New observables for future CMB anisotropy observations

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http://cosmologist.info/

Two things that should definitely be non-zero:

1. CMB bispectrum
2. non-Blackbody spectrum

University of Sussex

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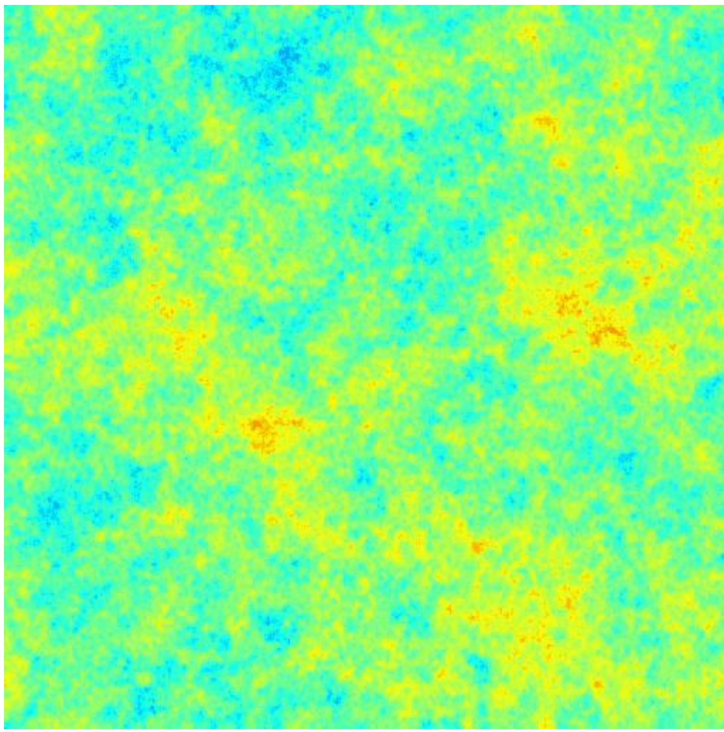
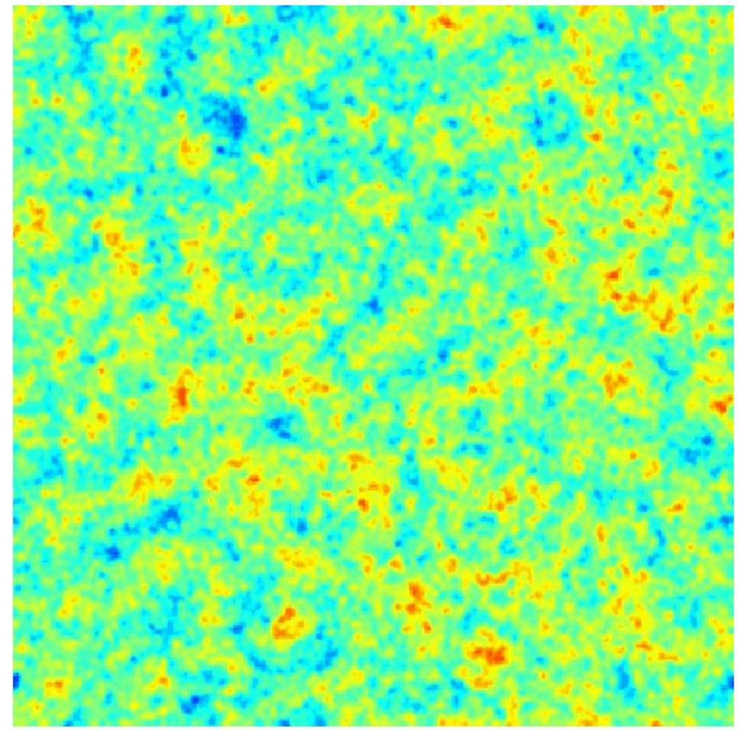
CMB temperature

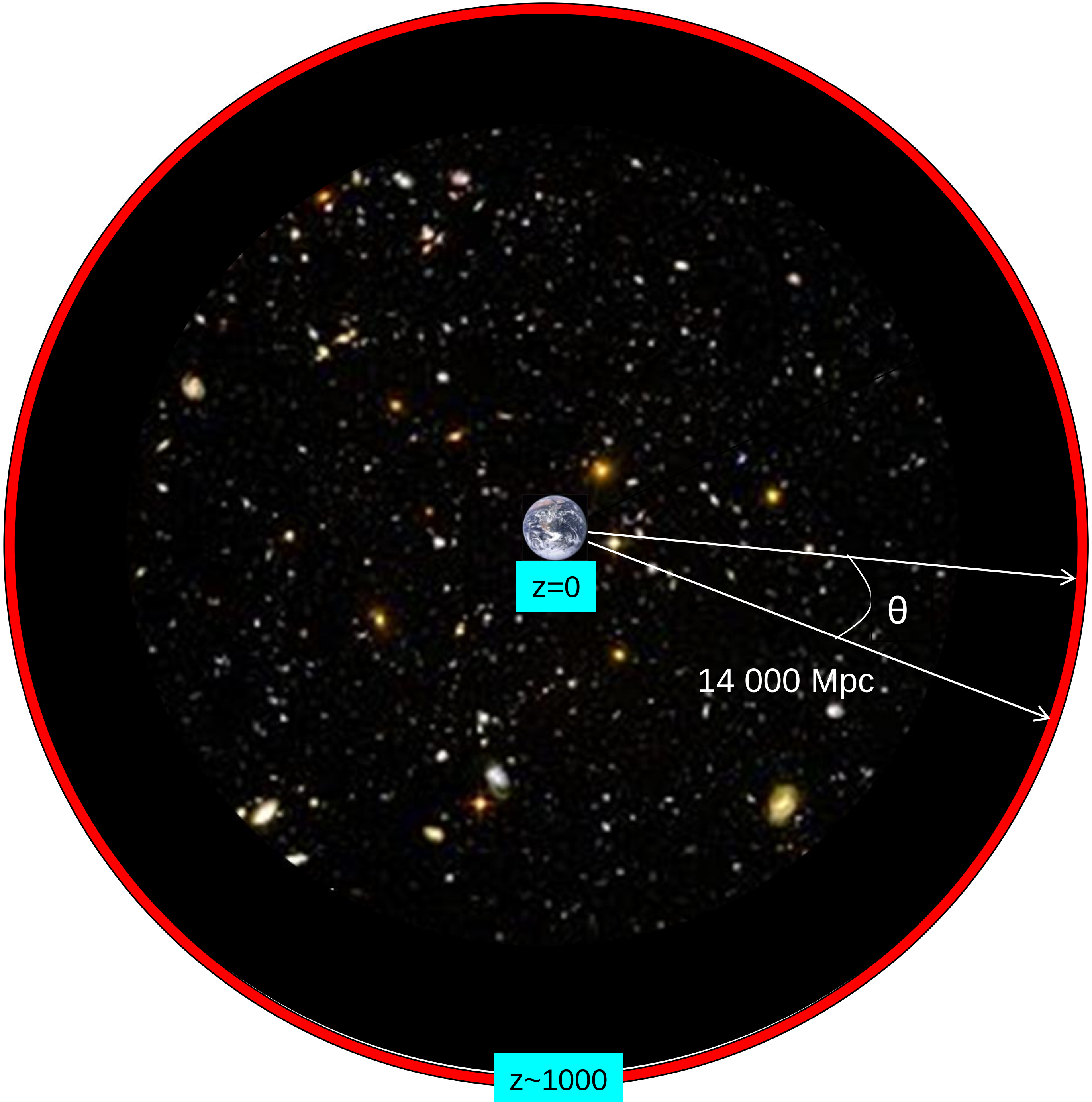
End of inflation



gravity+
pressure+
diffusion

Last scattering surface

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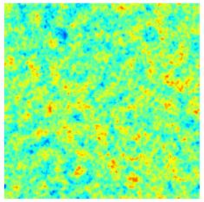
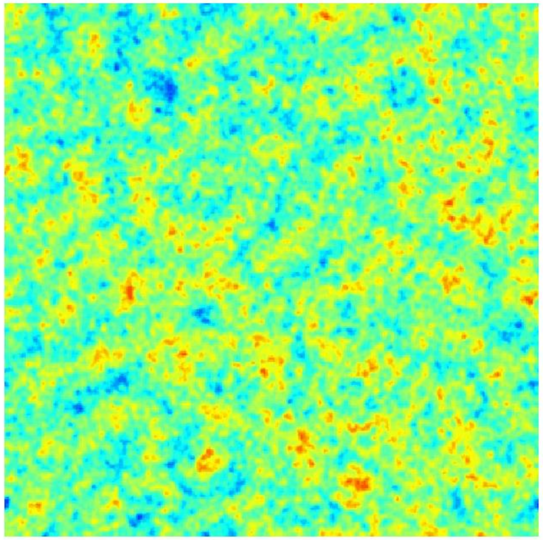
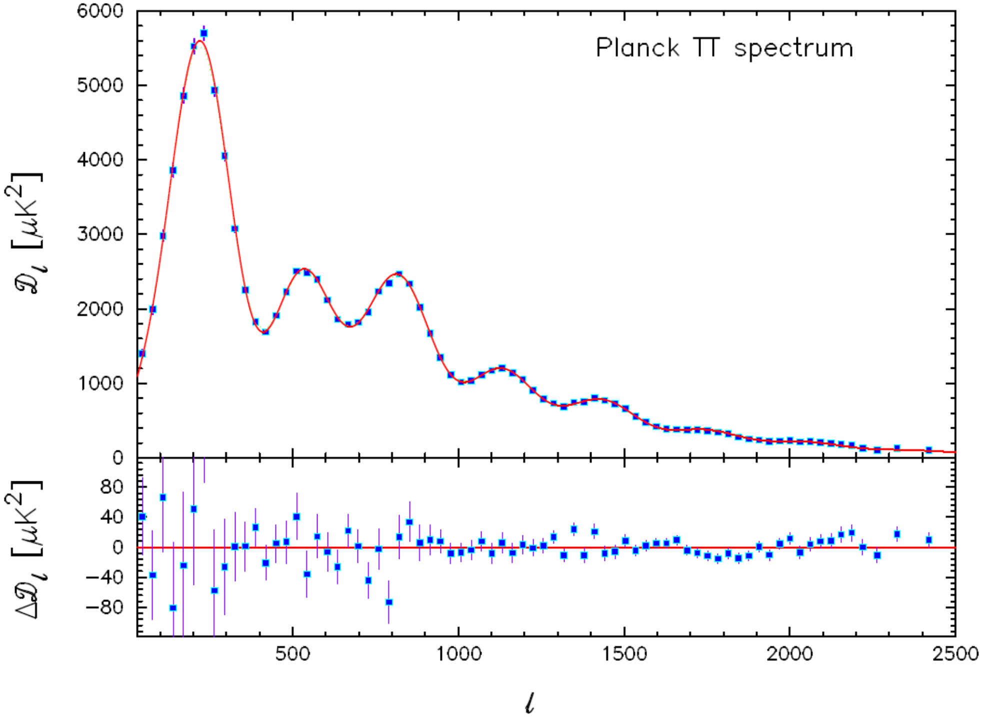


Observed CMB blackbody power spectrum





Observations

Constrain theory of early universe
+ evolution parameters and geometry

1. Beyond Gaussianity – general possibilities

Flat sky approximation: $\Theta(x) = \frac{1}{2\pi} \int d^2 l \, \Theta(l) e^{ix \cdot l}$ $(\Theta = T)$

Gaussian + statistical isotropy

$$\langle \Theta(l_1) \Theta(l_2) \rangle = \delta(l_1 + l_2) C_l$$

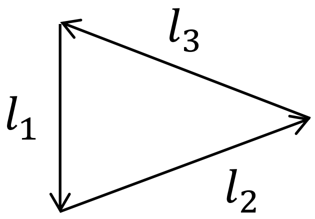
- power spectrum encodes all the information
- modes with different wavenumber are independent

