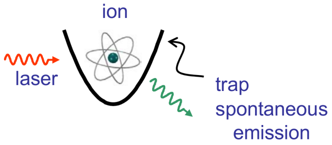


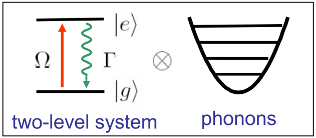


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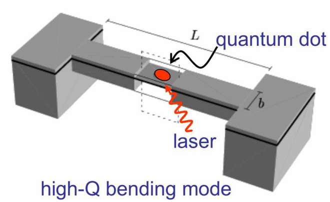


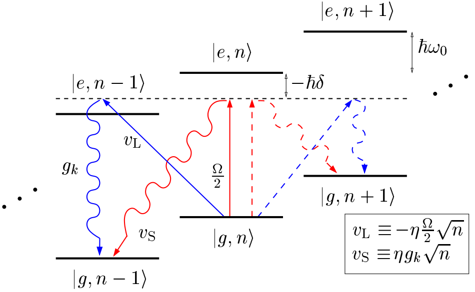


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# Electron-phonon interactions

Deformation potential coupling:

$$H_{DP} = \int d\bar{r}^3 \left[ D_c \hat{\rho}_e(\bar{r}) - D_v \hat{\rho}_h(\bar{r}) \right] \nabla \cdot \hat{\bar{u}}(\bar{r})$$

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- Continuum (thin-rod) elasticity theory
- $\lambda_p \gg$ QD size $\rightarrow \nabla \cdot \hat{\bar{u}}(\bar{r}) \approx \nabla \cdot \hat{\bar{u}}(\bar{r})|_{\bar{r}=\bar{r}_{\mathrm{QD}}}$
- Small QD (height $\sim$ 3nm, diameter $\sim$ 20nm) and $T \lesssim$ 10K $\rightarrow$ electronic degrees of freedom treated as TLS: $|e\rangle$ (single exciton), $|g\rangle$ (electronic ground state)

$$H_{\mathrm{QD-ph}} = (D_c - D_v)\, |e\rangle\langle e|\, \nabla \cdot \hat{\bar{u}}(\bar{r})|_{\bar{r}=\bar{r}_{\mathrm{QD}}}$$

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Coupling between QD exciton and resonator mode:

$$\hbar g_0 |e\rangle\langle e| (b_0 + b_0^\dagger)$$

# Electron-phonon interactions

Flexural mode:

$$\frac{g_0^{(f)}}{2\pi} = \frac{0.83\,(1-2\sigma)\,(D_c - D_v)}{(1+\sigma)^{1/4}\sqrt{\hbar\rho c_T}} \times \frac{x_{\mathrm{QD}}}{t\sqrt{w}L^{3/2}} \times X_0''\,(k_0 z_{\mathrm{QD}})$$

First factor is a materials parameter $\sim 10\,\mu^2\mathrm{MHz}$ and optimal parameters with $w = 0.1\,\mu$, $L = 1\,\mu \rightarrow \frac{g_0}{2\pi} = 10\,\mathrm{MHz}$

Bulk compressional mode:

$$\frac{g_0^{(c)}}{2\pi} = \frac{0.24\,(1-2\sigma)\,(D_c - D_v)}{(1+\sigma)^{1/4}\sqrt{\hbar\rho c_T}} \times \frac{1}{\sqrt{twL}} \times \sin\frac{\pi z_{\mathrm{QD}}}{L}$$

$$\frac{g_0^{(c)}}{g_0^{(f)}} \sim \sqrt{\frac{L}{t}} \qquad x_{\mathrm{ZPF}}^{(c)} \sim \frac{1}{\sqrt{tw}} \qquad x_{\mathrm{ZPF}}^{(f)} \sim \frac{1}{t}\sqrt{\frac{L}{w}}$$

