Probing the inflationary Universe with gravitational waves

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What we want to know about the inflationary Universe

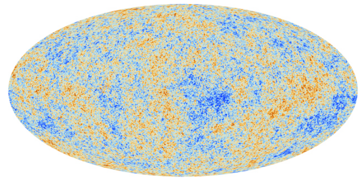
- What is the inflaton?
  - Shape of the potential?
  - Structure of the kinetic term?
  - Number of fields?

- What is the origin of density fluctuations?
  - Inflaton?
  - Some other field (e.g., curvaton)?

- What is the thermal history after inflation?
  - Reheating temperature?

⋮

We obtain the information through obs. of fluctuations.



- Primordial (scalar) fluctuation (primordial curvature perturbation ζ)

-- Power spectrum

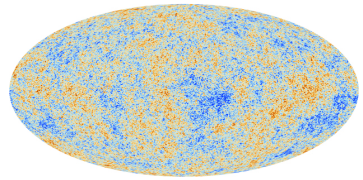
$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \right\rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_\zeta(k_1)$$

-- Non-Gaussianity (bispectrum, trispectrum)

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3).$$

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

We obtain the information through obs. of fluctuations.



- Primordial (scalar) fluctuation (primordial curvature perturbation ζ)

-- Power spectrum (amplitude, scale-dependence)

$$\mathcal{P}_\zeta(k) = A_s(k_{\rm ref})\left(\frac{k}{k_{\rm ref}}\right)^{n_s-1}$$

-- Non-Gaussianity (bispectrum, trispectrum)

$$B_\zeta(k_1,k_2,k_3) \longrightarrow f_{\rm NL}$$

$$T_\zeta(k_1,k_2,k_3,k_4) \longrightarrow \tau_{\rm NL} \quad g_{\rm NL}$$

(defined for some specific forms)

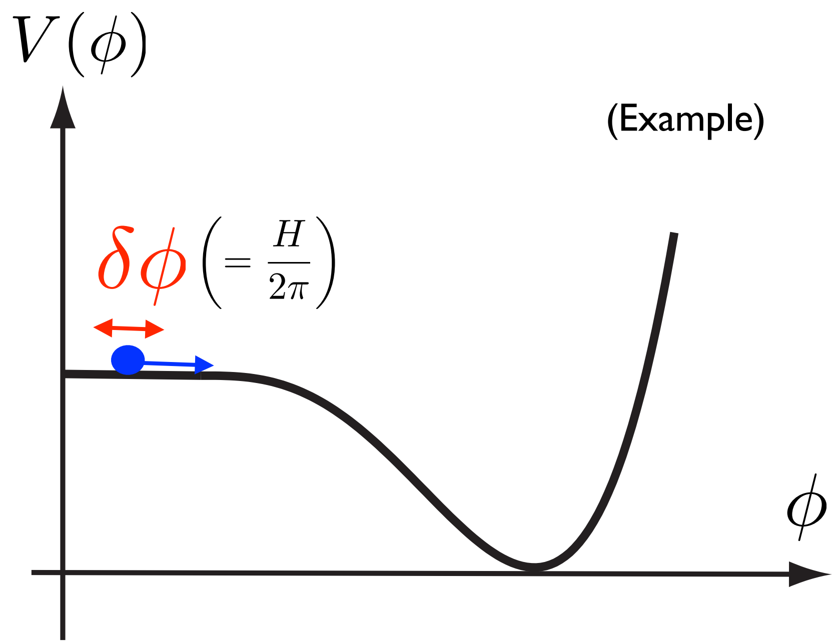
Characterizing the inflaton potential

- Curvature perturbation

$$\zeta = -\frac{H}{\dot{\phi}}\delta\phi$$

- Spectral index

$$n_s = 1 - 6\epsilon + 2\eta$$

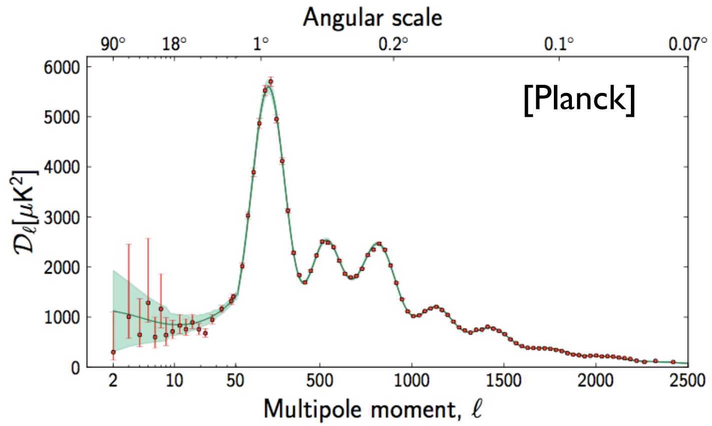


(Slow-roll parameters)

$$\epsilon = \frac{1}{2}M_{\rm pl}^2\left(\frac{V_\phi}{V}\right)^2 \qquad \eta = M_{\rm pl}^2\frac{V_{\phi\phi}}{V}$$

where $V_\phi \equiv \frac{dV}{d\phi}$

Power spectrum

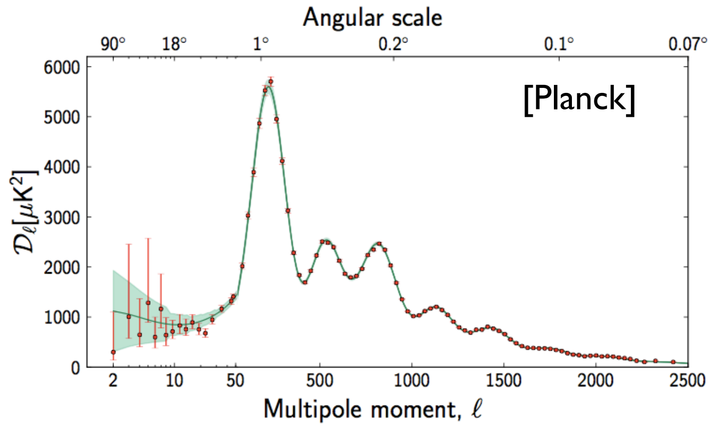


Primordial (scalar) power spectrum:

$$\mathcal{P}_\zeta(k) = A_s(k_{\mathrm{ref}}) \left(\frac{k}{k_{\mathrm{ref}}}\right)^{n_s - 1}$$

- Amplitude: $\ln(10^{10} A_s) = 3.089^{+0.024}_{-0.027}$ (68% CL) [Planck, Ade et al 2013]
- Spectral index: $n_s = 0.9603 \pm 0.0073$

Power spectrum

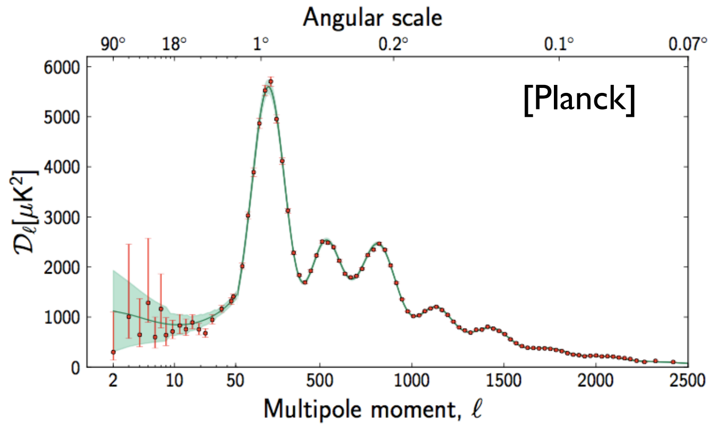


Primordial (scalar) power spectrum:

$$\mathcal{P}_\zeta(k) = A_s(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}}\right)^{n_s - 1 + \frac{1}{2}\alpha \ln(k/k_{\rm ref})}$$

- Amplitude: $\ln(10^{10} A_s) = 3.089^{+0.024}_{-0.027}$ (68% CL) [Planck, Ade et al 2013]
- Spectral index: $n_s = 0.9603 \pm 0.0073$
- Running of ns: $\alpha = -0.0134 \pm 0.0090$ ~O(SR^2)

Power spectrum



Primordial (scalar) power spectrum:

$$\mathcal{P}_\zeta(k) = A_s(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}}\right)^{n_s - 1 + \frac{1}{2}\alpha \ln(k/k_{\rm ref}) + \frac{1}{6}\beta \ln^2(k/k_{\rm ref})}$$

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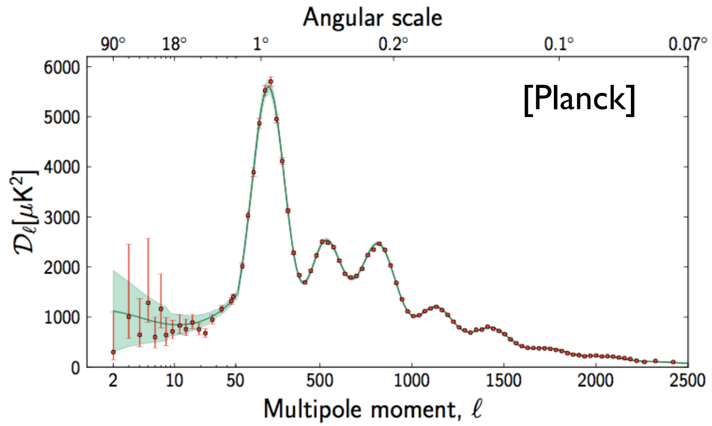
Power spectrum



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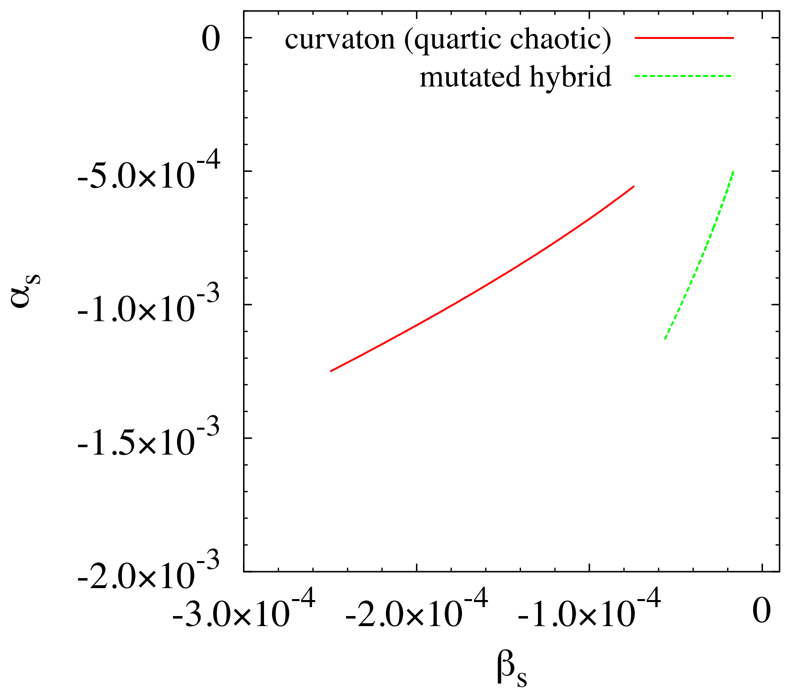
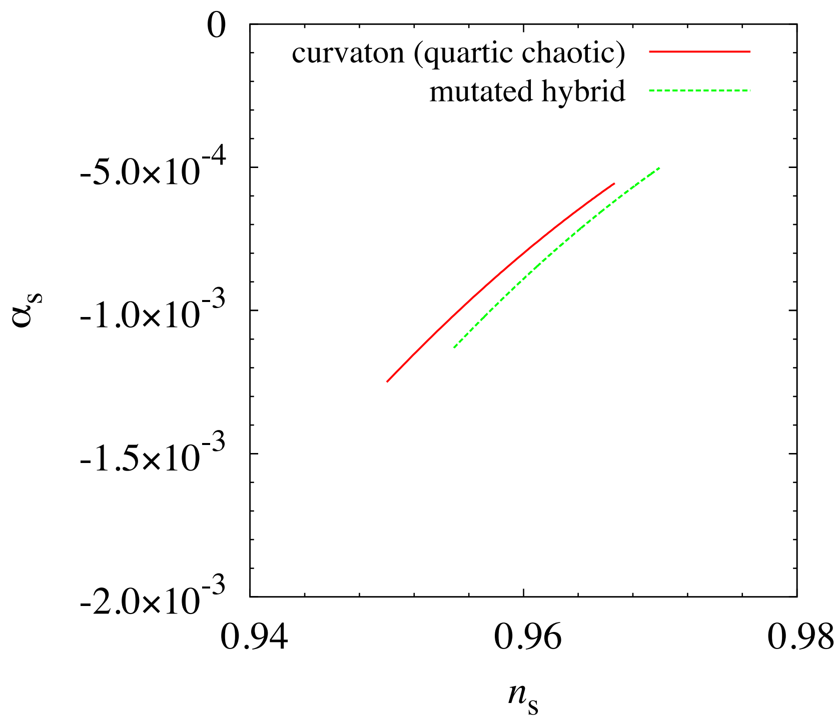
(Future 21cm observations may probe $\beta = \mathcal{O}(10^{-4})$). [Kohri, Oyama, Sekiguchi, TT 2013]

Running of running: useful ?