Higher-point correlations

Gaussian: can be written in terms of C_l

Non-Gaussian: non-zero connected n-point functions

Bispectrum



$$\boldsymbol{l_1} + \boldsymbol{l_2} + \boldsymbol{l_3} = \mathbf{0}$$

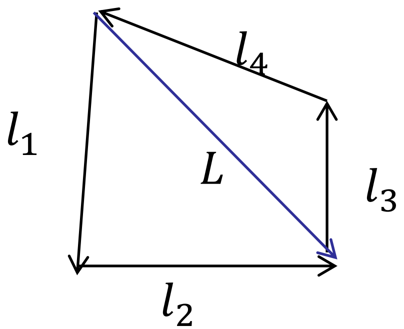
Flat sky approximation: $\langle \Theta(l_1)\Theta(l_2)\Theta(l_3)\rangle = \frac{1}{2\pi}\delta(l_1 + l_2 + l_3) b_{l_1 l_2 l_3}$

If you know $\Theta(l_1), \Theta(l_2)$, sign of $b_{l_1 l_2 l_3}$ tells you which sign of $\Theta(l_3)$ is more likely

Trispectrum

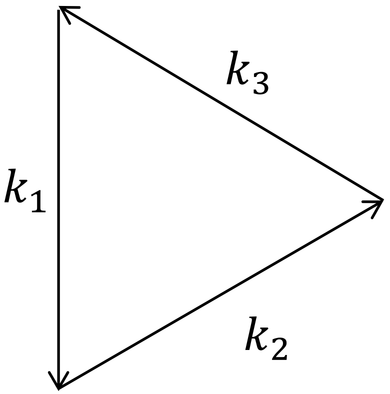
$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4)\rangle_C = (2\pi)^{-2}\delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{l}_3 + \mathbf{l}_4) T(\mathbf{l}_1, \mathbf{l}_2, \mathbf{l}_3, \mathbf{l}_4)$$

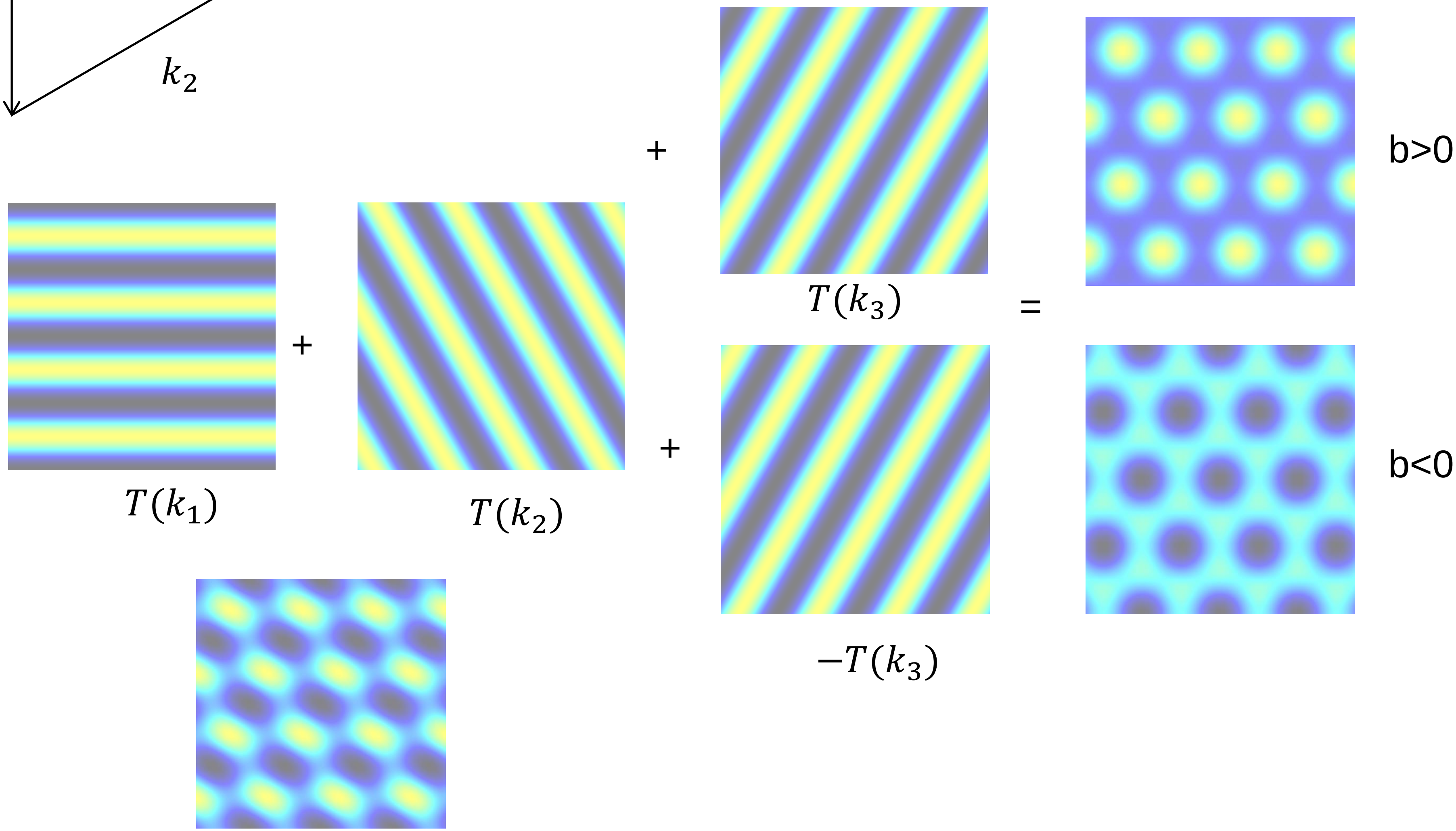
$$\langle \Theta(\mathbf{l}_1)\Theta(\mathbf{l}_2)\Theta(\mathbf{l}_3)\Theta(\mathbf{l}_4)\rangle_C = \frac{1}{2}\int \frac{d^2\mathbf{L}}{(2\pi)^2}\delta(\mathbf{l}_1 + \mathbf{l}_2 + \mathbf{L})\delta(\mathbf{l}_3 + \mathbf{l}_4 - \mathbf{L})\mathbb{T}^{(\ell_1\ell_2)}_{(\ell_3\ell_4)}(L) + \text{perms.}$$



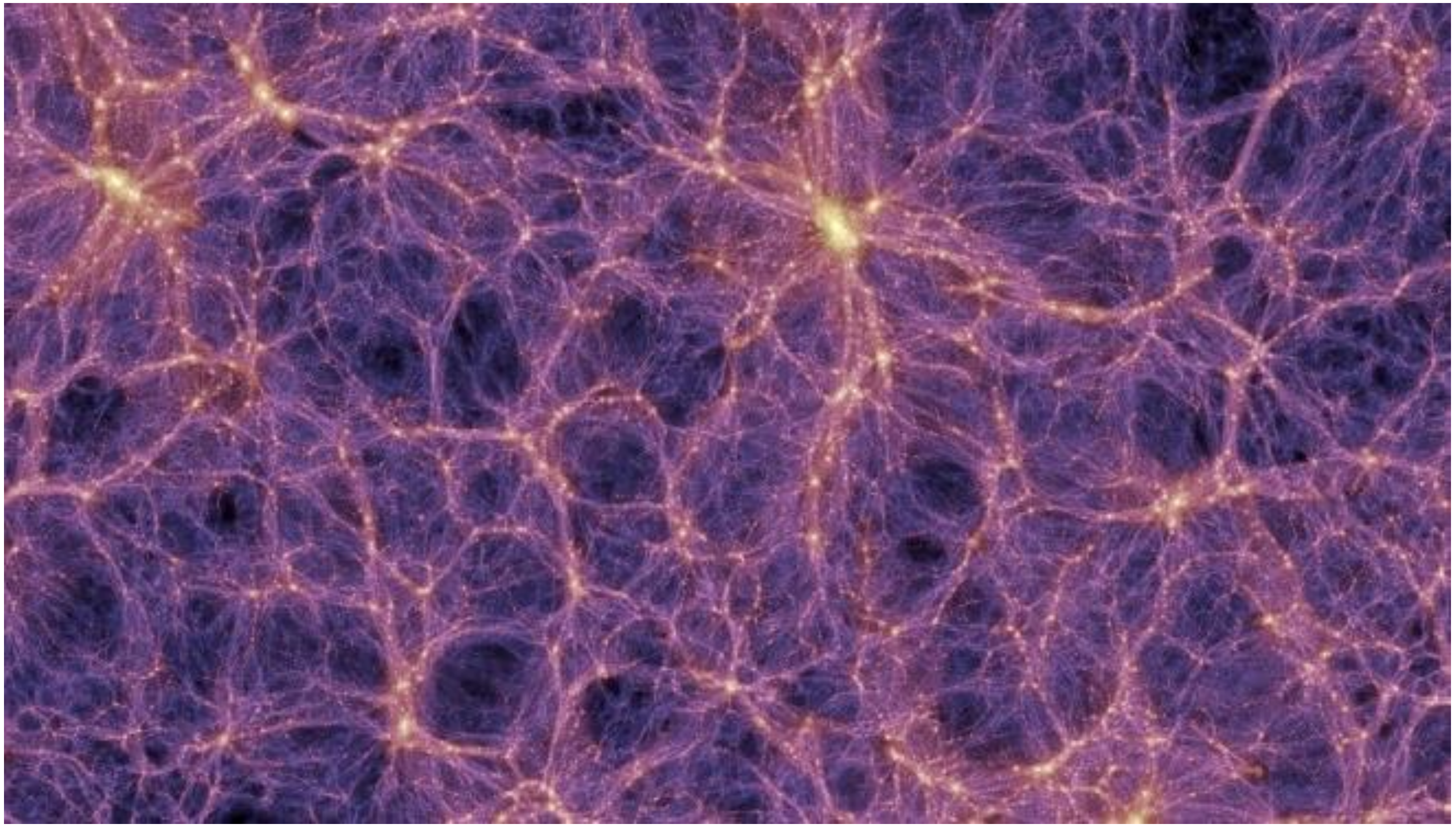
N-spectra…

Equilateral $k_1 + k_2 + k_3 = 0, |k_1| = |k_2| = |k_3|$



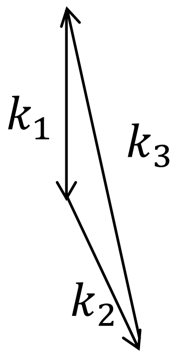


AL: The Real Shape of Non-Gaussianities, arXiv:1107.5431

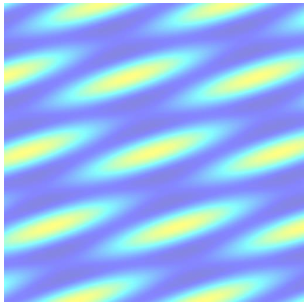


Millennium simulation

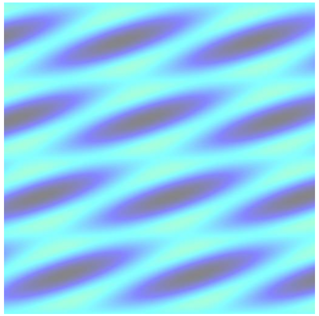
Near-equilateral to flattened:



