N-body Methods for Relativistic Cosmology

based on Phys. Rev. **D88** 103527 (arXiv:1308.6524), arXiv:1408.3352 (**today!**),

and work in progress with D. Daverio, R. Durrer, and M. Kunz

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Modern Cosmology: Early Universe, CMB and LSS

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Let’s go relativistic!

Strategy

- choose ansatz for the metric (perturbed FLRW)

$$ds^2 = a^2(\tau)\left[-(1+2\Psi)\,d\tau^2 + (1-2\Phi)\,\delta_{ij}dx^i dx^j \right. \\ \left. + h_{ij}dx^i dx^j - 2B_i dx^i d\tau\right]$$

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- stress-energy tensor is determined by solving the EOM's of all sources of stress-energy

$$T^{\mu\nu}_{\rm m} = \sum_n m_{(n)} \frac{\delta^{(3)}(\mathbf{x}-\mathbf{x}_{(n)})}{\sqrt{-g}} \left(-g_{\alpha\beta}\frac{dx^\alpha_{(n)}}{d\tau}\frac{dx^\beta_{(n)}}{d\tau}\right)^{-\frac{1}{2}} \frac{dx^\mu_{(n)}}{d\tau}\frac{dx^\nu_{(n)}}{d\tau}$$

Einstein’s equations at leading order

$$\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi = -4\pi G a^2 \delta T^0_0$$

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Anisotropic stress: $\Pi_{ij} \doteq \delta_{ik}T^k_j - \frac{1}{3}\delta_{ij}T^k_k$

$$\left(\frac{\partial^2}{\partial x^i \partial x^j} - \frac{1}{3}\delta_{ij}\Delta\right)(\Phi - \Psi) = 8\pi G a^2 \Pi^{(S)}_{ij}$$

$$B'_{(i,j)} + 2\mathcal{H}B_{(i,j)} = 8\pi G a^2 \Pi^{(V)}_{ij}$$

$$\frac{1}{2}h''_{ij} + \mathcal{H}h'_{ij} - \frac{1}{2}\Delta h_{ij} = 8\pi G a^2 \Pi^{(T)}_{ij}$$

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Geodesic equation at leading order ($\left|\frac{dx^i}{d\tau}\right| \ll 1$)

$$\frac{d^2 x^i}{d\tau^2} + \mathcal{H}\frac{dx^i}{d\tau} + \delta^{ij}\left(\Psi_{,j} - \mathcal{H} B_j - B'_j\right) = 0$$

Including "shortwave corrections" . . .

$$(1+4\Phi)\,\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Psi + \tfrac{3}{2}\delta^{ij}\Phi_{,i}\Phi_{,j} = -4\pi G a^2 \delta T^0_0$$

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$$\begin{gathered}\left(\frac{\partial^2}{\partial x^i \partial x^j} - \tfrac{1}{3}\delta_{ij}\Delta\right)\left[(\Phi-\Psi)\,(1+\Phi-\Psi)+\Phi^2\right]+ \\ B'_{(i,j)} + 2\mathcal{H}B_{(i,j)} + \\ \tfrac{1}{2}h''_{ij} + \mathcal{H}h'_{ij} - \tfrac{1}{2}\Delta h_{ij} + \\ 2\Psi\Phi_{,ij} - \tfrac{2}{3}\delta_{ij}\Psi\Delta\Phi - (\Phi-\Psi)_{,i}\,(\Phi-\Psi)_{,j} + \\ \tfrac{1}{3}\delta_{ij}\delta^{kl}\,(\Phi-\Psi)_{,k}\,(\Phi-\Psi)_{,l} = 8\pi G a^2 \Pi_{ij}\end{gathered}$$

Including “shortwave corrections” ...