(mutated hybrid)

[Kohri, Oyama, Sekiguchi, TT 2013]

$$V(\phi) = V_0 \left[1 - \left(\frac{\mu}{\phi} \right)^q + \cdots \right]$$

Running of running may be useful in some cases

Non-Gaussianity

Bispectrum: $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3).$

$$f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$$

$$f_{\rm NL}^{\rm equil} = -42 \pm 75$$

(68% CL)

$$f_{\rm NL}^{\rm orth} = -25 \pm 39$$

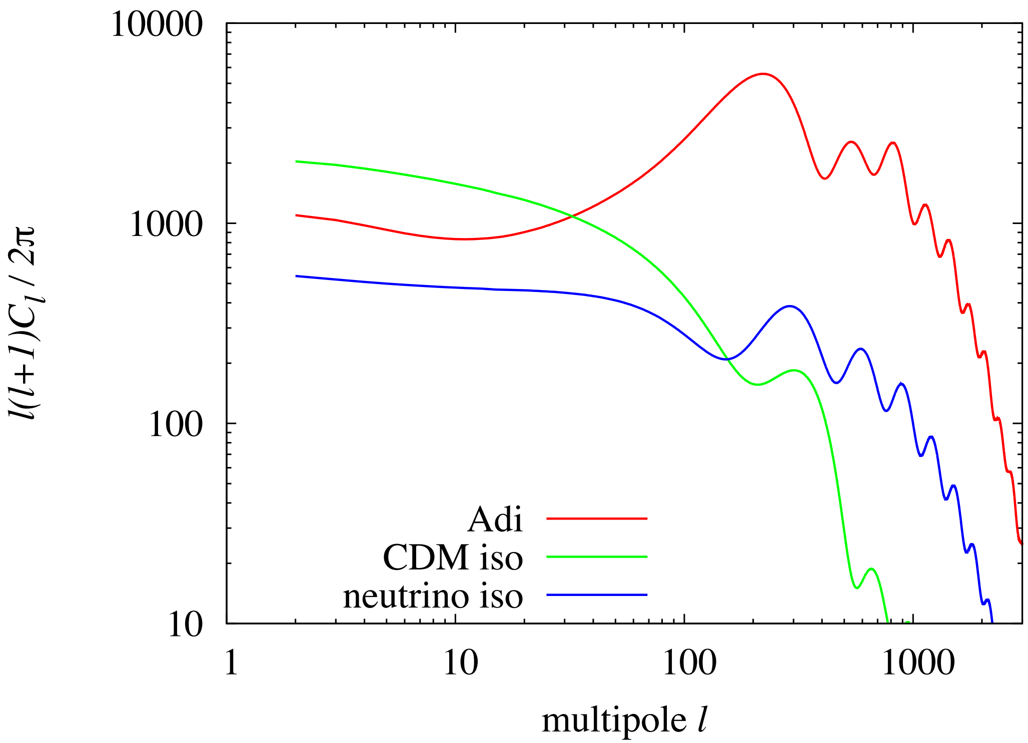
[Planck, Ade et al 2013]

Trispectrum: $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$

$\tau_{\rm NL} < 2800$ (95% CL) [Planck, Ade et al 2013]

$-7.7 \times 10^5 < g_{\rm NL} < 1.1 \times 10^5$ (95% CL) [Sekiguchi, Sugiyama 2013, WMAP9]

Yet another probe: Adiabaticity (isocurvature fluctuations)



Fluctuations should almost be adiabatic, but some fractional contributions from isocurvature fluc. are possible.

Constraints on the adiabaticity

- Constraints for the fraction parameter: $\alpha \equiv \frac{P_{\mathcal{S}}(k_0)}{P_\zeta(k_0) + P_{\mathcal{S}}(k_0)} = \frac{P_S(k_0)}{P_{\rm total}(k_0)}$

($P_S \sim < S^2 > \quad P_\zeta \sim < \zeta^2 >$)

  - $\alpha_{\rm uncorr} < 0.036 \quad (95\% \ {\rm CL})$ [Planck, Ade et al 2013]

    (for uncorrelated CDM isocurvature, e.g. axion)

  - $\alpha_{\rm corr} < 0.0025 \quad (95\% \ {\rm CL})$ [Planck, Ade et al 2013]

    (for correlated CDM isocurvature, e.g. curvaton)

Constraints on isocurvature non-G.

We can also define fNL for isocurvature fluctuations.

$$S(\vec{x}) = S_{\rm G}(\vec{x}) + f_{\rm NL}^{\rm (ISO)}\left(S_{\rm G}(\vec{x})^2 - \langle S_{\rm G}(\vec{x})^2\rangle\right)$$

- Constraint from WMAP

(no constraint from Planck)

$$\alpha^2 f_{\rm NL}^{\rm (iso)} = 40 \pm 66$$ [1σ] (from WMAP7, bispectrum)

[Hikage, Kawasaki, Sekiguchi, TT, 1211.1095]

(For other types of isocurvature fluctuations, see [Hikage, Kawasaki, Sekiguchi, TT, 1212.6001])

Probes of the inflationary Universe

- Primordial (scalar) fluctuation

  -- Power spectrum (amplitude, scale-dependence)

  -- Non-Gaussianity (bispectrum, trispectrum)

- Primordial tensor fluctuations (Gravitational waves)

$$ds^2 = -dt^2 + a(t)^2 \left[1 + h_{ij}\right] \, dx^i dx^j$$

$$h_{ij} = \sqrt{8\pi G} \sum_{A=+,\times} \int d^3k \; e^{i\boldsymbol{k}\cdot\boldsymbol{x}} h_k(t) e^A_{ij}(\boldsymbol{n})$$

$$\Rightarrow \quad \langle h_{k_1} h_{k_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \mathcal{P}_T(k_1)$$

Probes of the inflationary Universe

- Primordial (scalar) fluctuation

  -- Power spectrum (amplitude, scale-dependence)

  -- Non-Gaussianity (bispectrum, trispectrum)

- Primordial tensor fluctuations (Gravitational waves)

  -- Tensor power spectrum (amplitude, scale-dep.)

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}} \right)^{n_T}$$

Tensor power spectrum

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_{\rm ref})\left(\frac{k}{k_{\rm ref}}\right)^{n_T}$$

- Tensor-to-scalar ratio $r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta}$

Planck:

$$r < 0.11 \ (95\ \%\ \text{C.L.})$$

[Planck, Ade et al.1303.5082]

BICEP2:

$$r = 0.20^{+0.07}_{-0.05} \quad (68\%\ \text{C.L.})$$

[Ade et al, BICEP2 1403.3985]

Tensor power spectrum

$$\mathcal{P}_T(k) = \mathcal{P}_T(k_{\rm ref})\left(\frac{k}{k_{\rm ref}}\right)^{n_T}$$

- Tensor spectral index

BICEP2 alone:

$$n_T = 1.36 \pm 0.83 \quad (68\% \text{ C.L.})$$

BICEP2 +TT prior:

$$n_T = 1.67 \pm 0.53 \quad (68\% \text{ C.L.})$$

[Gerbino et al. 1403.5732]

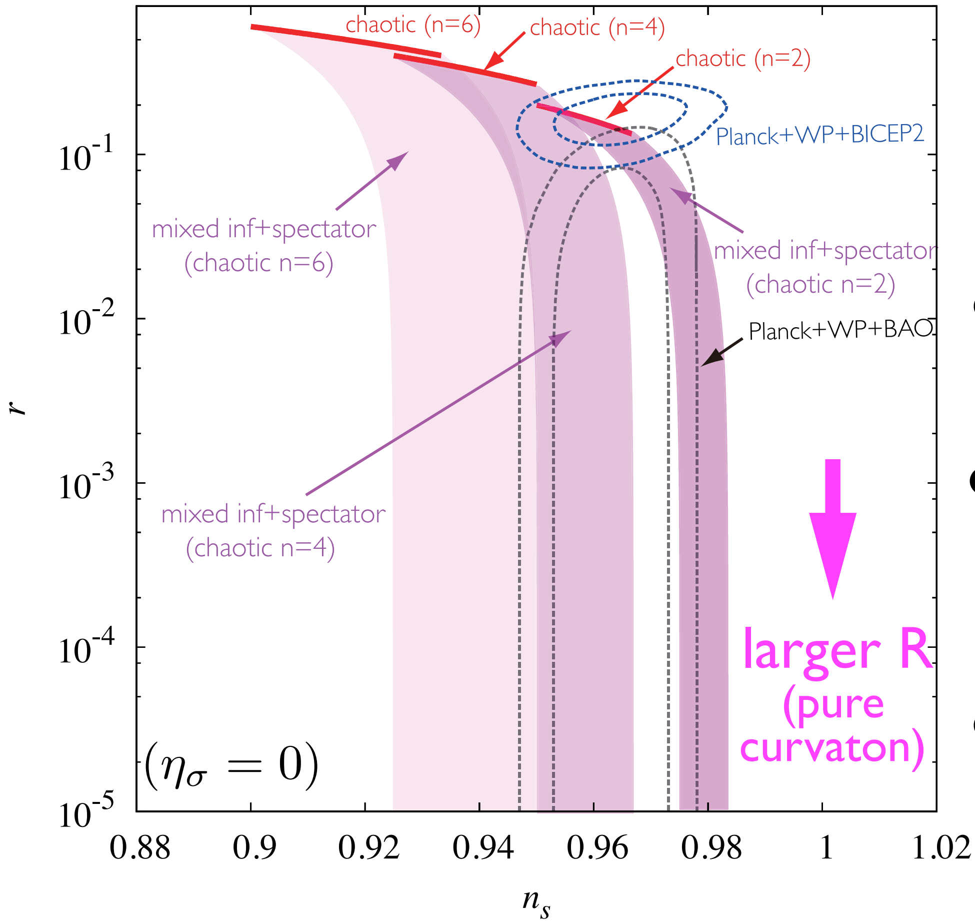
Inflationary observables

- Some of the inflationary observables are well-measured

$$A_s \qquad n_s \qquad f_{\rm NL} \qquad (r)$$

→ ... may NOT enough to pin down the inflationary model.

Mixed inflaton and curvaton model (ns-r plane)



[Updated from Enqvist, TT 2013]

$$R \equiv \frac{\mathcal{P}_\zeta^{(\sigma)}}{\mathcal{P}_\zeta^{(\phi)}}$$

R→0: (pure) inflaton case

R→∞: (pure) curvaton case

- Pure curvaton limit is excluded.
- Mixed inflaton+curvaton is allowed.
- fNL does not help to exclude this model.

Consistency test of inflation

- Standard case

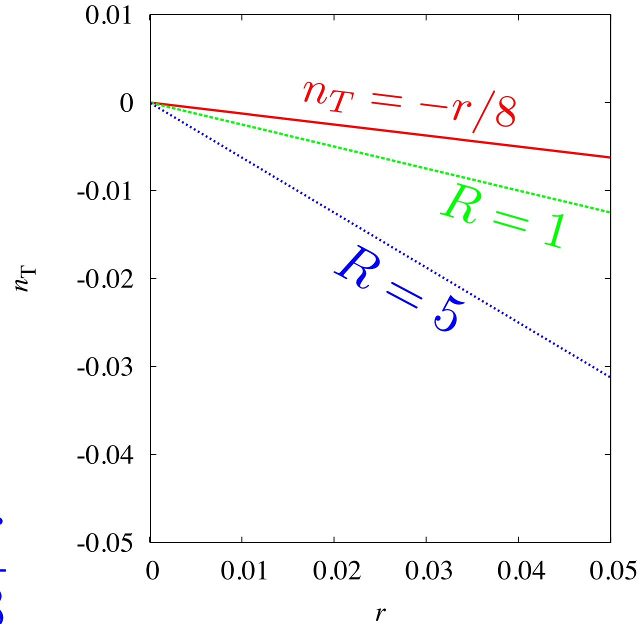
$$n_T = -2\epsilon$$
$$r = 16\epsilon$$

$\Rightarrow$ $n_T = -\dfrac{r}{8}$

- Curvaton case

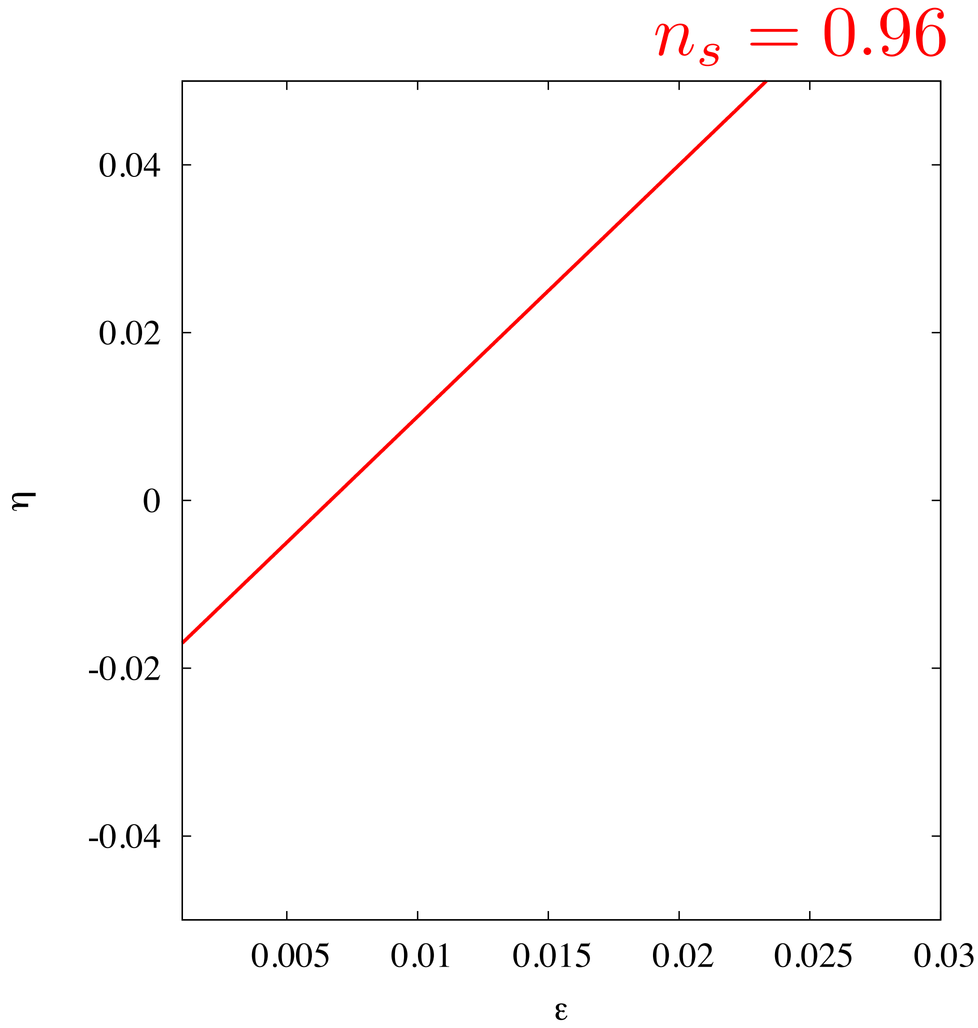
$$n_T = -2\epsilon$$
$$r = \frac{16\epsilon}{1+R}$$