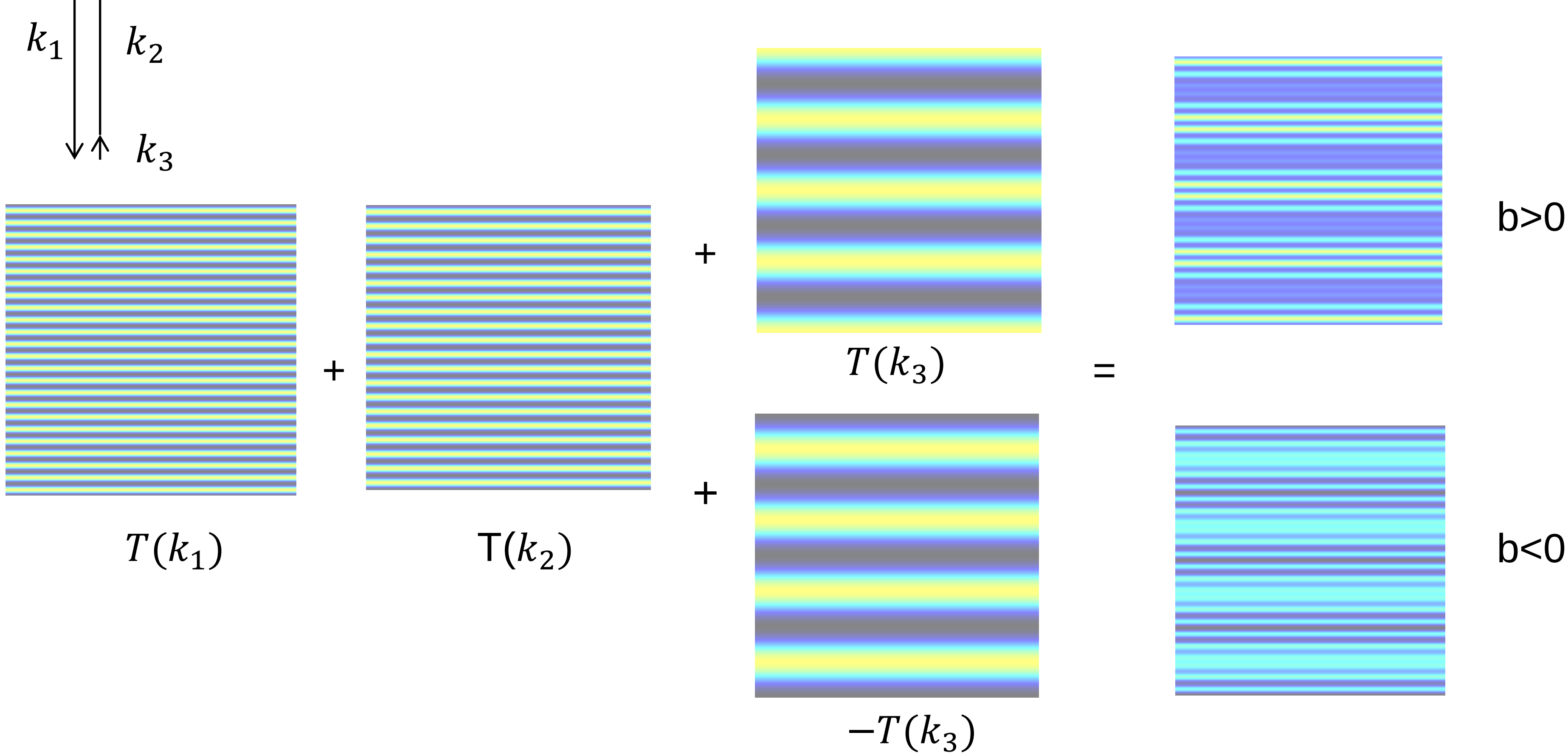
b>0



b<0



Local (squeezed) $k_1 + k_2 + k_3 = 0, \quad k_1 \ll k_2, k_3 \quad k_2 \sim -k_3$

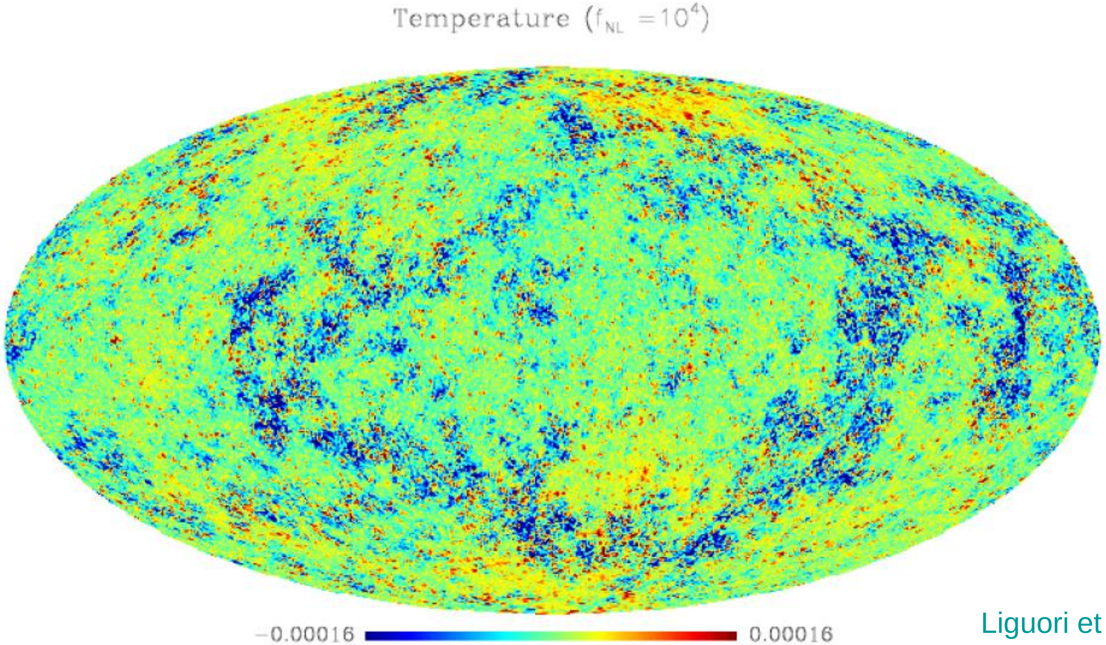


Squeezed bispectrum is a *correlation* of small-scale power with large-scale modes

Primordial local non-Gaussianity

e.g. $\zeta = \zeta_0 \left(1 + \frac{6}{5} f_{NL} \zeta_{0,l}\right)$

$$\Rightarrow T \sim T_g (1 + \frac{6}{5} f_{NL} \zeta_{*,l})$$

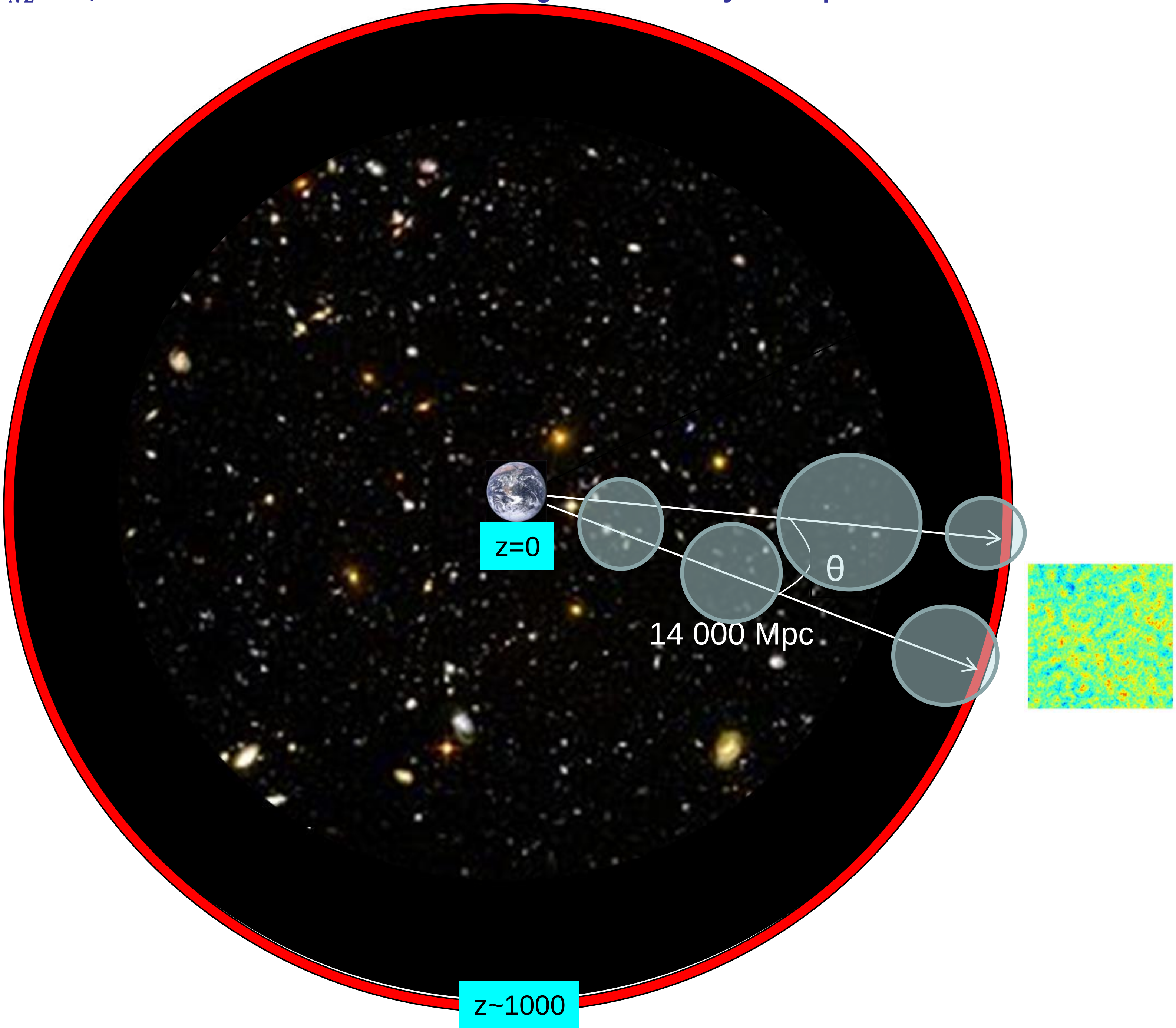


Liguori et al 2007

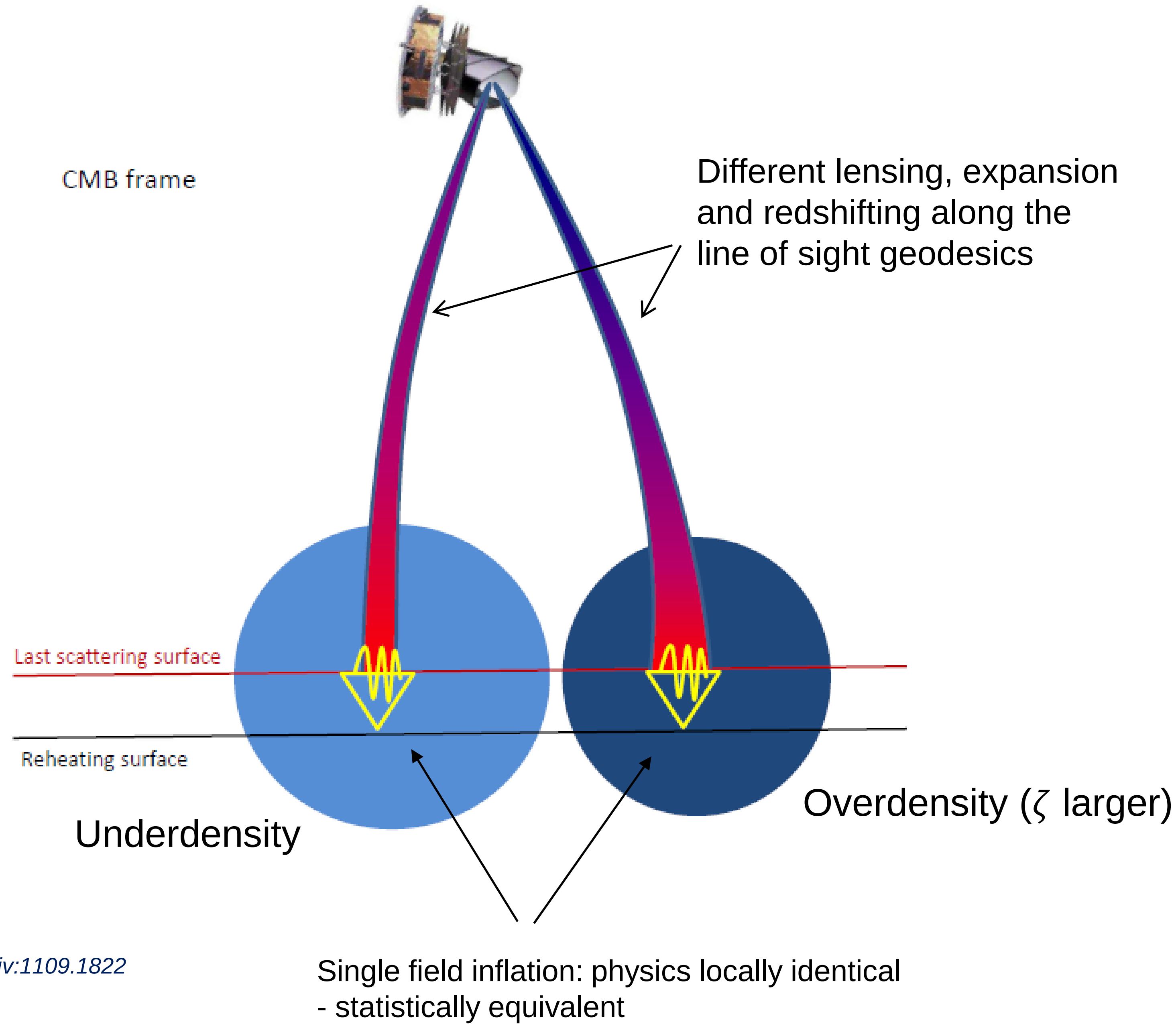
Single-field slow-roll inflation: $f_{NL} \sim 0$

