The curvaton scenario: Pre and post BICEP2/Planck1

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CB, Cortes & Liddle 2014 + many older papers

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The curvaton scenario

- An alternative model to single-field inflation for the origin of structures
- The inflaton drives inflation while the curvaton generates curvature perturbations (hence the name)
- This “liberates” the inflaton, at the expense of making inflation less predictive
- We now have two light degrees of freedom during inflation, sensitive to two potentials and initial conditions.

- The curvaton is a light field which

1. has a subdominant energy density during inflation
2. Is long lived (compared to the inflaton)
3. Generates the primordial curvature perturbation

- We will often drop assumption 3, and consider the mixed inflaton-curvaton scenario

Curvaton phenomenology

- Adding one extra field allows for interesting new phenomenology which single-field inflation cannot generate

1. Large local non-Gaussianity (breaking the Maldacena consistency relation between the squeezed bispectrum and the power spectrums spectral index)

2. Isocurvature perturbations - the relative energy density of different components (e.g. radiation and cold dark matter) is a function of position on all scales

3. A suppressed tensor-to-scalar ratio r

Observations don't (currently) require a second field, but high energy theories might

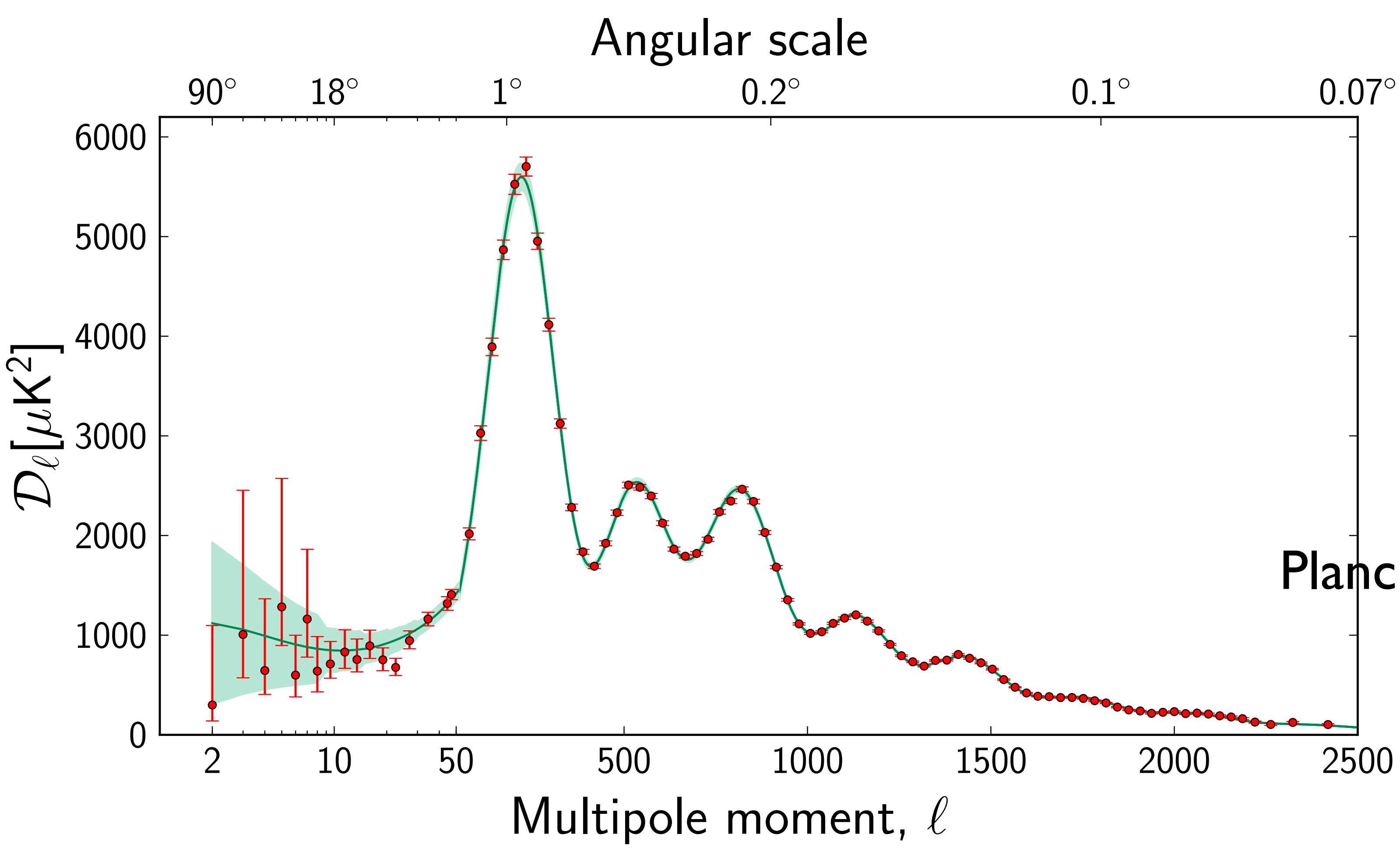
A brief history: The usual suspects from 2001: Enqvist and Sloth, Lyth and Wands (who created the name and got ~900 citations), Moroi and Takahashi.

Plus two related older papers, Linde and Mukhanov (1996), Mollerach (1990)

Related models

- Several other models predict essentially identical phenomenology (local non-Gaussianity, isocurvature perturbations and suppressed tensor perturbations)
- For example
  1. Modulated reheating (the efficiency of reheating is a function of position)
  2. Inhomogeneous end of inflation (inflation ends later in some positions)
  3. Models with a subdominant field curving the trajectory during inflation
- This is not a coincidence, all models are tracking the conversion of an initial isocurvature perturbation (corresponding to a light and subdominant field) into the adiabatic perturbation after inflation
- Wait for the next two talks by Ewan and Joe
- The models are physically different, and detailed predictions for the simplest realisations do vary
- However the curvaton is the earliest and perhaps the simplest to study of these cases, extremely popular

Planck measured power spectrum



Planck collab 2013 XV

Figure 37. The 2013 *Planck* CMB temperature angular power spectrum. The error bars include cosmic variance, whose magnitude is indicated by the green shaded area around the best fit model. The low-ℓ values are plotted at 2, 3, 4, 5, 6, 7, 8, 9.5, 11.5, 13.5, 16, 19, 22.5, 27, 34.5, and 44.5.

