Quantum State Tomography

quant-ph
1407.4759

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Benasque, July 10, 2014

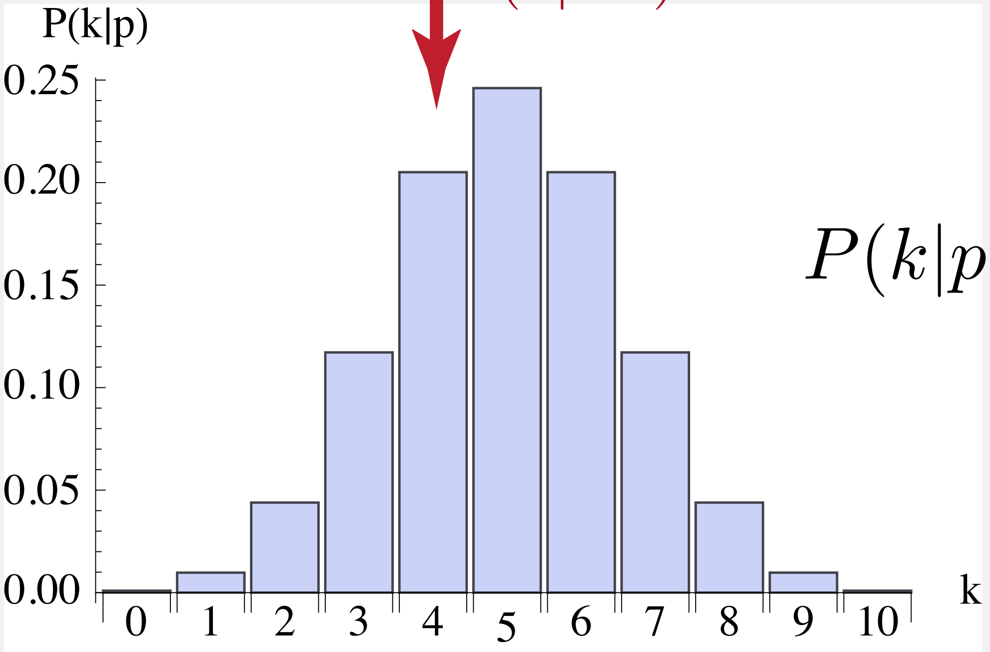
Coin flip tomography

N=10 coin flips

k=4 tails

→ estimate the coin tail probability p

$P(4|0.5) = 0.205$



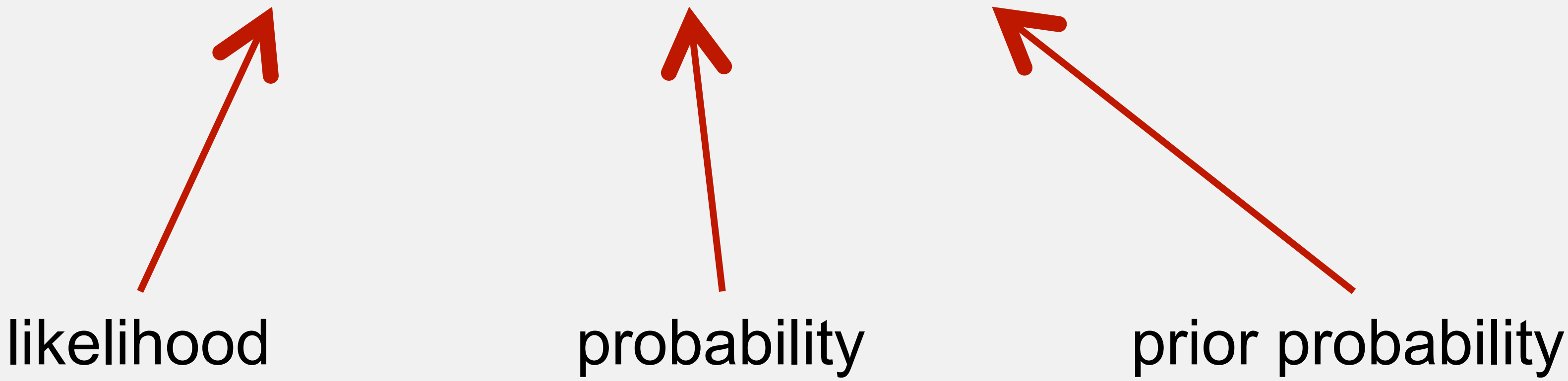
$$P(k|p) = \binom{N}{k} p^k (1-p)^{N-k}$$