$\Rightarrow$ $n_T = -(1+R)\dfrac{r}{8}$



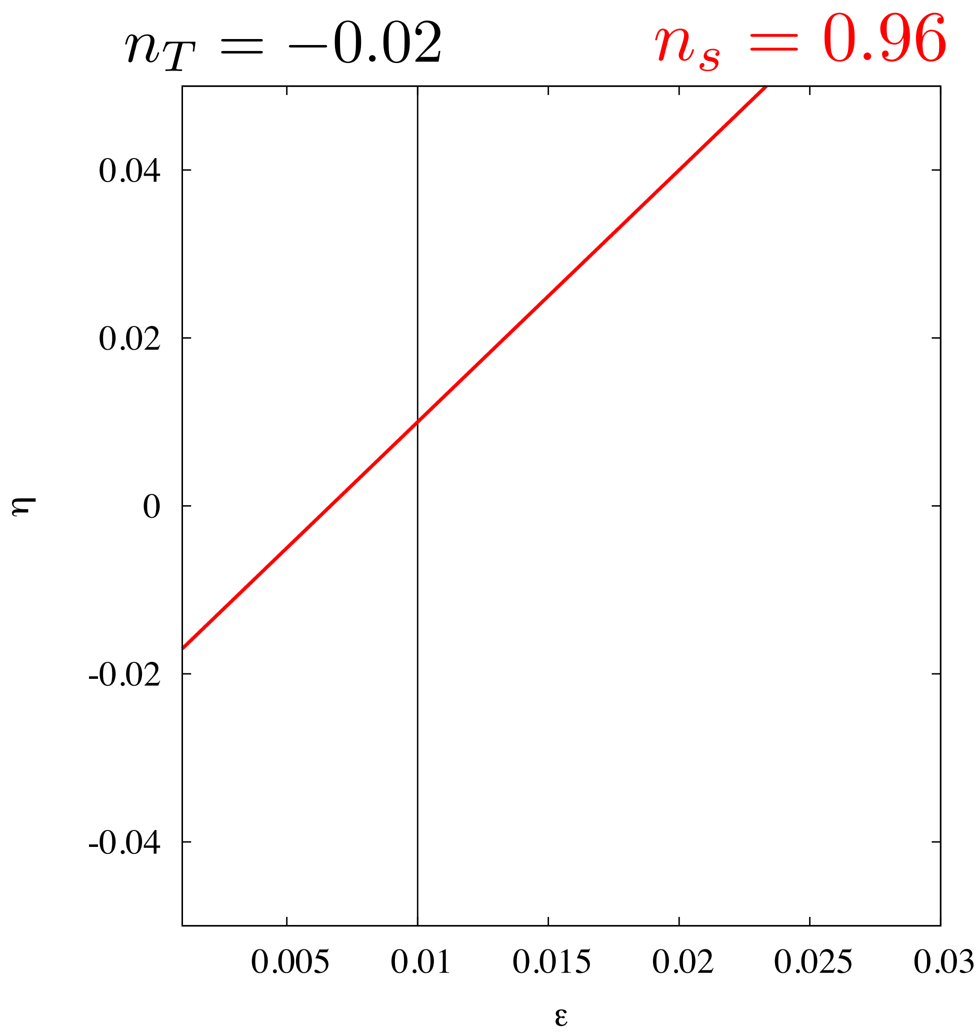
$\Rightarrow$ Tensor scale-dependence should give important test of the inflationary Universe

Yet another consistency test?



$$n_s = 1 - 6\epsilon + 2\eta$$

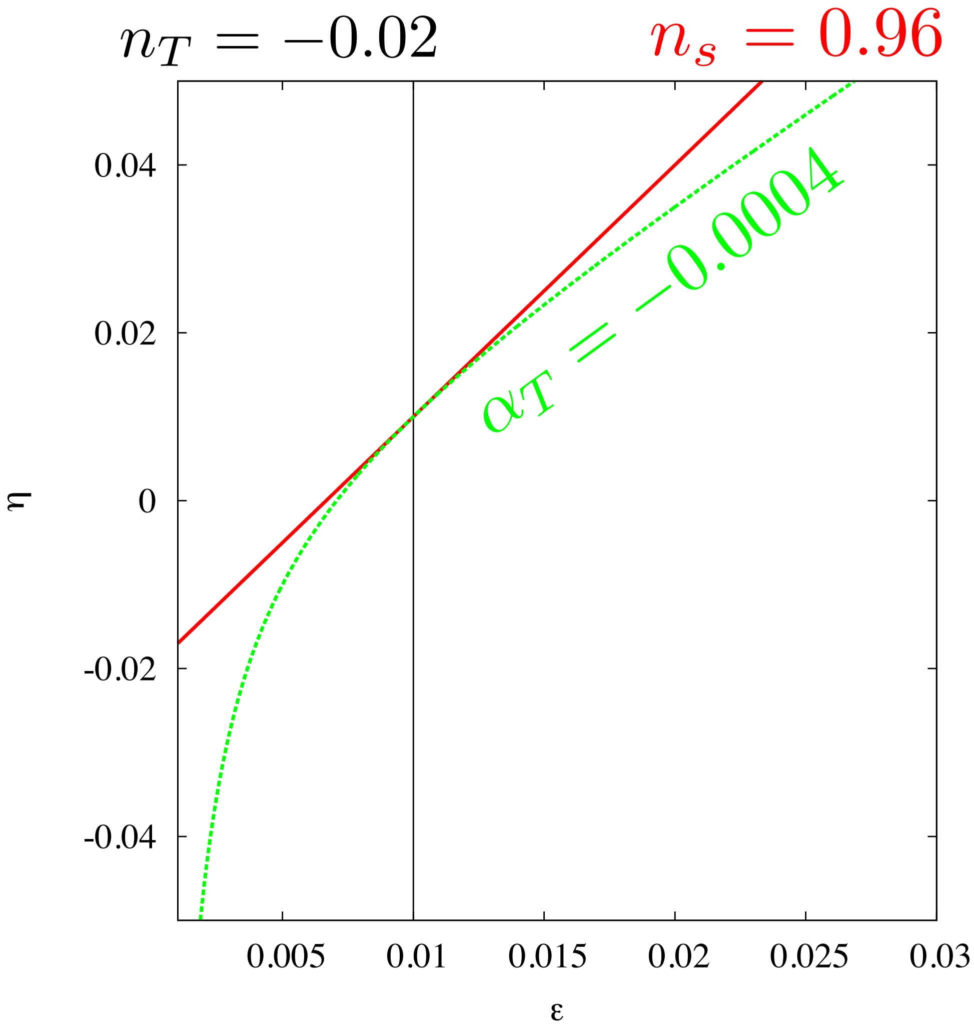
Yet another consistency test?



$$n_s = 1 - 6\epsilon + 2\eta$$

$$n_T = -2\epsilon$$

Yet another consistency test?



$$n_s = 1 - 6\epsilon + 2\eta$$

$$n_T = -2\epsilon$$

$$\alpha_T = -4\epsilon(2\epsilon - \eta)$$

If three lines cross at one point, we can check the consistency of the predictions.

Tensor scale-dependence would be useful

- It can give consistency check of the inflationary Universe.

- How can we probe the scale-dependence

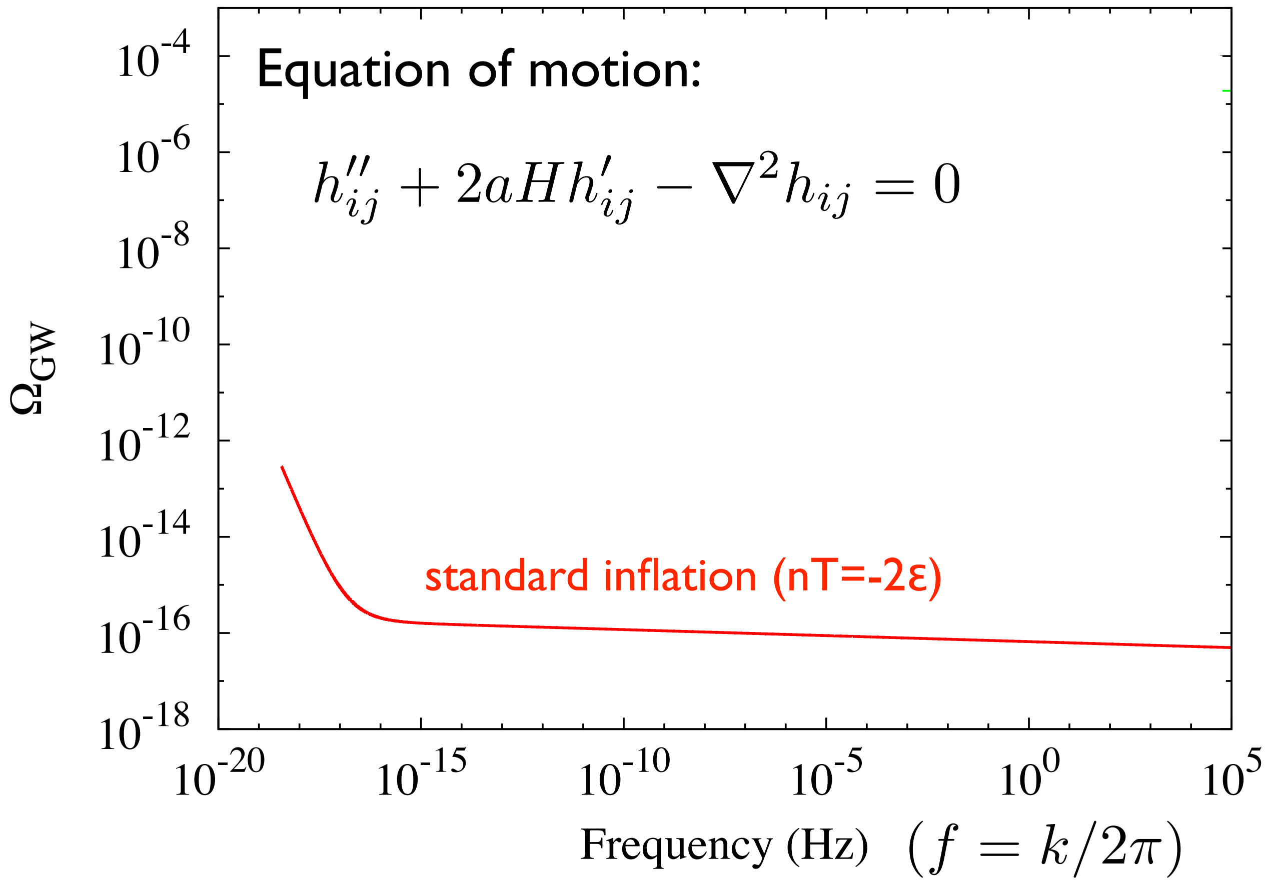
  -- CMB

  -- Direct detection

  (Interferometer, ...)

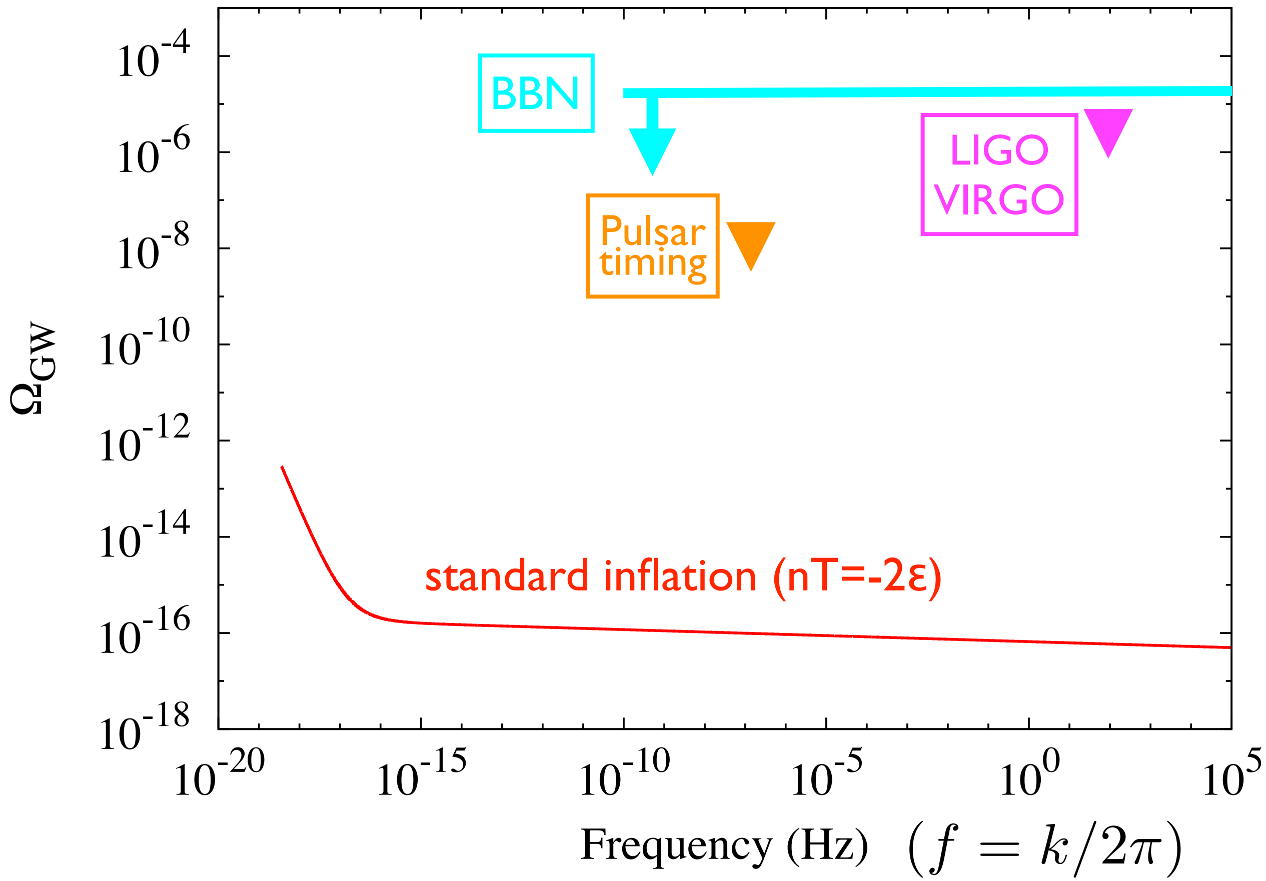
GW spectrum: $\Omega_{\rm GW}(k) \equiv \frac{1}{\rho_{\rm crit}} \frac{d\rho_{\rm GW}}{d\ln k}$

$$\rho_{\rm GW} = \frac{1}{64\pi G a^2} \left\langle (\partial_\tau h_{ij})^2 + (\nabla h_{ij})^2 \right\rangle$$



Current bound on GWs

Current bound is very weak...