Looks complicated, but all this can be fit by a primordial power law spectrum with just two input parameters

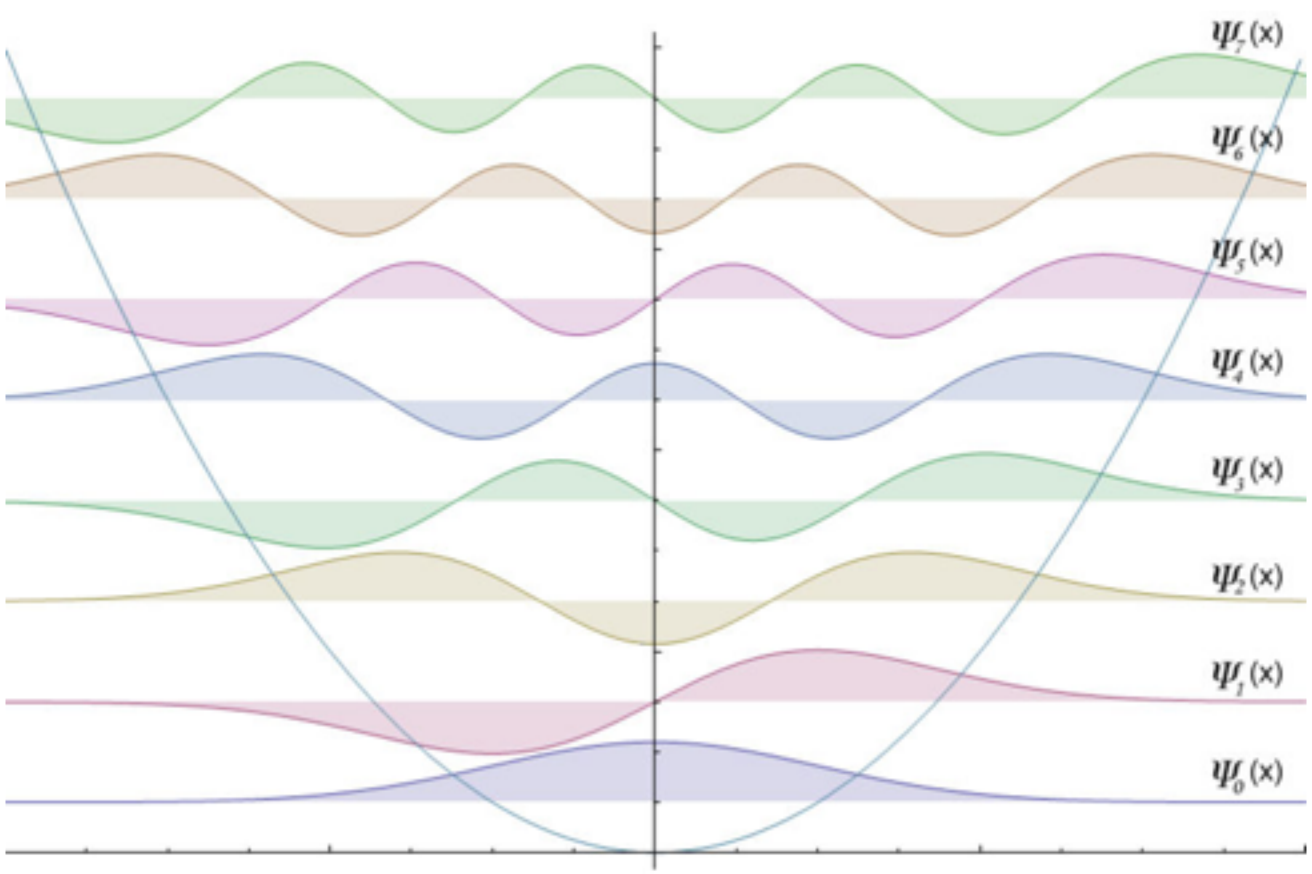
The range of scales probed is $2500/2=10^3=e^7$ - corresponds to about 7 efoldings of inflation

Enormous data compression

- Planck observes ~10^7 pixels in the CMB sky
- Reduced to ~10^3 Cl
- Further reduced to A and n_s-1
- Can only be justified if the perturbations are Gaussian
- Then by Wicks theorem, the odd point correlators are zero, the even ones are reducible to products of two points functions - i.e. all information is contained in the power spectrum

Why Gaussian perturbations?

- Gaussian perturbations are found everywhere in nature
- Often due to the central limit theorem
- The ground state of the simple harmonic oscillator is Gaussian - quantum origin of perturbations
- The initial curvaton field perturbation is expected to be Gaussian



Curvaton evolution

$$V = \frac{1}{2} m_\sigma^2 \sigma^2$$

- For simplicity, we initially assume a quadratic potential for the curvaton, most papers in the literature do so

$$\ddot{\sigma} + 3H\dot{\sigma} + V_{,\sigma} = 0,$$
$$\ddot{\delta\sigma} + 3H\dot{\delta\sigma} + V_{,\sigma\sigma}\delta\sigma = 0.$$

- Just for a quadratic potential, the two evolution equations are the same. This implies that the ratio of the two solutions is constant in time. The second equation neglects back reaction from gravity, accurate as long as its energy is subdominant

Curvaton density perturbations

$$\frac{\delta\rho_\sigma}{\rho_\sigma} \simeq \frac{V(\sigma+\delta\sigma)-V(\sigma)}{V(\sigma)} = 2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma}\right)^2$$

- This is a constant
- The truncation at second order follows because we assumed a quadratic potential
- The above formula follows the local model, and if the above was the final result for zeta we would have $f_{NL}\sim1$
- Gravity is non-linear, so further non-Gaussianities will be generated in all models, this also generates $f_{NL}\sim1$, but with a different shape which can be observationally distinguished - Antony Lewis's talk
- The above form of non-Gaussianity, Gaussian + Gaussian squared is known as the local form of non-Gaussianity
- However, we should consider that the curvaton is not the only component of the universe