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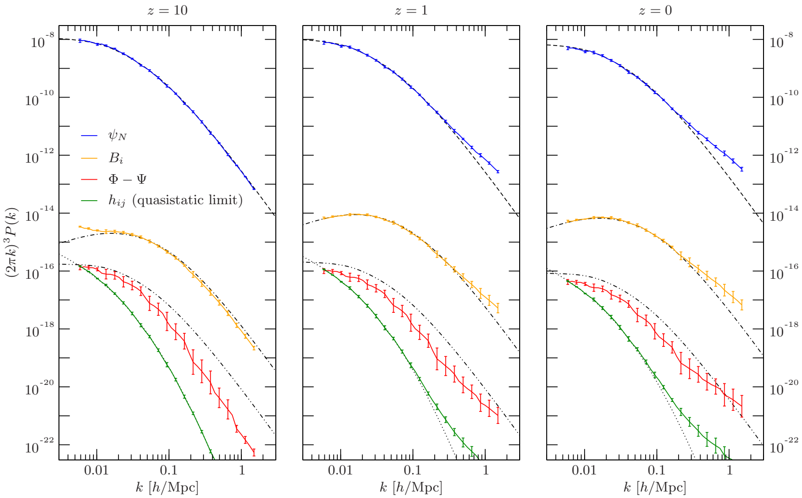
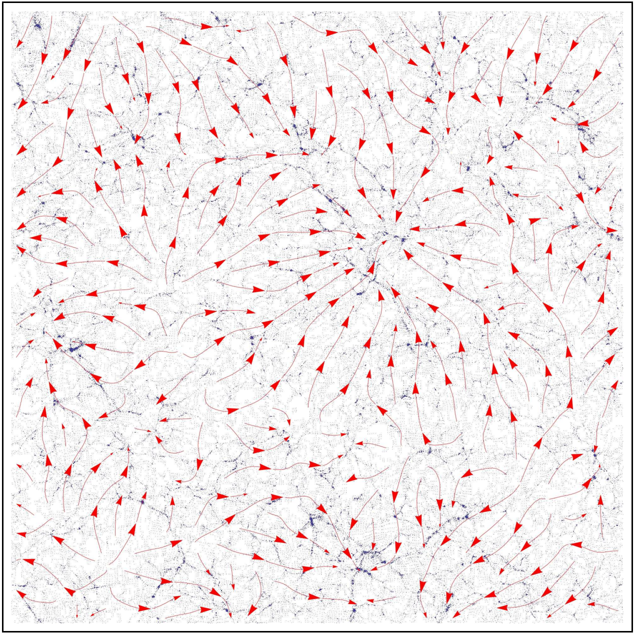
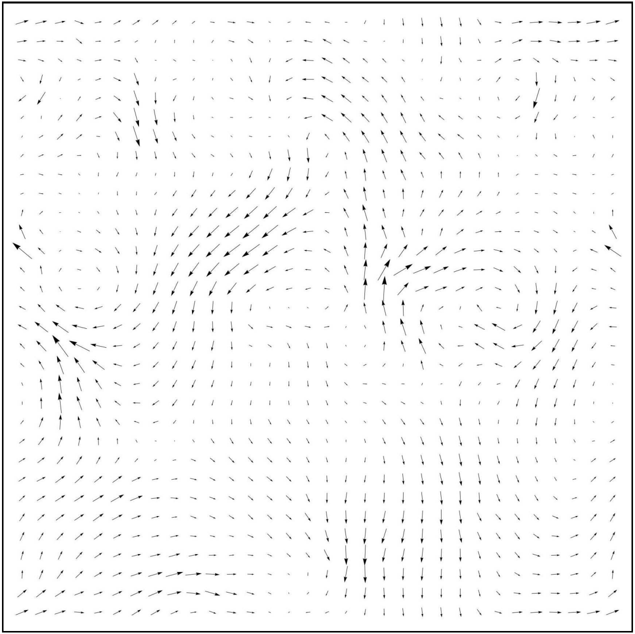
$$\left(\tfrac{\partial^2}{\partial x^i \partial x^j} - \tfrac{1}{3}\delta_{ij}\Delta\right)\left[(\Phi-\Psi)(1+\Phi-\Psi)+\Phi^2\right] +$$
$$B'_{(i,j)} + 2\mathcal{H}B_{(i,j)} +$$
$$\tfrac{1}{2}h''_{ij} + \mathcal{H}h'_{ij} - \tfrac{1}{2}\Delta h_{ij} +$$
$$2\Psi\Phi_{,ij} - \tfrac{2}{3}\delta_{ij}\Psi\Delta\Phi - (\Phi-\Psi)_{,i}(\Phi-\Psi)_{,j} +$$
$$\tfrac{1}{3}\delta_{ij}\delta^{kl}(\Phi-\Psi)_{,k}(\Phi-\Psi)_{,l} = 8\pi G a^2 \Pi_{ij}$$

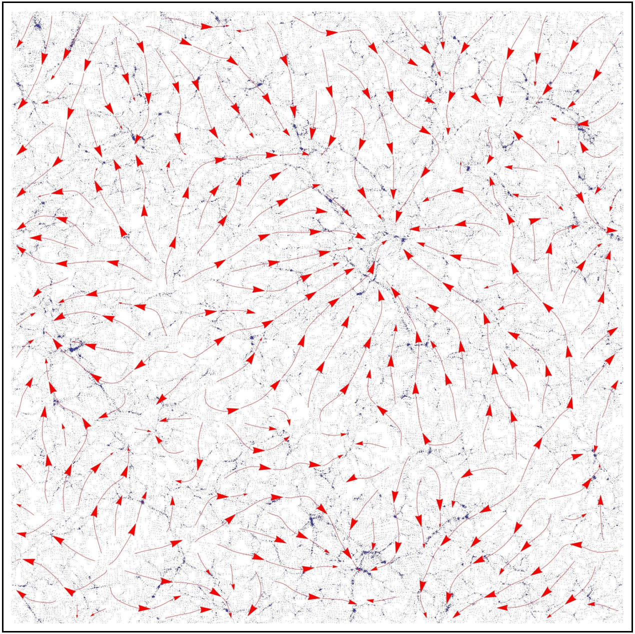
$$\tfrac{d^2x^i}{d\tau^2} + \mathcal{H}\tfrac{dx^i}{d\tau} + \delta^{ij}\left(\Psi_{,j} - \mathcal{H}B_j - B'_j - 2B_{[j,k]}\tfrac{dx^k}{d\tau}\right) = 0$$

