Islands of Stability beyond the Land of Horndeski

Towards the most general stable scalar-tensor theories

Miguel Zumalacárregui

Instut für Theoretische Physik - University of Heidelberg

Benasque Modern Cosmology Workshop - August 2014

What is the most general theory of gravity?

(that deserves our attention)

Ostrogradski's Theorem (1850)

Theories with $L \supset \dfrac{\partial^n q}{\partial t^n},\ n \geq 2$ are unstable*

$$L(q(t), \dot{q}, \ddot{q}) \rightarrow \frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \boxed{\frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}}} = 0$$

$$q, \dot{q}, \ddot{q}, \dddot{q} \rightarrow Q_1, Q_2, P_1, P_2$$

$$H = \boxed{\mathbf{P_1 Q_2}} + \text{terms independent of } P_1$$

* Assumes $\dddot{q}, \ddot{q} \leftrightarrow P_2, Q_2$

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loophole for Degenerate Theories:

- 2^{nd} order equations
- Implicit constraints / reduced phase space

Horndenski's Theory (1974)

$g_{\mu\nu}$ + $\boxed{\phi}$ + Local + 4-D + Lorentz Theory with 2^{nd} order Eqs.

$$\mathcal{L}_H = G_2(X,\phi) - G_3(X,\phi)\Box\phi$$
$$+G_4 R + G_{4,X}\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$
$$+ G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{;\nu}\phi_{;\nu}{}^{;\lambda}\phi_{;\lambda}{}^{;\mu}\right]$$

4 × free functions of ϕ, $X \equiv -\frac{1}{2}\phi_{,\mu}\phi^{,\mu}$

- Jordan-Brans-Dicke: $G_4 = \frac{\phi}{16\pi G}$, $G_2 = \frac{X}{\omega(\phi)} - V(\phi)$

Provides general framework to study gravity and cosmology

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- Deriv. couplings $G_4(X)$, $G_5 \neq 0$

Provides general framework to study gravity and cosmology

General Disformal Relation - Bekenstein (PRD 1992)

Matter sector $\sqrt{-\tilde{g}}\mathcal{L}_m(\tilde{g}_{\mu\nu}, ...)$ with

$$\tilde{g}_{\mu\nu} = \underbrace{C(X,\phi)g_{\mu\nu}}_{\text{conformal}} + \underbrace{D(X,\phi)\phi_{,\mu}\phi_{,\nu}}_{\text{disformal}}$$

$\Rightarrow$ 2^{nd} order eqs. for ϕ (automatically) $X = -\frac{1}{2}(\partial\phi)^2$

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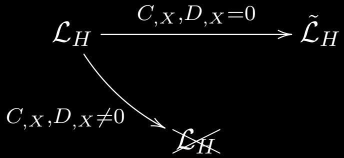

Original frame $\rightarrow$ second order

Jordan frame $\rightarrow$ Non-Horndeski theory $\rightarrow$ higher order!

Bettoni & Liberati (PRD 2013)

Is there even a Jordan Frame?

Non-trivial dependence:

$$\tilde{g}_{\mu\nu} = C(X,\phi)g_{\mu\nu} + D(X,\phi)\phi_{,\mu}\phi_{,\nu}$$

$$X = -\frac{1}{2}g_{\alpha\beta}\phi^{,\alpha}\phi^{,\beta}$$

Map between metrics:

$$\tilde{g}_{\mu\nu}\colon\ \mathbb{R}^{10} \to \mathbb{R}^{10}$$
$$g_{\mu\nu} \mapsto \tilde{g}_{\mu\nu}$$

Inverse function theorem:

$$\exists g_{\mu\nu}(\tilde{g}_{\mu\nu}) \Leftrightarrow \left|\frac{\partial \tilde{g}_{\mu\nu}}{\partial g_{\alpha\beta}}\right| \neq 0$$

The Jacobian - MZ, García-Bellido 1308.4685

$$g_{\mu\nu} \mapsto \tilde{g}_{\mu\nu} = C(X,\phi) g_{\mu\nu} + D(X,\phi)\phi_{,\mu}\phi_{,\nu}$$

⋆ Diagonalize Jacobian: $\left|\frac{\partial \tilde{g}_{\mu\nu}}{\partial g_{\alpha\beta}}\right| = \prod \lambda_i$

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Eigenvalues & Eigentensors

$$\left(\frac{\partial \tilde{g}_{\mu\nu}}{\partial g_{\alpha\beta}} - \lambda_i \mathbb{I}\right) \xi^i_{\alpha\beta} = 0$$