# Quantum emitter optomechanics

"Polaronic" representation canonical transformation eliminates QD-resonator coupling

$$H = \hbar\omega_0 b_0^\dagger b_0 - \hbar\delta\frac{\sigma_z}{2} + \hbar\left[\frac{\Omega}{2}\sigma_+ B^\dagger + \sum_k g_k \sigma_+ B^\dagger a_k + \frac{\sigma_z}{2}\sum_q \lambda_q b_q \right.$$
$$\left. + \left(b_0 + b_0^\dagger\right)\sum_q \zeta_q b_q + \text{h.c.}\right] + \hbar\sum_q \omega_q b_q^\dagger b_q + \hbar\sum_k \left(\omega_k - \omega_L\right) a_k^\dagger a_k$$

$\eta = \frac{g_0}{\omega_0}$ $\quad$ $B = e^{\eta\left(b_0 - b_0^\dagger\right)}$ $\quad$ $\eta_{\text{eff}}^2 \equiv \eta^2(\langle b_0^\dagger b_0\rangle + 1)$

# Quantum emitter optomechanics

$$H = \omega_0 b_0^\dagger b_0 + \frac{g_0}{2}\sigma_z \left(b_0^\dagger + b_0\right) + \frac{\Omega}{2}\sigma_x - \frac{\delta}{2}\sigma_z$$

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- Ultra-strong coupling regime of Rabi model: $g_0 \sim \omega_0$, but "single mode approximation" breaks down
- Subohmic spin-boson model: $g_0 \gg \omega_0 \to$ phonon environment with $J(\omega) = \sqrt{\omega_* \omega}$ where $\omega_* \sim \eta^4 \omega_0$.

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Defect and spin realisations:

- NV centres (other colour centres)
  (S. J. M. Habraken et al. NJP 2012, A. Albrecht et al. NJP 2013, K. V. Kepesidis et al. PRB 2014).
- Raman schemes or spin-phonon interactions
  (for NV centres or doped QDs) (S. D. Bennett et al. PRL 2013).
- Two-level fluctuators
  (T. Ramos, V. Sudhir, K. Stannigel, P. Zoller, and T. J. Kippenberg PRL 2013).
- Donor transitions in Si (O. O. Soykal, R. Ruskov, and C. Tahan, PRL 2011).

# Carbon-nanotube realisation

Quantum regime of
nanomechanical resonators

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Some critical requirements:

(i) quantum limited measurement of the transduced output
(ii) low effective masses
(iii) strong nonlinearities & interactions
(iv) high mechanical quality

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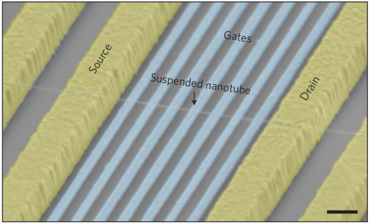


Waissman et al., Nature Nanotech. 2013

- Nanotubes as ultimate candidates to meet (ii), (iii) & (iv) ($\frac{\omega_0}{2\pi} Q \sim 10^{15}$Hz) (A.K. Hüttel et al., Nano Lett. 2009).
- Optomechanical schemes $\rightarrow$ (i) given shot-noise limited photodetection.

# Carbon-nanotube realisation

Optomechanics with suspended nanotubes

↓

Major challenge due to
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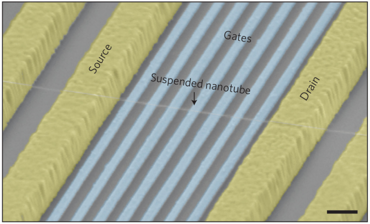


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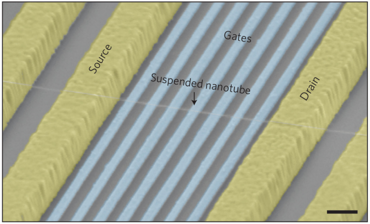