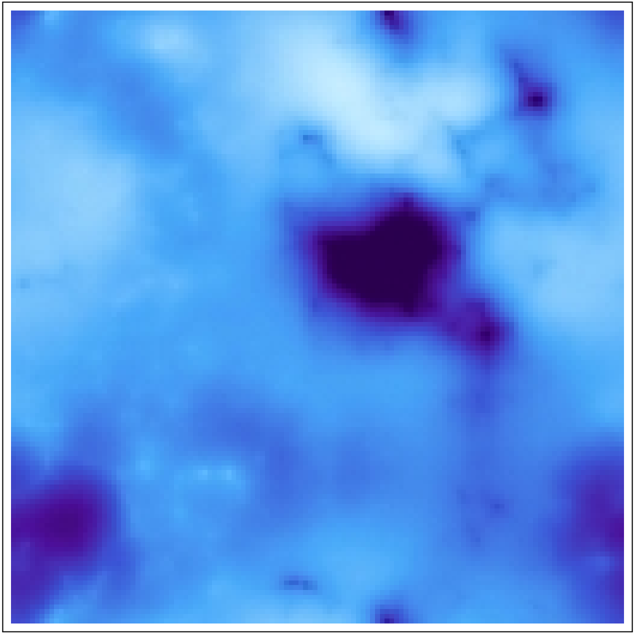


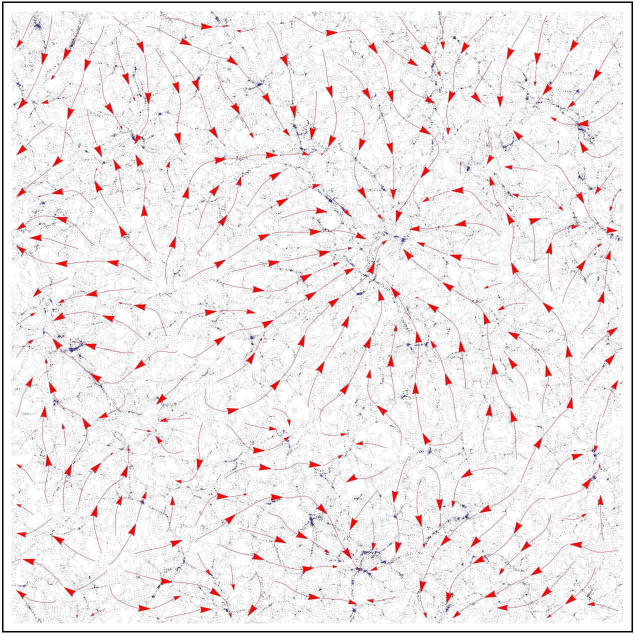
$z = 10$
$z = 1$
$z = 0$
$(2\pi k)^3 P(k)$
ψ_N
B_i
$\Phi - \Psi$
h_{ij} (quasistatic limit)
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-22}
0.01
0.1
1
k [h/Mpc]

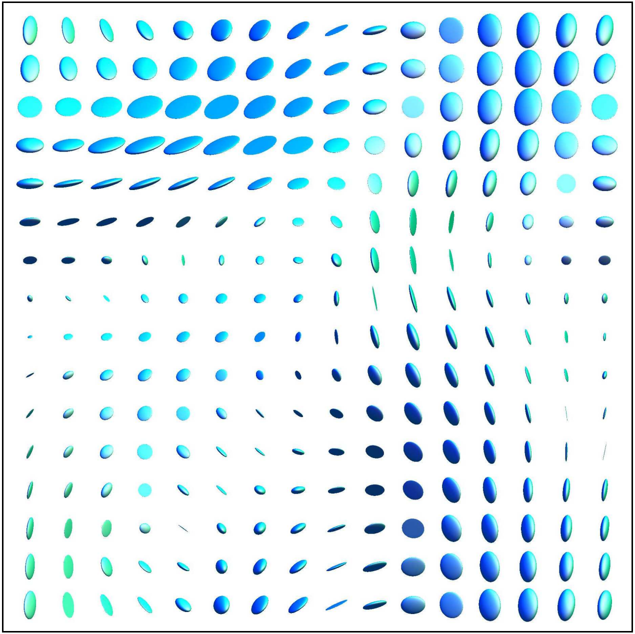












Summary

- N-body simulations within a GR framework are feasible
- unified relativistic treatment is a clear, logical and transparent way to address the most general observables with minimal restrictions on the cosmological model
- technology should be useful for simulations with relativistic sources (dynamical DE, neutrinos, . . .)