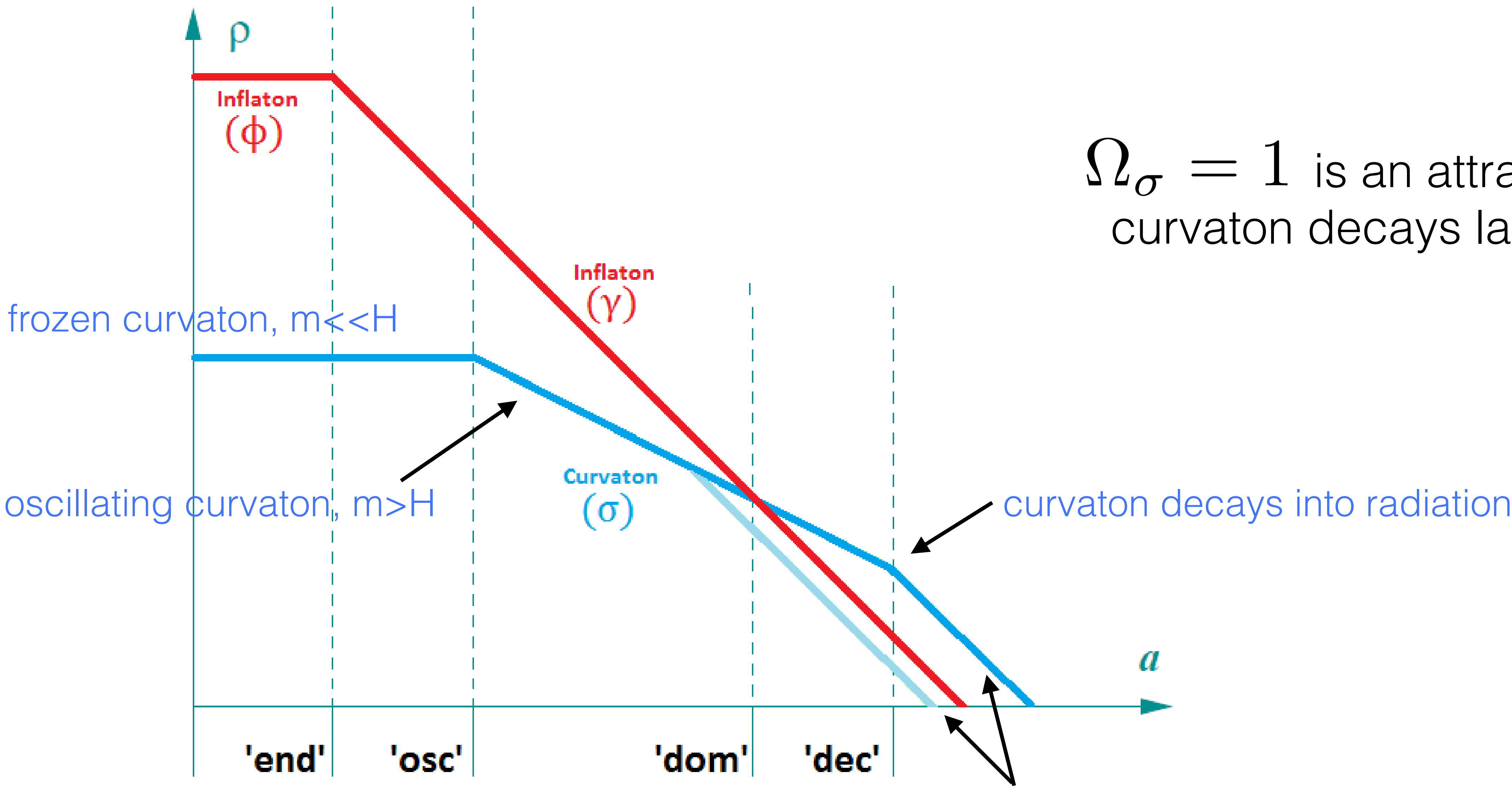
$$\zeta = \Omega_\sigma \zeta_\sigma \qquad \Omega_\sigma = \rho_\sigma/\rho_{tot}$$

$$f_{\rm NL} \propto \zeta^{(2)}/\zeta^{(1)2} \propto 1/\Omega_\sigma$$

Curvaton background evolution:

