# Light Propagation in the Averaged Universe

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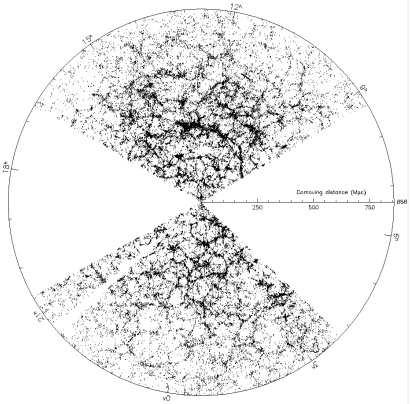
# Outline

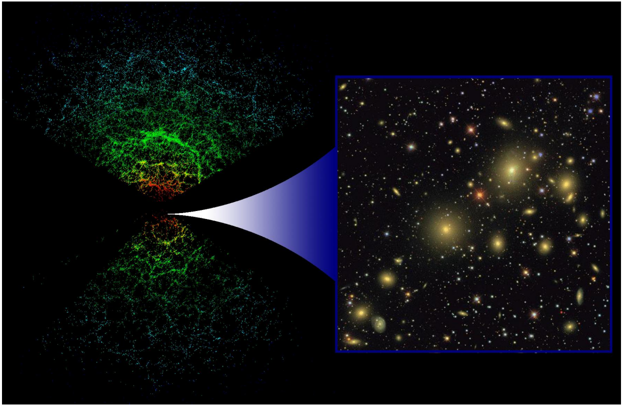
# Introduction

We observe the homogeneous matter distribution of the Universe, on scales larger than $100Mpc$.




[D. Hogg et al., 2001 (SDSS analysis)]
[C. Blake et al., 2007 (WiggleZ Dark Energy Survey)]

On small scales we see the lumpy universe with inhomogeneous distribution of matter.



# Effect of Inhomogeneities

- Matter distribution of real lumpy universe (on small scale)
  $T_{\mu\nu}^{(local)} = T_{\mu\nu}^{(discrete)}$

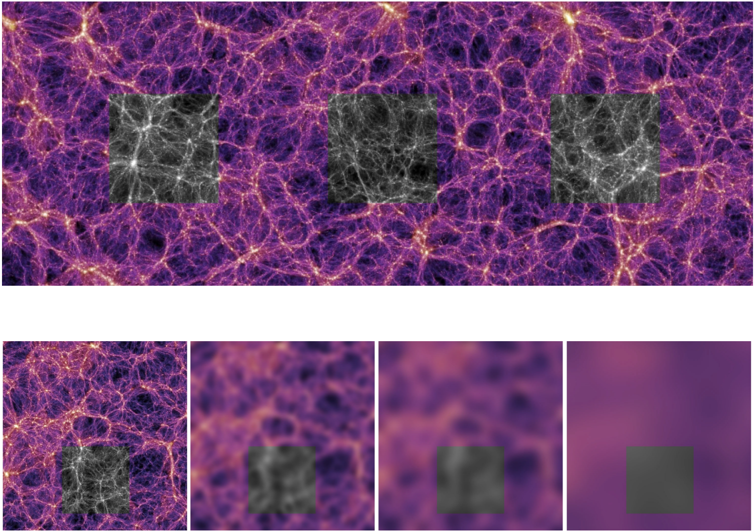
$$R_{\mu\nu}^{(local)} - \frac{1}{2} g_{\mu\nu}^{(local)} R^{(local)} = -k T_{\mu\nu}^{(local)}$$

- Matter distribution of homogeneous universe (on large scale)
  $T_{\mu\nu} = \langle T_{\mu\nu}^{(discrete)} \rangle$

$$T_{\mu\nu}^{(discrete)} \longrightarrow \langle T_{\mu\nu}^{(discrete)} \rangle$$

$$\langle R_{\mu\nu} \rangle - \frac{1}{2} \langle g_{\mu\nu} \rangle \langle g^{\alpha\beta} \rangle \langle R_{\alpha\beta} \rangle + Q_{\mu\nu} = -k \langle T_{\mu\nu}^{(discrete)} \rangle$$

where $Q_{\mu\nu}$ is the backreaction term.



Structure in simulation on the top pannel, and the averaging problem in the bottom row. [C. Clarkson et. al. 2010]

# The Averaging Problem

- The gravitational field equations on cosmological scales are obtained by averaging the Einstein field equations of general relativity.