$\Rightarrow$ Any significant detection would rule out large classes of inflation models

But even with $f_{NL} = 0$, we observe CMB at last scattering modulated by other perturbations



Last-scattering is modulated by the large-scale perturbations



arXiv:1204.5018

Also: astro-ph/0405428, arXiv:1109.1822

⇒ observable CMB bispectrum even with single-field inflation

Linear-short leg approximation for nearly-squeezed shapes ($l_1 < l_2, l_3$):

$$\langle \tilde{T}_{l_1 m_1} \tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \rangle \approx C_{l_1}^{T X_i} \left\langle \frac{\delta}{\delta X^*_{i, l_1 m_1}} \left(\tilde{T}_{l_2 m_2} \tilde{T}_{l_3 m_3} \right) \right\rangle$$

Weyl lensing bispectrum – from correlation of T with lensing via ISW

$$b_{l_1 l_2 l_3} = \frac{1}{2} \left[\left(l_1(l_1+1) + l_2(l_2+1) - l_3(l_3+1) \right) \right] C_{l_1}^{T\psi} \tilde{C}_{l_2} \text{ + perms}$$

Squeezed limit ($l_1 \ll l$)

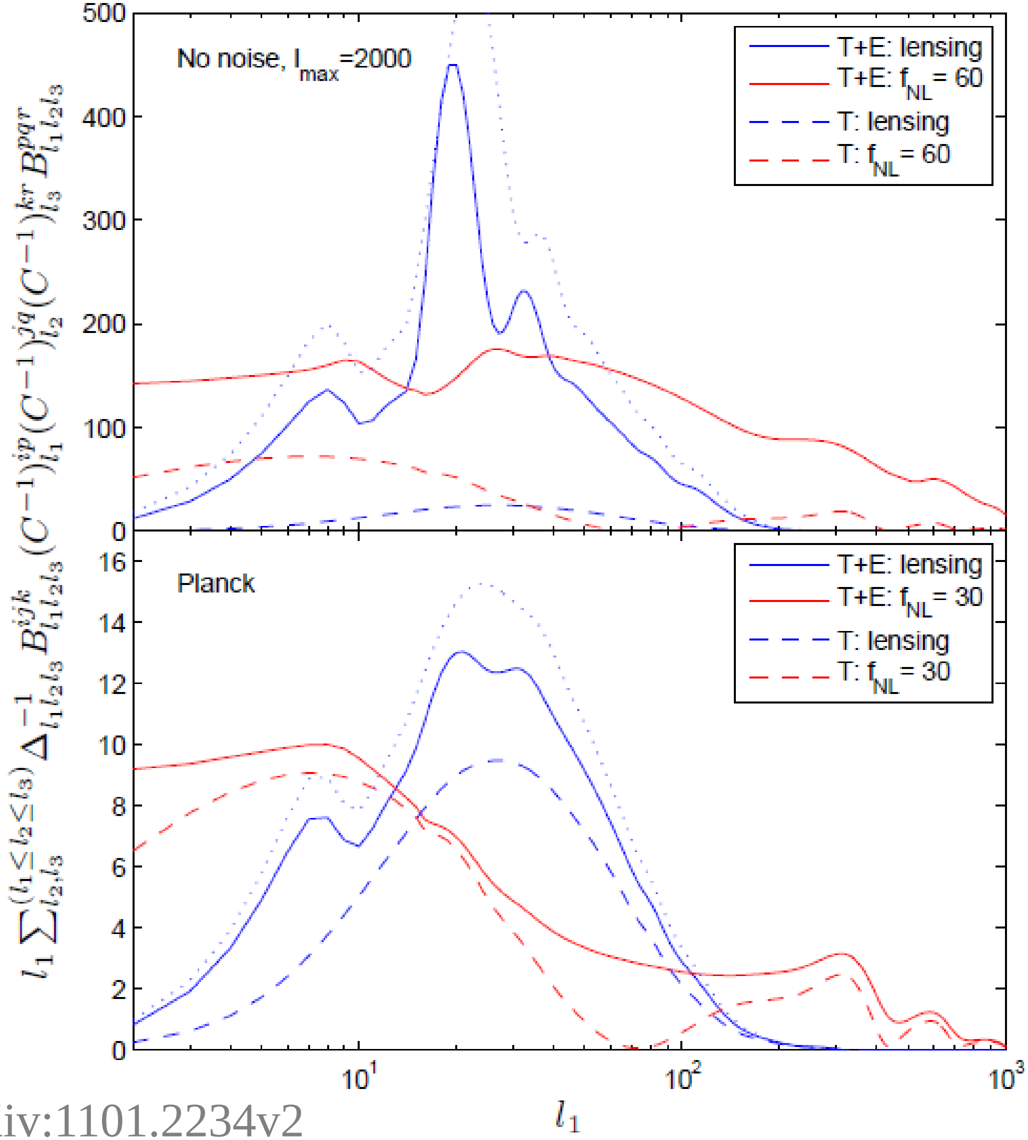
$$b_{l_1 l_2 l_3} \approx C_{l_1}^{T\kappa} \left[\frac{1}{l^2} \frac{\mathrm{d}(l^2 \tilde{C}_l)}{d \ln l} + \cos 2\phi_{l_1 l} \frac{\mathrm{d}\tilde{C}_l}{\mathrm{d} \ln l} \right]$$

$$\mathbf{l} \equiv (\mathbf{l}_2 - \mathbf{l}_3)/2$$

Similar result for polarization

Lensing signal-to-noise variance as a function of the largest-scale mode

(T+E; $l_{\max} = 2000$; $\sigma = 1/\sqrt{\text{area under curve}}$)



Perfect Planck $\sim 5\sigma$;
Cosmic Variance $\sim 9\sigma$

Already detected by Planck TT at $\sim 2.5\sigma$
arXiv:1303.5077

arXiv:1101.2234v2

Other effects of super-horizon modes:

Ricci focussing (anisotropic dilation) bispectrum

Squeezed limit ($l_1 \ll l$) $$b_{l_1 l_2 l_3} \approx -C_{l_1}^{T\zeta_0^*} \frac{1}{l^2} \frac{\mathrm{d}}{\mathrm{d}\ln l}(l^2 \tilde{C}_l)$$

Anisotropic redshifting bispectrum

$$b_{l_1 l_2 l_3} \approx C_{l_1} \left(\tilde{C}_{l_2} + \tilde{C}_{l_3} \right)$$

Allowing for polarization ($X, Y, Z = T, E$), total is

$$b^{\mathrm{sq},XYZ}_{\ell_1 \ell_2 \ell_3} = -\frac{1}{2} C^{X\zeta}_{\ell_1} \left[\frac{\mathrm{d}\,({\ell_2}^2\, C^{YZ}_{\ell_2})}{\ell_2\, \mathrm{d}\,\ell_2} + \frac{\mathrm{d}\,({\ell_3}^2\, C^{YZ}_{\ell_3})}{\ell_3\, \mathrm{d}\,\ell_3} \right]$$

$$+ C^{XT}_{\ell_1} \left[\delta_{ZT}\, C^{YT}_{\ell_2} + \delta_{YT}\, C^{ZT}_{\ell_3} \right] ,$$

This term dominates:
depends on peakiness
of C_l: boost from polarization

What about sub-horizon effects? Need full second-order Boltzmann code

Impact of polarisation on the intrinsic CMB bispectrum

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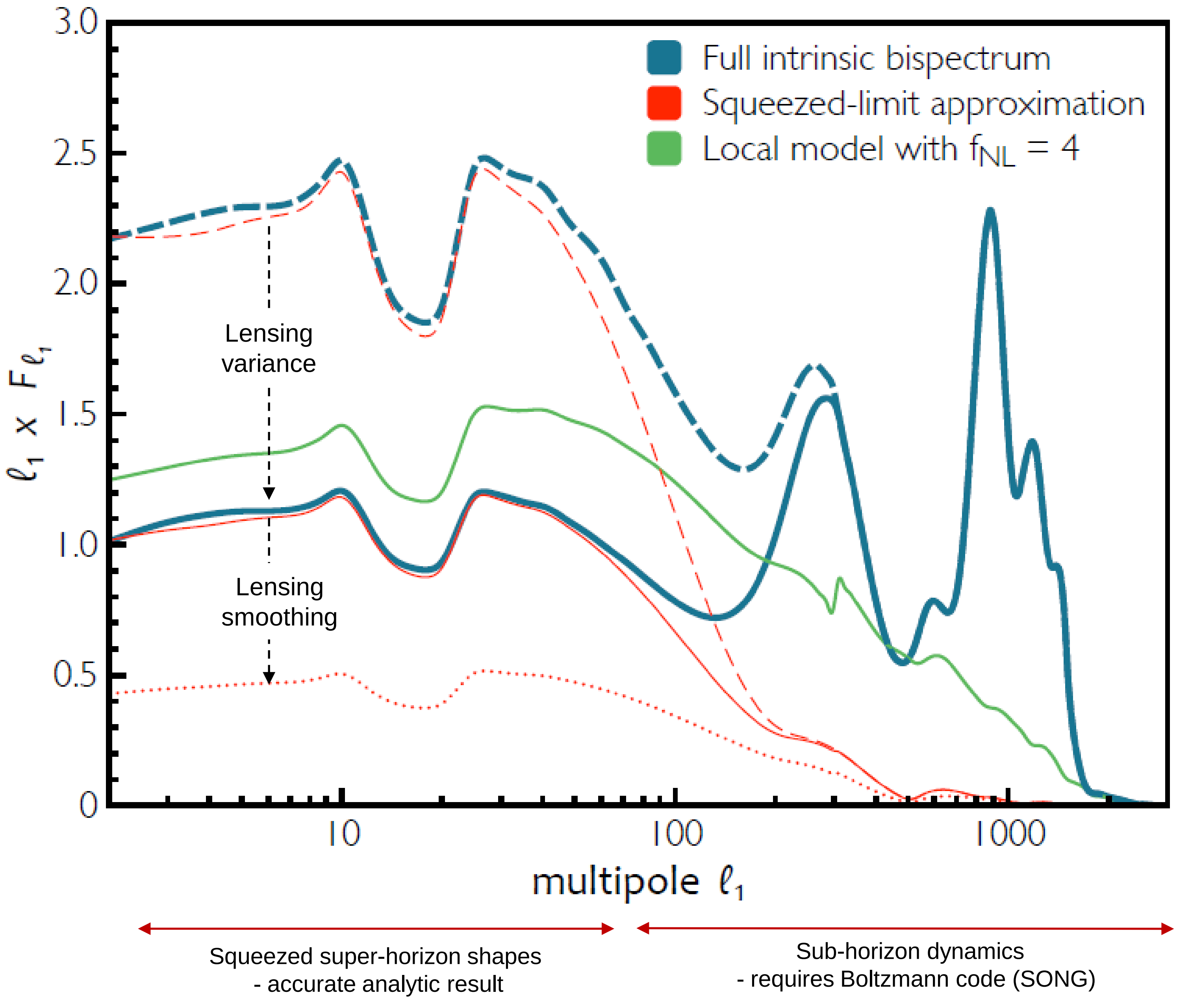
(Dated: June 12, 2014)