Log of scale factor versus log of energy density

$\Omega_\sigma = 1$ is an attractor if the curvaton decays late enough

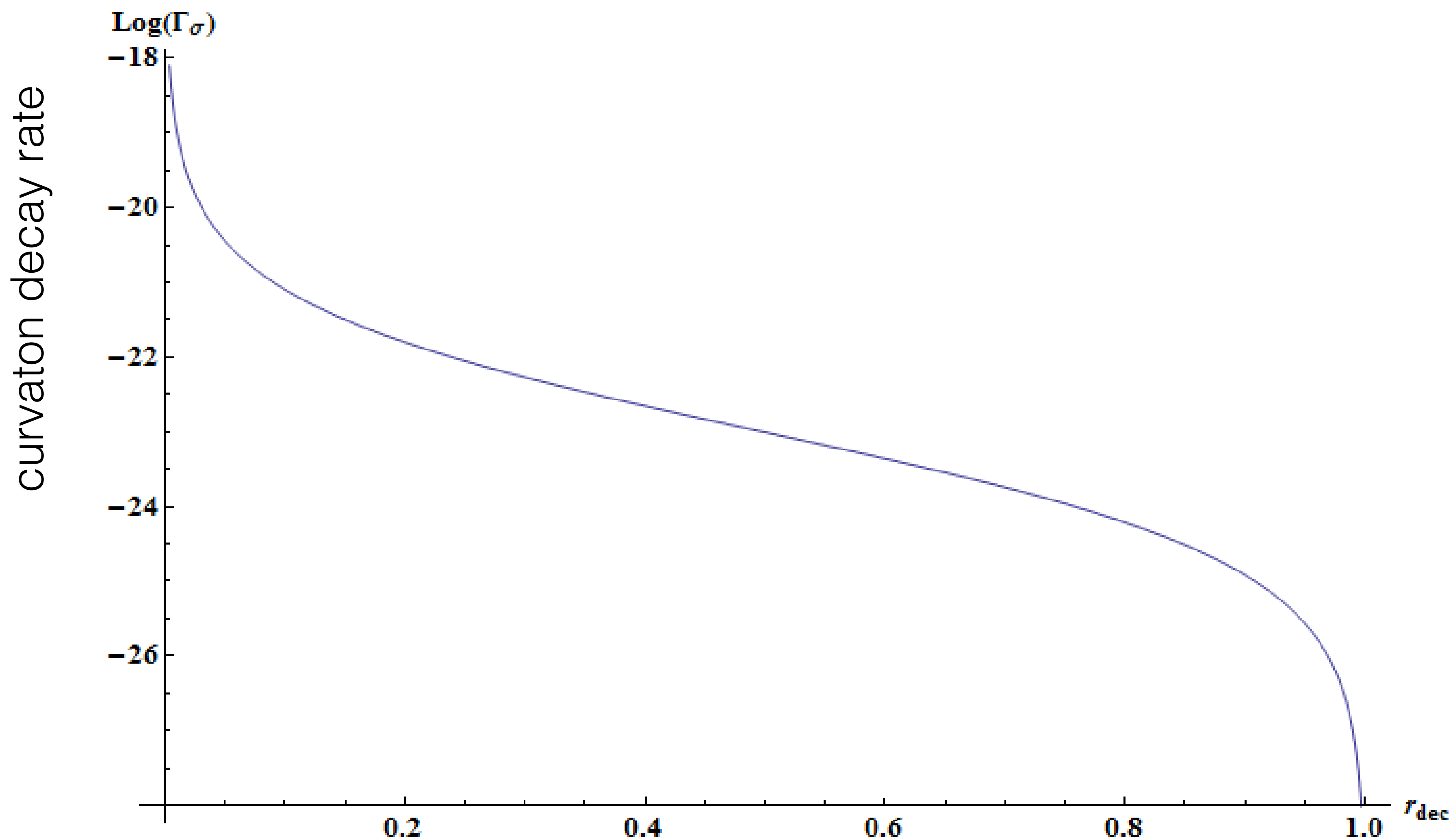


The curvaton may decay before or after it becomes dominant

Here we assume that the curvaton and inflaton decay instantaneously into radiation
The longer the curvaton lives, the larger its relative energy density becomes, as measured by r_{dec}

Curvaton decay rate vs r_{dec}

The curve will shift for different choices of masses and initial curvaton vev. But the shape remains the same.
For small curvaton decay rates, r_{dec}->1



Credit: Robert Hardwick

$$r_{\rm dec} \equiv \left.\frac{3\rho_\sigma}{4\rho_\gamma + 3\rho_\sigma}\right|_{\rm decay}$$

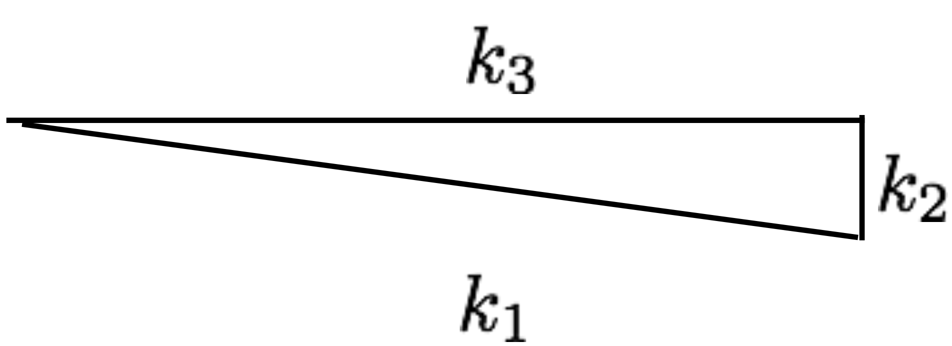
The local model of non-Gaussianity

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL}(\zeta_G^2(\mathbf{x}) - \langle\zeta_G^2(\mathbf{x})\rangle)$$

- The local model which arises from super-horizon evolution of the curvature perturbation
- Zeta is conserved in single-field models on large scales, therefore this model only arises in models with multiple light fields present during inflation
- The Planck constraint (and WMAP9 in brackets) are

$$f_{\rm NL} = 2.7 \pm 5.8 \qquad (37.2 \pm 19.9)$$

- Using the power spectrum amplitude, we see that the CMB is at least 99.9% Gaussian for this model.
- This shape has its largest signal in the squeezed limit, when one wavelength is very large
- Because a detection of a squeezed limit bispectrum would rule out all single-field models, the local model has been studied in great depth



$$k_2 \ll k_1 \simeq k_3$$

Komatsu et al; Decadel review 2009

Corrections to f_{NL}

- The basic result is correct, the less efficient the transfer from the curvaton perturbation to total curvature perturbation, the larger the non-Gaussianity becomes. This holds quite generally

- The "full" result is

$$f_{NL} = \frac{5}{4r_{\rm dec}} - \frac{5}{3} - \frac{5}{6} r_{\rm dec} \qquad r_{\rm dec} \equiv \left. \frac{3\rho_\sigma}{4\rho_\gamma + 3\rho_\sigma} \right|_{\rm decay}$$

- If f_{NL} is large, $f_{\rm NL} \propto \frac{1}{r_{\rm dec}} \propto \frac{1}{\Omega_\sigma}$

- The Planck constraint, $f_{NL}<10$, tells us $r_{dec}>0.1$. A priori, 10^{-5} was possible.