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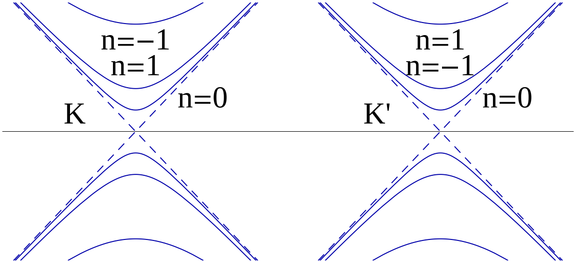
Alternative: Deformation potential coupling to localised excitons
IWR, C. Galland, W. Zwerger and A. Imamoglu, New J. Phys. 2012

Disorder-induced localisation → antibunching
(A. Hoegele et al., PRL 2008; M.S. Hofmann et al., Nat. Nanotech. 2013)

Controlled localisation → optically-active nanotube QD

# Nanotube excitons

Electronic states:




$$H_D = \hbar v_F \left( k_{x'} \hat{\sigma}_{x'} + k_{y'} \hat{\tau}_3 \hat{\sigma}_{y'} \right)$$

(i) graphene sheet rolled into a cylinder $\rightarrow$ $k \cdot p$ at "Dirac points" $K$ and $K'$

(ii) envelope function approximation within each 1D subband
—cf. Capaz et al. PRB 2006 but Bloch function as determined by (i)

# Nanotube excitons

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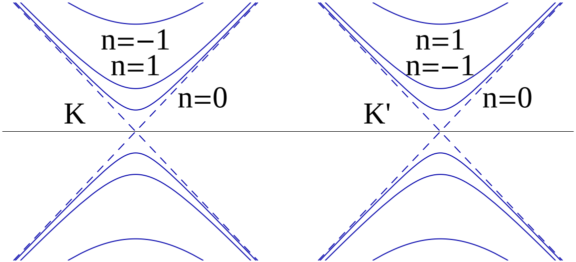


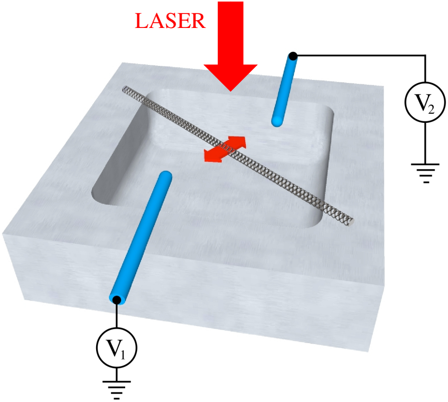

Envelope function approximation for singlet direct excitons:

$$|\psi_{nm\pm}\rangle = \frac{1}{2}\left(|K_{n,+}K^{*}_{m,-}\rangle \otimes |F_{nm}\rangle \pm |K'_{-m,+}K'^{*}_{-n,-}\rangle \otimes |F'_{nm}\rangle\right) \otimes (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \qquad n = m = 0 \rightarrow E_{11}$$

Weak axial magnetic field renders $|\psi_{00-}\rangle$ bright $\rightarrow$ tune $\Gamma$

# Excitonic nanotube quantum dot

Exciton localization via static inhomogeneous electric field.



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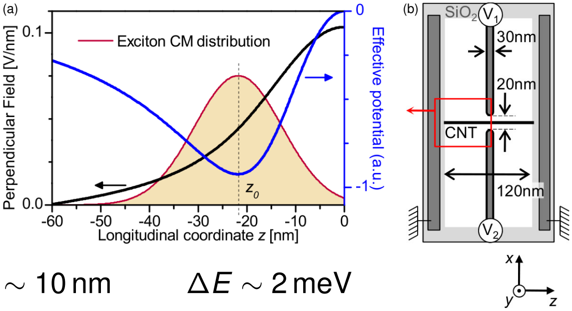


