Late time quantum backreaction in cosmology

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DG, T. Prokopec, V. Prymidis
PRD **89** (2014) 024024 [arXiv:1308.5954 [gr-qc]]

DG, T. Prokopec, D. van der Woude
[arXiv:1408.????]

Motivation

- All matter is quantum and quantum fluctuations generally carry some energy
- Gravitation is sourced by all energy
- Quantum-corrected Einstein equation:

$$G_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu}^{(cl)} + \left\langle T_{\mu\nu}^{(Q)} \right\rangle \right) \tag{1}$$

- Possible secular (time-dependent) effects in cosmology
- What are these effects? (Dark Energy)

Theoretical setting

- Semiclassical equation – numerical problem
- Initially backreaction is small (quantum effect), but maybe it builds up over time
- There must be a regime where it can be treated as a perturbation $\rightarrow$ study test fields living on FLRW
- Where perturbation becomes of order one compared to the background interesting things are to be expected
- Cannot predict what these effects are without solving self-consistently, but it is possible to see where to look for them (if any)

FLRW spacetime

- FLRW:

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2 = a^2(\eta)[-d\eta^2 + d\boldsymbol{x}^2], \quad dt = ad\eta \tag{2}$$

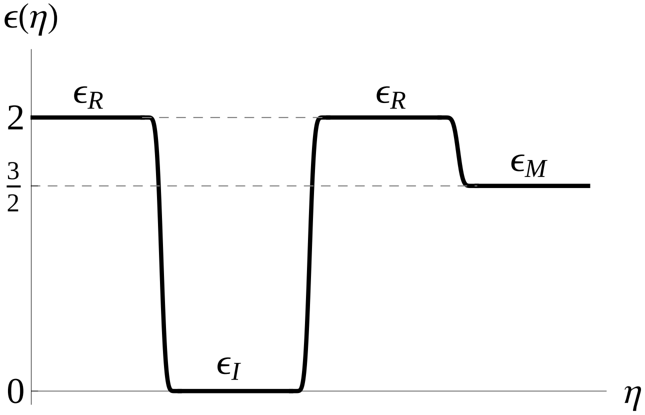
- Hubble rate: $H = \frac{\dot{a}}{a}$, conformal Hubble rate: $\mathcal{H} = \frac{a'}{a} = aH$.
- Friedmann equations:

$$\left(\frac{\mathcal{H}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i\,, \quad \frac{\mathcal{H}' - \mathcal{H}^2}{a^2} = -4\pi G \sum_i (\rho_i + p_i) \tag{3}$$

- Ideal fluids: $p_i = w_i \rho_i$, $\rho_i \sim a^{-3(1+w_i)}$
- Dominance of one fluid $(w) \rightarrow$ constant ϵ parameter

$$\epsilon = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} = \frac{3}{2}(1+w) \tag{4}$$

History of the Universe



Initial radiation period – IR regulator
(Ford, Parker PRD **16** (1977) 245)

Quantization of a scalar field on FLRW

$$S = \int d^D x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{\xi}{2} R \phi^2 \right] \tag{5}$$

- Equation of motion:

$$\phi'' + (D-2)\mathcal{H}\phi - \nabla^2 \phi + \xi (D-1)[2\mathcal{H}' + (D-2)\mathcal{H}^2]\phi = 0 \tag{6}$$

- Commutation relations:

$$\pi = a^{D-2}\phi' \quad , \quad [\hat{\phi}(\eta, \boldsymbol{x}), \hat{\pi}(\eta, \boldsymbol{y})] = i\delta^D(\boldsymbol{x} - \boldsymbol{y}) \tag{7}$$

- Expansion in Fourier modes:

$$\phi(\eta, \boldsymbol{x}) = a^{\frac{2-D}{2}} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \left[e^{i\boldsymbol{k}\cdot\boldsymbol{x}} U(k, \eta) \hat{b}(\boldsymbol{k}) + h.c. \right] \tag{8}$$

$$[\hat{b}(\boldsymbol{k}), \hat{b}^\dagger(\boldsymbol{q})] = (2\pi)^{D-1} \delta^{D-1}(\boldsymbol{k} - \boldsymbol{q}) \tag{9}$$

- Vacuum: $b(\boldsymbol{k})|\Omega\rangle = 0$

Mode function

- Equation of motion

$$\boxed{U'' + [k^2 + f(\eta)]U(k,\eta) = 0} \tag{10}$$

$$f(\eta) = -\tfrac{1}{4}[D-2-4\xi(D-1)][2\mathcal{H}' + (D-2)\mathcal{H}^2] \tag{11}$$