- If the curvaton dominates before it decays $f_{NL}=-5/4$

Mixed inflaton-curvaton scenario

All light fields are perturbed during inflation, we will now include the inflaton field perturbations

The power spectra due to the two fields is

$$P_\phi \sim \frac{1}{\epsilon}\left(\frac{H_*}{2\pi}\right)^2, \qquad P_\sigma \sim \Omega_\sigma^2 \frac{1}{\sigma_*^2}\left(\frac{H_*}{2\pi}\right)^2,$$

and the total power spectrum is

$$P_\zeta = P_\phi + P_\sigma.$$

The bispectrum is unchanged from the pure curvaton limit

$$B_\zeta = B_\sigma = \frac{1}{\Omega_\sigma} P_\sigma^2$$

but $f_{\rm NL}$ is reduced because the power spectrum is enhanced by the Gaussian inflaton field perturbations

$$f_{\rm NL} \sim \frac{B_\zeta}{P_\zeta^2} = \frac{B_\sigma}{P_\zeta^2} = \frac{1}{\Omega_\sigma}\frac{P_\sigma^2}{P_\zeta^2}.$$

The tensor-to-scalar ratio is also reduced

$$r = 16\epsilon \frac{P_\phi}{P_\zeta}.$$

Higher-order non-Gaussianity

$$\zeta = \zeta_G + \frac{3}{5} f_{NL} \left(\zeta_G^2 - \langle \zeta_G^2 \rangle \right) + \frac{9}{25} g_{NL} \zeta_G^3$$

$$\frac{\delta \rho_\sigma}{\rho_\sigma} \simeq \frac{V(\sigma + \delta\sigma) - V(\sigma)}{V(\sigma)} = 2\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma} \right)^2$$

- For a quadratic potential, we may truncate at second order, which implies $g_{NL}=0$. Quadratic potentials are simple to calculate with, so g_{NL} has been unfairly neglected.
- $|g_{NL}| >> f_{NL}^2$ is possible with non-quadratic potentials
- g_{NL} is hard to constrain. The current bound is $|g_{NL}| < 10^6$, Planck has not yet produced a constraint

Non-Gaussianity summary

- All single-source models must obey a relation between one trispectrum parameter and f_{NL}

$$\tau_{\rm NL} = \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- If multiple-fields contribute to zeta (eg the curvaton and inflaton), then

$$\tau_{\rm NL} \geq \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- A large g_{NL} would signal a non-quadratic potential for the curvaton
- f_{NL} will be scale dependent unless the curvaton potential is quadratic and the inflaton fluctuations are negligible
- An explicit example of how much we could learn from non-Gaussianity, it may contain lots of information

Isocurvature perturbations

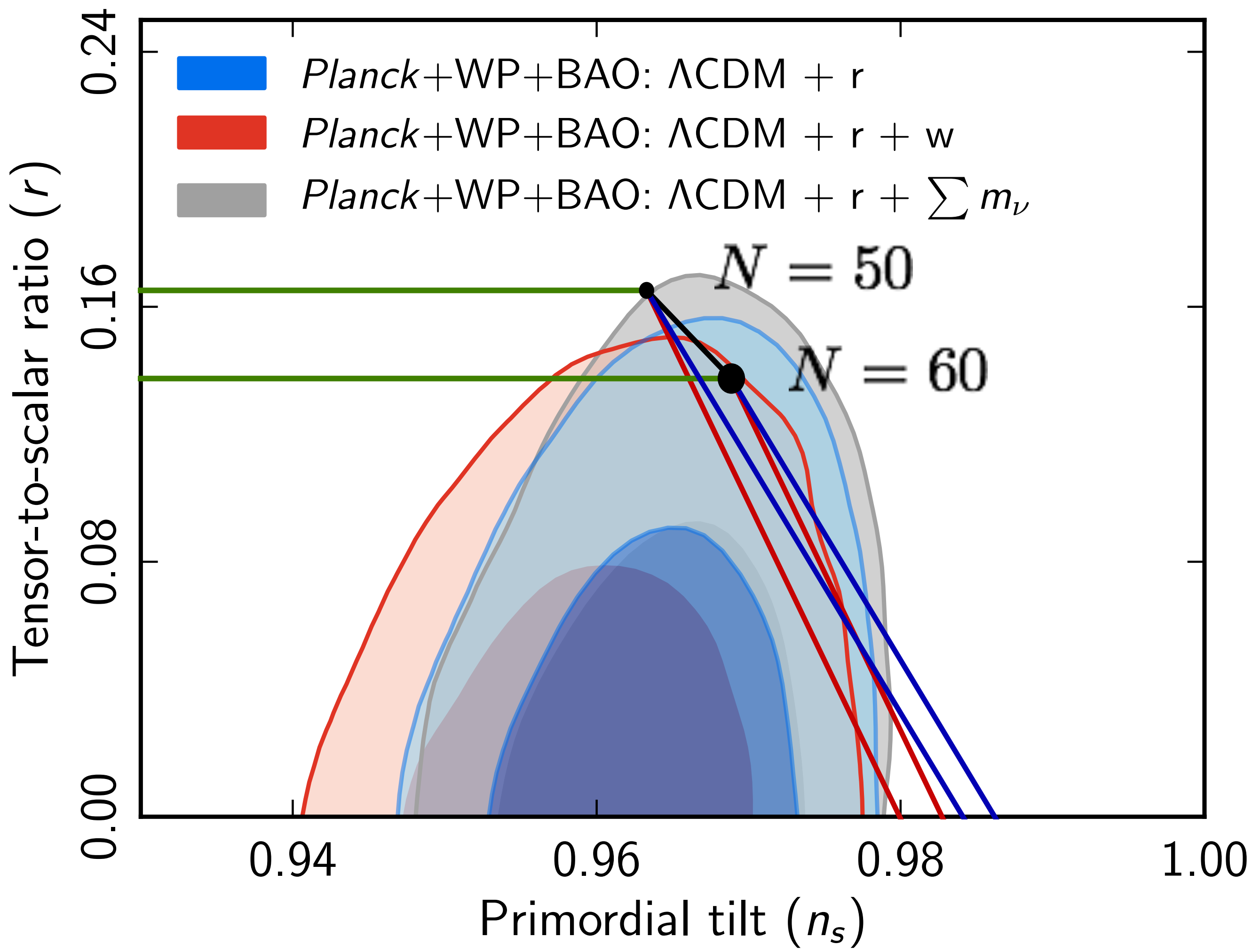
- Cosmological perturbations may be of two classes, adiabatic or isocurvature
- Adiabatic perturbations mean that locally all parts of the universe look the same, so e.g. the ratio of photons to baryons to CDM is the same everywhere
- The curvaton can generate isocurvature perturbations (most multi-field models can, single-field models never can), but if the universe thermalises after curvaton decay then none will survive. The tight Planck constraints are not a problem for the curvaton scenario, unless you have specified the reheating process (which a complete model needs)
- Planck polarisation data should significantly improve isocurvature bounds this year
- Theorists are not really able to interpret the 1% level isocurvature constraints in terms of early universe models, the thermal history of the universe prior to BBN is poorly understood