We compute the bispectrum induced in the cosmic microwave background (CMB) temperature and polarisation by the evolution of the primordial density perturbations using the second-order Boltzmann code SONG. We show that adding polarisation increases the signal-to-noise ratio by a factor four with respect to temperature alone and we estimate the observability of this intrinsic bispectrum and the bias it induces on measurements of primordial non-Gaussianity. When including all physical effects except the late-time non-linear evolution, we find for the intrinsic bispectrum a signal-to-noise of $S/N = 3.8$, 2.9, 1.6 and 0.5 for, respectively, an ideal experiment with an angular resolution of $\ell_{\rm max} = 3000$, the proposed CMB surveys PRISM and COrE, and Planck's polarised data; the bulk of this signal comes from the E-polarisation and from squeezed configurations. We discuss how CMB lensing is expected to reduce these estimates as it suppresses the bispectrum for squeezed configurations and contributes to the noise in the estimator. We find that the presence of the intrinsic bispectrum will bias a measurement of primordial non-Gaussianity of local type by $f_{\rm NL}^{\rm intr} = 0.66$ for an ideal experiment with $\ell_{\rm max} = 3000$. Finally, we verify the robustness of our results by reproducing the analytical approximation for the squeezed-limit bispectrum in the general polarised case.

arXiv:1406.2981

Intrinsic signal-to-noise variance as a function of the largest-scale mode

(ideal experiment, $l_{\max} = 3000$, T+E; $\sigma = 1/\sqrt{\text{area under curve}}$)



Lensing bispectrum is detectable at about 5σ

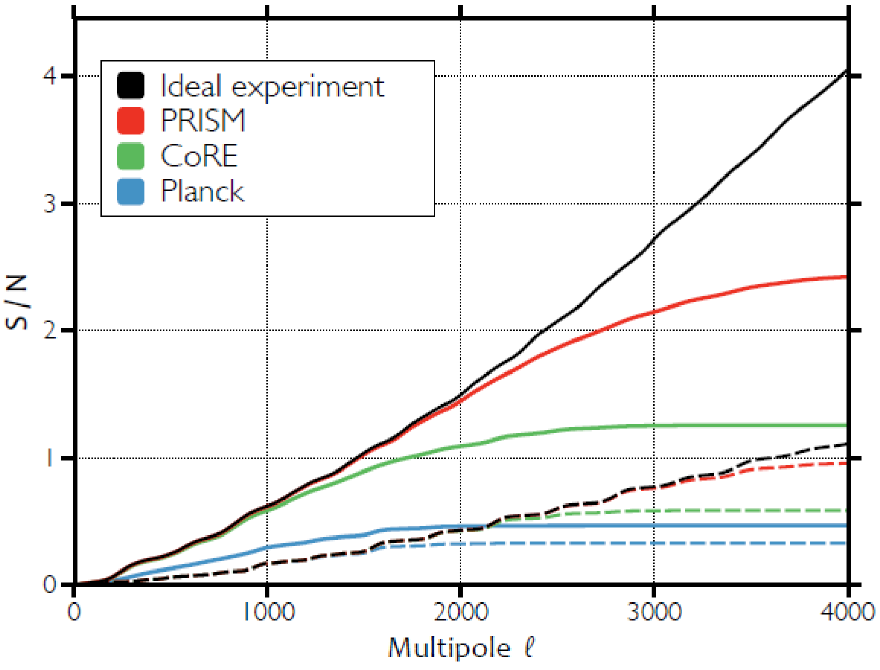
Is the intrinsic (non-lensing) bispectrum also observable?

Ideal experiment, straight 2-nd order calculation with no other NG: 3.8σ - YES!

BUT

Accounting for lensing variance: 2.7σ

+ accounting for lensing smoothing (lensing of intrinsic bispectrum): $\sim 2\sigma$



(includes lensing variance only)

Not a huge signal, but definitely an important bias for any similar shapes

For local f_{NL} bias is 0.4-0.6, small but not negligible

2. Rayleigh scattering
blue sky thinking for future CMB observations

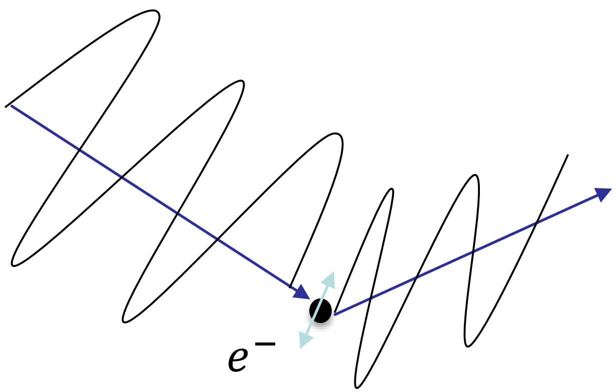
arXiv:1307.8148; previous work: Takahara et al. 91, Yu, et al. astro-ph/0103149

Classical dipole scattering

Oscillating dipole $\boldsymbol{p} = p_0 \sin(\omega t)\, \hat{\boldsymbol{z}} \Rightarrow$ radiated power $\propto \omega^4 p_0^2 \sin^2\theta\, \mathrm{d}\Omega$

Thomson Scattering

$$m_e \ddot{z} = -eE_z \sin\omega t$$
$$\Rightarrow \boldsymbol{p} = \frac{-e^2 E_z}{m_e \omega^2} \sin\omega t\ \hat{\boldsymbol{z}}$$



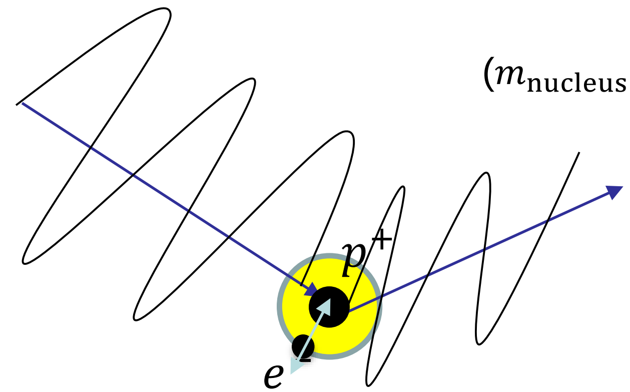
Frequency independent
Given by fundamental constants:

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)^2$$

Rayleigh Scattering

$$m_e \ddot{z} = -eE_z \sin\omega t - m_e \omega_0 z$$
$$\Rightarrow \boldsymbol{p} = \frac{-e^2 E_z}{m_e(\omega^2 - \omega_0^2)} \sin\omega t\ \hat{\boldsymbol{z}}$$

$(m_{\mathrm{nucleus}} \gg m_e)$



Frequency dependent
Depends on natural frequency ω_0 of target

$$\sigma_R \approx \frac{\omega^4}{\omega_0^4} \sigma_T \qquad (\omega \ll \omega_0)$$

http://farside.ph.utexas.edu/teaching/em/lectures/node95.html

Photon scattering rate

Total cross section ≈ $\Gamma(\nu) = n_e\sigma_T + \sigma_R(\nu)\left[n_H + R_{He}n_{He}\right]$

$$\sigma_R(\nu) = \left[\left(\frac{\nu}{\nu_{\rm eff}}\right)^4 + \frac{638}{243}\left(\frac{\nu}{\nu_{\rm eff}}\right)^6 + \dots\right]\sigma_T$$

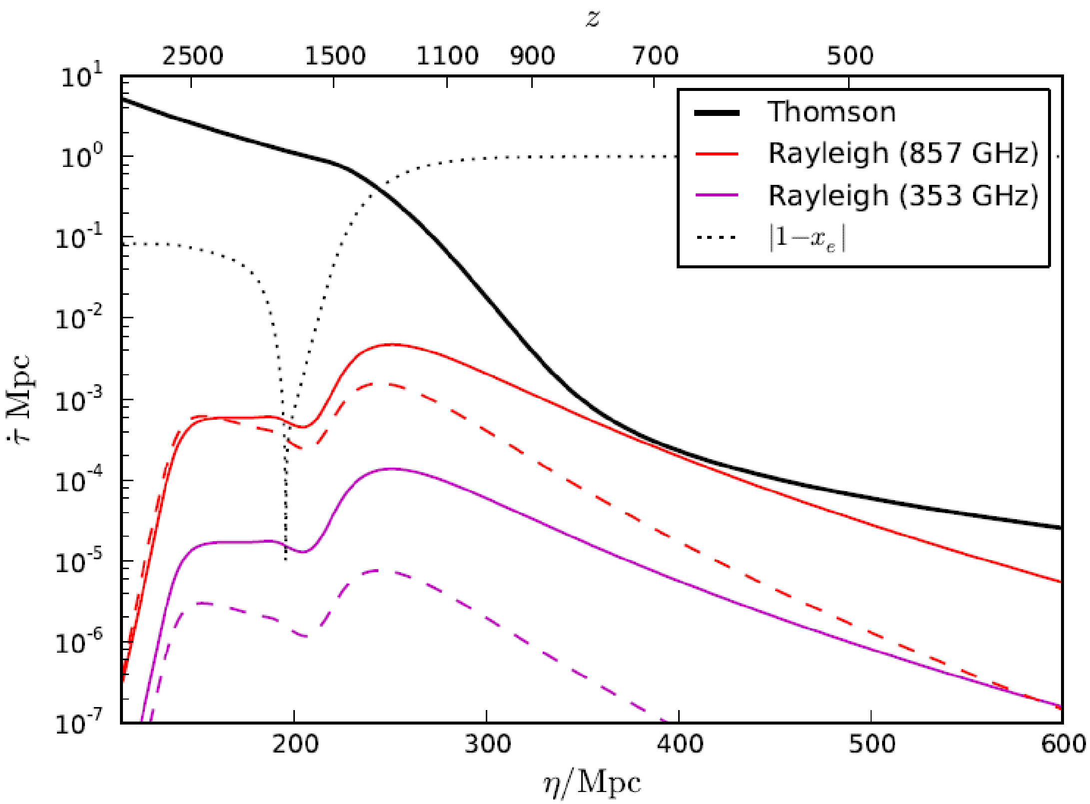
$$\nu_{\rm eff} \equiv \sqrt{\frac{8}{9}}\, c\, R_A \approx 3.1\times 10^6 \text{GHz} \quad , R_{He} \approx 0.1$$

(Lee 2005: Non-relativistic quantum calculation, for energies well below Lyman-alpha)