- Constant ϵ period Bunch-Davies mode functions:

$$u(k) = \sqrt{\frac{\pi}{4|\epsilon-1|\mathcal{H}}}\, H_\nu^{(2)}\left(\frac{k}{|\epsilon-1|\mathcal{H}}\right) \tag{12}$$

$$\nu^2 = \frac{1}{4} + \frac{D-2\epsilon}{4(1-\epsilon)^2}[D-2-4\xi(D-1)] \tag{13}$$

- Full solution:

$$U(k,\eta) = \alpha(k)u(\eta,k) + \beta(k)u^*(\eta,k) \tag{14}$$

- Bogolyubov coefficients:

$$|\alpha|^2 - |\beta|^2 = 1 \tag{15}$$

Energy-momentum tensor

- Operator:

$$\hat{T}_{\mu\nu} = \partial_\mu \hat{\phi} \partial_\nu \hat{\phi} - \frac{1}{2} g_{\mu\nu} \partial^\mu \hat{\phi} \partial_\mu \hat{\phi} + \xi \Big[G_{\mu\nu} - \nabla_\mu \nabla_\nu + g_{\mu\nu} \Box \Big] \hat{\phi}^2 \tag{16}$$

- Energy density and pressure:

$$\rho_Q = \frac{a^{-D}}{(4\pi)^{\frac{D-1}{2}} \Gamma(\frac{D-1}{2})} \int_0^\infty dk\, k^{D-2} \Bigg\{ 2k^2 |U|^2 + \frac{1}{2} \frac{\partial^2}{\partial \eta^2} |U|^2 - \frac{1}{2} [D - 2 - 4\xi(D-1)] \Big(\mathcal{H}' + \mathcal{H} \frac{\partial}{\partial \eta} \Big) |U|^2 \Bigg\} \tag{17}$$

(dimensional regularization and renormalization with R^2 counterterms)

- Equation of state parameter $w_Q = p_Q / \rho_Q$

Minimally coupled scalar

- DG, Prokopec, Prymidis: PRD **89** (2014) 024024 [arXiv:1308.5954 [gr-qc]]
- B-D initial state in inflation
- Fast transitions between cosmological periods – instantaneous transitions good approximation only in the IR (not in UV)
- Bogolyubov coefficients for an instantaneous transition at η_1 between period of constant ϵ_1 to period of constant ϵ_2

$$\alpha = -i\Big[U_1(\eta_1)u_2^{*\prime}(\eta_1) - U_1'(\eta_1)u_2^*(\eta_1)\Big] \tag{18}$$

$$\beta = -i\Big[U_1(\eta_1)u_2{}'(\eta_1) - U_1'(\eta_1)u_2(\eta_1)\Big] \tag{19}$$

- In matter era backreaction scales just as non-relativistic matter

$$\frac{\rho_Q}{\rho_B} \sim 10^{-13} \tag{20}$$

- Aoki, Iso, Sekino [arXiv:1403.1392 [quant-ph]] – confirm result, possible to change the amplitude by adding pre-inflationary periods, but not the scaling

Non-minimally coupled scalar

- DG, Prokopec, van der Woude [arXiv:1408:????]
- Cannot solve EMT integrals explicitly
- Split integrand into UV-divergent part (Bunch-Davies) + UV-finite part (contains Bogolyubov coefficients):

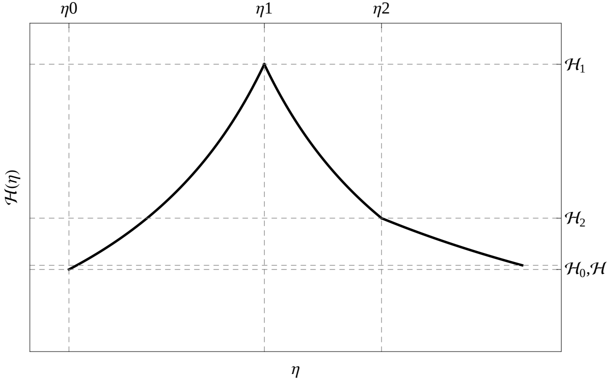
$$|U|^2 = |u|^2 + 2|\beta|^2|u|^2 + \alpha\beta^* u^2 + \alpha^*\beta u^{*2} \tag{21}$$