Coin flip tomography

N=10 coin flips
k=4 tails
→ estimate the coin tail probability p

Bayes' theorem:

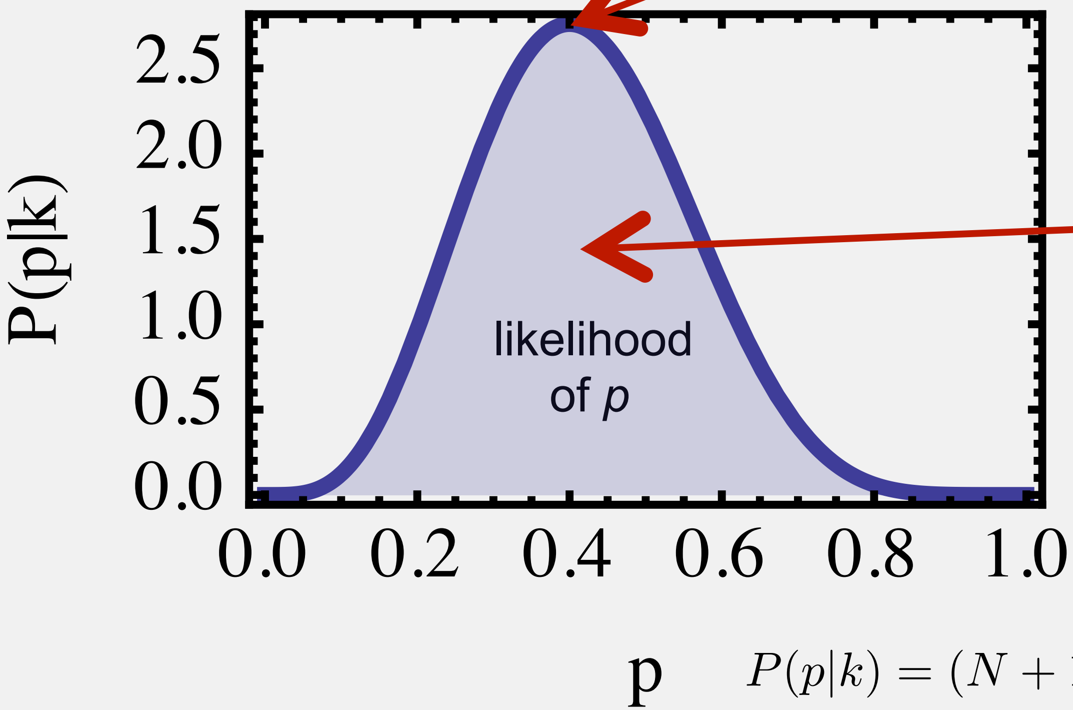
$$P(p|k) \propto P(k|p)P(p)$$



Coin flip tomography

N=10 coin flips
k=4 tails
→ estimate the coin tail probability p

maximum: $\hat{p} = 0.4$



$p = 0.42 \pm 0.14$

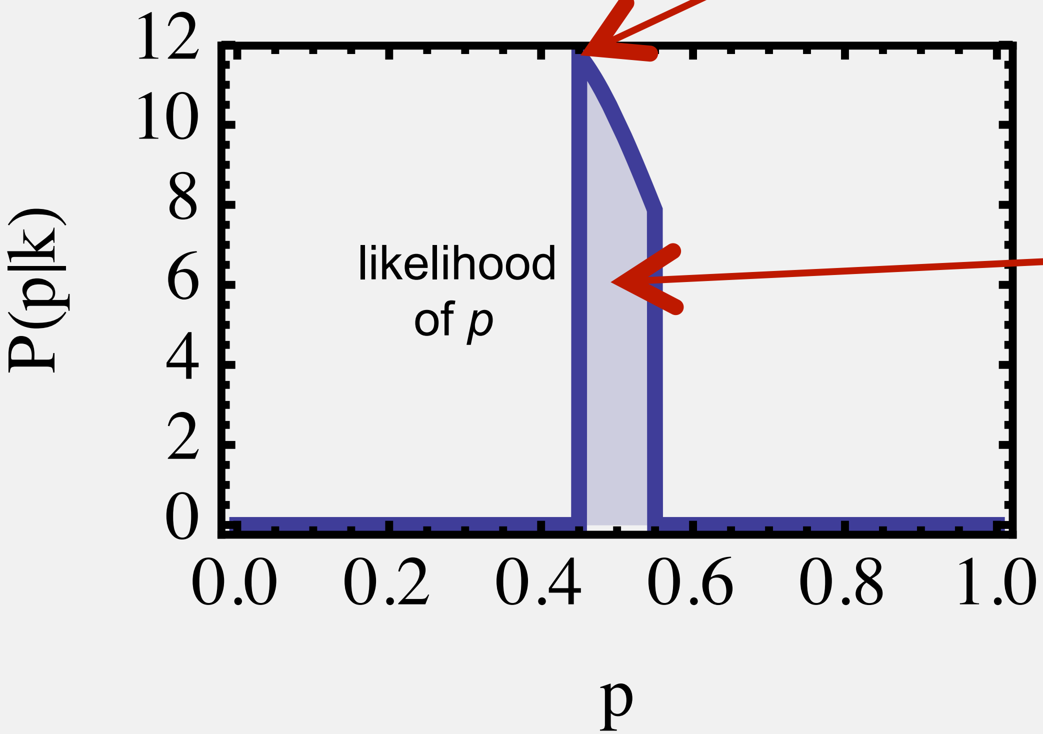
prior?