However, what if it’s blue?

- Blue-tilted GWs may resolve the tension between Planck and BICEP2.

- Blue-tilted tensor spectrum may be favored.

BICEP2 alone:

$$n_T = 1.36 \pm 0.83 \quad (68\% \text{ C.L.})$$

BICEP2 +TT prior:

$$n_T = 1.67 \pm 0.53 \quad (68\% \text{ C.L.})$$

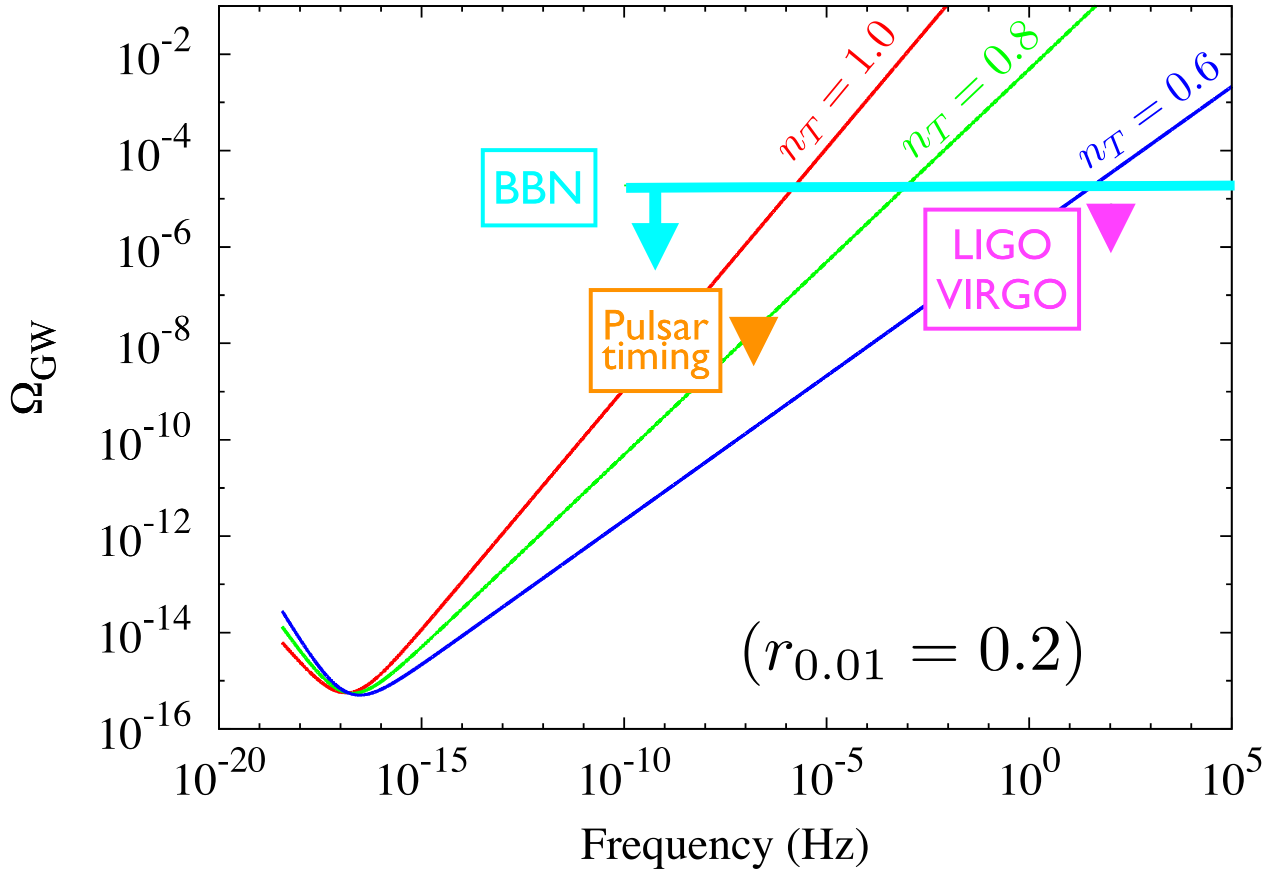
[Gerbino et al. 1403.5732]

- Some models (String gas, G-inflation, super-inflation, particle production during inflation, ...) give blue-tilted one.

May be worth investigating the constraint on blue-tilted GWs

Constraints on Blue-tilted GWs

[Stewart, Brandenberger 0711 4602; Kuroyanagi, TT, Yokoyama 1407.4785]



(BBN)

$$n_T < 0.43$$

(LIGO+VIRGO)

$$n_T < 0.54$$

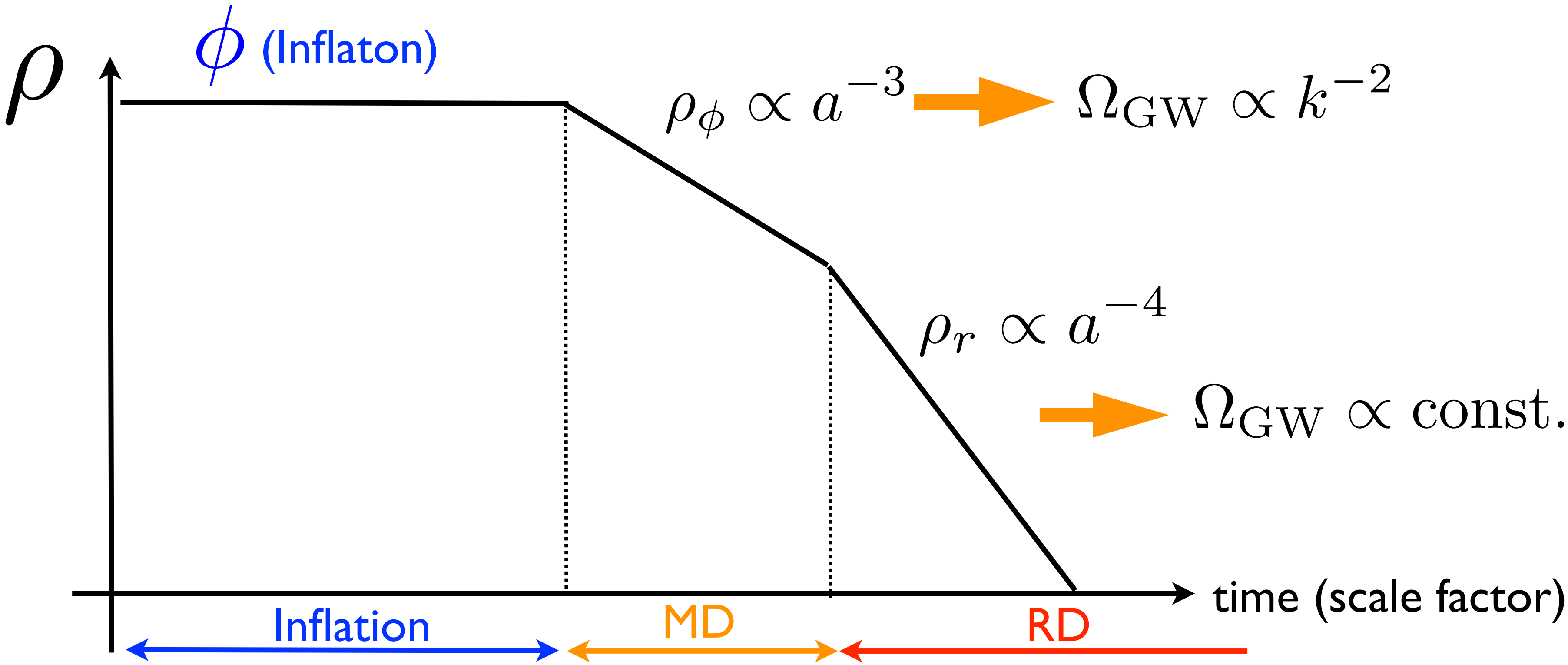
(Pulsar)

$$n_T < 0.87$$

[95% CL]

GW spectrum and thermal history

- Taking into account the thermal history (e.g., reheating after inflation), GW spectrum changes.



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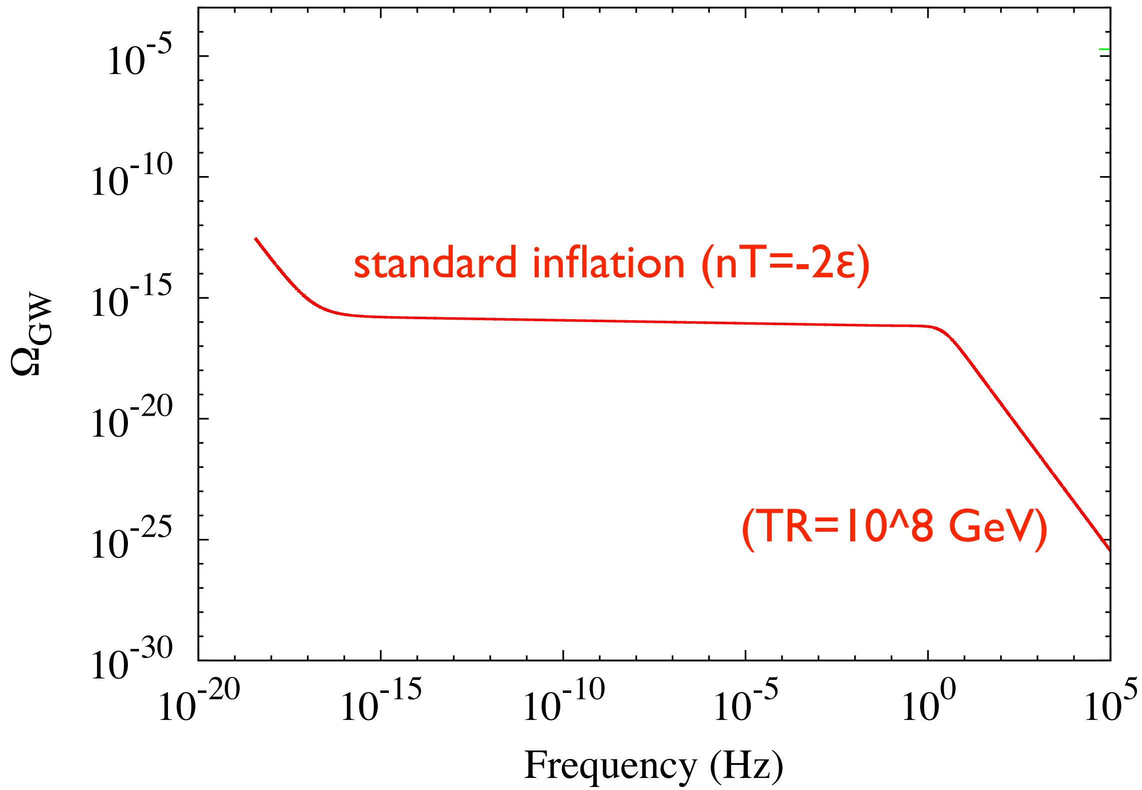

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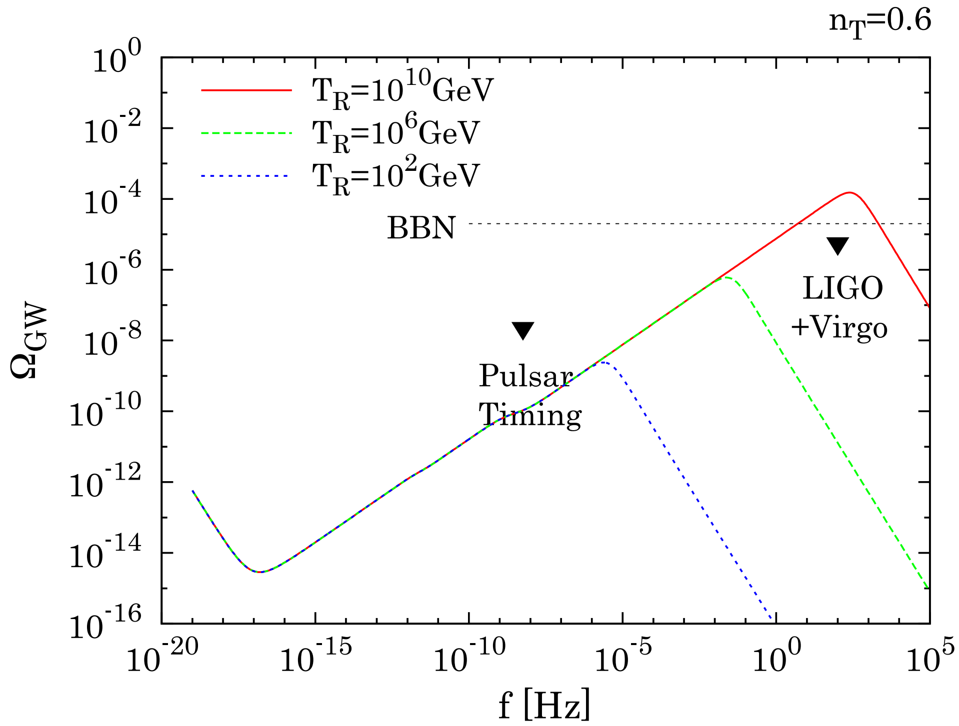
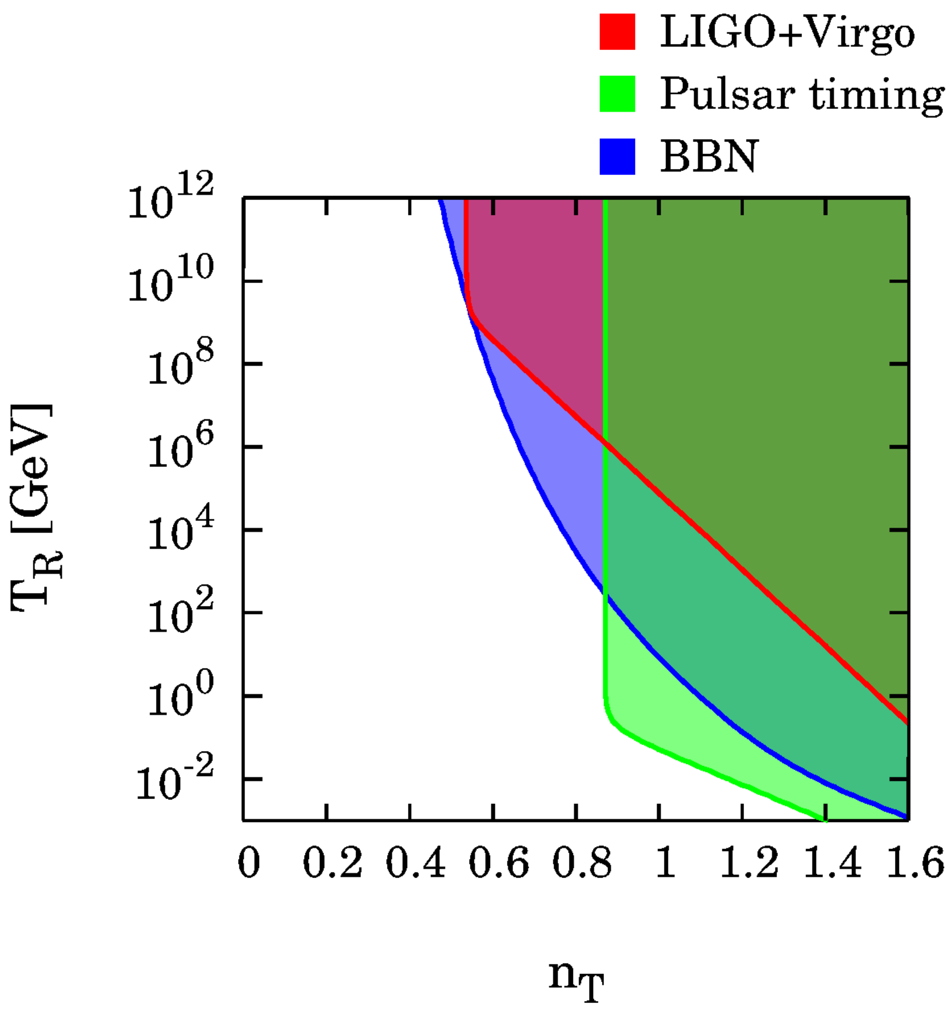