$\sigma_{\mathrm{CM}} \sim 10\,\mathrm{nm}$ $\Delta E \sim 2\,\mathrm{meV}$

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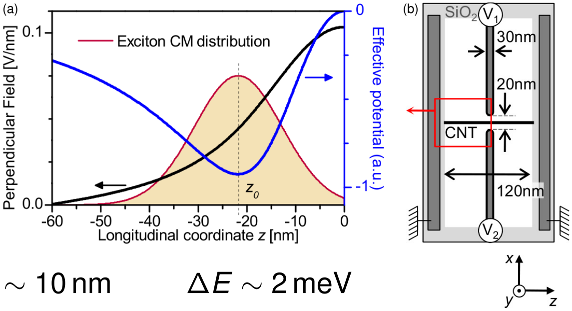


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Exciton-phonon coupling:

$$\langle \psi_{00\pm} | \hat{H}_{\mathrm{X-ph}} | \psi_{00\pm} \rangle \approx 2\nu g_2 (1+\sigma) \cos 3\theta \, \langle F_{00} | \frac{\partial \hat{\phi}_c}{\partial z}(\hat{z}_e) | F_{00} \rangle$$
$$+ 2\xi R \, [g_1 (1-\sigma) + \nu\zeta g_2 (1+\sigma) \cos 3\theta] \, \langle F_{00} | \frac{\partial^2 \hat{\phi}_f}{\partial z^2}(\hat{z}_e) E_\perp(\hat{z}_e) | F_{00} \rangle$$

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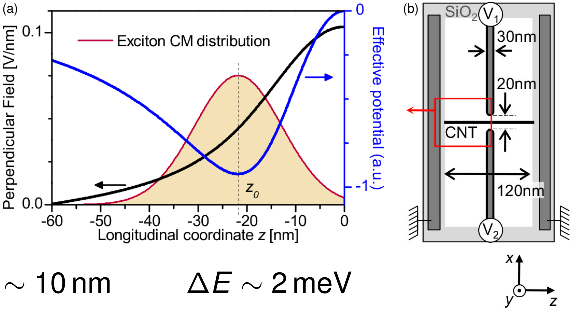


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QD-flexural mode coupling:

$$H_{\rm QD-R} = \hbar g_0 \left( b_0 + b_0^\dagger \right) \sigma_{ee}$$

$$\frac{g_0}{\omega_0} \approx 2^{3/4}(1-\sigma)\frac{g_1 \sigma_G^{1/4} \xi \mathcal{E}_\perp}{R E_G^{3/4}(q_0 L)} \sqrt{\frac{L}{\pi \hbar}}$$

$$\xi = eR/2\epsilon_\perp (E_{13} - E_{11})$$

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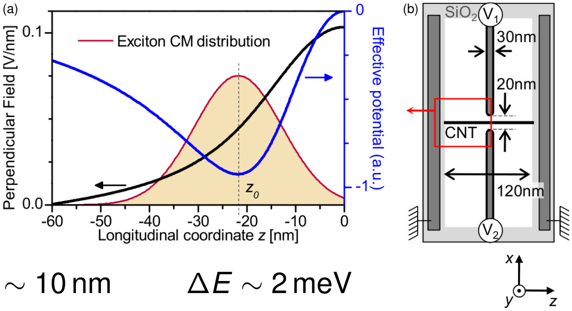


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QD-flexural mode coupling:

Parameters: $(n, m) = (9, 4)$, $L = 120$ nm, $Q = 1.5 \times 10^5$,
$\mathcal{E}_\perp = 36.8\mathrm{V}\mu^{-1} \rightarrow \frac{\omega_0}{2\pi} = 1.67\,\mathrm{GHz}$, $\frac{g_0}{\omega_0} = 0.086$, $\frac{g_0}{2\pi} = 144\,\mathrm{MHz}$

# Conclusions

- Optomechanics based on deformation-potential electron-phonon interactions
- Optical nanotransducer via homodyne measurements
- Optomechanical analogue of cavity-QED
- Strong (tunable) exciton-phonon coupling $\sim 10 - 100$MHz

IWR, C. Galland, W. Zwerger and A. Imamoglu, New J. Phys. 2012