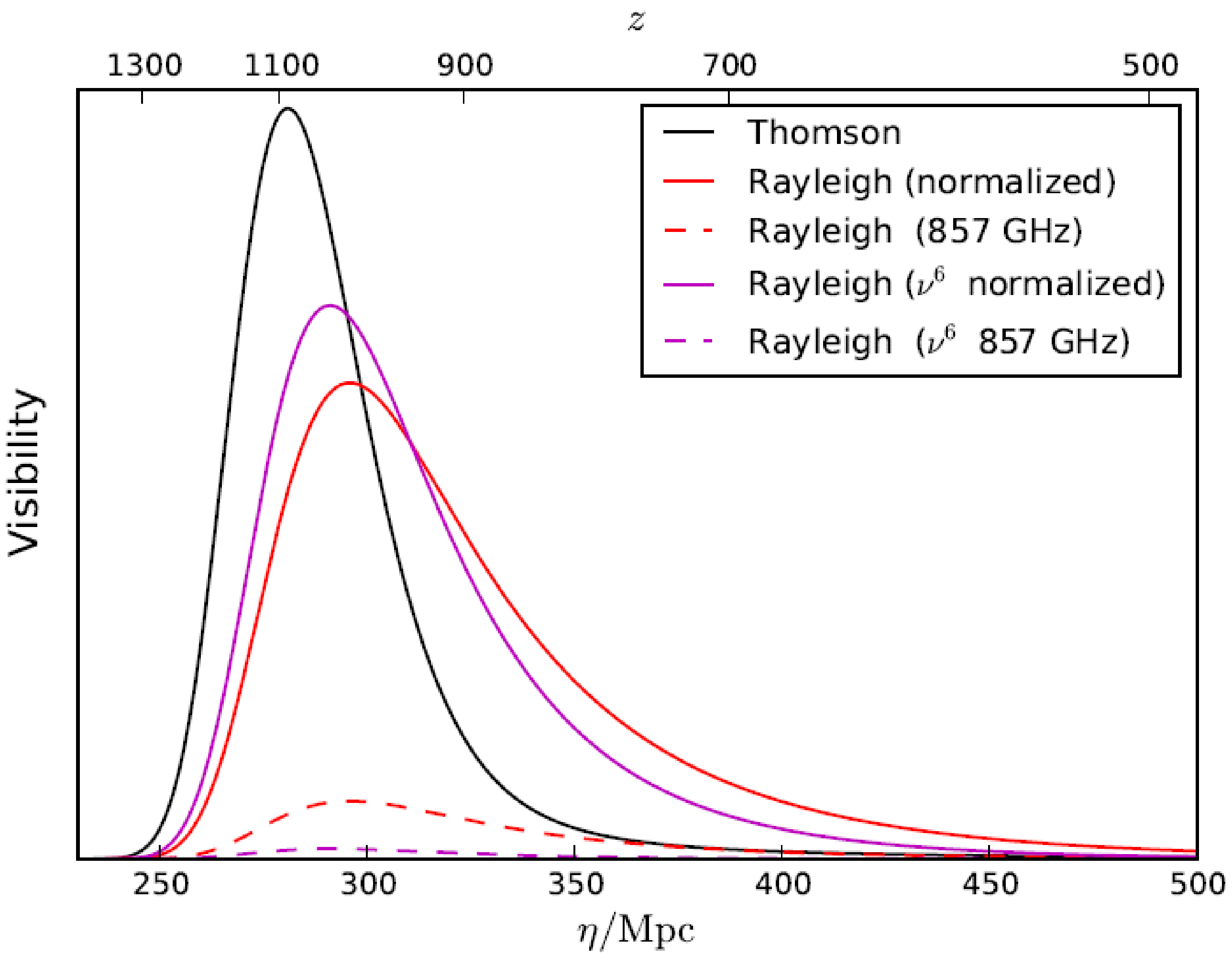
$$\dot{\tau} = \Gamma/(1+z)$$

Rayleigh only negligible compared to Thomson for

$$n_H\left(\frac{(1+z)\nu_{\rm obs}}{3\times 10^6\,\text{GHz}}\right)^4 \ll n_e$$



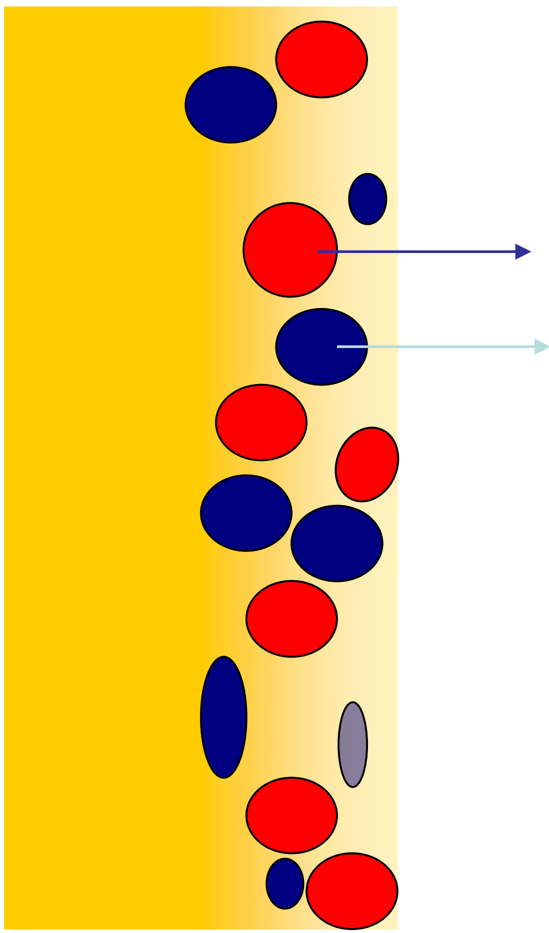
Visibility



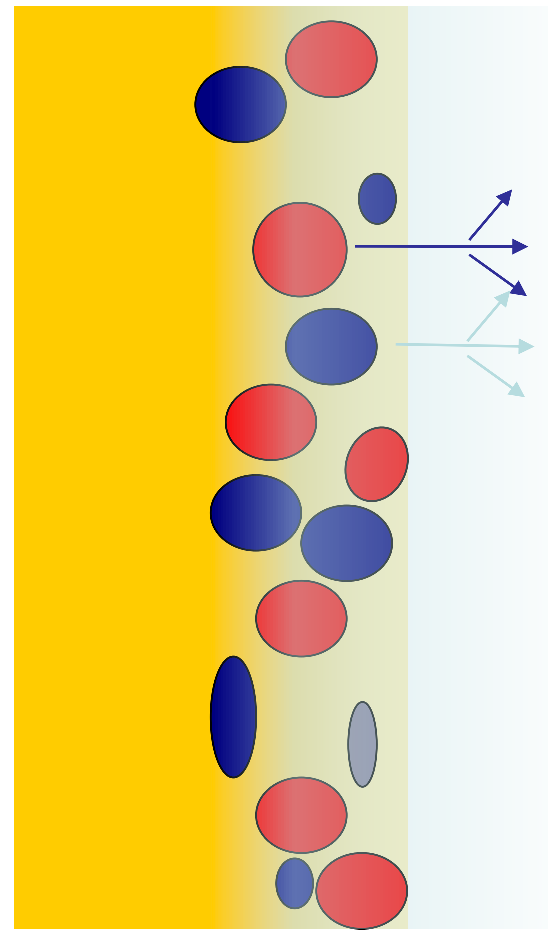
z
1300
1100
900
700
500
Thomson
Rayleigh (normalized)
Rayleigh (857 GHz)
Rayleigh (ν^6 normalized)
Rayleigh (ν^6 857 GHz)
Visibility
250
300
350
400
450
500
η/Mpc

Small-scale CMB

Primary signal

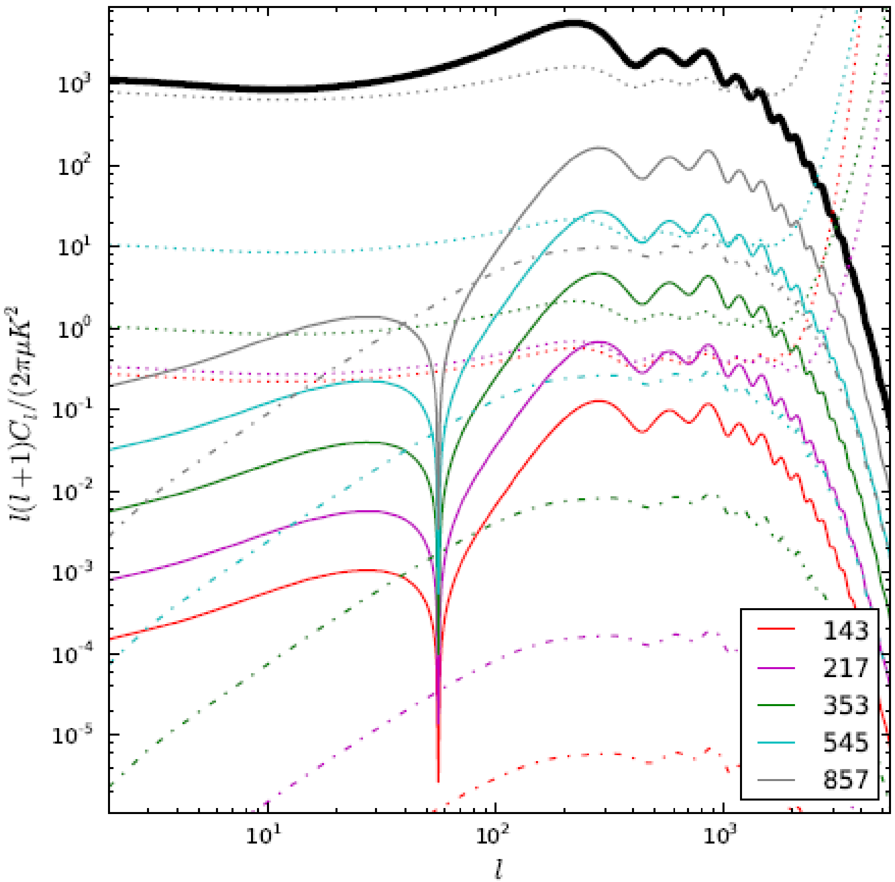


Primary + Rayleigh signal



Rayleigh temperature power spectrum

$$(\text{Primary+Rayleigh})^2 = \text{Primary}^2 + 2\ \text{Primary} \times \text{Rayleigh} + \text{Rayleigh}^2$$



Solid: Rayleigh × Primary
Dot-dashed: Rayleigh × Rayleigh
Dots: naïve Planck sensitivity

Small-scale signal is highly correlated to primary

Can hope to isolate using
Low frequency × High frequency

Note: not limited by cosmic variance of primary anisotropy
– multi-tracer probe of same underlying perturbation realization

Test of expansion and ionization history at recombination

Small-scale CMB cont.