- Averaging can lead to some effects in GR, because of

non-comutativity with time evolution

$$[\partial_t, \langle \quad \rangle] \neq 0$$

non-linearity of Einstein's equations

$$G_{\mu\nu}(\langle g_{\mu\nu} \rangle) \neq \langle G_{\mu\nu}(g_{\mu\nu}) \rangle$$

# procedures

The averaging involves integration of tensor operation.
Changing the coordinates will change the results in an arbitrary way.

Two procejures to handle this problem:

1- Defining a covriant averaging of tensors, via bilocal operators. Proposed by Zalaletdinov (1993)

2- Using only field equations involving averaged scalars. Proposed by Buchert (2000)

# Approach 1

Zalaletdinov's macroscopic gravity:
Arbitrary tensors can be averaged here.

$$\langle P^a(x)\rangle = \frac{1}{V_\Sigma}\int_\Sigma d^4x'\sqrt{-g'}\mathcal{A}^a_{a'}(x,x')p^{a'}(x')$$

Where $A^a_{a'}(x,x') = e^a_I(x)e^I_{a'}(x')$ is a bilocal operator.

The averaged Einstein equations reduce to Macroscopic Field Equations:

$$\langle g^{\beta\epsilon}\rangle M_{\epsilon\beta} - \frac{1}{2}\delta^\epsilon_\gamma\langle g^{\mu\nu}\rangle M_{\mu\nu} = 8\pi G\langle T^\epsilon_\gamma\rangle + C^\epsilon_\gamma$$

$\langle T^\epsilon_\gamma\rangle$ is the averaged energy-momentum tensor,

$C^\epsilon_\gamma$ is the correction term, given by $C^\epsilon_\gamma = \langle g^{\mu\nu}\rangle(Z^\epsilon_{\mu\nu\gamma} - \frac{1}{2}\delta^\epsilon_\gamma Z^\alpha_{\mu\nu\alpha})$

Where $Z^\alpha_{\mu\nu\beta} = 2Z^\alpha_{\mu\epsilon}{}^\epsilon{}_{\nu\beta}$

Details

# Approach 2

Buchert's spatial averaging of scalars:

The metric can be written in the synchronous gauge

$$ds^2 = dt^2 -^{(3)} g_{\mu\nu} dx^\mu dx^\nu$$

where ${}^{(3)}g_{\mu\nu}$ is the metric on hypersurface of constant $t$.

The spatial average of a scalar quantity $f$

$$\langle f \rangle(t, x) = \frac{1}{V_D} \int_D d^3x f(t, x) \sqrt{-g}$$

The obtained Bucherts equations are

$$3\frac{\ddot{a}_D}{a_D} = -4\pi G_N \langle \rho \rangle + \mathcal{Q}$$

$$3\frac{\dot{a}_D^2}{a_D^2} = 8\pi G_N \langle \rho \rangle - \frac{1}{2}\langle R \rangle - \frac{1}{2}\mathcal{Q}$$

Details

## Brief Overview:

- Problem of covarinat averaging
- Can't average tensor fields in covariant way (coordinate dependent results)

- Can use bitensors for curvature and matter,
  but not for metric itself,
  and leads to complex set of equations
  (Zalaletdinov).

- A less ambitious road: averaging scalars only,
  It seems to retain the main features,
  gives modified Friedmann equations
  (Buchert).

# Introduction

Effects of inhomogeneities on the cosmological expansion:

1. Effect on the dynamics through backreaction or correlation terms.
   - Applying Zalaletdinov or Buchert's approach on Einstein equations and Friedmann equations
     A. Coley et al. (2002), S. Räsänen (2004), E. Kolb (2005), A. Paranjape, T. Singh (2006), N.Li, D. Schwarz (2007)
2. Effect on light propagation.
   - Investigating geodesic equation of motion in an averaged universe
     S. Räsänen (2008), C. Clarkson (2008), A. Coley (2009)

# Geodesic Equation

Considering geodesic equation for a vector field $k^\mu$

$$k^\mu_{,\nu} k^\nu + \Gamma^\mu_{\nu\rho} k^\nu k^\rho = 0$$

$$(k^\mu_{,\nu} g_{\mu\alpha} + k^\mu g_{\mu\alpha,\nu} + k^\mu_{,\alpha} g_{\mu\nu}) k^\nu = 0$$