➡ Constraints on nT depend on thermal history

Constraint on nT and TR

[Kuroyanagi, TT, Yokoyama 1407.4785]



[TR = 10^6 GeV]

(BBN)

$n_T < 0.43$ → $n_T < 0.64$

(LIGO+VIRGO)

$n_T < 0.54$ → $n_T < 0.89$

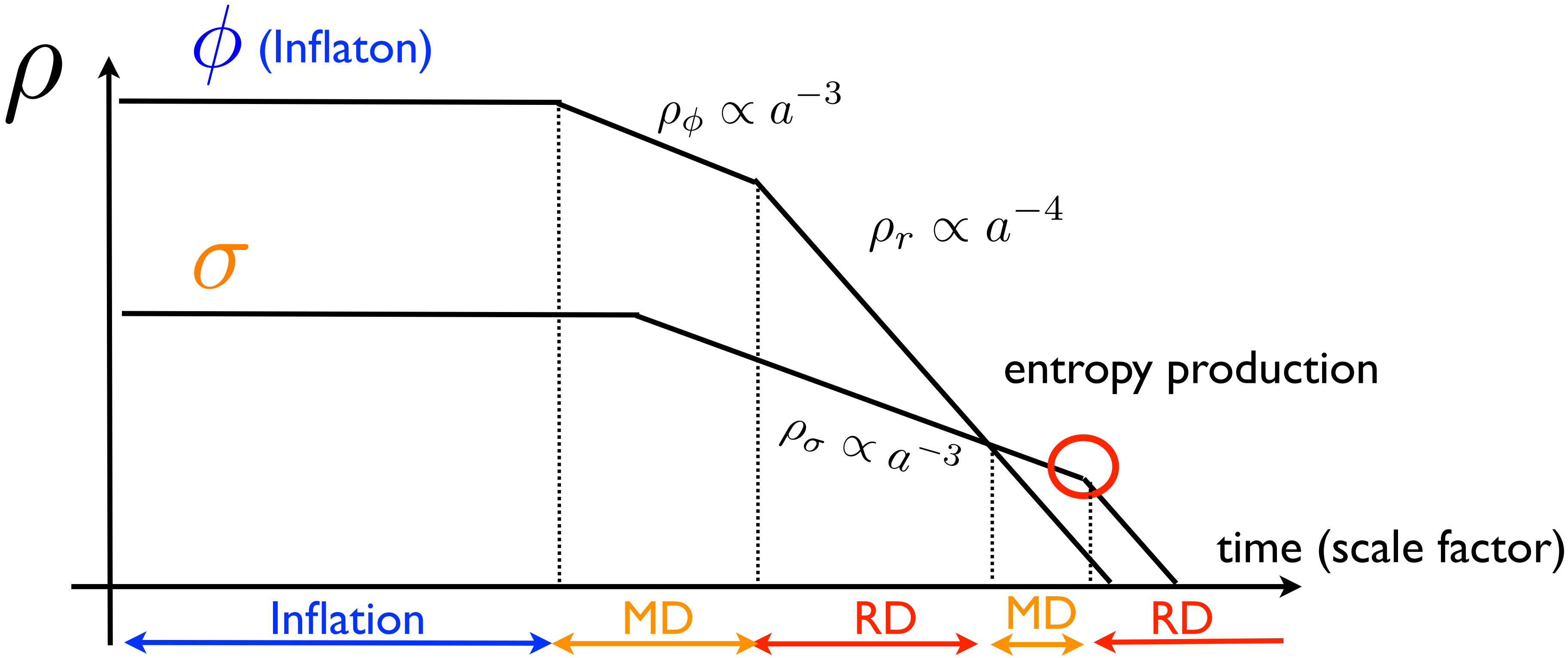
(Pulsar)

$n_T < 0.87$ → $n_T < 0.87$

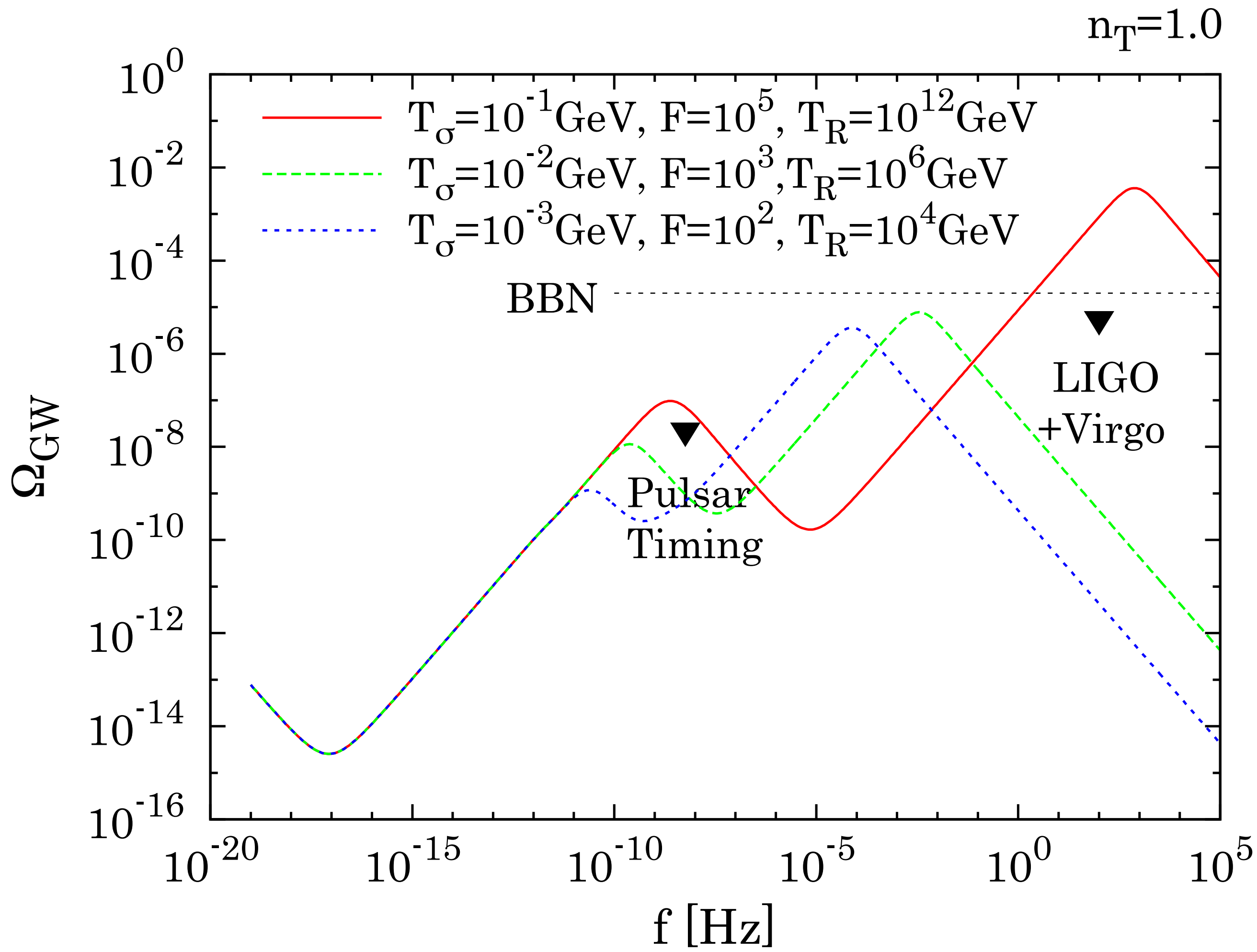
- Low reheating temperature allows more blue GWs.

GW with late-time entropy production

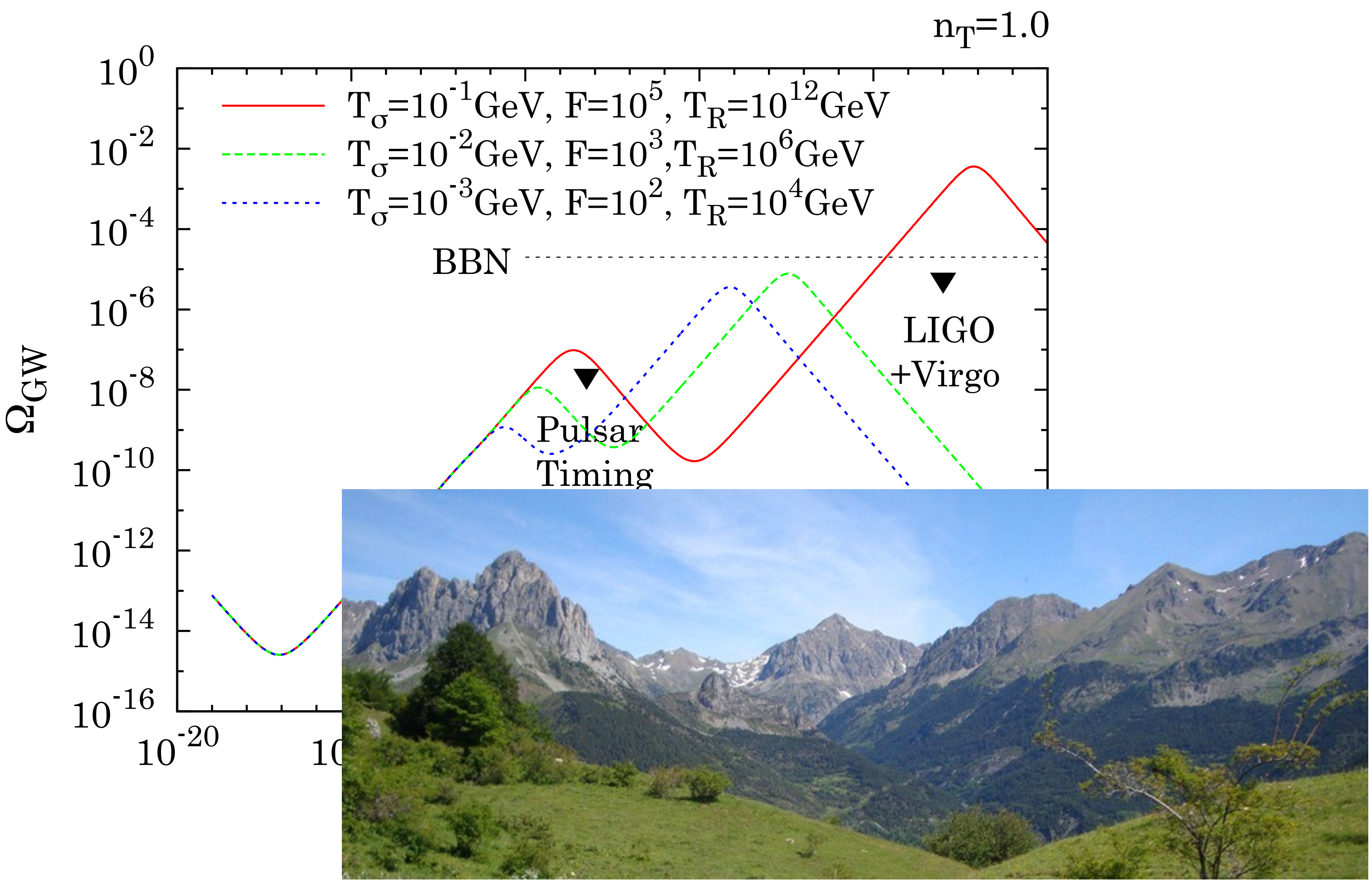
- If some scalar field dominates the Universe and produces entropy at late time, GW spectrum is further affected.