Hot spots are red, cold spots are blue

Polarization is scattered and is red too

Rayleigh difference signal: (photons scattered in to line of sight) – (scattered out)

$\sim \tau_R \, \Delta T$ ⇨ very correlated to primary ΔT

Large-scale CMB temperature

Rayleigh signal only generated by sub-horizon scattering
(no Rayleigh monopole background to distort by anisotropic photon redshifting)

$$\frac{\Delta T_0}{T}(\hat{\mathrm{n}}) = \underbrace{\frac{\Delta_\gamma(\eta_*)}{4}}_{} + \underbrace{\Psi(\eta_*) - \Psi_0}_{\text{Sachs-Wolfe}} + \underbrace{\hat{\mathrm{n}} \cdot (\mathrm{v}_o - \mathrm{v})}_{\text{Doppler}} + \underbrace{\int_{\eta_*}^{\eta_0} d\eta (\Psi' + \Phi')}_{\text{ISW}}$$

Temperature perturbation at recombination
(Newtonian Gauge)

Sachs-Wolfe

Doppler

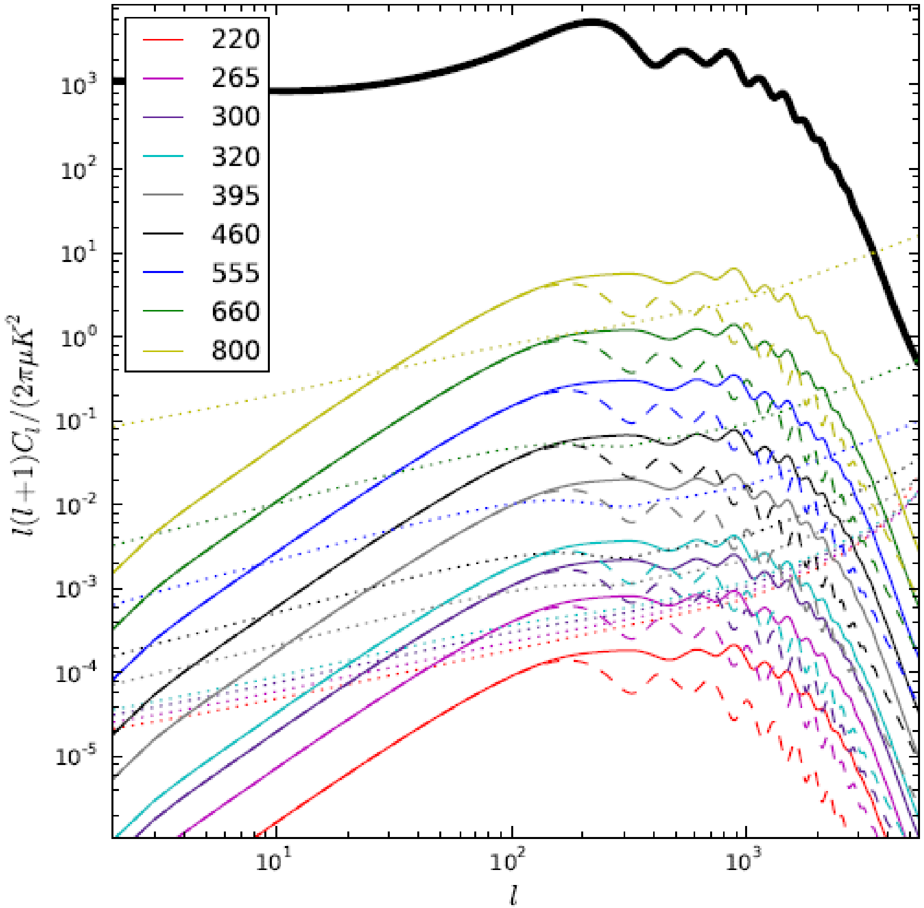
ISW

⇨ Rayleigh scattering probes Doppler terms independently of SW/ISW

Measure new primordial modes with Rayleigh×Rayleigh spectrum?

In principle could double number of modes compared to T+E!

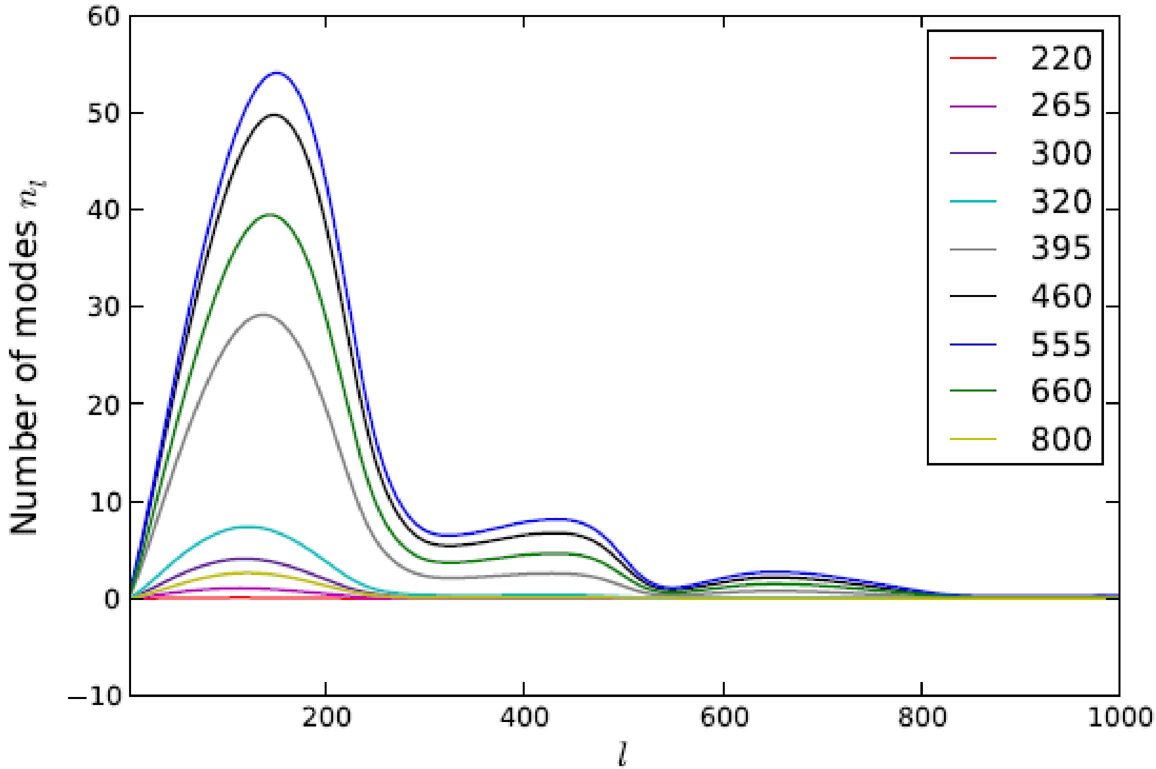
BUT: signal highly correlated to primary on small scales; need the uncorrelated part



Solid: Rayleigh× Rayleigh total; Dashed: uncorrelated part; Dots: error per $\frac{\Delta l}{l} = 10$ bin a from PRISM

Number of new inflation modes with PRISM-like future CMB

Define $n_l \equiv (2l+1) f_{\rm sky} {\rm Tr}\left[\left([\boldsymbol{C}_l + \boldsymbol{N}_l]^{-1}\boldsymbol{C}_l\right)^2\right]$

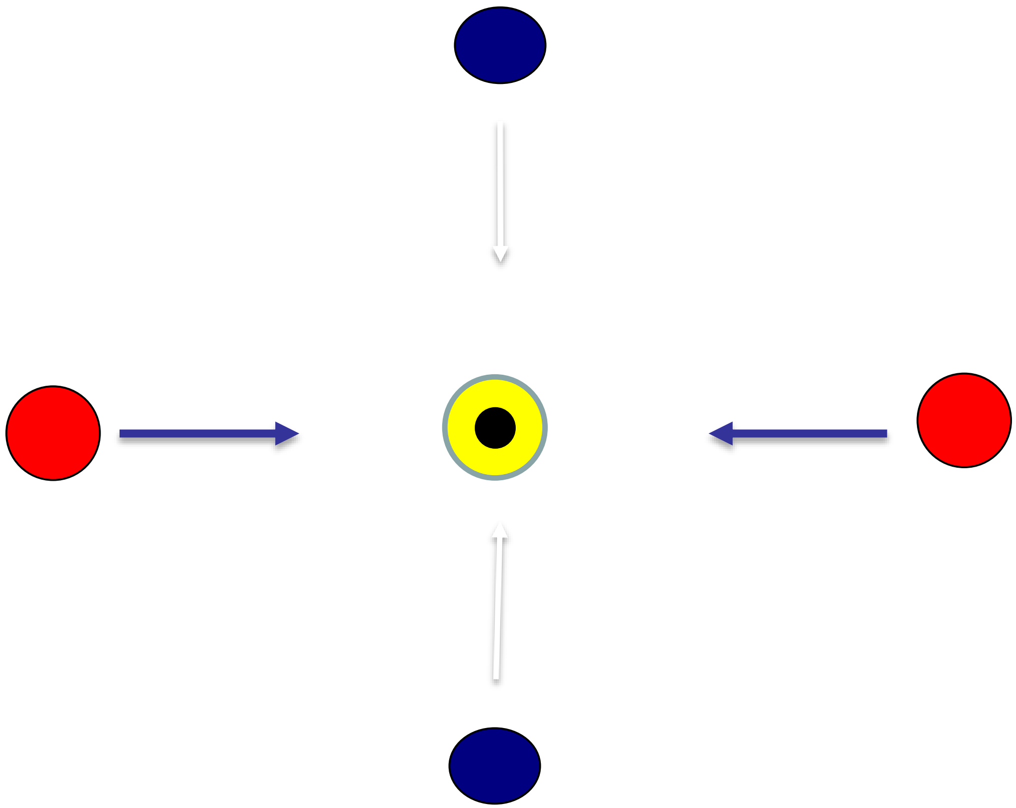


New modes almost all in the $l \leq 500$ temperature signal: total $\approx 10\,000$ extra modes

More horizon-scale information (disentangle Doppler and Sachs-Wolfe terms)

Polarization

Generated by scattering of anisotropic unpolarised light at recombination

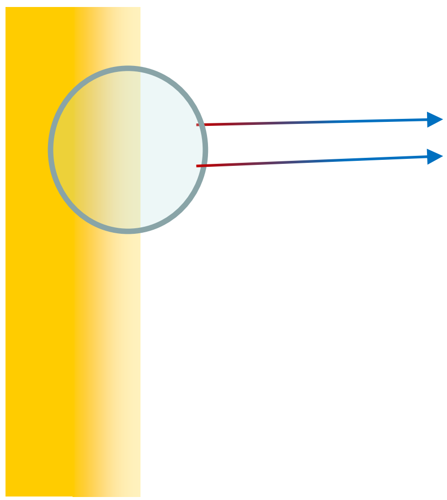


Rayleigh polarization signal highly correlated to the blackbody because scattering the same quadrupole

High-energy scattered more ⇒ large-scale polarization becomes blue

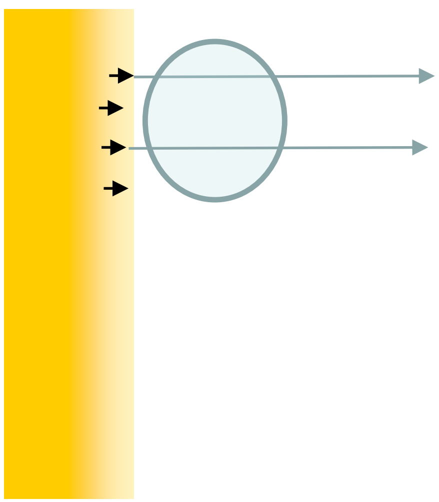
On horizon scales three nearly-independent perturbation modes being probed

$\frac{\Delta T}{T} + \Phi + ISW$
(anisotropic redshifting to constant temperature recombination surface)