⋆ $\lambda_C = C,$ $\xi^C_{\mu\nu}\phi^{,\mu} = 0$ conformal

⋆ $\boxed{\lambda_K = C - C_{,X} X + 2D_{,X} X^2}$ $\xi^K_{\mu\nu} = \partial \tilde{g}_{\mu\nu}/\partial X$ kinetic

$\lambda_K,\ \lambda_C \neq 0 \Rightarrow \exists$Jordan frame

Questions for the audience

Path integral:

$$\int \underbrace{\mathcal{D}g_{\mu\nu}\mathcal{D}\phi\mathcal{D}\psi_M}_{\left|\frac{\partial\tilde{g}_{\mu\nu}}{\partial g_{\alpha\beta}}\right|}\ e^{-\frac{i}{h}\int d^4x\mathcal{L}[g_{\mu\nu},\phi,\psi_M]}$$

- Quantum equivalence of physical frames?
- Any other use?

Conformal: Jordan Frame Action

$$\sqrt{-g}\left(\frac{R}{16\pi G}+\mathcal{L}_\phi\right)+\sqrt{-\tilde{g}}\tilde{\mathcal{L}}_m\,,$$

$$\downarrow$$

$$\frac{\sqrt{-g}}{16\pi G}\left(\Omega^2 R+\boxed{6\Omega_{,\alpha}\Omega^{;\alpha}}\right)+\sqrt{-g}\left(\tilde{\mathcal{L}}_\phi+\mathcal{L}_m\right)$$

conformal coupling:

$\tilde{g}_{\mu\nu}=Cg_{\mu\nu}$

inverse map:

$g_{\mu\nu}\to\Omega^2 g_{\alpha\beta}$

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$$\underbrace{\nabla_\mu\Big((\Omega R-6\Box\Omega)}_{\sim\partial^4\phi,\,\partial^3 g_{\mu\nu}}\Omega_{,X}\phi^{,\mu}\Big)+\Omega_{,\phi}(\Omega R-6\Box\Omega)+\frac{1}{2}\frac{\delta\mathcal{L}_\phi}{\delta\phi}=0$$

$$\Omega^2 G_{\mu\nu}+2\Omega\underbrace{\left(g_{\mu\nu}\Box\Omega-\Omega_{;\mu\nu}\right)}_{\partial^3\phi}$$

$$-\underbrace{(\Omega R-6\Box\Omega)}_{\partial^3\phi}\Omega_{,X}\phi_{,\mu}\phi_{,\nu}-g_{\mu\nu}\Omega_{,\alpha}\Omega^{,\alpha}+4\Omega_{,\mu}\Omega_{,\nu}=8\pi G T_{\mu\nu}$$

Conformal: Jordan frame equations

Scalar:

$$\nabla_\mu\Big((\Omega R - 6\Box\Omega)\Omega_{,X}\phi^{,\mu}\Big) + \Omega_{,\phi}(\Omega R - 6\Box\Omega) + \frac{1}{2}\frac{\delta\mathcal{L}_\phi}{\delta\phi} = 0$$

Metric $\rightarrow$ Take trace with $g^{\mu\nu}$

$$2\Omega(g_{\mu\nu}\Box\Omega - \Omega_{;\mu\nu}) + \Omega^2 G_{\mu\nu}$$

$$- g_{\mu\nu}\Omega_{,\alpha}\Omega^{,\alpha} + 4\Omega_{,\mu}\Omega_{,\nu} - (\Omega R - 6\Box\Omega)\Omega_{,X}\phi_{,\mu}\phi_{,\nu} = 8\pi G T_{\mu\nu}$$

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Metric $\rightarrow$ Take trace with $g^{\mu\nu}$

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Scalar:

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Implicit Constraint - MZ, García-Bellido 1308.4685

$$-(\Omega R - 6\Box\Omega) = \boxed{\frac{8\pi G\Omega_{,X}T}{\Omega - 2\Omega_{,X}X} \equiv \mathcal{T}_{\rm K}} \sim \partial\phi$$

Trace of metric eqs $\rightarrow$ solves high derivs!