By averaging the space time, we notice

$$\langle g_{\mu\alpha} \rangle_{,\nu} \neq \langle g_{\mu\alpha,\nu} \rangle$$

In the averaged form we have

$$(k^\mu_{,\nu} \langle g_{\mu\alpha} \rangle + k^\mu_{,\alpha} \langle g_{\mu\nu} \rangle + k^\mu \langle g_{\mu\alpha} \rangle_{,\nu}) k^\nu = (\langle g_{\mu\alpha} \rangle_{,\nu} - \langle g_{\mu\alpha,\nu} \rangle) k^\mu k^\nu$$

The modified term:

$$T_{\mu\alpha\nu} = \langle g_{\mu\alpha} \rangle_{,\nu} - \langle g_{\mu\alpha,\nu} \rangle$$

$$T_{\mu\alpha\nu} k^\mu k^\nu = \mathcal{I}_\alpha$$

# Geodesic Equation

Property (1):
$T^{sym}_{\mu\alpha\nu} = (T_{\mu\alpha\nu} + T_{\nu\alpha\mu})/2$ is a symmetric tensor.

Property (2):
$\mathcal{I}_\alpha k^\alpha = 0.$

Assume Flat Friedmann-Lemaître Model:

- averaged geometry $\langle g_{\mu\nu} \rangle = \bar{g}_{\mu\nu}$
- four-velocity $\bar{u}_\mu = (-1, 0)$
- and frequency $\omega = -\bar{u}_\mu k^\mu$ $\quad (k^0 = \omega$ and $k^i = \omega e^i / a)$

The general algebraic structure for $T^{sym}_{\mu\alpha\nu}$:

$$T^{sym}_{\mu\alpha\nu} = \frac{f_1}{2}(\bar{g}_{\mu\alpha}\bar{u}_\nu + \bar{g}_{\nu\alpha}\bar{u}_\mu) + f_2\bar{u}_\mu\bar{u}_\alpha\bar{u}_\nu + f_3\bar{g}_{\mu\nu}\bar{u}_\alpha$$

$f_1$, $f_2$ and $f_3$ are functions of cosmic time.

$$\mathcal{I}_\alpha = k^\mu T^{\mathrm{sym}}_{\mu\alpha\nu} k^\nu = f_1(-\omega)\bar{g}_{\mu\alpha}k^\mu + f_2\omega^2\bar{u}_\alpha$$

The contraction

$$\mathcal{I}_\alpha k^\alpha = -f_2\omega^3$$

must vanish as shown before. Thus $f_2 \equiv 0$.

Thus the inhomogeneity of the light propagation equation:

$$\mathcal{I}_\alpha = -f_1\omega\bar{g}_{\mu\alpha}k^\mu$$

# Geodesic Equation

- Geodesic equation (background level)

$$(k^{\mu}_{,\nu} g_{\mu\alpha} + k^{\mu} g_{\mu\alpha,\nu} + k^{\mu}_{,\alpha} g_{\mu\nu}) k^{\nu} = 0$$

- Geodesic equation (averaged form)

$$(k^{\mu}_{,\nu} \langle g_{\mu\alpha} \rangle + k^{\mu}_{,\alpha} \langle g_{\mu\nu} \rangle + k^{\mu} \langle g_{\mu\alpha} \rangle_{,\nu}) k^{\nu} = T_{\mu\nu} k^{\mu} k^{\nu} \equiv \mathcal{I}_{\alpha}$$

By considering isotropic homogeneous spatially flat FL universe as the background, we have two sets of equations for $\alpha = 0$ and $\alpha = i$.

# First Set of Equations ($\alpha = 0$)

$$(-2a^2 k^0_{,0} - 2a^2 \mathcal{H} k^0) k^0 + (-a^2 k^0_{,j} + a^2 \delta_{ij} k^i_{,0}) k^j = 0$$

$$(-2a^2 k^0_{,0} - 2a^2 H k^0) k^0 + (-a^2 k^0_{,j} + a^2 \delta_{ij} k^i_{,0}) k^j = T_{000} k^0 k^0 + T_{i0j} k^i k^j$$

where $H = \frac{\dot{a}}{a}$.

Averaged geodesic equation (time component)

$$(-\omega)(\dot{\omega} + \omega_{,i} e^i / a + H\omega) = \mathcal{I}_0 = f_1 \omega^2$$

The equation in terms of the affine parameter $\frac{d\omega}{d\lambda} = k^\mu \frac{\partial \omega}{\partial x^\mu}$:

$$\frac{d\omega}{d\lambda} + H_{eff} \omega^2 = 0$$

$H_{eff} \equiv H + f_1$ is the effective Hubble rate.