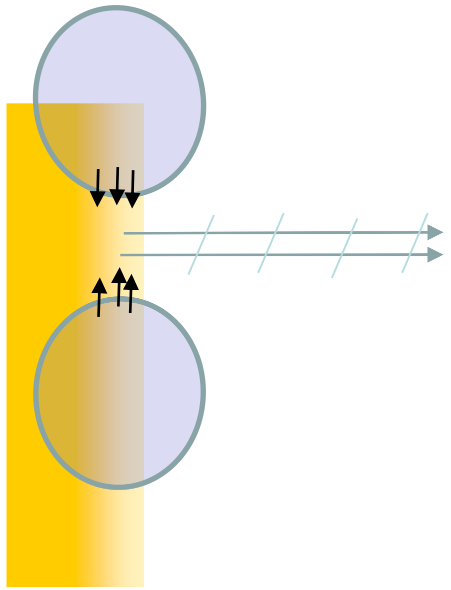
Primary

$\hat{n} \cdot v_b$: Doppler



Rayleigh, Primary

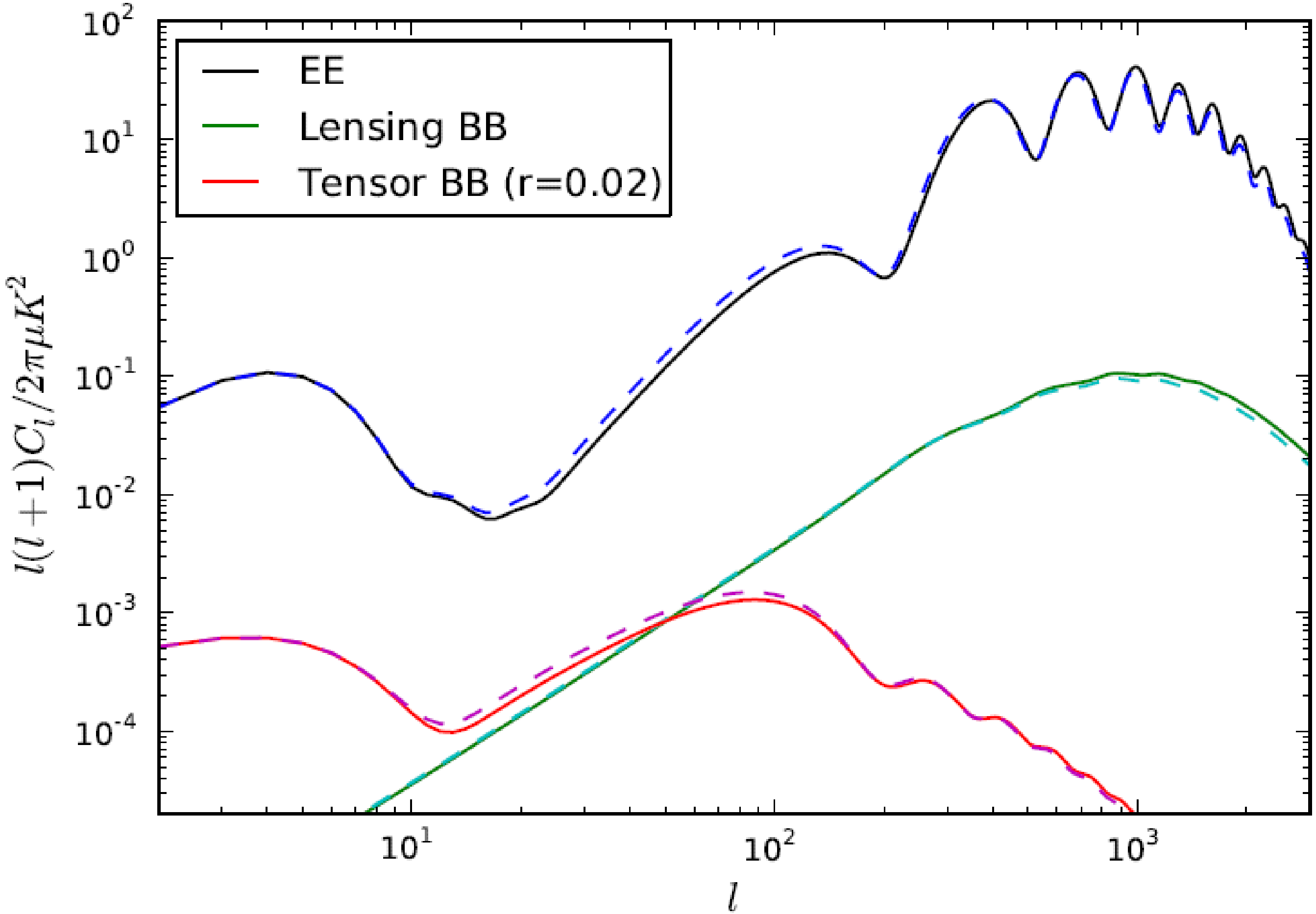
Polarization from quadrupole scattering



Rayleigh, Primary

Rayleigh polarization power spectra

Solid: primary   Dashed: primary + Rayleigh (857GHz)



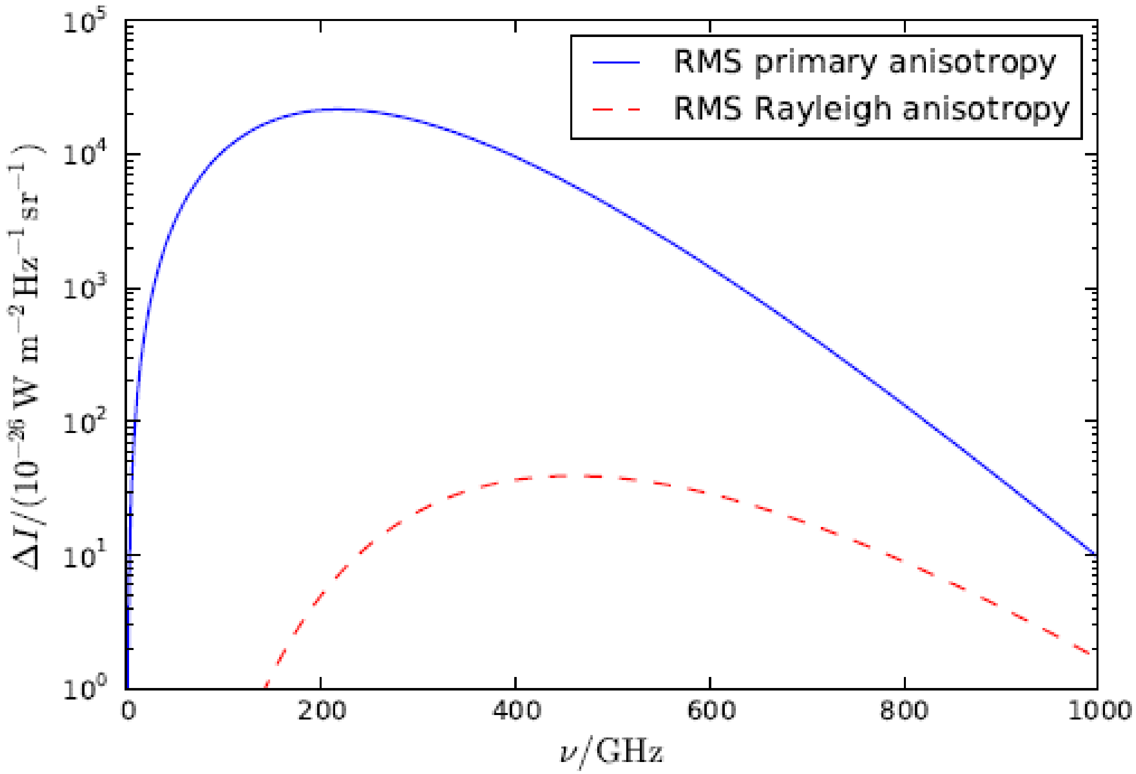
Large-scale polarization from scattering into the line of sight ⇒ polarized CMB sky is blue

but same quadrupole, so highly correlated to primary

Expected signal as function of frequency

Zero order: uniform blackbody not affected by Rayleigh scattering (elastic scattering, photons conserved)

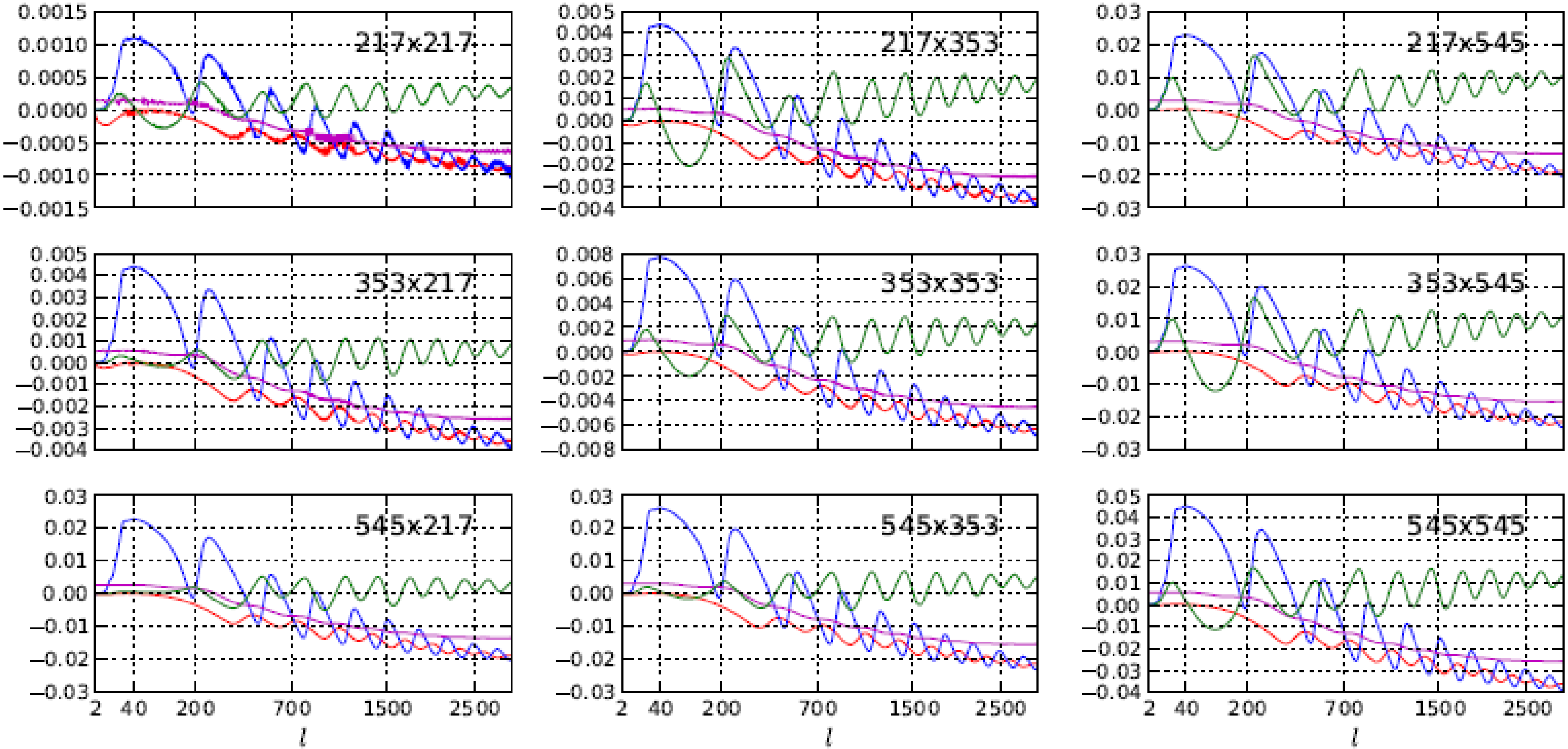
1[st] order: anisotropies modified, no longer frequency independent



Need sensitivity at $200\mathrm{GHz} \leq \nu \leq 800\mathrm{GHz}$

(+probably higher for foreground separation efficiency; very hard above 350GHz from ground)

Fractional total C_l differences at realistic frequencies



TT, EE, BB: $\frac{\Delta C_l}{C_l}$ TE: $\frac{\Delta C_l^{TE}}{\sqrt{C_l^{EE} C_l^{TT}}}$

Rayleigh summary

- Significant Rayleigh signal at $\nu \geq 200$ GHz; several percent on T, E at $\nu \geq 500$GHz

- Non-blackbody signal in the anisotropies (but no spectral distortion in monopole)

- Strongly correlated to primary signal on small scales (mostly damping)
  – robust detection via cross-correlation?

- Powerful test of recombination physics/expansion

- Boosts large-scale polarization (except B modes from lensing)
  - consistency test that large-scale B modes are from scattering at recombination
  - in principle separate lensing and tensors at the power spectrum level

- Multi-tracer probe of last-scattering
  - limited by noise/foregrounds, not cosmic variance

- May be able to provide additional primordial information (10,000+ modes)
  - mostly horizon-scale T modes at recombination from Doppler signal