GW with late-time entropy production



GW with late-time entropy production



[http://www.hikepyrenees.co.uk]

GW with late-time entropy production

- Late time entropy production scenario can be characterized by two parameters:

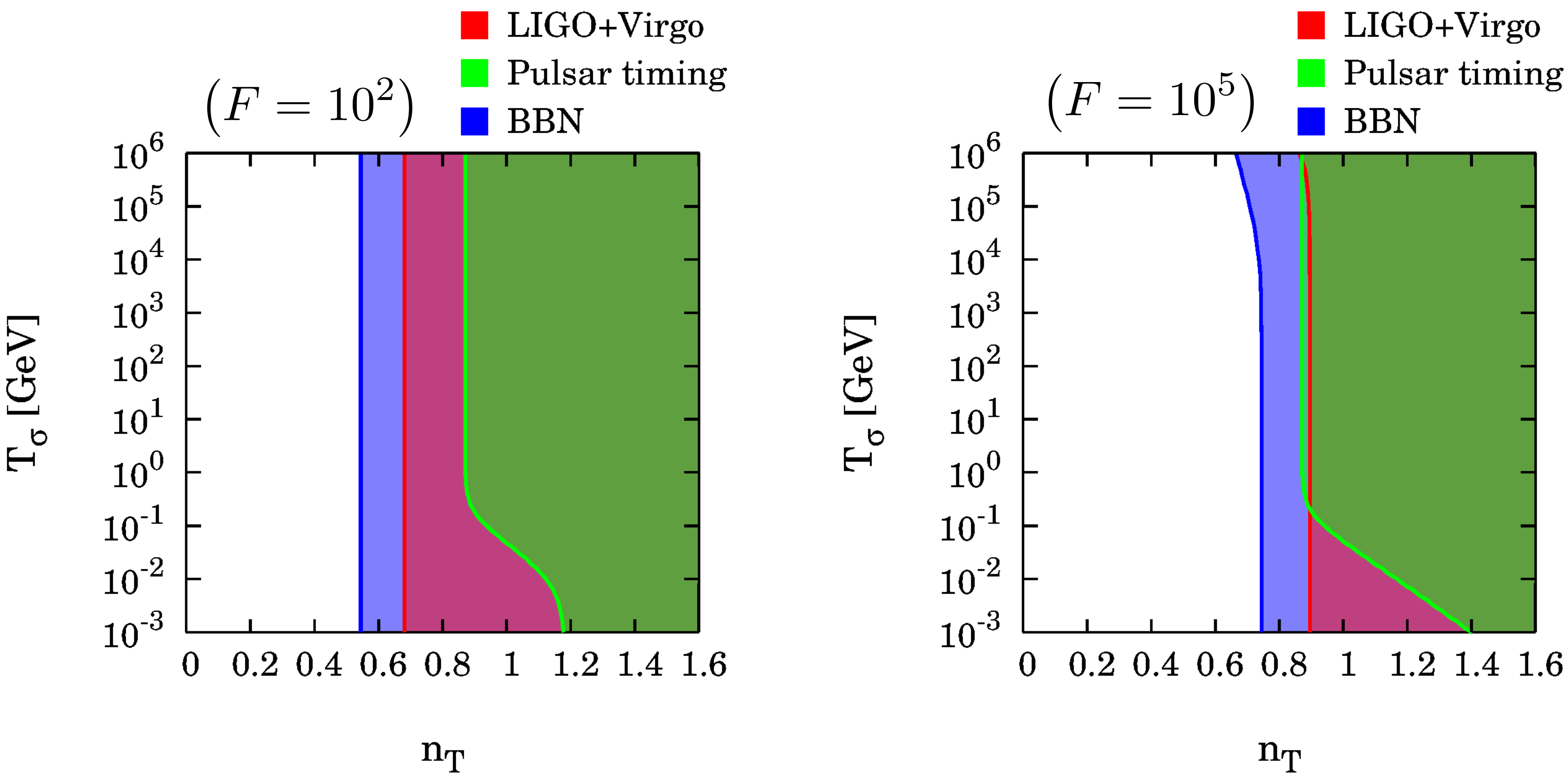
-- Temperature at which the entropy produced: T_σ

(the time when the scalar field decays)

-- Amount of entropy produced: $F \equiv \dfrac{s(T_\sigma)a^3(T_\sigma)}{s(T_R)a^3(T_R)}$

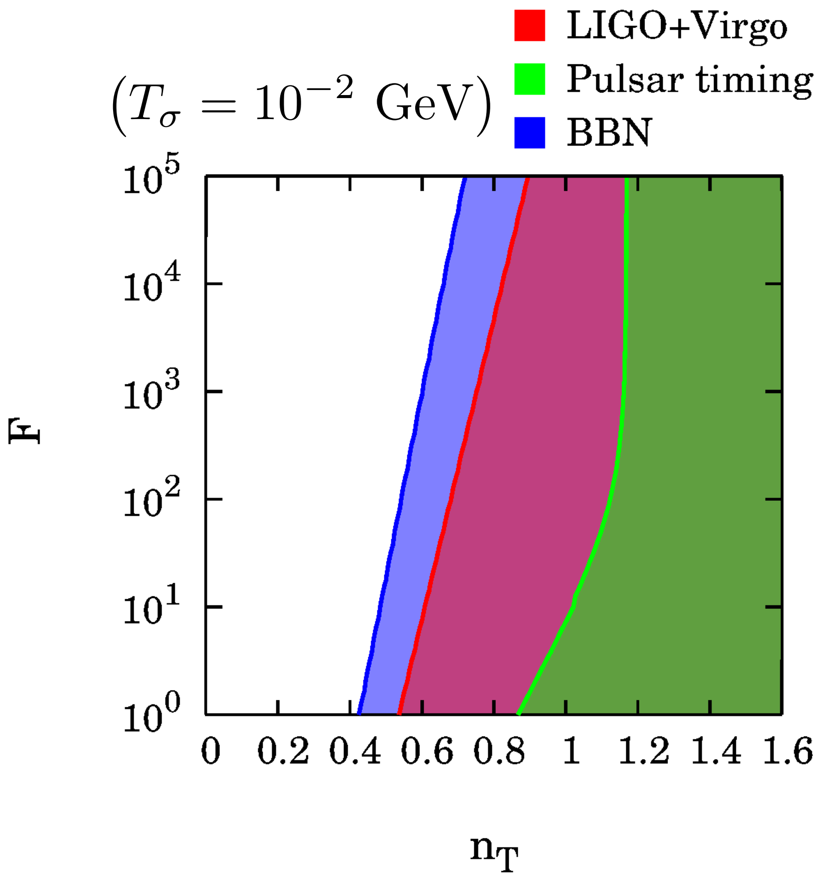
Constraint on nT and Tσ

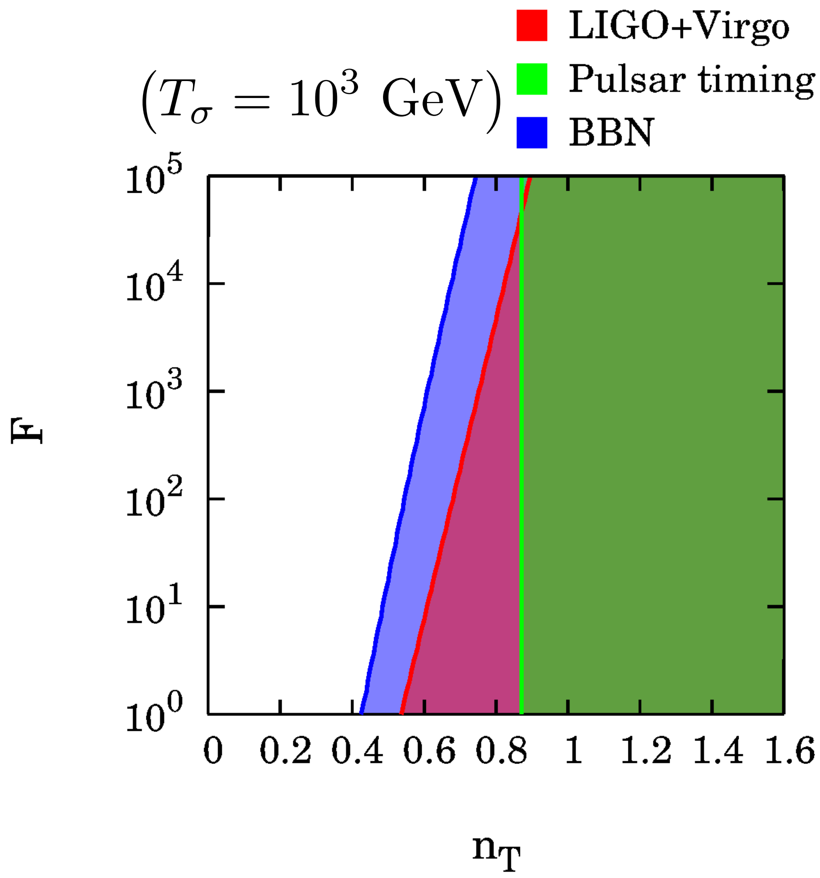
[Kuroyanagi, TT, Yokoyama 1407.4785]



Constraint on nT and F

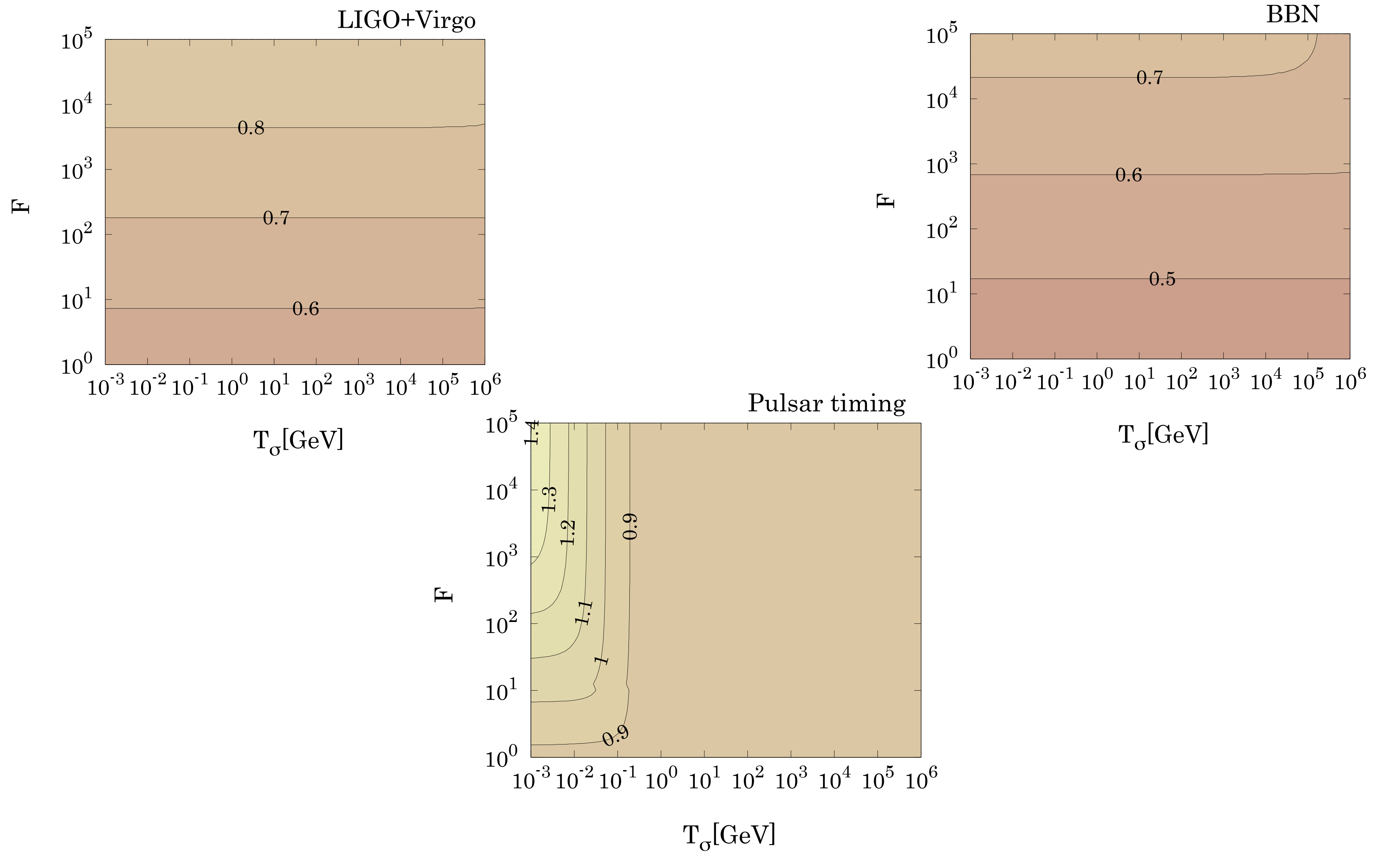
[Kuroyanagi, TT, Yokoyama 1407.4785]





Constraint on nT (2σ upper bound)

[Kuroyanagi, TT, Yokoyama 1407.4785]



More general case

[Kuroyanagi, TT, Yokoyama 1407.4785]

- In some models, the tensor spectral index changes from nT --> nT' at some frequency.

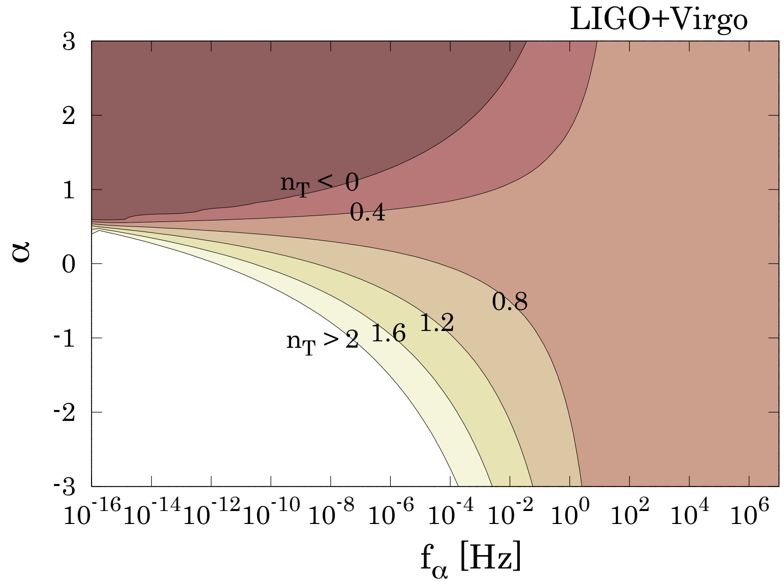
- The Universe may experience non-RD (non-MD) phase (such as kination epoch).