$$P(p|k) = (N+1)\binom{N}{k} p^k (1-p)^{N-k}$$

Coin flip tomography

N=10 coin flips
k=4 tails
→ estimate the coin tail probability p

maximum: $\hat{p} = 0.45$



$p = 0.50 \pm 0.03$

prior: $0.45 \leq p \leq 0.55$

Spin-half tomography

$$\hat{\rho} = \frac{\mathbb{1} + \vec{r}\cdot\hat{\vec{\sigma}}}{2} = \frac{\mathbb{1} + x\hat{\sigma}_x + y\hat{\sigma}_y + z\hat{\sigma}_z}{2} = \begin{pmatrix} \frac{1+z}{2} & \frac{x-iy}{2} \\ \frac{x+iy}{2} & \frac{1-z}{2} \end{pmatrix}$$

$$(r = \sqrt{x^2 + y^2 + z^2} \leq 1)$$

Measurements:

$29 \times \lvert x\uparrow\rangle$	$1 \times \lvert x\downarrow\rangle$
$25 \times \lvert y\uparrow\rangle$	$5 \times \lvert y\downarrow\rangle$
$15 \times \lvert z\uparrow\rangle$	$15 \times \lvert z\downarrow\rangle$

Likelihood:

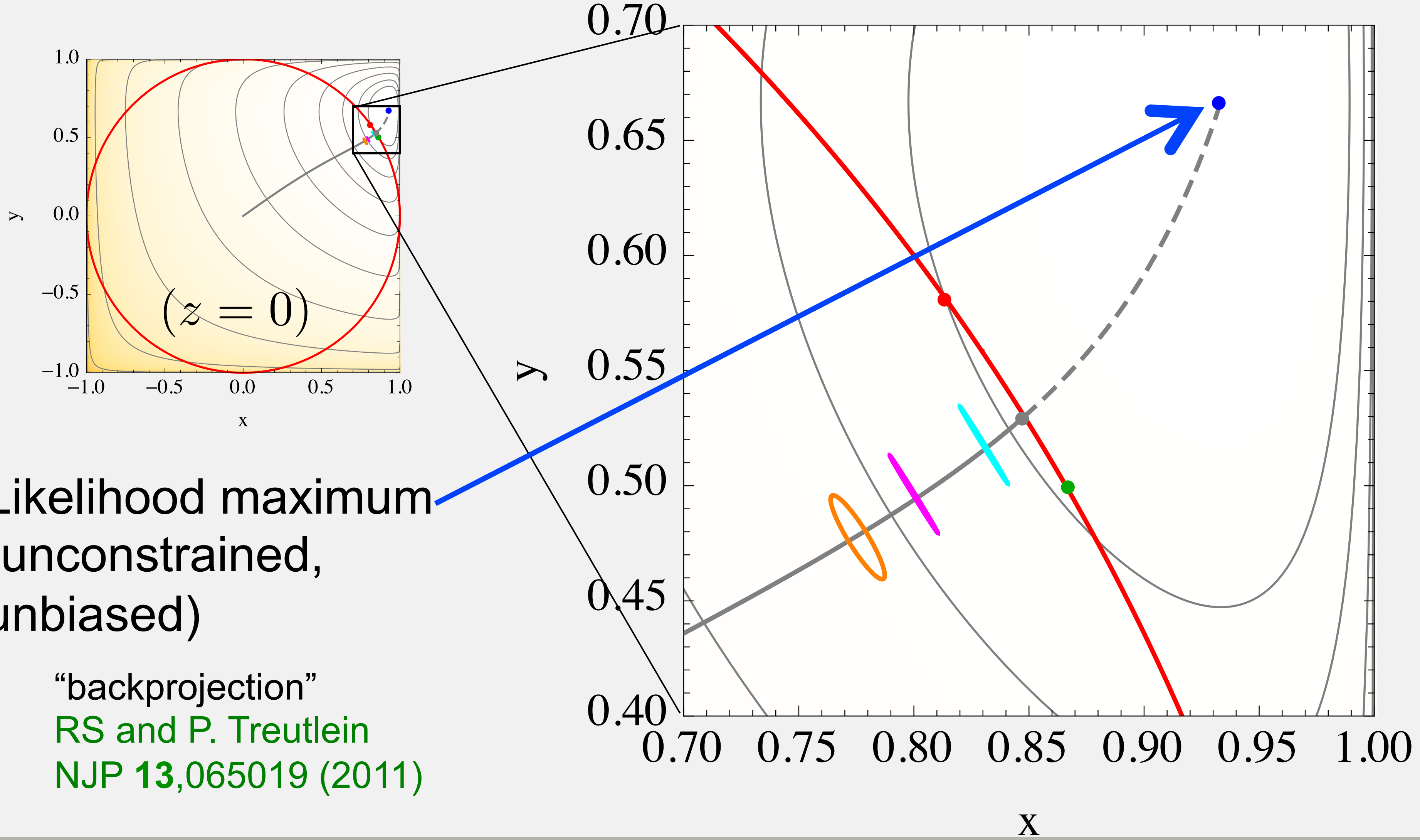
$$P(\hat{\rho}|\mathrm{data}) = \left(\frac{1+x}{2}\right)^{29}\left(\frac{1-x}{2}\right)^{1}\left(\frac{1+y}{2}\right)^{25}\left(\frac{1-y}{2}\right)^{5}\left(\frac{1+z}{2}\right)^{15}\left(\frac{1-z}{2}\right)^{15} \times P(\hat{\rho})$$

Prior is likely a Haar measure: $P(\hat{\rho}) = P(r)$

Find $\hat{\rho}$!

Spin-half tomography

$$P(\hat{\rho}|\text{data}) = \left(\frac{1+x}{2}\right)^{29}\left(\frac{1-x}{2}\right)^{1}\left(\frac{1+y}{2}\right)^{25}\left(\frac{1-y}{2}\right)^{5}\left(\frac{1+z}{2}\right)^{15}\left(\frac{1-z}{2}\right)^{15} \times P(\hat{\rho})$$



Likelihood maximum (unconstrained, unbiased)