- Calculate only UV-finite part ($\mu \gg \mathcal{H}$)

$$\int_0^\infty dk\, k^n |\beta|^2 |u|^2 = \left(\cancel{\int_\mu^\infty} + \int_0^\mu \right) dk\, k^n |\beta(k/\mathcal{H}_i)|^2 |u(k/\mathcal{H})|^2 \tag{22}$$

- IR part well approximated by sudden transitions – expand in appropriate ratios of Hubble rates and isolate cut-off independent leading contributions

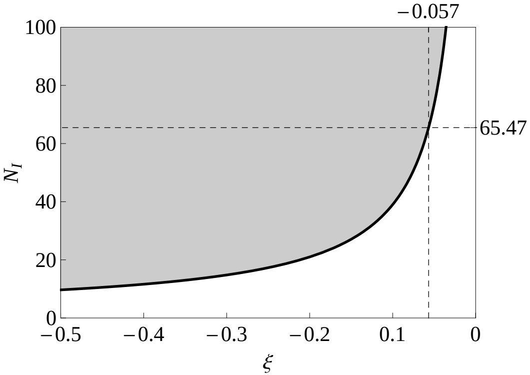
Hierarchy of scales



$\mathcal{H}_0, \mathcal{H} \ll \mu \ll \mathcal{H}_2 \ll \mathcal{H}_1 \quad \Rightarrow \quad$ expand in $k/\mathcal{H}_1$ and $k/\mathcal{H}_2$

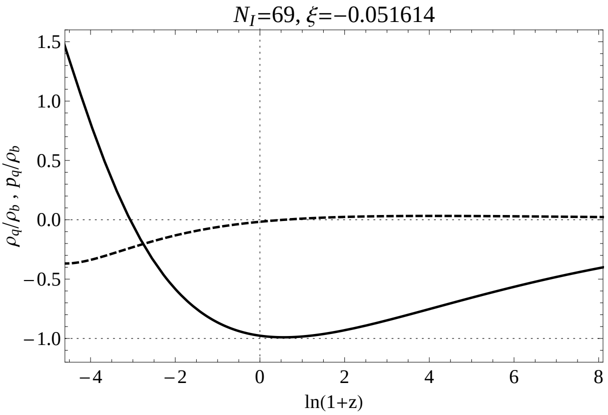
Backreaction during inflation ($\epsilon_I \approx 0.01$)

- Janssen, Prokopec: PRD **83** (2011) 084035, [arXiv:0906.0666 [gr-qc]]
- Backreaction should not start dominating in inflation



Backreaction during matter era

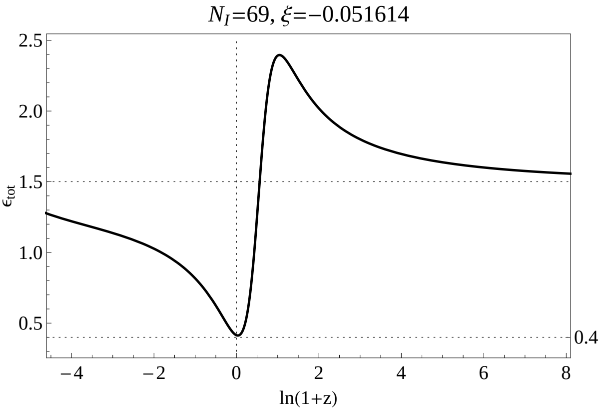
- Generally transient effects around $\mathcal{H} \sim \mathcal{H}_0$ (from negative to positive energy)



- For these values: $\rho_Q/\rho_B \sim 0.4$ at the end of inflation, and constant $\rho_Q/\rho_B \sim 0.5$ during radiation.

Total (quantum corrected) ϵ parameter

$$\epsilon_{tot} = \frac{3}{2}\left[1 + \frac{p_Q}{\rho_B + \rho_Q}\right] \tag{23}$$



Not to be taken too seriously as prediction!

Conclusions and outlook

- Backreaction from quantum fluctuations can indeed become important
- Backreaction grows during inflation for $\xi < 0$ (dynamical screening of cosmological constant), and is constant during radiation
- Possibly interesting transient phenomena for $\mathcal{H} \sim \mathcal{H}_0$ that naively looks like dark energy – self-consistent solution to semiclassical equations required! (problem for numerics)
- Slow transitions?
- Any other quantum fields?