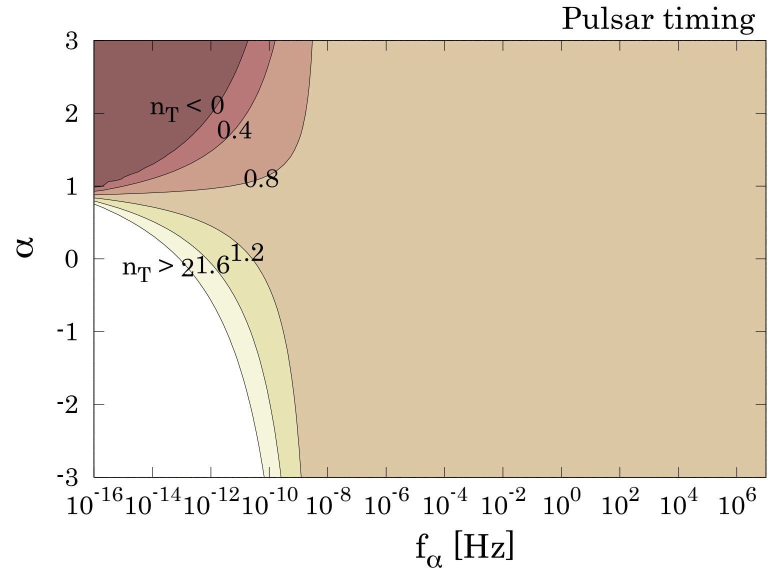
general modeling of the spectrum

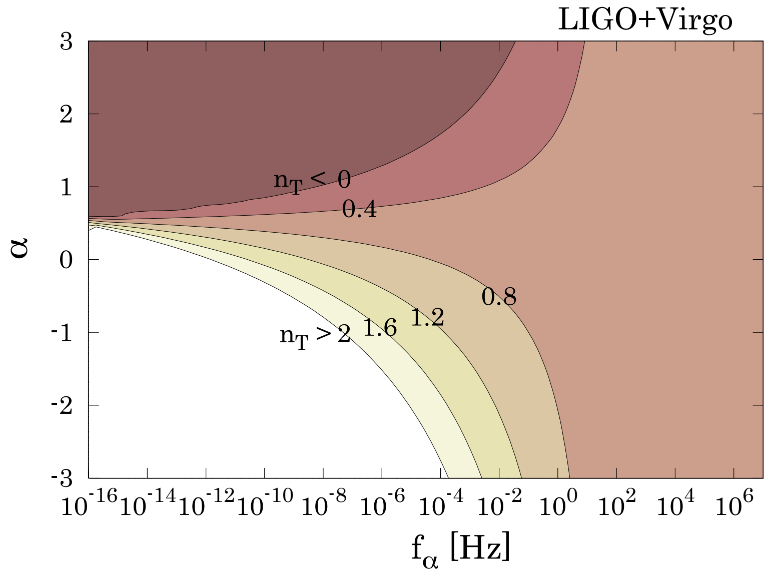
$$\Omega_{\rm GW}(k) = \begin{cases} \frac{1}{12}\left(\frac{k}{aH}\right)^2 T_T^2(k) A_T(k_{\rm ref}) \left(\frac{k}{k_{\rm ref}}\right)^{n_T} & (k < k_\alpha), \\ \frac{1}{12}\left(\frac{k}{aH}\right)^2 T_T^2(k) A_T(k_{\rm ref}) \left(\frac{k_\alpha}{k_{\rm ref}}\right)^{n_T} \left(\frac{k}{k_\alpha}\right)^{\alpha} & (k > k_\alpha), \end{cases}$$

Constraint on nT (2σ upper bound)

[Kuroyanagi, TT, Yokoyama 1407.4785]





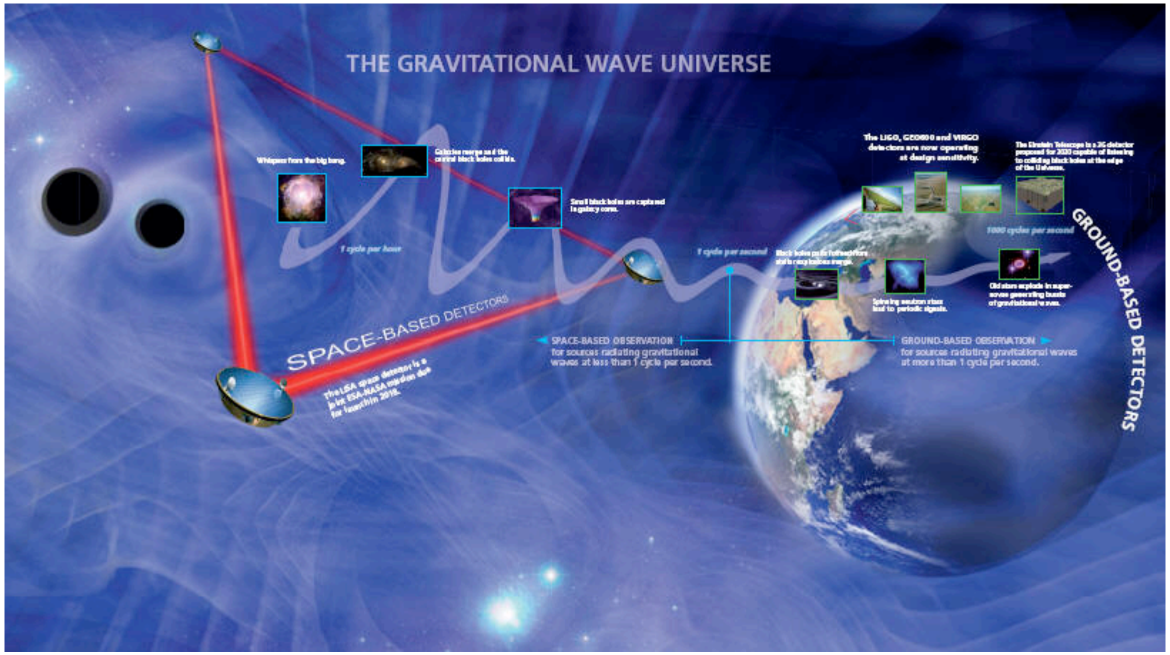


What about the standard case?

- Future space-based interferometer experiments

-- BBO

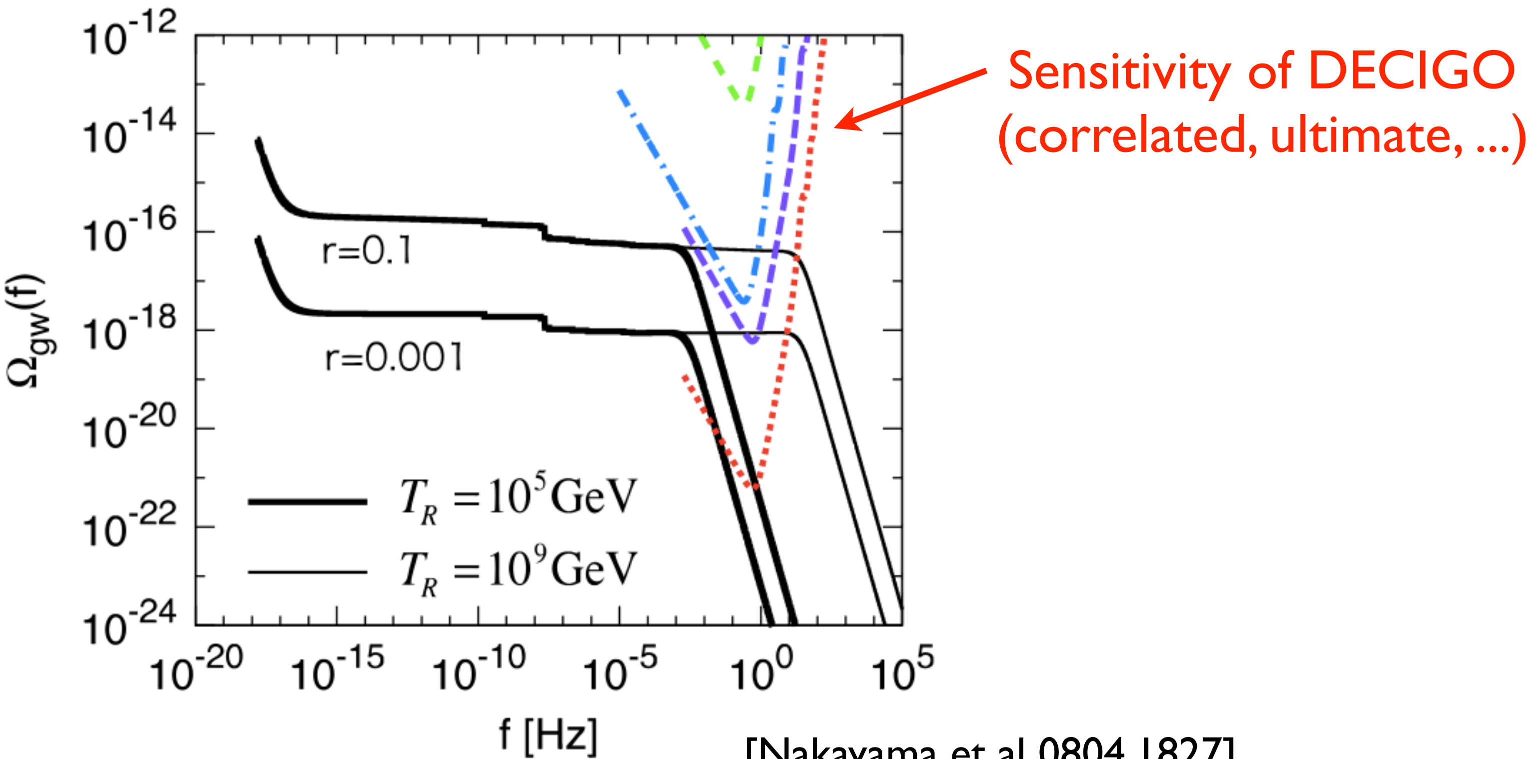
-- DECIGO



[http://www.personal.soton.ac.uk/nils/rsweb/thefacts.htm]

Probing inflation with future direct detection of GW

- Future space-based exp. are sensitive at f ~ 1 Hz.



[Nakayama et al 0804.1827]

Probing inflation with future direct detection of GW

- Future space-based exp. are sensitive at f ~ 1 Hz.

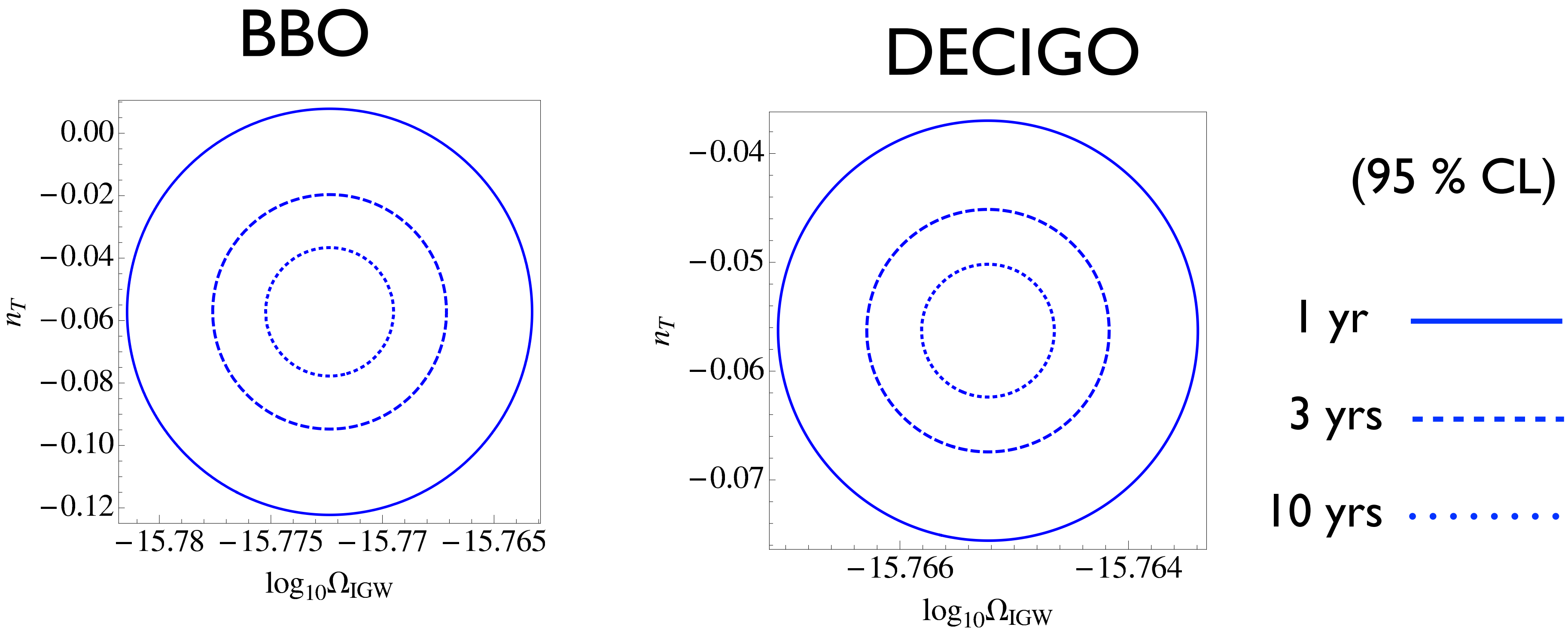
- We may be able to probe n_T very precisely.

  (Even the running α_T ?)

- We may be able to probe the reheating temperature.