The simplest curvaton scenario

$$V(\phi,\sigma) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\sigma^2\sigma^2$$

- Parameter constraints were originally made by Bartolo and Liddle (2002), the data allowed so much freedom they restricted the model to i) the Gaussian case ii) negligible inflaton perturbations
- CB, Cortes and Liddle (2014) revisited the model with Planck data. Even dropping those two assumptions we find the model is close to being ruled out. Observational data has improved a lot.
- We also allow the inflating curvaton scenario, in which the curvaton drives a second period of inflation. Applies when sigma>M_{Pl}.

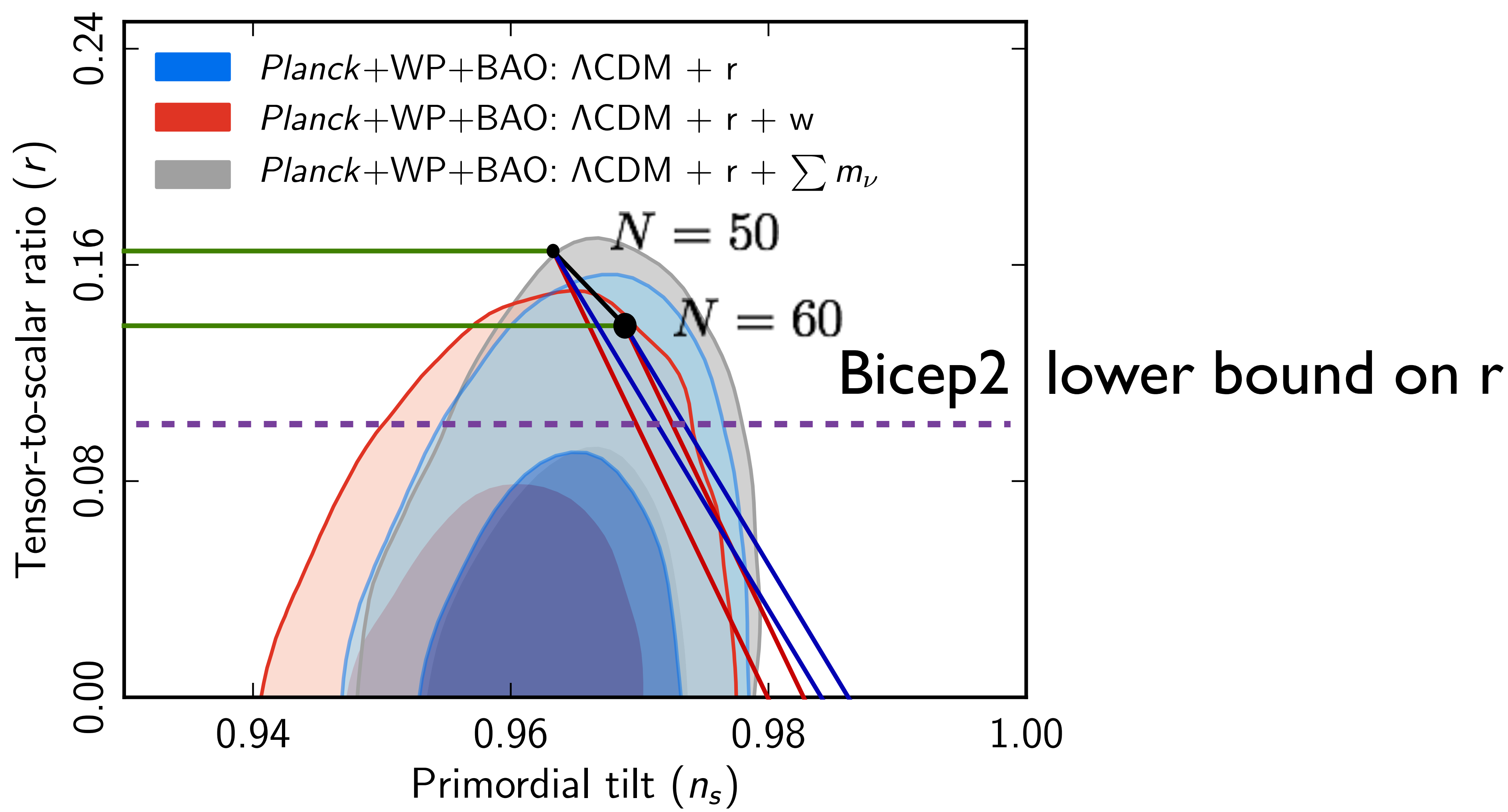
Curvaton post Planck



Red lines are for negligible curvaton mass, blue lines have m_sigma=m_phi/2. Green lines are the inflating curvaton regime, where it drives a second period of inflation.

Curvaton scenario has a lower bound on r_{dec} from the Planck satellite via f_{NL}. But only a detection of f_{NL}<-5/4 would rule it out. However, the simplest curvaton scenario, where both it and the inflaton field have quadratic fields may soon be ruled out. Changing the inflaton potential changes the quadratic curvaton predictions.

Curvaton post Planck and BICEP2



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BICEP2 adds a lower bound on the tensor to scalar ratio, which requires that the inflaton perturbations contribute at least 50% of the total curvature perturbation (talk to Tomo about a caveat). If confirmed, this rules out the original curvaton scenario, in which the inflaton perturbations and hence r are negligible.

A difficult time for curvaton fans?