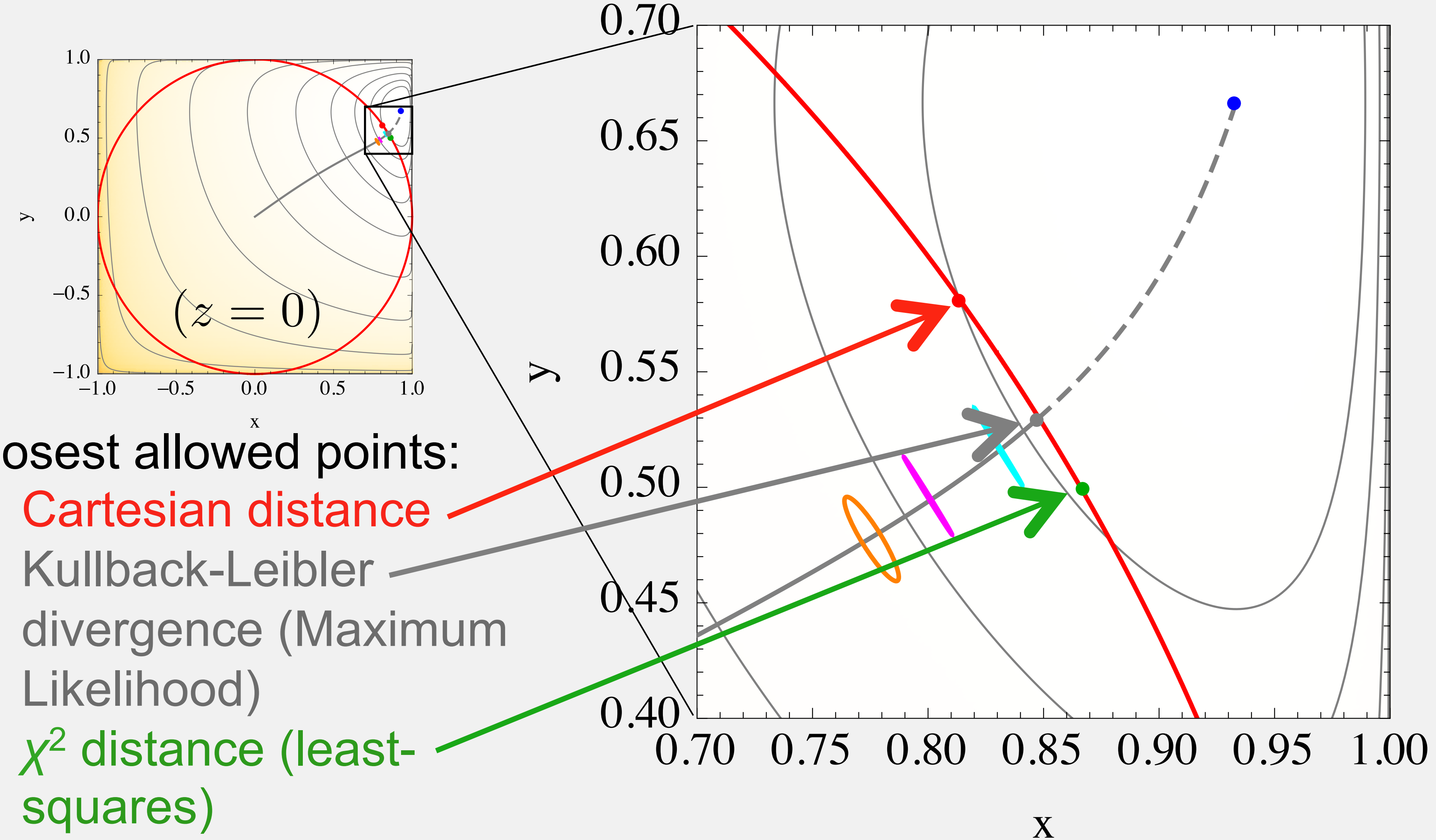
"backprojection"

RS and P. Treutlein

NJP **13**,065019 (2011)

Spin-half tomography

$$P(\hat{\rho}|\text{data}) = \left(\frac{1+x}{2}\right)^{29}\left(\frac{1-x}{2}\right)^{1}\left(\frac{1+y}{2}\right)^{25}\left(\frac{1-y}{2}\right)^{5}\left(\frac{1+z}{2}\right)^{15}\left(\frac{1-z}{2}\right)^{15} \times P(\hat{\rho})$$



closest allowed points:

- Cartesian distance
- Kullback-Leibler divergence (Maximum Likelihood)
- χ^2 distance (least-squares)

Bayesian-mean estimate

$$P(\hat\rho|\text{data}) = \left(\frac{1+x}{2}\right)^{29}\left(\frac{1-x}{2}\right)^{1}\left(\frac{1+y}{2}\right)^{25}\left(\frac{1-y}{2}\right)^{5}\left(\frac{1+z}{2}\right)^{15}\left(\frac{1-z}{2}\right)^{15} \times P(\hat\rho)$$

$$\langle\hat\rho\rangle = \int \hat\rho\, P(\hat\rho|\text{data})\mathcal{D}\hat\rho$$

measure
(contains prior)