Expected sensitivity

[Jinno, Moroi, TT 1406.1666]

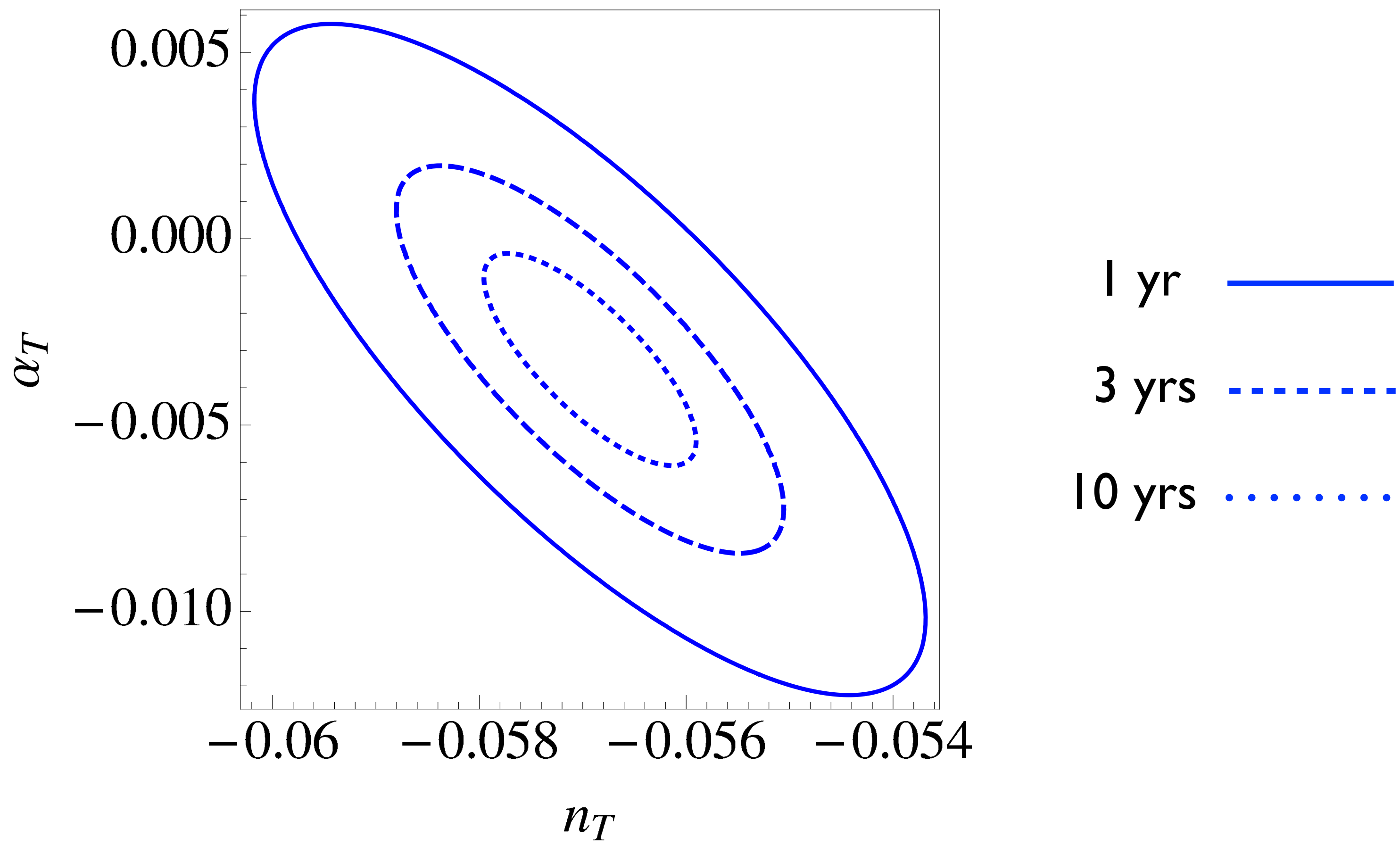


(Fiducial model: quadratic chaotic inflation)

(We have chosen the pivot scale such that correlation between Omega_GW and nT vanishes.)

nT and its running αT

[Jinno, Moroi, TT 1406.1666]

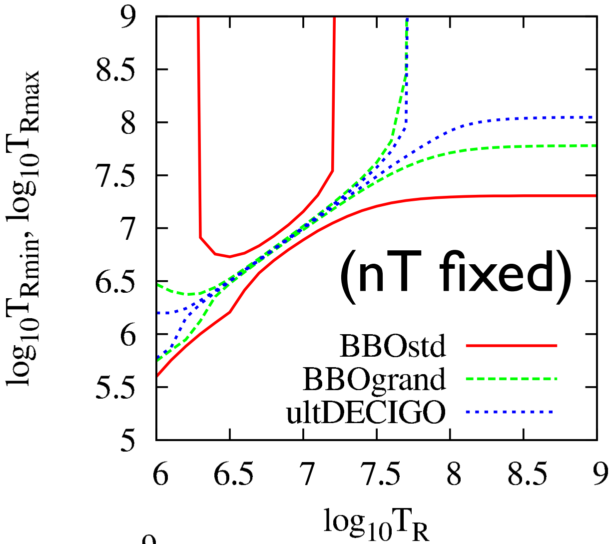


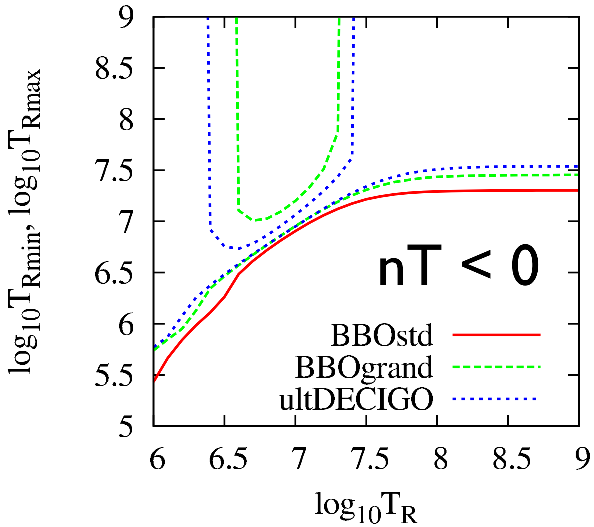
(We have assumed "ultimate" setup for DECIGO.)

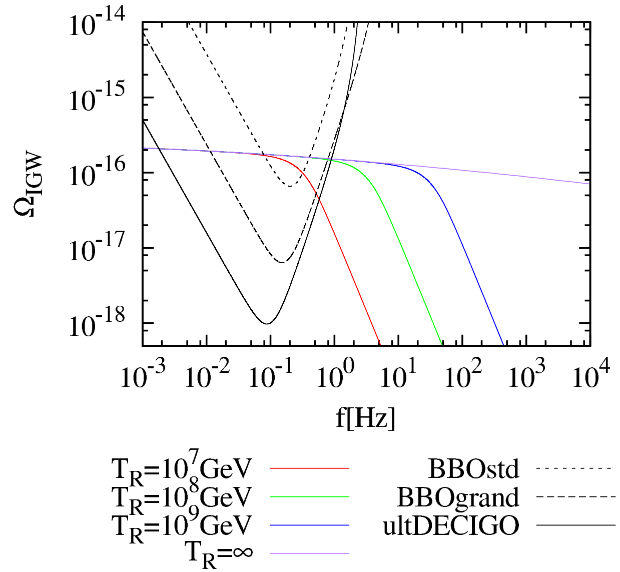
Reheating temperature

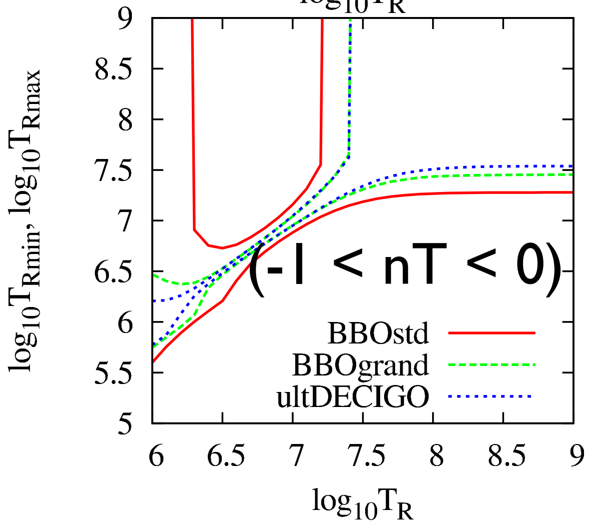
[Jinno, Moroi, TT 1406.1666]

- We may be able to obtain upper/lower bound for T_R



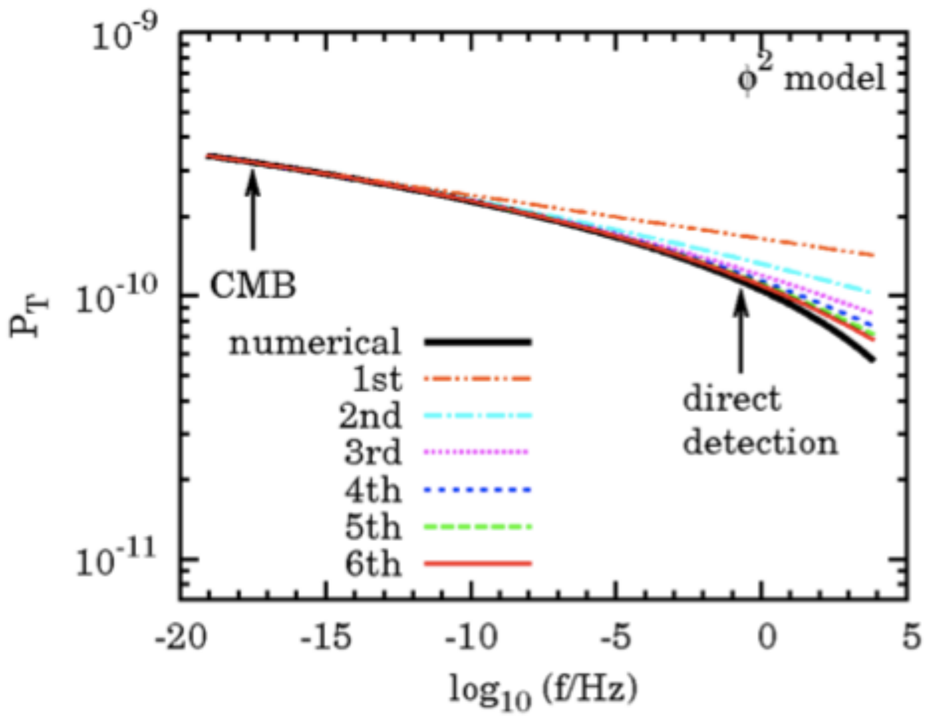






CMB + space-based GW exp.

- CMB and future space-based GW scales are so different. Need some care when we use both CMB and direct detection GW obs.

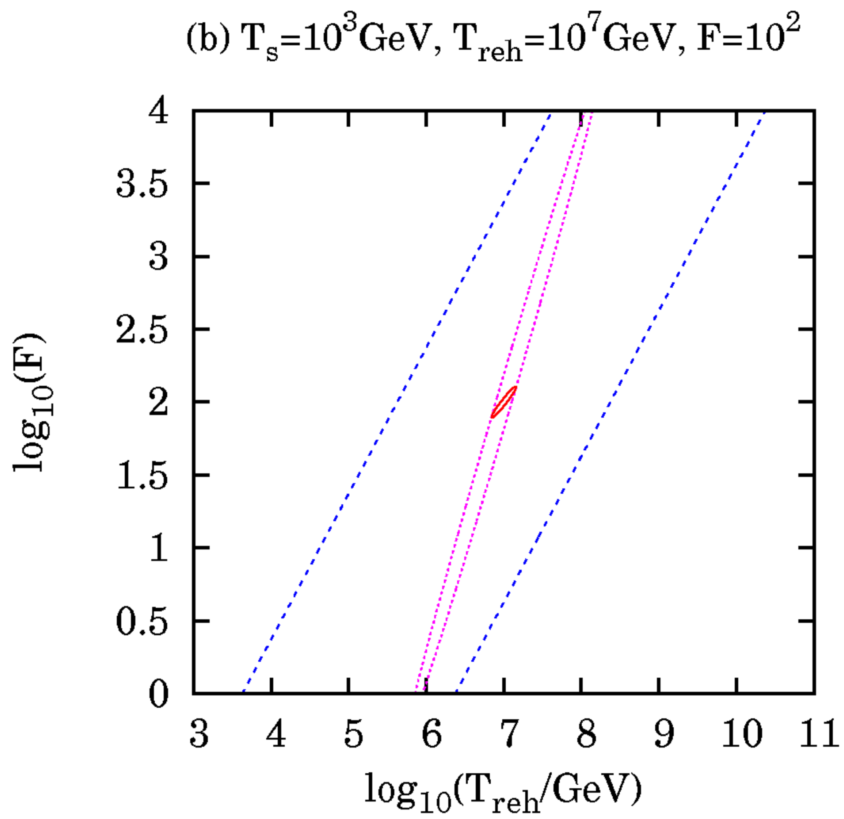


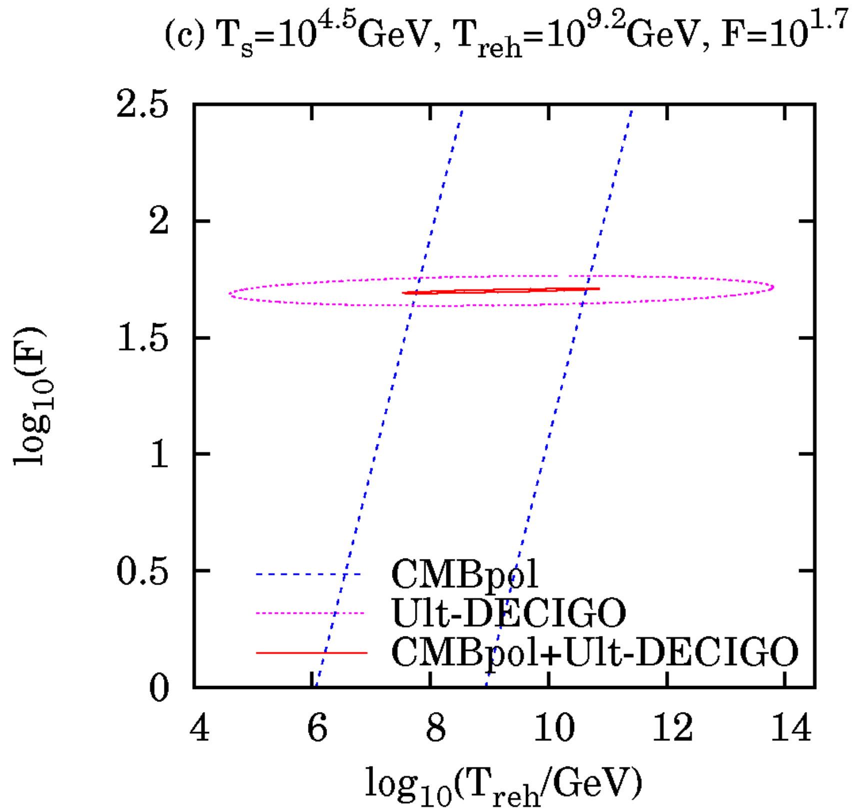
[Kuroyanagi, TT 1106.3437]

Synergy between CMB and direct GWs

[Kuroyanagi, Ringeval, TT 1301.1778]

- CMB and direct detection exp. of GWs are complementary.





Summary

- Gravitational waves would be very useful to probe the inflationary Universe.

  (If confirmed to be sizable amplitude in any observations.)

- Probing the tensor spectral index would give crucial consistency test of the inflationary Universe

- Future direct interferometer experiments may probe the rehearing temperature.

- Gravitational waves are the key to precisely understand the inflationary Universe.