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- Is the BICEP2 detection correct? The mood is swinging strongly against it, but we need to wait for new data and Planck dust maps
- Right or wrong, mixed scenarios in which both the inflaton and curvaton contribute to the primordial curvature perturbation can never be ruled out by a detection of tensors
- We may take a positive view, either a large negative running of the curvaton (Sloth 2014) or anti-correlated isocurvature modes (Kawasaki & Yokoyama 2014) as means to suppress the large scale power and alleviate possible Planck/BICEP tension

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- We may take a positive view, either a large negative running of the curvaton (Sloth 2014) or anti-correlated isocurvature modes (Kawasaki & Yokoyama 2014) as means to suppress the large scale power and alleviate possible Planck/BICEP tension
- In addition, Planck did significantly improve the constraints on both local non-Gaussianity and isocurvature perturbations, but there was no detection of either. This makes the curvaton phenomenology less interesting.
- However, the curvaton does not in any way require the existence of isocurvature perturbations today, and a natural limit of non-Gaussianity is local f_{NL}=-5/4. So Planck data does not come close to ruling it out all curvaton scenarios.
- Planck data alone puts pressure on the simplest curvaton scenario

The curvaton on a knife-edge?

Few people still believe BICEP
The mixed scenario can never be ruled out
But Planck has ruled out a lot of interesting parameter space

Can we ever learn the truth?

- The curvaton scenario really is different from single-field inflation
- During inflation we have a second, perturbed degree of freedom
- From the end of inflation until after the curvaton decays, the universe behaves very differently. Both at the homogeneous and the perturbed level.
- Because the perturbations are so tiny, $f_{NL}=-5/4$ is a small perturbation. This might be the only surviving observational signature
- The predictions are not similar because of fine tuning, and the curvaton is not a perturbative correction to single-field inflation
- There is a good motivation to distinguish $f_{NL}\sim 1$ from 0

Conclusions

- If confirmed, BICEP2 has ruled out the original curvaton scenario in which the inflaton perturbations can be neglected
- Ignoring BICEP2, Planck has put pressure on the simplest curvaton scenario (quadratic inflaton and curvaton potentials), due to a combination of the spectral index and r.
- The above is true even if we allow an arbitrary proportion of the perturbations to come from the inflaton (we also allow the curvaton to drive a second period of inflation). The data is good enough to start ruling out two-field scenarios
- Non-Gaussianity constrains the curvaton to not be too subdominant, but are a long way from testing the f_{NL}=-5/4 limit. If non-G is detected, we could learn a lot.
- Without a detection of local non-Gaussianity or isocurvature perturbations we will never need a curvaton type mechanism, but this does not imply the curvaton didn't exist. How should we proceed?

A general test of single-source models

- For all models in which only one field generates the primordial curvature perturbation (other than the inflaton), there is a consistency relation between one term of the trispectrum and bispectrum

$$\tau_{\rm NL} = \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- In models where multiple fields contribute there is instead the Suyama-Yamaguchi inequality

$$\tau_{\rm NL} \geq \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- For the mixed inflaton curvaton scenario

$$\tau_{\rm NL} = \frac{P_\zeta}{P_\sigma}\left(\frac{6 f_{\rm NL}}{5}\right)^2 \geq \left(\frac{6 f_{\rm NL}}{5}\right)^2$$

- From Planck, $\mathrm{tau}_{\rm NL}$<2800 (95% confidence)

Scale-dependence of f_{NL}

$$n_{f_{NL}} \equiv \frac{\partial \log |f_{NL}|}{\partial \log k}$$

- Analogously to the power spectrum, f_{NL} is expected to have some scale dependence. This reflects evolution during inflation, e.g. it ends

- It can distinguish between different non-Gaussian scenarios, not just between Gaussian and non-Gaussian models
- The amplitude of f_{NL} can be tuned in most non-Gaussian models, so a precise measurement of f_{NL} wont do this

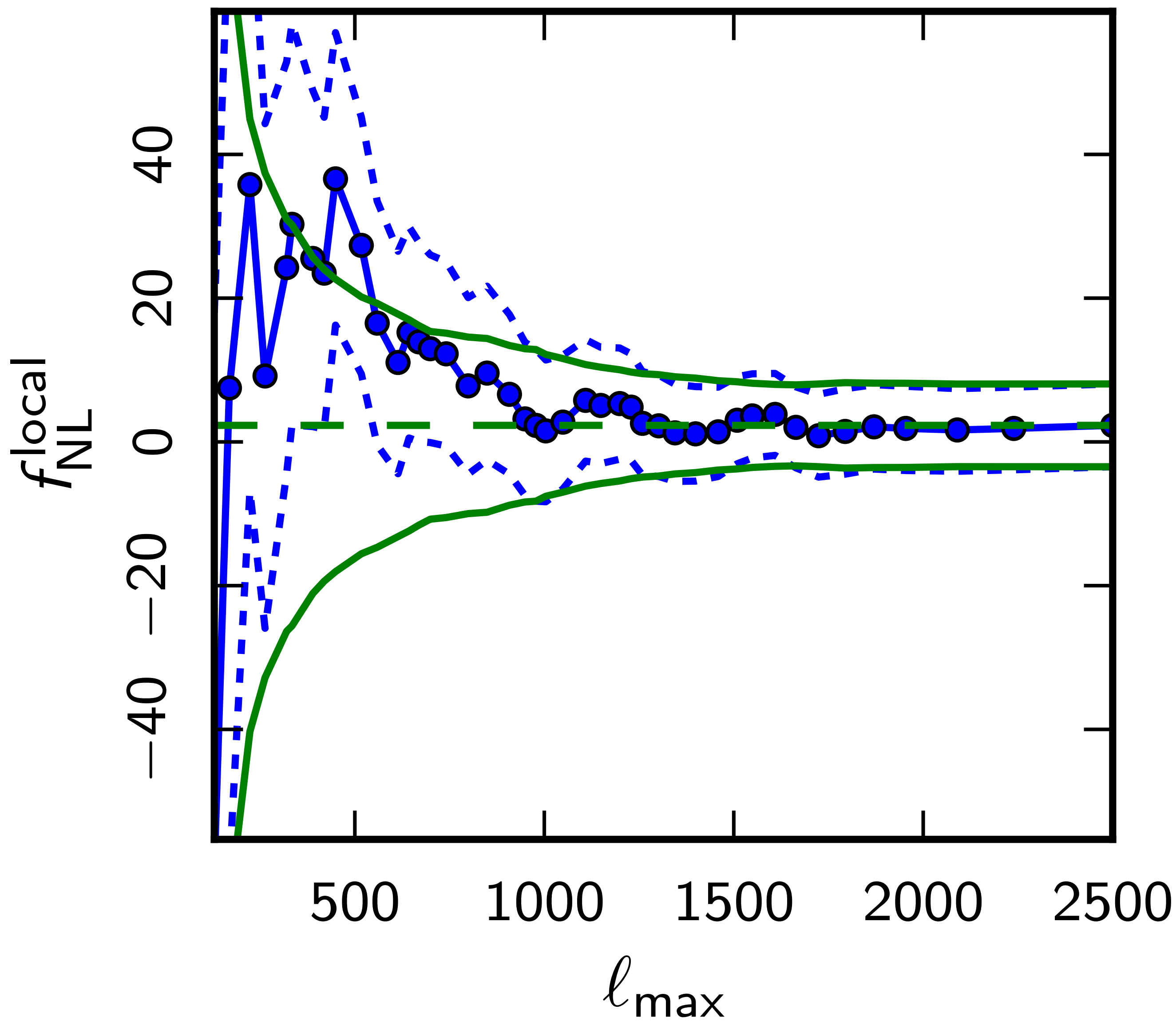
- In contrast, the scale dependence often can not be tuned independently of:
    1. f_{NL}
    2. spectral index of the power spectrum