# First Set of Equations ($\alpha = 0$)

$$(-2a^2 k^0_{,0} - 2a^2 \mathcal{H} k^0) k^0 + (-a^2 k^0_{,j} + a^2 \delta_{ij} k^i_{,0}) k^j = 0$$

$$(-2a^2 k^0_{,0} - 2a^2 H k^0) k^0 + (-a^2 k^0_{,j} + a^2 \delta_{ij} k^i_{,0}) k^j = T_{000} k^0 k^0 + T_{i0j} k^i k^j$$

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$H_{eff} \equiv H + f_1$ is the effective Hubble rate.

# Second Set of Equations ($\alpha = i$)

$$k^j_{,0} k^0 \bar{g}_{ij} + k^j_{,l} k^l \bar{g}_{ij} + k^0_{,i} k^0 \bar{g}_{00} + k^j_{,i} k^l \bar{g}_{jl} + k^j k^0 \bar{g}_{ij,0} + k^j k^l \bar{g}_{ij,l} = 0 and \neq 0$$

$$a^2 (k^j_{,0} k^0 \delta_{ij} + k^j_{,l} k^l \delta_{ij} + k^j_{,i} k^l \delta_{jl} + 2Hk^j k^0 \delta_{ij} - k^0_{,i} k^0) = 2T_{ij0} k^0 k^j$$

Modified light propagation equation (spatial components)

$$a\gamma_{ij} e^j \omega(\dot{\omega} + \frac{e^i}{a}\omega_{,i} + H\omega) + a\omega^2 \gamma_{ij}(\dot{e}^j + e^j_{|k} e^k) = \mathcal{I}_i = -\omega^2 a \gamma_{ij} e^j f_1$$

where $\bar{g}_{ij} = a^2 \gamma_{ij}$,
and $|$ denotes a covariant derivative with respect to the 3-metric $\gamma_{ij}$.

$$\dot{e}^j + e^j_{|k} e^k = 0$$

$$\frac{De^i}{D\lambda} = 0$$

i.e. light rays propagate alonge straight lines.

# Estimation for $f_1(t)$

$$ds^2 = -e^{2\phi}dt^2 + a^2(t)e^{-2\psi}\gamma_{ij}dx^i dx^j$$

$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$, by construction $\langle \delta g_{\mu\nu} \rangle = 0$

$$T_{000}^{\mathrm{sym}} = -\langle \delta g_{00,0} \rangle = 2\langle e^{2\phi}\dot{\phi} \rangle$$

$$f_1 = 2\langle e^{2\phi}\dot{\phi} \rangle$$

$\dot{\phi} \neq 0$ (ISW effect) $\rightarrow \langle e^{2\phi}\dot{\phi} \rangle \neq 0$.

$$H_{\mathrm{eff}} = H + 2\langle e^{\phi}\dot{\phi} \rangle$$

Following $\delta(\mathbf{r}, t) \equiv \frac{\rho(\mathbf{r},t) - \bar{\rho}(t)}{\bar{\rho}(t)}$

- For an over-dense region $(\delta > 0) \Longrightarrow \dot{\phi} < 0 \quad \rightsquigarrow H_{eff} < H$.
- For an underdense region $(\delta < 0) \Longrightarrow \dot{\phi} > 0 \quad \rightsquigarrow H_{eff} > H$.

# Summary

- A cosmological solution obtained by averaging underlying inhomogeneous Universe. Buchert's and Zalaletdinov's averaging approaches, leads to space time corrections to the standard cosmological equations.

- The aim of our work was to investigate the effects of averaging on light propagation. Our central result is a modification of null geodesic equation, which is a fully covariant vector equation.

- By applying this light propagation equation to a flat, spatially isotropic and homogeneous averaged metric (compatible with the standard model of cosmology), This modification can be expressed as an effective Hubble rate

- So far we studied a single light ray. The next logical step is to study the equation of geodesic deviation to find a modification to the luminosity and angular diameter distances.

# Macroscopic Gravity (MG)

## Zalaletdinov's macroscopic gravity:

Arbitrary tensors can be averaged here.

$$\langle P^a(x) \rangle = \frac{1}{V_\Sigma} \int_\Sigma d^4x' \sqrt{-g'} \mathcal{A}^a_{a'}(x, x') p^{a'}(x')$$

Where $A^a_{a'}(x, x') = e^a_l(x) e^l_{a'}(x')$ is a bilocal operator.