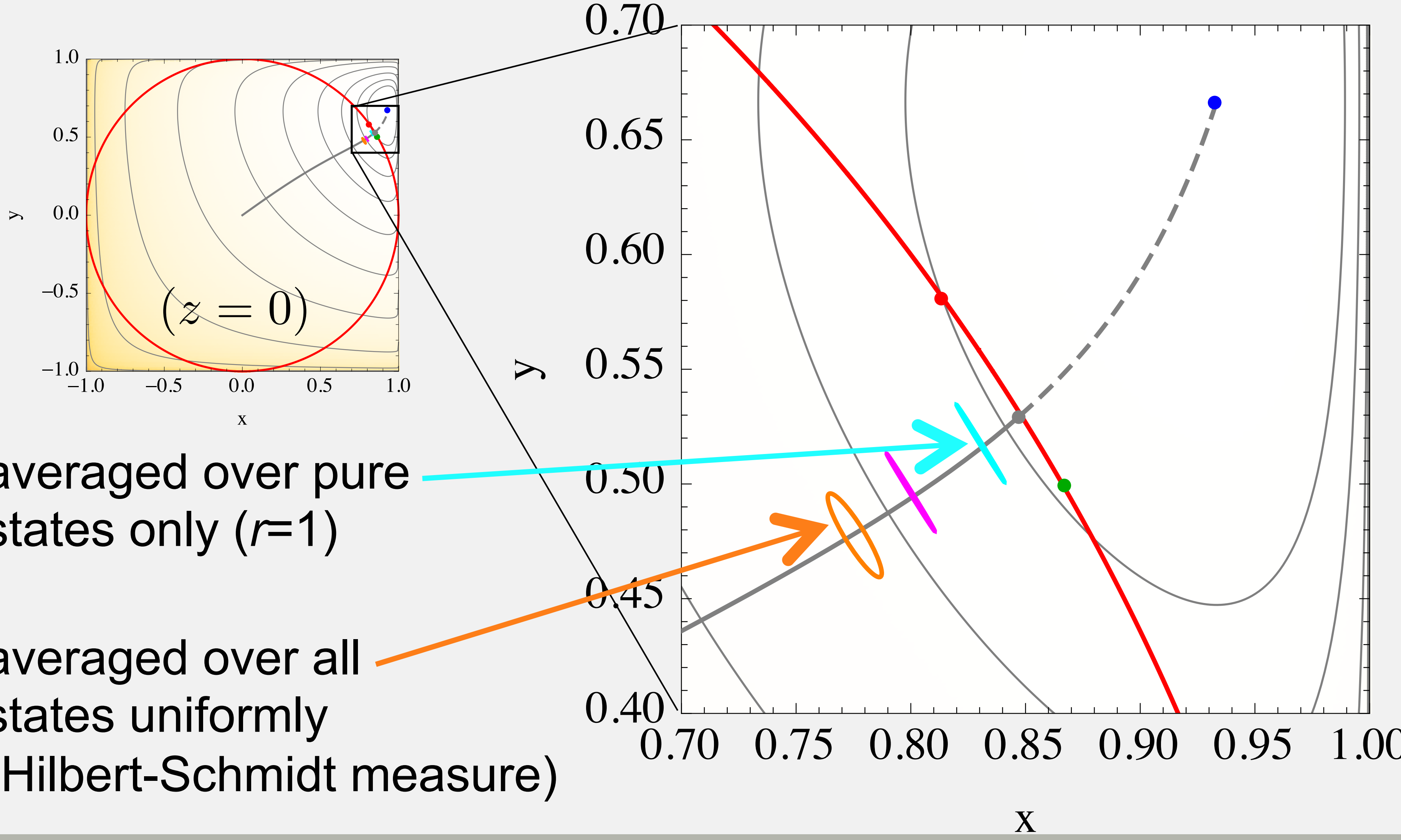
uncertainty:

$$(\Delta\hat\rho)^2 = \int [\hat\rho - \langle\hat\rho\rangle]^2\, P(\hat\rho|\text{data})\mathcal{D}\hat\rho$$

R. Blume-Kohout, NJP **12**,043034 (2010)

Spin-half tomography

$$P(\hat{\rho}|\text{data}) = \left(\frac{1+x}{2}\right)^{29}\left(\frac{1-x}{2}\right)^{1}\left(\frac{1+y}{2}\right)^{25}\left(\frac{1-y}{2}\right)^{5}\left(\frac{1+z}{2}\right)^{15}\left(\frac{1-z}{2}\right)^{15}\times P(\hat{\rho})$$