- Scale dependence arises from either multiple fields contributing to zeta, or due to self-interactions in the potential (leading to non-linear equations)

$$n_{f_{NL}} \sim \frac{\sqrt{r_T}}{f_{NL}} \frac{V'''}{3H^2} \qquad r_T = \frac{P_T}{P_\zeta}$$

CB et al 2010

Planck and scale dependence



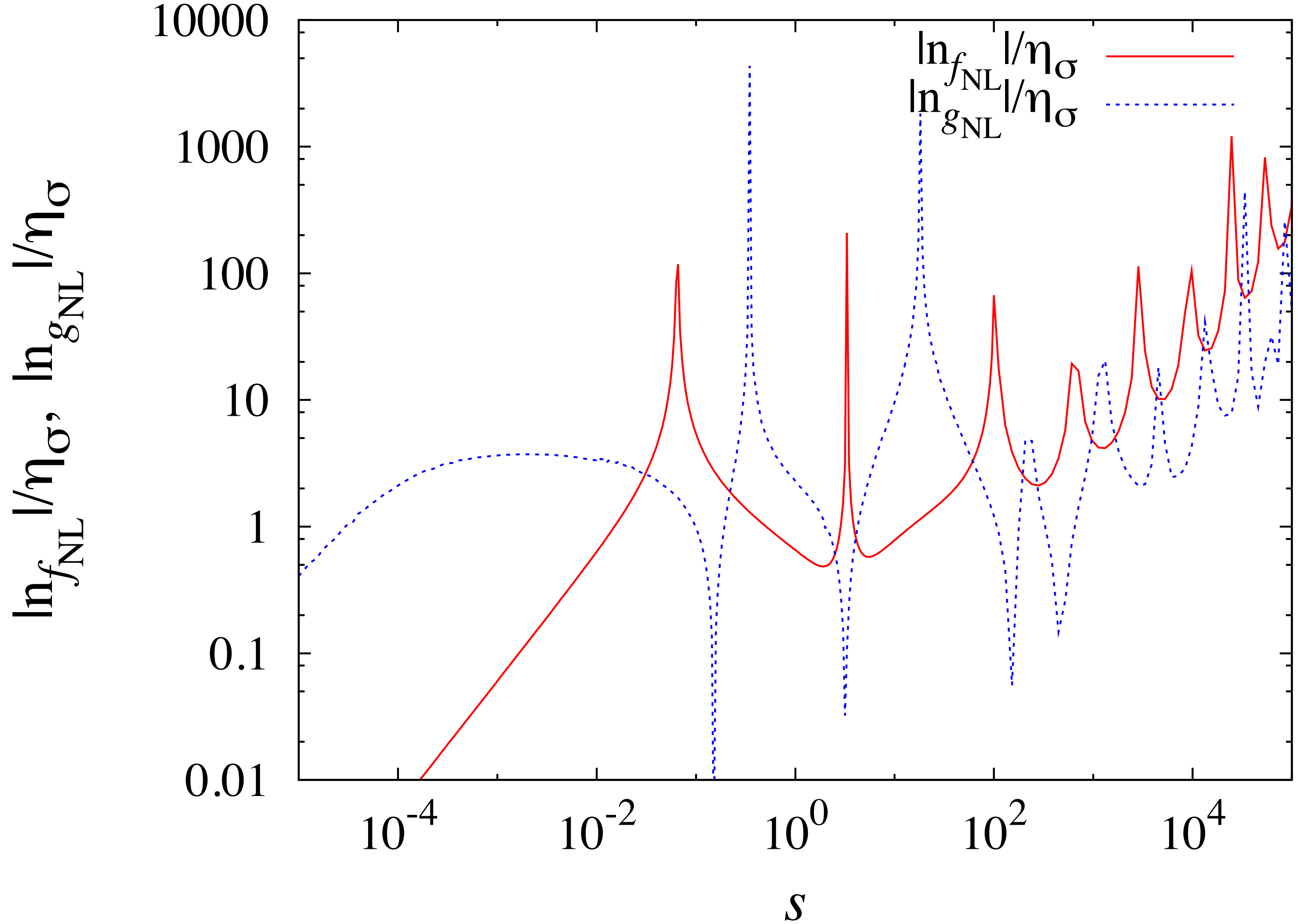
WMAP had consistently found a preference for positive f_{NL}. Planck is consistent with this, because the low l modes do prefer a positive value

Large f_{NL} on large scales from a self-interacting curvaton model could also help to explain the power spectrum dipole asymmetry

Potentially large scale dependence of f_{NL}

Curvaton potential: V=quadratic + higher-order monomial

s=(higher-order monomial)/quadratic at horizon crossing
=“self-interaction” strength



The scale dependence can be much larger than the slow-roll parameters, even for small self-interactions

CB, Enqvist, Nurmi & Takahashi 2011