Implicit Constraint - MZ, García-Bellido 1308.4685

$$\mathcal{T}_{\mathrm{K}} \equiv \frac{8\pi G \Omega_{,X} T}{\Omega - 2\Omega_{,X} X} = -(\Omega R - 6\Box\Omega)$$

Scalar Field eqs:

$$\nabla_\mu \left(\phi^{,\mu} \mathcal{T}_{\mathrm{K}}\right) + \frac{\Omega_{,\phi}}{\Omega_{,X}} \mathcal{T}_{\mathrm{K}} - \frac{1}{2}\frac{\delta \mathcal{L}_\phi}{\delta \phi} = 0$$

Metric eqs:

$$\frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} \supset g_{\mu\nu}\Box\Omega - \Omega_{;\mu\nu} = \left(\begin{array}{c|c} g^{k\alpha}\Omega_{;k\alpha} & -\Omega_{;0i} \\ \hline -\Omega_{;0i} & g_{ij}\Box\Omega - \Omega_{;ij} \end{array}\right)$$

- ✓ No higher time derivatives
- ✓ Procedure works for general disformal theories

Healthy theories beyond Horndeski

Proposed extension

characterized by 2 new free functions of X, ϕ

$$\mathcal{L}_{H+BH} = \mathcal{A}_4 R + \mathcal{B}_4 \left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$

$$+\mathcal{A}_5 G_{\mu\nu}\phi^{;\mu\nu} - \mathcal{B}_5/6\left[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{;\nu}\phi_{;\nu}{}^{;\lambda}\phi_{;\lambda}{}^{;\mu}\right]$$

Gleyzes, Langlois, Piazza & Vernizzi (1404.6495 and 1408.1952)

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  $\supset$ general disformal, but <u>not conformal</u> couplings

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- Hamiltonian analysis $\Rightarrow$ <u>no extra D.o.F.</u>
- kinetic mixing <u>with matter</u>: $L_{\rm int} \propto \delta\dot{\phi}\delta\rho_m$ (perturbed FRW)

  $\sim \nabla T$ terms in the field equation (implicit constraint)

Conclusions

- $\exists$ a third generation of Scalar-Tensor theories
- non-trivial frame transformations/field redefinitions
- ~~no-go theorems~~ $\Rightarrow$ interesting opportunities!
- Kinetic mixing with matter $\Rightarrow$ distinctive features
- LOTS of things to do
  - ⋆ Phenomenology: Viable? Interesting?
  - ⋆ Most general ST theory?

Backup Slides

Jordan frame - General case

$$\bar{g}_{\mu\nu} = A(X,\phi)g_{\mu\nu} + B(X,\phi)\phi_{,\mu}\phi_{,\nu}$$

$$\delta\left(\sqrt{-\bar{g}}\bar{R}\right) \supset -\sqrt{-\bar{g}}\,\bar{G}^{\mu\nu}\delta\bar{g}_{\mu\nu}$$

$$\supset -\sqrt{-\bar{g}}\,\bar{G}^{\mu\nu}\Big(\underbrace{\frac{\partial\bar{g}_{\alpha\beta}}{\partial g_{\mu\nu}}}_{\text{Jacobian}}\,\delta g_{\alpha\beta} - \underbrace{\frac{\partial\bar{g}_{\mu\nu}}{\partial X}}_{\text{Eigentensor}}\,\phi^{,\alpha}(\delta\phi)_{,\alpha}\cdots\Big)$$

Recall:

$$\frac{\partial\bar{g}_{\alpha\beta}}{\partial g_{\mu\nu}}\cdot\frac{\partial\bar{g}_{\mu\nu}}{\partial X} = \underbrace{(A - A_{,X}X + 2B_{,X}X^2)}_{\text{Eigenvalue}}\frac{\bar{g}_{\alpha\beta}}{\partial X}$$

Implicit Constraint - MZ, García-Bellido 1308.4685

$$\bar{G}^{\mu\nu}\frac{\partial\bar{g}_{\mu\nu}}{\partial X} = \sqrt{\frac{g}{\bar{g}}}\frac{8\pi G\,T^{\mu\nu}\,\frac{\partial\bar{g}_{\mu\nu}}{\partial X}}{(A - A_{,X}X + 2B_{,X}X^2)} \equiv \mathcal{T}_{\mathrm{K}}$$

Other uses of the Jacobian

Path integral:

$$\int \underbrace{\mathcal{D}g_{\mu\nu}\mathcal{D}\phi\mathcal{D}\psi_M}_{\left|\frac{\partial\tilde{g}_{\mu\nu}}{\partial g_{\alpha\beta}}\right|}\; e^{-\frac{i}{h}\int d^4x\mathcal{L}[g_{\mu\nu},\phi,\psi_M]}$$

$\Rightarrow$ Quantum equivalence of physical frames

Energy momentum in different frames:

$$T^{\mu\nu} = \sqrt{\frac{\tilde{g}}{g}}\frac{\partial\tilde{g}_{\alpha\beta}}{\partial g_{\mu\nu}}\tilde{T}^{\alpha\beta}$$