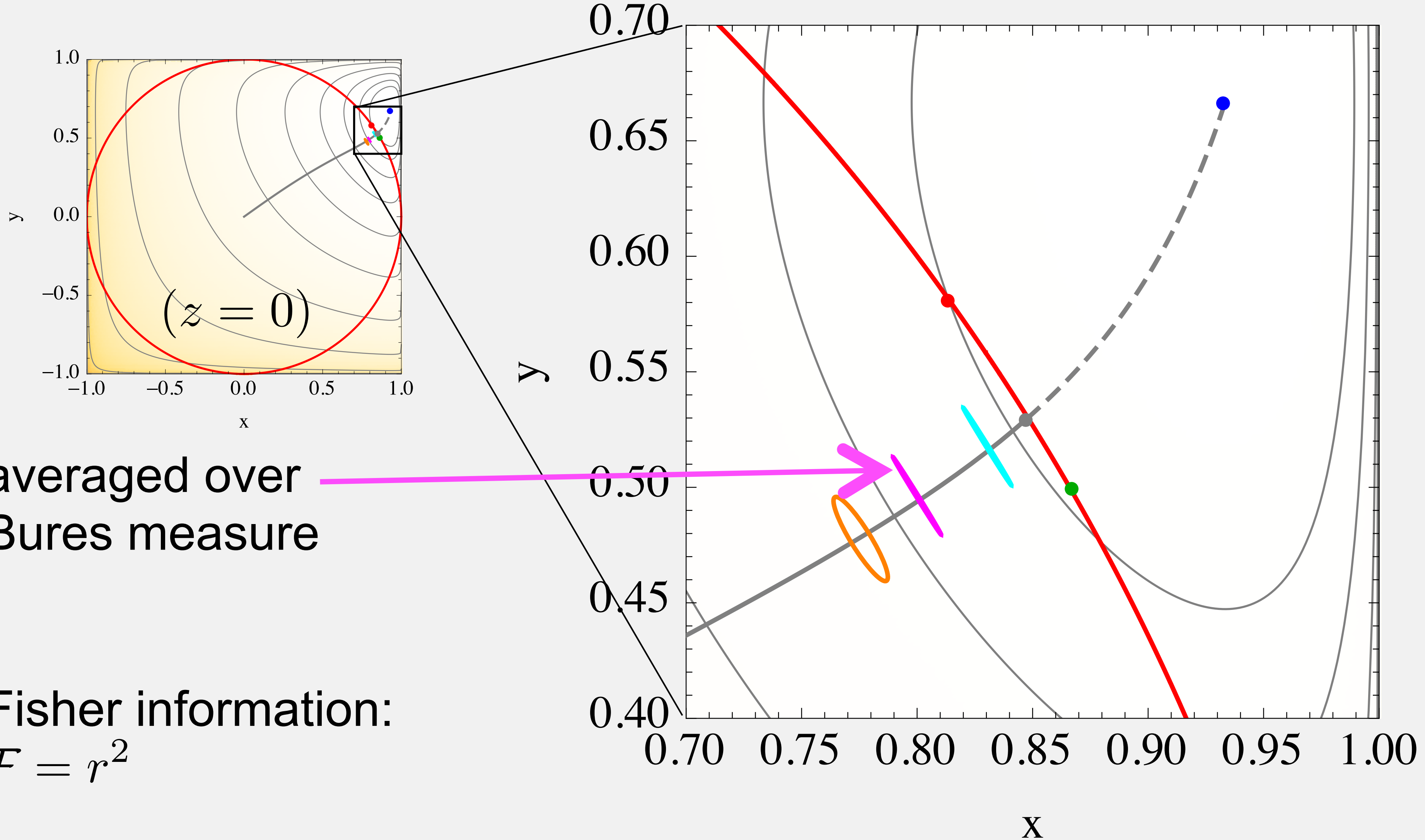


averaged over pure states only (r=1)

averaged over all states uniformly (Hilbert-Schmidt measure)

Spin-half tomography

$$P(\hat{\rho}|\text{data}) = \left(\frac{1+x}{2}\right)^{29}\left(\frac{1-x}{2}\right)^{1}\left(\frac{1+y}{2}\right)^{25}\left(\frac{1-y}{2}\right)^{5}\left(\frac{1+z}{2}\right)^{15}\left(\frac{1-z}{2}\right)^{15}\times P(\hat{\rho})$$



averaged over Bures measure

Fisher information:
$\mathcal{F} = r^2$

Larger systems

- projective total-spin measurements along many axes
- fewer (independent) measurements than unknowns in the density matrix (d^2-1) extra information?
- almost always negative eigenvalues in backprojected density matrix
- constraining leads to low-rank states → too "special" → exaggerated structure
  - ➢ underestimate entropy
  - ➢ overestimate Fisher information
- Bayesian mean: smoother results?