- The averaged of the connection $\Gamma^\mu_{\rho\nu}$ on $M$ is by definition taken to be the connection $\langle \Gamma^\mu_{\rho\nu} \rangle$ of the averaged manifold $\bar{M}$.
- The averaged metric $\langle g_{\mu\nu} \rangle$ can be chosen as the metric on averaged manifold.

The macroscopic Riemann tensor is $M^{\mu}_{\nu\alpha\beta}$

$$M^{\mu}_{\nu\alpha\beta} = \partial_{\alpha}\langle\Gamma^{\mu}_{\nu\beta}\rangle - \partial_{\beta}\langle\Gamma^{\mu}_{\nu\alpha}\rangle + \langle\Gamma^{\mu}_{\sigma\alpha}\rangle\langle\Gamma^{\sigma}_{\nu\beta}\rangle - \langle\Gamma^{\mu}_{\sigma\beta}\rangle\langle\Gamma^{\sigma}_{\nu\alpha}\rangle$$

$M^{\mu}_{\nu\alpha\beta} \neq \langle R^{\mu}_{\nu\alpha\beta}\rangle$ where $R^{\mu}_{\nu\alpha\beta}$ is the average of the microscopic Riemann tensor.

connection correlation tensor defined by

$$Z^{\alpha}_{\beta[\gamma}{}^{\mu}{}_{\underline{\nu}\sigma]} = \langle\Gamma^{\alpha}_{\beta[\gamma}\Gamma^{\mu}_{\underline{\nu}\sigma]}\rangle - \langle\Gamma^{\alpha}_{\beta[\gamma}\rangle\langle\Gamma^{\mu}_{\underline{\nu}\sigma]}\rangle$$

The averaged Einstein equations reduce to Macroscopic Field Equations:

$$\langle g^{\beta\epsilon}\rangle M_{\epsilon\beta} - \frac{1}{2}\delta^{\epsilon}_{\gamma}\langle g^{\mu\nu}\rangle M_{\mu\nu} = 8\pi G\langle T^{\epsilon}_{\gamma}\rangle + C^{\epsilon}_{\gamma}$$

$\langle T^{\epsilon}_{\gamma}\rangle$ is the averaged energy-momentum tensor,

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Where $Z^{\alpha}_{\mu\nu\beta} = 2Z^{\alpha}_{\mu\epsilon}{}^{\epsilon}{}_{\nu\beta}$

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# Buchert's spatial averaging approach

## Buchert's spatial averaging of scalars:

The metric can be written in the synchronous gauge

$$ds^2 = dt^2 - ^{(3)} g_{\mu\nu} dx^\mu dx^\nu$$

where $^{(3)}g_{\mu\nu}$ is the metric on hypersurface of constant $t$.

The spatial average of a scalar quantity $f$

$$\langle f \rangle(t, x) = \frac{1}{V_D} \int_D d^3x f(t, x) \sqrt{-g}$$

where $V_D = \int_D d^3x \sqrt{-g}$.
The starting point is expansion and averaging do not commut:

$$\partial_t \langle f \rangle_D - \langle \partial_t f \rangle_D = \langle \Theta f \rangle_D - \langle \Theta \rangle_D \langle f \rangle_D$$

where $\Theta$ is a expansion rate.

$$\Theta^\mu_\nu = \frac{1}{2} g^{\mu\rho} \partial_t g_{\rho\nu}; \quad \Theta = \Theta^\mu_\mu$$

The averaged scale factor is defined as $a_D \propto V_D^{\frac{1}{3}}$,
which ensures $\langle \Theta \rangle_D = 3\partial_t ln(a_D) = 3H_D$.

The obtained Bucherts equations are

$$3\frac{\ddot{a}_D}{a_D} = -4\pi G_N \langle \rho \rangle + \mathcal{Q}$$

$$3\frac{\dot{a}_D^2}{a_D^2} = 8\pi G_N \langle \rho \rangle - \frac{1}{2}\langle R \rangle - \frac{1}{2}\mathcal{Q}$$

where $\mathcal{Q}$ is the backreaction term

$$\mathcal{Q} = \frac{2}{3}[\langle \Theta^2 \rangle_D - \langle \Theta \rangle_D^2] - 2\langle \sigma^2 \rangle_D$$

and $\sigma$ is a shear scalar $\sigma^\mu_\nu = \Theta^\mu_\nu - \frac{1}{3}\delta^\mu_\nu \Theta; \quad \sigma^2 = \frac{1}{2}\sigma^\mu_\nu \sigma^\nu_\mu$.

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